

# Recent Developments in the Physics of PBHs

Antonio Riotto  
University of Geneva

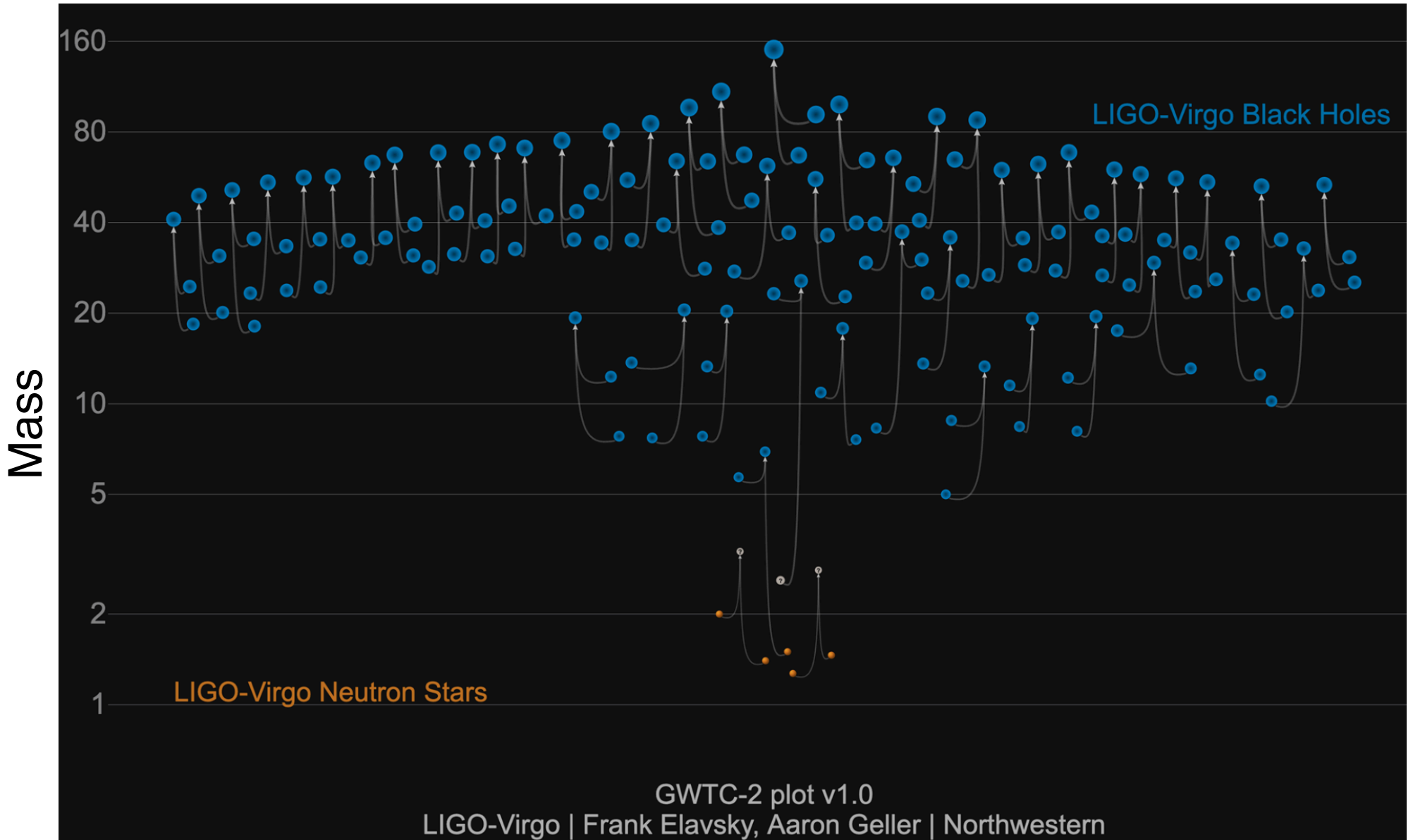
# Plan of the talk

- PBHs and GWTC-2
- PBHs and NANOGrav 12.5 yr

In collaboration with V. Baibhav, E. Berti, V. De Luca,  
V. Desjacques, G. Franciolini, P. Pani, K. Wong

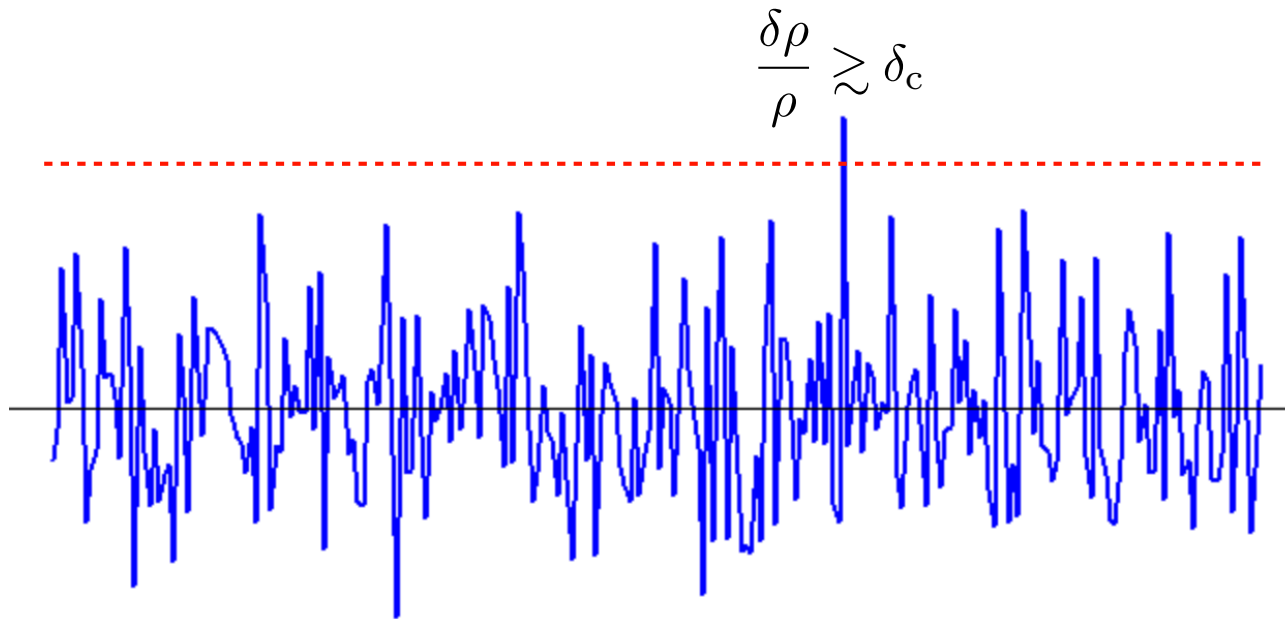
2009.04731 & 2011.01865

# Motivation



## Assumption:

PBHs are originated from peaks of the density contrast



PBHs are rare events, tail of the distribution

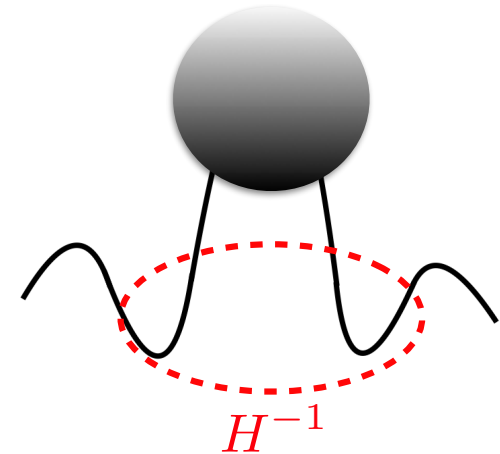
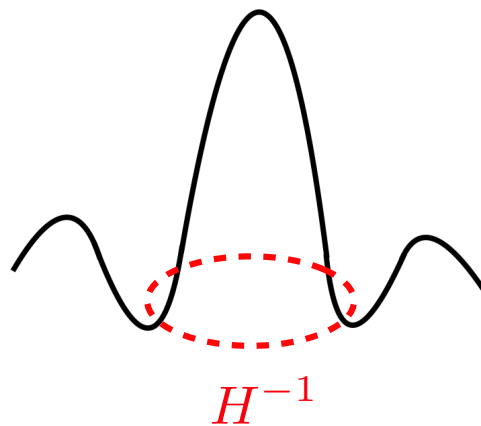
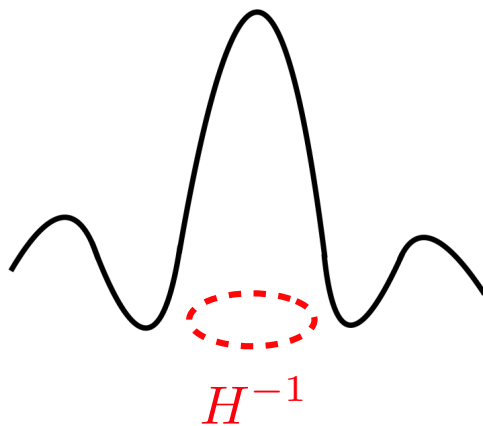
One possible mechanism: large fluctuations from inflation

## Assumption:

PBHs are originated from peaks of the density contrast

$$\frac{\delta\rho}{\rho} \gtrsim \delta_c$$

$$M_{\text{PBH}} \sim M_H$$



$$\beta(M) = \int_{\delta_c}^{\infty} \frac{d\delta}{\sqrt{2\pi}\sigma_\delta} e^{-\delta^2/2\sigma_\delta^2} \quad \sigma_\delta^2 = \int_0^\infty d \ln k W^2(k, R_H) \mathcal{P}_\delta(k)$$

Lot of work to go beyond the standard Gaussian lore

See I. Musco et al. (2020) for a simple prescription to calculate the threshold, including horizon crossing non-linear effects

# Properties of PBHs at formation

# The PBH mass function at formation

Mass distribution dependent on the curvature perturbation spectrum and statistical properties

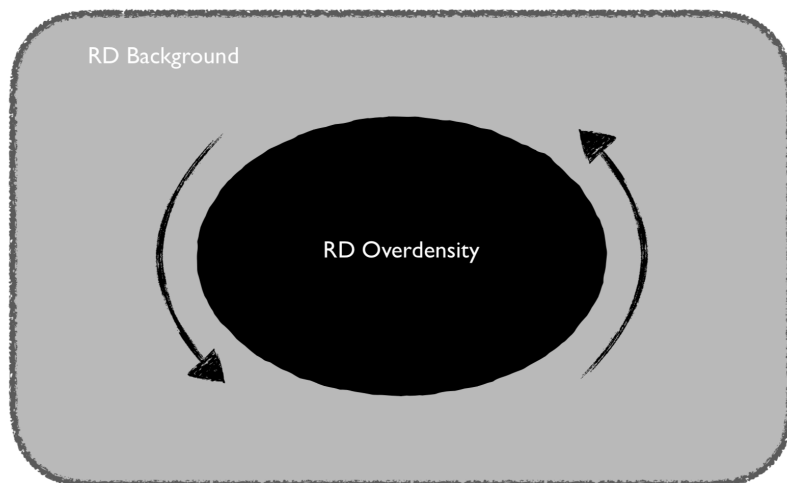
Standard parametrisation

$$\psi(M_{\text{PBH}}) = \frac{1}{\sqrt{2\pi} M_{\text{PBH}} \sigma} \exp\left(-\frac{\ln^2(M_{\text{PBH}}/M_c)}{2\sigma^2}\right)$$

May have different forms, e.g. if the curvature perturbation is broad, but still peaked at a given mass

# The spin of PBHs at formation is small

- PBHs originate from peaks, that is from *maxima* of the local density contrast. Need peak theory to obtain the probability distribution of the spin
- The spin results from the action of the torques generated by the gravitational tidal forces upon horizon crossing
- It is a *first-order* effect in perturbation theory when accounting for the fact that the collapse is not spherical



