Primordial Black Holes from a ‘tiny Bump/Dip’ in the Inflaton potential

LTPDM - 2020 @ICTS

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Inter-University Centre for Astronomy and Astrophysics(IUCAA), Pune

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In collaboration with **Prof. Varun Sahni**, IUCAA, Pune.

13 November, 2020

Prologue

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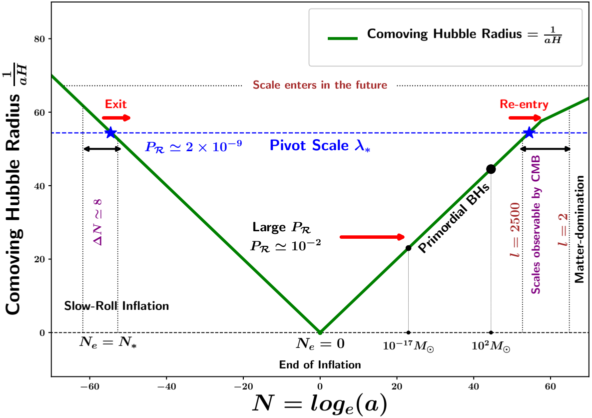
- PBH formation takes place in the **radiation dominated epoch**.
- Standard 'short' reheating history.

Details of the Mechanism



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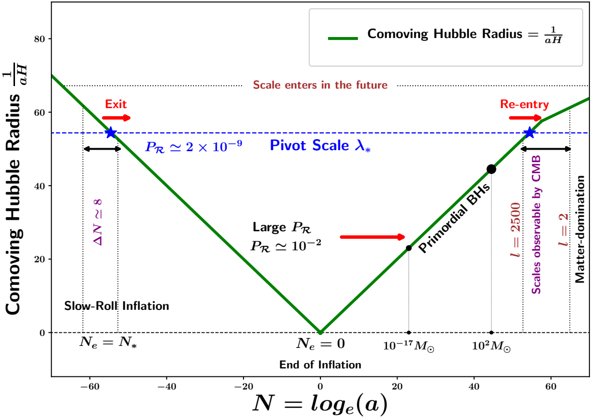

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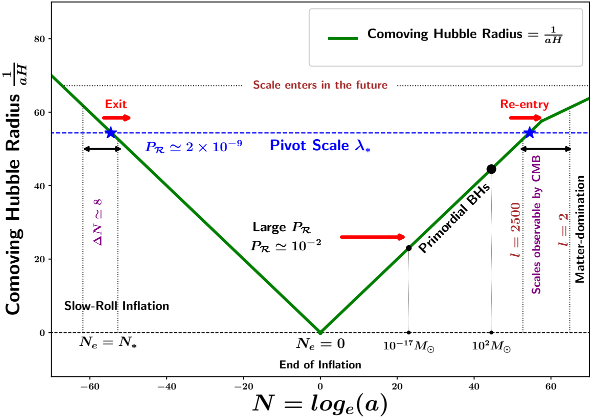

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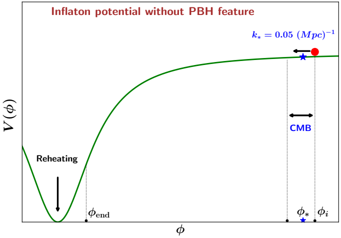
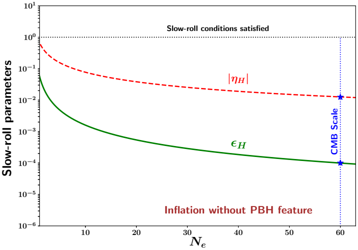


Slow-roll conditions $\epsilon_H, \eta_H \ll 1$



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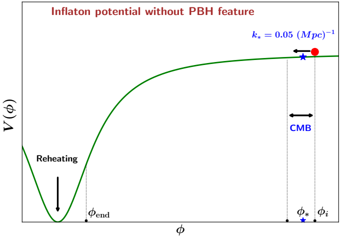

