XYZTP spectroscopy using lattice QCD

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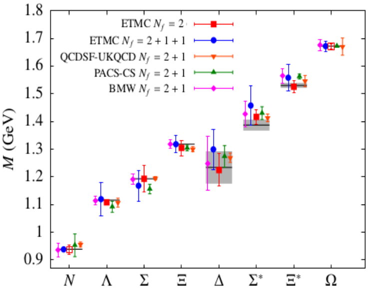
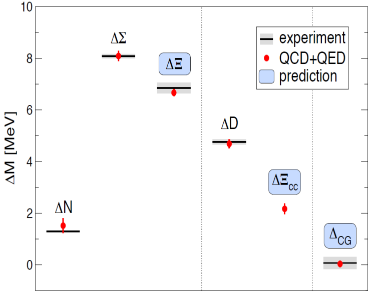
QEICIII 2024 ICTS-TIFR

Hadron masses from lattice QCD



ETMC PRD 2014



BMW Science 2014

(ii) Emergence of mass and spin: The investigation of the emergence of mass and spin in all visible composite matter is of utmost importance in our understanding of nature. Lattice QCD methods provide a unique tool to study it from first principles with controlled systematics. Lattice QCD methods can already calculate the nucleon mass with an accuracy at a percent level. However, it is still intriguing how the collective interactions of tiny quarks and gluons emerge into a massive hadron. It is of fundamental interest to find how the mass of a composite subatomic particle, through the dynamics of strong interactions, decomposes into various parts

MSV2035 document

Nucleons are not alone

There is a big family of partilces observed in nature, of which nucleon is just a member.

Baryons

p	$1/2^+$	****
n	$1/2^+$	****
$N(1440)$	$1/2^+$	****
$N(1520)$	$3/2^-$	****
$N(1535)$	$1/2^-$	****
$N(1650)$	$1/2^-$	****
$N(1675)$	$5/2^-$	****
$N(1680)$	$5/2^+$	****
$N(1700)$	$3/2^-$	***
$N(1710)$	$1/2^+$	****
$N(1720)$	$3/2^+$	****
$N(1860)$	$5/2^+$	**
$N(1875)$	$3/2^-$	***
$N(1880)$	$1/2^+$	***
$N(1895)$	$1/2^-$	****
$N(1900)$	$3/2^+$	****
$N(1990)$	$7/2^+$	**
$N(2000)$	$5/2^+$	**
$N(2040)$	$3/2^+$	*
$N(2060)$	$5/2^-$	***
$N(2100)$	$1/2^+$	***
$N(2120)$	$3/2^-$	***
$N(2190)$	$7/2^-$	****
$N(2220)$	$9/2^+$	****
$N(2250)$	$9/2^-$	****
$N(2300)$	$1/2^+$	**
$N(2570)$	$5/2^-$	**
$N(2600)$	$11/2^-$	***
$N(2700)$	$13/2^+$	**

$\Delta(1232)$	$3/2^+$	****
$\Delta(1600)$	$3/2^+$	****
$\Delta(1620)$	$1/2^-$	****
$\Delta(1700)$	$3/2^-$	****
$\Delta(1750)$	$1/2^+$	*
$\Delta(1900)$	$1/2^-$	***
$\Delta(1905)$	$5/2^+$	****
$\Delta(1910)$	$1/2^+$	****
$\Delta(1920)$	$3/2^+$	***
$\Delta(1930)$	$5/2^-$	***
$\Delta(1940)$	$3/2^-$	**
$\Delta(1950)$	$7/2^+$	****
$\Delta(2000)$	$5/2^+$	**
$\Delta(2150)$	$1/2^-$	*
$\Delta(2200)$	$7/2^-$	***
$\Delta(2300)$	$9/2^+$	**
$\Delta(2350)$	$5/2^-$	*
$\Delta(2390)$	$7/2^+$	*
$\Delta(2400)$	$9/2^-$	**
$\Delta(2420)$	$11/2^+$	****
$\Delta(2750)$	$13/2^-$	**
$\Delta(2950)$	$15/2^+$	**
Λ	$1/2^+$	****
Λ	$1/2^-$	**
$\Lambda(1405)$	$1/2^-$	****
$\Lambda(1520)$	$3/2^-$	****
$\Lambda(1600)$	$1/2^+$	****
$\Lambda(1670)$	$1/2^-$	****
$\Lambda(1690)$	$3/2^-$	****
$\Lambda(1710)$	$1/2^+$	*
$\Lambda(1800)$	$1/2^-$	***
$\Lambda(1810)$	$1/2^+$	***
$\Lambda(1820)$	$5/2^+$	****
$\Lambda(1830)$	$5/2^-$	****
$\Lambda(1890)$	$3/2^+$	****
$\Lambda(2000)$	$1/2^-$	*
$\Lambda(2050)$	$3/2^-$	*
$\Lambda(2070)$	$3/2^+$	*
$\Lambda(2080)$	$5/2^-$	*
$\Lambda(2085)$	$7/2^+$	**
$\Lambda(2100)$	$7/2^-$	****
$\Lambda(2110)$	$5/2^+$	***
$\Lambda(2325)$	$3/2^-$	*
$\Lambda(2350)$	$9/2^+$	***
$\Lambda(2585)$		**

Σ^+	$1/2^+$	****
Σ^0	$1/2^+$	****
Σ^-	$1/2^+$	****
$\Sigma(1385)$	$3/2^+$	****
$\Sigma(1580)$	$3/2^-$	*
$\Sigma(1620)$	$1/2^-$	*
$\Sigma(1660)$	$1/2^+$	***
$\Sigma(1670)$	$3/2^-$	****
$\Sigma(1750)$	$1/2^-$	***
$\Sigma(1775)$	$5/2^-$	****
$\Sigma(1780)$	$3/2^+$	*
$\Sigma(1880)$	$1/2^+$	**
$\Sigma(1900)$	$1/2^-$	**
$\Sigma(1910)$	$3/2^-$	***
$\Sigma(1915)$	$5/2^+$	****
$\Sigma(1940)$	$3/2^+$	*
$\Sigma(2010)$	$3/2^-$	*
$\Sigma(2030)$	$7/2^+$	****
$\Sigma(2070)$	$5/2^+$	*
$\Sigma(2080)$	$3/2^+$	*
$\Sigma(2100)$	$7/2^-$	*
$\Sigma(2160)$	$1/2^-$	*
$\Sigma(2230)$	$3/2^+$	*
$\Sigma(2250)$		***
$\Sigma(2455)$		**
$\Sigma(2620)$		**
$\Sigma(3000)$		*
$\Sigma(3170)$		*

Ξ^0	$1/2^+$	****
Ξ^-	$1/2^+$	****
$\Xi(1530)$	$3/2^+$	****
$\Xi(1620)$		*
$\Xi(1690)$		***
$\Xi(1820)$	$3/2^-$	***
$\Xi(1950)$		***
$\Xi(2030)$	$\geq \frac{5}{2}^?$	***
$\Xi(2120)$		*
$\Xi(2250)$		**
$\Xi(2370)$		**
$\Xi(2500)$		*
Ω^-	$3/2^+$	****
$\Omega(2012)^-$	$?^-$	***
$\Omega(2250)^-$		***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**
Λ_c^+	$1/2^+$	****
$\Lambda_c(2595)^+$	$1/2^-$	***
$\Lambda_c(2625)^+$	$3/2^-$	***
$\Lambda_c(2765)^+$		*
$\Lambda_c(2860)^+$	$3/2^+$	***
$\Lambda_c(2880)^+$	$5/2^+$	***
$\Lambda_c(2940)^+$	$3/2^-$	***
$\Sigma_c(2455)$	$1/2^+$	****
$\Sigma_c(2520)$	$3/2^+$	***
$\Sigma_c(2800)$		***
Ξ_c^+	$1/2^+$	***
Ξ_c^0	$1/2^+$	****
$\Xi_c'^+$	$1/2^+$	***
$\Xi_c'^0$	$1/2^+$	***
$\Xi_c(2645)$	$3/2^+$	***
$\Xi_c(2790)$	$1/2^-$	***
$\Xi_c(2815)$	$3/2^-$	***
$\Xi_c(2930)$		**
$\Xi_c(2970)$		***
$\Xi_c(3055)$		***
$\Xi_c(3080)$		***
$\Xi_c(3123)$		*
Ω_c^0	$1/2^+$	***
$\Omega_c(2770)^0$	$3/2^+$	***
$\Omega_c(3000)^0$		***
$\Omega_c(3050)^0$		***
$\Omega_c(3065)^0$		***
$\Omega_c(3090)^0$		***
$\Omega_c(3120)^0$		***