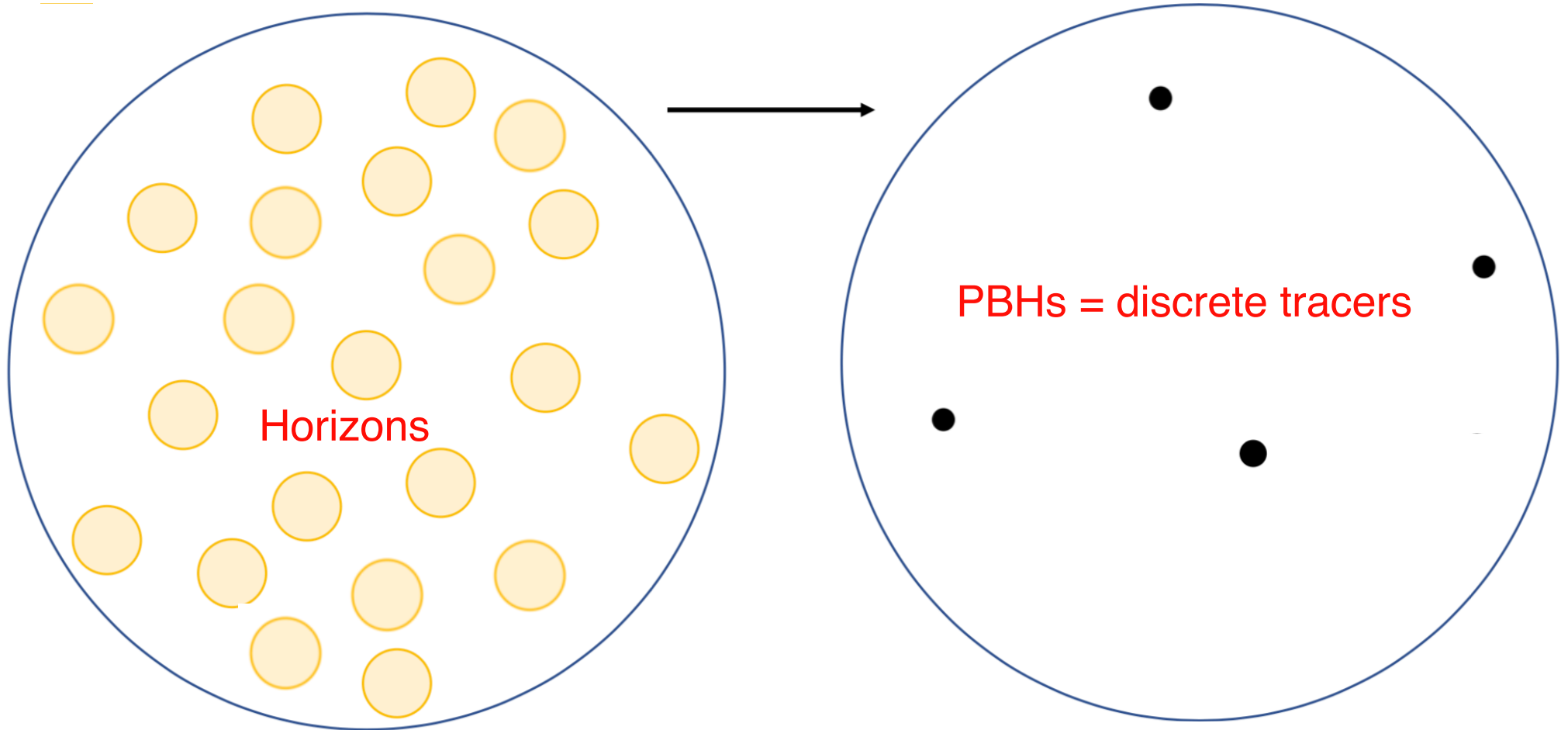
$$\vec{\chi} = \vec{S} / G_N M_{\text{PBH}}^2$$

$$\chi_i \sim 10^{-2} \sqrt{1 - \gamma^2}$$

Shape of the density power spectrum

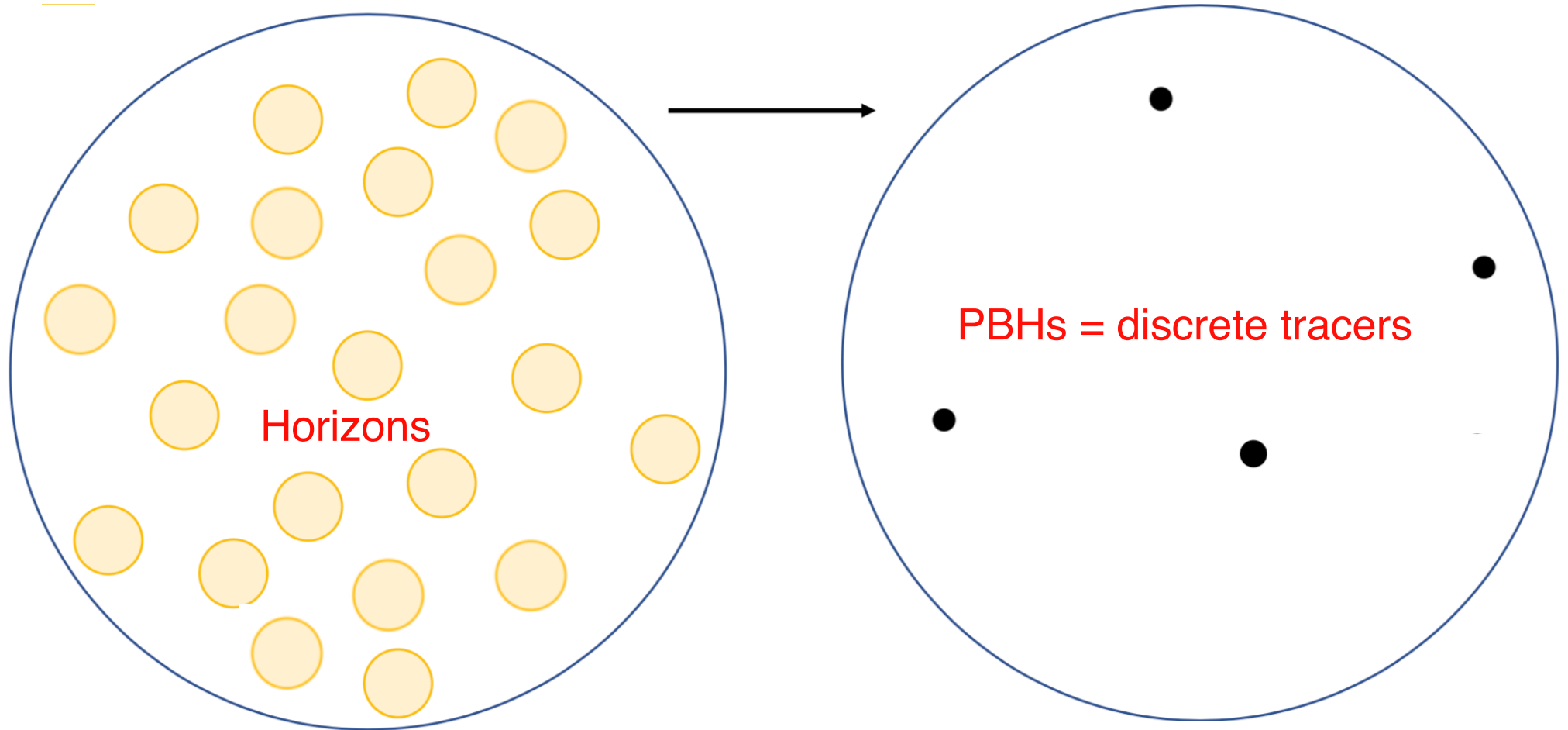


# PBHs are not clustered at formation



$$\left\langle \frac{\delta\rho_{\text{PBH}}(\vec{x}, z)}{\bar{\rho}_{\text{DM}}} \frac{\delta\rho_{\text{PBH}}(0, z)}{\bar{\rho}_{\text{DM}}} \right\rangle = \frac{f_{\text{PBH}}^2}{n_{\text{PBH}}} \delta_{\text{D}}(\vec{x}) + \xi(x, z)$$

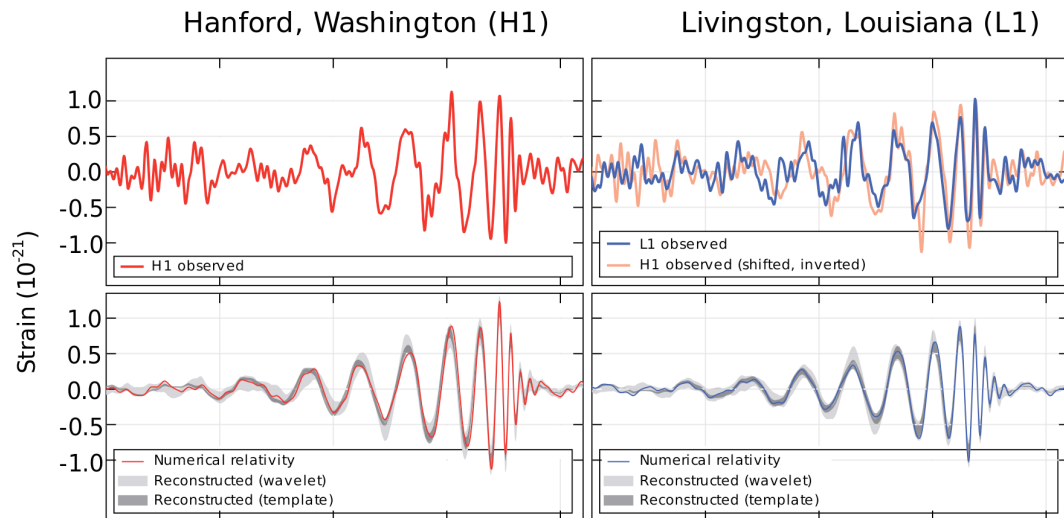
# PBHs are not clustered at formation



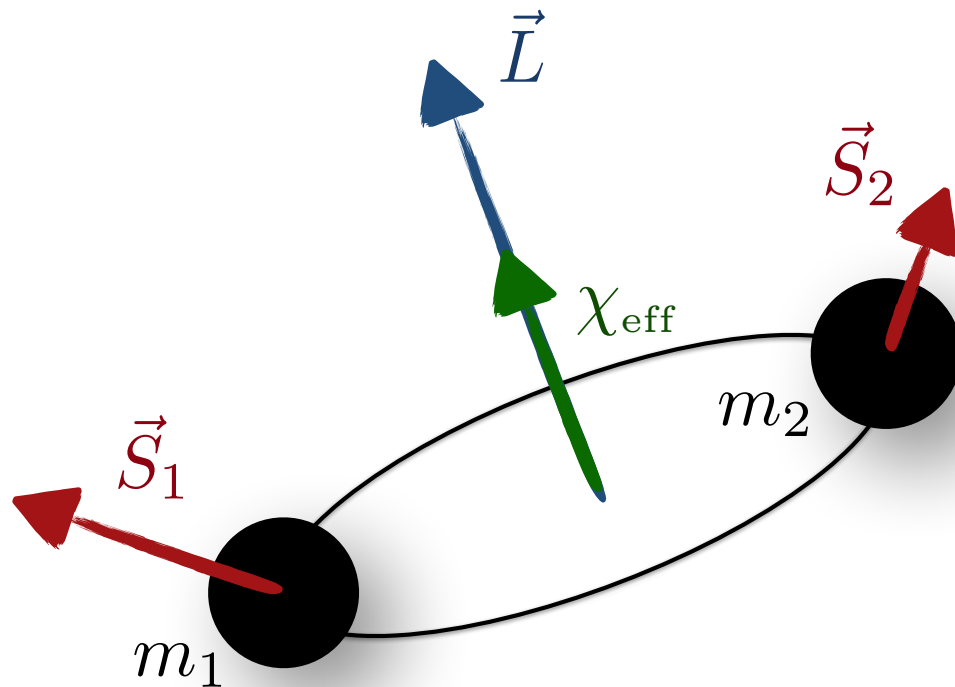
True also if the PBH mass function is broad  
(understandable from the excursion set theory)

# GWTC-2 and PBHs

# BH binary



GW150914, LIGO (2016)



Waveforms dependent on the binary event parameters

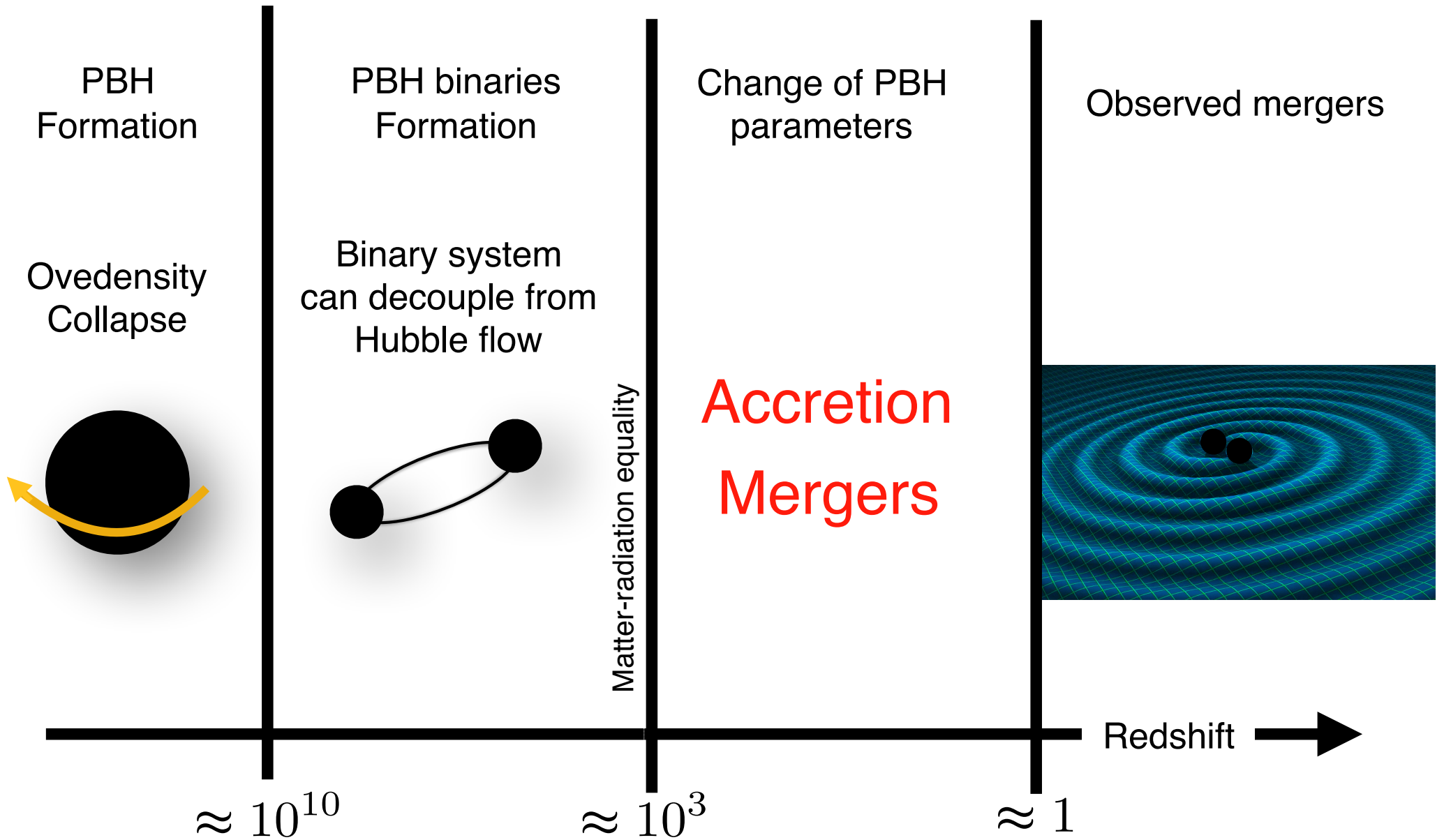
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$q = m_2 / m_1$$

$$\chi_{\text{eff}} = \frac{\vec{S}_1 / m_1 + \vec{S}_2 / m_2}{m_1 + m_2} \cdot \hat{L}$$

...

