Flavor asymmetry of light sea quarks in the proton

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Sea quarks

Fock states of the proton

$$|P\rangle = |uud\rangle, |uudg\rangle, |uudq\bar{q}\rangle + ...$$

• It is not possible to distinguish any individual down or up quark as a valence or sea quark.
• All the anti quarks in the proton belongs to sea quarks only.

$$u_{sea}(x) = \bar{u}_{sea}(x) = \bar{u}(x), \quad d_{sea}(x) = \bar{d}_{sea}(x) = \bar{d}(x)$$

Flavor symmetry: The sea are generated predominantly by gluons splitting into quark-antiquark pairs $g \to q\bar{q}$ and the splitting is flavor independent,

$$\bar{u}(x) = \bar{d}(x)$$

Flavor asymmetry in light sea quarks

- The Gottfried sum rule written in terms of proton and neutron structure functions:

$$\begin{aligned} S_G &= \int_0^1 \left[F_2^p(x) - F_2^n(x)\right] \frac{\mathrm{d}x}{x} \\ &= \frac{1}{3}\int_0^1 \left[u_v(x) - d_v(x)\right] \mathrm{d}x - \frac{2}{3}\int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] \mathrm{d}x \\ &= \frac{1}{3} - \frac{2}{3}\int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] \mathrm{d}x \end{aligned}$$

-K. Gottfried, Phys. Rev. Lett. 18 (1967) 1174.

- In light sea quark flavor symmetry: $\bar{d}_p(x) = \bar{u}_p(x)$ and $S_G = \frac{1}{3}$.

But NMC Collaboration reported a value of $S_G = 0.235 \pm 0.026$

$$\int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] \mathrm{d}x = 0.147 \pm 0.039,$$

-New Muon Collaboration,Phys.Rev.Lett. 66, 2712-2715 (1991)

$$\bar{d}_p(x) \neq \bar{u}_p(x)$$

- Discussed in the lectures by Sanghwa Park and Christine Aidala Talk

Light front dynamics

Light-front dynamics describes how a relativistic system changes along a light-front direction.

- In light front,

LF time $x^+ = x^0 + x^3$
$x^- = x^0 - x^3$
$x^\perp = (x^1, x^2)$.
LF energy $p^- = p^0 - p^3$

- No square root in energy dispersion relation $k^2 = k^+ k^- - k_\perp^2 = m^2$.

$$H_{LF}|\psi\rangle = M^2|\psi\rangle$$

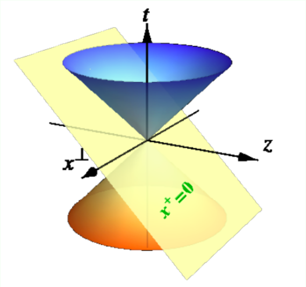


Figure: Leutwyler 1978

In LF, Solving nonperturbative QCD is equivalent to solving the Hamiltonian eigenvalue problem.

Light front wave functions (LFWFs)

• In LF, the hadron state $|\psi\rangle$ is expanded in multi-particle fock states $|n\rangle$ of free LF Hamiltonian $|\psi\rangle = \sum_n \psi_n |n\rangle$, where

$$|n\rangle = |uud\rangle, |uudg\rangle, |uudq\bar{q}\rangle$$

• Fock component ψ_n is known as light front wave function.
• LFWFs $\psi_n(x_i, k_{\perp,,i}, \lambda_i)$ depend only on the relative longitudinal, transverse momentum and spin of the parton.

• Momentum conservation
$\sum_{i=1}^n x_i = 1, \sum_{i=1}^n k_{\perp,i} = 0$

• Overlap of LFWFs: PDFs, TMDs, GPDs...

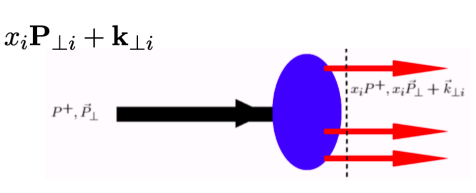


Figure: S Brodsky

• LFWFs are frame independent object and provide intrinsic information of the structure of hadrons

Sea quarks spectator model

• Assume proton as an effective system of five particle fock state $|uudq\bar{q}\rangle$, consider one of sea quark (spin-$\frac{1}{2}$) is active and rest of the system (spectator) is in spin-0 state.