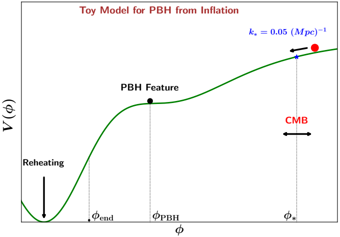

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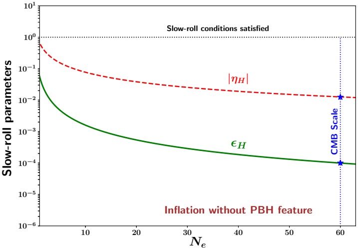

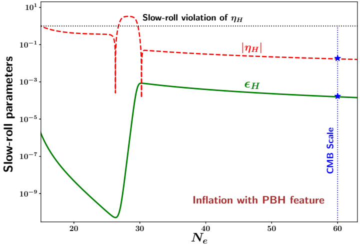

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Equation of motion of each Fourier mode

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with $\epsilon_1 = \epsilon_H$ and $\epsilon_{n+1} = -\frac{d\ln\epsilon_n}{dN_e}$.

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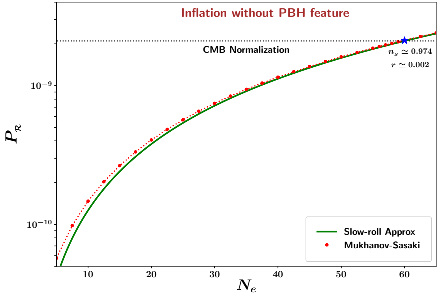
$$P_{\mathcal{R}} = \frac{1}{8\pi^2}\left(\frac{H}{m_p}\right)^2\frac{1}{\epsilon_H}\Big|_{k=aH} = \left(\frac{H}{2\pi}\right)^2\left(\frac{H}{m_p}\right)^2\frac{m_p^2}{\dot{\phi}^2}$$

Power spectrum : Slow-roll Inflation vs PBH formation



Slow-roll approx. holds.

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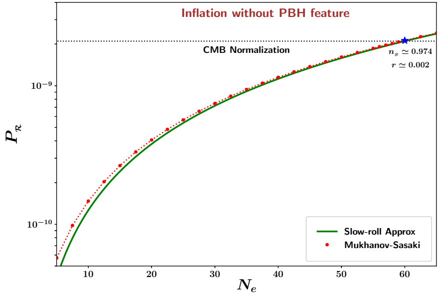
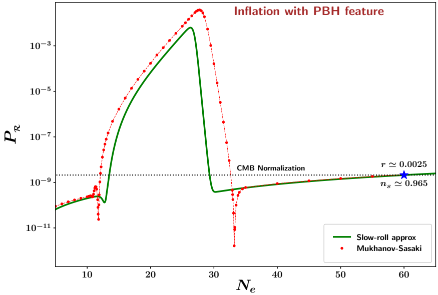
Motohashi, Hu 2017





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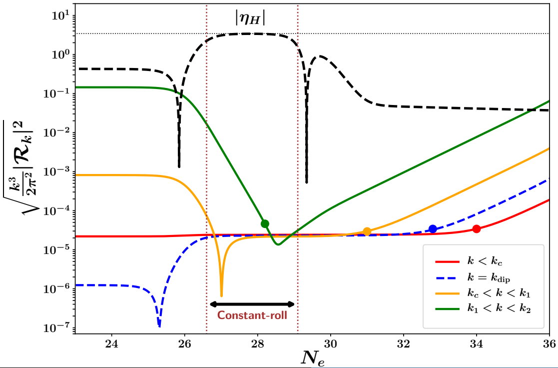
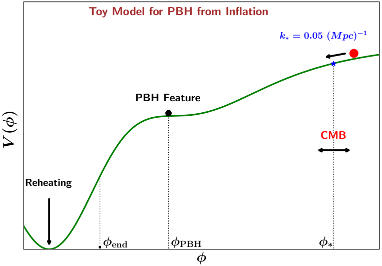


PBH feature: ‘Near inflection point’ vs ‘tiny local Bump‘



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- Not fully possible to separate PBH scale and CMB scale.
- Higher mass PBH formation affects CMB scale Physics.

