Foundations of quantum mechanics & neutrino oscillations

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Understanding the Universe through Neutrinos, ICTS, Bengaluru, 01 May 2024

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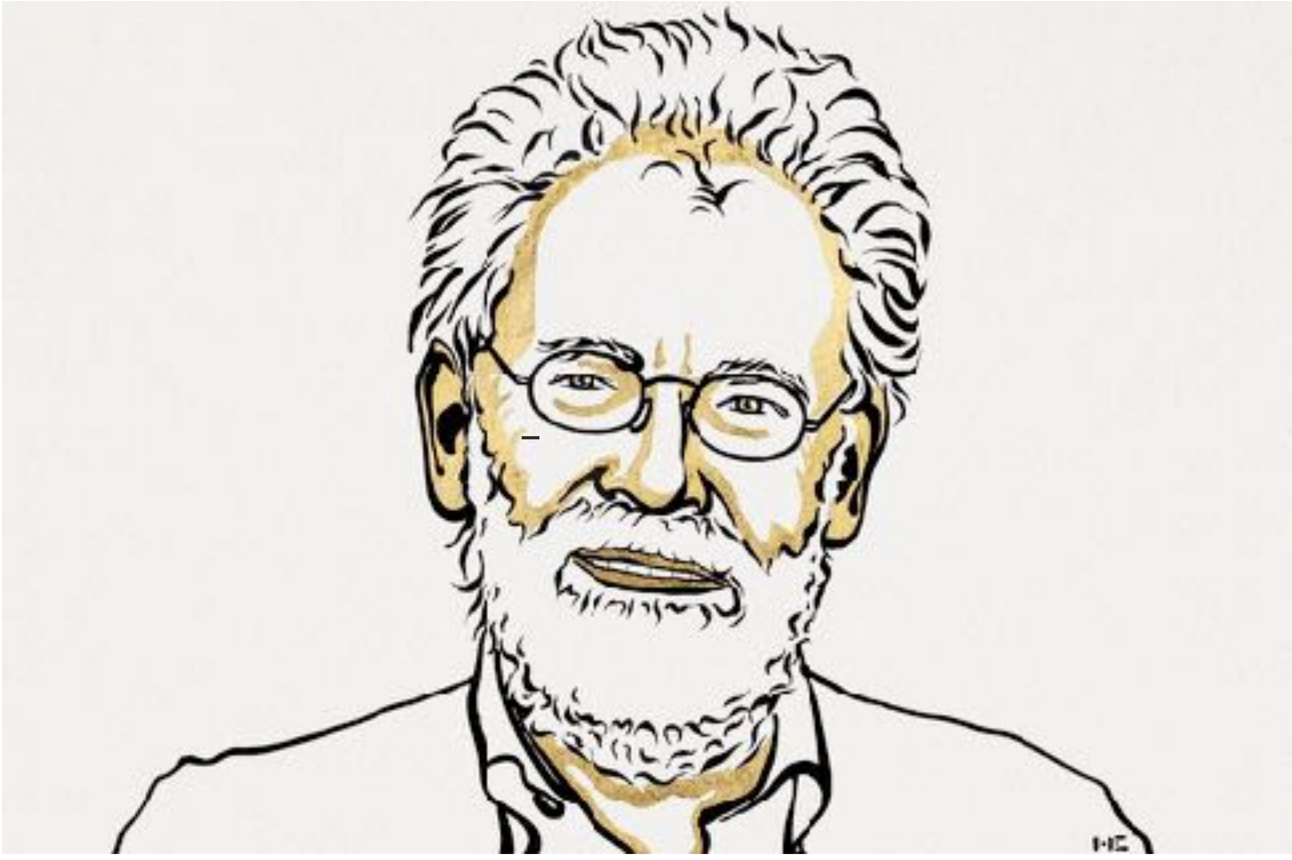
2022 Nobel

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

ALAIN ASPECT

JOHN F CLAUSER



ANTON ZEILINGER

What can precision neutrino experiments reveal about foundational aspects of QM ?

Plan

- Foundational aspects of quantum mechanics
  - spatial and temporal correlations
- High energy physics context - neutrino oscillations
  - mapping of two state neutrinos to a two-level quantum system
- Temporal correlations (Leggett-Garg inequalities) in neutrino oscillations
  - enhancement
  - damping
- Temporal correlations (Leggett-Garg inequalities) in neutrino oscillations plus decay
  - Dirac versus Majorana
- Experiments confronting the LGI tests
  - MINOS and Daya Bay
- Some ideas

Primer on foundational aspects of QM

From EPR to Bell and CHSH and to LG

1920-1930

Viewpoint - classical or quantum

- The point of view offered by classical physics tells us that the physical properties of a given object exist independent of observation. The measurement process simply discloses the physical properties of that object.
- However, quantum mechanics states that no physical property exists independent of observation. Rather, such physical properties arise as a consequence of measurements performed upon the system.
- For example, according to quantum mechanics a qubit does not possess definite properties of ‘spin in the z direction, σz’, and ‘spin in the x direction, σx’, each of which can be revealed by performing the appropriate measurement. Rather, quantum mechanics gives a set of rules which specify, given the state vector, the probabilities for the possible measurement outcomes when the observable σz is measured, or when the observable σx is measured.

1935

The work of EPR

A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? Phys. Rev., 47:777–780, 1935.

- In 1935, Albert Einstein, Boris Podolsky, and Nathan Rosen challenged the quantum viewpoint and posed the question "Can the quantum-mechanical description of reality be considered complete ?".
- Einstein had said "The real factual situation of the system S2 is independent of what is done with the system S1, which is spatially separated from the former." This locality principle is motivated by special relativity, which prohibits instantaneous action at a distance. Such a principle was implicitly invoked in the EPR argument when it was asserted that a measurement on particle #1 cannot affect the condition of the spatially separated particle #2, since there is no interaction between the particles.
- However, it was found that one could predict (prior to measurement) with certainity the outcome of measurement on the second particle by making a measurement on the first. They used the term "element of reality" to describe a physical property such that it is possible to predict with certainity its value, just before the measurement.
- This contradicted the view of quantum mechanics according to which a particle would not have a definite value of a property prior to measurement.

1951

Realistic experiment - Bohm

D. Bohm. Quantum Theory. Prentice- Hall, Englewood Cliffs, New Jersey, 1951 ; see also D. Bohm and Y. Aharonov, Phys. Rev. 108, 1070 (1957)

- System of two atoms, each having spin half prepared in a state of total spin zero. e.g. unstable excited states of certain diatomic molecules
- Singlet spin state vector :

spin down

$$|\Psi_0\rangle = (\langle|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle) \sqrt{\frac{1}{2}}$$

spin up

- The particles are allowed to separate, and when they are well beyond the range of interaction we can measure the z component of spin of particle #1. Because the total spin is zero, we can predict with certainty, and without in any way disturbing the second particle, that the z component of spin of particle #2 must have the opposite value.
- The value of σz (2) is an element of reality, according to the EPR criterion and so are any number of spin components…
- Quantum state description is not a complete description of physical reality.

Spin correlations in arbitrary directions

- Singlet spin state vector :

spin up in z direction

$$|\Psi_0\rangle = (\langle|+\rangle \otimes |-\rangle - |-\rangle \otimes |+\rangle) \sqrt{\frac{1}{2}}$$

spin down in z direction

$$\langle\Psi_0|\sigma_a \otimes \sigma_b|\Psi_0\rangle = -\cos(\theta_{ab})$$

where σ_a is the component of Pauli spin operator in direction of unit vector a.

- The correlation depends upon the angle between directions a and b.
- If a is chosen to be along the z direction,

$$\langle\Psi_0|\sigma_a \otimes \sigma_b|\Psi_0\rangle = \frac{1}{2}(\langle-|\sigma_b|-\rangle - \langle+|\sigma_b|+\rangle) = -\cos(\theta_{ab})$$

1964

Bell’s Inequalities

J. S. Bell. On the Einstein-Podolsy- Rosen paradox. Physics, 1:195–200, 1964. Reprinted in J. S. Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Cambridge, 1987 ; see also M. A. Nielson and I. A. Chuang (2010)

- John Bell revisited the EPR experiment in 1964 and came up with a set of inequalities which allow us to test the ideas of EPR. The idea was based on two assumptions :
  - Realism : Physical properties have definite value independent of observation.
  - Locality : Any measurement performed on A does not affect the result of measurement of B.

Together, these are referred to as local realism.

- To illustrate the idea proposed by Bell, let us consider the following set-up involving three observers : Aspect, Brout and Clauser.
- Clauser prepares two particles and sends one to Aspect and other one to Brout.
- Both perform two distinct measurements of the respective particles they recieve.

1964

Bell’s Inequalities

J. S. Bell. On the Einstein-Podolsy- Rosen paradox. Physics, 1:195–200, 1964. Reprinted in J. S. Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Cambridge, 1987 ; see also . F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969) and M. A. Nielson and I. A. Chuang (2010)

- Physical properties measured by Aspect are denoted by PQ and PR and by Brout by PS and PT. Values are denoted by Q,R,S,T which (for simplicity) can have outcome +1 or 1.
- Algebraically, we obtain

$$QS + RS + RT - QT = \pm 2$$

- If p(q, r, s, t) is the probability that, before the measurements are performed, the system is in a state given by Q = q, R = r, S = s, and T = t and E(.) denotes the mean value of a quantity, then it can be shown that

$$E(QS) + E(RS) + E(RT) - E(QT) \leq 2$$

where, Aspect and Brout can determine the quantities such as E(QS) etc.by repeating the experiment multiple times.

- This is the generalised form of Bell’s inequality, also referred to as the Clauser-Horne-Shimony-Holt (CHSH) inequality.

1964

Generalized Bell's Inequalities

J. S. Bell. On the Einstein-Podolsy- Rosen paradox. Physics, 1:195–200, 1964. Reprinted in J. S. Bell, Speakable and Unspeakable in Quantum Mechanics, Cambridge University Press, Cambridge, 1987 ; F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969); B. S. Cirelson, Lett. Math. Phys. 4, 93 (1980). and M. A. Nielson and I. A. Chuang (2010)

- Singlet state example

$$Q = Z_1 \quad ; \quad S = \frac{-Z_2 - X_2}{\sqrt{2}}$$

$$R = X_1 \quad ; \quad T = \frac{Z_2 - X_2}{\sqrt{2}}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle QS \rangle = \frac{1}{\sqrt{2}} \; ; \quad \langle RS \rangle = \frac{1}{\sqrt{2}} \; ; \quad \langle RT \rangle = \frac{1}{\sqrt{2}} \; ; \quad \langle QT \rangle = -\frac{1}{\sqrt{2}}$$

$$\langle QS \rangle + \langle RS \rangle + \langle RT \rangle - \langle QT \rangle = 2\sqrt{2}$$

Maximum violation of CHSH inequalities, Tsirelson bound

QM is inconsistent with Bell's inequalities. Implies that we need to abandon either locality or realism.

1985

Leggett-Garg Inequalities

A. J. Leggett and A. Garg, Phys. Rev. Lett. 54, 857 (1985); see also C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. 77, 016001 (2013).

- In 1985, Leggett and Garg derived a class of inequalities which have the following assumptions:
  - Macroscopic realism (MR): A macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states.
  - Non-Invasive measurability (NIM): It is possible, in principle, to determine which of the states the system is in, without affecting the states itself or the system's subsequent dynamics.
- Whilst classical mechanics conforms with both of these assumptions, quantum mechanics certainly does not — the existence of a macroscopic superposition would violate the first, and its quantum- mechanical collapse under measurement, the second.
- Leggett-Garg Inequalities (LGI) bear strong formal analogies to Bell-inequalities. In a Bell-inequality one considers measurements occurring on two (or more) systems at spacelike separation, in a LGI, one considers repeated measurements, at different times, of a single observable, on a single system: a timelike, rather than a spacelike separation between measurements.

Formalism of LGI

C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. 77, 016001 (2013).

- Consider an experiment with two outcomes.
  We define a dichotomic observable:
  $$Q = \pm 1$$

- Two time correlation functions $C_{ij} = \langle Q(t_i) Q(t_j) \rangle$
  $\boxed{-1 \leq C_{ij} \leq 1}$
  $C_{ij} = 1 \rightarrow$ Perfectly correlated
  $C_{ij} = -1 \rightarrow$ Perfectly anti-correlated
  $C_{ij} = 0 \rightarrow$ No correlation
- Macrorealism restricts the following combination of two time correlation functions:

$$\begin{aligned} K_3 = C_{12} + C_{23} - C_{31} &= \langle Q_1 Q_2 \rangle + \langle Q_2 Q_3 \rangle - \langle Q_1 Q_3 \rangle \\ K_3 &= \langle Q_1 Q_2 \rangle + \langle [Q_2 - Q_1] \, Q_3 \rangle \end{aligned}$$

$$K_3 = \begin{cases} 1 + 0 = 1 \\ -1 + (\pm 2) = 1 \quad \text{or} - 3 \end{cases}$$

This gives the condition

$$-3 \leq K_3 \leq 1$$

which is the simplest LGI. Similarly

$$-2 \leq K_4 \leq 2$$

- Violation of this inequality implies that any one of the assumptions (Macroscopic realism or non-invasive measurement) is not valid. Hence, the LGI parameter values lying outside the these limits are indicative of the quantumness.
- In general we have

$$\begin{aligned} -n \leq K_n \leq (n-2) \qquad & 3 \leq n, \text{odd}; \\ -(n-2) \leq K_n \leq (n-2) \qquad & 4 \leq n, \text{even} \end{aligned}$$

Two state quantum system

Correlators :

$$C_{ij} = \frac{1}{2}\langle\{\hat{Q}_i \hat{Q}_j\}\rangle$$

If $\hat{Q}_i = \vec{a}_i \cdot \vec{\sigma}$ where $\vec{\sigma}$ denotes Pauli matrices, $\vec{a}_i$ is the unit vector, we obtain

$$\frac{1}{2}\langle\{\hat{Q}_i \hat{Q}_j\}\rangle \quad = \quad \vec{a}_i \cdot \vec{a}_j \langle\hat{\mathbb{I}}\rangle = \vec{a}_i \cdot \vec{a}_j$$

Using this we can express K_n as

$$K_n \quad = \quad \sum_{m=1}^{n-1} \cos\theta_m \tau - \left(\cos\sum_{m=1}^{n-1}\theta_m\right)\tau$$

where θ_m is the angle between $\vec{a}_m$ and $\vec{a}_{m+1}$.

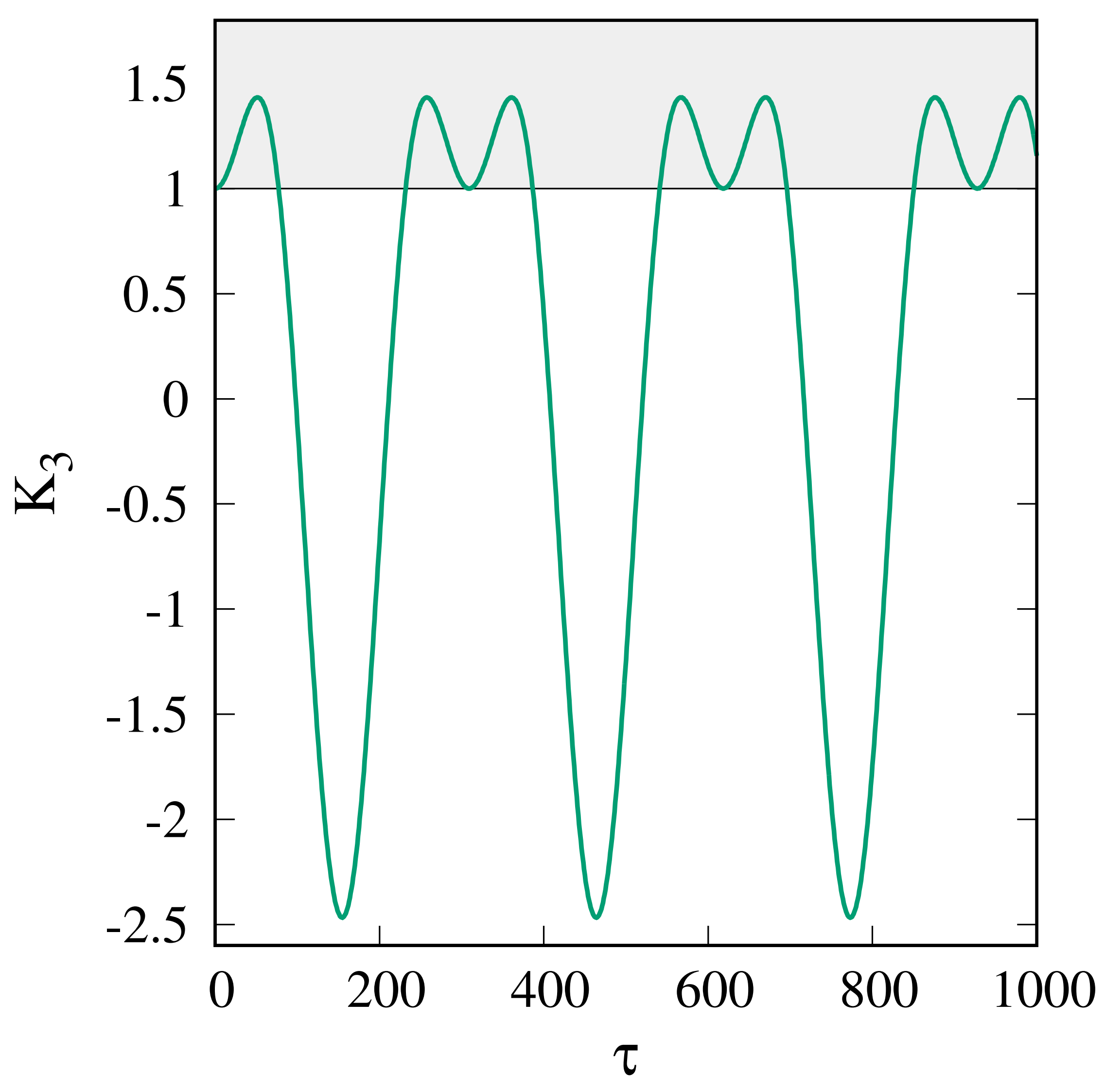
Maximum Violation :

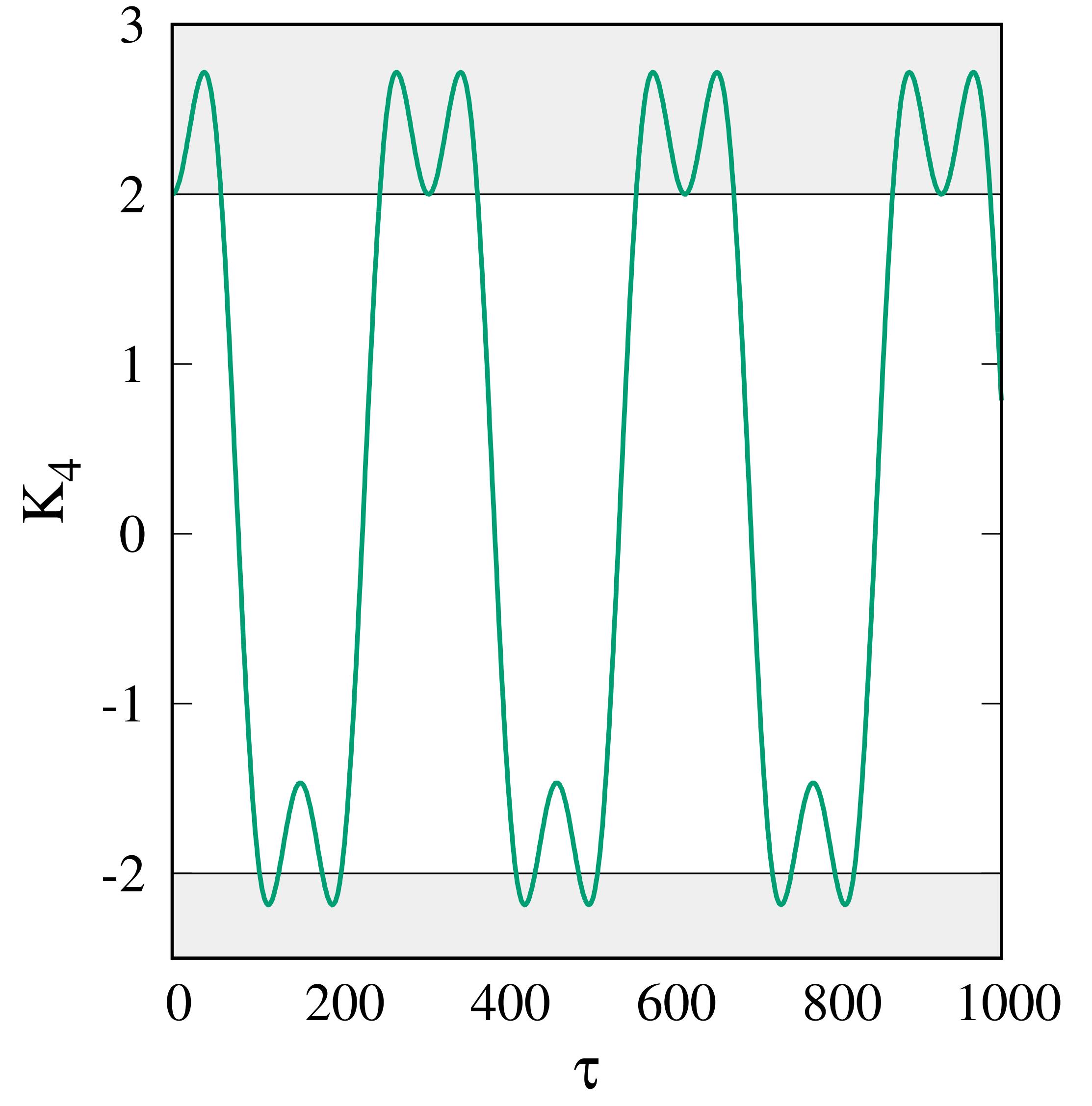
$$K_n^{max} = n\cos\frac{n}{2}$$

$$K_3^{max} = \frac{3}{2}; \quad K_4^{max} = 2\sqrt{2}; \quad K_5^{max} = \frac{5}{4}(1+\sqrt{5})$$

