Mitigating ecological tipping points via game-environment feedback

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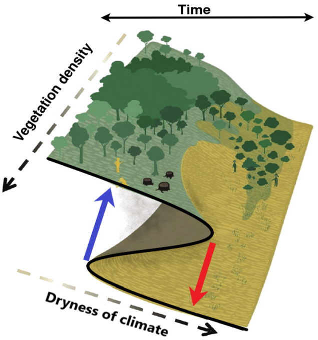
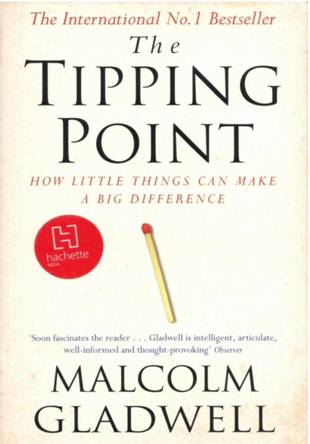
Critical Transitions or Tipping Points

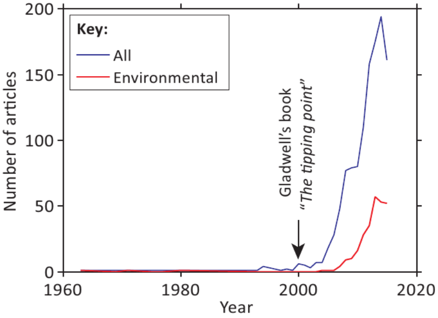
In complex systems, **"tipping"** or **"critical transitions"** are **sudden**, **large**, often **irreversible** changes in the state of a system, under the influence of small stochastic perturbations.



Bathiany, Sebastian, et al. Simple tipping or complex transition? Lessons from a green Sahara. PAGES Magazine 24 (2016): 20-21.

How little things can make a big difference!





Surge in research on tipping points

Scheffer et al. (2001). Catastrophic shifts in ecosystems. Nature, 413, 591–596.
Van Nes et al. (2016). What Do You Mean, 'Tipping Point'? TREE, 31, 902.

Catastrophic and Non-catastrophic transitions: Temporal systems

- Catastrophic Transitions: Sudden, large, and often irreversible changes in the state of a system.
- Non-catastrophic Transitions: Smooth and reversible, and characterized by quantitatively similar dynamics prior and post transition.

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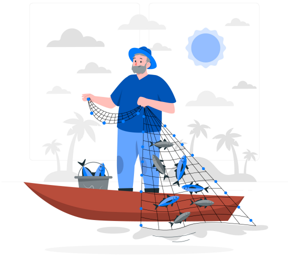
The dynamics of the resource (m) under harvesting stress are governed by:

$$\frac{dm}{dt} = rm\left(1 - \frac{m}{K}\right) - qe\psi_i(m), \tag{1}$$

where e is the harvesting effort.



Catastrophic and non-catastrophic transitions under harvesting stress

- Type-I ($\psi_1(m) = m$): Non-catastrophic extinction
- Type-II ($\psi_2(m) = \dfrac{m}{h + m}$): Catastrophic extinction
- Type-III ($\psi_3(m) = \dfrac{m^2}{h^2 + m^2}$): Catastrophic collapse to low biomass

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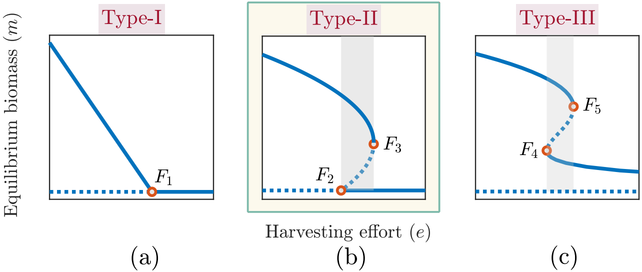

Figure: Changes in the resource equilibrium biomass for varying harvesting effort.

Scheffer et al., Catastrophic regime shifts in ecosystems: linking theory to observation, Trends in ecology & evolution, 18, 648–656, 2003.