$$|P;\uparrow(\downarrow)\rangle = \sum_{\bar{q}} \int \frac{\mathrm{d}x\, \mathrm{d}^2\mathbf{k}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \Big[\psi^{\uparrow(\downarrow)}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp)|+\frac{1}{2},0;xP^+,\mathbf{k}_\perp\rangle + \psi^{\uparrow(\downarrow)}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp)|-\frac{1}{2},0;xP^+,\mathbf{k}_\perp\rangle\Big].$$

$$\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp) = \varphi(x,\mathbf{k}^2_\perp), \qquad \psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp) = -\frac{k^1+ik^2}{\kappa}\,\varphi(x,\mathbf{k}^2_\perp),$$

$$\psi^{\downarrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp) = \frac{k^1-ik^2}{\kappa}\,\varphi(x,\mathbf{k}^2_\perp), \qquad \psi^{\downarrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp) = \varphi(x,\mathbf{k}^2_\perp),$$

where $\varphi(x,\mathbf{k}^2_\perp)$ is the modified soft wall AdS/QCD wave function

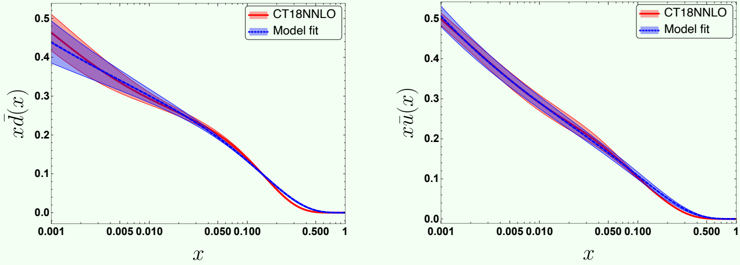
$$\varphi(x,\mathbf{k}_\perp) = \sqrt{A}\frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}\,x^{\alpha}(1-x)^{3+\beta}\exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2 x(1-x)}\mathbf{k}^2_\perp\right],$$

- PC, D Chakrabarti ,C Mondal e-Print:**2312.01484**.

Fitting of model parameters

We fixed our model parameters by fitting the unpolarized PDF $f_1^{\bar{q}}(x)$ with the CTEQ18 NNLO global analysis data.

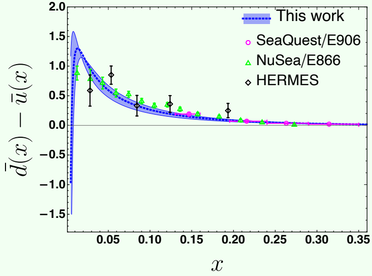
$$f_1^{\bar{q}}(x) = x\bar{q}(x) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 + |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 \right]$$

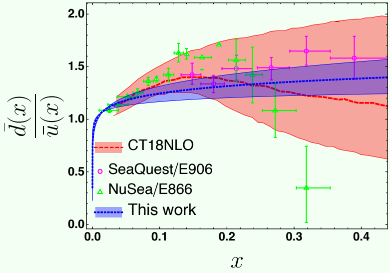


	A	α	β
$\overline{u}$	$0.055^{0.003}_{-0.002}$	$-0.611^{0.001}_{0.001}$	$-1.33^{-0.131}_{-0.181}$
$\overline{d}$	$0.082^{-0.006}_{0.007}$	$-0.572^{-0.014}_{0.016}$	$-1.33^{-0.132}_{0.163}$

$$\langle x \rangle_{\bar{d}} = \int_{0.001}^{1} \mathrm{d}x\, x\bar{d}(x) = 0.039 \pm 0.002, \quad \langle x \rangle_{\bar{u}} = \int_{0.001}^{1} \mathrm{d}x\, x\bar{u}(x) = 0.032 \pm 0.003\,.$$