Ξ_{cc}^{++}		***
Λ_b^0	$1/2^+$	***
$\Lambda_b(5912)^0$	$1/2^-$	***
$\Lambda_b(5920)^0$	$3/2^-$	***
$\Lambda_b(6146)^0$	$3/2^+$	***
$\Lambda_b(6152)^0$	$5/2^+$	***
Σ_b	$1/2^+$	***
Σ_b^*	$3/2^+$	***
$\Sigma_b(6097)^+$		***
$\Sigma_b(6097)^-$		***
Ξ_b^0, Ξ_b^-	$1/2^+$	***
$\Xi_b'(5935)^-$	$1/2^+$	***
$\Xi_b(5945)^0$	$3/2^+$	***
$\Xi_b(5955)^-$	$3/2^+$	***
$\Xi_b(6227)$		***
Ω_b^-	$1/2^+$	***
$P_c(4312)^+$		*
$P_c(4380)^+$		*
$P_c(4440)^+$		*
$P_c(4457)^+$		*

Mesons

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$
• $f_2(1270)$	$0^+(2^{++})$
• $f_1(1285)$	$0^+(1^{++})$
• $\eta(1295)$	$0^+(0^{-+})$
• $\pi(1300)$	$1^-(0^{-+})$
• $a_2(1320)$	$1^-(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$
• $\eta(1405)$	$0^+(0^{-+})$
• $h_1(1415)$	$0^-(1^{+-})$
$a_1(1420)$	$1^-(1^{++})$
• $f_1(1420)$	$0^+(1^{++})$
• $\omega(1420)$	$0^-(1^{--})$
$f_2(1430)$	$0^+(2^{++})$
• $a_0(1450)$	$1^-(0^{++})$
• $\rho(1450)$	$1^+(1^{--})$
• $\eta(1475)$	$0^+(0^{-+})$
• $f_0(1500)$	$0^+(0^{++})$
$f_1(1510)$	$0^+(1^{++})$
• $f_2'(1525)$	$0^+(2^{++})$
$f_2(1565)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$
$h_1(1595)$	$0^-(1^{+-})$
• $\pi_1(1600)$	$1^-(1^{-+})$
• $a_1(1640)$	$1^-(1^{++})$
$f_2(1640)$	$0^+(2^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$
• $\omega(1650)$	$0^-(1^{--})$
• $\omega_3(1670)$	$0^-(3^{--})$
• $\pi_2(1670)$	$1^-(2^{-+})$
• $\phi(1680)$	$0^-(1^{--})$
• $\rho_3(1690)$	$1^+(3^{--})$
• $\rho(1700)$	$1^+(1^{--})$
• $a_2(1700)$	$1^-(2^{++})$
• $f_0(1710)$	$0^+(0^{++})$
$\eta(1760)$	$0^+(0^{-+})$
• $\pi(1800)$	$1^-(0^{-+})$
$f_2(1810)$	$0^+(2^{++})$
$X(1835)$	$?^?(0^{-+})$
• $\phi_3(1850)$	$0^-(3^{--})$
• $\eta_2(1870)$	$0^+(2^{-+})$
• $\pi_2(1880)$	$1^-(2^{-+})$
$\rho(1900)$	$1^+(1^{--})$
$f_2(1910)$	$0^+(2^{++})$
$a_0(1950)$	$1^-(0^{++})$
• $f_2(1950)$	$0^+(2^{++})$
• $a_4(1970)$	$1^-(4^{++})$
$\rho_3(1990)$	$1^+(3^{--})$
$\pi_2(2005)$	$1^-(2^{-+})$
• $f_2(2010)$	$0^+(2^{++})$
$f_0(2020)$	$0^+(0^{++})$
• $f_4(2050)$	$0^+(4^{++})$
$\pi_2(2100)$	$1^-(2^{-+})$
$f_0(2100)$	$0^+(0^{++})$
$f_2(2150)$	$0^+(2^{++})$
$\rho(2150)$	$1^+(1^{--})$
• $\phi(2170)$	$0^-(1^{--})$
$f_0(2200)$	$0^+(0^{++})$
$f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$\eta(2225)$	$0^+(0^{-+})$
$\rho_3(2250)$	$1^+(3^{--})$
• $f_2(2300)$	$0^+(2^{++})$
$f_4(2300)$	$0^+(4^{++})$
$f_0(2330)$	$0^+(0^{++})$
• $f_2(2340)$	$0^+(2^{++})$
$\rho_5(2350)$	$1^+(5^{--})$
$f_6(2510)$	$0^+(6^{++})$

OTHER LIGHT

Further States

STRANGE ($S = \pm1, C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
• $K_0^*(700)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2300)^0$	$1/2(0^+)$
$D_0^*(2300)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(?^?)$
$D_J^*(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2740)^0$	$1/2(?^?)$
$D_3^*(2750)$	$1/2(3^-)$
$D(3000)^0$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}^*(2573)$	$0(2^+)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{s1}^*(2860)^\pm$	$0(1^-)$
$D_{s3}^*(2860)^\pm$	$0(3^-)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$-baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
• $B_1(5721)^+$	$1/2(1^+)$
• $B_1(5721)^0$	$1/2(1^+)$
$B_J^*(5732)$	$?(?^?)$
• $B_2^*(5747)^+$	$1/2(2^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$
$B_J(5840)^+$	$1/2(?^?)$
$B_J(5840)^0$	$1/2(?^?)$
• $B_J(5970)^+$	$1/2(?^?)$
• $B_J(5970)^0$	$1/2(?^?)$

BOTTOM, STRANGE ($B = \pm1, S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
$X(5568)^\pm$	$?(?^?)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• B_c^+	$0(0^-)$
$B_c(2S)^\pm$	$0(0^-)$

$c\overline{c}$ (+ possibly non-$q\overline{q}$ states)

• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$0^-(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$

$c\overline{c}$ continued

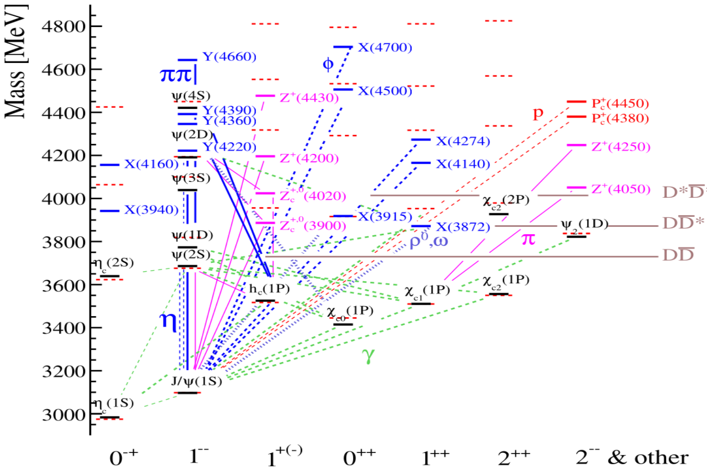
	$I^G(J^{PC})$
• $\psi(3770)$	$0^-(1^{--})$
• $\psi_2(3823)$	$0^-(2^{--})$
• $\psi_3(3842)$	$0^-(3^{--})$
$\chi_{c0}(3860)$	$0^+(0^{++})$
• $\chi_{c1}(3872)$	$0^+(1^{++})$
• $Z_c(3900)$	$1^+(1^{+-})$
• $X(3915)$	$0^+(0/2^{++})$
• $\chi_{c2}(3930)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
• $X(4020)^\pm$	$1^+(?^{?-})$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$1^-(?^{?+})$
$X(4055)^\pm$	$1^+(?^{?-})$
$X(4100)^\pm$	$1^-(?^{??})$
• $\chi_{c1}(4140)$	$0^+(1^{++})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$Z_c(4200)$	$1^+(1^{+-})$
• $\psi(4230)$	$0^-(1^{--})$
$R_{c0}(4240)$	$1^+(0^{--})$
$X(4250)^\pm$	$1^-(?^{?+})$
$\psi(4260)$	$0^-(1^{--})$
• $\chi_{c1}(4274)$	$0^+(1^{++})$
$X(4350)$	$0^+(?^{?+})$
• $\psi(4360)$	$0^-(1^{--})$
$\psi(4390)$	$0^-(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
• $Z_c(4430)$	$1^+(1^{+-})$
$\chi_{c0}(4500)$	$0^+(0^{++})$
• $\psi(4660)$	$0^-(1^{--})$
$\chi_{c0}(4700)$	$0^+(0^{++})$

