

Primordial Black Holes and Gravitational Waves

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inflationary model
construction

Curvature perturbation to PBH

- gradient expansion/separate universe approach

$$6H^2(t, x) + R^{(3)}(t, x) = 16\pi G\rho(t, x) + \dots \quad \text{Hamiltonian constraint (Friedmann eq.)}$$

$$\begin{aligned} \rightarrow R^{(3)} &\approx -\frac{4}{a^2} \nabla^2 \mathcal{R}_c \approx \frac{8\pi G}{3} \delta\rho_c & \rightarrow \frac{\delta\rho_c}{\rho} &\approx \mathcal{R}_c \quad \text{at} \quad \frac{k^2}{a^2} = H^2 \end{aligned}$$

$$\begin{array}{ccc} R^{(3)} \simeq 0 & \xleftrightarrow[H^{-1} = a/k]{R^{(3)} \sim H^2} & \text{formation of a closed universe} \end{array}$$

- If $R^{(3)} \sim H^2$ ($\Leftrightarrow \delta\rho_c / \rho \sim 1$), it collapses to form BH

Young, Byrnes & MS '14

$$M_{\text{PBH}} \sim \rho H^{-3} \sim 10^5 M_{\odot} \left(\frac{t}{1\text{s}}\right) \sim 20 M_{\odot} \left(\frac{k}{1\text{pc}^{-1}}\right)^{-2}$$

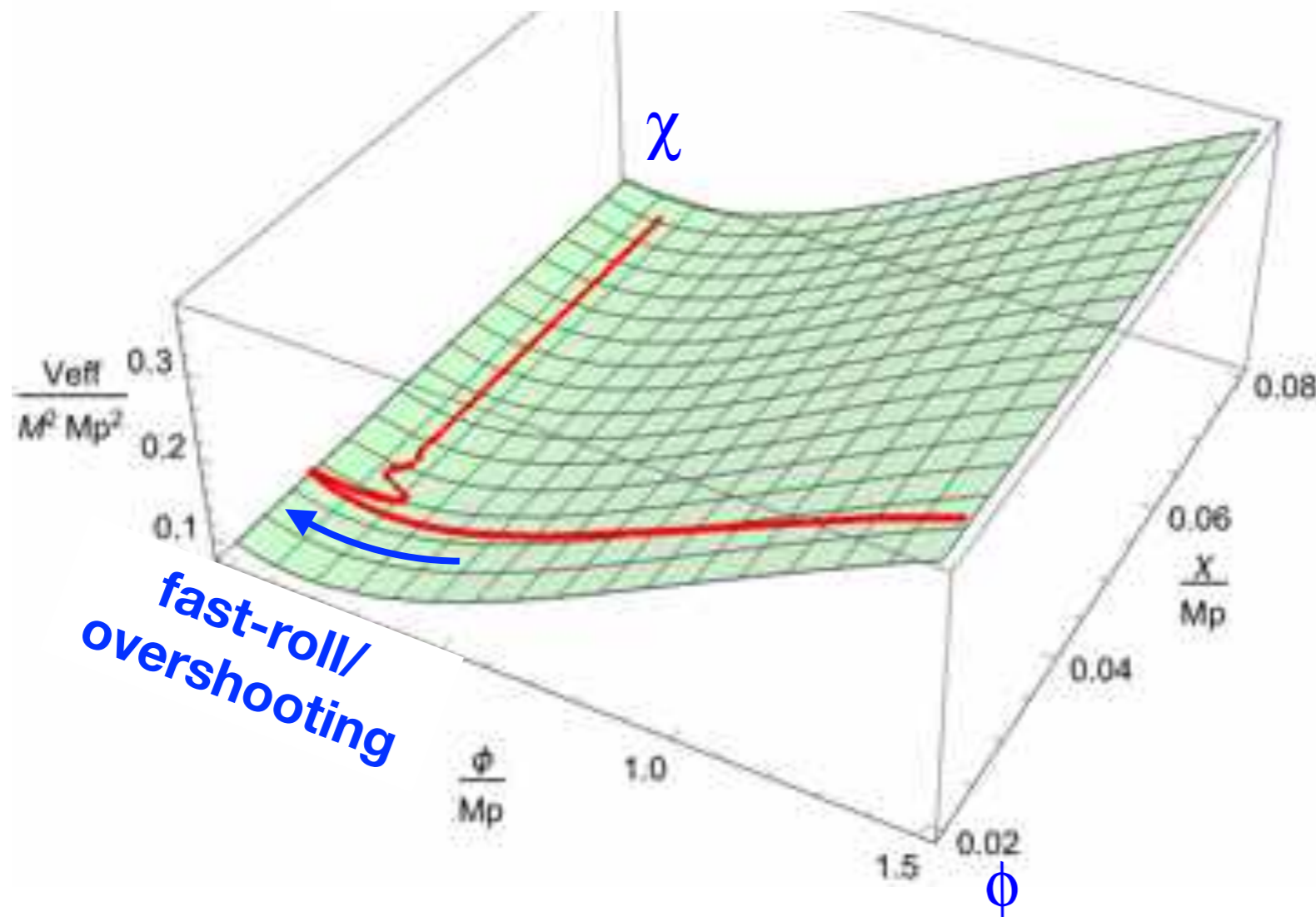
- Spins of PBHs are expected to be very small