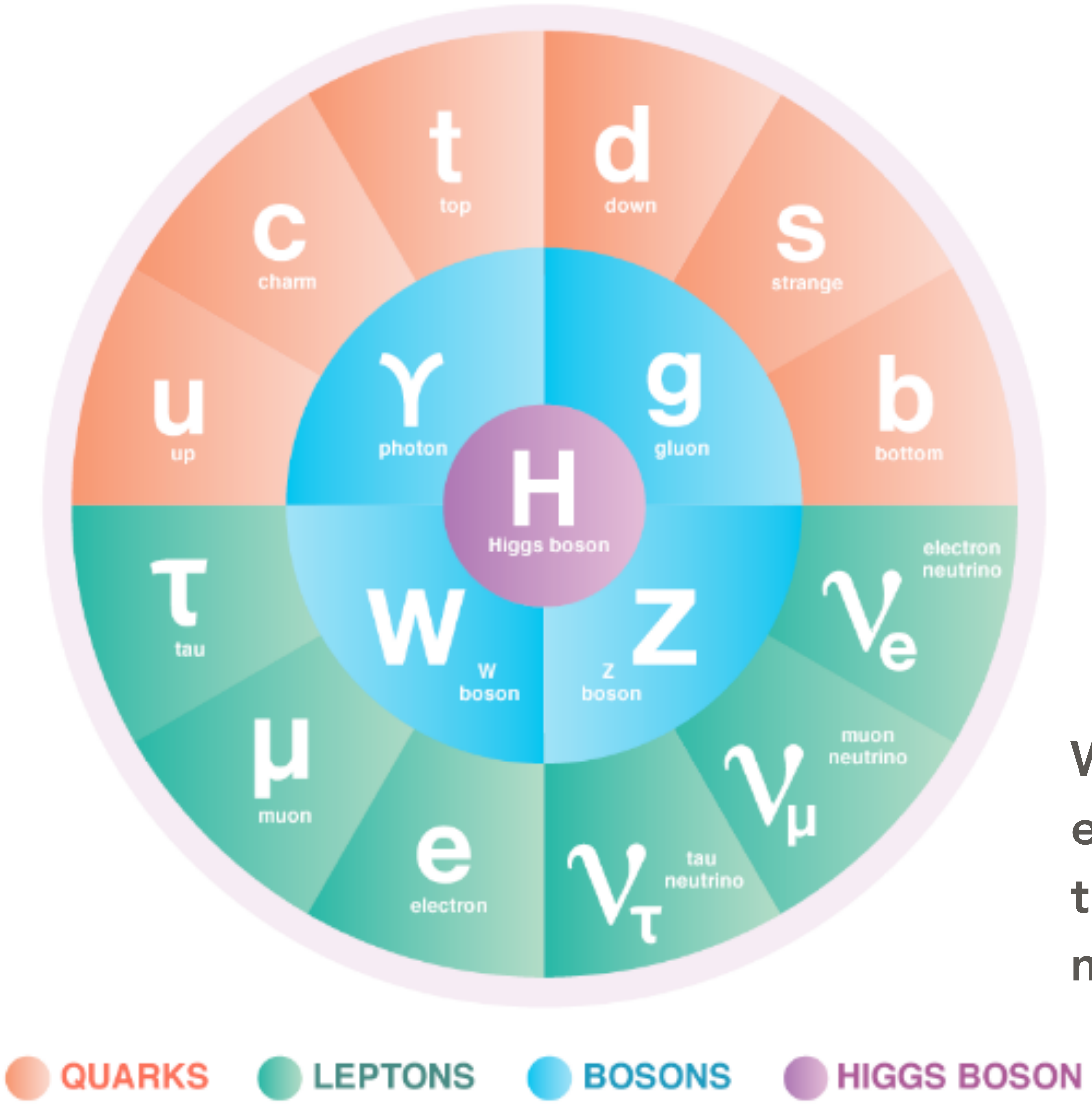
$$K_3 \quad = \quad 2\cos\Omega\tau - \cos 2\Omega\tau$$

$$K_4 \quad = \quad 3\cos\Omega\tau - \cos 3\Omega\tau$$



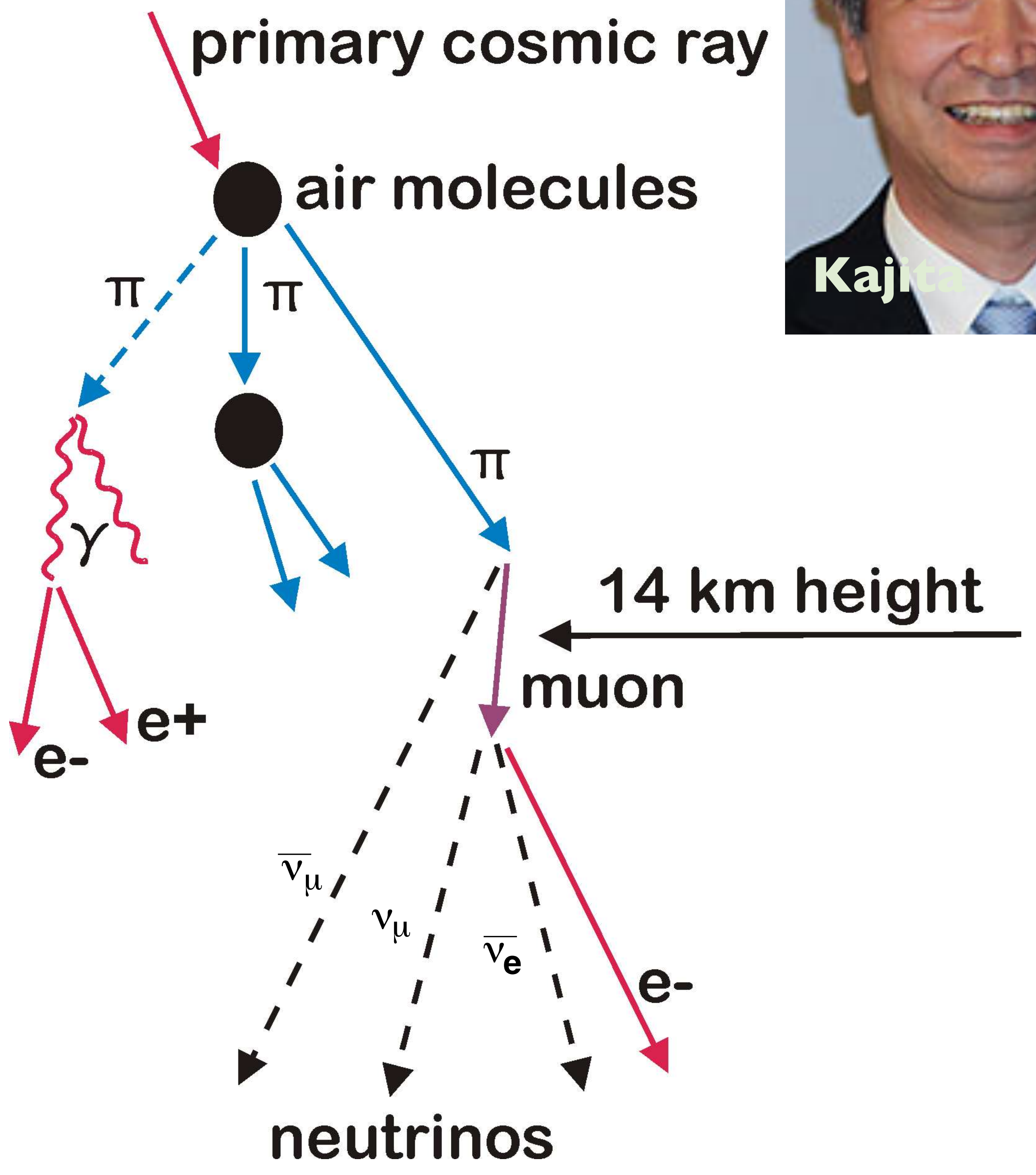


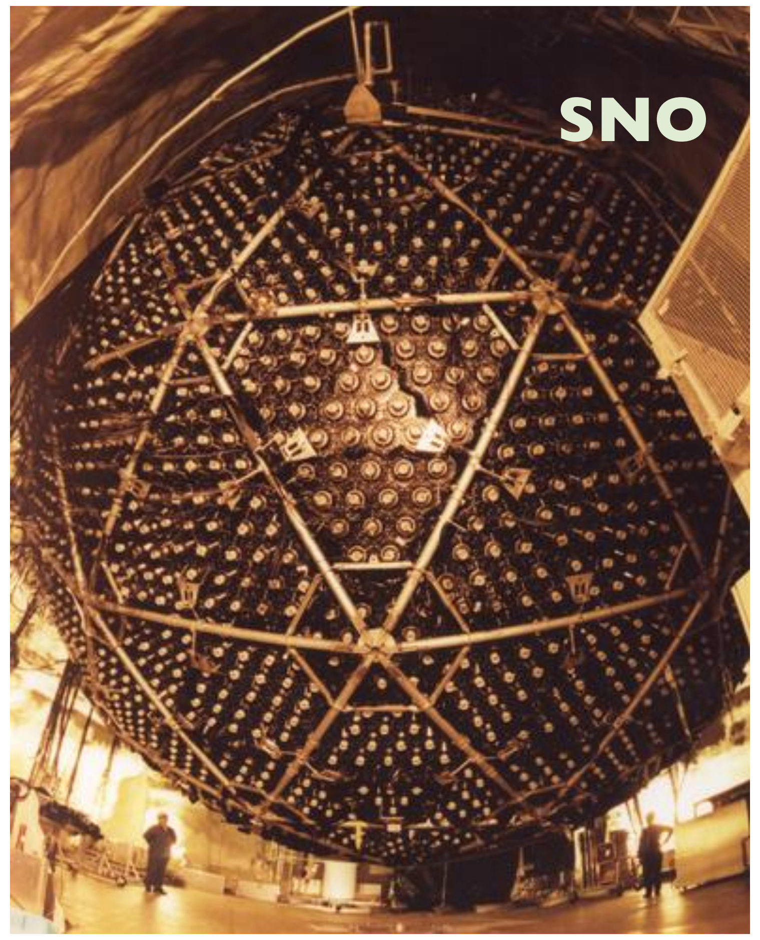
Context - Particle physics, neutrinos



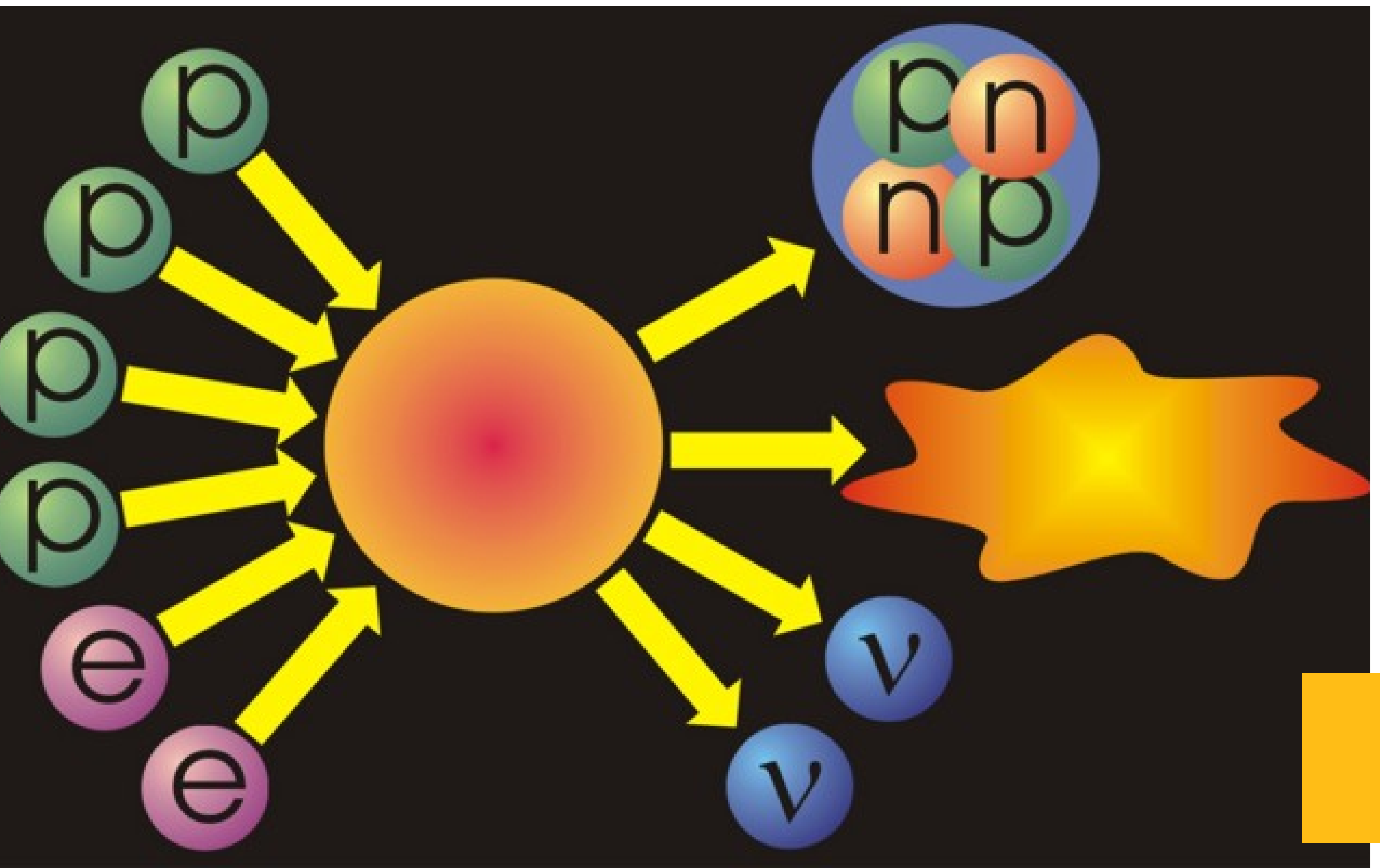
We will consider neutrinos to explore questions pertaining to foundations of quantum mechanics.

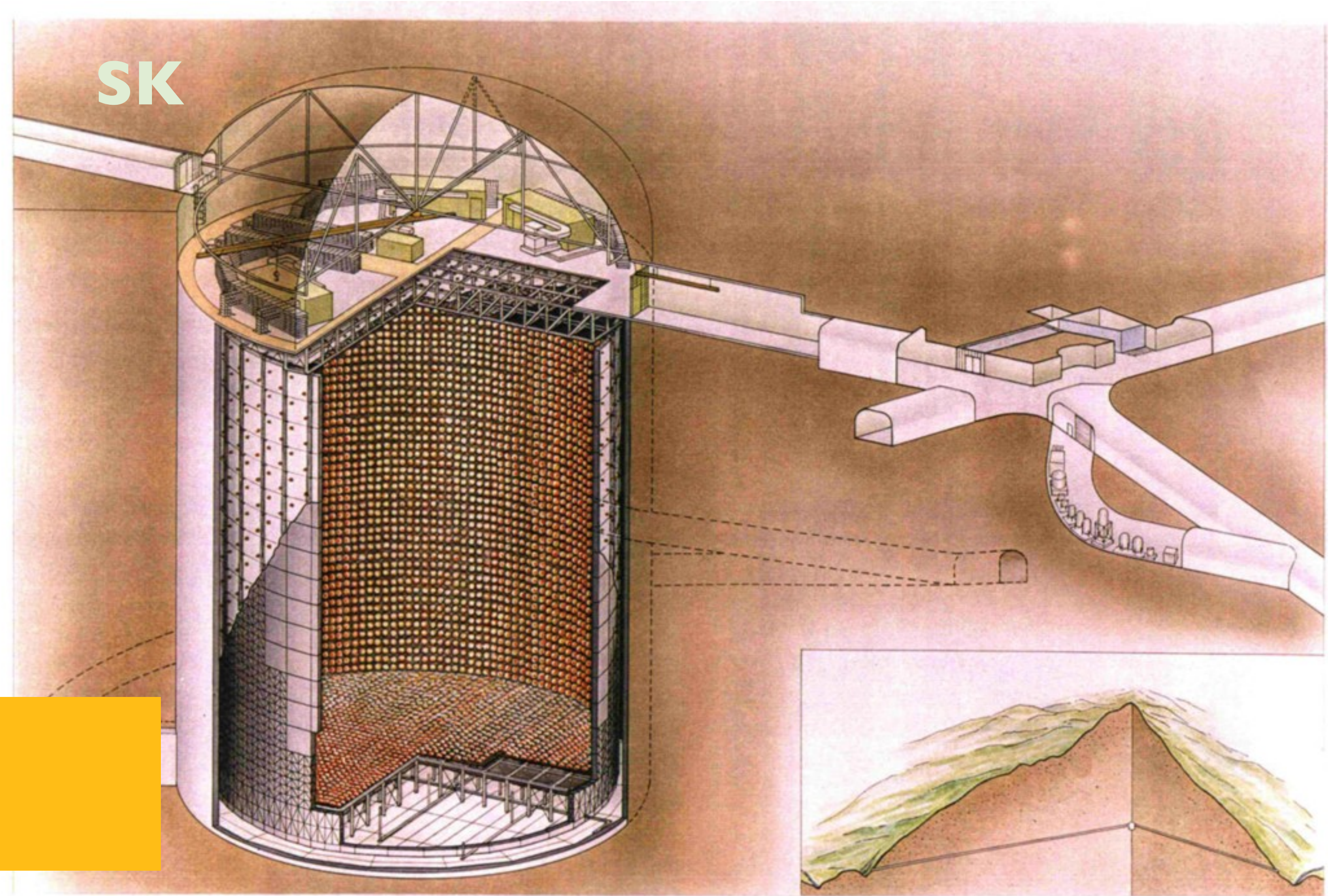
2015 Nobel





Neutrino oscillations





Two flavour case can be seen as two level quantum system

Origin of idea of neutrino oscillations

- 1957: Pontecorvo - Hadron-lepton symmetry => Leptonic analogue of the famous oscillation in the Kaon sector. $K_0 \leftrightarrow \bar{K}_0$
- Natural candidate - neutrino (only neutral lepton known at that time !) "....there exists the possibility of real neutrino to anti-neutrino transitions in vacuum provided lepton charge is not conserved…"

MESONIUM AND ANTIMESONIUM

B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor May 23, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 549-551 (August, 1957)

GELL-MANN and Pais[1] were the first to point out the interesting consequences which follow from the fact that K^0 and $\tilde{K}^0$ are not identical particles.[2] The possible $K^0 \rightarrow \tilde{K}^0$ transition, which is due to the weak interactions, leads to the necessity of considering neutral K-mesons as a superposition of particles K_1^0 and K_2^0 having a different combined parity.[3] In the present note the question is treated whether there exist other "mixed" neutral particles (not necessarily "elementary") besides the K^0-meson, which differ from their anti-particles and for which the particle → antiparticle transitions are not strictly forbidden.

INVERSE BETA PROCESSES AND NONCONSERVATION OF LEPTON CHARGE

B. PONTECORVO

Joint Institute for Nuclear Research

Submitted to JETP editor October 19, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 247-249 (January, 1958)

RECENTLY the question was discussed[1] whether there exist other "mixed" neutral particles beside the K^0 mesons,[2] i.e., particles that differ from the corresponding antiparticles, with the transitions between particle and antiparticle states not being strictly forbidden. It was noted that the neutrino might be such a mixed particle, and consequently there exists the possibility of real neutrino ⇄ antineutrino transitions in vacuum, provided that lepton (neutrino) charge[3] is not conserved. In the present note we make a more de-

Origin of idea of neutrino oscillations

- 1957: Pontecorvo - Hadron-lepton symmetry => Leptonic analogue of the famous oscillation in the Kaon sector.
- 1962: Maki, Nakagawa, Sakata - The first proposal of the concept of flavour mixing and oscillation involving 2 flavours of neutrinos.

Progress of Theoretical Physics, Vol. 28, No. 5, November 1962

Remarks on the Unified Model of Elementary Particles

Ziro MAKI, Masami NAKAGAWA and Shoichi SAKATA

Institute for Theoretical Physics
Nagoya University, Nagoya

(Received June 25, 1962)

A particle mixture theory of neutrino is proposed assuming the existence of two kinds of neutrinos. Based on the neutrino-mixture theory, a possible unified model of elementary particles is constructed by generalizing the Sakata-Nagoya model.*) Our scheme gives a natural explanation of smallness of leptonic decay rate of hyperons as well as the subtle difference of G_ν's between μ-e and β-decay.

Starting with this scheme, the possibility of K_{e3} mode with $\Delta S/\Delta Q=-1$ is also examined, and some bearings on the dynamical role of the B-matter, a fundamental constituent of baryons in the Nagoya model, are clarified.

Origin of idea of neutrino oscillations

- 1957: Pontecorvo - Hadron-lepton symmetry => Leptonic analogue of the famous oscillation in the Kaon sector.
- 1962: Maki, Nakagawa, Sakata - The first proposal of the concept of flavour mixing and oscillation involving 2 flavours of neutrinos.
- 1969: Gribov and Pontecorvo - Idea of flavour oscillations among the 2 known neutrino types after muon neutrino was known to exist.

Volume 28B, number 7 PHYSICS LETTERS 20 January 1969

NEUTRINO ASTRONOMY AND LEPTON CHARGE

V. GRIBOV* and B. PONTECORVO
Joint Institute for Nuclear Research, Dubna, USSR

Received 20 December 1968

It is shown that lepton nonconservation might lead to a decrease in the number of detectable solar neutrinos at the earth surface, because of $\nu_e \rightleftarrows \nu_\mu$ oscillations, similar to $K^0 \rightleftarrows \tilde{K}^0$ oscillations. Equations are presented describing such oscillations for the case when there exist only four neutrino states.

Why do neutrinos oscillate ?

- Neutrinos are produced and detected via weak interaction
- Weak (flavour) eigenstates differ from stationary (mass) states of the Hamiltonian. In fact, they are linear superpositions of the stationary mass states
- Leads to oscillation phenomena which is very similar to birefringence in optics - depends on properties of the medium
- Oscillations of neutrinos takes place even in vacuum - driven by non-zero mass splittings and non-zero mixing angles
- In matter, oscillations are still driven by mass splittings and mixing angles which get modified due to CC potential for coherent forward scattering of electron neutrino with electron
- Incoherent scattering cross section is negligible -> sustained coherence even over astrophysical length scales.

Two flavour neutrino oscillations

B. Pontecorvo, Sov. Phys. JETP 26, 984 (1968). [Zh. Eksp. Teor. Fiz. 53, 1717 (1967)] ; Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. 28, 870 (1962)

- Flavour states are connected to mass states by

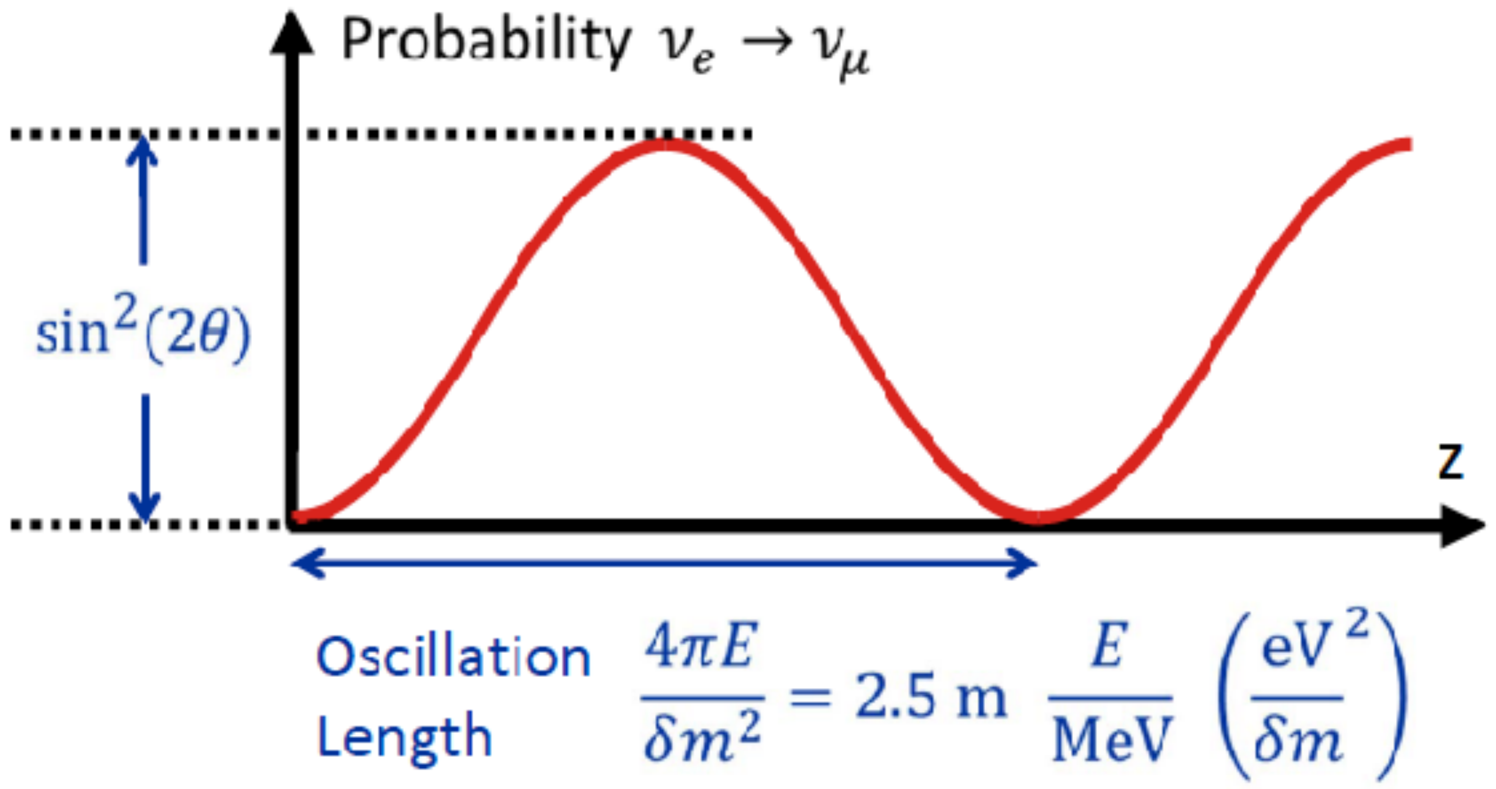
$$\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix}$$

- Each mass eigenstate propagates as

$$e^{ipz} \text{ with } p = \sqrt{E^2 - m^2} \simeq E - m^2/2E$$

$$\delta m^2 = m_2^2 - m_1^2$$

- Oscillation arises due to the phase difference $\frac{\delta m^2}{2E} z$



- Oscillation probability $P_{e\mu}(L/E) = \sin^2 2\theta \sin^2(\frac{\delta m^2 L}{4E})$

Visualizing oscillations

Mehta, PRD 2009, see also Kim, Sze and Nussinov, PRD35 (1987); Kim, Kim and Sze, PRD37 (1988).

