Blast and splash in a one-dimensional cold gas

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In collaboration with Subhadip Chakraborti, Abhishek Dhar, Santhosh Ganapa (2021)

Tibor Antal, Sid Redner (2008)

Hydrodynamics and Fluctuations — Microscopic Approaches in Condensed Matter Systems (September 2021)

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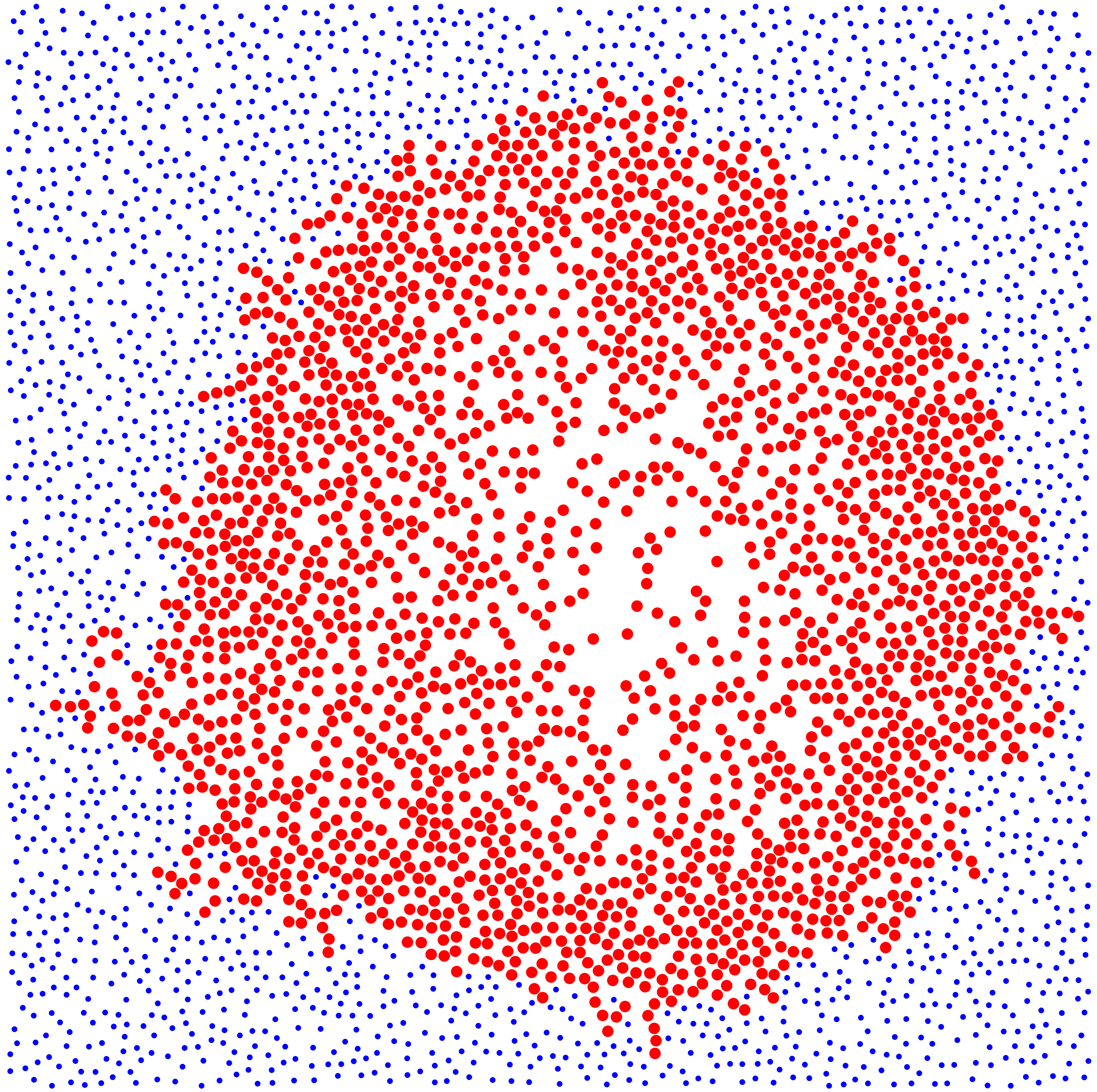
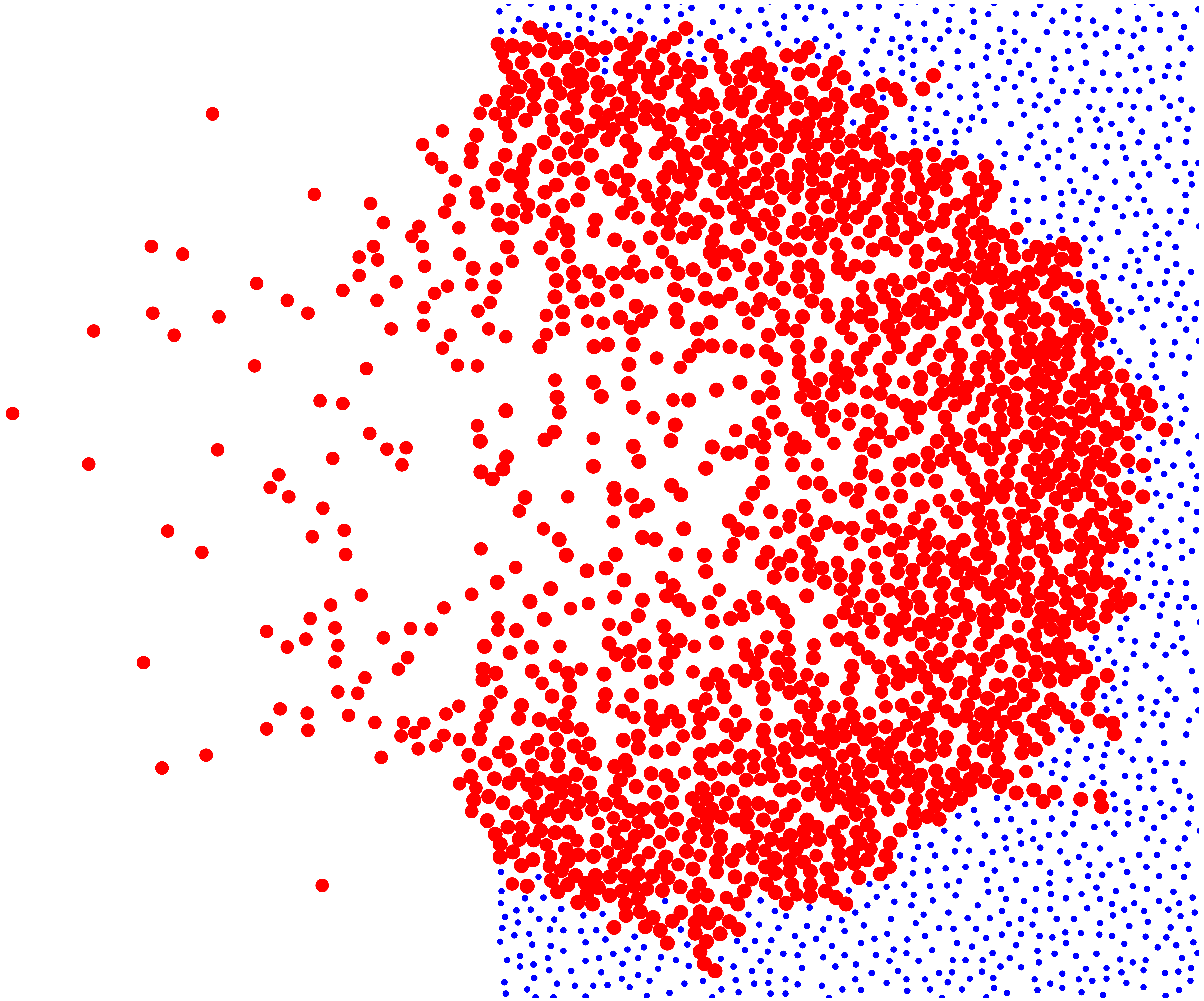
Basic system: An infinite (or semi-infinite) system of hard spheres at rest subjected to a sudden injection of energy, say hitting a single sphere (a kind of billiard)