de Luca et al. 2019, Harada et al. 2020, ...

Model 1: Scalaron+ χ model

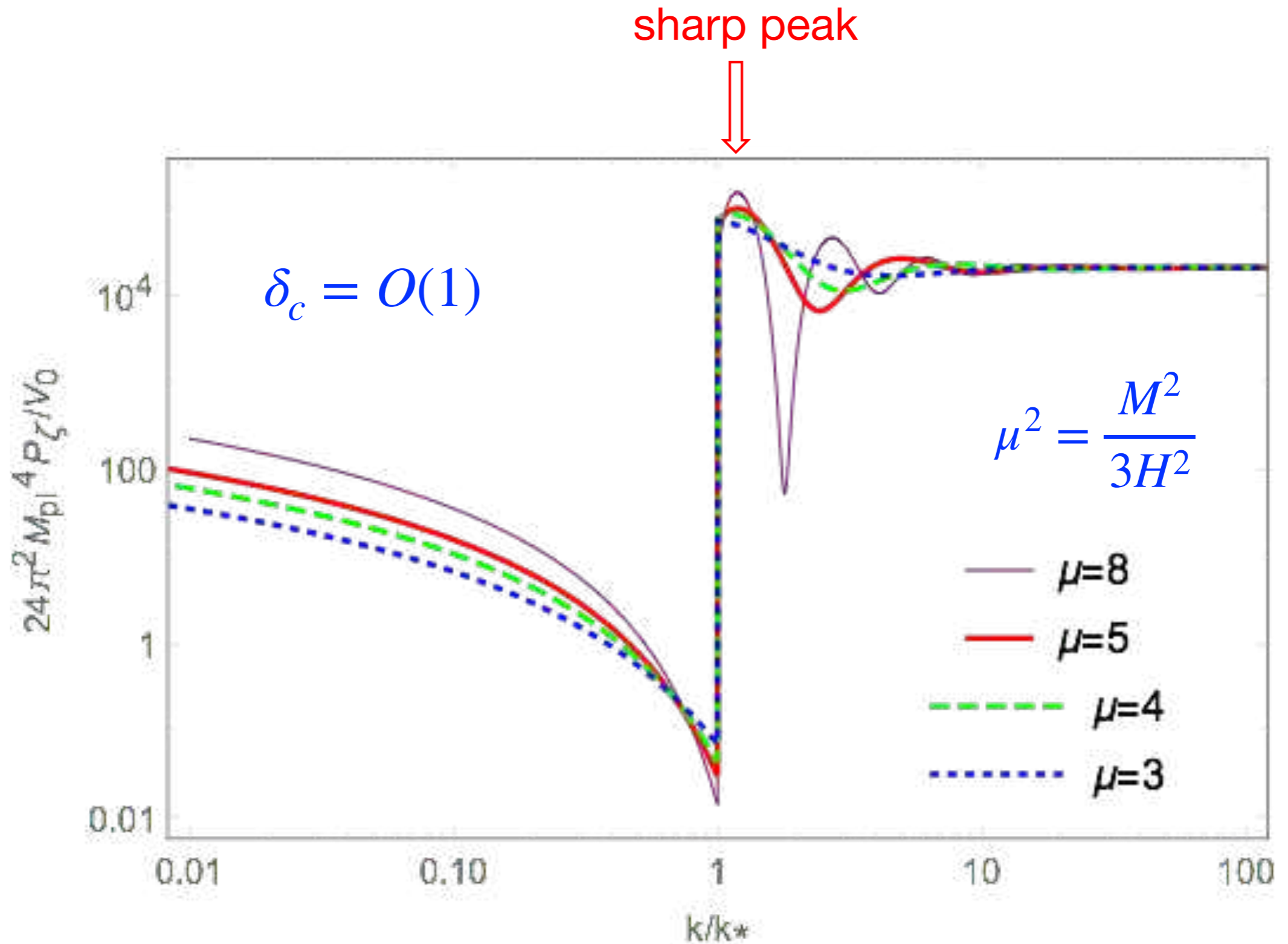
Pi, Zhang, Huang & MS '17

$$S_J = \int d^4x \sqrt{-g} \left\{ \frac{M_{\text{Pl}}^2}{2} \left(R + \frac{R^2}{6M^2} \right) - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) - \frac{1}{2} \xi R \chi^2 \right\}.$$