- Schrodinger-like equation in terms of flavour spinor (in the UR limit)

$$i\partial_t \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \mathbb{H} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\delta m^2}{2E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

- Neutrino flavour density matrix and commutator form

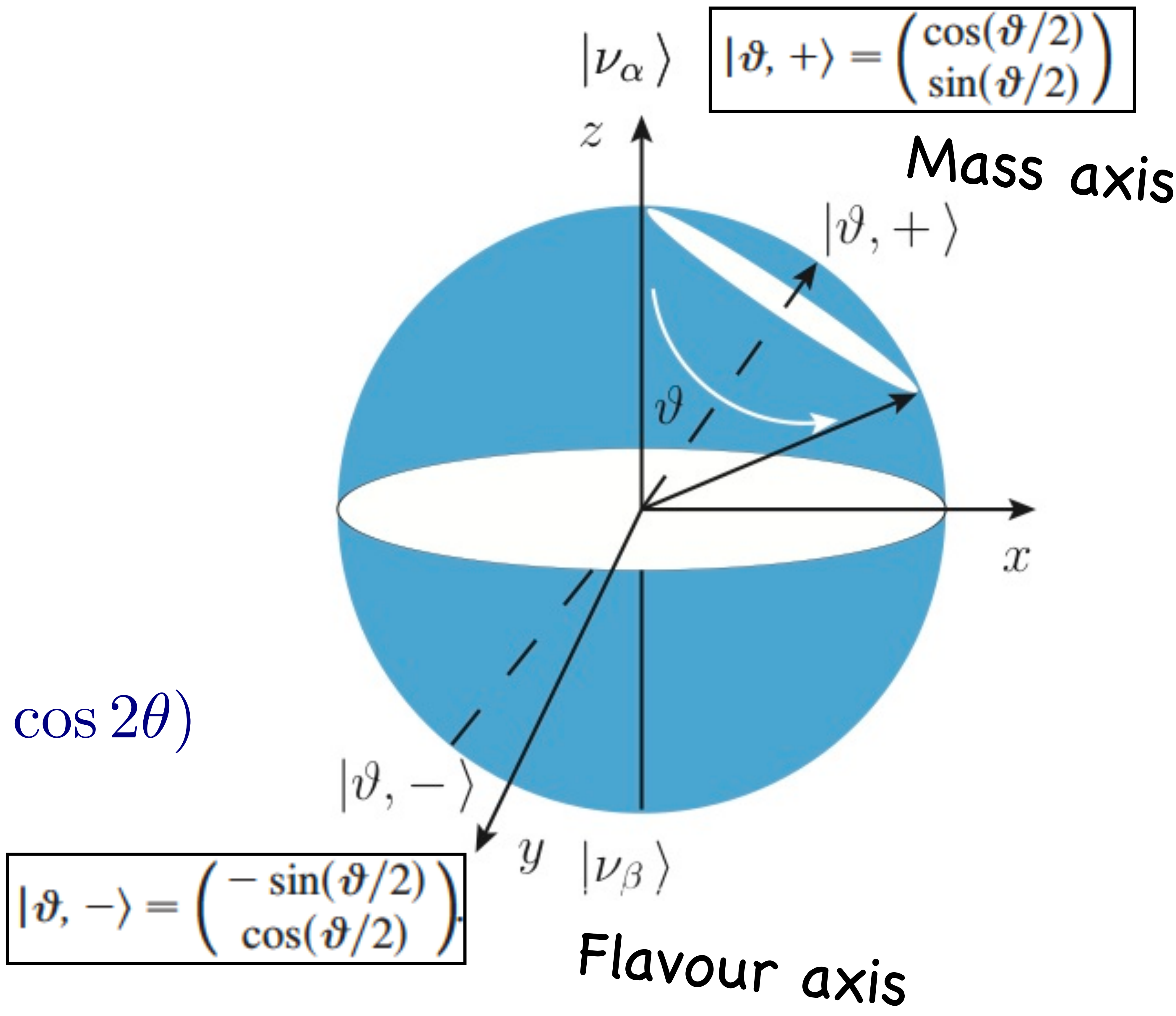
$$\rho = \begin{pmatrix} \langle \nu_e | \nu_e \rangle & \langle \nu_e | \nu_\mu \rangle \\ \langle \nu_\mu | \nu_e \rangle & \langle \nu_\mu | \nu_\mu \rangle \end{pmatrix} \qquad i\partial_t \rho = [\mathbb{H}, \rho]$$

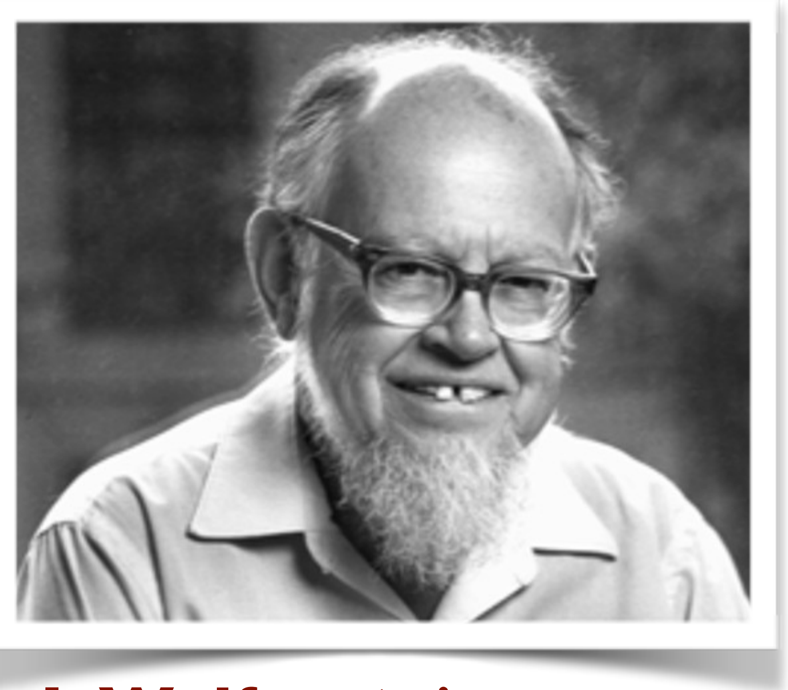
- Expand 2 by 2 Hermitian matrices in terms of Pauli matrices

$$\rho = \frac{1}{2}[\mathrm{Tr}(\rho) + \mathbf{P}\cdot\sigma] \qquad \mathbb{H} = \frac{\delta m^2}{2E}\mathbf{B}\cdot\sigma \qquad \mathbf{B} = (\sin 2\theta, 0, \cos 2\theta)$$

- Analogous to spin precession in a magnetic field

$$\dot{\mathbf{P}} = \omega \mathbf{B} \times \mathbf{P}$$





L.Wolfenstein

Standard interactions

Wolfenstein 1978, see also Nussiniov, PLB63, 201, 1976

- Neutrinos in matter suffer flavour-dependent refraction

$$V_{\rm weak} = \sqrt{2}G_F \times (N_e - N_n/2) \quad \text{for} \quad \nu_e$$
$$= \sqrt{2}G_F \times (-N_n/2) \quad \text{for} \quad \nu_\mu$$

- The potential changes sign for anti-neutrinos

- For typical Earth density ~ 5 g/cc

$$\Delta V_{\rm weak} \approx 2 \times 10^{-13}\ eV = 0.2\ peV$$

PHYSICAL REVIEW D VOLUME 17, NUMBER 9 1 MAY 1978

Neutrino oscillations in matter

L. Wolfenstein
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.

- Elastic forward scattering dominates at low E (real part)
- Incoherent scattering cross section is usually very small

S. Mikheyev

The MSW Effect

Mikheev and Smirnov, Sov Jour. Nucl Phys. 42, 913 (1985)

A. Smirnov

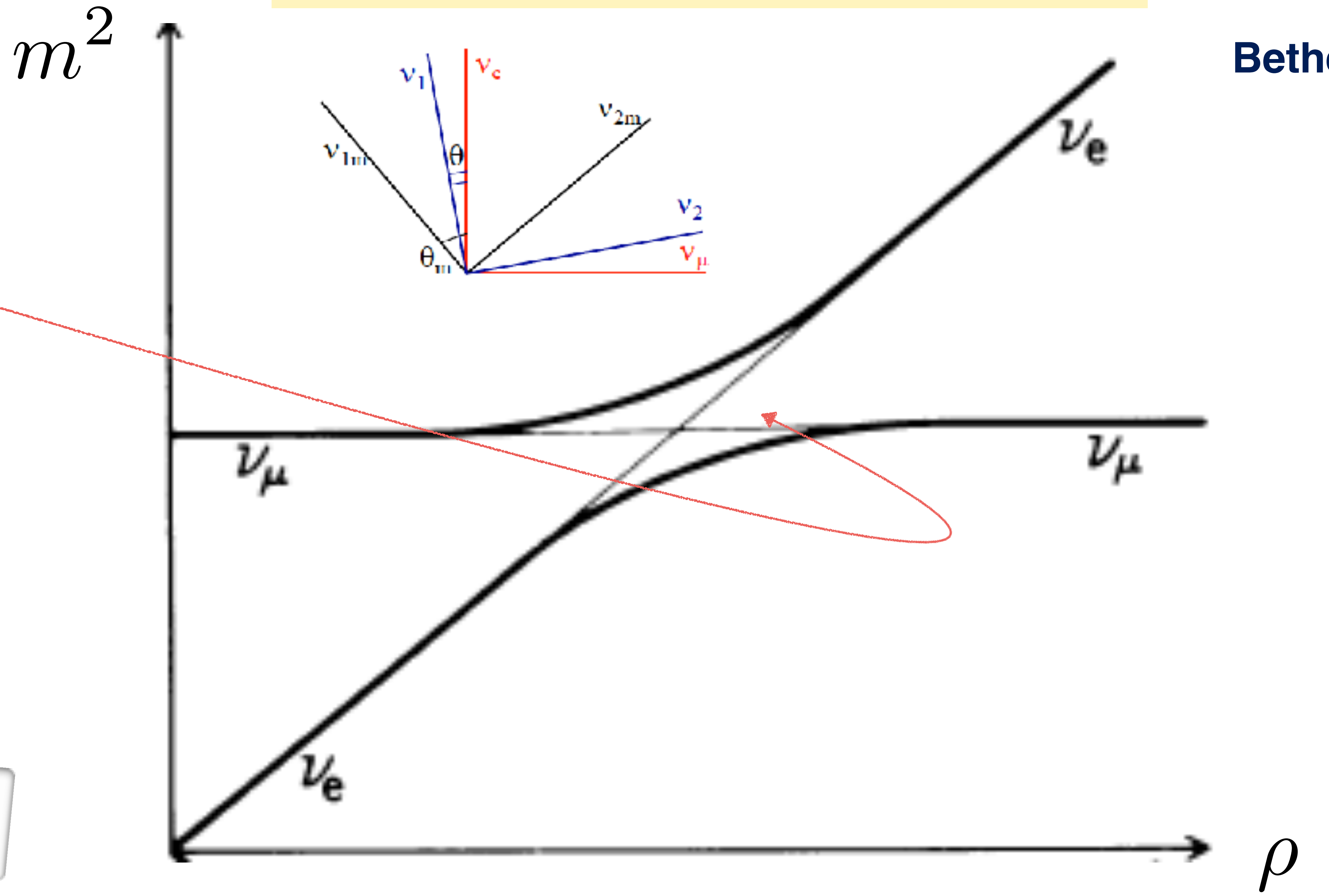
In electrically neutral matter, UR limit

$$\mathbb{H}_\nu = \left(p + \frac{m_1^2 + m_2^2}{4p} + \frac{V_C}{2} + V_N\right)\mathbb{I} + \frac{1}{2}\begin{pmatrix} V_C - \omega\cos 2\theta & \omega\sin 2\theta \\ \omega\sin 2\theta & -(V_C - \omega\cos 2\theta) \end{pmatrix}$$

$V_C = \sqrt{2}G_F n_e$ and $V_N = -\sqrt{2}G_F n_n/2$

Mixing becomes maximal when the diagonal elements vanish, i.e.

$$\frac{V_C}{\omega} = \cos 2\theta$$



Bethe (1986)

$$V_C = \sqrt{2}G_F n_e$$

$$\omega = \frac{\delta m^2}{2E}$$

Complete conversion in the adiabatic limit !

Optical effects and their counterparts in the neutrino system

Mehta, PRD 2009

Effect of a medium can be described in terms of

$$\mathbb{H} = D\mathbb{I} + A\sigma_x + B\sigma_y + C\sigma_z$$

D leads to overall phase while A, B, C generate non-trivial optical effects

Optical effects

- Circular birefringence (optical activity) : C, D non-zero
- Linear birefringence (wave plate) : A, D non-zero
- Elliptic birefringence (quartz plate) : A, B, C, D non-zero
- Dichroism (absorption) : H need not be Hermitian

Neutrino oscillations

- Oscillations in vacuum : A, C, D non-zero

$$A = \frac{\omega}{2}\sin 2\theta \ ; \ \ B = 0 \ ; \ \ C = -\frac{\omega}{2}\cos 2\theta$$

- Oscillations in matter : A, C, D non-zero

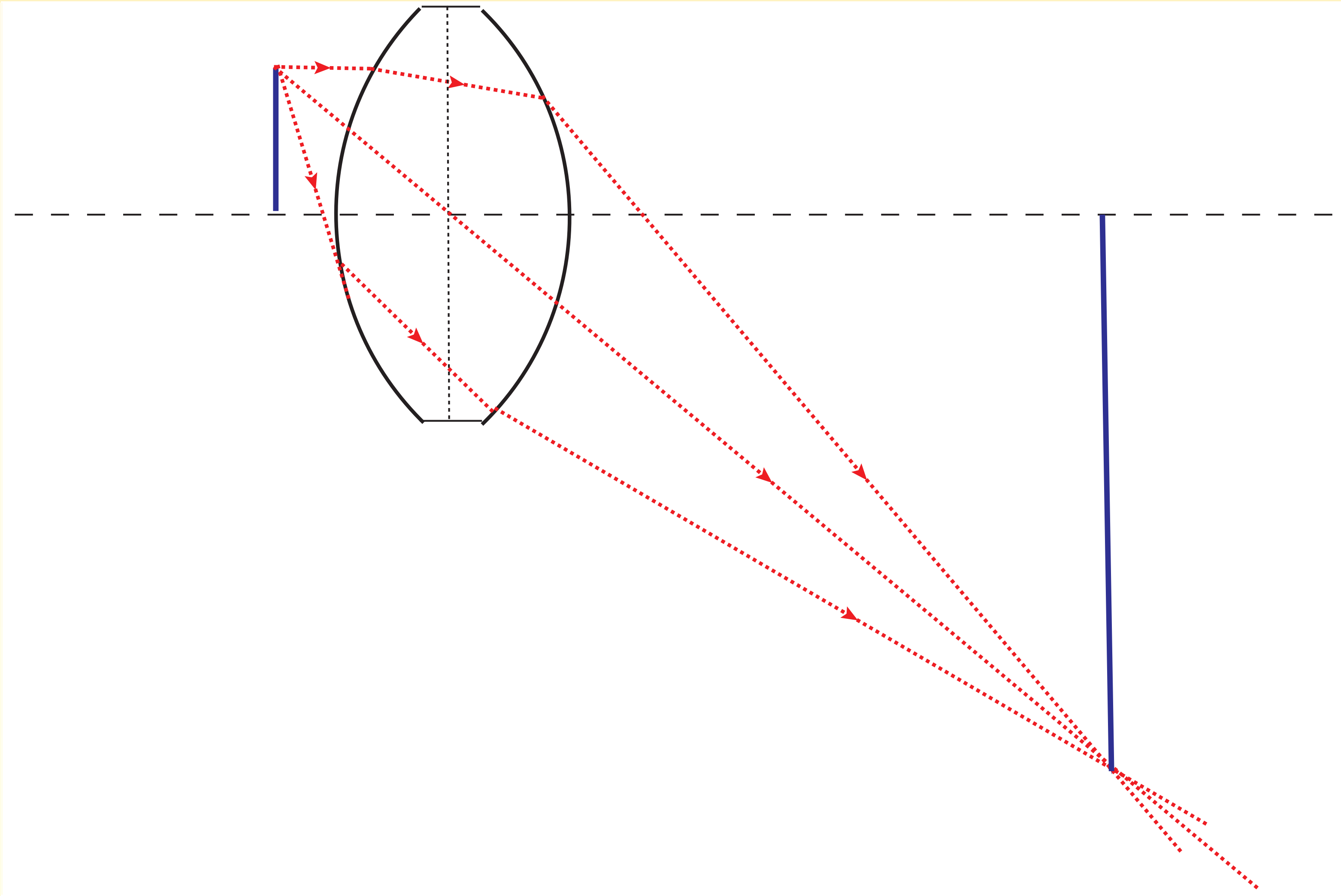
$$A = \frac{\omega}{2}\sin 2\theta \ ; \ \ B = 0 \ ; \ \ C = -\frac{\omega}{2}\sin 2\theta + \frac{1}{2}\sqrt{2}G_F n_e$$

- Dichroism (absorption) : negligible

- Can we make devices similar to the optical devices using reflective and refractive property of neutrinos ?
- If we take Sun as a lens, then the focal length is given by

$$f = \frac{1}{2}\frac{R_\odot}{(n_{refr} - 1)}$$

Lens Maker's formula (tiny n_{refr} limit)



- *For* 10 *MeV neutrinos passing through Sun with density* $\rho = 150$ *g cm*$^{-3}$*, one gets the focal length to be around* $10^{18} R_\odot \sim 10^5$ *size of our Galaxy.*
- Potentially observable effect of small refractive index is via neutrino oscillations !!

Three flavour neutrino oscillations

Pontecorvo, Sov. Phys. JETP, 6 (1957), p. 429 ; Maki, Nakagawa, Sakata, Prog. Theor. Phys., 28 (1962), p. 870

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{Atmospheric} \overbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}^{Reactor} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{Solar}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$ and δ is the Dirac-type CP phase

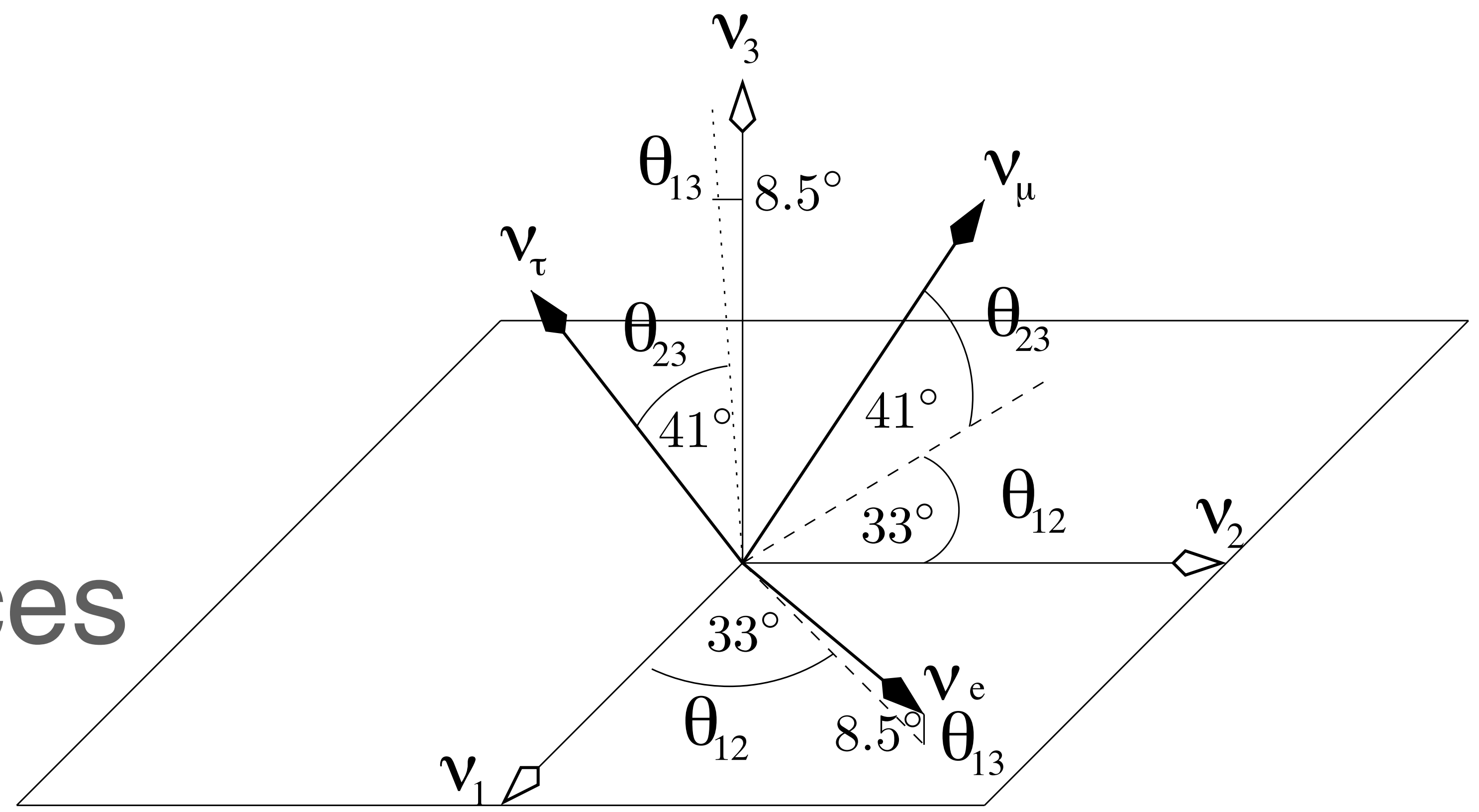
If Majorana - two additional phases appear, $U \rightarrow U\text{diag}(1, e^{i\kappa}, e^{i\zeta})$