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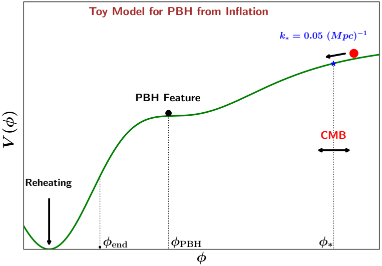

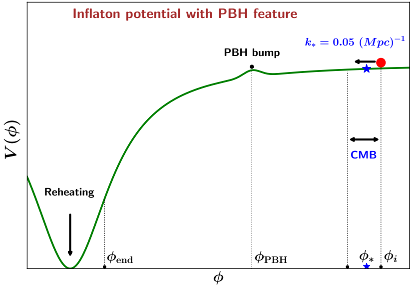

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- PBH feature is a tiny local correction to the base potential
- PBH scale and CMB scales are easily separated
- Higher mass PBHs are generated with equal ease as the lighter ones.

Proposed Model: PBH from a tiny Bump in the potential

$$V(\phi) = V_b(\phi)\,[1 + \varepsilon(\phi)]$$

V_b: **Base inflationary potential** (responsible for generating quantum fluctuations compatible with the CMB constraints on n_s, r.)

$\varepsilon(\phi) \ll 1$: **Tiny local bump** in the potential at ϕ_0 having height A, width σ,

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Bumps:

$$\varepsilon(\phi) = A \exp\left(-\frac{1}{2}\frac{(\phi-\phi_0)^2}{\sigma^2}\right)\ ,$$

$$\varepsilon(\phi) = A \cosh^{-2}\left(\frac{\phi-\phi_0}{\sigma}\right)$$

PBHs from a tiny Bump in the KKLT potential

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PBHs from a tiny Bump in the KKLT potential

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Parameters (Keeping $n_s = 0.9653$, $r = 0.0025$)

$M_{\rm PBH}$	A	σ (in m_p)	ϕ_0 (in m_p)
$6 \times 10^{-17}\ M_\odot$	1.876×10^{-3}	1.993×10^{-2}	2.005
$1.04 \times 10^{-13}\ M_\odot$	1.17×10^{-3}	1.59×10^{-2}	2.188
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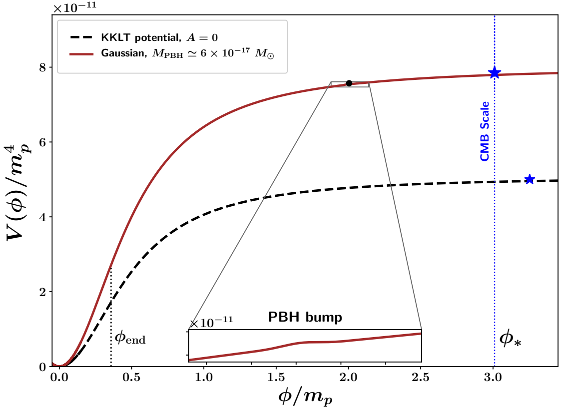

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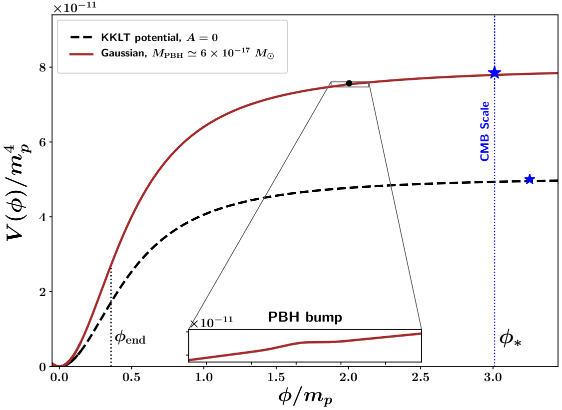

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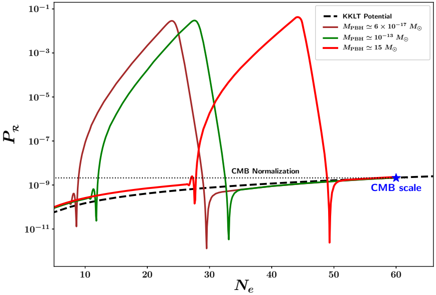
Due to presence of the bump, the CMB scale shifts towards the center.