An infinite system: Hard spheres initially at rest, uniformly distributed in the entire space (the blast problem)

- Orsay group (Barbier, Trizac, … 2015, 2016)
- Chennai group (Rajesh, Joy, Pathak, … 2017, 2019, 2021)
- Inelastic collisions have been studied as well (we assume elastic collisions)
- Perhaps the supernova explosion is the closest system in Nature. The released energy is immense, and the zero temperature approximation is reasonable. Also, the gas is essentially monoatomic (as hard spheres).
- Astrophysical studies rely on continuum approximation. Our system (particles initially at rest) is well suited to direct molecular dynamics simulations. In one dimension, it is infinitely diluted with known equation of state.

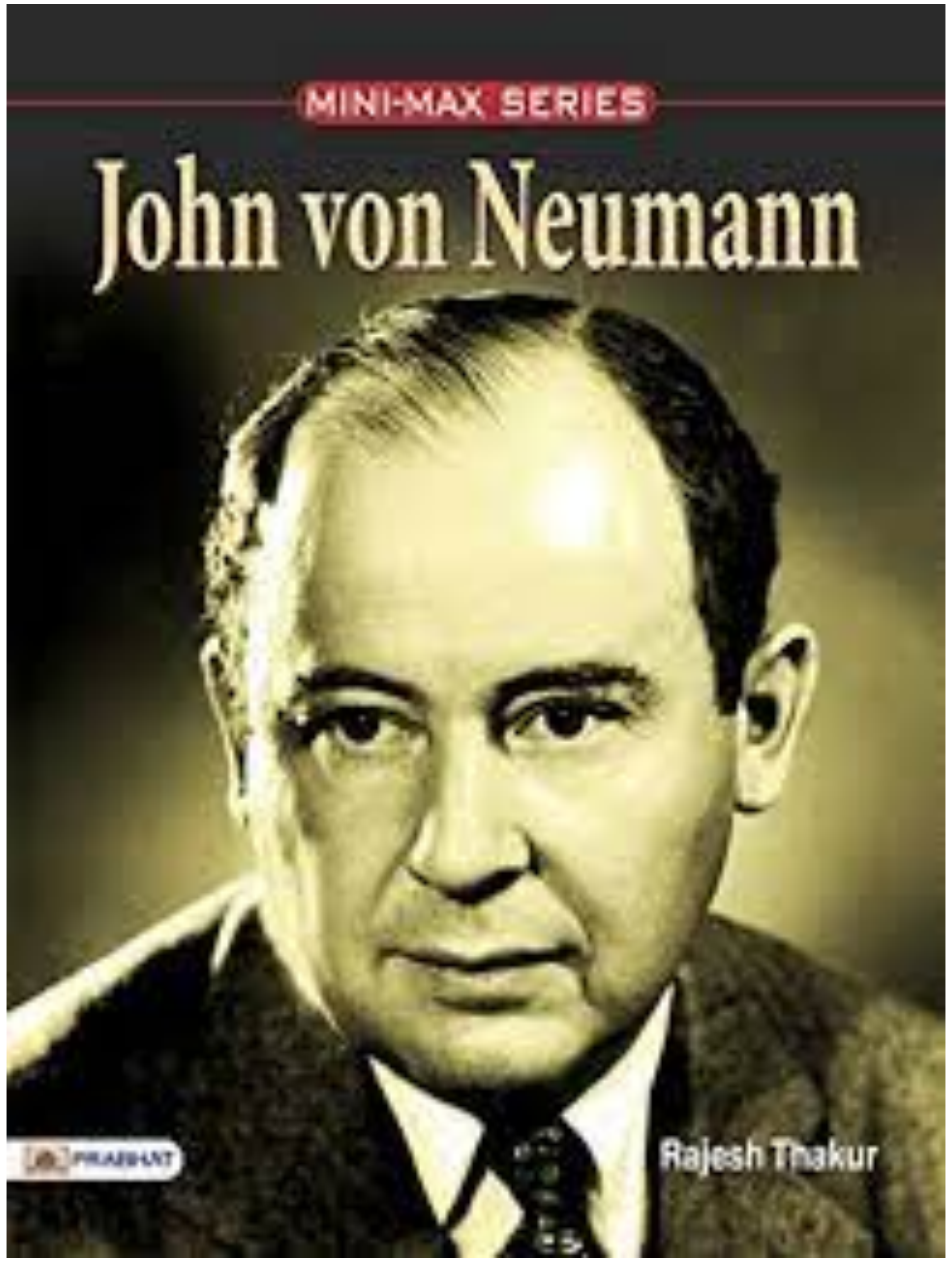
The blast caused by the instantaneous release of energy E in the small region creates a spherical shock wave propagating through the quiescent gas. The blast is infinitely strong if the pressure behind the shock wave can be neglected. In normal conditions, this is valid up to a certain time; for the gas at zero temperature, it remains valid forever. Below we consider an infinitely strong blast.

Dimensional analysis predicts the position of the shock wave $R = R(t)$ in terms of time t counted from the moment when the blast has occurred, the released energy E and the density ρ_∞ in front of the shock wave:

$$R(t) = \left(\frac{Et^2}{A\rho_\infty} \right)^{\frac{1}{d+2}}$$

Here d is the spatial dimension. The determination of the amplitude A requires an effort, the dependence of the position of the shock $R(t)$ on the basic parameters automatically follows from dimensional considerations.

Atomic explosion (Taylor, von Neumann, Sedov)



Continuous framework: Ideal compressible gas

Behind the shock wave, the radial velocity $v(r,t)$, density $\rho(r,t)$ and pressure $p(r,t)$ satisfy

$$\partial_t \rho + \partial_r(\rho v) + \frac{d-1}{r}\rho v = 0$$

$$(\partial_t + v\partial_r)\ln\frac{p}{\rho^\gamma} = 0$$

$$\rho(\partial_t + v\partial_r)v + \partial_r p = 0$$

where γ is the adiabatic index; for the monoatomic gas, $\gamma = 1 + 2/d$.