# PBH evolution



# Accretion onto isolated PBHs

For  $f_{\text{PBH}} < 1$  PBHs coexist with another DM component in the universe

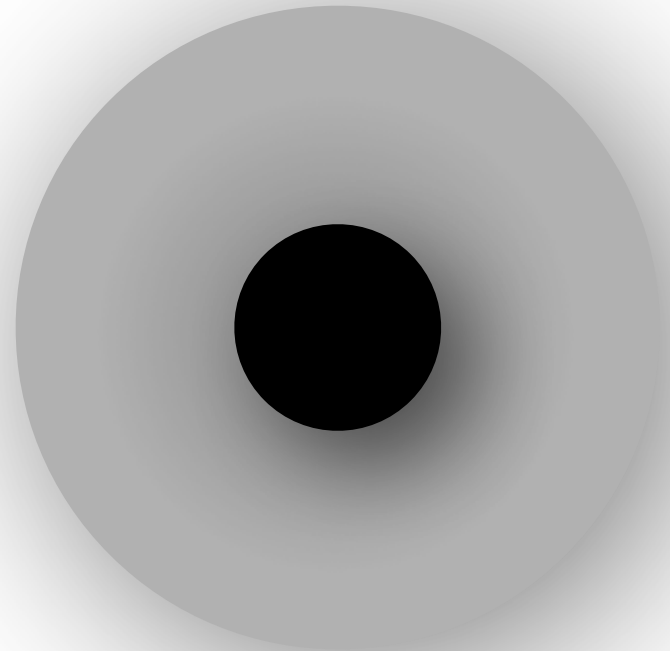
A DM halo builds up around the PBHs  
enhancing accretion

(larger gravitational potential well)

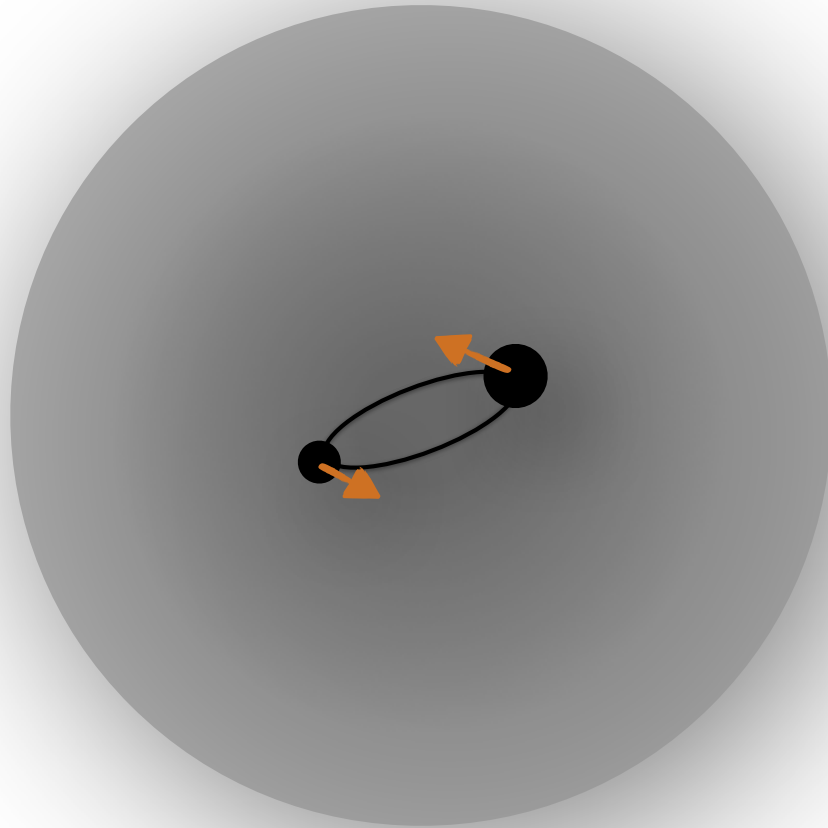
$$M_h(z) \approx 3M_{\text{PBH}} \left( \frac{1000}{1+z} \right)$$

Bondy-Hoyle accretion from the  
surrounding baryonic fluid

$$\dot{M} = 4\pi\lambda m_H n_{\text{gas}} v_{\text{eff}}^{-3} M^2$$



# Accretion onto PBH binaries



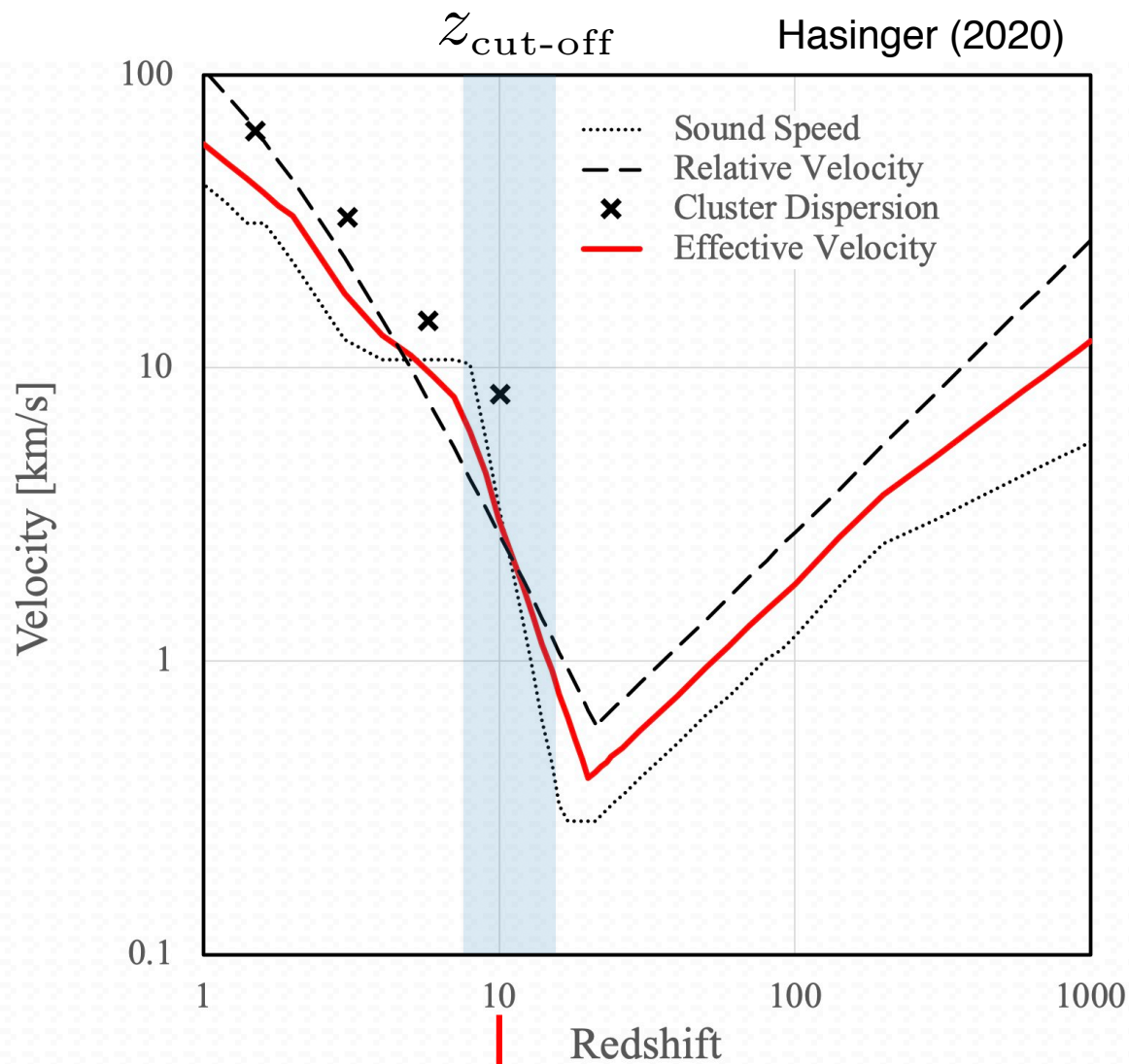
Accretion on the system enhances the gas density around the PBH binary

Accretion on the single PBH modulated by masses and orbital velocities

$$\dot{M}_1 = \dot{M} \frac{1}{\sqrt{2(1+q)}}$$

$$\dot{M}_2 = \dot{M} \sqrt{\frac{q}{2(1+q)}}$$

- The smaller PBH always experiences a larger relative accretion
- PBH can experience accretion for  $M \gtrsim \mathcal{O}(10)M_\odot$