PBHs from KKLT potential: Power spectrum and Abundance



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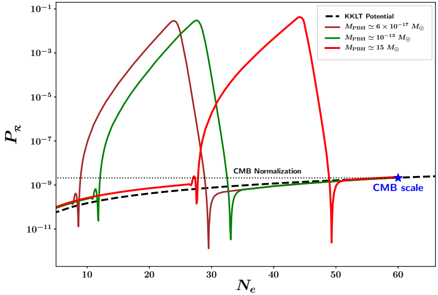

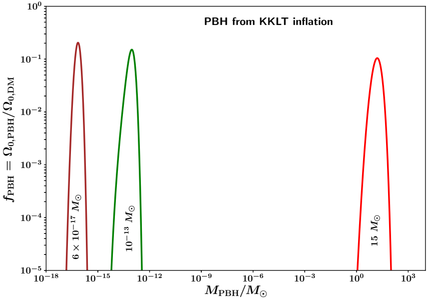

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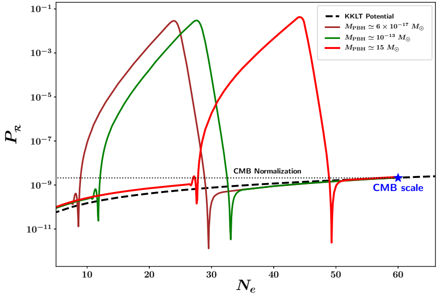

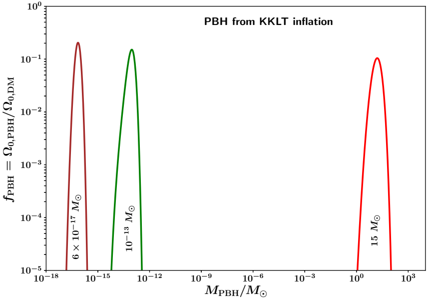

Assumptions: Gaussian fluctuations, Press-Schechter Theory, Threshold $\delta_{\rm th} = 0.414$, Gaussian Window function for smoothing.

PBHs from a tiny Bump in the α-attractor potential

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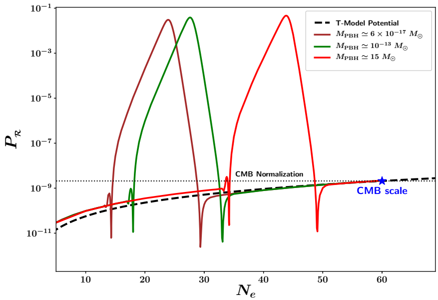

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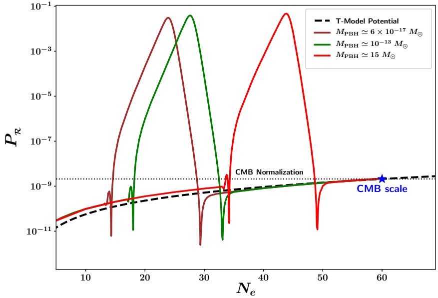

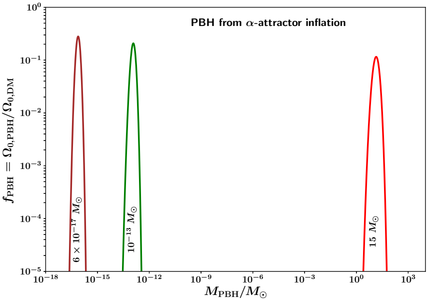

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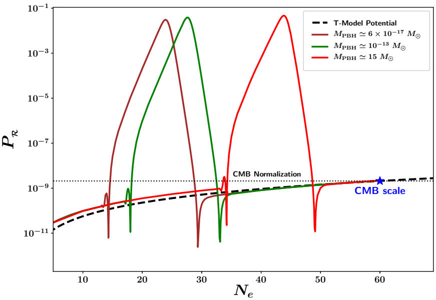