Lotka, AJ., Elements of physical biology. Reprinted 1956, New York: Dover, 1925 .

Game-environment feedback

- Resource dynamics incorporating harvesting strategies:

$$\frac{dm}{dt} = rm\left(1 - \frac{m}{K}\right) - \frac{q(e_L x + e_H(1-x))m}{h+m}. \tag{2}$$

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- Transformation of resource abundance to environmental state:

$$\frac{dn}{dt} = rn\left(1 - \frac{an}{K}\right) - \frac{q(e_L x + e_H(1-x))n}{h+an}, \tag{3}$$

where

$$a = \frac{1}{2}\left[(K-h) + \sqrt{(K+h)^2 - \frac{4qe_L K}{r}}\right].$$

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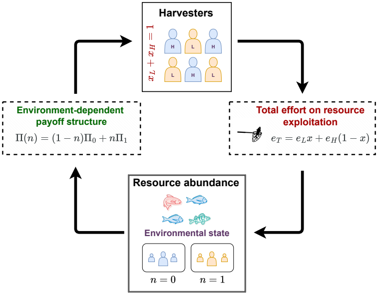

Weitz et al., An oscillating tragedy of the commons in replicator dynamics with game-environment feedback, PNAS, 113(47), E7518–E7525, 2016

Payoff structures capturing diverse social dilemmas

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$$\Pi(n) = (1-n)\begin{bmatrix} 1 & -D_{r_0} \\ 1 + D_{g_0} & 0 \end{bmatrix} + n\begin{bmatrix} 1 & -D_{r_1} \\ 1 + D_{g_1} & 0 \end{bmatrix}.$$

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$$\left.\begin{aligned} \pi_L(x,n) &= x-(1-x)\left[(1-n)D_{r_0}+nD_{r_1}\right], \\ \pi_H(x,n) &= x\left[1+(1-n)D_{g_0}+nD_{g_1}\right]. \end{aligned}\right\}$$

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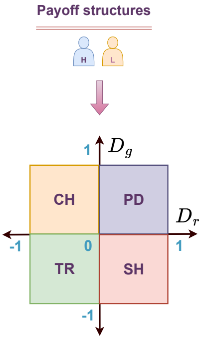
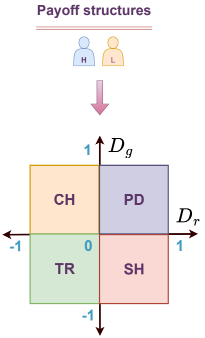


- PD vs. TR: A Contrast The combination of TR and PD presents the greatest disparity between individual self-interest and collective welfare.



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- CH+PD
- SH+PD

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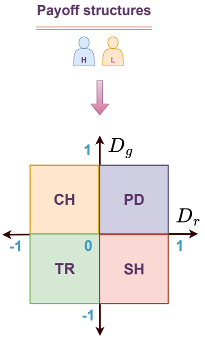

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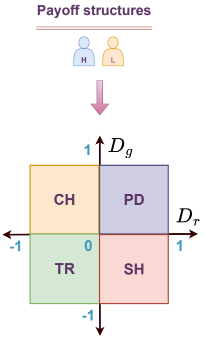

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Punishment of detected defectors and associated replicator equation

- Modified payoff matrix:

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- Coupled human–environment system:

$$\begin{aligned} \frac{dx}{dt} &= x(1-x)(\widehat{\pi}_L(x,n) - \widehat{\pi}_H(x,n)), \\ \frac{dn}{dt} &= rn\left(1 - \frac{an}{K}\right) - \frac{q(e_L x + e_H(1-x))n}{h+an}. \end{aligned} \tag{4}$$

Results: Increased harvesting effort with different game combinations has variable effects on environmental tipping points