Structure formation  
reionization epoch

- Virialised velocities
- Higher temperatures

$$\dot{M} \approx (v_{\text{rel}}^2 + c_s^2)^{-3/2}$$



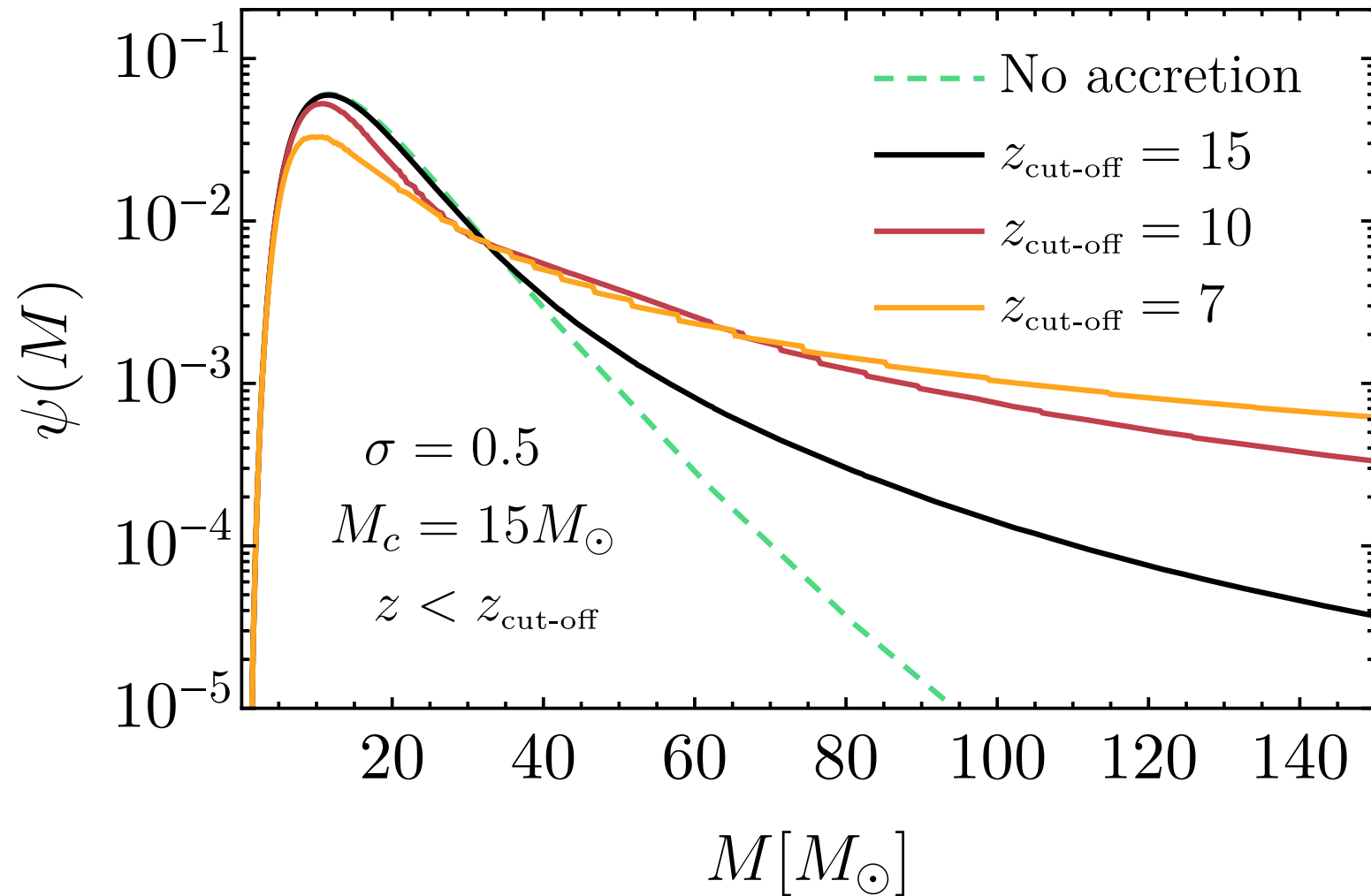
Strong suppression around

$z_{\text{cut-off}}$

Uncertainties  
in the accretion model  
accounted for by  
varying the cut-off



# PBH mass function evolution

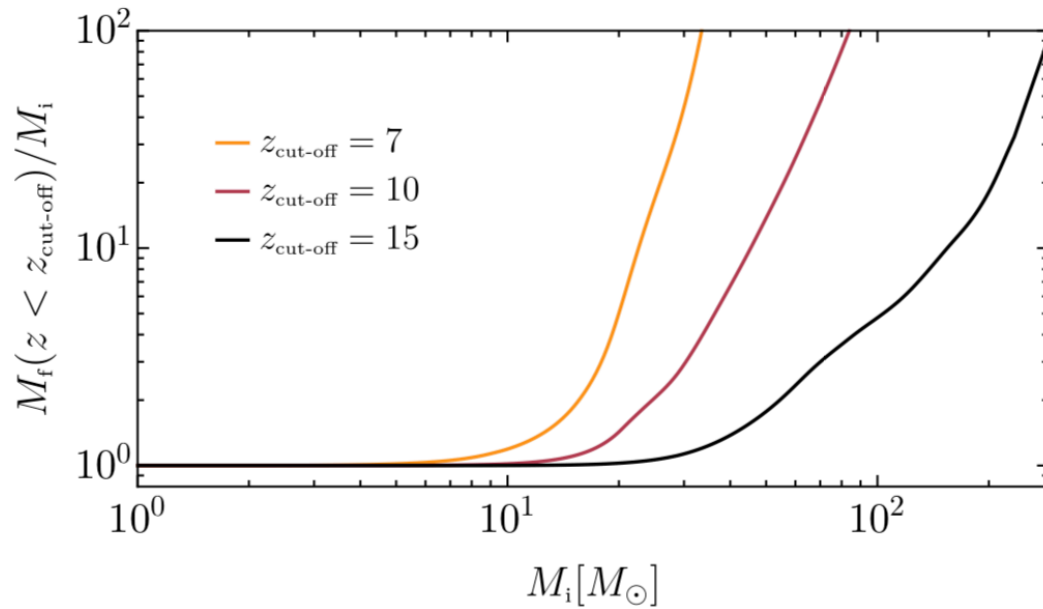


Non-linear mass evolution enhances large-mass tails

Fitting the GWTC-2 large mass tail

will push towards smaller central masses of the mass function

# PBH mass evolution in binaries

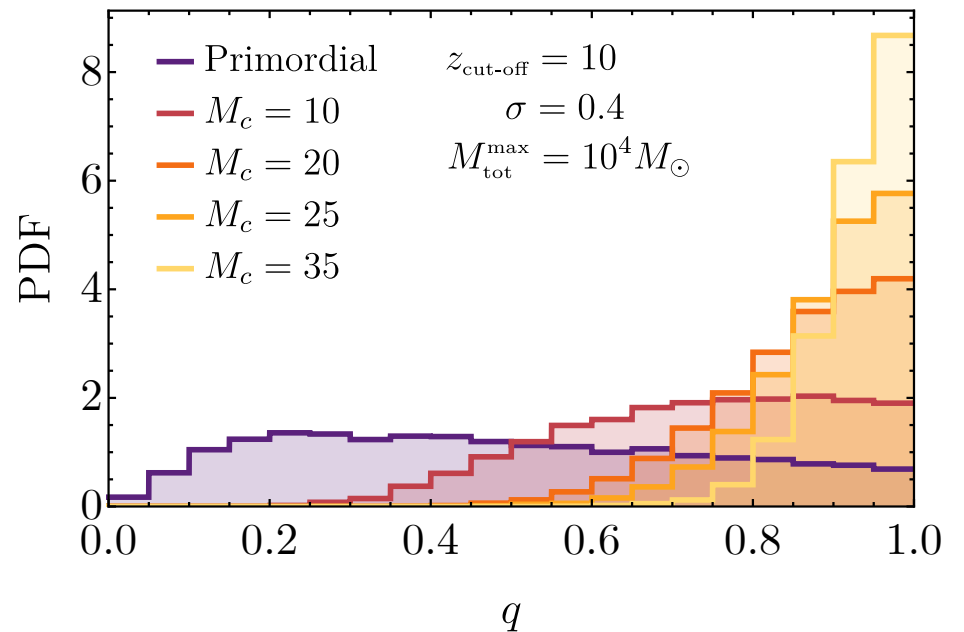


Large impact on masses, depending on the strength of accretion

$$\dot{q} = q \left( \frac{\dot{M}_2}{M_2} - \frac{\dot{M}_1}{M_1} \right)$$

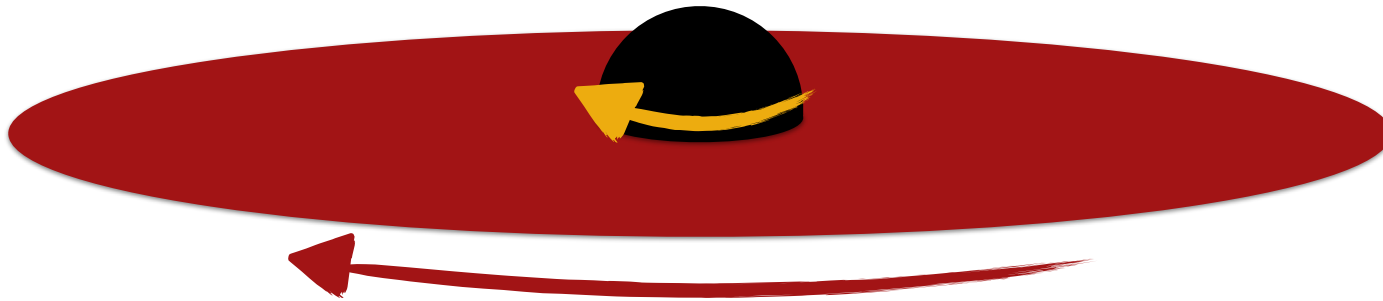
Evolution of the mass ratio distribution towards equal-masses

Same MF initial width, at larger masses



# PBH spin evolution

If matter angular momentum is large enough, an accreting disk forms, leading to a spin growth

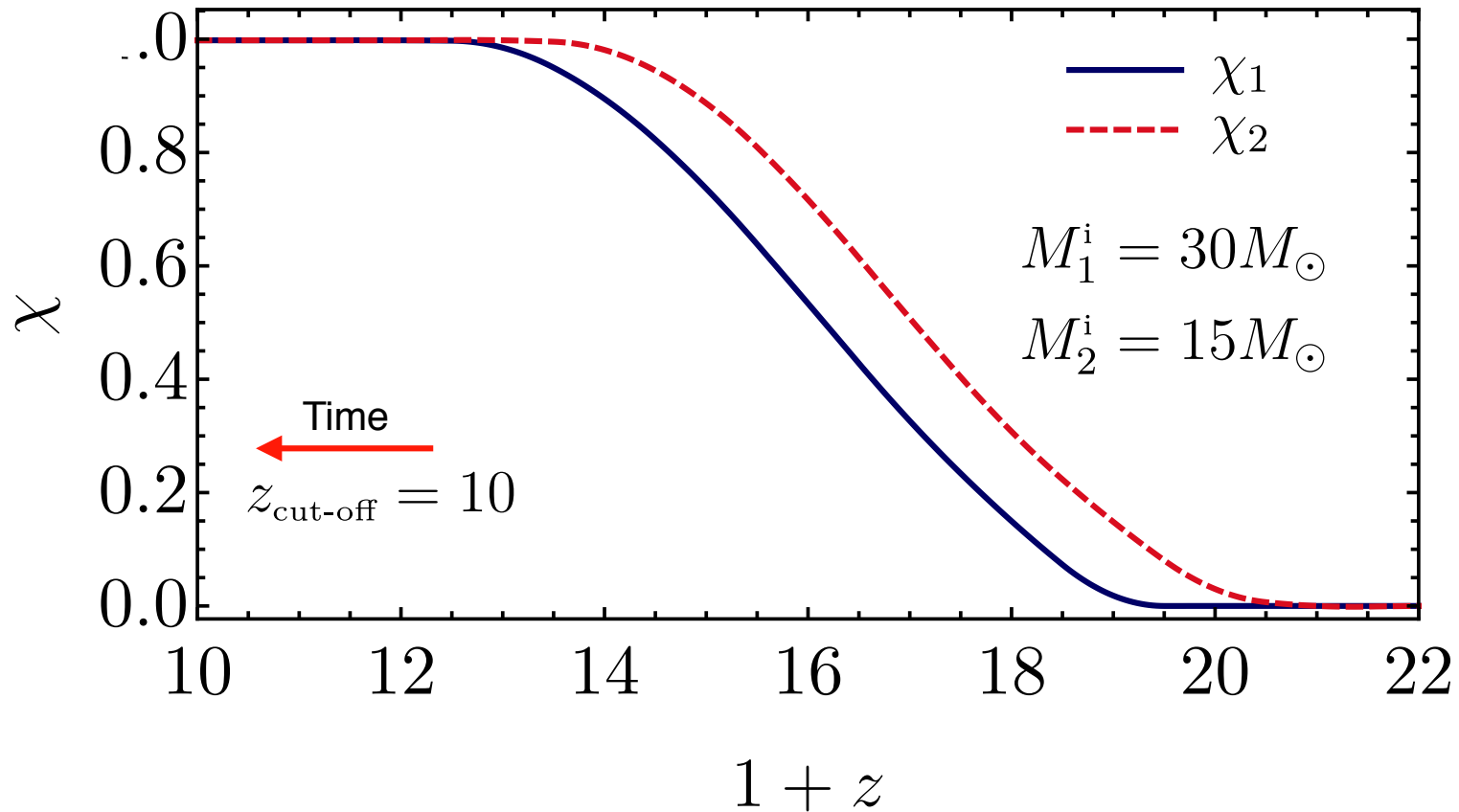


Angular momentum transfer between gas and PBH

$$\dot{\chi} = g(\chi) \frac{\dot{M}}{M}$$

by solving the geodesic model of disk accretion

## Spins pushed towards extremality



- Uncorrelated spin orientation
- Effective spin spreads around zero
- Accretion: low/large mass - low/large spin correlation

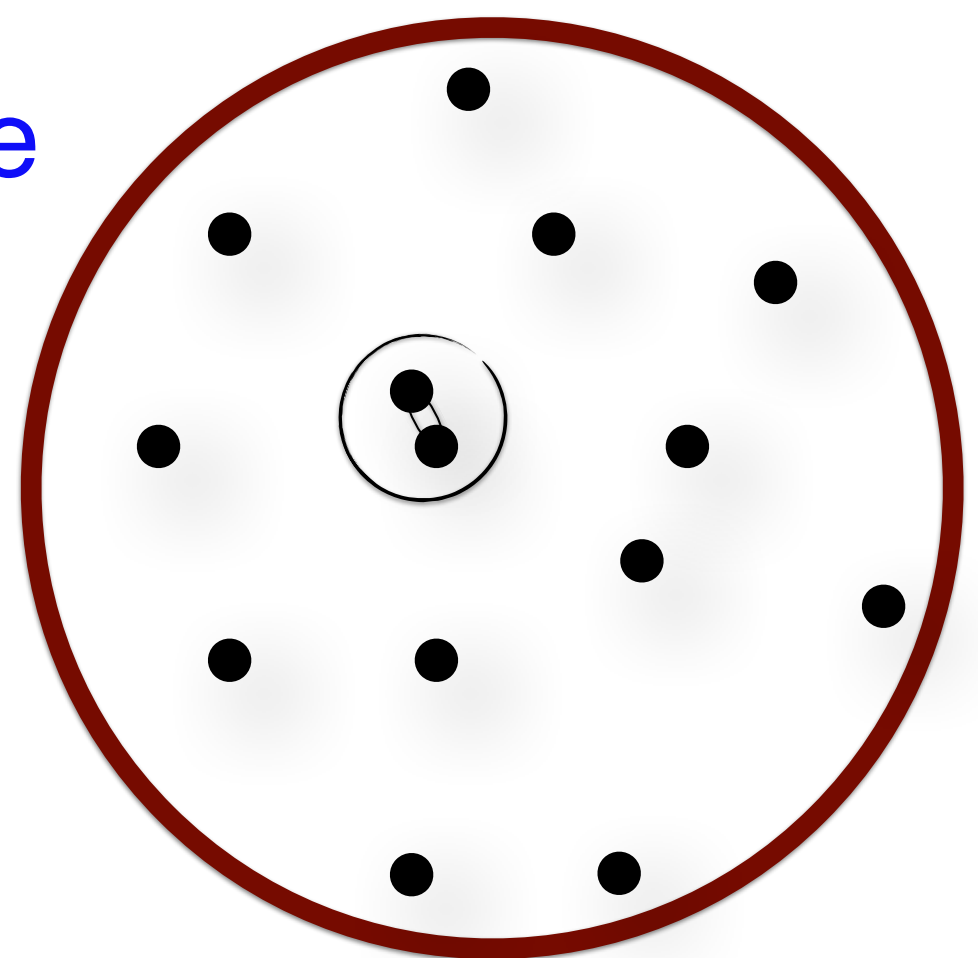
# Merger rate

- Initial spatial Poisson distribution
- Random decoupling of binary systems



Compute probability of decoupling and the binary initial geometry

- Semi-major axis
- Eccentricity



M. Raidal et al (2018)

$$\frac{dR}{dm_1 dm_2} = \frac{1.6 \times 10^6}{\text{Gpc}^3 \text{ yr}} f_{\text{PBH}}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \left(\frac{M_{\text{tot}}}{M_{\odot}}\right)^{-\frac{32}{37}} S(M_{\text{tot}}, f_{\text{PBH}}) \mathcal{A}_{\text{acc}}(m_j) \psi(m_1) \psi(m_2)$$