Unknowns

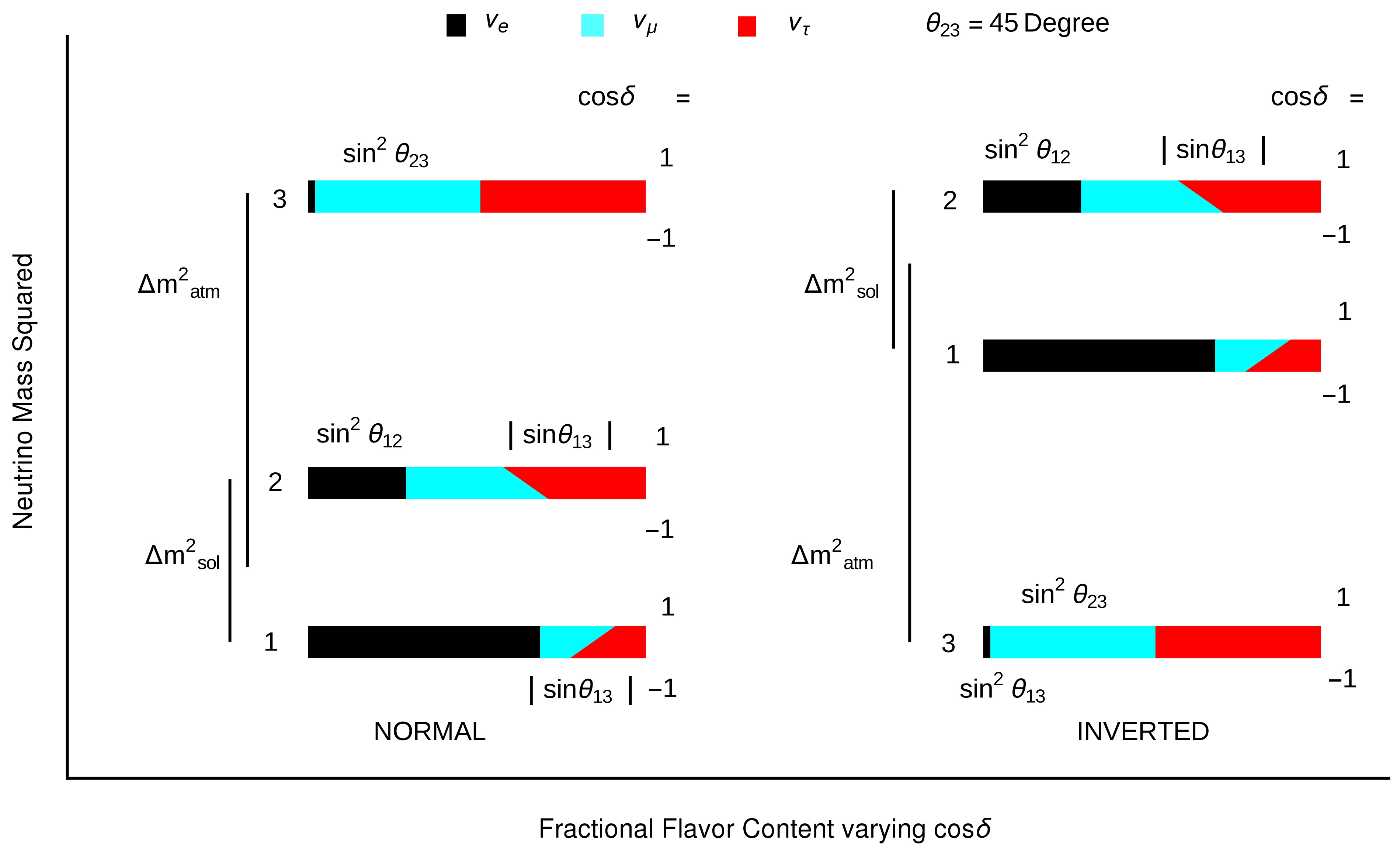
- CP violating phase
- Sign of larger mass-splitting
- Octant of theta 23

Parameters

- 3 angles
- 1 phase
- 2 mass-squared differences



Credit : King



Credit : Mena and Parke, 2004

Tests of LGI in neutrino oscillations

Work by other groups

Bell's inequalities, LGI in neutrino oscillation context

- D. Gangopadhyay, D. Home, A.S. Roy, Probing the Leggett-Garg inequality for oscillating neutral kaons and neutrinos. Phys. Rev. A 88, 022115 (2013)
- D. Gangopadhyay, A.S. Roy, Three-flavoured neutrino oscillations and the Leggett-Garg inequality. Eur. Phys. J. C 77, 260 (2017)
- S. Banerjee, A.K. Alok, R. Srikanth, B.C. Hiesmayr, A quantum-information theoretic analysis of three-flavor neutrino oscillations.
  Eur. Phys. J. C 75, 487 (2015)
- A.K.Alok, S.Banerjee, S. U. Sankar, Quantum correlations interms of neutrino oscillation probabilities. Nucl. Phys. B 909, 65 (2016). arXiv:1411.5536
- J. Naikoo, A.K. Alok, S. Banerjee, S.U. Sankar, G. Guarnieri, B.C. Hiesmayr, Leggett–Garg-type inequalities and the neutrino mass- degeneracy problem. Eur.Phys.J.C 78 (2018) 11, 914.
- K. Dixit, Javid Naikoo, Subhashish Banerjee, A.K.Alok,Study of coherence and mixedness in meson and neutrino systems, Eur.Phys.J.C 79 (2019) 2, 96
- J. Naikoo, A K Alok, S Banerjee, S. U Sankar, G Guarnieri, A quantum information theoretic quantity sensitive to the neutrino mass-hierarchy, Nucl.Phys.B 951 (2020) 114872.

Work by other groups

Bell's inequalities, LGI in neutrino oscillation context

- K. Dixit, A.K.Alok,New physics effects on quantum coherence in neutrino oscillations, Eur.Phys.J.Plus 136 (2021) 3, 334
- B. Yadav, K. Dixit, A.K.Alok, Can NSI affect non-local correlations in neutrino oscillations? Eur.Phys.J.C 82 (2022) 446
- A K Jha, S Mukherjee, B A. Bambah, Tri-Partite entanglement in Neutrino Oscillations Mod.Phys.Lett.A 36 (2021) 09, 2150056
- A K Jha, A Chatla, B A. Bambah, Quantum simulation of oscillating neutrino 2010.06458 [hep-ph]
- A K Jha, A Chatla, B A. Bambah, Neutrinos as Qubits and Qutrits 2203.13485 [hep-ph]
- A K Jha, A Chatla, Quantum studies of neutrinos on IBMQ processors, Eur.Phys.J.ST 231 (2022) 2, 141-149
- D. K. Chattopadhyay and A. Dighe, Quantum mismatch - a powerful measure of quantumness in neutrino oscillations PRD 2023
- L. Konwar, J. Vardani, B. Yadav, Violation of LGtI inequalities in the light of NOvA and T2K anomaly 2401.02886 [hep-ph]

LGI in two flavour case

Gangopadhyay, Home and Sinha Roy , Phys. Rev. A 88, 022115 (2013)

$$Q = \begin{cases} +1 & \text{for } \nu_\mu \\ -1 & \text{for } \nu_e \text{ or } \nu_\tau \end{cases}$$

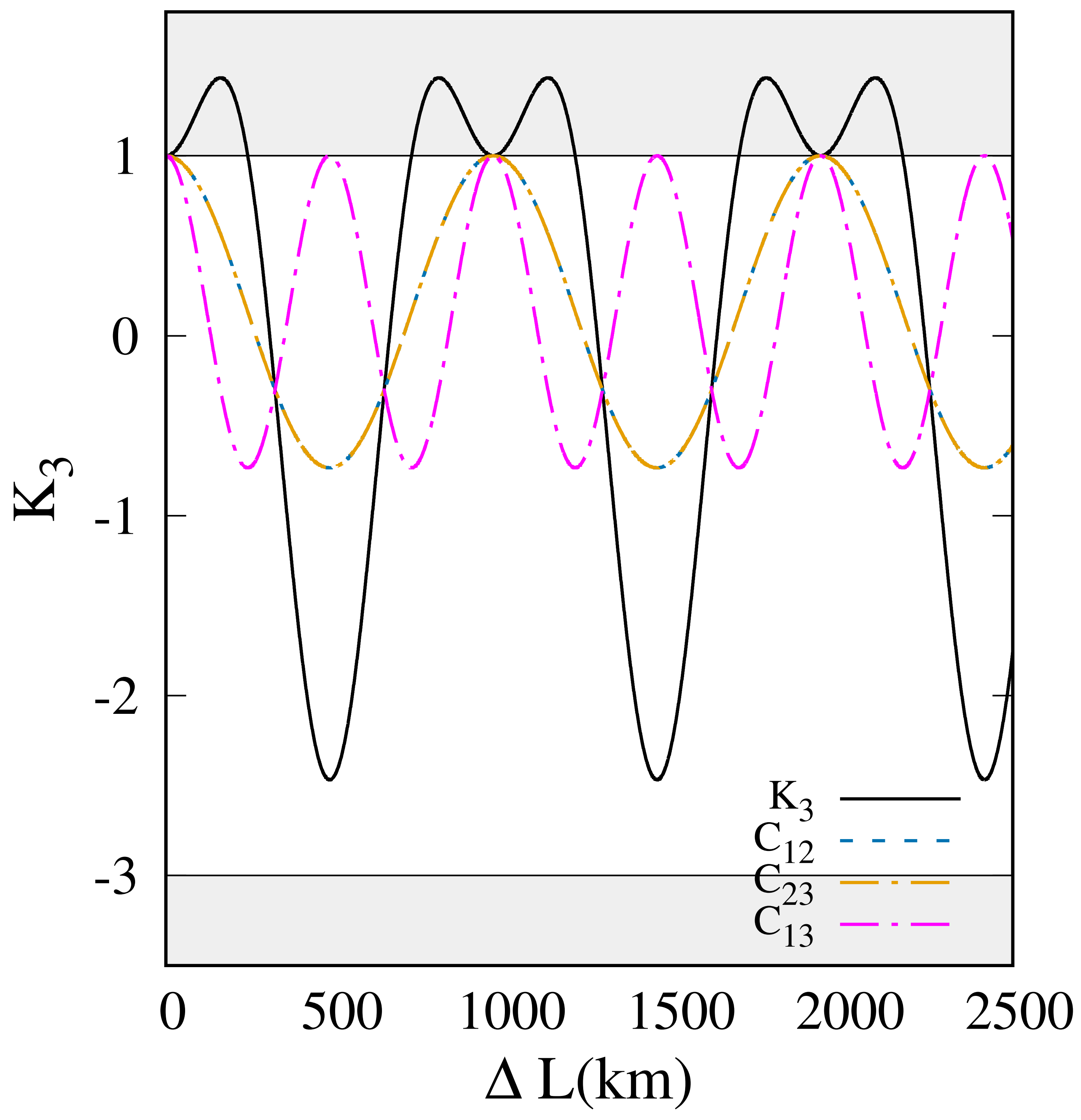
$$C_{12} = \mathbb{P}_{\nu_e\nu_e}(t_1,t_2) - \mathbb{P}_{\nu_e\nu_\mu}(t_1,t_2) - \mathbb{P}_{\nu_\mu\nu_e}(t_1,t_2) + \mathbb{P}_{\nu_\mu\nu_\mu}(t_1,t_2)$$

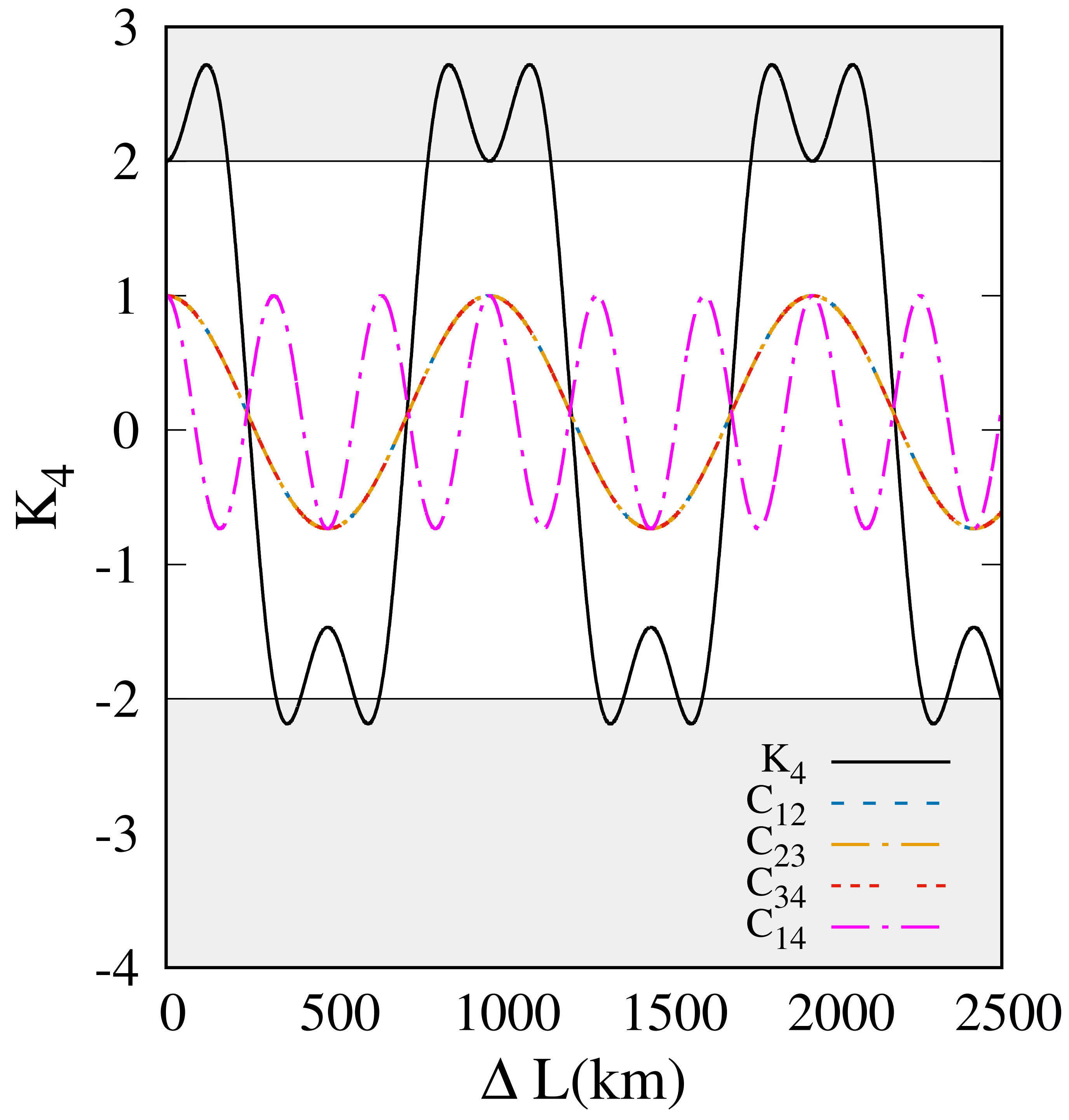
$$\mathbb{P}_{\nu_\alpha\nu_\beta}(t_1,t_2) = P_{\mu\alpha}(t_1)P_{\alpha\beta}(t_2)$$ Joint Probabilities

$$C_{12} = 1 - 2\sin^2 2\theta \sin^2\left(\frac{\Delta m^2 \Delta L}{4E}\right)$$

$$K_3 = 1 - 2\sin^2 2\theta\left[2\sin^2\frac{\Delta m^2\tau}{4E} - \sin^2\frac{2\Delta m^2\tau}{4E}\right]$$

$$K_4 = 2 - 2\sin^2 2\theta\left[3\sin^2\frac{\Delta m^2\tau}{4E} - \sin^2\frac{3\Delta m^2\tau}{4E}\right]$$





Shafaq and Mehta, J Phys. G 48, 085002 (2021)

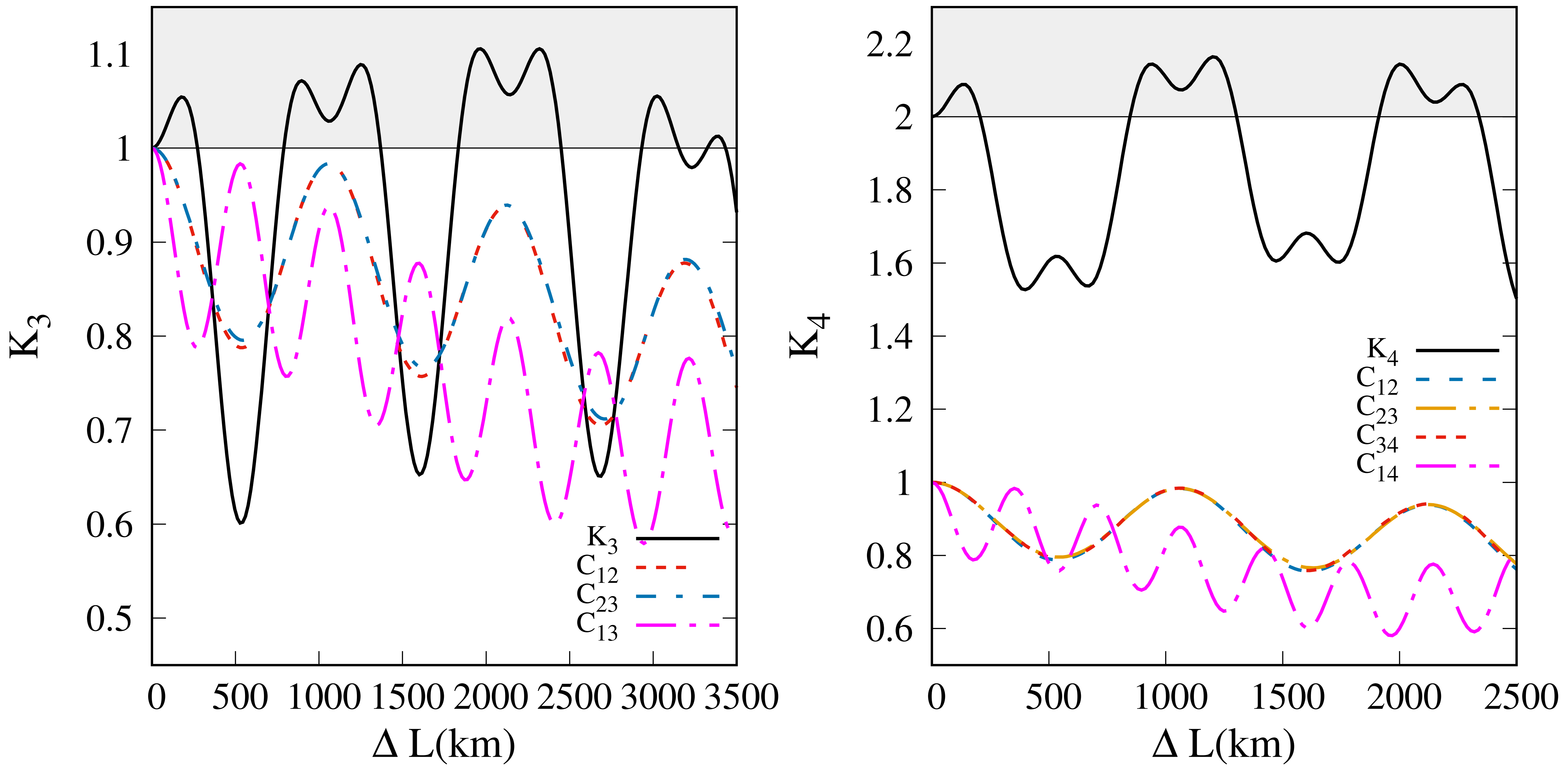
LGI in three flavour case

Gangopadhyay and Home,Eur. Phys. J. C 77, 260 (2017)

$$Q = \begin{cases} +1 & \text{for } \nu_\mu \\ -1 & \text{for } \nu_e \text{ or } \nu_\tau \end{cases}$$

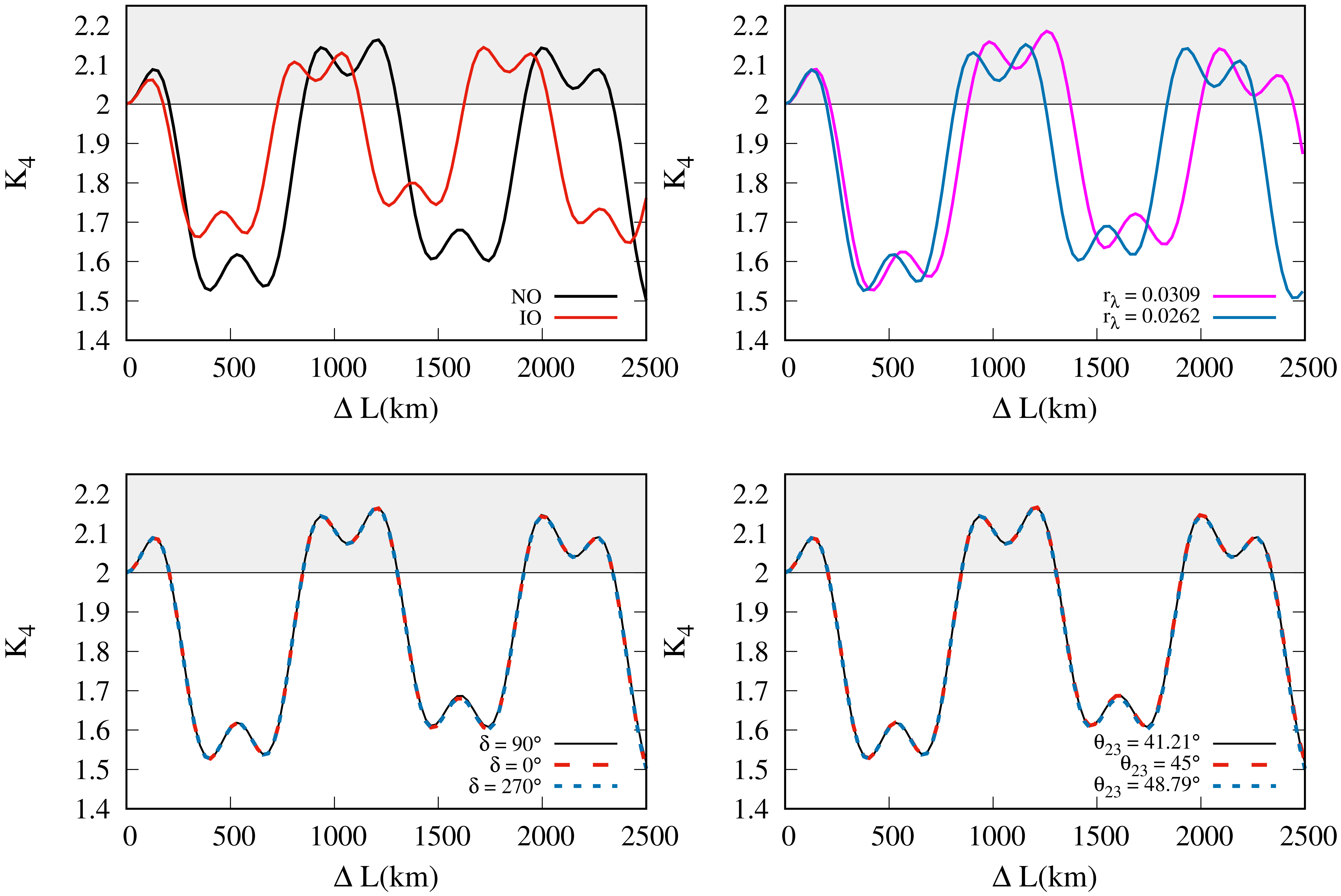
$$C_{12} = \mathbb{P}_{\nu_e\nu_e}(L_1,L_2) - \mathbb{P}_{\nu_e\nu_\mu}(L_1,L_2) - \mathbb{P}_{\nu_e\nu_\tau}(L_1,L_2) - \mathbb{P}_{\nu_\mu\nu_e}(L_1,L_2) + \mathbb{P}_{\nu_\mu\nu_\mu}(L_1,L_2) + \mathbb{P}_{\nu_\mu\nu_\tau}(L_1,L_2) - \mathbb{P}_{\nu_\tau\nu_e}(L_1,L_2) + \mathbb{P}_{\nu_\tau\nu_\mu}(L_1,L_2) + \mathbb{P}_{\nu_\tau\nu_\tau}(L_1,L_2)$$

$$\mathbb{P}_{\nu_\alpha\nu_\beta}(t_1,t_2) = P_{\mu\alpha}(t_1)P_{\alpha\beta}(t_2)$$



Shafaq and Mehta, J Phys. G 48, 085002 (2021)

LGI in three flavour case - standard unknowns



- Almost no dependence on CP phase
- Almost no dependence on theta 23
- There is some dependence on mass hierarchy as well as on mass ordering parameter

Shafaq and Mehta, J Phys. G 48, 085002 (2021)

- Study of temporal correlations in the form of LGI has attracted significant attention in recent times in the context neutrino oscillations. It should be noted that while different dichotomic observables have been employed in these studies, the neutrino matter interactions have been considered to be standard in these studies.
- Non-standard interactions are currently one of the most widely studied new physics topics in the context of neutrino oscillations as these are well motivated both theoretically and experimentally. Moreover there can be other kinds of new physics effects like decoherence and decay that could leave distinct imprints on neutrino oscillation probability.
- Thus, it is worthwhile to investigate different physics scenarios beyond the SM and study their impact on oscillation probabilities.
- We invoke non-standard interactions and damping effects (including decoherence, decay) on oscillation probabilities and study their implications on the LGI.