The hydrodynamic variables acquire a self-similar form

$$v = \frac{r}{t}\, V, \qquad \rho = \rho_\infty G, \qquad \frac{p}{\rho} = \frac{r^2}{t^2}\, Z$$

with V, G, Z depending on $\xi = r/R$. The dimensionless quantities satisfy a system of three coupled ODEs which can be solved (as found by von Neumann and Sedov). In one dimension

$$R(t) = \left(\frac{1071}{152}\,\frac{Et^2}{\rho_\infty}\right)^{\frac{1}{3}}$$

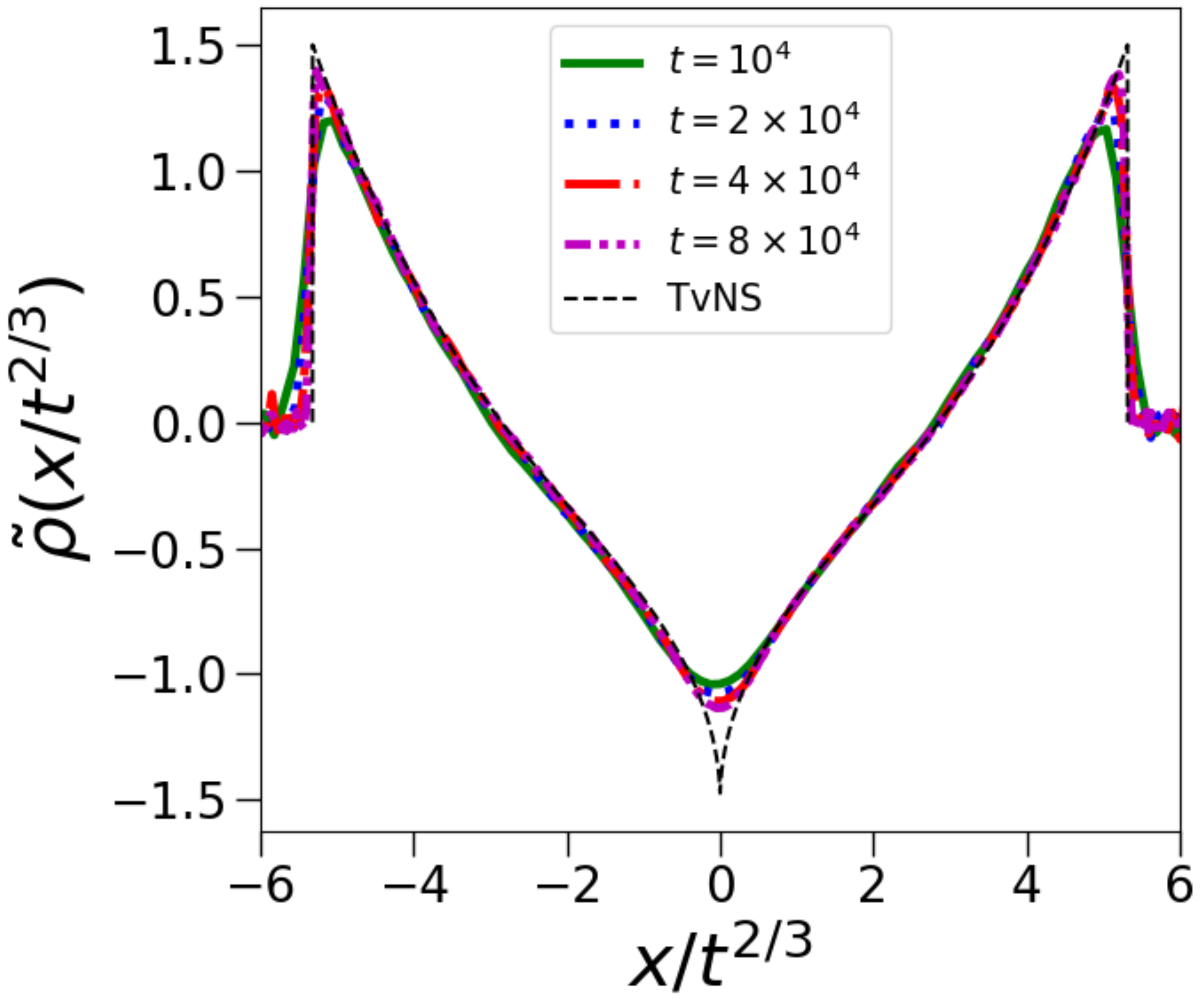
Blast in one dimension

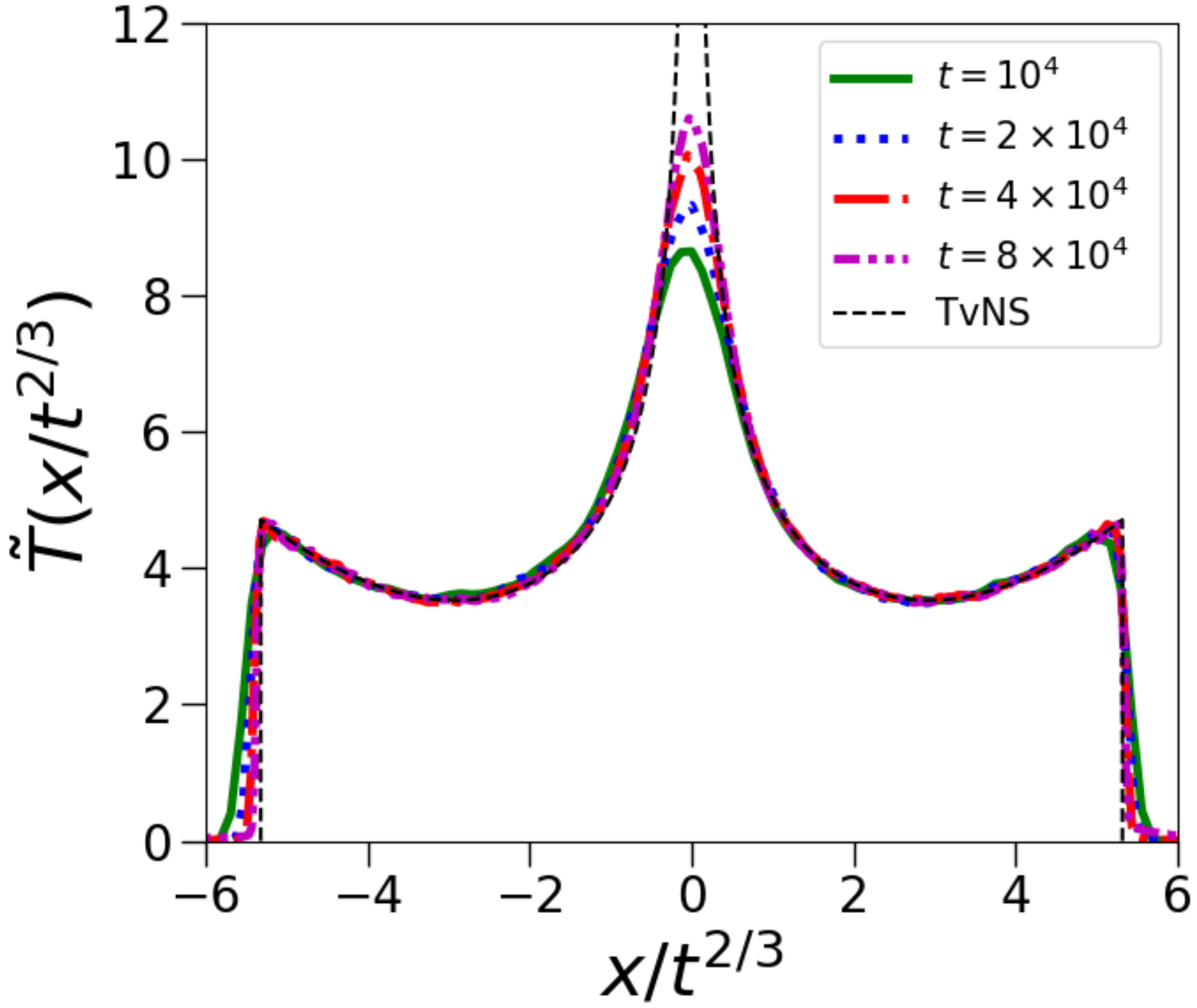
- Point particles with alternating masses: alternating hard particles (AHP). Mass dispersion is necessary to avoid pathology (no relaxation otherwise).