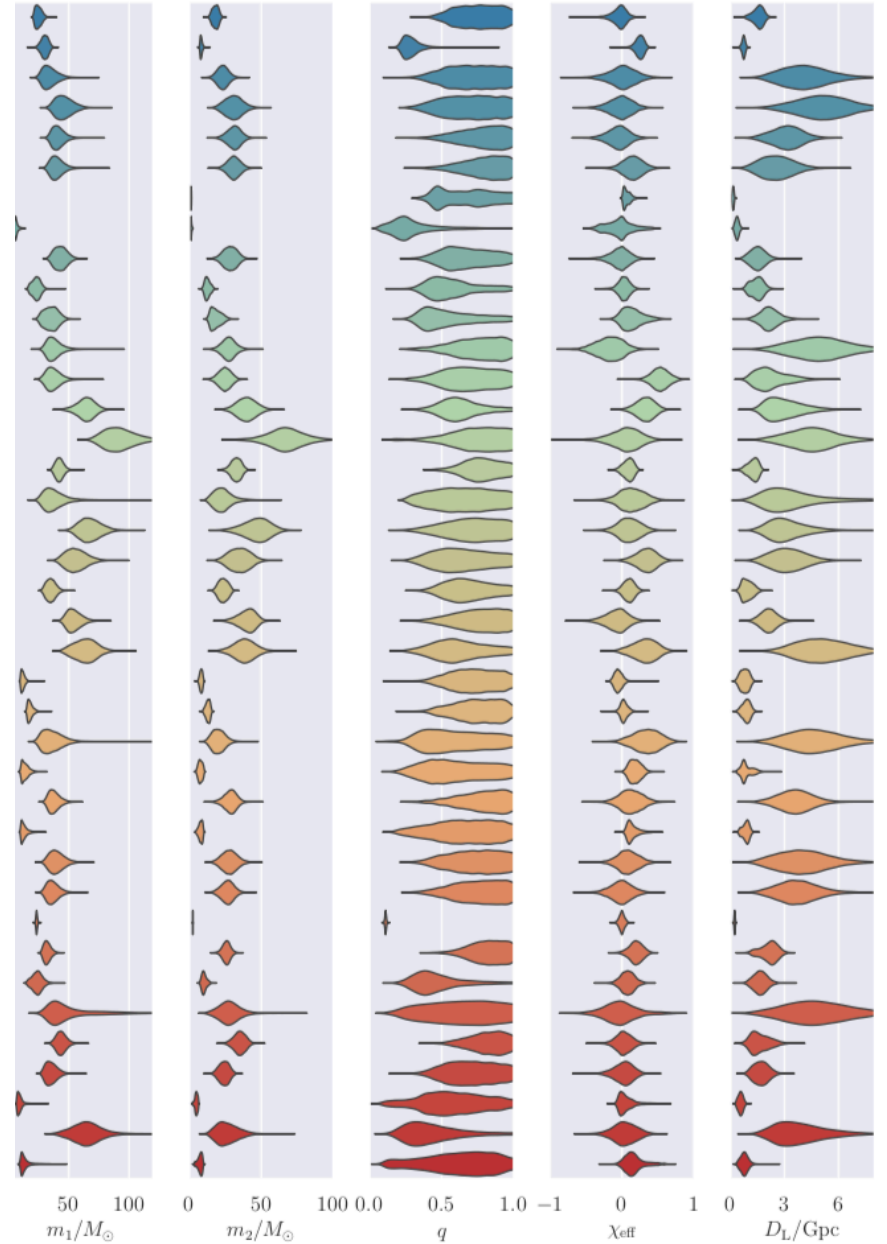
- Accretion hardens the binaries
- Larger masses leads to shorter mergers