$b\overline{b}$ (+ possibly non-$q\overline{q}$ states)

• $\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$0^-(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon_2(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$0^-(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_{b1}(3P)$	$0^+(1^{++})$
• $\chi_{b2}(3P)$	$0^+(2^{++})$
• $\Upsilon(4S)$	$0^-(1^{--})$
• $Z_b(10610)$	$1^+(1^{+-})$
• $Z_b(10650)$	$1^+(1^{+-})$
$\Upsilon(10753)$	$?^?(1^{--})$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

Summary tables taken from Particle Data Group website (2021): pdg.lbl.gov/

Understanding the spectrum at the core of understanding strong interactions.



Rich energy spectrum. XYZ states.

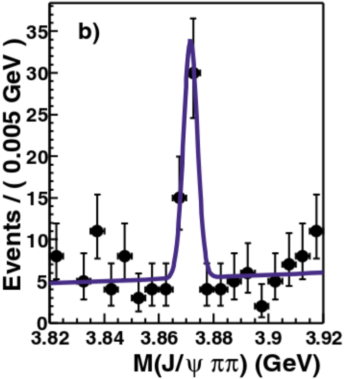
$\bar{c}c$ picture works well for states below open charm threshold.

Olsen *et al* 1708.04012

No single description for states above the open charm threshold.

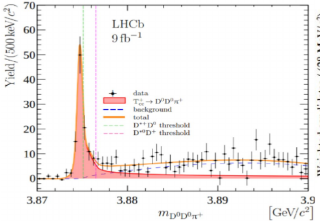
Experimental facts : X(3872)

- first observed in Belle 2003
  (Belle PRL 2003)
- Quantum numbers, $J^{PC} = 1^{++}$
  (LHCb, 2013)
- Appears within 1 MeV below
  $D^0 \bar{D}^{*0}$ threshold.
- Preferred strong decay modes
  $J/\psi\ \omega$ and $J/\psi\ \rho$

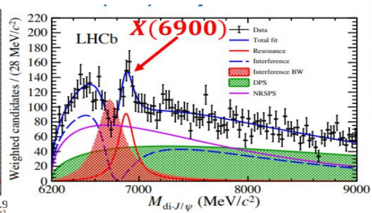


- The isospin still uncertain
  * nearly equal branching fraction to $J/\psi\ \omega$ and $J/\psi\ \rho$ decays.
  * No charge partner candidates observed.

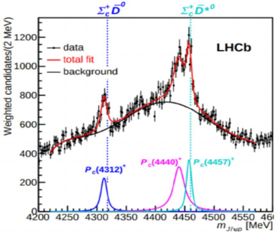
Beyond baryons and mesons in experiments



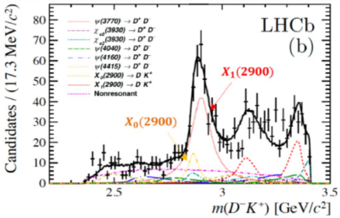
T_{cc} LHCb 2021



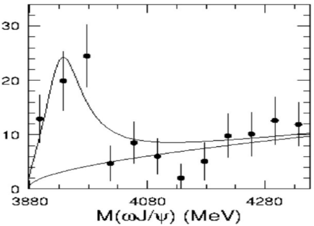
X(6900) LHCb 2020



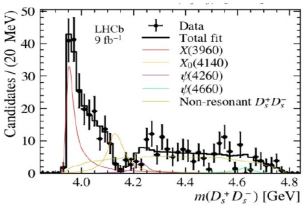
P_c LHCb 2019



X(2900) LHCb 2020



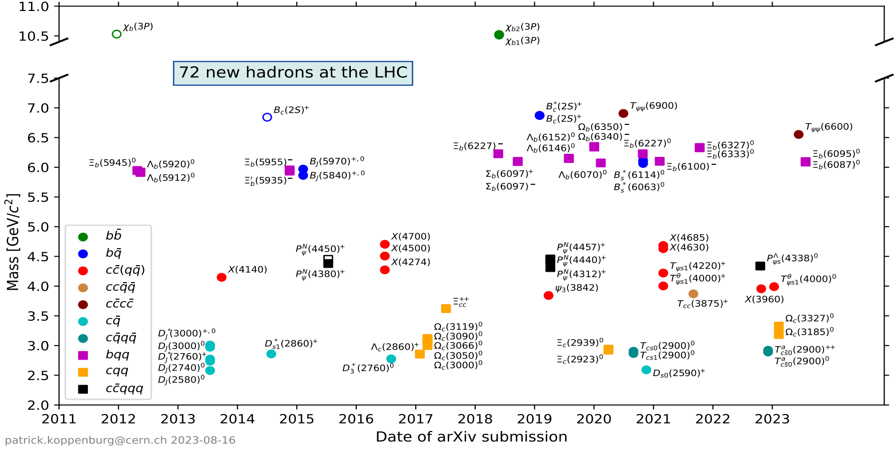
X(3915) Belle 2005



X(3960) LHCb 2021

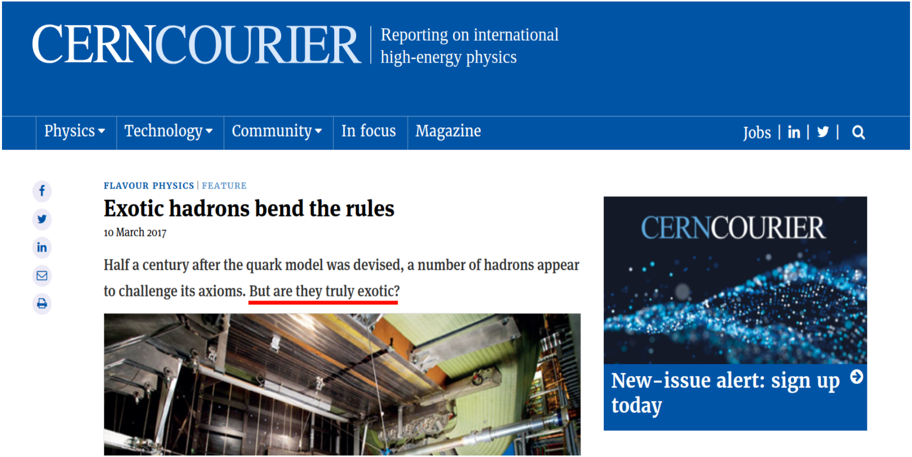
See a recent talk by Liming Zhang here

Summary of LHCb discoveries



https://www.nikhef.nl/∼pkoppenb/particles.html

See a recent talk by Liming Zhang here



Frank Close 2017, https://cerncourier.com/a/exotic-hadrons-bend-the-rules/

XYZ: exotic mesons, T: Tetraquarks, P: Pentaquarks

Mass, nature and the structure of exotics

- My talk only cover mass determinations and coupling to strong decays.
  Assumes isospin symmetry throughout the talk, $m_u = m_d$.

- EIC detections, if made, could complement other experiments like
  BESIII, BelleII, and LHCb.