3 flavour neutrino oscillations and non-standard neutrino oscillations

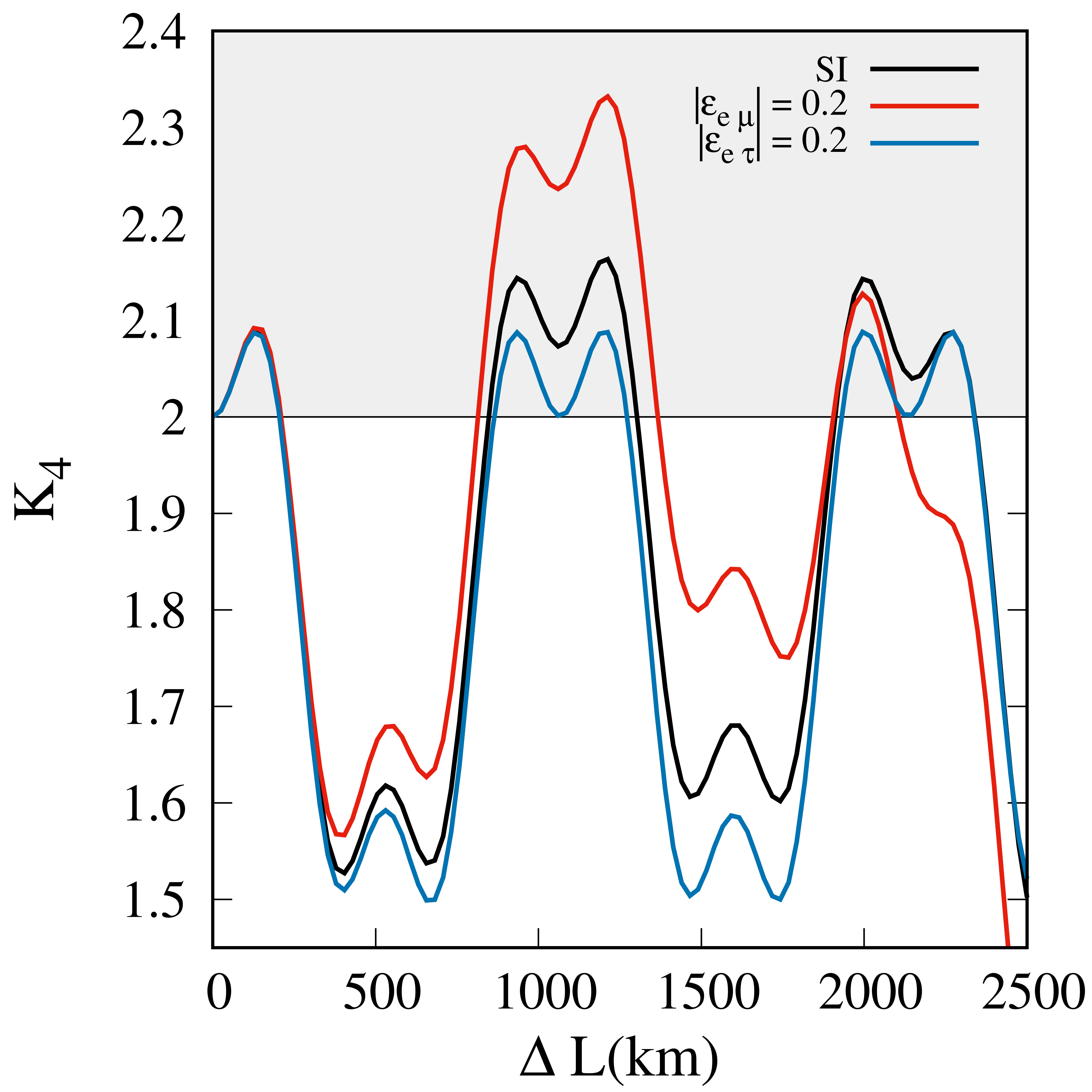
Ref: Wolfenstein (1978), Grossman (1995), Berezhiani, Rossi (2002), Davidson et al. (2003) , Ohlsson, Tortola and Farzan

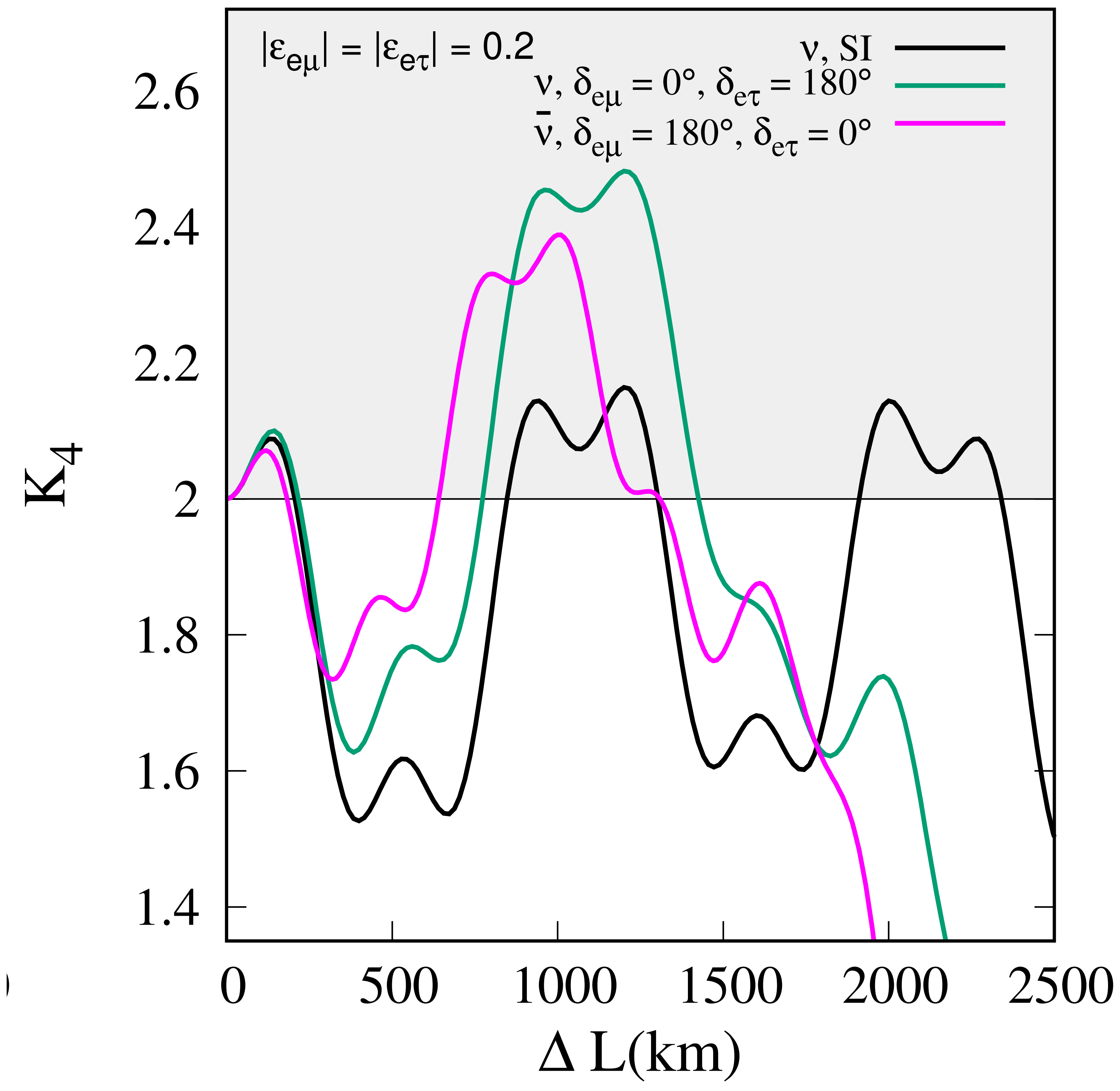
$$\mathcal{L}_{NSI} = -2\sqrt{2}G_F\epsilon^{f\,C}_{\alpha\beta}\;[\bar{\nu}_\alpha\gamma^\mu P_L\nu_\beta]\;[\bar{f}\gamma_\mu P_C f]\;, \qquad P_C = (1\pm\gamma_5)/2.$$

$$\mathcal{H} = \frac{1}{2E}\left\{\mathcal{U}\begin{pmatrix} 0 & & \\ & \delta m^2_{21} & \\ & & \delta m^2_{31}\end{pmatrix}\mathcal{U}^\dagger + A(x)\begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}{}^\star & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}{}^\star & \epsilon_{\mu\tau}{}^\star & \epsilon_{\tau\tau}\end{pmatrix}\right\},$$

- Oscillation parameters such as the mixing angles and mass-squared splittings have been measured with great precision
- New physics interactions were initially proposed to provide an alternative to the oscillation formalism. However, this is now ruled out and we can study new physics effects as sub-leading effects in the discussion of oscillation formalism
- The new physics effects can impact determination of standard oscillation parameters and lead to more complicated parameter degeneracies

NSI induced Enhancement in K4





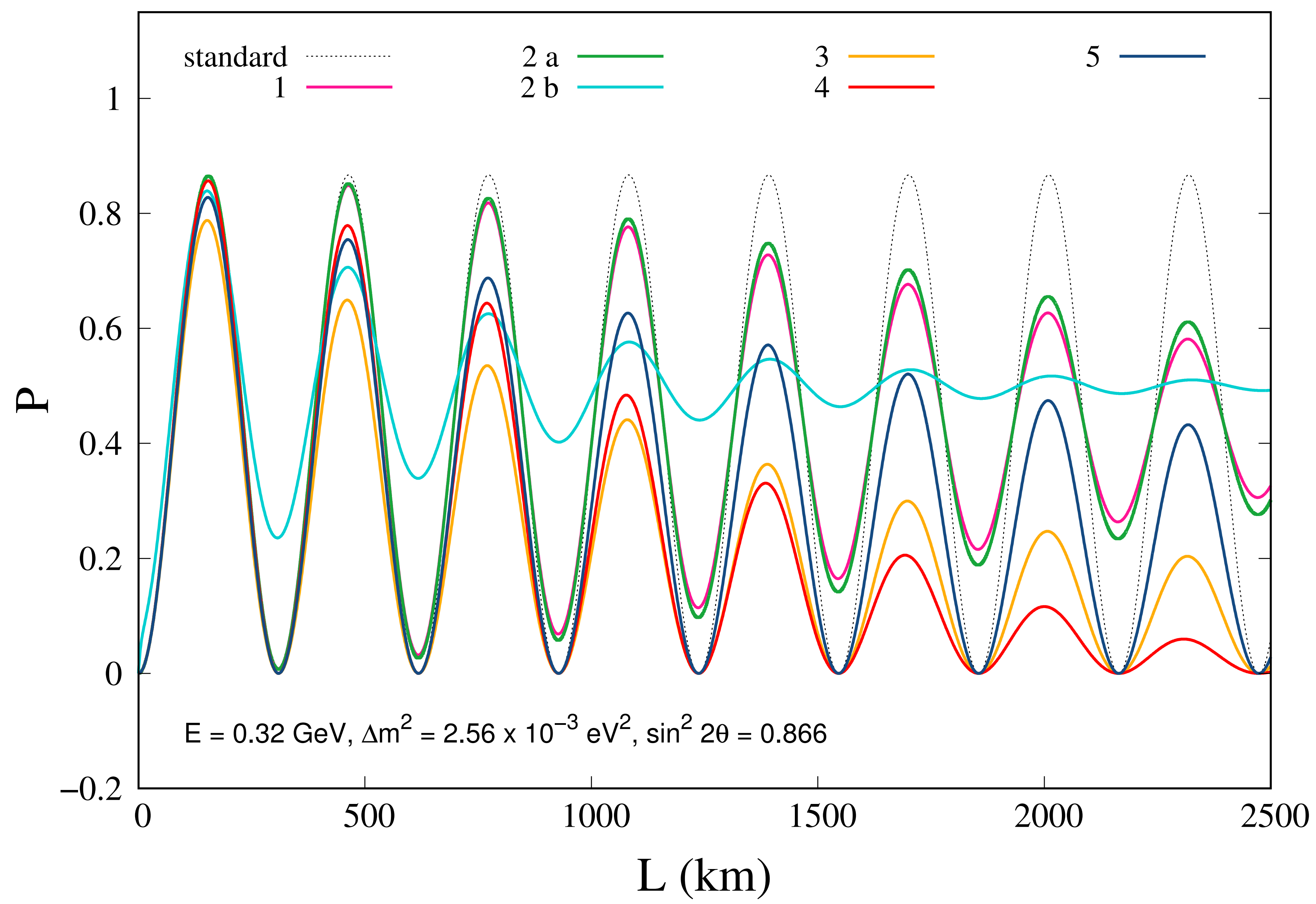
Shafaq and Mehta, J Phys. G 48, 085002 (2021)

Damped oscillations and LGI

Blennow, Ohlsson and Winter, JHEP (2005)

$$P_{\alpha\beta} = \sum_{i,j=1}^{3} U_{\alpha j} U^*_{\beta j} U^*_{\alpha i} U_{\beta i} \exp(-\mathrm{i}2\Delta_{ij}) D_{ij}$$

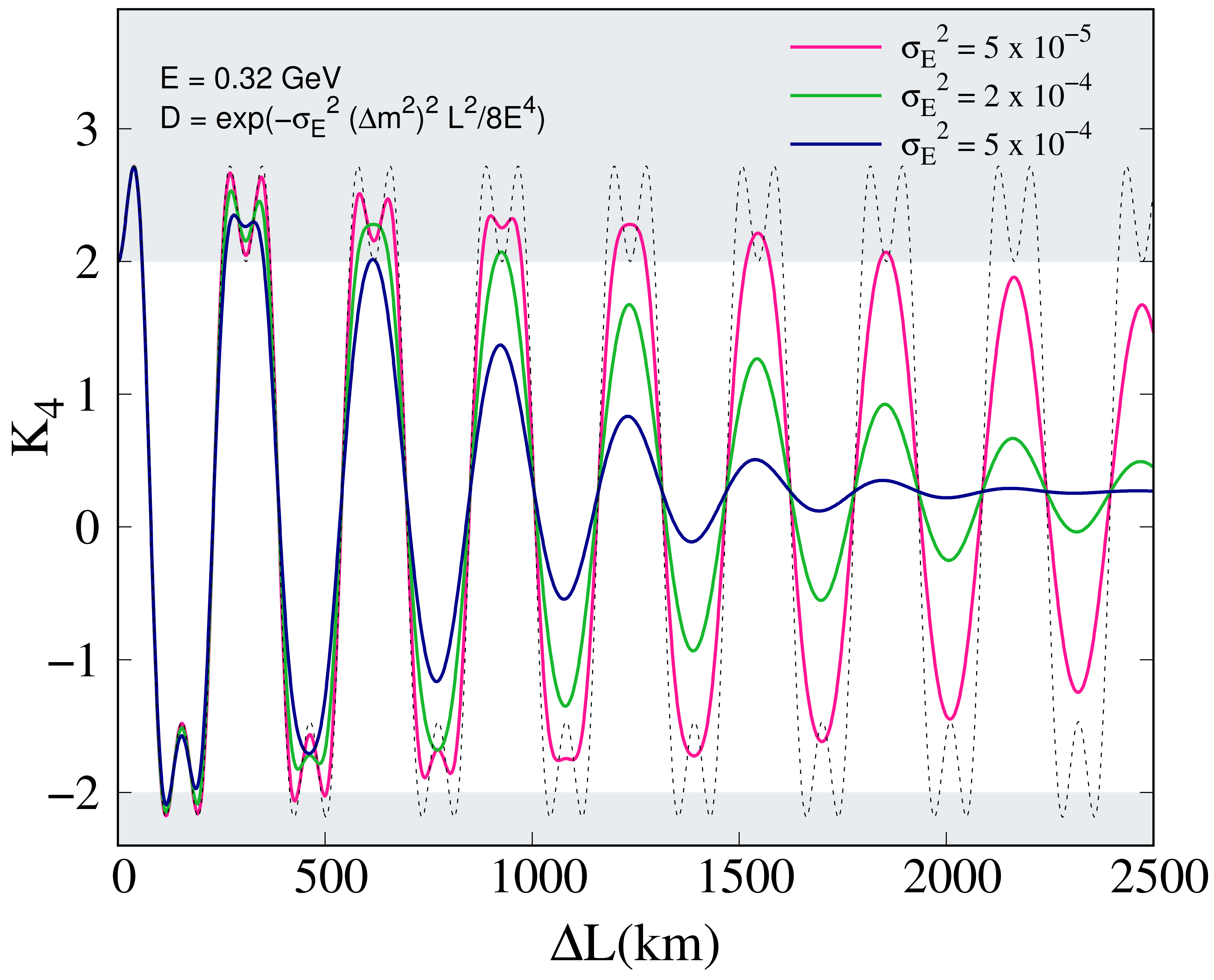
$$= \sum_{i=1}^{3} J^{\alpha\beta}_{ii} D_{ii} + 2 \sum_{1\le i<j\le 3} |J^{\alpha\beta}_{ij}| D_{ij} \cos(2\Delta_{ij} + \arg J^{\alpha\beta}_{ij})$$

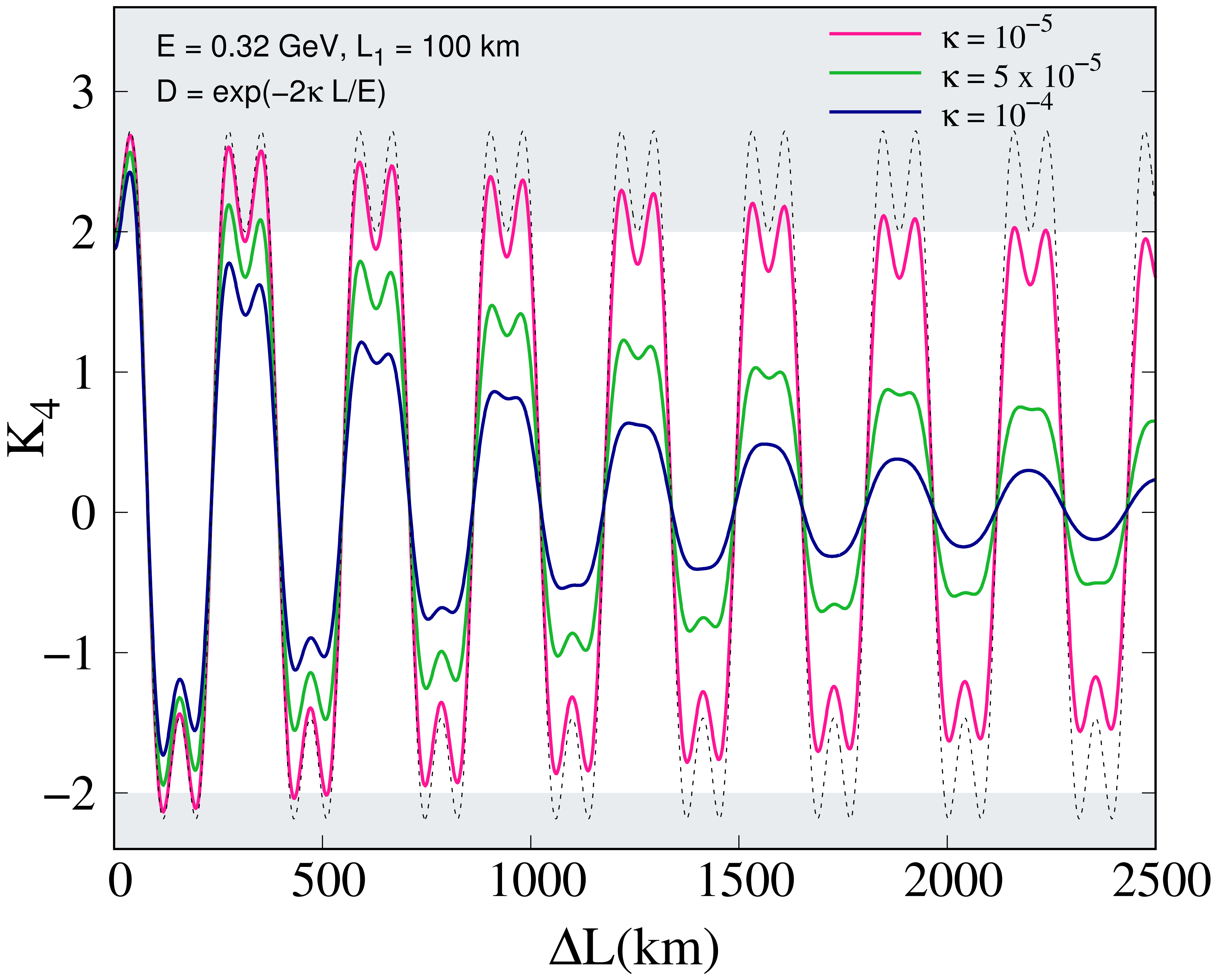


S. No.	Damping Scenario	$D_{ij} = \exp\left(-\kappa_{ij} \frac{\lvert\Delta m^2_{ij}\rvert^{\xi} L^{\beta}}{E^{\gamma}}\right)$	κ (units)
	Decoherence like $\xi \neq 0$		
1	Intrinsic wave packet decoherence	$\exp\left(-\sigma_E^2 \frac{(\Delta m^2_{ij})^2 L^2}{8E^4}\right)$	$\frac{\sigma_E^2}{8}$ (GeV^2)
2	Quantum decoherence	$\exp\left(-\kappa \frac{(\Delta m^2_{ij})^2 L^2}{E^2}\right)$	κ (dimensionless)
	Decay like $\xi = 0$		
3	Invisible neutrino decay	$\exp\left(-\kappa \frac{L}{E}\right)$	κ (GeV $\cdot$ km^{-1})
4	Oscillations into sterile neutrino	$\exp\left(-\epsilon \frac{L^2}{(2E)^2}\right)$	ϵ (eV^4)
5	Neutrino absorption	$\exp(-\kappa L E)$	κ (GeV^{-1} $\cdot$ km^{-1})

Shafaq, Kushwaha and Mehta, 2021

Damping induced Suppression in K4





Shafaq, Kushwaha and Mehta, 2021

2 GENERATION CASE

The W-interactions (19) are given in the mass eigenbasis by

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \left(\overline{u_L} \quad \overline{c_L} \right) \gamma^\mu (V_{uL} V_{dL}^\dagger) \begin{pmatrix} d_L \\ s_L \end{pmatrix} W_\mu^+ + \text{h.c.} \tag{24}$$

The matrix $(V_{uL} V_{dL}^\dagger)$ is the mixing matrix for 2 quark generations. It is a 2×2 unitary matrix. As such, it generally contains 4 parameters, of which one can be chosen as a real angle, θ_C, and 3 are phases:

$$(V_{uL} V_{dL}^\dagger) = \begin{pmatrix} \cos\theta_C \, e^{i\alpha} & \sin\theta_C \, e^{i\beta} \\ -\sin\theta_C \, e^{i\gamma} & \cos\theta_C \, e^{i(-\alpha+\beta+\gamma)} \end{pmatrix}. \tag{25}$$