V. De Luca et al. (2020)

# GWTC-2=O1+O2+O3a

Event	$M$ ( $M_\odot$ )	$\mathcal{M}$ ( $M_\odot$ )	$m_1$ ( $M_\odot$ )	$m_2$ ( $M_\odot$ )	$\chi_{\text{eff}}$	$D_L$ (Gpc)	$z$	$M_{\text{f}}$ ( $M_\odot$ )	$\chi_{\text{f}}$	$\Delta\Omega$ (deg <sup>2</sup> )	SNR
GW190408_181802	42.9 <sup>+4.1</sup> <sub>-2.9</sub>	18.3 <sup>+1.8</sup> <sub>-1.2</sub>	24.5 <sup>+5.1</sup> <sub>-3.4</sub>	18.3 <sup>+3.2</sup> <sub>-3.4</sub>	-0.03 <sup>+0.13</sup> <sub>-0.19</sub>	1.58 <sup>+0.40</sup> <sub>-0.59</sub>	0.30 <sup>+0.06</sup> <sub>-0.10</sub>	41.0 <sup>+3.8</sup> <sub>-2.7</sub>	0.67 <sup>+0.06</sup> <sub>-0.07</sub>	140	15.3 <sup>+0.2</sup> <sub>-0.3</sub>
● GW190412	38.4 <sup>+3.8</sup> <sub>-3.7</sub>	13.3 <sup>+0.4</sup> <sub>-0.3</sub>	30.0 <sup>+4.7</sup> <sub>-4.7</sub>	8.3 <sup>+1.6</sup> <sub>-0.9</sub>	0.25 <sup>+0.08</sup> <sub>-0.11</sub>	0.74 <sup>+0.14</sup> <sub>-0.17</sub>	0.15 <sup>+0.03</sup> <sub>-0.03</sub>	37.3 <sup>+3.9</sup> <sub>-3.9</sub>	0.67 <sup>+0.05</sup> <sub>-0.05</sub>	21	18.9 <sup>+0.2</sup> <sub>-0.3</sub>
GW190413_052954	56.9 <sup>+13.1</sup> <sub>-8.9</sub>	24.0 <sup>+5.4</sup> <sub>-3.7</sub>	33.4 <sup>+12.4</sup> <sub>-7.4</sub>	23.4 <sup>+6.7</sup> <sub>-6.3</sub>	0.01 <sup>+0.29</sup> <sub>-0.33</sub>	4.10 <sup>+2.41</sup> <sub>-1.89</sub>	0.66 <sup>+0.30</sup> <sub>-0.27</sub>	54.3 <sup>+12.4</sup> <sub>-8.4</sub>	0.69 <sup>+0.12</sup> <sub>-0.13</sub>	1400	8.9 <sup>+0.4</sup> <sub>-0.8</sub>
GW190413_134308	76.1 <sup>+15.9</sup> <sub>-10.6</sub>	31.9 <sup>+7.3</sup> <sub>-4.6</sub>	45.4 <sup>+13.6</sup> <sub>-9.6</sub>	30.9 <sup>+10.2</sup> <sub>-9.6</sub>	-0.01 <sup>+0.24</sup> <sub>-0.28</sub>	5.15 <sup>+2.44</sup> <sub>-2.34</sub>	0.80 <sup>+0.30</sup> <sub>-0.31</sub>	72.8 <sup>+15.2</sup> <sub>-10.3</sub>	0.69 <sup>+0.10</sup> <sub>-0.12</sub>	520	10.0 <sup>+0.4</sup> <sub>-0.5</sub>
GW190421_213856	71.8 <sup>+12.5</sup> <sub>-8.6</sub>	30.7 <sup>+5.5</sup> <sub>-6.6</sub>	40.6 <sup>+10.4</sup> <sub>-8.2</sub>	31.4 <sup>+7.5</sup> <sub>-8.2</sub>	-0.05 <sup>+0.23</sup> <sub>-0.26</sub>	3.15 <sup>+1.37</sup> <sub>-1.42</sub>	0.53 <sup>+0.18</sup> <sub>-0.21</sub>	68.6 <sup>+11.7</sup> <sub>-8.1</sub>	0.68 <sup>+0.10</sup> <sub>-0.11</sub>	1000	10.7 <sup>+0.2</sup> <sub>-0.4</sub>
GW190424_180648	70.7 <sup>+13.4</sup> <sub>-9.8</sub>	30.3 <sup>+5.7</sup> <sub>-6.9</sub>	39.5 <sup>+10.9</sup> <sub>-9.8</sub>	31.0 <sup>+7.4</sup> <sub>-7.3</sub>	0.15 <sup>+0.22</sup> <sub>-0.22</sub>	2.55 <sup>+1.56</sup> <sub>-1.33</sub>	0.45 <sup>+0.22</sup> <sub>-0.21</sub>	67.1 <sup>+12.5</sup> <sub>-9.2</sub>	0.75 <sup>+0.08</sup> <sub>-0.09</sub>	26000	10.4 <sup>+0.2</sup> <sub>-0.4</sub>
● GW190425	3.4 <sup>+0.3</sup> <sub>-0.1</sub>	1.44 <sup>+0.02</sup> <sub>-0.02</sub>	2.0 <sup>+0.6</sup> <sub>-0.3</sub>	1.4 <sup>+0.3</sup> <sub>-0.3</sub>	0.06 <sup>+0.11</sup> <sub>-0.05</sub>	0.16 <sup>+0.07</sup> <sub>-0.07</sub>	0.03 <sup>+0.01</sup> <sub>-0.02</sub>	-	-	9900	12.4 <sup>+0.3</sup> <sub>-0.4</sub>
GW190426_152155	7.2 <sup>+3.5</sup> <sub>-1.5</sub>	2.41 <sup>+0.08</sup> <sub>-0.08</sub>	5.7 <sup>+4.0</sup> <sub>-2.3</sub>	1.5 <sup>+0.8</sup> <sub>-0.5</sub>	-0.03 <sup>+0.33</sup> <sub>-0.30</sub>	0.38 <sup>+0.19</sup> <sub>-0.16</sub>	0.08 <sup>+0.04</sup> <sub>-0.03</sub>	-	-	1400	8.7 <sup>+0.5</sup> <sub>-0.6</sub>
GW190503_185404	71.3 <sup>+9.3</sup> <sub>-8.0</sub>	30.1 <sup>+4.2</sup> <sub>-4.0</sub>	42.9 <sup>+9.2</sup> <sub>-7.8</sub>	28.5 <sup>+7.5</sup> <sub>-7.9</sub>	-0.02 <sup>+0.20</sup> <sub>-0.26</sub>	1.52 <sup>+0.71</sup> <sub>-0.66</sub>	0.29 <sup>+0.11</sup> <sub>-0.11</sub>	68.2 <sup>+8.7</sup> <sub>-7.5</sub>	0.67 <sup>+0.09</sup> <sub>-0.12</sub>	94	12.4 <sup>+0.2</sup> <sub>-0.3</sub>
GW190512_180714	35.6 <sup>+3.9</sup> <sub>-3.4</sub>	14.5 <sup>+1.3</sup> <sub>-1.0</sub>	23.0 <sup>+5.4</sup> <sub>-5.7</sub>	12.5 <sup>+3.5</sup> <sub>-2.5</sub>	0.03 <sup>+0.13</sup> <sub>-0.13</sub>	1.49 <sup>+0.53</sup> <sub>-0.59</sub>	0.28 <sup>+0.09</sup> <sub>-0.10</sub>	34.2 <sup>+3.9</sup> <sub>-3.4</sub>	0.65 <sup>+0.07</sup> <sub>-0.07</sub>	230	12.2 <sup>+0.2</sup> <sub>-0.4</sub>
GW190513_205428	53.6 <sup>+8.6</sup> <sub>-5.9</sub>	21.5 <sup>+3.6</sup> <sub>-3.9</sub>	35.3 <sup>+9.6</sup> <sub>-9.0</sub>	18.1 <sup>+7.3</sup> <sub>-4.2</sub>	0.12 <sup>+0.29</sup> <sub>-0.18</sub>	2.16 <sup>+0.94</sup> <sub>-0.80</sub>	0.39 <sup>+0.14</sup> <sub>-0.13</sub>	51.3 <sup>+8.1</sup> <sub>-5.8</sub>	0.69 <sup>+0.14</sup> <sub>-0.12</sub>	490	12.9 <sup>+0.3</sup> <sub>-0.4</sub>
GW190514_065416	64.2 <sup>+16.6</sup> <sub>-9.6</sub>	27.4 <sup>+6.9</sup> <sub>-4.3</sub>	36.9 <sup>+13.4</sup> <sub>-7.3</sub>	27.5 <sup>+8.2</sup> <sub>-7.7</sub>	-0.16 <sup>+0.28</sup> <sub>-0.32</sub>	4.93 <sup>+2.76</sup> <sub>-2.41</sub>	0.77 <sup>+0.34</sup> <sub>-0.33</sub>	61.6 <sup>+16.0</sup> <sub>-9.2</sub>	0.64 <sup>+0.11</sup> <sub>-0.14</sub>	2400	8.2 <sup>+0.3</sup> <sub>-0.6</sub>
● GW190517_055101	61.9 <sup>+10.0</sup> <sub>-9.6</sub>	26.0 <sup>+4.0</sup> <sub>-4.2</sub>	36.4 <sup>+11.8</sup> <sub>-7.8</sub>	24.8 <sup>+6.9</sup> <sub>-7.1</sub>	0.53 <sup>+0.20</sup> <sub>-0.19</sub>	2.11 <sup>+1.79</sup> <sub>-1.00</sub>	0.38 <sup>+0.26</sup> <sub>-0.16</sub>	57.8 <sup>+9.4</sup> <sub>-9.1</sub>	0.87 <sup>+0.05</sup> <sub>-0.07</sub>	460	10.7 <sup>+0.4</sup> <sub>-0.6</sub>
● GW190519_153544	104.2 <sup>+14.5</sup> <sub>-14.9</sub>	43.5 <sup>+6.8</sup> <sub>-6.8</sub>	64.5 <sup>+11.3</sup> <sub>-13.2</sub>	39.9 <sup>+11.0</sup> <sub>-19.5</sub>	0.33 <sup>+0.19</sup> <sub>-0.22</sub>	2.85 <sup>+2.02</sup> <sub>-1.14</sub>	0.49 <sup>+0.27</sup> <sub>-0.17</sub>	98.7 <sup>+13.5</sup> <sub>-11.2</sub>	0.80 <sup>+0.07</sup> <sub>-0.12</sub>	770	15.6 <sup>+0.2</sup> <sub>-0.3</sub>
● GW190521	157.9 <sup>+37.4</sup> <sub>-20.4</sub>	66.9 <sup>+15.5</sup> <sub>-9.2</sub>	91.4 <sup>+29.3</sup> <sub>-17.5</sub>	66.8 <sup>+20.7</sup> <sub>-20.7</sub>	0.06 <sup>+0.31</sup> <sub>-0.37</sub>	4.53 <sup>+2.30</sup> <sub>-2.13</sub>	0.72 <sup>+0.29</sup> <sub>-0.29</sub>	150.3 <sup>+35.8</sup> <sub>-20.0</sub>	0.73 <sup>+0.11</sup> <sub>-0.14</sub>	940	14.2 <sup>+0.3</sup> <sub>-0.3</sub>
GW190521_074359	74.4 <sup>+6.8</sup> <sub>-4.6</sub>	31.9 <sup>+3.1</sup> <sub>-2.4</sub>	42.1 <sup>+5.9</sup> <sub>-4.9</sub>	32.7 <sup>+5.4</sup> <sub>-6.2</sub>	0.09 <sup>+0.10</sup> <sub>-0.13</sub>	1.28 <sup>+0.38</sup> <sub>-0.57</sub>	0.25 <sup>+0.06</sup> <sub>-0.10</sub>	70.7 <sup>+6.4</sup> <sub>-4.2</sub>	0.72 <sup>+0.05</sup> <sub>-0.07</sub>	500	25.8 <sup>+0.1</sup> <sub>-0.2</sub>
GW190527_092055	58.5 <sup>+27.9</sup> <sub>-10.6</sub>	24.2 <sup>+11.9</sup> <sub>-4.4</sub>	36.2 <sup>+9.5</sup> <sub>-9.5</sub>	22.8 <sup>+12.7</sup> <sub>-8.1</sub>	0.13 <sup>+0.29</sup> <sub>-0.28</sub>	3.10 <sup>+4.85</sup> <sub>-1.64</sub>	0.53 <sup>+0.61</sup> <sub>-0.25</sub>	55.9 <sup>+26.4</sup> <sub>-10.4</sub>	0.73 <sup>+0.12</sup> <sub>-0.16</sub>	3800	8.1 <sup>+0.4</sup> <sub>-1.0</sub>
GW190602_175927	114.1 <sup>+18.5</sup> <sub>-15.7</sub>	48.3 <sup>+8.6</sup> <sub>-8.0</sub>	67.2 <sup>+16.0</sup> <sub>-12.6</sub>	47.4 <sup>+13.4</sup> <sub>-16.6</sub>	0.10 <sup>+0.25</sup> <sub>-0.25</sub>	2.99 <sup>+2.02</sup> <sub>-1.26</sub>	0.51 <sup>+0.27</sup> <sub>-0.19</sub>	108.8 <sup>+17.2</sup> <sub>-14.8</sub>	0.71 <sup>+0.10</sup> <sub>-0.13</sub>	720	12.8 <sup>+0.2</sup> <sub>-0.3</sub>
● GW190620_030421	90.1 <sup>+17.3</sup> <sub>-12.1</sub>	37.5 <sup>+7.8</sup> <sub>-5.7</sub>	55.4 <sup>+15.8</sup> <sub>-12.0</sub>	35.0 <sup>+11.6</sup> <sub>-11.4</sub>	0.34 <sup>+0.21</sup> <sub>-0.25</sub>	3.16 <sup>+1.67</sup> <sub>-1.43</sub>	0.54 <sup>+0.22</sup> <sub>-0.21</sub>	85.4 <sup>+15.9</sup> <sub>-11.4</sub>	0.80 <sup>+0.08</sup> <sub>-0.14</sub>	6700	12.1 <sup>+0.3</sup> <sub>-0.4</sub>
GW190630_185205	58.8 <sup>+4.7</sup> <sub>-4.8</sub>	24.8 <sup>+2.1</sup> <sub>-2.0</sub>	35.0 <sup>+6.9</sup> <sub>-5.7</sub>	23.6 <sup>+5.2</sup> <sub>-5.1</sub>	0.10 <sup>+0.12</sup> <sub>-0.13</sub>	0.93 <sup>+0.56</sup> <sub>-0.40</sub>	0.19 <sup>+0.10</sup> <sub>-0.07</sub>	56.1 <sup>+4.5</sup> <sub>-4.6</sub>	0.70 <sup>+0.06</sup> <sub>-0.07</sub>	1300	15.6 <sup>+0.2</sup> <sub>-0.3</sub>
● GW190701_203306	94.1 <sup>+11.6</sup> <sub>-9.3</sub>	40.2 <sup>+5.2</sup> <sub>-4.7</sub>	53.6 <sup>+11.7</sup> <sub>-7.8</sub>	40.8 <sup>+8.3</sup> <sub>-11.5</sub>	-0.06 <sup>+0.23</sup> <sub>-0.28</sub>	2.14 <sup>+0.79</sup> <sub>-0.73</sub>	0.38 <sup>+0.12</sup> <sub>-0.12</sub>	90.0 <sup>+10.8</sup> <sub>-8.6</sub>	0.67 <sup>+0.09</sup> <sub>-0.12</sub>	45	11.3 <sup>+0.2</sup> <sub>-0.4</sub>
GW190706_222641	101.6 <sup>+17.9</sup> <sub>-13.5</sub>	42.0 <sup>+8.4</sup> <sub>-6.2</sub>	64.0 <sup>+15.2</sup> <sub>-15.2</sub>	38.5 <sup>+12.5</sup> <sub>-12.4</sub>	0.32 <sup>+0.25</sup> <sub>-0.30</sub>	5.07 <sup>+2.57</sup> <sub>-2.11</sub>	0.79 <sup>+0.31</sup> <sub>-0.28</sub>	96.3 <sup>+16.7</sup> <sub>-13.2</sub>	0.80 <sup>+0.08</sup> <sub>-0.17</sub>	610	12.6 <sup>+0.2</sup> <sub>-0.4</sub>
GW190707_093326	20.0 <sup>+1.9</sup> <sub>-1.3</sub>	8.5 <sup>+0.6</sup> <sub>-0.4</sub>	11.5 <sup>+3.3</sup> <sub>-1.7</sub>	8.4 <sup>+1.4</sup> <sub>-1.6</sub>	-0.05 <sup>+0.10</sup> <sub>-0.08</sub>	0.80 <sup>+0.37</sup> <sub>-0.38</sub>	0.16 <sup>+0.07</sup> <sub>-0.07</sub>	19.2 <sup>+1.9</sup> <sub>-1.3</sub>	0.66 <sup>+0.03</sup> <sub>-0.04</sub>	1300	13.3 <sup>+0.2</sup> <sub>-0.4</sub>
GW190708_232457	30.8 <sup>+2.5</sup> <sub>-1.8</sub>	13.1 <sup>+0.9</sup> <sub>-0.6</sub>	17.5 <sup>+4.7</sup> <sub>-2.3</sub>	13.1 <sup>+2.0</sup> <sub>-2.7</sub>	0.02 <sup>+0.10</sup> <sub>-0.08</sub>	0.90 <sup>+0.33</sup> <sub>-0.40</sub>	0.18 <sup>+0.06</sup> <sub>-0.07</sub>	29.4 <sup>+2.5</sup> <sub>-1.7</sub>	0.69 <sup>+0.04</sup> <sub>-0.04</sub>	14000	13.1 <sup>+0.2</sup> <sub>-0.3</sub>
● GW190719_215514	55.8 <sup>+16.3</sup> <sub>-10.0</sub>	22.7 <sup>+5.9</sup> <sub>-3.7</sub>	35.2 <sup>+16.9</sup> <sub>-9.9</sub>	20.2 <sup>+8.1</sup> <sub>-6.5</sub>	0.35 <sup>+0.28</sup> <sub>-0.32</sub>	4.61 <sup>+2.84</sup> <sub>-2.17</sub>	0.73 <sup>+0.35</sup> <sub>-0.30</sub>	52.9 <sup>+15.6</sup> <sub>-9.5</sub>	0.80 <sup>+0.10</sup> <sub>-0.16</sub>	2300	8.3 <sup>+0.3</sup> <sub>-1.0</sub>
● GW190720_000836	21.3 <sup>+4.3</sup> <sub>-2.3</sub>	8.9 <sup>+0.5</sup> <sub>-0.8</sub>	13.3 <sup>+6.6</sup> <sub>-3.9</sub>	7.8 <sup>+2.2</sup> <sub>-2.2</sub>	0.18 <sup>+0.14</sup> <sub>-0.12</sub>	0.81 <sup>+0.71</sup> <sub>-0.33</sub>	0.16 <sup>+0.12</sup> <sub>-0.06</sub>	20.3 <sup>+4.5</sup> <sub>-2.3</sub>	0.72 <sup>+0.06</sup> <sub>-0.05</sub>	510	11.0 <sup>+0.3</sup> <sub>-0.3</sub>
GW190727_060333	65.8 <sup>+10.9</sup> <sub>-7.4</sub>	28.1 <sup>+4.9</sup> <sub>-3.4</sub>	37.2 <sup>+9.4</sup> <sub>-5.9</sub>	28.8 <sup>+5.6</sup> <sub>-7.9</sub>	0.12 <sup>+0.26</sup> <sub>-0.25</sub>	3.60 <sup>+1.56</sup> <sub>-1.51</sub>	0.60 <sup>+0.20</sup> <sub>-0.22</sub>	62.6 <sup>+10.2</sup> <sub>-7.0</sub>	0.73 <sup>+0.10</sup> <sub>-0.10</sub>	860	11.9 <sup>+0.3</sup> <sub>-0.5</sub>
● GW190728_064510	20.5 <sup>+4.5</sup> <sub>-1.3</sub>	8.6 <sup>+0.5</sup> <sub>-0.3</sub>	12.2 <sup>+7.1</sup> <sub>-2.2</sub>	8.1 <sup>+1.7</sup> <sub>-2.6</sub>	0.12 <sup>+0.19</sup> <sub>-0.07</sub>	0.89 <sup>+0.25</sup> <sub>-0.37</sub>	0.18 <sup>+0.05</sup> <sub>-0.07</sub>	19.5 <sup>+4.6</sup> <sub>-1.3</sub>	0.71 <sup>+0.04</sup> <sub>-0.04</sub>	410	13.0 <sup>+0.2</sup> <sub>-0.4</sub>
GW190731_140936	67.1 <sup>+15.3</sup> <sub>-10.2</sub>	28.4 <sup>+6.8</sup> <sub>-4.5</sub>	39.3 <sup>+11.8</sup> <sub>-8.2</sub>	28.0 <sup>+8.9</sup> <sub>-8.4</sub>	0.08 <sup>+0.24</sup> <sub>-0.30</sub>	3.97 <sup>+2.56</sup> <sub>-2.07</sub>	0.65 <sup>+0.32</sup> <sub>-0.30</sub>	63.9 <sup>+14.4</sup> <sub>-9.8</sub>	0.71 <sup>+0.10</sup> <sub>-0.12</sub>	3000	8.6 <sup>+0.2</sup> <sub>-0.5</sub>
GW190803_022701	62.7 <sup>+11.8</sup> <sub>-8.4</sub>	26.7 <sup>+5.2</sup> <sub>-3.8</sub>	36.1 <sup>+10.2</sup> <sub>-6.7</sub>	26.7 <sup>+7.1</sup> <sub>-7.6</sub>	-0.01 <sup>+0.25</sup> <sub>-0.26</sub>	3.69 <sup>+2.04</sup> <sub>-1.69</sub>	0.61 <sup>+0.26</sup> <sub>-0.24</sub>	59.9 <sup>+11.2</sup> <sub>-7.9</sub>	0.69 <sup>+0.10</sup> <sub>-0.11</sub>	1500	8.6 <sup>+0.3</sup> <sub>-0.5</sub>
● GW190814	25.8 <sup>+1.0</sup> <sub>-0.9</sub>	6.09 <sup>+0.06</sup> <sub>-0.06</sub>	23.2 <sup>+1.1</sup> <sub>-1.0</sub>	2.59 <sup>+0.08</sup> <sub>-0.09</sub>	0.00 <sup>+0.06</sup> <sub>-0.06</sub>	0.24 <sup>+0.04</sup> <sub>-0.05</sub>	0.05 <sup>+0.009</sup> <sub>-0.010</sub>	25.6 <sup>+1.0</sup> <sub>-0.9</sub>	0.28 <sup>+0.02</sup> <sub>-0.02</sub>	19	24.9 <sup>+0.1</sup> <sub>-0.2</sub>
● GW190828_063405	57.5 <sup>+7.5</sup> <sub>-4.4</sub>	24.8 <sup>+3.3</sup> <sub>-2.0</sub>	31.8 <sup>+5.8</sup> <sub>-3.9</sub>	25.9 <sup>+4.4</sup> <sub>-4.6</sub>	0.19 <sup>+0.15</sup> <sub>-0.16</sub>	2.22 <sup>+0.63</sup> <sub>-0.95</sub>	0.40 <sup>+0.09</sup> <sub>-0.15</sub>	54.5 <sup>+6.9</sup> <sub>-4.0</sub>	0.76 <sup>+0.06</sup> <sub>-0.07</sub>	520	16.2 <sup>+0.2</sup> <sub>-0.3</sub>
GW190828_065509	34.1 <sup>+5.5</sup> <sub>-4.5</sub>	13.3 <sup>+1.2</sup> <sub>-0.9</sub>	23.8 <sup>+7.2</sup> <sub>-7.0</sub>	10.2 <sup>+3.5</sup> <sub>-2.1</sub>	0.08 <sup>+0.16</sup> <sub>-0.16</sub>	1.66 <sup>+0.63</sup> <sub>-0.61</sub>	0.31 <sup>+0.10</sup> <sub>-0.10</sub>	32.9 <sup>+5.7</sup> <sub>-4.5</sub>	0.65 <sup>+0.09</sup> <sub>-0.08</sub>	640	10.0 <sup>+0.3</sup> <sub>-0.5</sub>
GW190909_114149	71.2 <sup>+54.3</sup> <sub>-15.0</sub>	29.5 <sup>+17.5</sup> <sub>-6.2</sub>	43.2 <sup>+50.7</sup> <sub>-12.2</sub>	27.6 <sup>+13.0</sup> <sub>-10.9</sub>	-0.03 <sup>+0.44</sup> <sub>-0.36</sub>	4.77 <sup>+3.70</sup> <sub>-2.66</sub>	0.75 <sup>+0.45</sup> <sub>-0.37</sub>	68.3 <sup>+52.5</sup> <sub>-14.5</sub>	0.68 <sup>+0.16</sup> <sub>-0.18</sub>	4200	8.1 <sup>+0.4</sup> <sub>-0.7</sub>
GW190910_112807	78.7 <sup>+9.5</sup> <sub>-9.0</sub>	33.9 <sup>+4.3</sup> <sub>-3.9</sub>	43.5 <sup>+7.6</sup> <sub>-6.2</sub>	35.1 <sup>+6.3</sup> <sub>-7.0</sub>	0.02 <sup>+0.19</sup> <sub>-0.18</sub>	1.57 <sup>+1.07</sup> <sub>-0.64</sub>	0.29 <sup>+0.17</sup> <sub>-0.11</sub>	75.0 <sup>+8.7</sup> <sub>-8.5</sub>	0.70 <sup>+0.08</sup> <sub>-0.07</sub>	10000	14.1 <sup>+0.2</sup> <sub>-0.3</sub>
GW190915_235702	59.5 <sup>+7.5</sup> <sub>-6.2</sub>	25.1 <sup>+3.1</sup> <sub>-2.6</sub>	34.9 <sup>+9.5</sup> <sub>-6.2</sub>	24.4 <sup>+5.5</sup> <sub>-6.0</sub>	0.03 <sup>+0.19</sup> <sub>-0.24</sub>	1.70 <sup>+0.71</sup> <sub>-0.64</sub>	0.32 <sup>+0.11</sup> <sub>-0.11</sub>	56.8 <sup>+7.1</sup> <sub>-5.8</sub>	0.71 <sup>+0.09</sup> <sub>-0.11</sub>	380	13.6 <sup>+0.2</sup> <sub>-0.3</sub>
GW190924_021846	13.9 <sup>+5.1</sup> <sub>-0.9</sub>	5.8 <sup>+2.0</sup> <sub>-1.2</sub>	8.8 <sup>+7.0</sup> <sub>-2.0</sub>	5.0 <sup>+1.3</sup> <sub>-1.9</sub>	0.03 <sup>+0.30</sup> <sub>-0.09</sub>	0.57 <sup>+0.22</sup> <sub>-0.30</sub>	0.12 <sup>+0.04</sup> <sub>-0.05</sub>	13.3 <sup>+5.2</sup> <sub>-1.9</sub>	0.67 <sup>+0.05</sup> <sub>-0.05</sub>	380	11.5 <sup>+0.3</sup> <sub>-0.4</sub>
GW190929_012149	90.6 <sup>+21.2</sup> <sub>-14.1</sub>	34.3 <sup>+8.6</sup> <sub>-6.5</sub>	64.7 <sup>+22.4</sup> <sub>-18.9</sub>	25.7 <sup>+14.4</sup> <sub>-9.7</sub>	0.03 <sup>+0.27</sup> <sub>-0.27</sub>	3.68 <sup>+2.98</sup> <sub>-1.68</sub>	0.61 <sup>+0.38</sup> <sub>-0.24</sub>	87.5 <sup>+20.7</sup> <sub>-14.1</sub>	0.64 <sup>+0.17</sup> <sub>-0.23</sub>	1800	9.8 <sup>+0.8</sup> <sub>-0.6</sub>
GW190930_133541	20.3 <sup>+0.9</sup> <sub>-1.5</sub>	8.5 <sup>+0.5</sup> <sub>-0.5</sub>	12.3 <sup>+2.5</sup> <sub>-2.3</sub>	7.8 <sup>+1.7</sup> <sub>-3.3</sub>	0.14 <sup>+0.31</sup> <sub>-0.15</sub>	0.78 <sup>+0.37</sup> <sub>-0.33</sub>	0.16 <sup>+0.07</sup> <sub>-0.06</sub>	19.3 <sup>+9.3</sup> <sub>-1.5</sub>	0.72 <sup>+0.07</sup> <sub>-0.06</sub>	1800	9.5 <sup>+0.3</sup> <sub>-0.5</sub>