- Different production mechanisms at EIC:
  Exclusive photoproduction of charmonium-like states — Albaladejo *et. al.* 2008.01001
  SemiInclusive electroproduction of XYZs — Shi *et. al.* 2208.02639

- Heavy quarks hadrons as important probes of nuclear medium.
  Flipping around the idea: Medium to probe the poorly understood hadrons.
  A new way to reveal the nature and structure of exotics.
  Zhang *et. al.* 2004.00024, Esposito *et. al.* 2006.15044

- The nature of exotics from quark mass dependence.
  Collins, Nefediev, MP, Prelovsek 2402.xxxxx

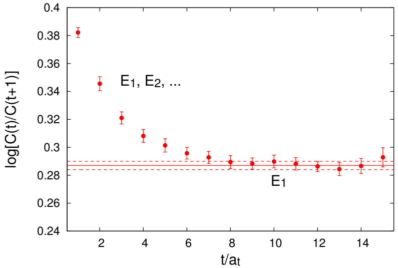
QCD spectrum from Lattice QCD

- Aim : to extract the physical states of QCD.

- Euclidean two point current-current correlation functions

$$C_{ji}(t_f - t_i) = \langle 0|\mathcal{O}_j(t_f)\bar{\mathcal{O}}_i(t_i)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$$

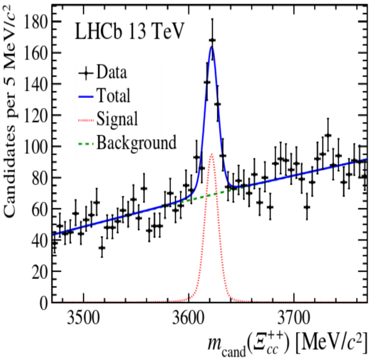
where $\mathcal{O}_j(t_f)$ and $\bar{\mathcal{O}}_i(t_i)$ are the desired interpolating operators and $Z_j^n = \langle 0|\mathcal{O}_j|n\rangle$.

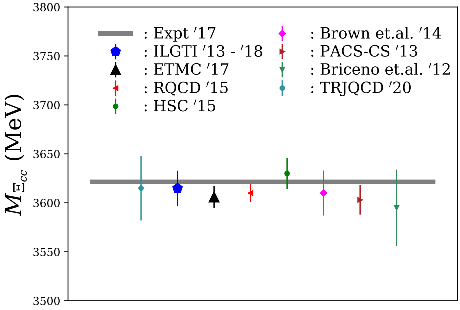


- Effective mass defined as $\log[\frac{C(t)}{C(t+1)}]$

- The ground state : from the exponential fall off at large times.
  Non-linear fitting techniques.

The first doubly charm baryon : Ξ_{cc}





Ξ_{cc} isospin splitting (LQCD), 2.16(11)(17) MeV : BMW Science**347** 1452 '15
SELEX measurement (3519 MeV) : Mattson *et al.* PRL**89** 112001 '02

All lattice calculations disfavors SELEX peak to be a doubly charm baryon.

More on heavy baryon interactions in this link.

Excited hadron spectrum

- Most hadrons labelled exotic: close to strong decay thresholds.

- Challenges include extracting densly populated spectra.

  Extracting densly populated states
  Extracting radial and orbital excitations
  Extracting excitations with spin $> 3/2$
  Systematic spin identification
  Multiple scattering channels affecting the single hadron spectra

- Scattering parameters from finite volume energy shifts.
  Lüscher's formalism and its generalizations.

  Lüscher 1991, Briceño 2014 and references and references therein

- Encouraging achievements in the light and heavy hadron spectra.

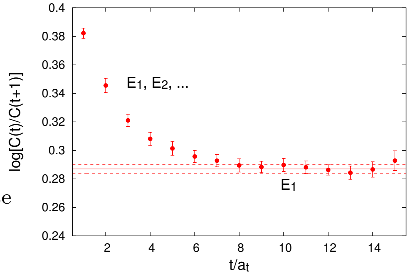
  c.f. yearly Lattice conference proceedings

Remember the correlation function

$$C_{ji}(t_f - t_i) = \langle 0|\mathcal{O}_j(t_f)\bar{\mathcal{O}}_i(t_i)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2m_n} e^{-m_n(t_f - t_i)}$$

- The operator can in principle couple with all the states that have its q. #s.

- The strength of coupling Z_n determines the quality of signal.
- The excited state information in early time slices.
- Limited # time slices to extract excited state energies from multi-exponential fits.



- Extraction of energy degenerate states is impossible this way.

Excited state information in $C_{ji}(t)$

- Instead let us build a matrix of correlatiion functions:

$$C_{ji}(t) = \langle 0|\Phi_j(t)\bar{\Phi}_i(0)|0\rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2E_n} e^{-E_n(t)}$$

  where $\Phi_j(t)$ and $\bar{\Phi}_i(0)$ are the desired interpolating operators.
  $Z_j^n = \langle 0|\Phi_j|n\rangle$ are the operator-state overlaps.

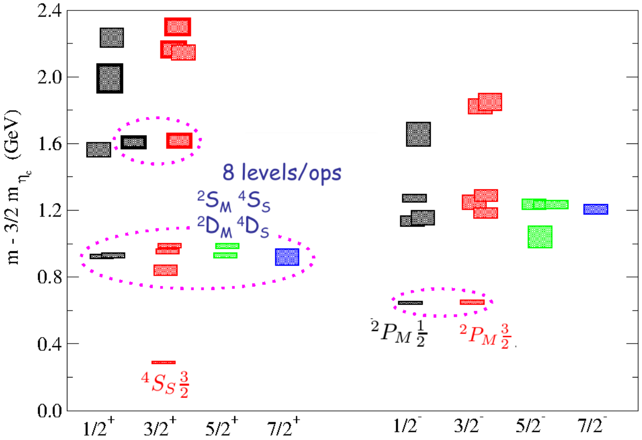
- Operators can have any structure that obey the desired quantum no.s.
  Ask me if you are interested in the details of operators $\Phi_j(t)$.

- $C_{ji}(t)$ is Hermitian by construction.
  The eigensystem is automatically orthogonal.

- Diagonalization of a Hermitian matrix $\Rightarrow$ Unitary transformation.
  The physical states represented by the eigensystem are linear combination of elements in the correlation matrix.

Michael 1985, Cohen-Tanoudji-Diu-Laloë QM textbook

Ω_{ccc} spectrum



MP *et al* (HSC) 2013

Consistent with $SU(3)_F \otimes SU(2)_S \otimes O(3)$ expectations

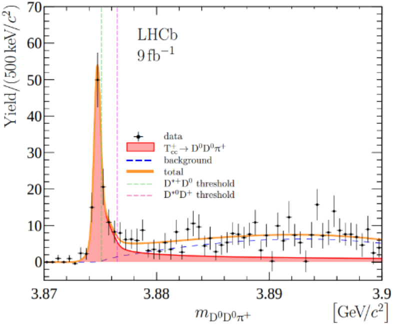
Equivalent calculations of light baryons, Singly charm baryons, doubly charm baryons and tripy bottom baryons.

See this link.

Excited state studies achievements and caveats

- Systematic extraction of various radial and orbital excitations.
- Systematic methodology for spin identification.
- Broadly consistent with nonrelativistic quark model.
- No “freezing degrees of freedom”; no parity doubling.
- Main caveat: excited energy states open to strong decay thresholds.
- **Next crucial step : Study the effects of two-hadron interpolators.**
  Rest of my talk.
- More experimental results can motivate lattice practitioners to take up these challenges.