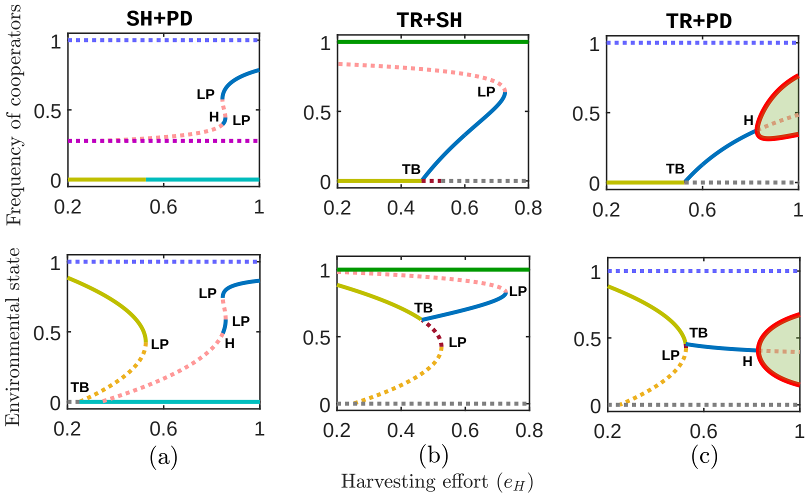
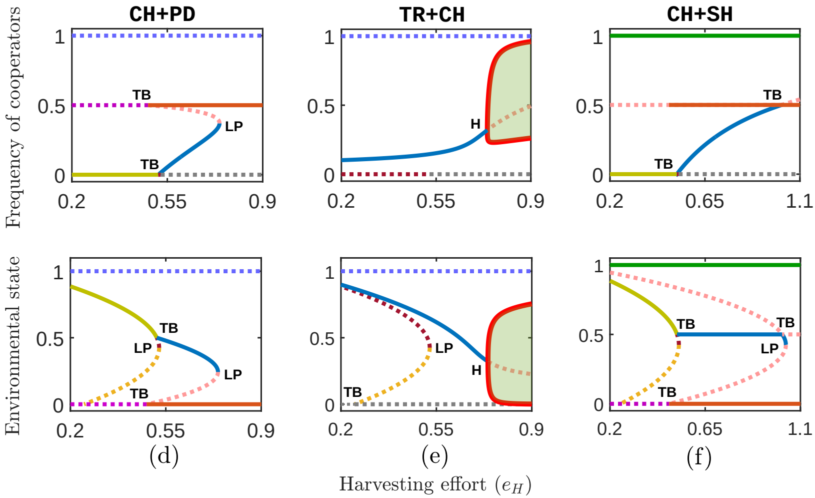


Figure: Without monitoring system, bifurcation diagrams of the coupled system for different game combinations.



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Oscillatory tragedy of the commons

The **"tragedy of the commons"** illustrates a significant dilemma in which individuals, lured by the prospect of seeking advantages over others, unwittingly contribute to exhaustion of shared resources. This, in turn, leads to a decrease in the collective well-being of the entire community.

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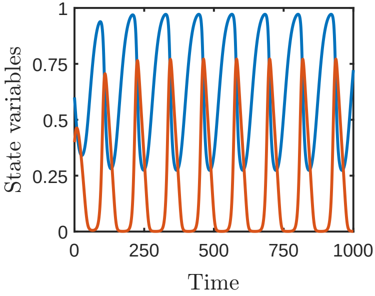
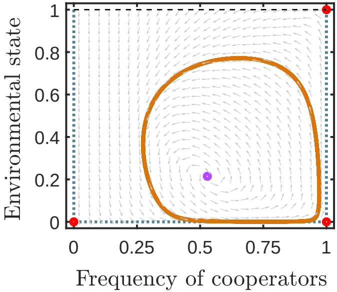


(a)



(b)

Figure: For the combination of TR and CH, oscillatory behaviors of both strategies and the environment.