- In one dimension the emergence of the standard hydrodynamic behavior is questionable (Fourier's law of heat conduction is violated). Still, we observe good agreement between direct simulations and the TvNS solution. The major discrepancy occurs in the core region where dissipative effects matter.

- The predictions for the size of the core region, and hydrodynamic variables in the core region, agree with simulations.

- The NSF equations fairly well agree with direct simulations in the core region. The coefficient of thermal conductivity proportional to (Temperature)^(1/2) times (density)^(1/3).





S.Ganapa, S.Chakraborti, PK, A.Dhar PRL (2021) & Phys. Fluids (2021)

According to the TvNS, the temperature near the center of explosion diverges while the density vanishes

$$T \sim |x|^{-\frac{1}{2}}\, t^{-\frac{1}{3}}, \qquad \rho \sim |x|^{1/2}/t^{1/3}$$

The divergence of temperature is unphysical. Direct simulations show that the temperature at the center of the explosion remains finite and decreases with time. To rectify the analysis one must take into account the dissipative processes.

The entropy equation near the origin

$$\rho^3(\partial_t + v\partial_x)\left[T/\rho^2\right] = \partial_x\left(T^{1/2}\,\rho^{1/3}\partial_x T\right)$$

from which

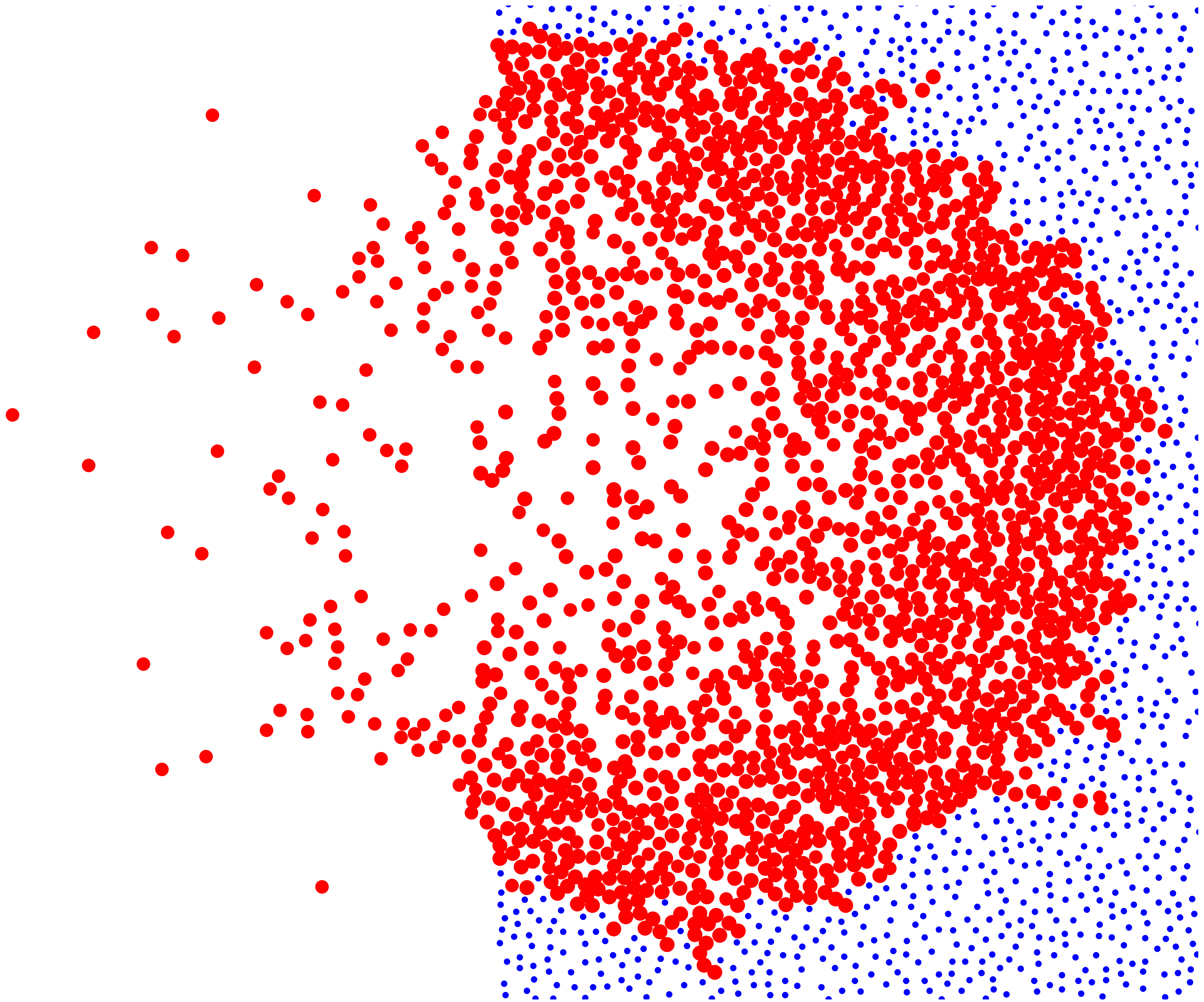
$$\rho\,\frac{T}{t} \sim \frac{T^{3/2}}{x^2}\,\rho^{1/3} \Longrightarrow T \sim |x|^{\frac{14}{3}}\,t^{-\frac{22}{9}}$$