Flavor asymmetries of light sea quarks



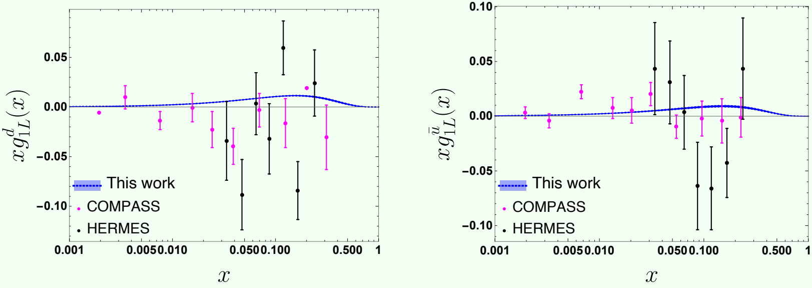


Model/Experiments	x-range	$\int \mathrm{d}x \left[\bar{d}(x) - \bar{u}(x)\right]$
This work	$0.13 < x < 0.45$	0.015 ± 0.004
SeaQuest/E906	$0.13 < x < 0.45$	0.015 ± 0.003
This work	$0.015 < x < 0.35$	0.069 ± 0.015
NuSea/E866	$0.015 < x < 0.35$	0.0803 ± 0.011
NMC	$0.004 < x < 0.80$	0.148 ± 0.039
HERMES	$0.020 < x < 0.30$	0.16 ± 0.03

- PC, D Chakrabarti ,C Mondal e-Print:**2312.01484**.

Sea quarks helicity

$$g_{1L}^{\bar{q}}(x) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 - |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 \right]$$



$$\Delta\Sigma^{\bar{d}} = \int_{0.001}^{1} \mathrm{d}x g_{1L}^{\bar{d}}(x) = 0.031 \pm 0.001\,, \quad \Delta\Sigma^{\bar{u}} = \int_{0.001}^{1} \mathrm{d}x g_{1L}^{\bar{u}}(x) = 0.026 \pm 0.002\,.$$

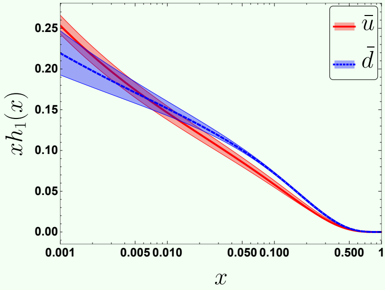
Transversity

Transversity measures the distribution of quark transverse spin in a transverse polarized proton.

$$h_1(x) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \psi^{\uparrow\dagger}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp)\psi^{\downarrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp)\,,$$
$$= Ax^{2\alpha}(1-x)^{7+2\beta}\,.$$

Sea quark tensor charges at the scale $Q_0 = 2$ GeV as

$$\delta\bar{d} = \int_{0.001}^{1} \mathrm{d}x\, h_1^{\bar{d}}(x) = 0.75 \pm 0.05\,,$$
$$\delta\bar{u} = \int_{0.001}^{1} \mathrm{d}x\, h_1^{\bar{u}}(x) = 0.73 \pm 0.04\,.$$



Generalized Parton distributions (GPDs)

• GPDs contain important information about the nucleon mass, angular momentum and mechanical properties of the proton.

• The off forward matrix elements of bilocal vector and axial vector current as parameterized in terms of chiral even GPDs:

$$\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' \,|\bar{q}(-z)\gamma^+ q(z),\, |p, \lambda\rangle\Big|_{z^+=0,\, \mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p', \lambda')\left[H^q\,\gamma^+ + E^q\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\right]u(p, \lambda)$$

$$-\frac{i}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p', \lambda' |\, \bar{q}(-z)\gamma^+\gamma_5 q(z)\,|p, \lambda\rangle\Big|_{z^+=0,\, \mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p', \lambda')\left[\tilde{H}^q\,\gamma^+\gamma_5 + \tilde{E}^q\,\frac{\gamma_5\Delta^+}{2M}\right]u(p, \lambda),$$

- Meissner, Metz, Goeke Phys.Rev.D 76 (2007) 034002

At zero skewness $\tilde{E}^q = 0$, only three chiral even GPDs survive.

GPDs in terms of LFWFs

$$H^{\bar{q}}(x,-\Delta_\perp^2) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\left[\psi^{\dagger\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}'_\perp) + \psi^{\dagger\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}'_\perp)\right]$$

$$\tilde{H}^{\bar{q}}(x,-\Delta_\perp^2) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\left[\psi^{\dagger\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}'_\perp) - \psi^{\dagger\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}'_\perp)\right]$$

$$E^{\bar{q}}(x,-\Delta_\perp^2) = \frac{-2M_N}{\Delta_\perp^1 - i\Delta_\perp^2}\int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\left[\psi^{\dagger\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}''_\perp)\psi^{\downarrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}'_\perp) + \psi^{\dagger\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}''_\perp)\psi^{\downarrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}'_\perp)\right]$$

where, transverse momentum of the the final and initial struck quark

$$\mathbf{k}''_\perp = \mathbf{k}_\perp + (1-x)\frac{\Delta_\perp}{2}, \qquad \mathbf{k}'_\perp = \mathbf{k}_\perp - (1-x)\frac{\Delta_\perp}{2}\,.$$