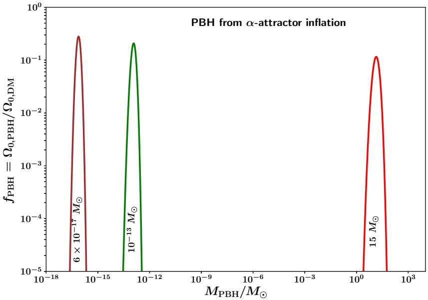

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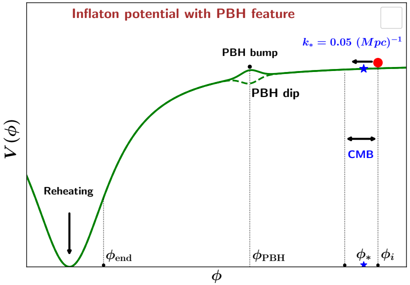

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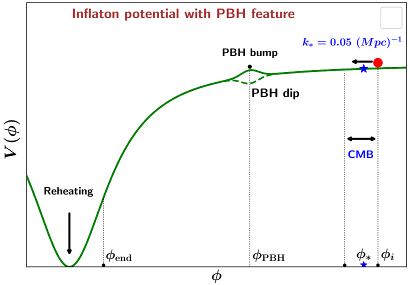

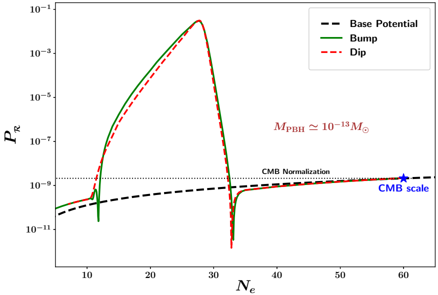

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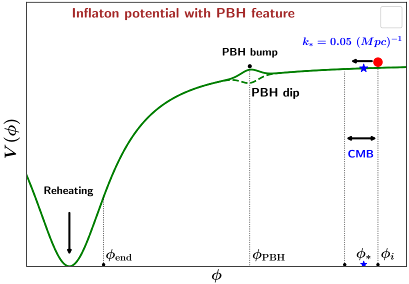

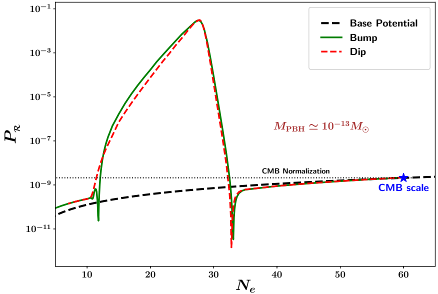

Symmetry: Dip/Bump symmetry in the potential correspond to the ascend and descend of the scalar field through the bump and the dip.

Steepest Growth Bound

[Byrnes, Cole and Patil, 2019]

Steepest possible growth: $n_s - 1 \leq 4$

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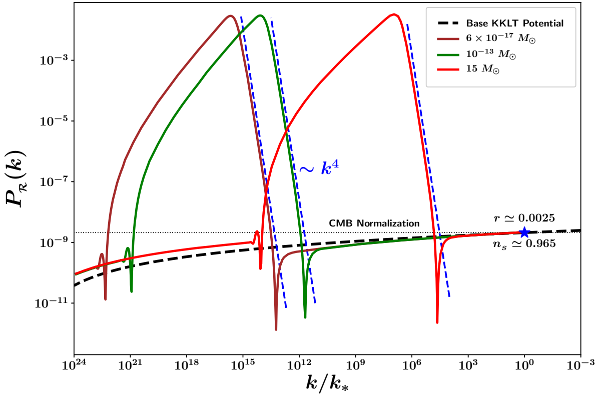
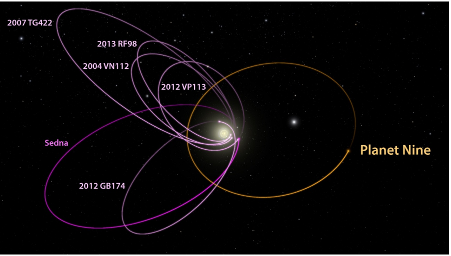
[Byrnes, Cole and Patil, 2019]

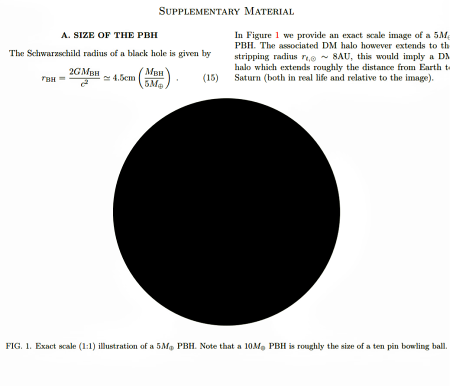
Steepest possible growth: $n_s - 1 \leq 4$



Unusual alignment of trans-Netpune Objects suggests the existence of a **5-10 Earth Mass object**.