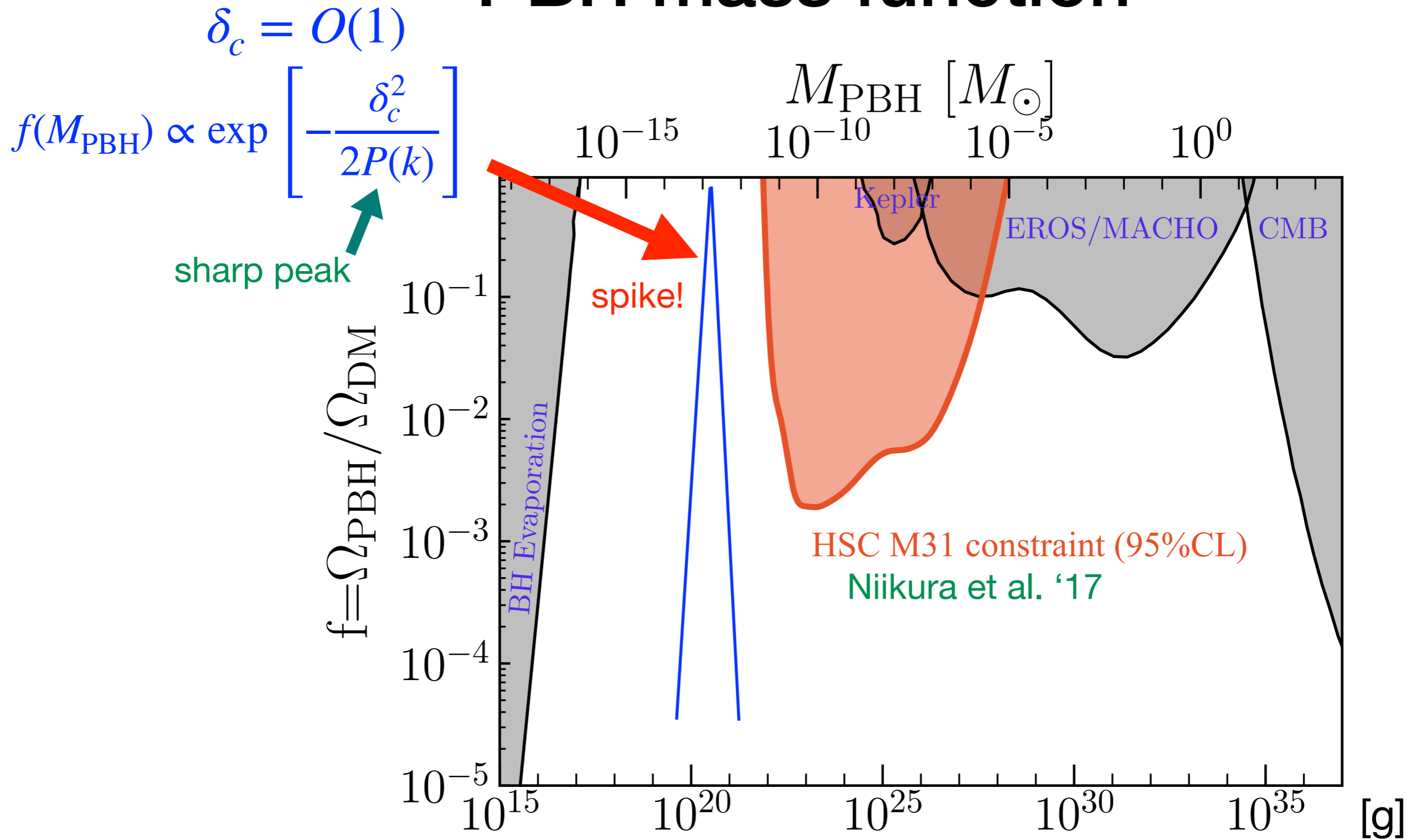
2-stage inflation

- Scalaron ϕ becomes massive at the end of the 1st stage.
- Field χ plays the role of inflaton at the 2nd stage.



scalaron + x model can produce a sharp