Equating two estimates gives the size of the core region

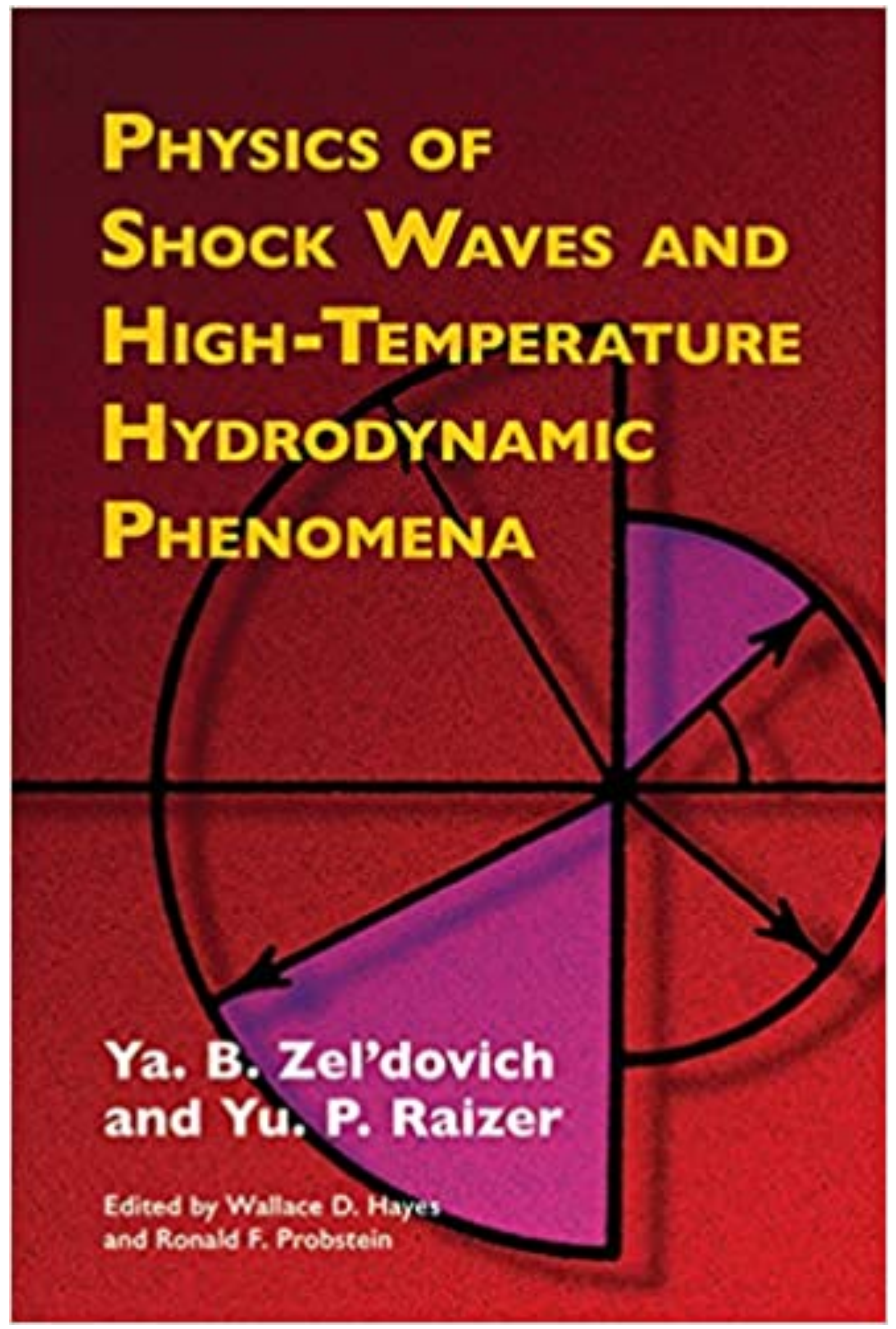
$$X \sim t^{\frac{38}{93}}\,, \qquad T_0 \sim t^{-\frac{50}{93}}\,, \qquad \rho_0 \sim t^{-\frac{4}{31}}$$

Anomalous Fourier law (Dhar, Grassberger, Casati, Prozen, Politi,…); what we use advocated by Hurtado & Garrido

Semi-infinite system: hard spheres initially at rest in the half-space (splash problem)



Zeldovich, von Weizsacker, Raiser



We study the one-dimensional splash problem

- The left-most particle, say at $x=0$, suddenly starts moving into the half-line $x>0$ with other AHPs initially at rest.
- Q: What is the total energy of particles in the initially occupied half-line ?

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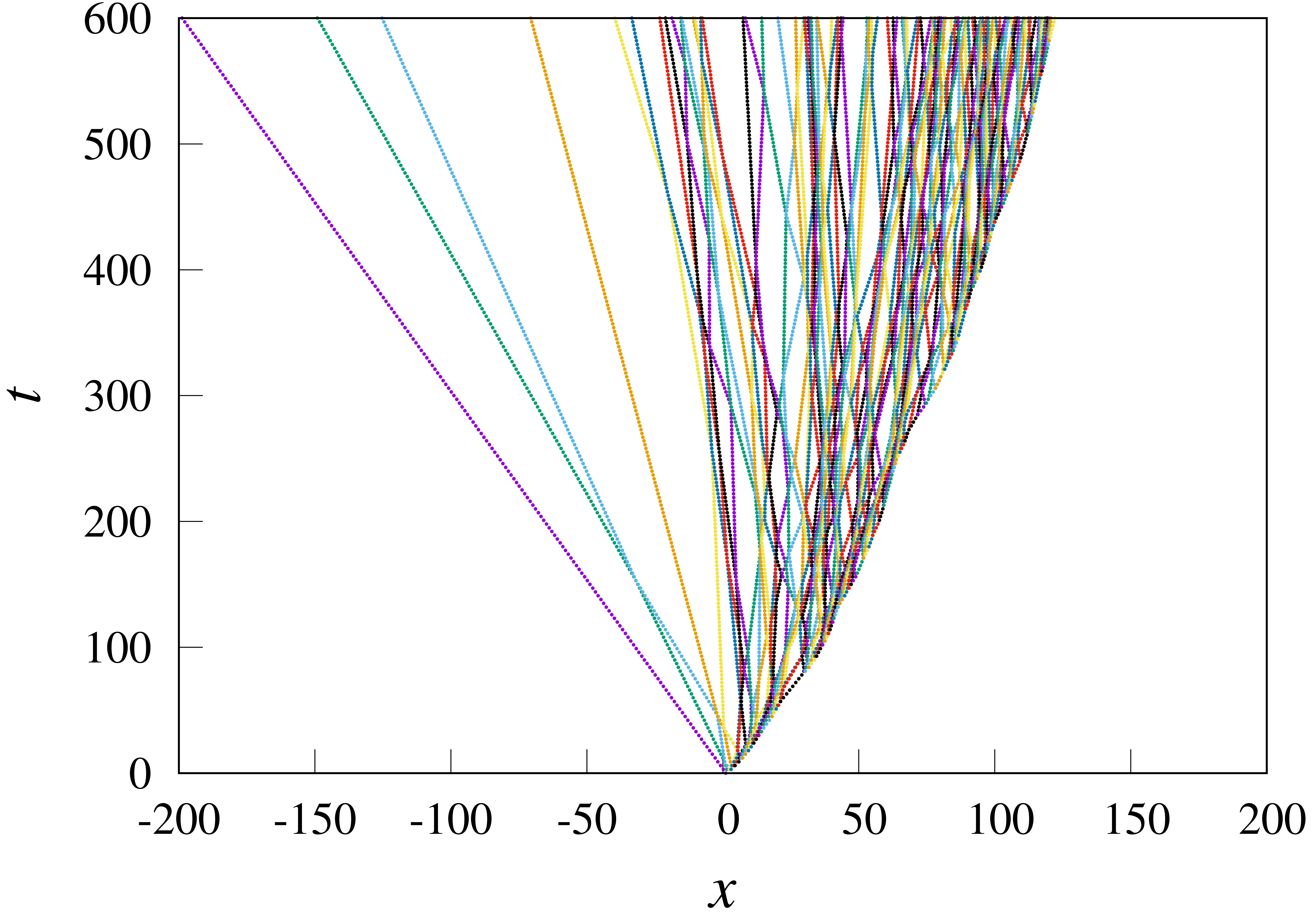
- Q: Total numbers of moving particles on the left and right ?
- A: Comparable, both grow as $t^{(2-b)/3} = t^{0.62795\ldots}$