[Scholtz, Unwin 2019], [Ed. Witten 2020],[Arbey, Auffinger 2020]



SUPPLEMENTARY MATERIAL

A. SIZE OF THE PBH

The Schwarzschild radius of a black hole is given by

$$r_{\rm BH} = \frac{2GM_{\rm BH}}{c^2} \simeq 4.5{\rm cm}\left(\frac{M_{\rm BH}}{5M_\oplus}\right) . \qquad (15)$$

In Figure 1 we provide an exact scale image of a $5M_\oplus$ PBH. The associated DM halo however extends to th stripping radius $r_{t,\odot} \sim 8$AU, this would imply a DM halo which extends roughly the distance from Earth t Saturn (both in real life and relative to the image).

FIG. 1. Exact scale (1:1) illustration of a $5M_\oplus$ PBH. Note that a $10M_\oplus$ PBH is roughly the size of a ten pin bowling ball.

Summary

- A tiny **local bump** or a **dip** in the base inflaton potential can easily amplify scalar power by 10^7 orders.
- Local nature of the bump does not affect CMB scale potential directly.
- Hence **Heavy mass** black holes with $M_{\rm PBH} \sim 10^2\ M_{\odot}$ can be produced with the **same ease** as **the lighter ones** with $M_{\rm PBH} \sim 10^{-17}\ M_{\odot}$ without compromising CMB scale n_s and r.
- **Simple functional form** of the proposed model treats the **PBH feature** to be a **small correction to the base inflaton potential**.

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- **Simple functional form** of the proposed model treats the **PBH feature** to be a **small correction to the base inflaton potential**.

S. S. Mishra and V. Sahni, "*Primordial Black Holes from a tiny bump in the Inflaton potential*," **JCAP 04 (2020) 007[arXiv:1911.00057 [gr-qc]]**.

Ongoing Work

- Effects of Non-Gaussianity, Quantum Diffusion

Ongoing Work

- Effects of Non-Gaussianity, Quantum Diffusion
- Broad Mass Function

Ongoing Work

- Effects of Non-Gaussianity, Quantum Diffusion
- Broad Mass Function
- Extended period of reheating

Article Link

astrobites — About · Latest Research · Be

They see me rolling... inflating

by Philippa Cole | Dec 16, 2019 | Daily Paper Summaries | 1 comment

Title: Primordial Black Holes from a tiny bump in the Inflaton potential

Authors: Swagat S. Mishra and Varun Sahni

Authors' Institution: Inter-University Centre for Astronomy and Astrophysics, Ganeshkhind, Pune, India

Status: Open access on arXiv

We're still on the hunt for what dark matter actually is. The most popular candidates are usually some sort of particle, like the WIMP or the axion. However, an idea that was first considered by Stephen Hawking (among others), is that tiny black holes which formed right after the Big Bang could also do the trick. There have been many searches carried out for these littl'uns, known as primordial black holes, but there's been no success just yet. However, whether there's enough of them floating around to make up some or all of the dark matter or not, we still need to work out how they could have been produced in the first place. For that, we'll need to go back 13.8 billions years...