peak in the curvature perturbation spectrum at small scale
 non-Gaussianity is found to be small in this model

PBH mass function

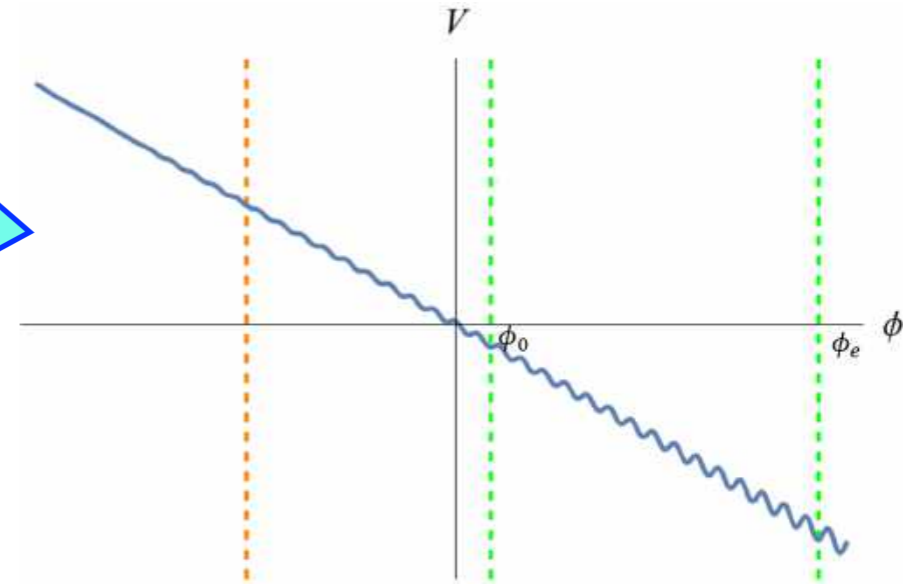
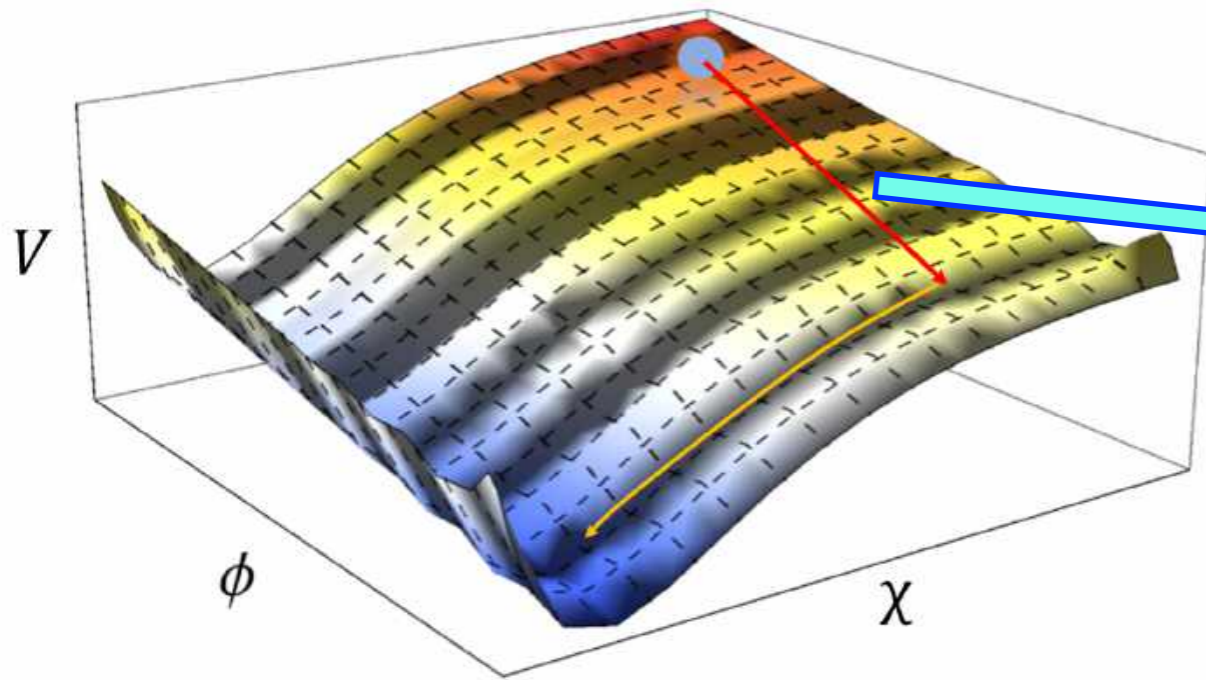