Doubly heavy tetraquarks: T_{cc}^+



LHCb: 2109.01038, 2109.01056

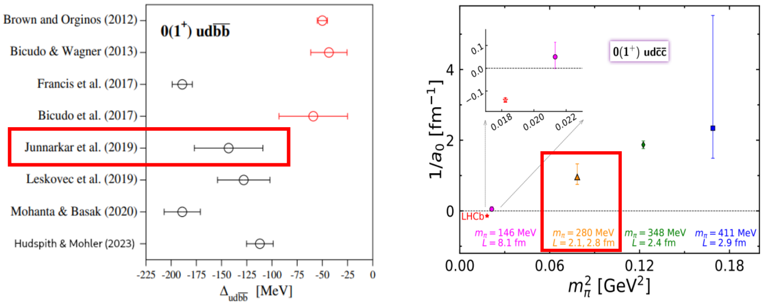
$$\delta m \equiv m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0})$$

$$\delta m_{\rm pole} = -360 \pm 40^{+4}_{-0}\ {\rm keV}/c^2\,,$$
$$\Gamma_{\rm pole} = 48 \pm 2^{+0}_{-14}\,{\rm keV}\,.$$

- The doubly charmed tetraquark T_{cc}^+, $I = 0$ and favours $J^P = 1^+$. Nature Phys., Nature Comm. 2022
  Striking similarities with the longest known heavy exotic, X(3872).
- No features observed in $D^0D^+\pi^+$: possibly not $I = 1$.
- Many more exotic tetraquark candidates discovered recently, T_{cs}, $T_{c\bar{s}}$, $X(6900)$.
  Prospects also for T_{bc} in the near future. See talk by Ivan Polyakov at Hadron 2023
- Doubly heavy tetraquarks: theory proposals date back to 1980s.

c.f. Ader&Richard PRD25(1982)2370

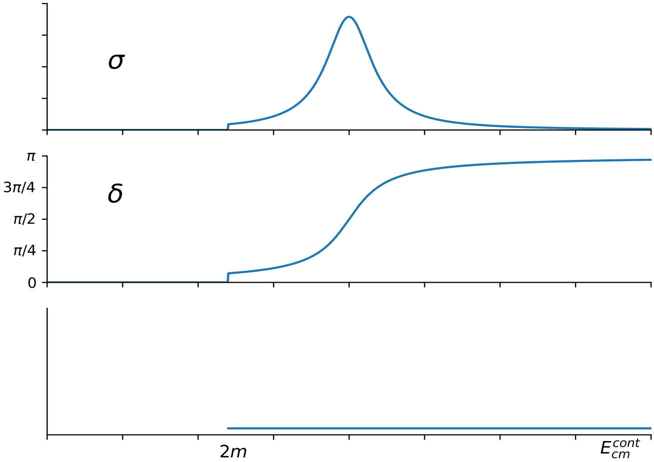
Motivation from lattice, T_{bb} and T_{cc}



- Isoscalar axialvector channel $I(J^P) = 0(1^+)$.
- Deeper binding in doubly bottom tetraquarks $\mathcal{O}(100MeV)$. Fig: Hudspith&Mohler 2023
  Red box: Our (ILGTI) work on QQ tetraquarks: Junnarkar, Mathur, MP PRD 2019
- Shallow bound state in doubly charm tetraquarks $\mathcal{O}(100keV)$. Fig: HALQCD 2023
  Red box: T_{cc} (RQCD) and its quark mass dependence, an upcoming work: stay tuned.
- No conclusive results in the bottom-charm tetraquark sector.
  A summary of different lattice investigations → see review by Pedro Bicudo, 2212.07793

The challenge on lattice: Resonances in the infinite volume continuum

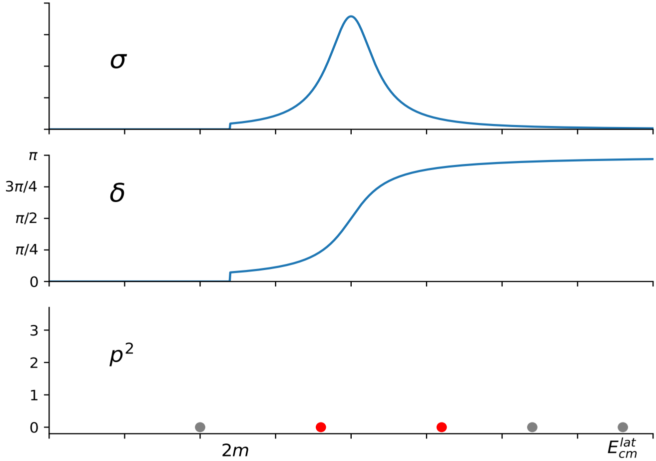
Scattering cross sections, phase shifts, branch cuts, Riemann sheets.



Schematic picture for illustration. Should not be taken quantitatively.

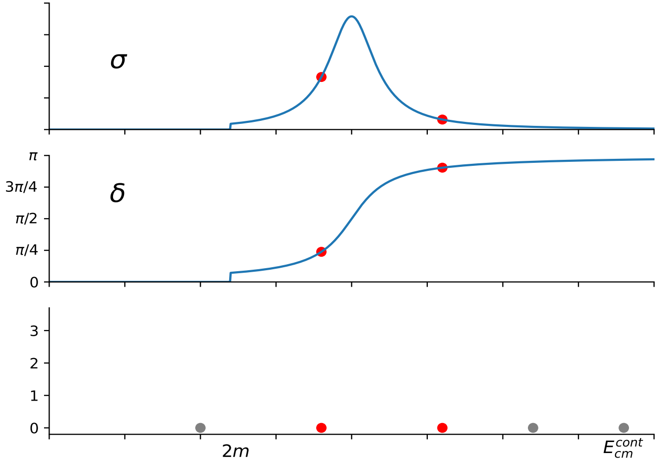
Resonances on the lattice (elastic) : ??

Discrete spectrum: No branch cuts, no Riemann sheets, no resonances!



Maiani-Testa no-go theorem [1990]

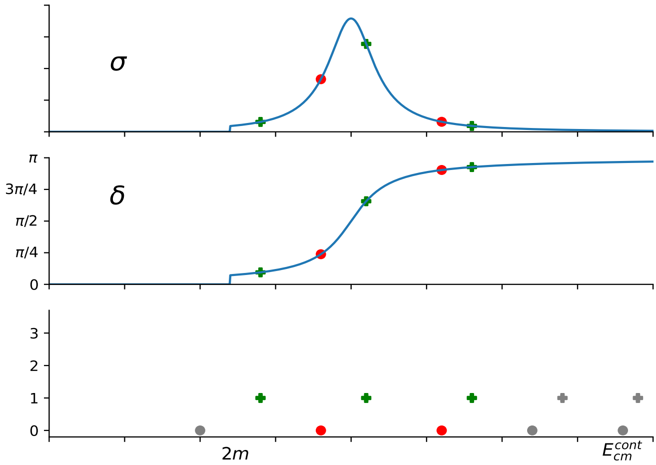
Infinite volume scattering amplitudes ⇔ Finite volume spectrum



Lüscher [1991]

Resonances on the lattice (elastic) : Lüscher (1991)

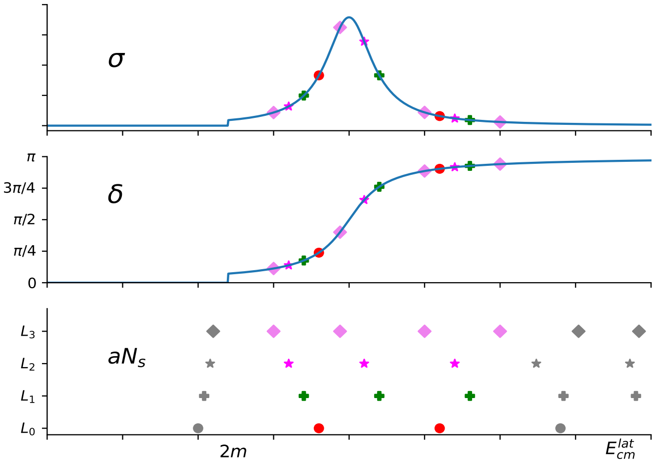
Infinite volume scattering amplitudes ⇔ Finite volume spectrum



Different inertial frames can be utilized to extract more information

Resonances on the lattice (elastic) : Lüscher (1991)

Infinite volume scattering amplitudes ⇔ Finite volume spectrum



Multiple physical volumes can also be utilized to extract more information.

For generalizations of Lüscher framework, *c.f.* Briceño, Hansen 2014-15

Finite volume spectrum and infinite volume physics

- On a finite volume Euclidean lattice : Discrete energy spectrum
  Cannot constrain infinite volume scattering amplitude away from threshold.

Maiani-Testa 1990

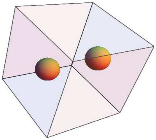
- Non-interacting two-hadron levels are given by

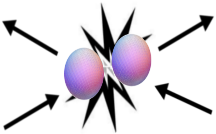
$$E(L) = \sqrt{m_1^2 + \vec{k}_1^2} + \sqrt{m_2^2 + \vec{k}_2^2} \text{ where } \vec{k}_{1,2} = \tfrac{2\pi}{L}(n_x, n_y, n_z).$$

- Switching on the interaction: $\vec{k}_{1,2} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$. *e.g.* in 1D $\quad \vec{k}_{1,2} = \frac{2\pi}{L} n + \frac{2}{L}\delta(k)$.