By the transformation

$$(V_{uL} V_{dL}^\dagger) \to V = P_u (V_{uL} V_{dL}^\dagger) P_d^*, \tag{26}$$

with

$$P_u = \begin{pmatrix} e^{-i\alpha} & \\ & e^{-i\gamma} \end{pmatrix}, \quad P_d = \begin{pmatrix} 1 & \\ & e^{i(-\alpha+\beta)} \end{pmatrix}, \tag{27}$$

we eliminate the three phases from the mixing matrix. (We redefine the mass eigenstates $u_{L,R} \to P_u u_{L,R}$ and $d_{L,R} \to P_d d_{L,R}$, so that the mass matrices remain unchanged. In particular, they remain real.) Notice that there are three independent phase differences between the elements of P_u and those of P_d, and three phases in $(V_{uL} V_{dL}^\dagger)$. Consequently, there are no physically meaningful phases in V, and hence no CP violation:[4]

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}.$$

NO CP PHASE IN 2 GENERATIONS

Y. Nir, 1991 TASI LECTURE

PT symmetric non-Hermitian Hamiltonians

C. M. Bender and S. Boettcher, Phys. Rev. Lett., 80, 5243–5246, 1998; T. Ohlsson and S. Zhou, J. Math. Phys., 61, 052104, 2020; T. Ohlsson and S. Zhou, J. Math. Phys., 62, 042104, 2021

- General Form of PT symmetric non-Hermitian Hamiltonian

$$\mathcal{H} = \begin{pmatrix} \rho e^{i\psi} & \sigma e^{i\phi} \\ \sigma e^{-i\phi} & \rho e^{-i\psi} \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} \rho e^{i\psi} & \sigma \\ \sigma & \rho e^{-i\psi} \end{pmatrix}$$

symmetric

- Eigenvalues

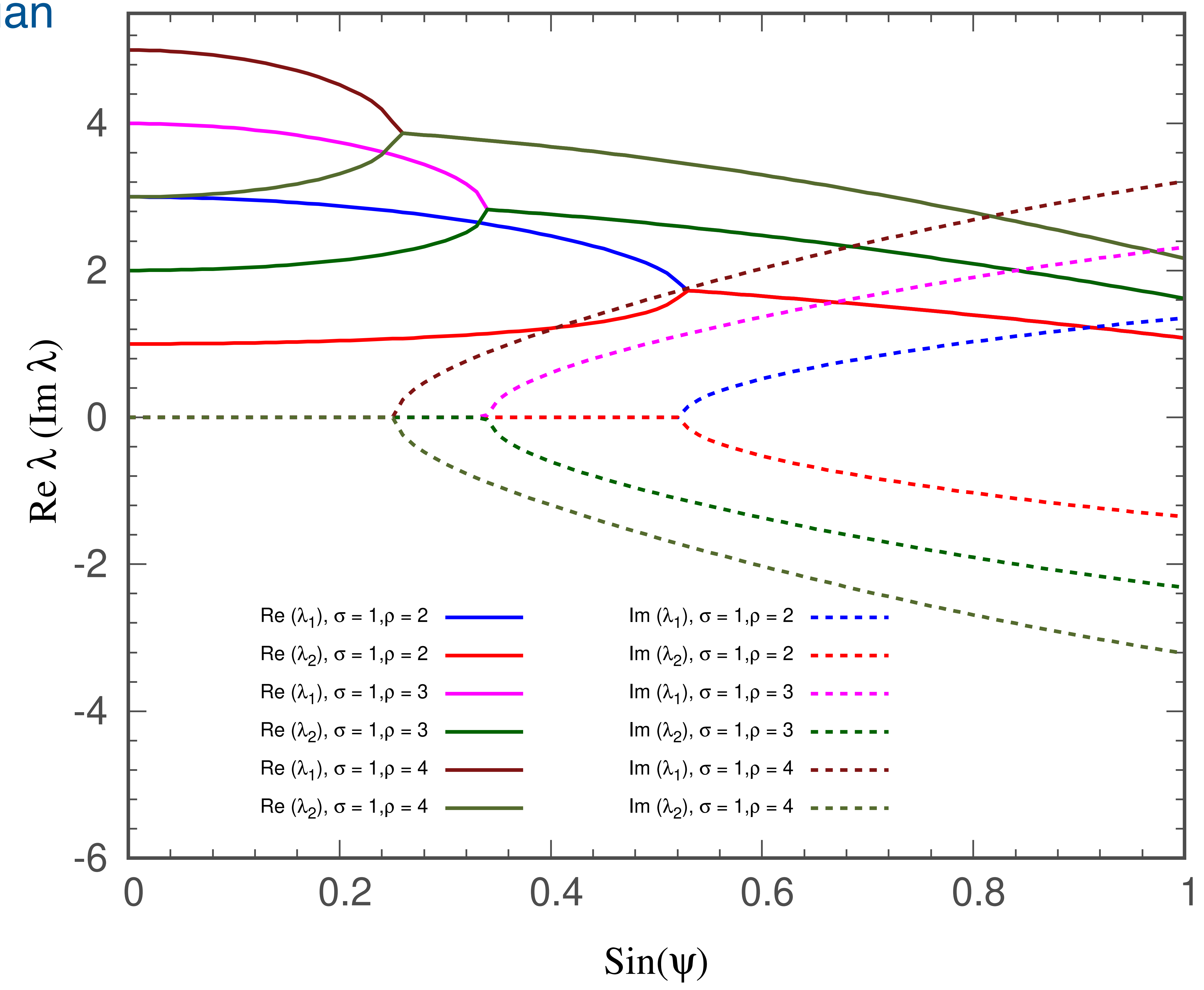
$$\lambda_\pm = \rho \cos\psi \pm \sqrt{\sigma^2 - \rho^2 \sin^2\psi}\ ,$$

$$\lambda_\pm = \rho\sqrt{1 - \sin^2\psi} \pm \sqrt{\sigma^2 - \rho^2 \sin^2\psi}.$$

- Eigenvectors

$$v_\pm = \frac{1}{\sqrt{2\cos\alpha}} \begin{pmatrix} e^{\pm i\alpha/2} \\ \pm e^{\mp i\alpha/2} \end{pmatrix}$$

$$\sin\alpha = \frac{\rho \sin\psi}{\sigma}.$$



Rushiya, Parveen and Mehta

Neutrino oscillations plus decay - two flavour case

C. M. Bender and S. Boettcher, Phys. Rev. Lett., vol. 80, pp. 5243–5246, 1998; T. Ohlsson and S. Zhou, J. Math. Phys., vol. 61, no. 5, p. 052104, 2020;
Dixit, Pradhan, Uma Sankar, Phys. Rev. D, 107, 013002, 2023.

- 2 x 2 mixing matrix

$$U = \begin{pmatrix} \cos\theta e^{i\omega_1} & \sin\theta e^{i(\omega_1+\phi)} \\ -\sin\theta e^{i(\omega_2-\phi)} & \cos\theta e^{i\omega_2} \end{pmatrix}$$

$$U = \begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ -\sin\theta & \cos\theta e^{i\phi} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

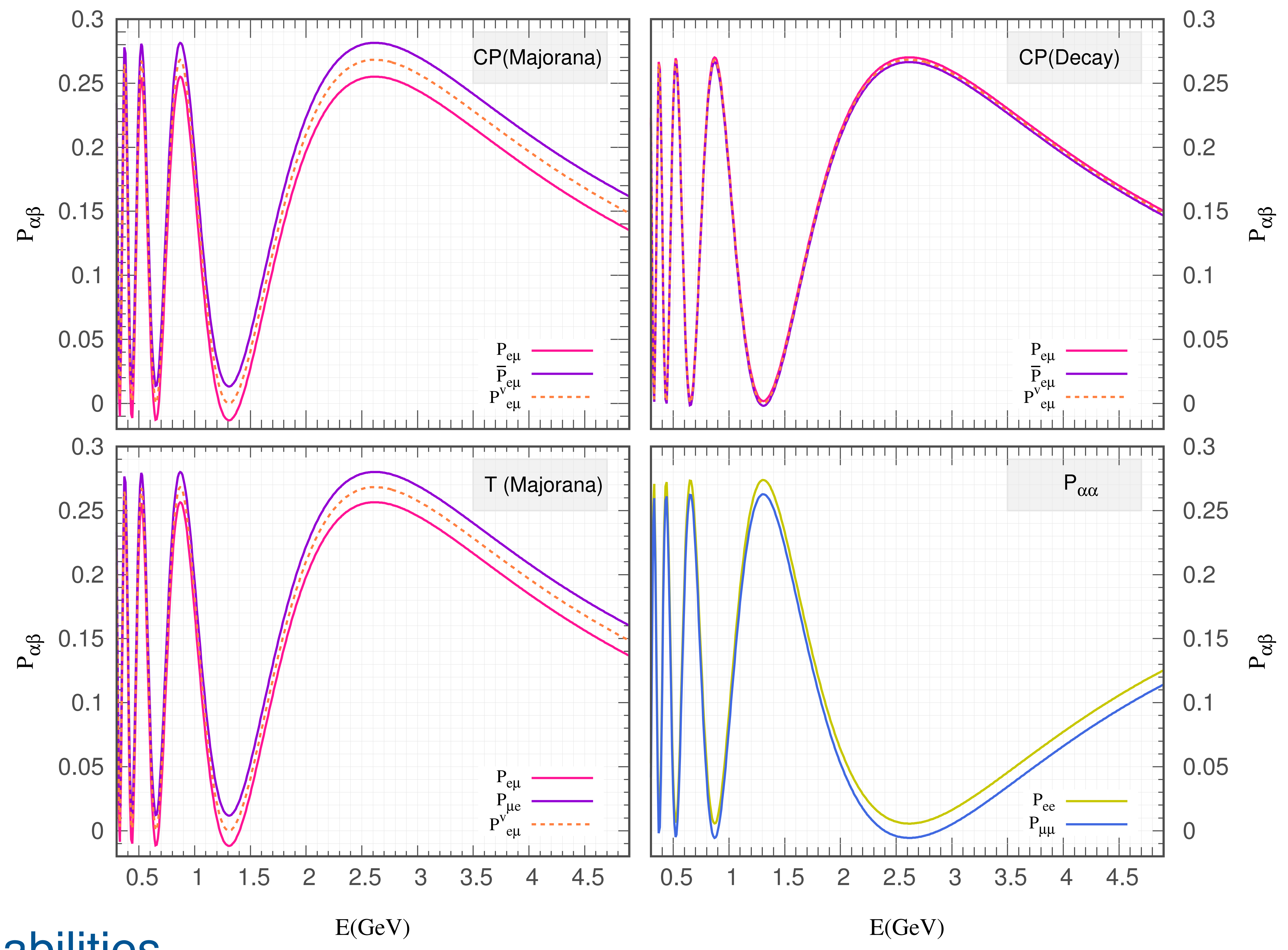
No Dirac phase but one Majorana phase

- Non-hermitian case :

$$\mathcal{H} = \mathcal{M} - i\Gamma/2,$$

$$\mathcal{M} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}, \quad \Gamma/2 = \begin{pmatrix} b_1 & \frac{1}{2}\eta e^{i\xi} \\ \frac{1}{2}\eta e^{-i\xi} & b_2 \end{pmatrix}$$

$$\mathcal{H} \;=\; \left[\frac{(a_1+a_2)}{2}\sigma_0 - \frac{(a_2-a_1)}{2}\sigma_z - \frac{i}{2}\left((b_1+b_2)\sigma_0 + \vec{\sigma}.\vec{\Gamma}\right)\right]$$

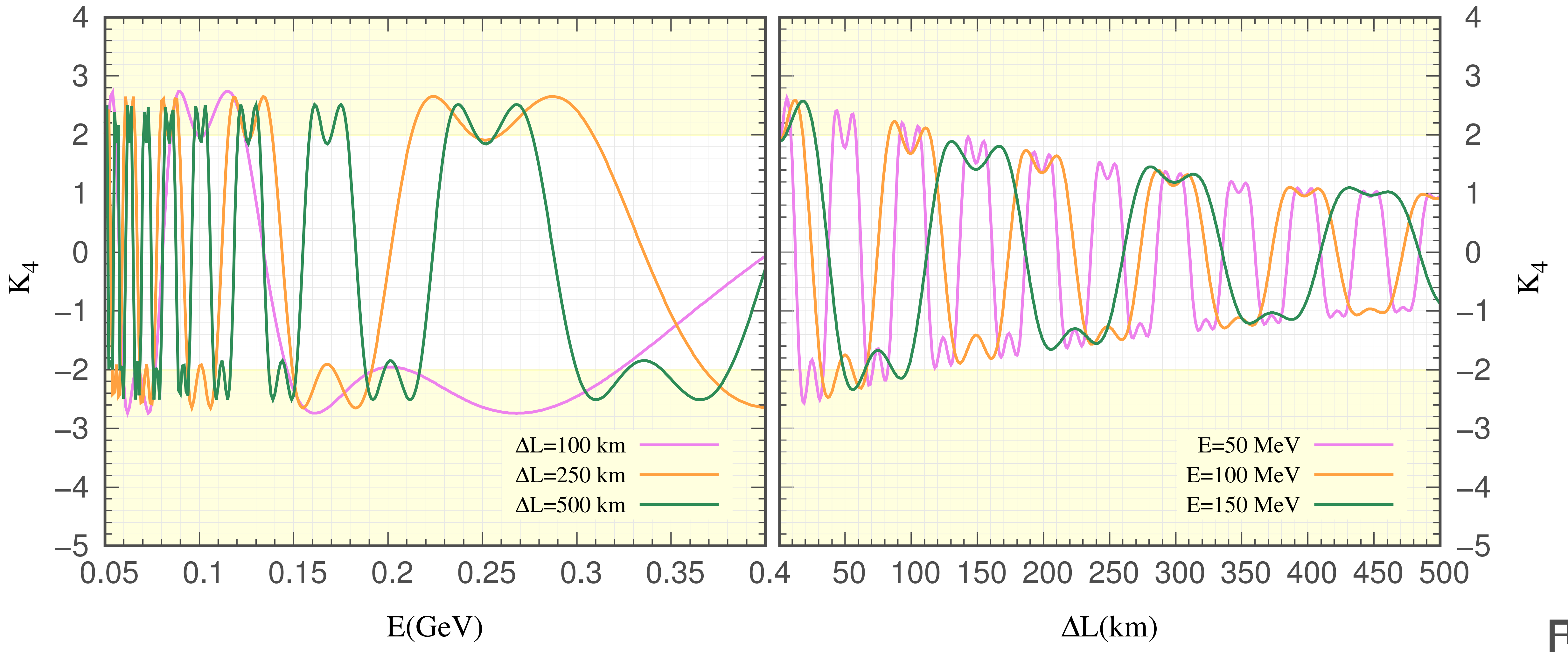


- Majorana phase appears at the level of oscillation probabilities

Soni, Shafaq and Mehta, 2307.04496

Dirac versus Majorana via LGI violation

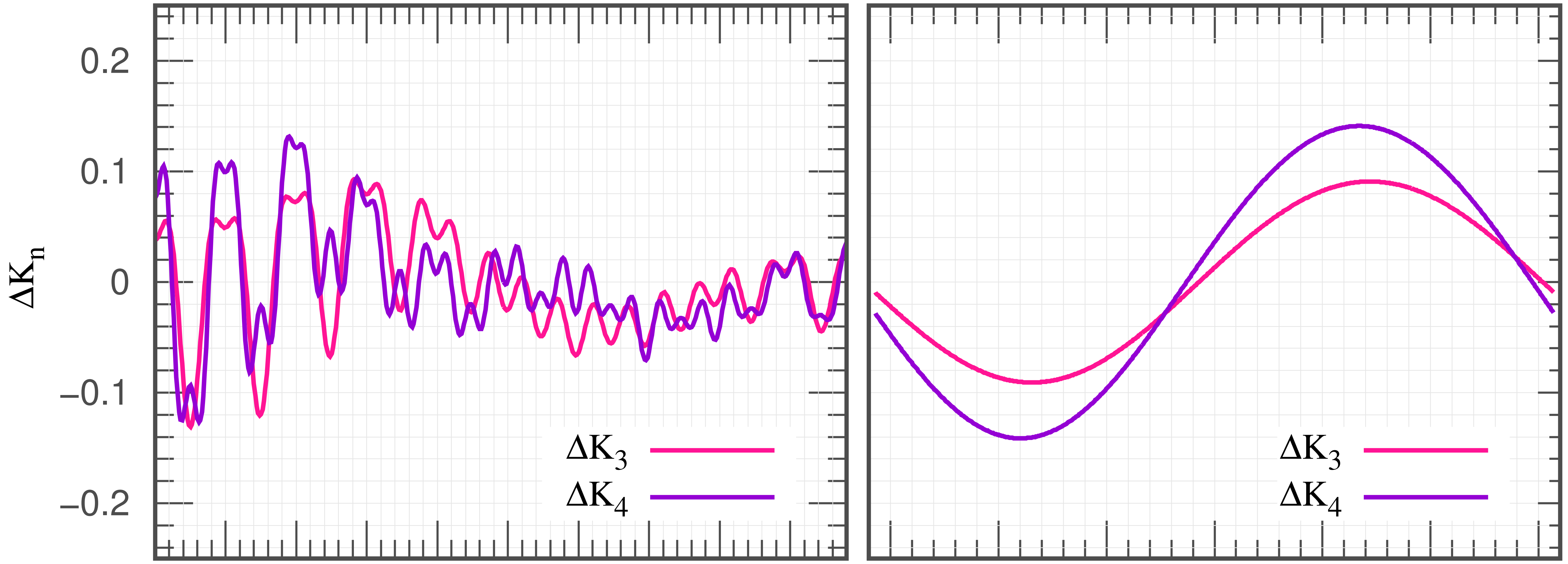
C. M. Bender and S. Boettcher, Phys. Rev. Lett., vol. 80, pp. 5243–5246, 1998; T. Ohlsson and S. Zhou, J. Math. Phys., vol. 61, no. 5, p. 052104, 2020;
Dixit, Pradhan, Uma Sankar, Phys. Rev. D, 107, 013002, 2023.



K3 and K4 are different from standard case

$$K_3 = 1 - 2\sin^2 2\theta \left[2\sin^2 \frac{\Delta m^2 \tau}{4E} - \sin^2 \frac{2\Delta m^2 \tau}{4E} \right]$$

$$K_4 = 2 - 2\sin^2 2\theta \left[3\sin^2 \frac{\Delta m^2 \tau}{4E} - \sin^2 \frac{3\Delta m^2 \tau}{4E} \right]$$



$$\Delta K_n = K_n^{\nu,M} - K_n^{\nu,D}$$

K4 allows for better (~15%) discrimination between Dirac and Majorana.

Soni, Shafaq and Mehta, 2307.04496

Precision tests at neutrino experiments

Neutrino sources

Credit : Sabila Parveen, adapted from 1903.04333 [astro-ph.HE], 1911.05088, PRD 2020

2016

LGI test at MINOS

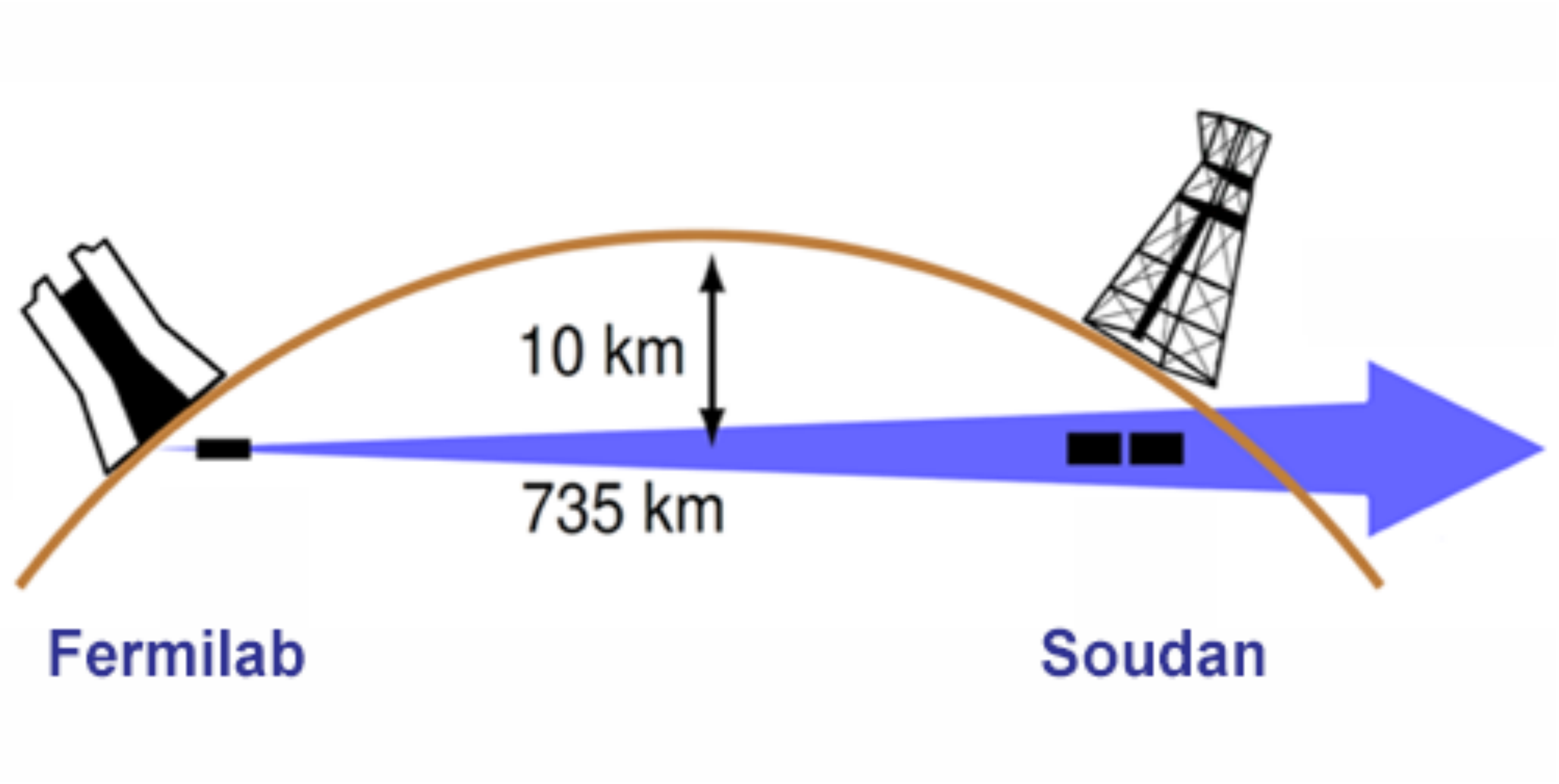
J. A. Formaggio, D. I. Kaiser, M. M. Murskyj, and T. E. Weiss, Phys. Rev. Lett. 117, 050402 (2016) ; M. Schirber, Physics 9, s81 (2016)

PRL **117,** 050402 (2016) PHYSICAL REVIEW LETTERS week ending 29 JULY 2016

Violation of the Leggett-Garg Inequality in Neutrino Oscillations

J. A. Formaggio,* D. I. Kaiser, M. M. Murskyj, and T. E. Weiss
Massachusetts Institute of Technology, Cambridge, Massachusetts, USA
(Received 8 February 2016; published 26 July 2016)

The Leggett-Garg inequality, an analogue of Bell's inequality involving correlations of measurements on a system at different times, stands as one of the hallmark tests of quantum mechanics against classical predictions. The phenomenon of neutrino oscillations should adhere to quantum-mechanical predictions and provide an observable violation of the Leggett-Garg inequality. We demonstrate how oscillation phenomena can be used to test for violations of the classical bound by performing measurements on an ensemble of neutrinos at distinct energies, as opposed to a single neutrino at distinct times. A study of the MINOS experiment's data shows a greater than 6σ violation over a distance of 735 km, representing the longest distance over which either the Leggett-Garg inequality or Bell's inequality has been tested.