# Hierarchical Bayesian inference and ML

Event parameters  $\vec{\theta}$

$m_1$        $m_2$        $\chi_{\text{eff}}$        $z$

Population Hyperparameters  $\vec{\lambda}$

$M_c$        $\sigma$        $f_{\text{PBH}}$        $z_{\text{cut-off}}$

$$p(\vec{\lambda}|\vec{d}) \propto p(\vec{\lambda}) \int d\vec{\theta} p(\vec{d}|\vec{\theta}) p_{\text{pop}}(\vec{\theta}|\vec{\lambda})$$

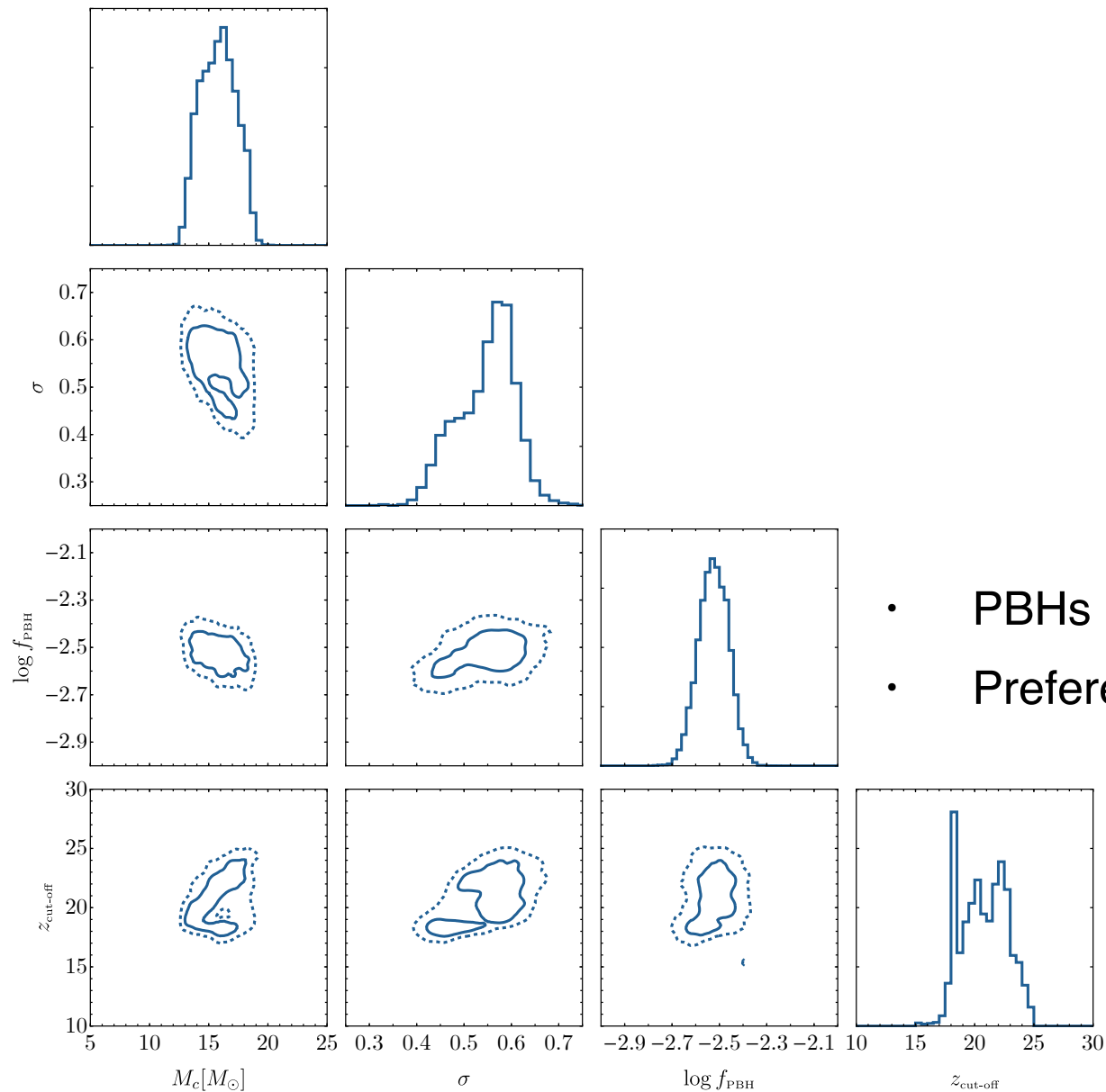
Posterior  
distribution

Hyperparameter  
prior

Single event  
likelihood

Population  
likelihood (ML)