Extra Slides

Examples of inflection point potentials

1.

$$V(\phi) = V_0 \frac{1 + a\phi^2 + b\phi^4 + c\phi^6}{(1 + d\phi^2)^2}$$

Jain, Bhaumik 2019

2.

$$V(\phi) = V_0 \left[1 + a_1 - \exp\left(-a_2 \tanh\frac{\phi}{\sqrt{6\alpha}}\right) - a_1 \exp\left(-a_3 \tanh^2\frac{\phi}{\sqrt{6\alpha}}\right)\right]^2$$

Mehbub 2019

3.

$$V(\phi) = V_0 \left[c_0 + c_1 \tanh\frac{\phi}{\sqrt{6\alpha}} + c_2 \tanh^2\frac{\phi}{\sqrt{6\alpha}} + c_3 \tanh^3\frac{\phi}{\sqrt{6\alpha}}\right]^2$$

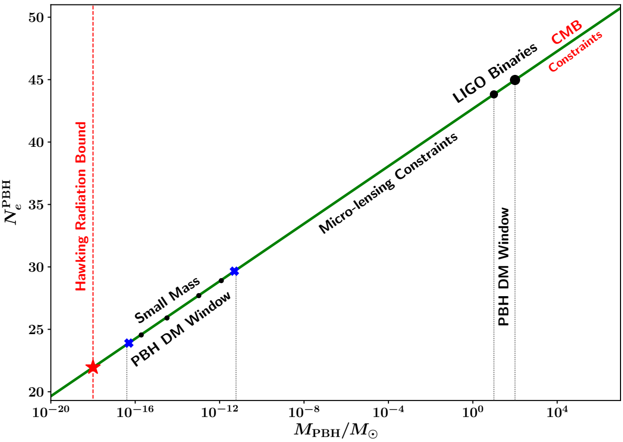
Dalianis et. al 2018

Extra Slides

Black Hole Mass-Radius Relation

$$R_s = \frac{2GM_{\rm BH}}{c^2} = 2.95 \times \left(\frac{M_{\rm BH}}{M_\odot}\right)\ \rm km$$

Black Holes	**Mass** $M_{\rm BH}$	**Radius** R_s
SMBH M87	$6.3 \times 10^9\ M_\odot$	1.86×10^{10} km
Solar Mass	2×10^{30} kg	2.95 km
Earth Mass	5.97×10^{24} kg	8.85 mm
Lunar Mass	7.35×10^{22} kg	0.11mm
Asteroid Mass	3×10^{21} kg	4.45 μm
Himalaya Mass	10^{12} kg	1.5×10^{-15} m



N_e^{PBH}
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25
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10^{-20}
10^{-16}
10^{-12}
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10^{0}
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$M_{\mathrm{PBH}}/M_{\odot}$
Hawking Radiation Bound
Small Mass
PBH DM Window
Micro-lensing Constraints
LIGO Binaries
PBH DM Window
CMB
Constraints

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Mass of the formed PBHs are roughly the Hubble-mass at horizon re-entry i.e

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Hence

$$\frac{M_{\rm PBH}}{M_\odot} = 1.55 \times 10^{24} \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{106.75}\right)^{1/6} (1+z)^{-2} \; ,$$

or alternatively

$$\frac{M_{\rm PBH}}{M_\odot} = 1.13 \times 10^{15} \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{106.75}\right)^{-1/6} \left(\frac{k_{\rm PBH}}{k_*}\right)^{-2} \; .$$

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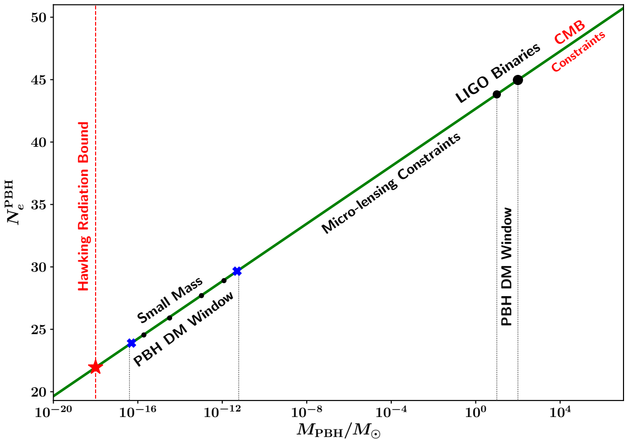
Similarly

$$N_* - N_e^{\rm PBH} = 17.33 + \frac{1}{2} \ln \frac{\gamma}{0.2} - \frac{1}{12} \ln \frac{g_*}{106.75} - \frac{1}{2} \ln \frac{M_{\rm PBH}}{M_\odot} \;\; .$$