- MINOS measures the survival probabilities of oscillating muon neutrinos produced in the NuMI accelerator complex.
- The accelerator provides a source of neutrinos with a fixed baseline and an energy spectrum that peaks at a point corresponding to $\delta L/E\nu \sim 250$ km/GeV, close to the region where the survival probability $P_{\mu\mu}$ reaches its first minimum.
- This experimental design provides an ideal phase space to test for LGI violations.

Difficulty in performing LGI measurements

- One needs a minimum of three time measurements (for K3).
- This means that one requires at least three baselines with identical detection possibilities to infer the simplest of LGI parameters, K3.
- However, it is practically impossible to realize the three baseline measurement experimentally.
- The authors used the fact that in the phase factor one has two experimental handles - one is the L and other one is the E which can be independently tuned. One can mimic the change in L by a corresponding change in E.
- This is how the collaboration performed a test of LGI using data from MINOS experiment with L = 735 km, by selecting various energies Ea for measurements such that the phases obeyed a certain sum rule.

2016

LGI test at MINOS

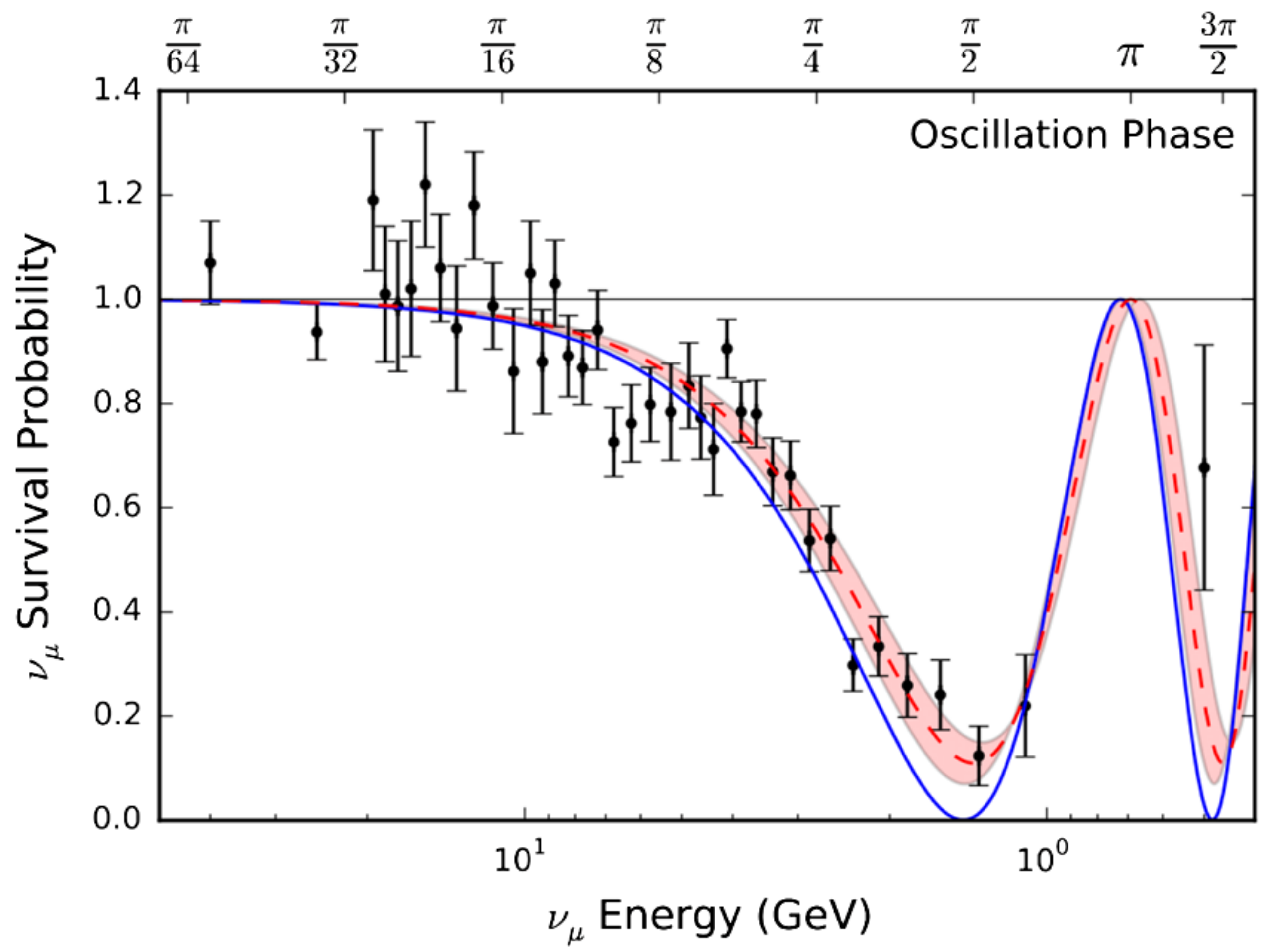
J. A. Formaggio, D. I. Kaiser, M. M. Murskyj, and T. E. Weiss, Phys. Rev. Lett. 117, 050402 (2016) ; M. Schirber, Physics 9, s81 (2016)

$$\psi_{a;ij} \simeq \frac{\omega_a}{2}(t_j - t_i) = \frac{1}{4E_a}(m_2^2 - m_1^2)(t_j - t_i).$$

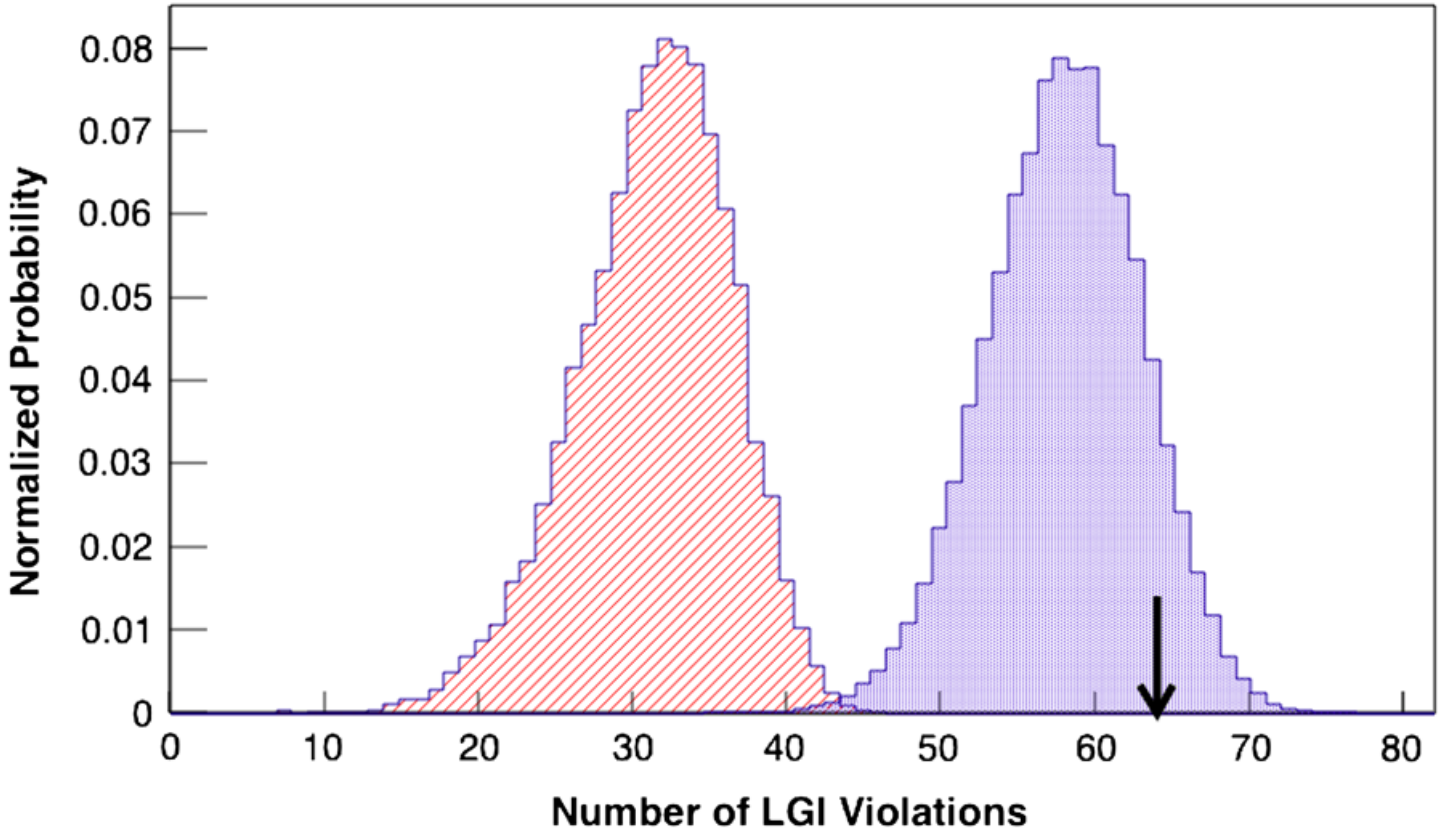
$$\sum_{i=1}^{n-1} \psi_{a;i,i+1} = \psi_{a;1n}$$

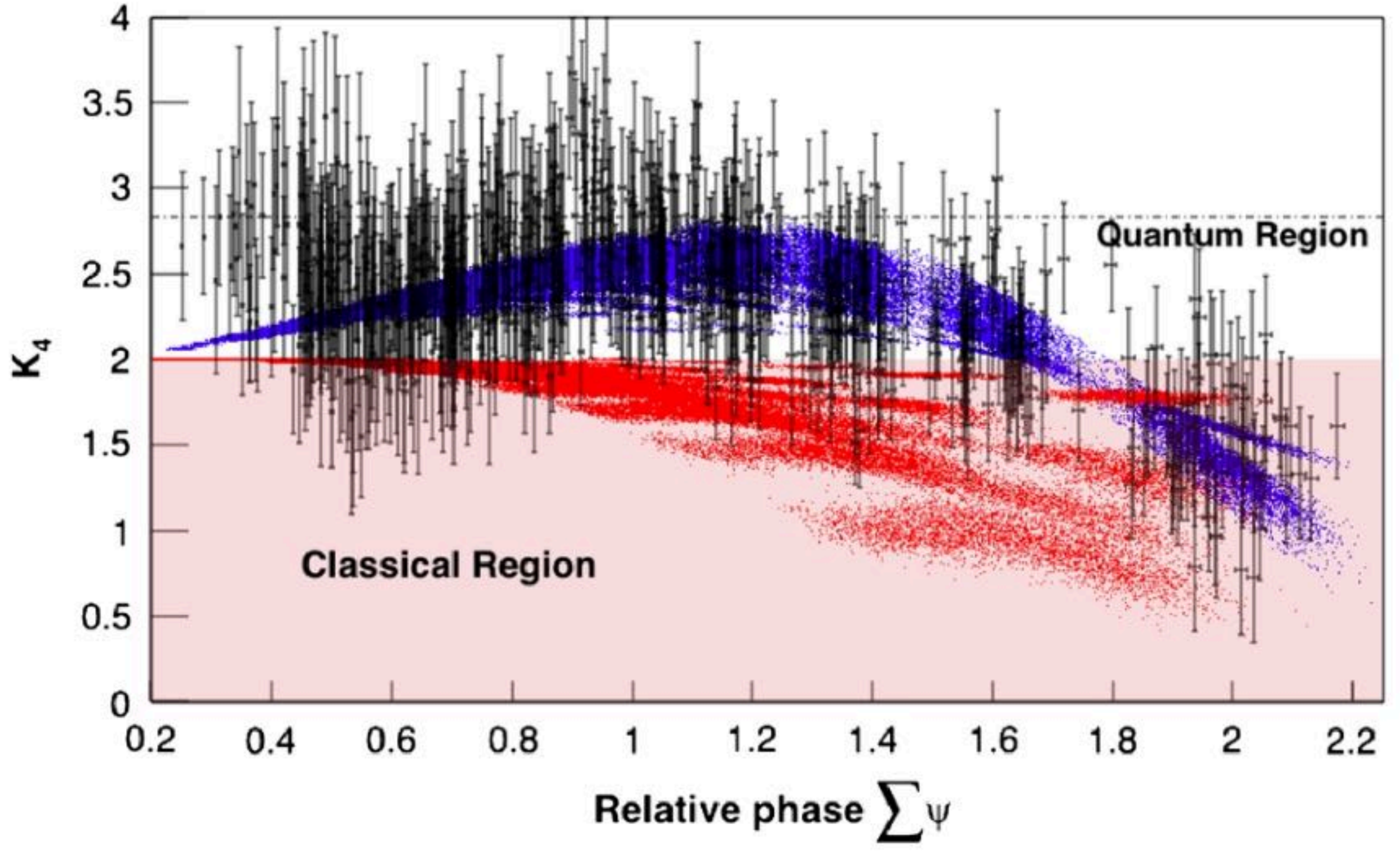
$$\mathcal{C}_{ij}(\omega_a) = 1 - 2\sin^2 2\theta \sin^2 \psi_{a;ij}.$$

$$K_n^Q = (2-n) + 2\sum_{a=1}^{n-1} P_{\mu\mu}(\psi_a) - 2P_{\mu\mu}\left(\sum_{a=1}^{n-1}\psi_a\right)$$



- This violation occurs over a distance of 735 km, providing the longest range over which a Bell-like test of quantum mechanics has been carried out to date.





- The number of K3 values that violate the LGI bound. The red curve indicates the expected classical distribution, while the indigo curve indicates the quantum expectation. The arrow indicates the observed number of violations.
- The observed number of LGI violations (64 out of 82) represents a 6.2σ deviation from the number of violations one would expect to arise from an underlying classical distribution.

- A total of 577 (out of 715) violations of the LGI were observed for K4.
- Clear discrepancy between the observed number of violations and the classical prediction. The K4 data are inconsistent with the realistic prediction at confidence 7σ.

J. A. Formaggio, D. I. Kaiser, M. M. Murskyj, and T. E. Weiss, Phys. Rev. Lett. 117, 050402 (2016)

2017

LGI test at Daya Bay

Fu and Chen, Eur. Phys. Jour. C 77, 775 (2017)

Eur. Phys. J. C (2017) 77:775
https://doi.org/10.1140/epjc/s10052-017-5371-y

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Testing violation of the Leggett–Garg-type inequality in neutrino oscillations of the Daya Bay experiment

Qiang Fu[1,2,3,a], **Xurong Chen**[1,b]

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[2] Lanzhou University, Lanzhou 730000, China
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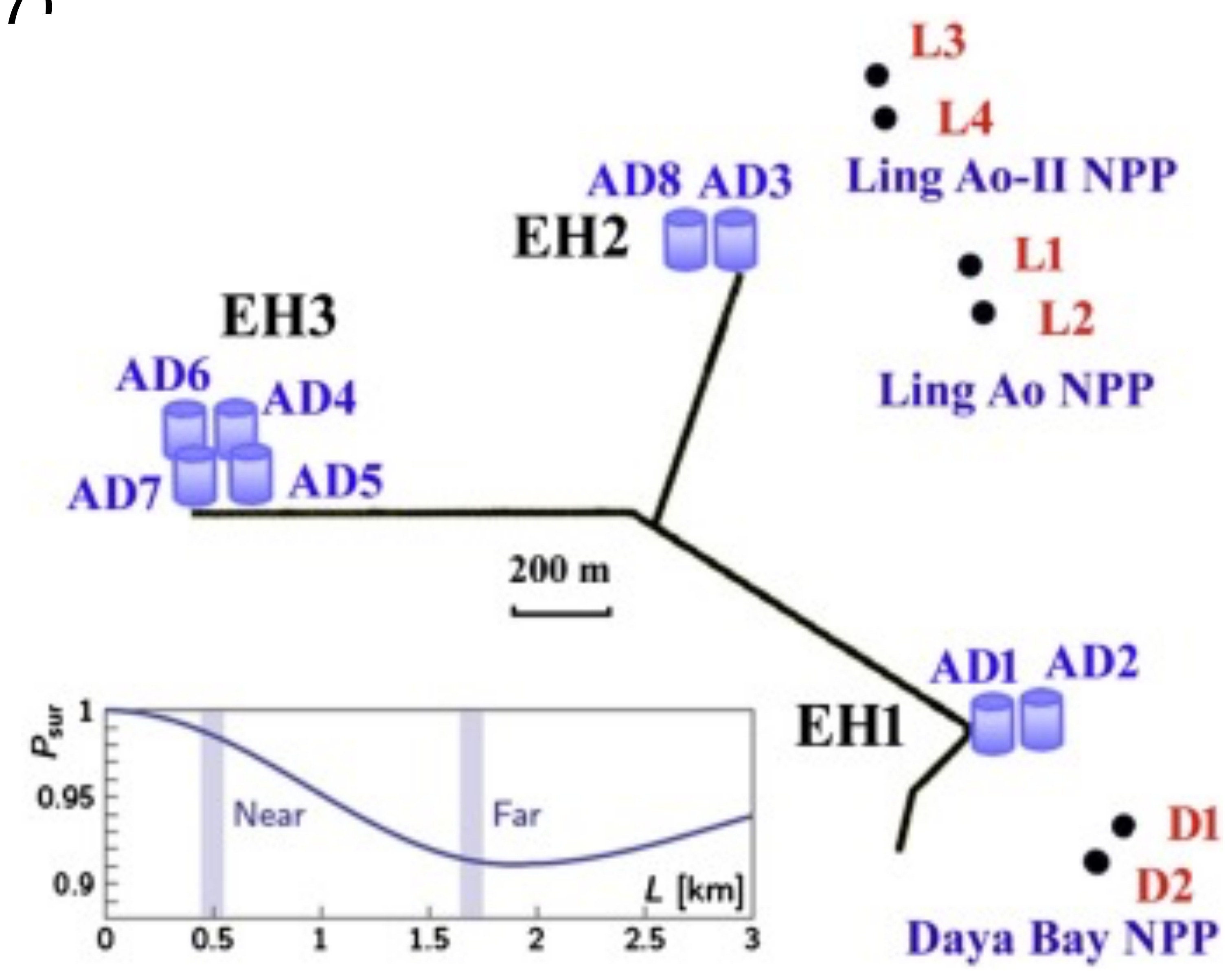


Cao and Luk, Nucl. Phys. B 908, 62 (2016)

- Daya Bay measures the survival probabilities of oscillating electron antineutrinos produced by nuclear power plants (NPP).
- The Daya Bay experiment consists of three underground experimental halls (EHs) connected with horizontal tunnels.
- Eight antineutrino detectors (ADs) are installed in the three halls, with two in EH1, two in EH2, and four in EH3. Each AD has 20-ton target mass to catch the reactor antineutrinos.

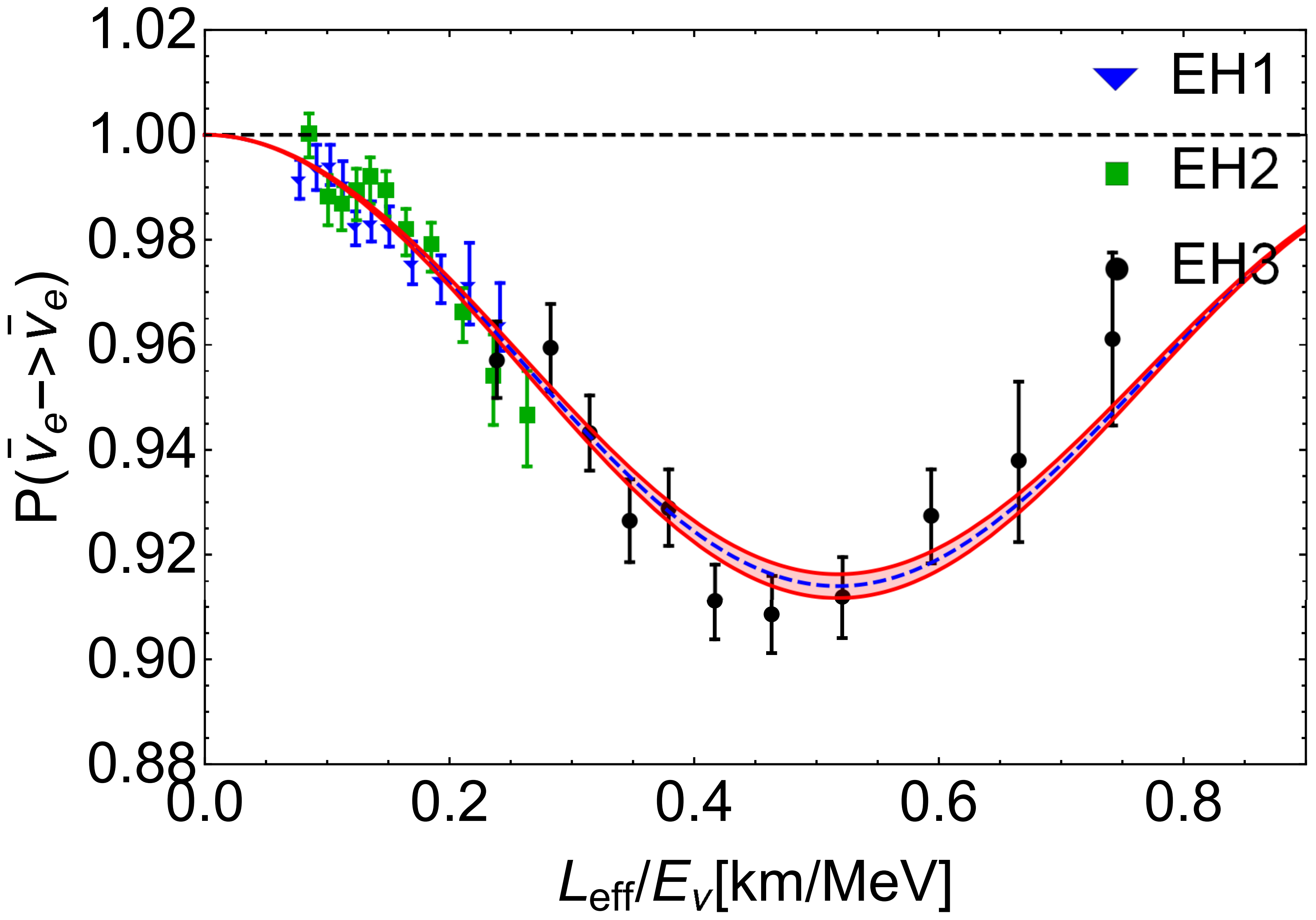
2017

LGI test at Daya Bay

Fu and Chen, Eur. Phys. Jour. C 77, 775 (2017)

$$C_{12} = 1 - \left[\sin 2\theta_{13} \sin\left(\frac{1.267\Delta m_{ee}^2}{E} c(t_2 - t_1)\right)\right]^2$$
$$= 2P_{\overline{\nu}_e\to\overline{\nu}_e}(t_2 - t_1) - 1.$$

$$K_n^Q = -2 + 2\sum_{a=1}^{n-1} P_{ee}(\psi_a) - 2P_{ee}\left(\sum_{a=1}^{n-1}\psi_a\right)$$