600
500
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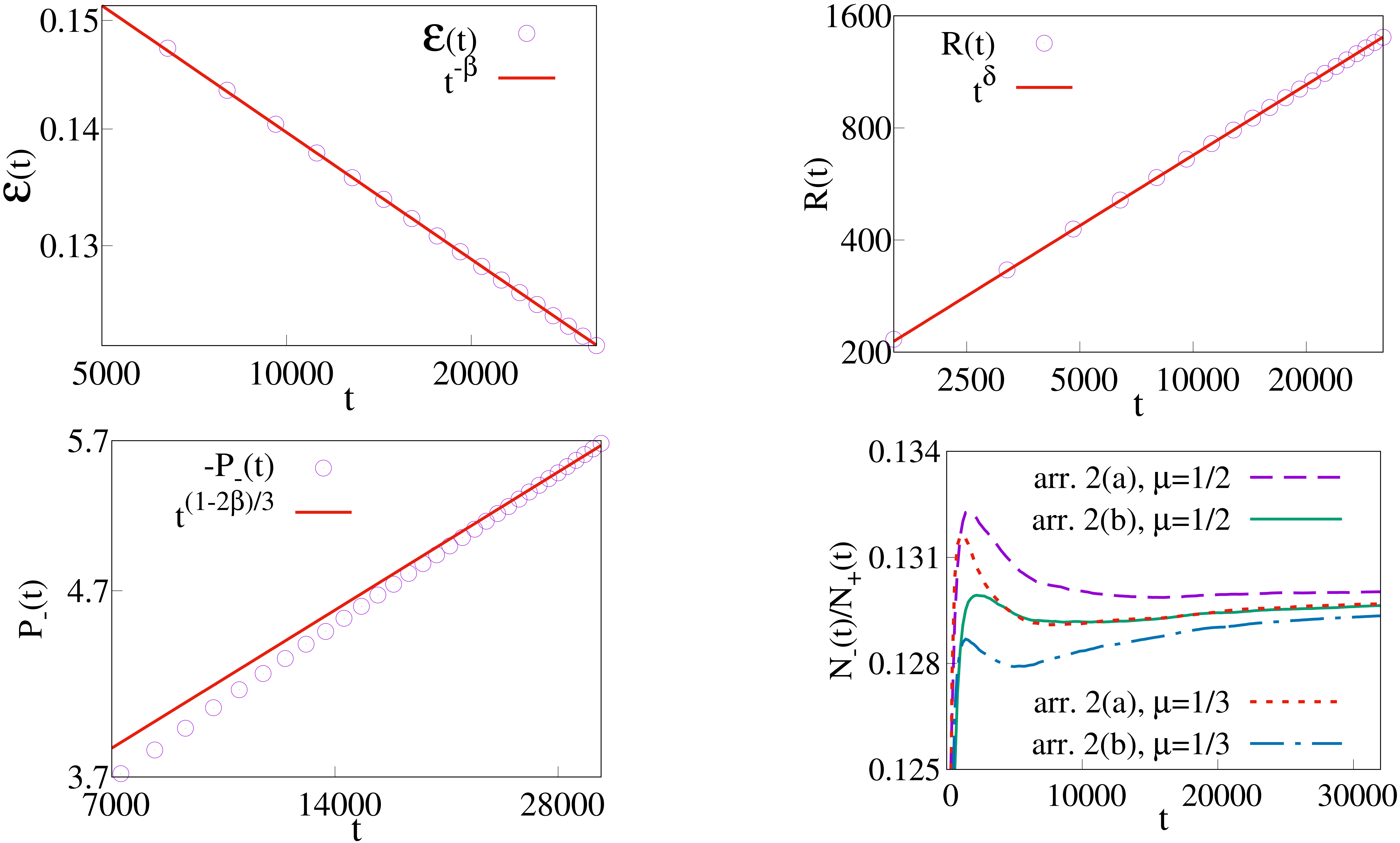

0.15
0.14
0.13
Ɛ(t)
Ɛ(t)
t^-β
5000
10000
20000
t
1600
800
400
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R(t)
R(t)
t^δ
2500
5000
10000
20000
t
5.7
4.7
3.7
P_(t)
-P_(t)
t^(1-2β)/3
7000
14000
28000
t
0.134
0.131
0.128
0.125
N_(t)/N_+(t)
arr. 2(a), μ=1/2
arr. 2(b), μ=1/2
arr. 2(a), μ=1/3
arr. 2(b), μ=1/3
0
10000
20000
30000
t

Splash problem: Hydrodynamic and billiard behaviors

- Hydrodynamic behavior on the right behind the shock wave at R(t), and on the left on distances of order R(t). But there is no well-defined boundary separating the hydro part from the splatter particles (that reached the ultimate speed and ceased to collide). The splatter particles constitute a growing non-hydro (billiard) leftover.

- The analysis of the hydro behavior is similar to the analysis of the blast problem. BUT: The exponent describing the growth of the position R of the shock waveis NOT fixed by dimension analysis. It is effectively an eigenvalue, the system of ODEs for the scaled velocity, density, and temperature has a consistent solution only at a specific exponent. This gives $b = 0.11614383675\ldots$

- The splatter particles form a (growing) fan and move ballistically.

- Each particle eventually joins the fan. This plausible plausible assertion is a conjecture.

Splash problem: Self-similar solutions of the second kind

- In the realm of the macroscopic (hydro) approach the position of the shock wave may depend on the energy, density and time. The only quantity with dimension of length that can be formed from these three quantities is the same as in the blast problem. This implies the $(\text{time})^{2/3}$ growth.

- Thus in the macroscopic framework it is paradoxical to find anything else !

- Whenever it happened, yet the solution remained scaling (self-similar in hydro language), the solution was termed a self-similar (SS) solution of the 2nd kind.

- The microscopic framework has an extra parameter with dimension of length, the inter particle distance. So the scaling $(\text{time})^{(2-b)/3} (\text{length})^{b/3}$ is consistent.