scalaron+x model can realize

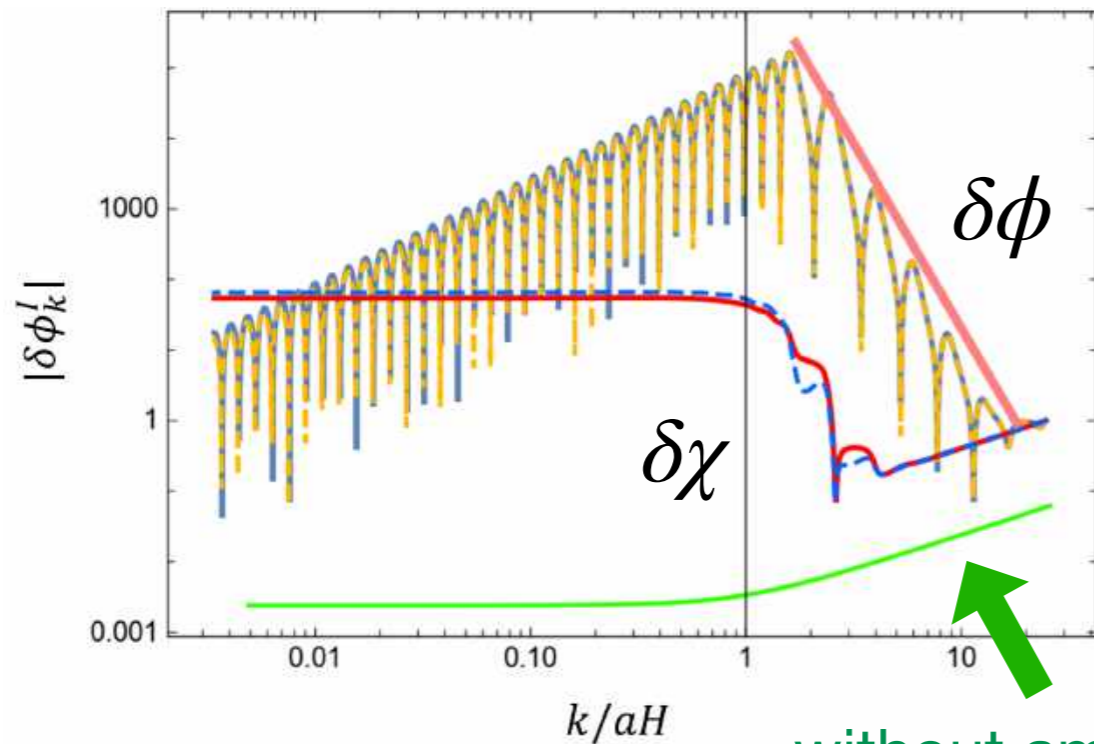
PBH=CDM scenario with a monochromatic PBH mass!