- Lüscher's formalsim: **finite volume level shifts ⇔ infinite volume phase shifts.**

Lüscher 1991





- Generalizations of Lüscher's formalism: *c.f.* Briceño 2014
  Quite complex problem: inelastic resonances $(R \to H_1 H_2, H_3 H_4)$

Scattering amplitude parametrization

- Scattering amplitude: $S = 1 + i\frac{4k}{E_{cm}}t$

- For an elastic scattering, and assuming only S-wave,

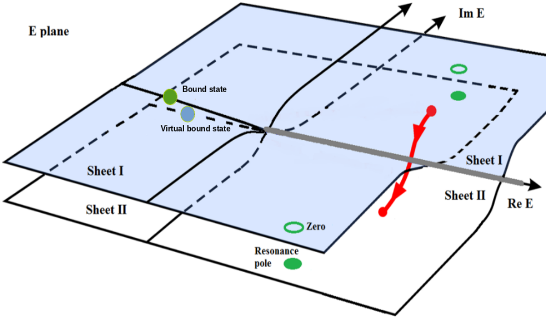
$$t^{-1} = \frac{2\tilde{K}^{-1}}{E_{cm}} - i\frac{2k}{E_{cm}}, \quad \text{with} \quad \tilde{K}^{-1} = k.cot\delta(k)$$

(virtual/bound) state constraint below threshold: $k.cot\delta(k) = (+/-)\sqrt{-k^2}$

- Lüscher's prescription: $k.cot\delta(k) = \mathcal{F}(k)$, where $\mathcal{F}(k^2)$ is a known mathematical function. k^2 is determined from each extracted finite volume energy splittings.

- Parametrize $k.cot\delta(k)$ as different functions of k.
  Effective Range Expansion (ERE): $k.cot\delta(k) = a_0^{-1} + 0.5r_0k^2 + \beta_i k^{2i+4}$.
  The best fits and fit estimates determined to represent the energy dependence of the amplitude.

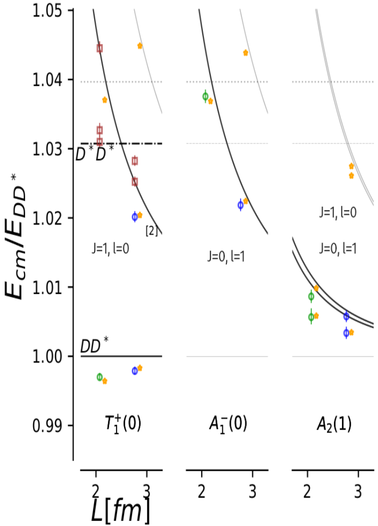
Virtual/bound states

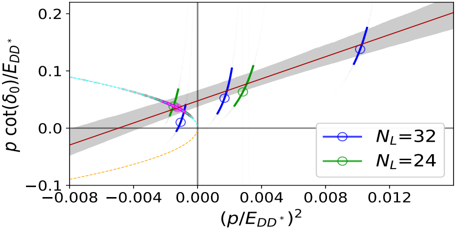


- $T \propto (pcot\delta_0 - ip)^{-1}$. Bound state is a pole in T with $p = i|p|$. Virtual bound state is a pole in T with $p = -i|p|$.

- An example for virtual bound state: spin-singlet dineutron.

DD^* scattering in $l = 0, 1$ @ $m_c^{(h)}$ with an ERE





MP, Prelovsek PRL 2022

$m_\pi \sim 280$ MeV

- Fit quality:
  $\chi^2/d.o.f. = 3.7/5.$
- Fit parameters:
  $a_0^{(1)} = 1.04(0.29)$ fm & $r_0^{(1)} = 0.96(^{+0.18}_{-0.20})$ fm
  $a_1^{(0)} = 0.076(^{+0.008}_{-0.009})$ fm^3 & $r_1^{(0)} = 6.9(2.1)$ fm^{-1}
- Binding energy:
  $\delta m_{T_{cc}} = -9.9(^{+3.6}_{-7.2})$ MeV.
- First evaluation of the DD^* amplitude in T_{cc} channel.

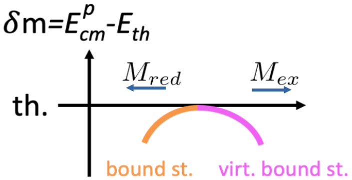
+/g refers to positive parity, -/u refers to negative parity.

Our observations and inferences with ERE approach

- A shallow virtual bound state pole in s-wave related to T_{cc}.

	m_D [MeV]	$\delta m_{T_{cc}}$ [MeV]	T_{cc}
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(h)}$)	1927(1)	$-9.9^{+3.6}_{-7.2}$	virtual bound st.
lat. ($m_\pi \simeq 280$ MeV, $m_c^{(l)}$)	1762(1)	$-15.0(^{+4.6}_{-9.3})$	virtual bound st.
exp.	1864.85(5)	$-0.36(4)$	bound st.

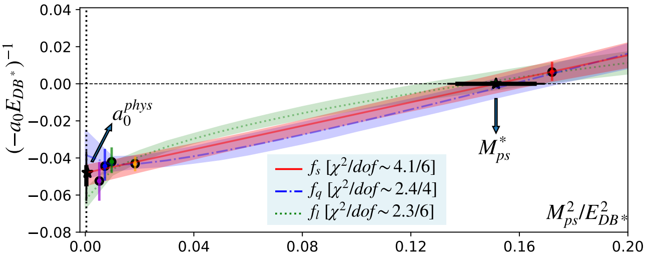
- For $m_\pi > m_\pi^{phys}$, T_{cc} is expected to become a virtual bound state. At $m_\pi \sim$ 280 MeV, we indeed find a shallow virtual bound state.
- Observations in line with the expected behaviour of a near-threshold molecular bound state pole in simple Quantum Mechanical potentials.



MP, Prelovsek PRL 2022. See a video demonstration at the end.

- $M_{red}(\propto m_c)$ is the reduced mass of the DD^* system.
- The mass of the particle exchanged during the interaction $M_{ex}(\propto m_{u/d})$.

T_{bc} $(I)J^P = (0)1^+$ bound state



MP *et al* 2307.14128, Archana Radhakrishnan's talk on Friday.

- Light quark mass ($m_{u/d}$ or M_{ps}) dependence
  indicates a real bound state at physical pion mass.

- DB^* scattering length[1] and binding energy (w.r.t. E_{DB^*}) in the continuum limit

$$a_0^{phys} = 0.57(^{+4}_{-5})(17)\text{ fm} \quad \text{and} \quad \delta m_{T_{bc}} = -43(^{+6}_{-7})(^{+14}_{-24})\text{ MeV}$$

- A more recent lattice investigation also suggesting attractive interactions.

Alexandrou *et al* 2312.02925

[1] Note the sign convention used: $[k cot\delta_0 \sim -1/a_0]$

Pion exchange cuts/left-hand cuts and shortcomings with an ERE and QC

- A two fold problem: (Unphysical pion masses used in lattice)
  ERE convergences fails at the left-hand cut.
  $2 \to 2$ Generalized LQC does not incorporate such lhc effects.