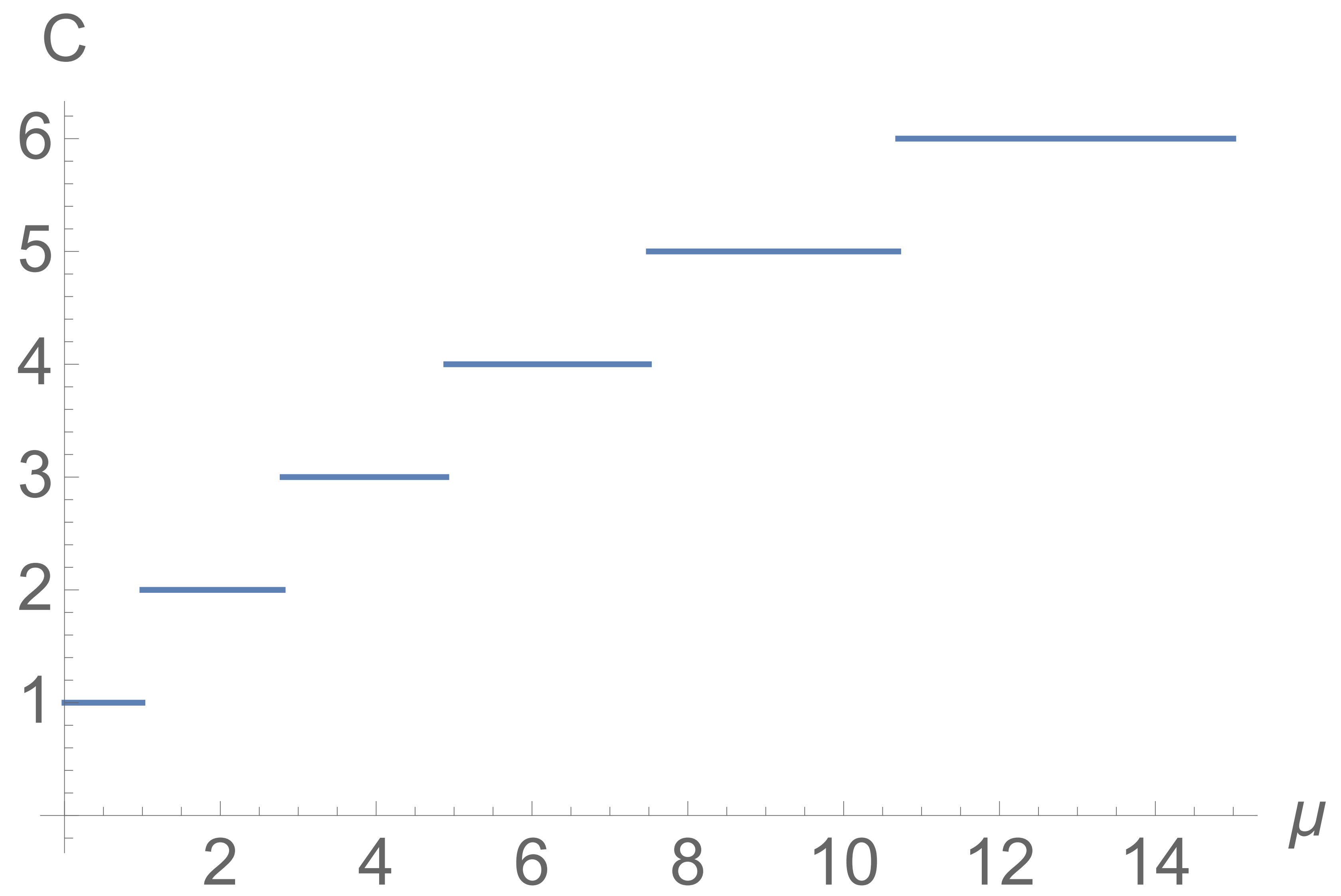
- A remarkable analogy between SS of the 1st kind <—> SS of the 2nd kind and descriptions of phase transitions Landau <—> Wilson (emphasized by Goldenfeld in his book).

Digression on Billiards: The maximal number of collisions

- Suppose we have a finite number of balls in R^d. How many collisions they may undergo before the collisions will cease?
- Sinai conjectured (in late 60s) that this number of collisions is finite. In late 70s he proved the conjecture in one dimension. Vaserstein (79) proved Sinai's conjecture for arbitrary dimension. This problem remains popular as known upper bounds seem far from tight.
- In the case of identical balls, the maximal number of collisions S(B) of B balls in three dimensions is known for B=2 and B=3. Obviously, S(2)=1; the proof that S(3)=4 is very complicated [Murphy & Cohen, 1993]. For 5 balls, S(5) is unknown.

Splash problem: Billiard features

- The total number of collisions grows without bounds, namely as t^{2(2-b)/3}. The dominant contribution comes from collisions in the hydrodynamic region.
- Each particle undergoes a finite number of collisions. This is an analog of Sinai's conjecture in the splash problem.
- Consider the left-most particle. How many collisions it is going to experience?
- We cannot even prove that it has a hypothetical monotonically increasing form



The left-most particle

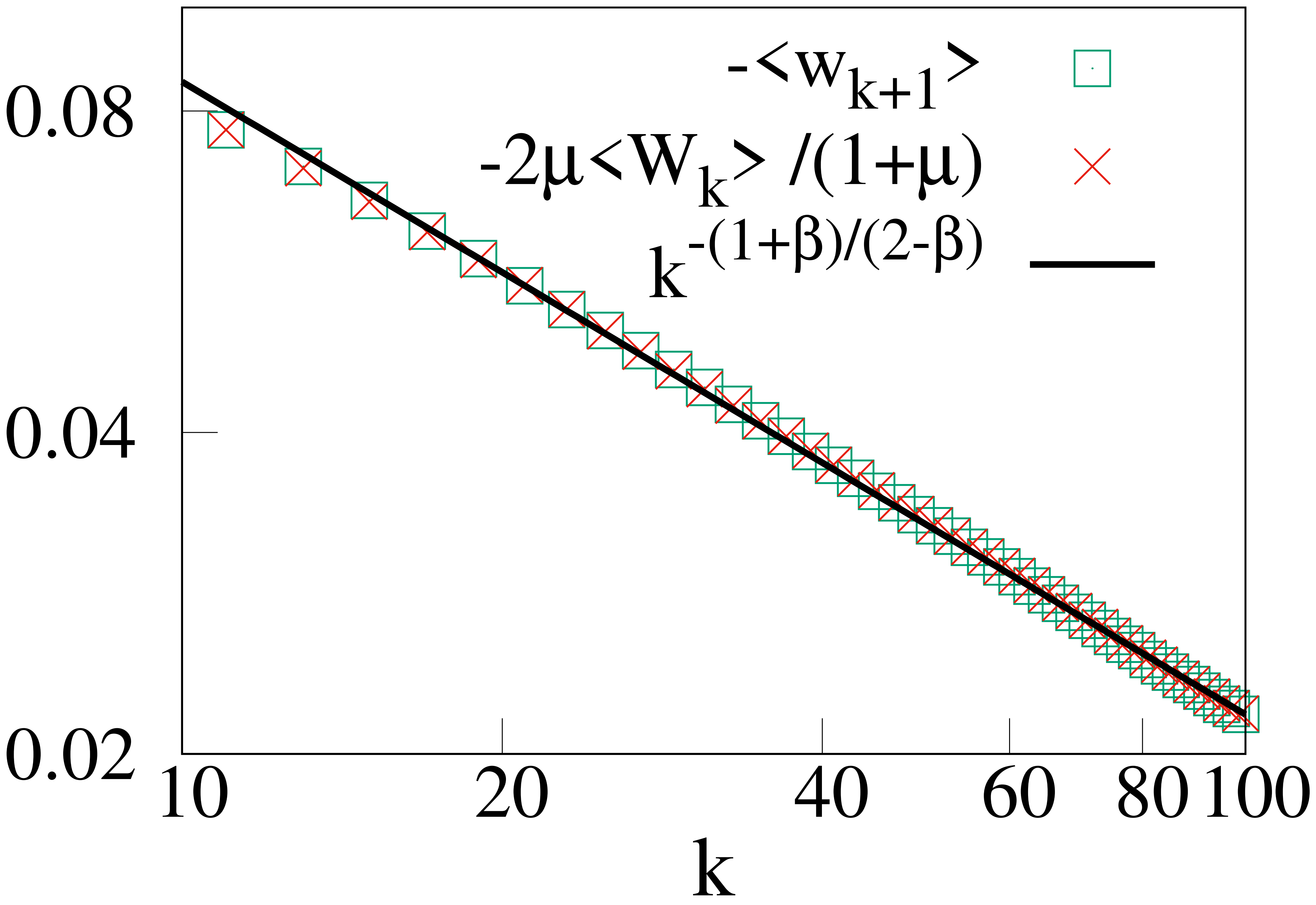
Initial (alternating) masses are $(\mu, 1, \mu, 1, \mu, 1, ...)$. Initial velocities are $(1, 0, 0, 0, \ldots)$. After the 1st collision