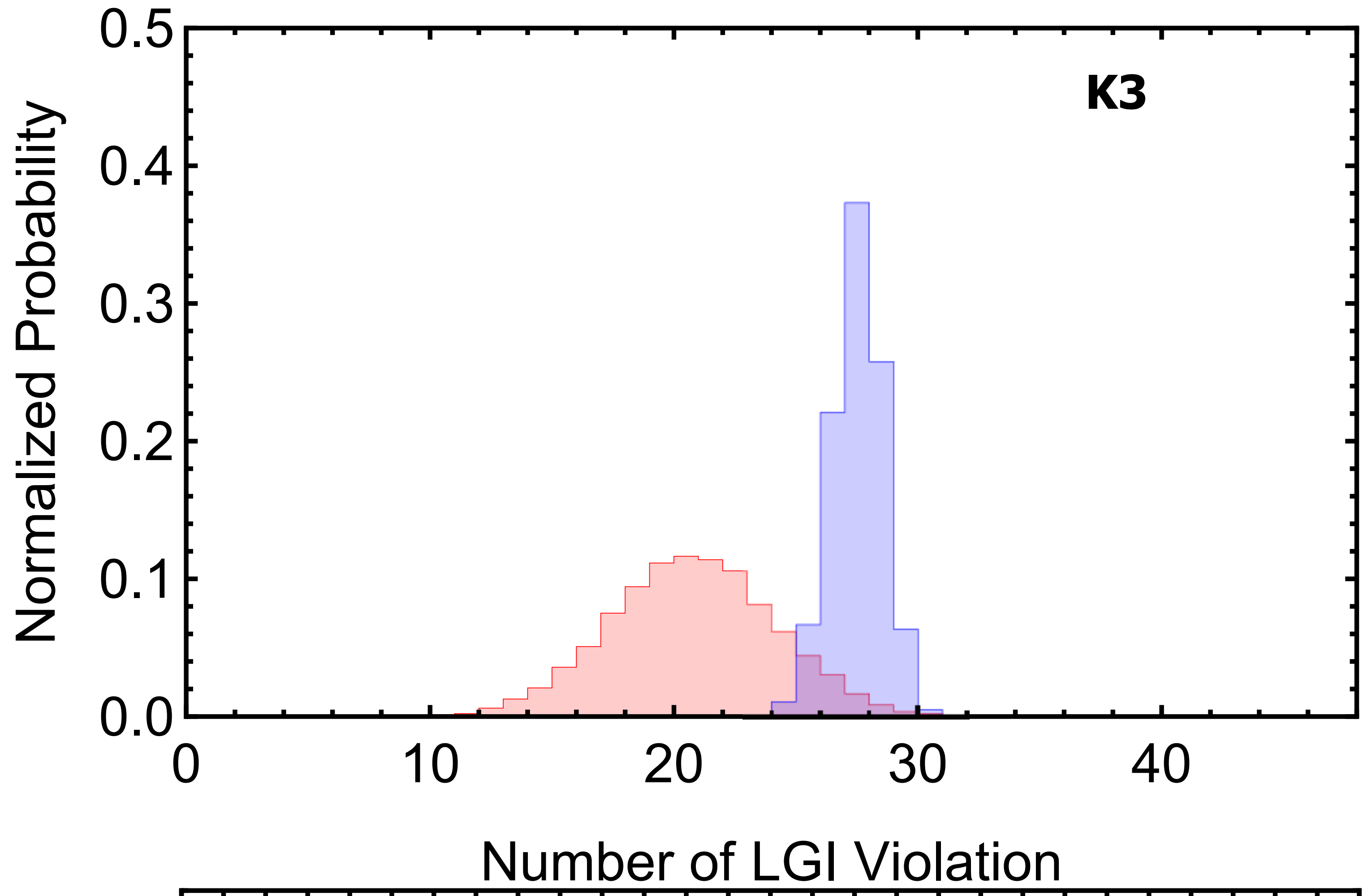


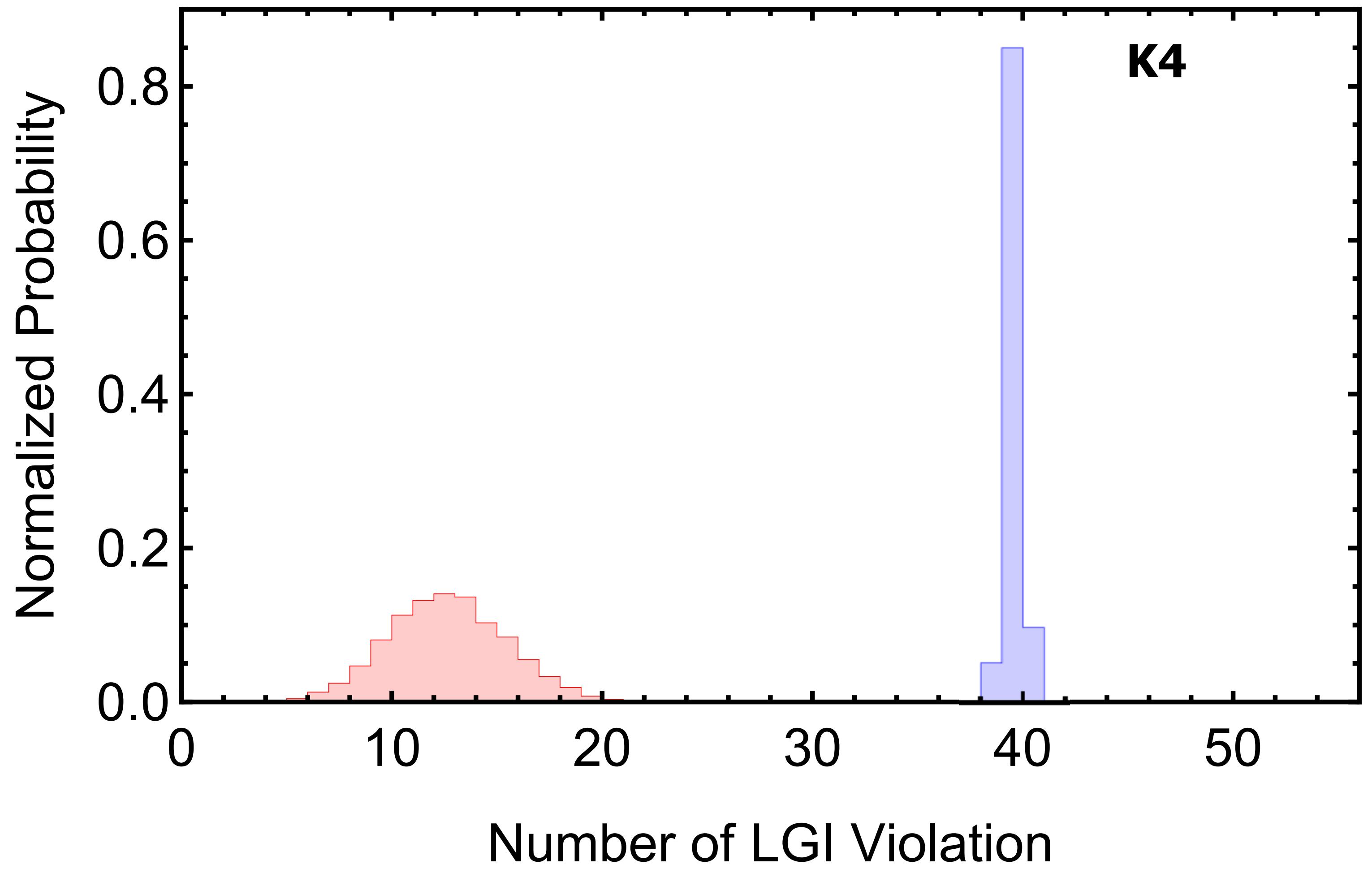
- The Daya Bay experiment covers an energy between 1 and 8 MeV.
- The ranges of effective baseline and energy correspond to a phase range of (0, 3/4π), within which the violations of LGI will be observed near the minimum point of the anti-neutrino survival probability.

2017

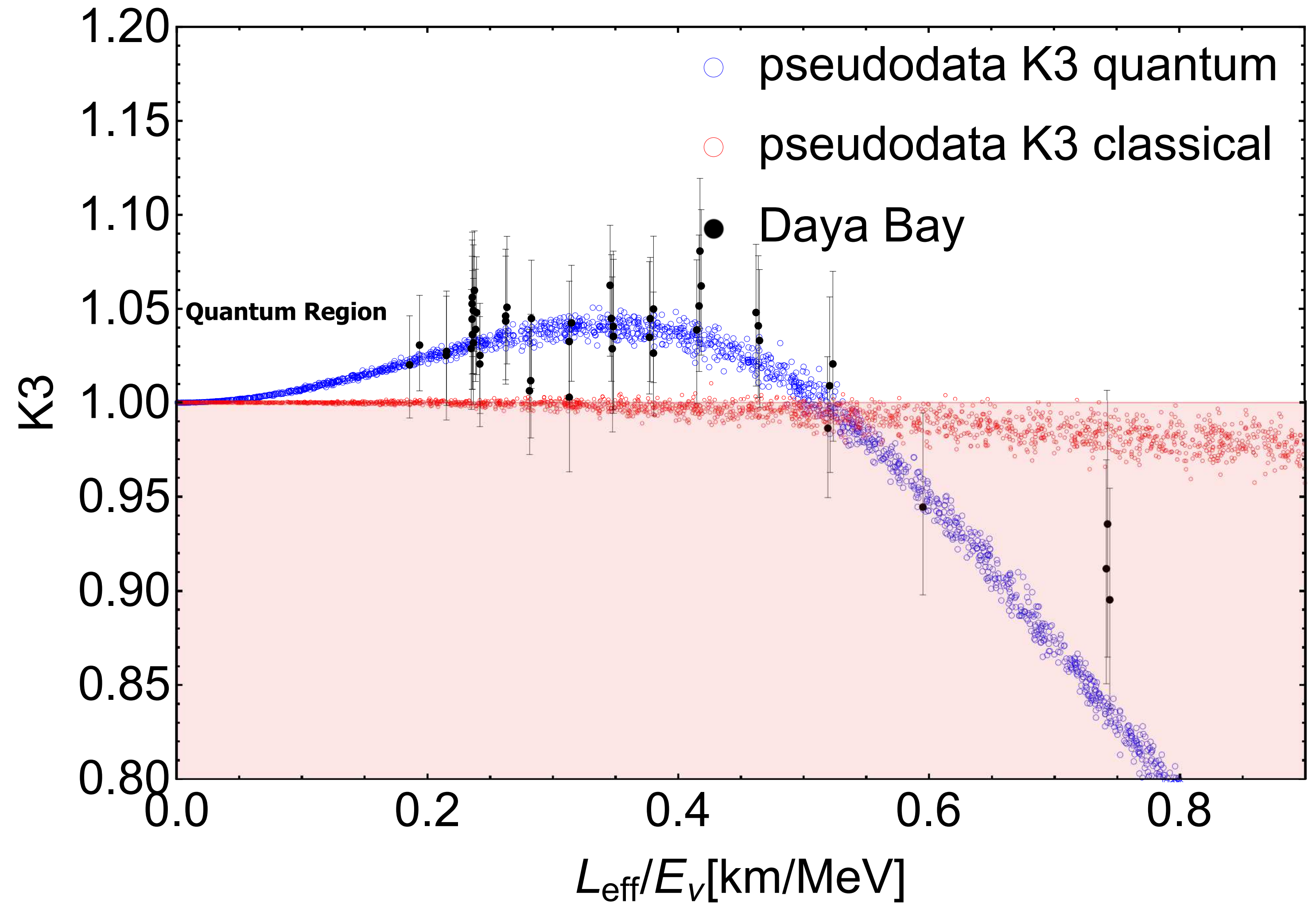
LGI test at Daya Bay

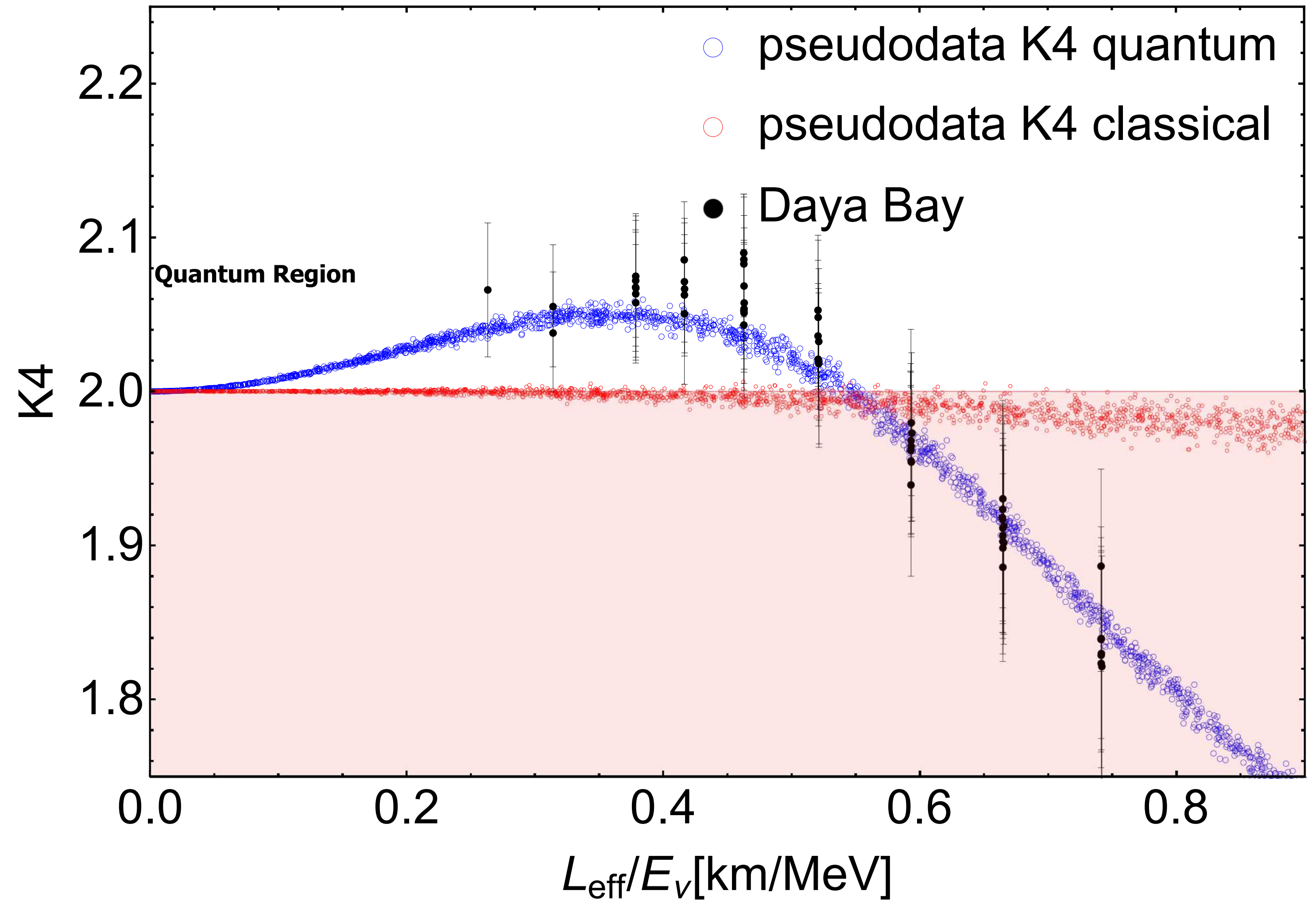
Fu and Chen, Eur. Phys. Jour. C 77, 775 (2017)





- For the actual number of LGI violations (41 in 48 data points), there exists a 6.1σ deviation from the expected distribution of the classical prediction.
- K4 data also possesses 6σ deviation from the classical prediction.





Recent works and some ideas…

Quantum mismatch

PHYSICAL REVIEW D **108,** 112013 (2023)

Quantum mismatch: A powerful measure of quantumness in neutrino oscillations

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The quantum nature of neutrino oscillations would be reflected in the mismatch between the neutrino survival probabilities with and without an intermediate observation. We propose this quantum mismatch as a measure of quantumness in neutrino oscillations. For two neutrino flavors, it inevitably performs better than the Leggett-Garg measure. For three flavors, we devise modified definitions of these two measures, which would be applicable for experiments that measure neutrino survival probabilities with negligible matter effects. The modified definitions can be used to probe deviations from expected classical behavior, even for systems with an unknown number of states. For neutrino experiments like DUNE, MINOS, and JUNO, we identify the energies where these modified measures can probe quantumness efficiently.

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No signaling in time

No-signaling-in-time as a condition for macrorealism: the case of neutrino oscillations

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We consider two necessary and sufficient conditions for macrorealism recently appeared in the literature, known as no-signaling-in-time and arrow-of-time conditions, respectively, and study them in the context of neutrino flavor transitions, within both the plane wave description and the wave packet approach. We then compare the outcome of the above investigation with the implication of various formulations of Leggett–Garg inequalities. In particular, we show that the fulfillment of the addressed conditions for macrorealism in neutrino oscillations implies the fulfillment of Leggett–Garg inequalities, whereas the converse is not true. Finally, in the framework of wave packet approach, we also prove that, for distances longer than the coherence length, the no-signaling-in-time condition is always violated whilst Leggett–Garg inequalities are not.

2211.16931v2 [hep-th]

Eur. Phys. J. C (2022) 82:799
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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Geuine tripartite entanglement in three-flavor neutrino oscillations

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Abstract The violation of Leggett–Garg inequalities tested the quantumness of neutrino oscillations (NOs) across macroscopic distances. The quantumness can be quantified by using the tools of the quantum resource theories. Recently, a new genuine tripartite entanglement measure (Xie et al. in Phys Rev Lett 127:040403, 2021), concurrence fill, is defined as the square root of the area of the concurrence triangle satisfying all genuine multipartite entanglement conditions. It has several advantages compared to other existing tripartite measures. Here, we focus on using concurrence fill to quantify the tripartite entanglement in three flavor NOs. Concurrence fill

ing, three different flavors of neutrino are electron e, muon μ, and tau τ leptons, in which the three flavor states are unitary linear combinations of three mass eigenstates [2,3]. NO shows that a given flavor may change into another flavor in the neutrino propagation. The probability of measuring a particular flavor for a neutrino varies periodically as it propagates through space, and can be measured at the arbitrary time. The values of the oscillations parameters have been measured and analyzed in both theory and experiment in recent years [4–8]. Remarkably, oscillation probabilities of neutrino can be used to study the different properties from classical to quan-

LGI in reactor & accelerator experiments

Evaluation of the Leggett-Garg inequality by means of the neutrino oscillations observed in reactor and accelerator experiments

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Abstract

We revisit the study of the violation of the Leggett-Garg inequality in neutrino oscillation data as a mean to test some of the fundamental aspects of quantum mechanics. In particular, we consider the results by the Daya Bay and RENO reactor experiments, and the MINOS and NOvA accelerator experiments. We

2401.00240v1 [hep-ph]

CPV from entanglement

- Minimization of concurrence (a measure of entanglement) leads to a prediction for the value of δ_{CP}
- Conjecture: minimum entanglement leads to PMNS parameters
- CP conservation favoured

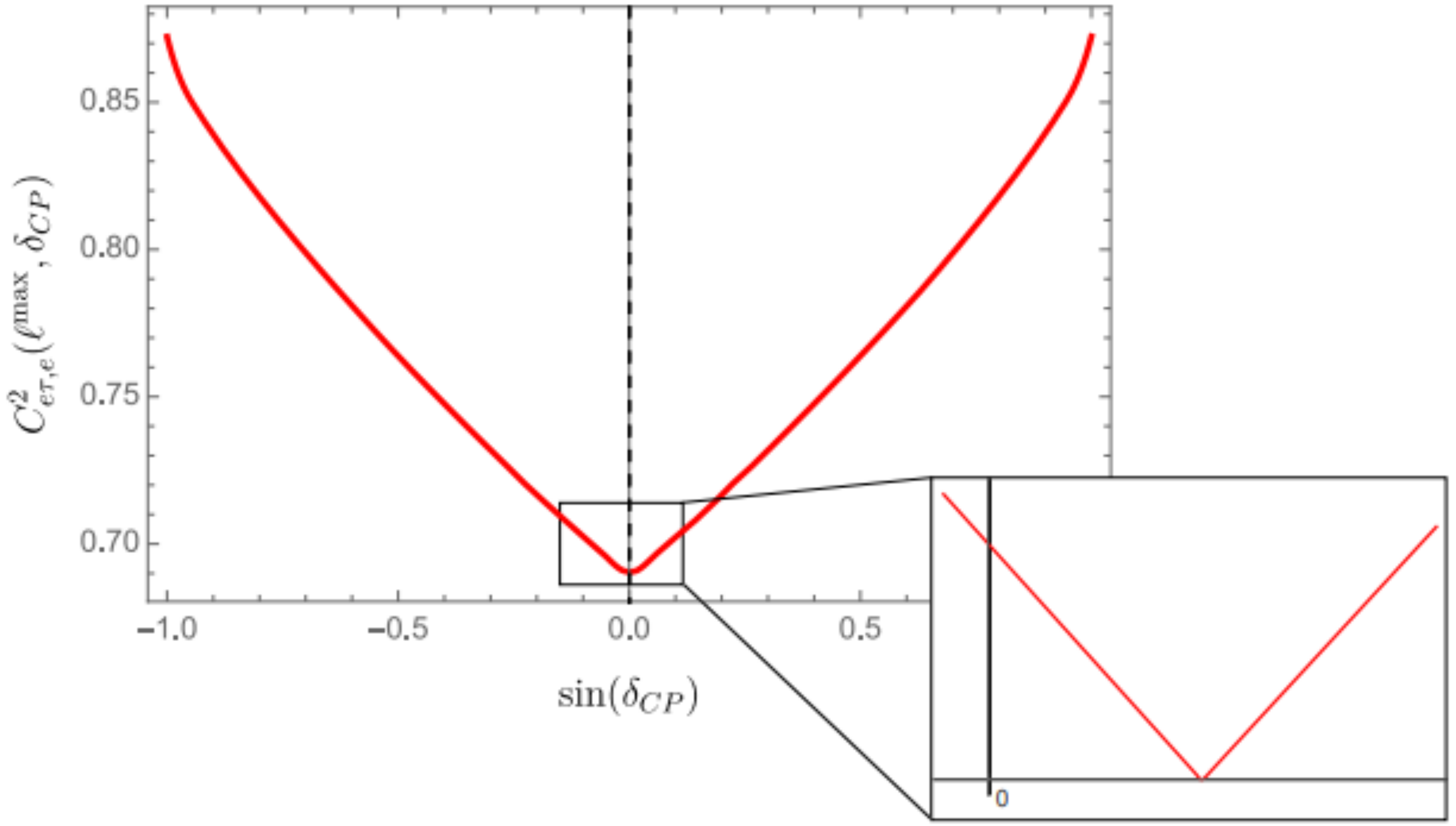

Figure 1. Numerical solution of equations (9) and (10) with respect to $\sin(\delta_{CP})$. The global minimum is unique and approximately equal to $\sin(\delta_{CP}) \approx 0.000474$. All free parameters apart from $\sin(\delta_{CP})$ are fixed according to the most recent experimental data from the Particle Data Book [19], using 1-sigma errors.

2207.03303: Quinta, Sousa, Omar

Performance in comparison with other measures

Measure	Smoothness	Discriminance	Extendable, $d > 3$
Generalised Geometric Measure (GGM)[1]	$\times$	$\times$	✓
Genuine Multipartite Concurrence (GMC)[2]	$\times$	$\times$	✓
Concurrence Fill (F)[3]	✓	✓	✓*
Geometric mean of Bipartite Concurrence (GBC)[4]	✓	✓	$\times$
Geometric mean of Bipartite Riemannian entanglement measures (GBR)	✓	✓	✓

1. Sen-De and Sen, Phys. Rev. A (81) 1, 2010
2. Eberly et. al, Phys. Rev. A (86) 6, 2012
3. Eberly and Xie, Phys. Rev. Lett. (127) 4, 2021
4. Shang and Li, Phys. Rev. Research (4) 2, 2022

Summary

- Foundations of quantum mechanics is an active area of research, widely studied in the optics context and electronic context.
- Neutrino oscillations provide an ideal platform to look for such violations at macroscopic distances that might not accessible be in other contexts.
- Foundational aspects and tests may allow for indirect tests for new physics scenarios such as non-standard neutrino interactions or effects that could cause damping effects.
- High energy physics experiments specifically neutrino oscillations have reached the level of precision to allow for stringent tests of temporal inequalities of the LGI type.
- LGI can allow for probing the nature of neutrinos if we consider non-hermitian scenario (neutrino oscillation plus decay)
- So far, the experimental efforts have used simple two flavour case. It would be interesting to use the data with three flavours to see the impact on violation of LGI.

Thank you