Figure taken from arXiv:2401.06609

- Unphysical pion masses ($m_\pi > \Delta M = M_{D^*} - M_D$, stable D^* meson):

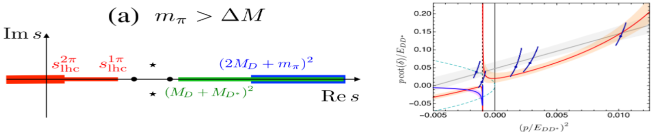
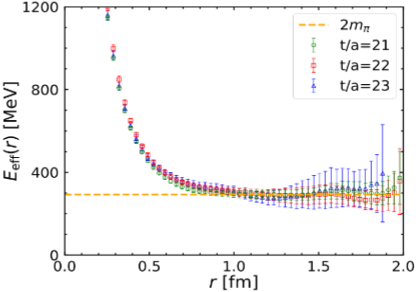


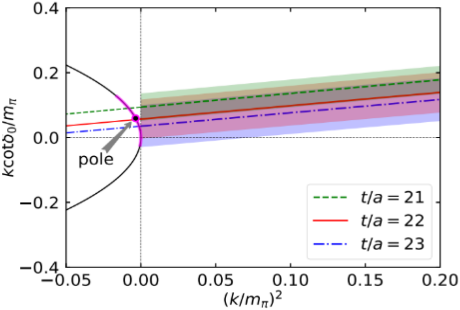
Figure taken from Meng-Lin Du *et al* arXiv:2303.09441[PRL]

Fits with a potential that incorporates the one pion exchange:
Virtual bound states ⇒ Virtual resonances

Alternatively: HALQCD approach @ near physical m_π

- DD^* s-wave scattering amplitudes from the lattice extracted DD^* potential.





$$E_{\rm eff}(r) = -\frac{\ln[V(r)r^2/a_3]}{r}$$

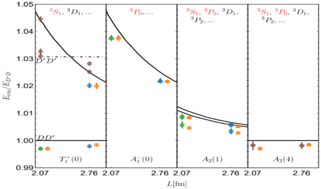
$$V^B_{\rm fit}(r; m_\pi) = \sum_{i=1,2} a_i \; e^{-(r/b_i)^2} + a_3 \left(1 - e^{-(r/b_3)^2}\right)^n V^n_\pi(r)$$

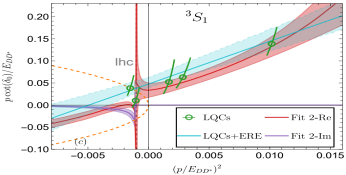
Lyu *et al* arXiv:2302.04505

Long distance potential dominated by two pion exchange, not OPE.
Phase shifts extracted from long distance behaviour.
Shallow virtual bound state turning to a real bound state at physical m_π

Solutions: A plane-wave approach and modified LQC

- An effective field theory incorporating OPE with a plane wave basis expansion.





Lu Meng *et al* arXiv:2312.01930

Virtual bound states ⇒ Virtual resonances [$m_\pi \sim$280 MeV]

- Modified 3-particle (Lüscher) Quantization Condition:

Hansen, Romero-Lopez, Sharpe, 2401.06609, Raposo, Hansen, 2311.18793

See a recent talk by Romero-Lopez here

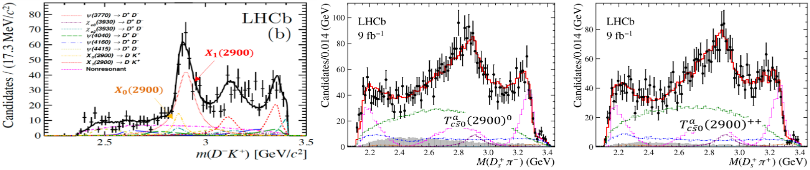
A rigorous procedure, but demands multiple lattice inputs.

- $D\pi$ finite volume spectrum up to the $D\pi\pi$ threshold.
- Isovector DD finite volume spectrum up to the $DD\pi$ threshold.
- Isoscalar $DD\pi$ finite volume spectrum up to the $DD\pi\pi$ threshold.

Excited charmed-light and charmed-strange mesons

- Scalar D_0^* a broad feature in the $D\pi$ amplitudes, whereas a narrow D_{s0}^* below the DK threshold.

- Recent [LHCb] discoveries of T_{cs} [$X_1(2900)$, $X_0(2900)$], $T_{c\bar{s}0}(2900)^{0/++}$.



See a recent talk by Liming Zhang here

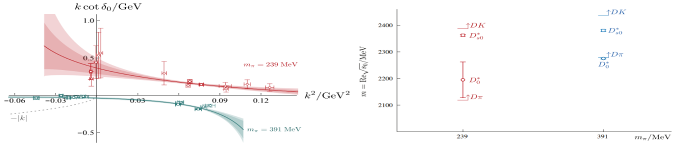
- A new framework of four quark systems with a charm quark and remaining light/strange quarks [$cs\bar{u}\bar{d}$, $cu\bar{s}\bar{d}$, $cd\bar{s}\bar{u}$]. LHCb discoveries

- A handful of lattice calculations (not explicitly exotic channels):

Mohler *et al* 1308.3175 (PRL), Lang *et al* 1403.8103, Bali *et al* 1706.01247, Gayer *et al* 2102.04973, Mohler *et al* 1208.4059, Moir *et al* 1607.07093, Gregory *et al* 2106.15391, Yan *et al* 2312.01078

Recent lattice investigations

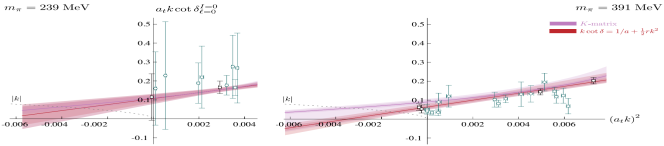
- Scalar charmed mesons and the $D\pi$ amplitudes,



Gayer *et al* 2102.04973

D_0^* pole real part consistently below that for D_{s0}^* for either m_π.

- Isoscalar $D\bar{K}$ scattering in s-wave (explicitly flavor exotic channel "$cs\bar{q}_1\bar{q}_2$"):



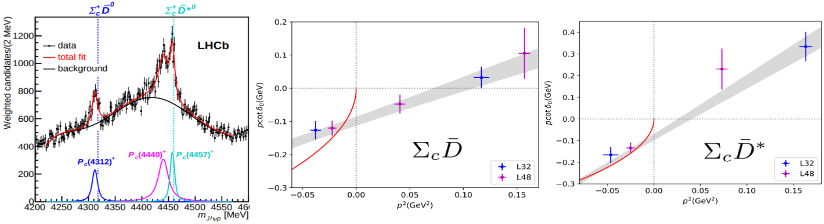
Cheung *et al* 2008.06432

Weak attraction indicating presence of a virtual state.

Pentaquarks, P_c in $J/\psi p$ final states

- Narrow pentaquark structures $P_c(4312)^+$, $P_c(4440)^+$, and $P_c(4457)^+$ in $J/\psi p$ final states. Features close below the $\Sigma_c \bar{D}$ and $\Sigma_c \bar{D}^*$

LHCb 1904.03947 (PRL)



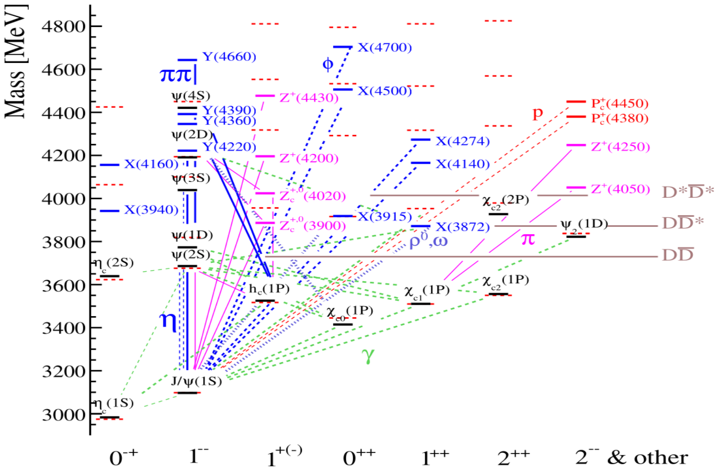
- Indications for shallow bound states in $\Sigma_c \bar{D}$ and $\Sigma_c \bar{D}^*$ from lattice. Coupling to $J/\psi p$ omitted in the analysis. $m_\pi \sim 294$ MeV.

Xing *et al* 2210.08555

- Evidence for $P_{cs}(4459)^0$ $(\bar{c}csud)$. No lattice investigation yet.