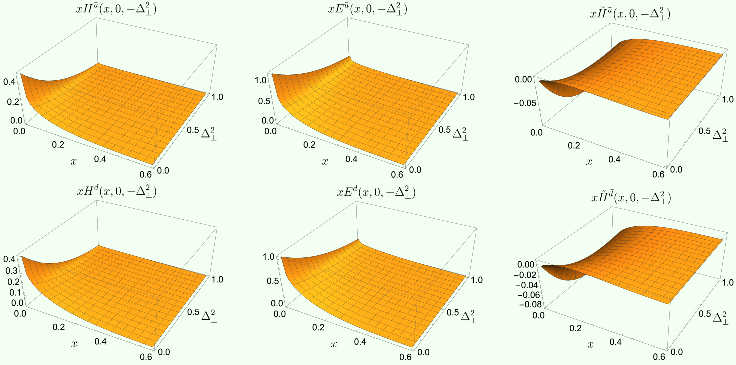
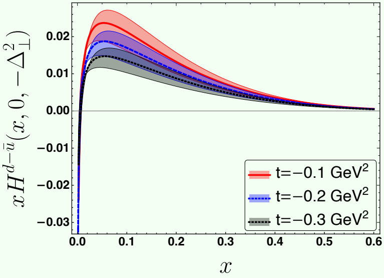
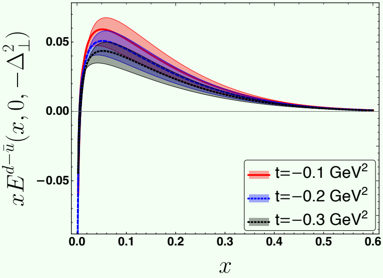


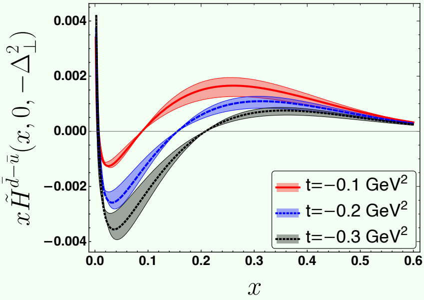
Figure: The sea quark chiral-even GPDs plotted in the range $0.001 < x < 0.6$ at zero skewness for the proton calculated within our model. The upper panel is for the $\bar{u}$ quark and the lower panel represents the distributions for the $\bar{d}$ quark.

Flavor asymmetry at non-zero momentum transfer





- Electric and Magnetic asymmetries are negative in $0.001 < x < 0.005$ independent of choice of $\Delta_\perp^2$.
- The helicity distribution in our model changes its sign with x and $\Delta_\perp^2$.



Quark Transverse Momentum Distributions (TMDs)

TMDs describe correlations between the transverse momentum of partons and the polarization of the partons and/or parent nucleon.

$$\Phi^{\Gamma}(x, \mathbf{k}_{\perp}; P, S) = \frac{1}{xP^{+}} \int \frac{d\xi^{-}}{2\pi} \frac{d^{2}\xi_{\perp}}{(2\pi)^{2}} e^{ik\cdot\xi} \langle P, S|\bar{\psi}(0)\mathcal{W}(0,\,\xi)\Gamma\psi(\xi)|P, S\rangle\Big|_{\xi^{+}=0},$$

- Meissner, Metz, Goeke Phys.Rev.D 76 (2007) 034002

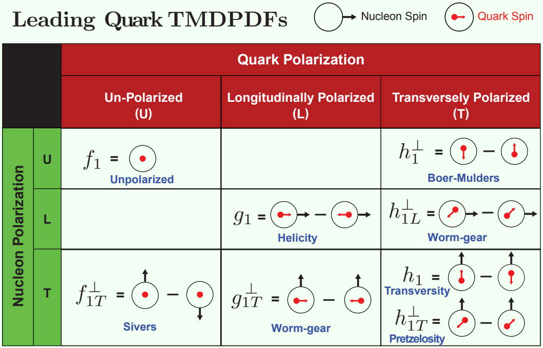


Figure: TMD handbook 2023

Transverse momentum distributions (TMDs)