Hardin et al., The tragedy of the commons: the population problem has no technical solution; it requires a fundamental extension in morality, Science, 162(3859), 1243–1248, 1968

Summary of the results of the effects of varying efforts on system dynamics

Table-1: Without system monitoring ($p = 0$), outcomes of the coupled human-environment system for different game combinations with increasing defector harvesting effort e_H, where IC-TOC: Initial condition dependent TOC, and O-TOC: Oscillatory TOC

Game combination		**Effect of e_H on tipping**	**Effect of e_H on TOC**
At $n = 0$	At $n = 1$		
SH	PD	Eluded	IC-TOC
TR	SH	Eluded	No TOC
TR	PD	Eluded	No TOC
CH	PD	Shifted	IC-TOC
TR	CH	Eluded	O-TOC
CH	SH	Shifted	IC-TOC

For varying intrinsic resource growth rate (r) and carrying capacity (K), evolutionary outcomes of the environmental state under a punishment-free scenario

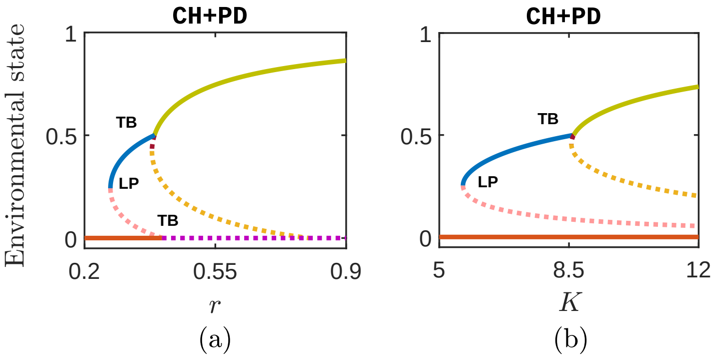


Figure: Environmental improvement in terms of resource growth rate and carrying capacity. While varying growth rate transitions the system from bistability to monostability, varying carrying capacity maintains the dependence of evolutionary outcomes on initial conditions.

Governance of the monitoring efficacy p on the evolutionary dynamics of the environmental state for different game combinations

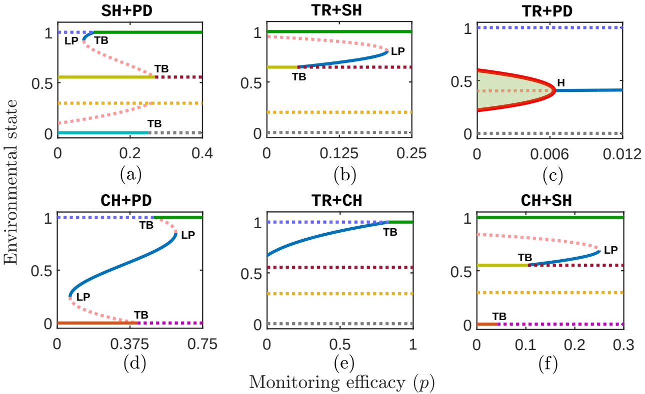


Figure: An improvement in system monitoring contributes to environmental enrichment. In (a) and (e), the environment undergoes smooth transitions to the replete state, while in (b), (d), and (f), the transition is non-smooth.

Chen et al., Punishment and inspection for governing the commons in a feedback-evolving game, PLoS Comput. Biol., 14(7), e1006347, 2018

Summary of the results of the effects of monitoring efficacy on system dynamics

Table-2: For fixed effort levels, outcomes of the coupled human-environment system for different game combinations with increasing monitoring efficacy (p)

Game combination		TOC without monitoring	Effect of p on tipping	Effect of p on TOC
At $n=0$	At $n=1$			
SH	PD	IC-TOC	Eluded	Eluded
TR	SH	No TOC	Eluded	No TOC
TR	PD	No TOC	No effect	No TOC
CH	PD	TOC	Eluded	Eluded
TR	CH	No TOC	Eluded	No TOC
CH	SH	IC-TOC	Eluded	Eluded

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- In games combining TR and CH or TR and PD, intensified efforts affect the system's stable configuration, leading to cyclic patterns in both the frequency of cooperators and environmental state.
- The tragedy of the commons can be mitigated through targeted punishment.

Mandal et al., Proc. R. Soc. A, 481, 2024.0915, 2025

Thank You!