LHCb Science Bulletin 2021

Charmonium



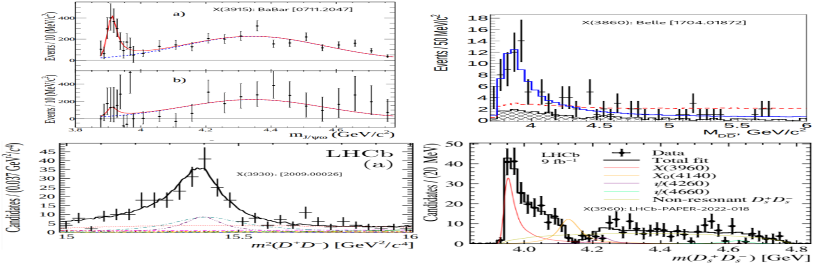
Rich energy spectrum. XYZ states.

$\bar{c}c$ picture works well for states below open charm threshold.

Olsen *et al* 1708.04012

No single description for states above the open charm threshold.

Focus: Scalar charmonium-like states



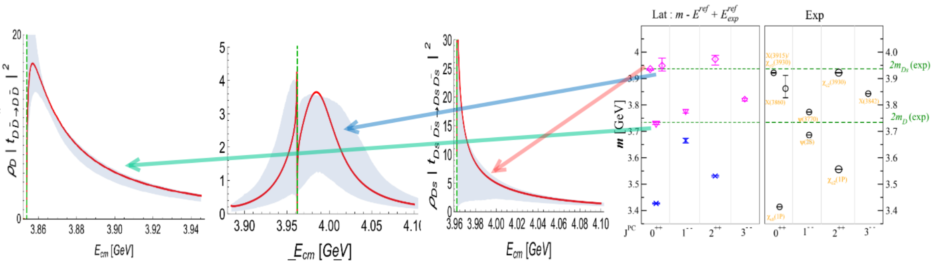
- Several likely related features, $X(3915)$, $X(3930)$, $X(3960)$.
  Proximity to the $\bar{D}_sD_s$ threshold: Possible hidden strange content [$\mathbf{cs\bar{c}\bar{s}}$]
  $\Rightarrow$ narrow width from $\bar{D}D$

- Several phenomenological studies supporting this:
  Lebed Polosa 1602.08421, Chen *et al* 1706.09731, Bayar *et al* 2207.08490

- Another feature named as X(3860) observed by Belle. No evidence from LHCb.

- Yet unknown $\bar{D}D$ bound state, predicted by models.
  Gamermann *et al* 0612179, Hidalgo-Duque *et al* 1305.4487, Baru *et al* 1605.09649

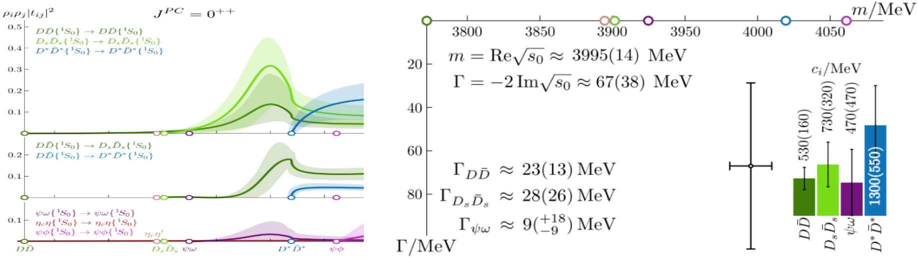
- Such a $\bar{D}D$ bound state is supported by re-analysis of the exp. data.
  Danilkin *et al* 2111.15033, Ji *et al* 2212.00631.



- First extraction of coupled $\bar{D}D$-$\bar{D}_sD_s$ scattering amplitude. $[\mathbf{\bar{c}c}, \mathbf{\bar{c}c\bar{q}q}; \mathbf{q} \to \mathbf{u}, \mathbf{d}, \mathbf{s}$, and $\mathbf{I} = \mathbf{0}]$.
- Lattice QCD ensembles : CLS Consortium
  $m_\pi \sim$ 280 MeV, $m_K \sim$ 467 MeV, $m_D \sim$ 1927 MeV, $a \sim 0.086$ fm
- In addition to conventional charmonium states, we observe candidates for three excited scalar charmonium states
  - ⇒ a yet unobserved shallow $\bar{D}D$ bound state.
  - ⇒ a $\bar{D}D$ resonance possibly related to X(3860).
  - ⇒ a narrow resonance just below and with large coupling to $\bar{D}_sD_s$ threshold.
    possibly related to X(3960) / X(3930) / X(3915).
- Our (RQCD) recent publications on charmonium:

Collins, Mohler, MP, Piemonte, Prelovsek 2111.02934, 2011.02541, 1905.03506.

Recent lattice investigation by HSC



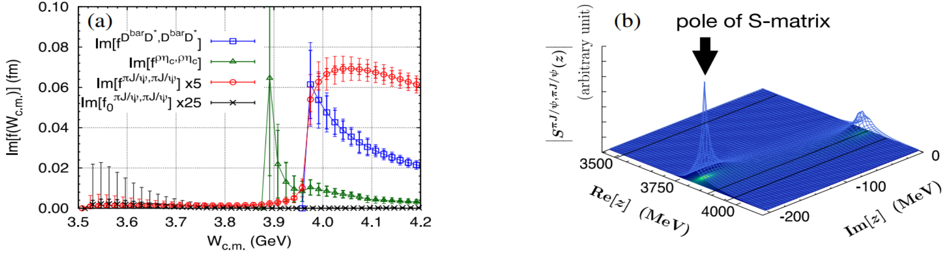
HSC 2309.14070, 2309.14071.

- Two-hadron channels considered: $\eta_c\eta$, $\eta_c\eta'$, $\bar{D}D$, $\bar{D}_sD_s$, $\psi\omega$, $\psi\phi$, $\bar{D}^*D^*$, $\chi_{c1}\eta$.

- Anisotropic lattice QCD ensembles : Hadron Spectrum Collaboration
  $m_\pi \sim$ 391 MeV, $m_K \sim$ 540 MeV, $m_D \sim$ 1852 MeV, $a_s \sim 0.12$ fm

- In addition to conventional charmonium states, only a single scalar resonance below 4 GeV
  $\Rightarrow$ with large coupling to all open charm channels.
  relation to X(3960) / X(3930) / X(3915) / $\chi_{c0}(3860)$ features ?

- Results in conflict with several other theoretical and experimental studies.
  Resolution: quark mass dependence ?



HALQCD 1602.03465 (PRL).

- Lattice calculations from two different fronts:
  Calculations based on Lüscher's formalism and using HALQCD approach

- HALQCD work: Coupled $J/\psi\pi$-$\rho\eta_c$-$\bar{D}D^*$ scattering.
  $m_\pi \sim$ 400-700 MeV, $a \sim 0.09$ fm
  Strong coupling between $\bar{D}D^*$ and other two channels.
  $Z_c(3900)$ not a usual resonance, but a threshold cusp

- Lüscher's formalism: no robust supporting/exluding remarks for such a near threshold state.
  Prelovsek *et al* 1405.7623, Chen *et al* 1403.1318, 1503.02371, CLQCD 1907.03371

Summary

- Reported on lattice spectroscopic calculations of various XYZTP.

- Elastic resonances and near-threshold states
  Several matured lattice determinations.
  New challenges related left hand cuts.

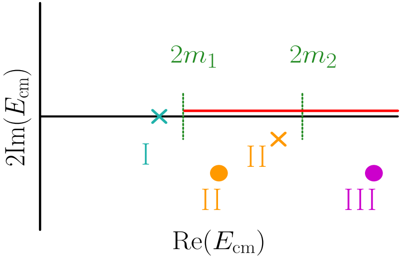
- Inelastic resonances, exotic hadrons, etc.
  Multiple channels: Still a complex problem.

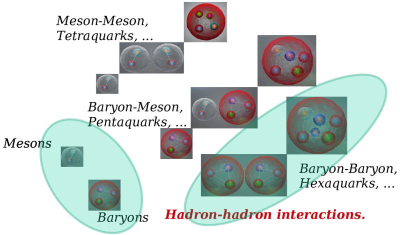
- Not covered: Baryon-baryon (heavy) scattering

  c.f. Junnarkar, Mathur PRL 2019, PRD 2022

  Mathur, MP, Chakraborty PRL 2023

  Lyu *et. al.* PRL 2021





Thank you

Quark mass dependence: a QuanMech understanding

$$R \propto M_{red} \propto 1/M_{ex}$$