For $\Gamma \equiv \gamma^+,\ \gamma^+\gamma^5,\ i\sigma^{j+}\gamma^5$.

$$\Phi^{[\gamma^+]}(x, k_\perp; \mathbf{S}) = f_1^q - \frac{\epsilon_\perp^{ij} k_\perp^i S_\perp^j}{m} f_{1T}^{\perp q},$$

$$\Phi^{[\gamma^+\gamma^5]}(x, k_\perp; \mathbf{S}) = S_z g_{1L}^q + \frac{k_\perp \cdot S_\perp}{m} g_{1T}^q$$

$$\Phi^{[i\sigma^{j+}\gamma^5]}(x, k_\perp; \mathbf{S}) = S_\perp^j h_1^q + S_z \frac{k_\perp^j}{m} h_{1L}^{\perp q} + S_\perp^i \frac{2k_\perp^i k_\perp^j - k_\perp^2 \delta^{ij}}{2m^2} h_{1T}^{\perp q} + \frac{\epsilon_\perp^{ji} k_\perp^i}{m} h_1^{\perp q}$$

which can be also expressed in terms of LFWFs:

$$f_1^{\bar{q}}(x, \mathbf{k}_\perp) = \frac{1}{16\pi^3}\left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 + |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x, \mathbf{k}_\perp)|^2\right],$$

$$g_{1L}^{\bar{q}}(x, \mathbf{k}_\perp) = \frac{1}{16\pi^3}\left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 - |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x, \mathbf{k}_\perp)|^2\right],$$

$$h_1^{\bar{q}}(x, \mathbf{k}_\perp^2) = \frac{1}{2(2\pi)^3}\psi^{\uparrow\dagger}_{\bar{q},+\frac{1}{2}}(x, \mathbf{k}_\perp)\psi^{\downarrow}_{\bar{q},-\frac{1}{2}}(x, \mathbf{k}_\perp),$$

TMDs

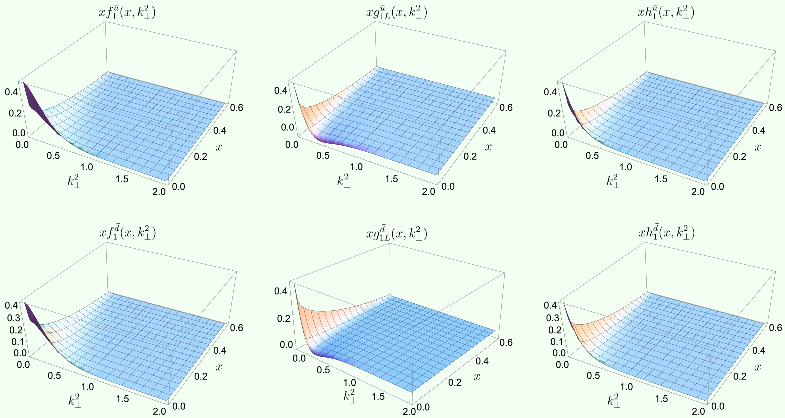
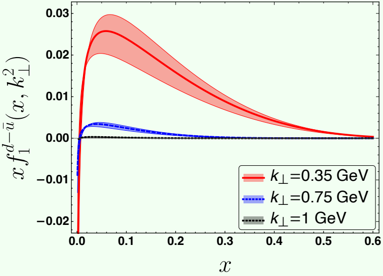
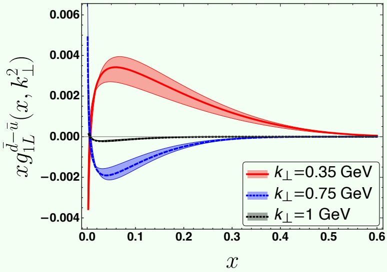