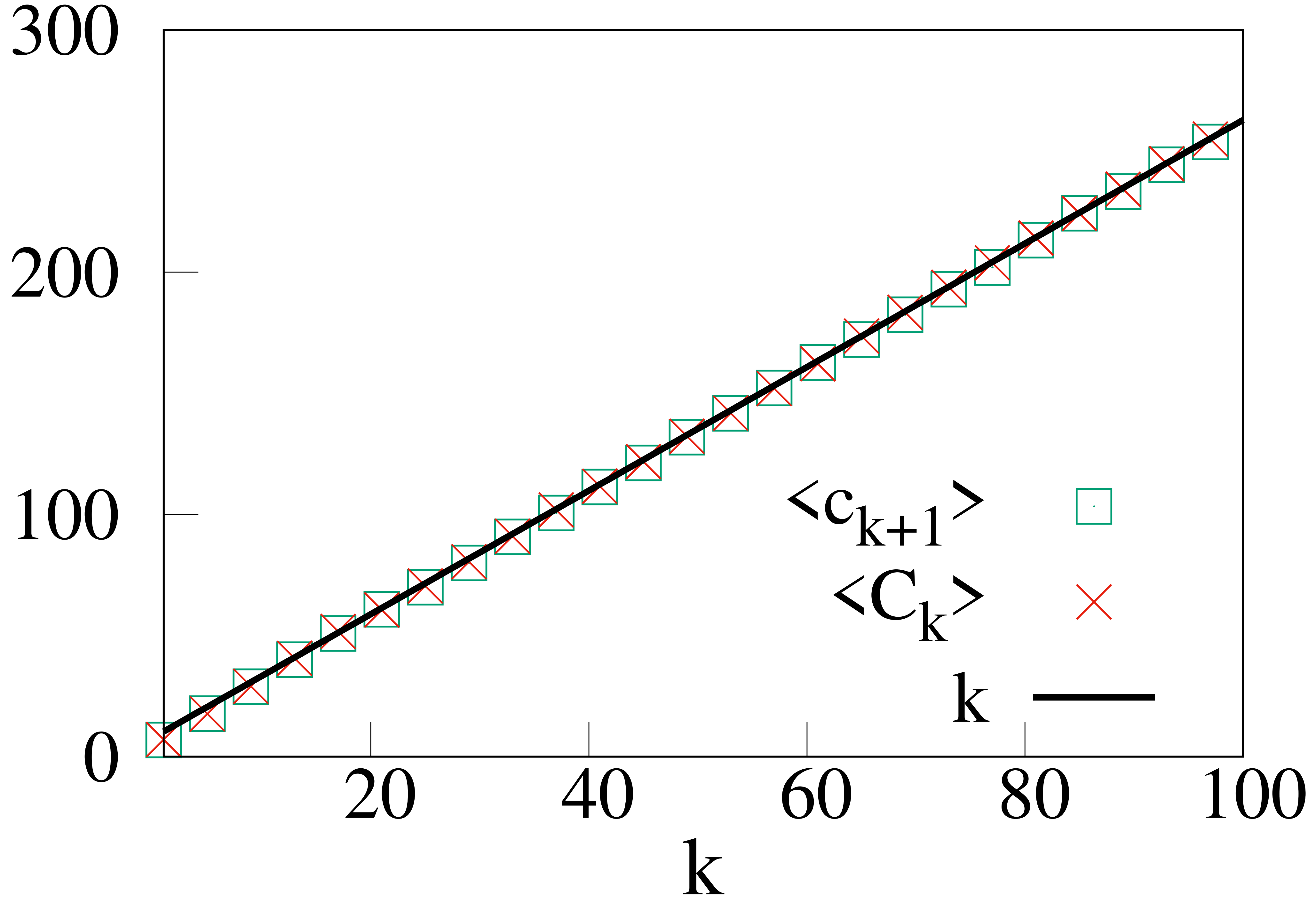
$$v_1 = \frac{\mu - 1}{\mu + 1} \qquad\qquad v_2 = \frac{2\mu}{\mu + 1}$$

When $\mu \leq 1$, the ultimate velocity of the 1st particle is *conjecturally* v_1. The ultimate speed of the 2nd particle cannot be larger than v_2, and $-v_1 > v_2$ is obeyed when $\mu < \frac{1}{3}$. Thus the conjecture is certainly true when $\mu < \frac{1}{3}$. It is also obviously true when $\mu = 1$. Thus the conjecture is very plausible.

The knowledge of energy and momentum of the splatter particles allows to guess the decay of the ultimate speeds

$$\sum_{i\geq 1} w_i^2 = O(1), \qquad \sum_{i=1}^{t^{(2-\beta)/3}} w_i \sim t^{(1-2\beta)/3} \qquad \Longrightarrow \qquad w_i \sim i^{-\frac{1+\beta}{2-\beta}}$$





Blast and splash problems: Challenges

- The agreement between direct molecular dynamics simulations and theory seems better in one dimension than in two and three dimensions. This is puzzling.

- The structure of the shock wave in the realm of kinetic theory is an old challenge. In our situation, the shock is infinitely strong, yet the solution of the Boltzmann equation in this case is still unknown. (The 1st attempt was made by Tamm (1946), also during the work on the atomic bomb.) The blast and splash problems are not the best settings, however; it is better to consider the piston problem.

- The surface of the blast region is spherical according to hydrodynamics. Fluctuations of the surface are unknown (and beyond the scope of deterministic hydrodynamics).

- Find the limit shape of boundary in the splash problem (when $d=2$ or $d=3$).

- Proof that each particle is eventually kicked out from the $x>0$ half-space and it joins the fan. (This is certainly correct even if hard to prove in one dimension; perhaps wrong in higher dimensions.)

The End