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Constraints

Calculation of PBH Abundance: Press-Schechter Theory

Fractional abundance at the present epoch, defined by

$$f_{\rm PBH} = \frac{\Omega_{\rm 0PBH}(M_{\rm PBH})}{\Omega_{\rm 0DM}} \ ,$$

is given by

$$f_{\rm PBH}(M_{\rm PBH}) = 1.68\times 10^{8} \left(\frac{\gamma}{0.2}\right)^{1/2} \left(\frac{g_*}{106.75}\right)^{-1/4} \left(\frac{M_{\rm PBH}}{M_\odot}\right)^{-1/2} \beta(M_{\rm PBH})\,.$$

Where the mass fraction $\beta(M_{\rm PBH})$ of PBHs at formation is defined by

$$\beta(M_{\rm PBH}) = \frac{\rho_{\rm PBH}}{\rho_{\rm tot}}\bigg|_{\rm formation} \ .$$

In the Press-Schechter theory, $\beta(M_{\rm PBH})$ is interpreted as

$$\beta(M_{\rm PBH}) = \gamma \int_{\delta_{\rm th}}^{1} P(\delta) d\delta \ ,$$

Calculation of PBH Abundance: Press-Schechter Theory

which is given by

$$\beta(M_{\rm PBH}) = \gamma \int_{\delta_{\rm th}}^{1} \frac{d\delta}{\sqrt{2\pi}\sigma_{_{M_{\rm PBH}}}} \exp\left[-\frac{\delta^2}{2\sigma^2_{_{M_{\rm PBH}}}}\right] \approx \gamma \frac{\sigma_{_{M_{\rm PBH}}}}{\sqrt{2\pi}\delta_{\rm th}} \exp\left[-\frac{\delta^2_{\rm th}}{2\sigma^2_{_{M_{\rm PBH}}}}\right] .$$

The variance of the density contrast coarse-grained over the comoving Hubble scale $R = 1/k_{\rm PBH} = 1/\left(aH\right)_{\rm PBH}$(or mass scale $M_{\rm PBH}$) is given by

$$\sigma^2_{_{M_{\rm PBH}}} = \int \frac{dk}{k} P_\delta(k) W^2(k, R) \ ,$$

where $W(k, R)$ is the Fourier transform of the Gaussian window function used for smearing the original Gaussian density contrast field over the comoving Hubble scale to obtain the coarse-grained density contrast δ and is given by

$$W(k, R) = \exp\left(-\frac{1}{2}k^2 R^2\right) \ .$$

Calculation of PBH Abundance: Press-Schechter Theory

The power spectrum for the density contrast P_δ is related, in the radiation dominated epoch, to the primordial comoving curvature power spectrum by the famous expression

$$P_\delta(k) = \frac{16}{81}\left(\frac{k}{aH}\right)^4 P_{\mathcal{R}}(k)\ .$$

So we get a final expression for the variance of the density contrast as

$$\sigma^2_{M_{\rm PBH}} = \frac{16}{81}\int \frac{dk}{k}\left(\frac{k}{k_{\rm PBH}}\right)^4 \exp\left(-\frac{k^2}{k^2_{\rm PBH}}\right) P_{\mathcal{R}}(k)\ .$$

$$\beta(M_{\rm PBH}) = \gamma \int_{\delta_{\rm th}}^{1} \frac{d\delta}{\sqrt{2\pi}\sigma_{M_{\rm PBH}}} \exp\left[-\frac{\delta^2}{2\sigma^2_{M_{\rm PBH}}}\right] \approx \gamma \frac{\sigma_{M_{\rm PBH}}}{\sqrt{2\pi}\delta_{\rm th}} \exp\left[-\frac{\delta^2_{\rm th}}{2\sigma^2_{M_{\rm PBH}}}\right]\ .$$

$$f_{\rm PBH}(M_{\rm PBH}) = 1.68\times 10^8 \left(\frac{\gamma}{0.2}\right)^{1/2} \left(\frac{g_*}{106.75}\right)^{-1/4} \left(\frac{M_{\rm PBH}}{M_\odot}\right)^{-1/2} \beta(M_{\rm PBH})\ .$$

Sensitivity to the threshold density contrast $\partial_{\rm th}$