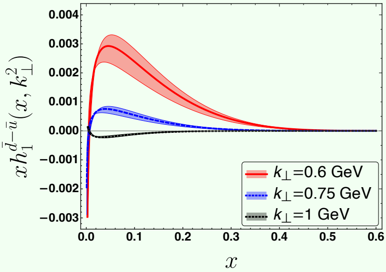
Figure: The sea quark T-even TMDs, $xf_1^{\bar{q}}(x, \mathbf{k}_\perp^2)$ (left panel), $xg_{1L}^{\bar{q}}(x, \mathbf{k}_\perp^2)$ (middle panel), and $xh_1^{\bar{q}}(x, \mathbf{k}_\perp^2)$ (right panel) calculated in the range $0.001 < x < 0.6$ and momentum transfer $0 < \mathbf{k}_\perp^2 < 2$ GeV^2 in the proton within our model. The upper panel is for the $\bar{u}$ quark and the lower panel represents the distributions for the $\bar{d}$ quark. $\mathbf{k}_\perp^2$ is in units of GeV^2.

Flavor asymmetries in terms of TMDs

- In the range $0.001 < x < 0.005$, $\bar{u} > \bar{d}$ independent of $k_\perp$.
- The sign of helicity changes with $k_\perp$.



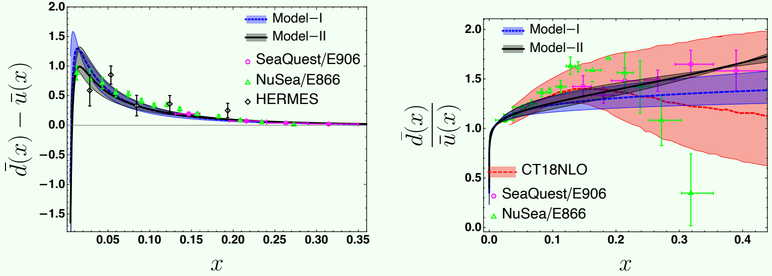




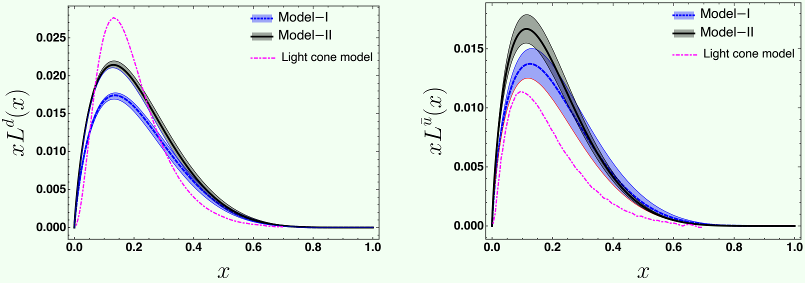
$$\varphi(x,\mathbf{k}_\perp^2) = \sqrt{A}\frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}x^{\alpha}(1-x)^{3+\beta}\exp\left[-\frac{\mathbf{k}_\perp^2}{2\kappa^2}\right].$$

-S Brodsky, Valery, Ivan Schmidt, Phys.Rev.D 106 (2022) 9, 094025

Using the same fitting methodology, we can obtain the model parameters.



$$L^{\bar{q}}(x) = \frac{1}{2}\left\{x\left[H^{\bar{q}/P}(x,0,0) + E^{\bar{q}/P}(x,0,0)\right] - \widetilde{H}^{\bar{q}/P}(x,0,0)\right\} .$$



	$L^{\bar{u}}$	$L^{\bar{d}}$	$\Delta\Sigma^{\bar{u}}/2$	$\Delta\Sigma^{\bar{d}}/2$
Model-I	0.040 ± 0.003	0.048 ± 0.002	0.013 ± 0.001	0.015 ± 0.001
Model-II	0.048 ± 0.003	0.059 ± 0.002	0.00	0.00
LC model	0.025	0.046	0.00	0.00

LC model- X Luan, Zhun Lu, Eur.Phys.J.C 83 (2023) 6, 504

Summary

- We constructed a scalar spectator model for light sea quarks of the proton and fitted the model parameters using the CTEQ18 NNLO data.
- We predicted flavor asymmetry in the form of $\bar{d} - \bar{u}$, $\frac{\bar{d}}{\bar{u}}$ and compared our results with the latest SeaQuest, NuSea and HERMES results.
- We also interpreted the flavor asymmetry at non-zero transverse momentum and non-zero momentum transfer.
- We have also explored the flavor asymmetry in a different phenomenological model.
- Our spectator model predicted non-zero helicity asymmetry while the phenomenological models it is zero.
- We have also discussed OAM results from the two models.

Thanks for joining....