# Population posterior distribution



$$M_c \simeq 16M_\odot$$

$$\sigma \simeq 0.56$$

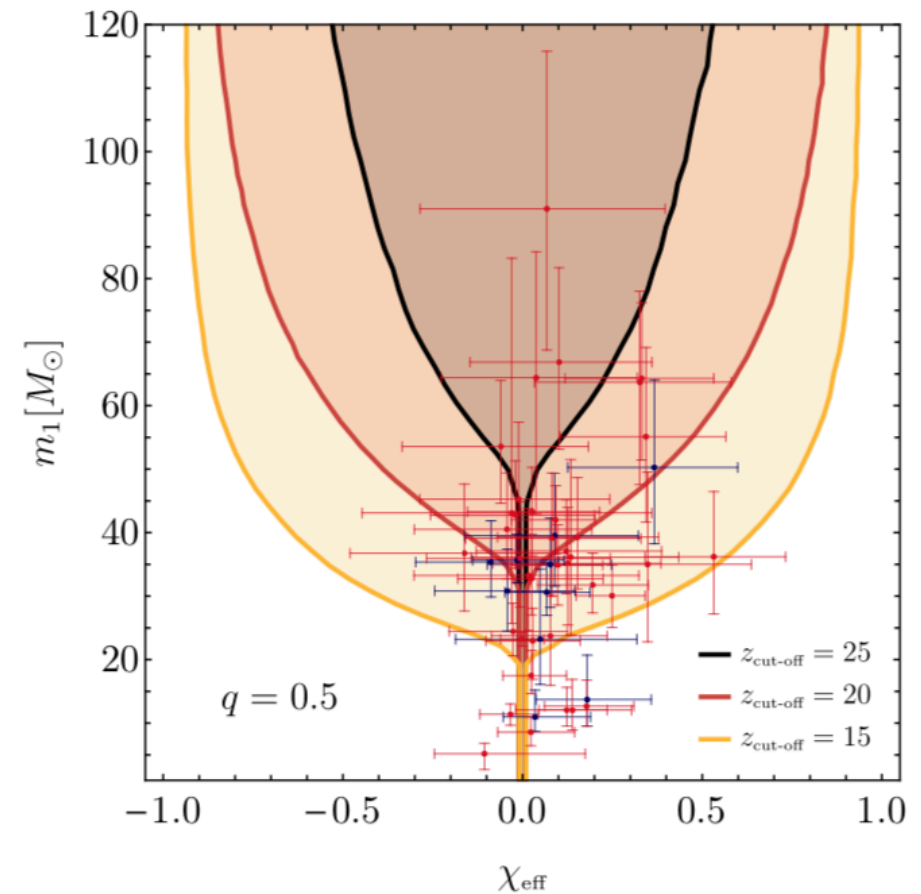
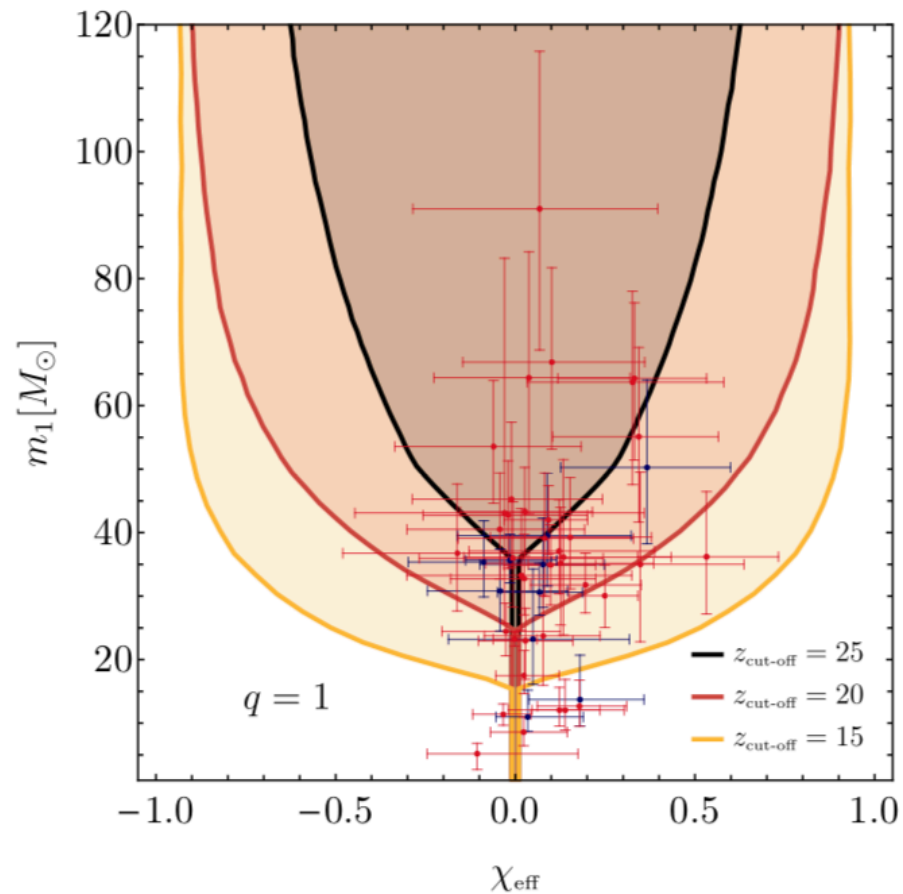
$$f_{\text{PBH}} \simeq 3 \cdot 10^{-3}$$

$$z_{\text{cut-off}} \simeq 20$$

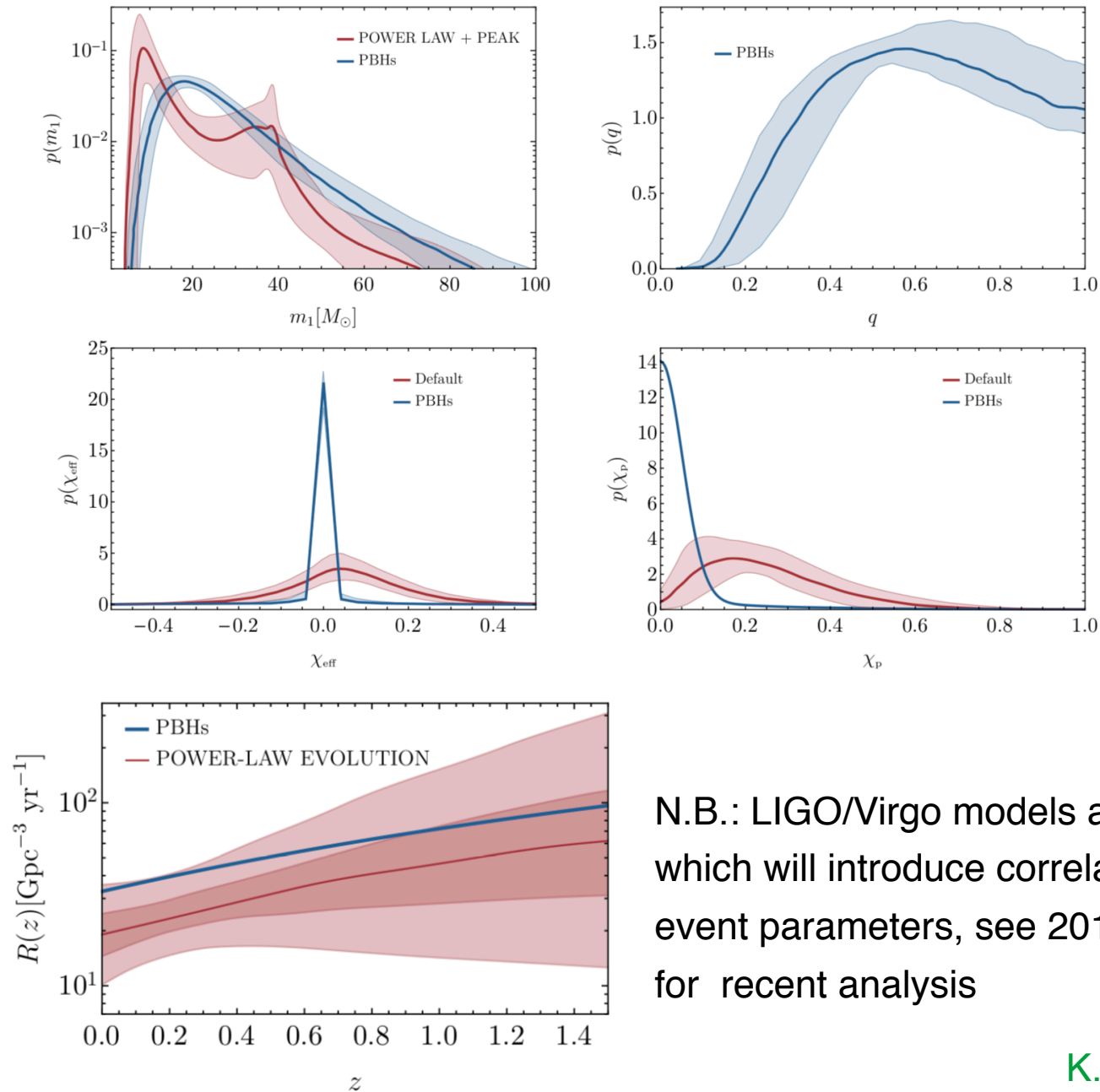
- PBHs are not the dark matter
- Preference for a mild accretion



# Prediction for the effective spin distribution

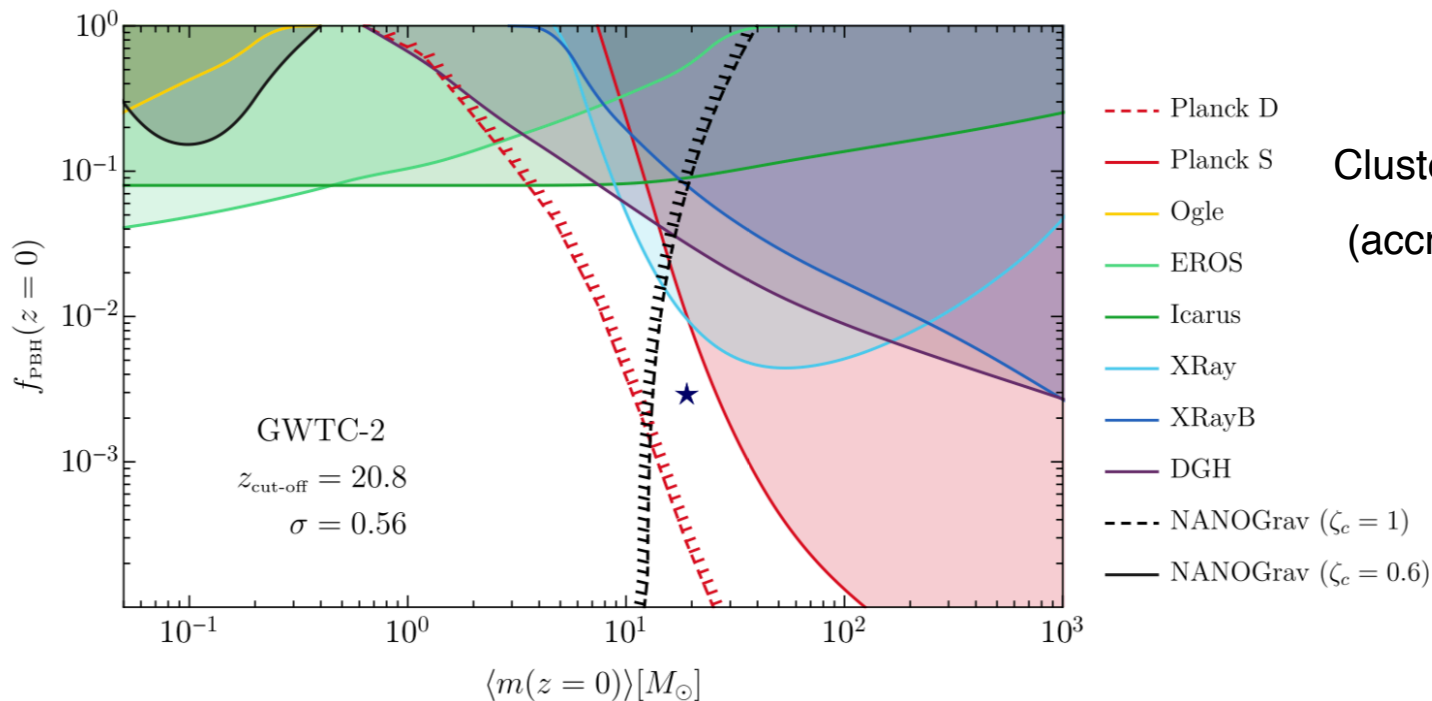
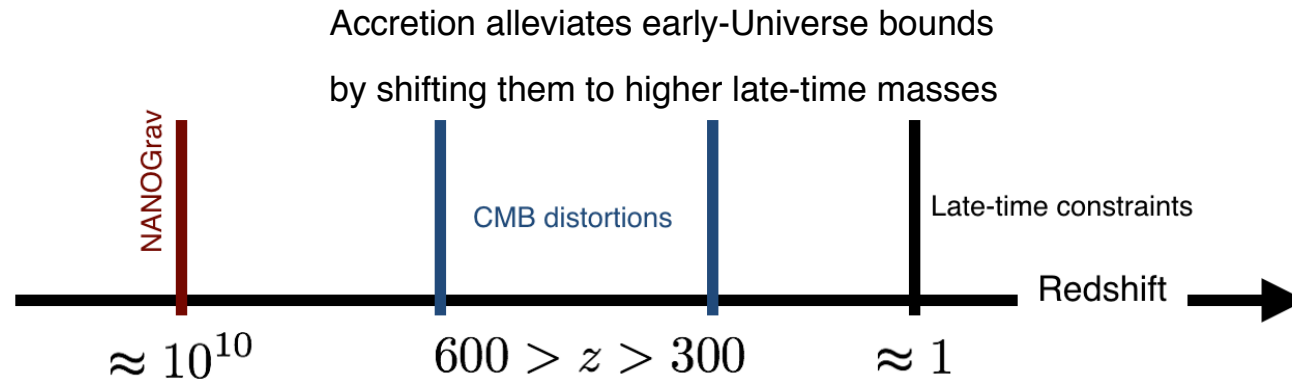


# PBH population properties



N.B.: LIGO/Virgo models are not *astrophysical*, which will introduce correlations among the event parameters, see 2011.03564, 2011.05332 for recent analysis

# The LIGO/Virgo events can be all PBHs not the dark matter in the universe



Clustering may not change the result  
(accretion bounds at high redshifts)

$$f_{\text{PBH}} \lesssim z \cdot 10^{-4}$$

# Conclusions

All GWTC-2 events may be PBHs, but PBHs are not the dark matter in the LIGO/Virgo mass band