Model 2: Resonant Amplification Model

Z. Zhou, J. Jiang, Y. Cai, MS & S. Pi, 2020



$$V(\phi) \sim \Lambda(\phi) \cos\left(\frac{\phi}{f_a}\right) \text{ with growing } \Lambda(\phi)$$



$\delta\phi$: amplified by oscillating potential

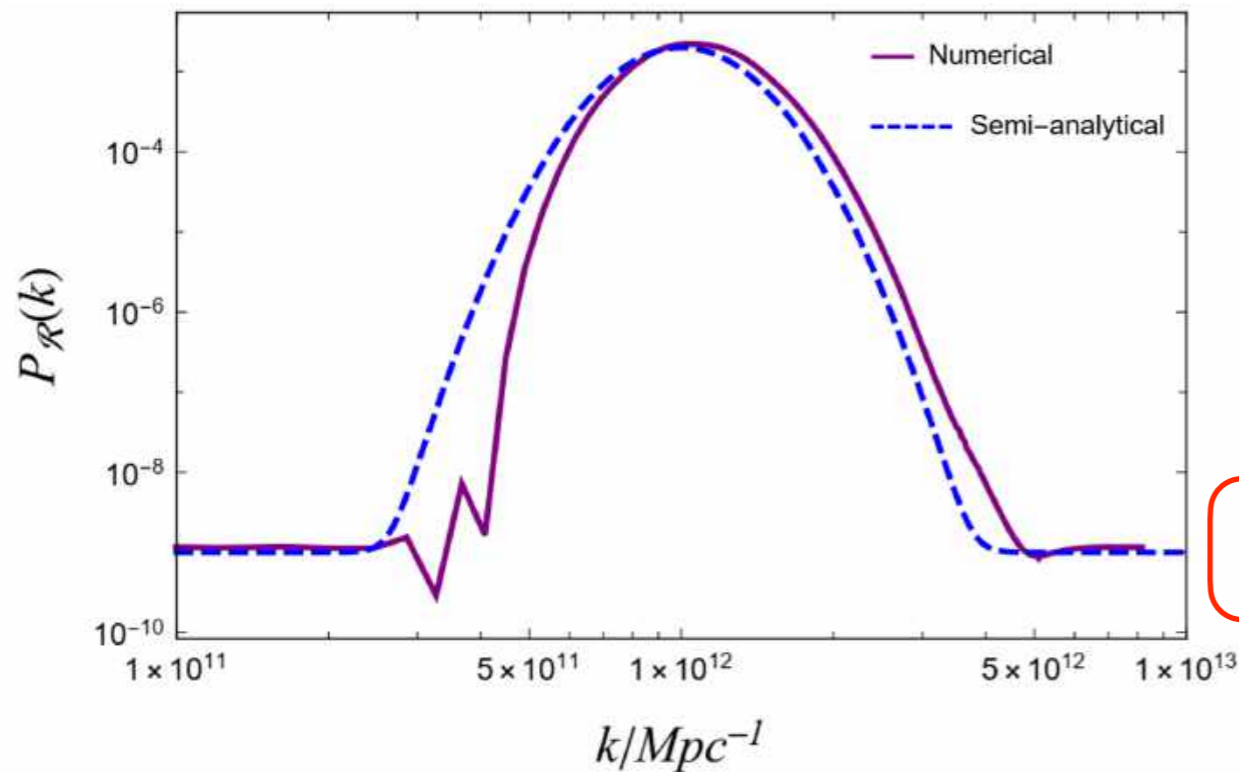
$\delta\chi$: amplified through coupling to $\delta\phi$



This leads to enhancement of curvature perturbation

without amplification

Curvature perturbation spectrum and PBH mass function: an example



$$P_{\mathcal{R}}(k) = \frac{H^2}{8\pi^2 M_p^2 \epsilon_{\chi\chi}} \mathcal{A}^2(k)$$

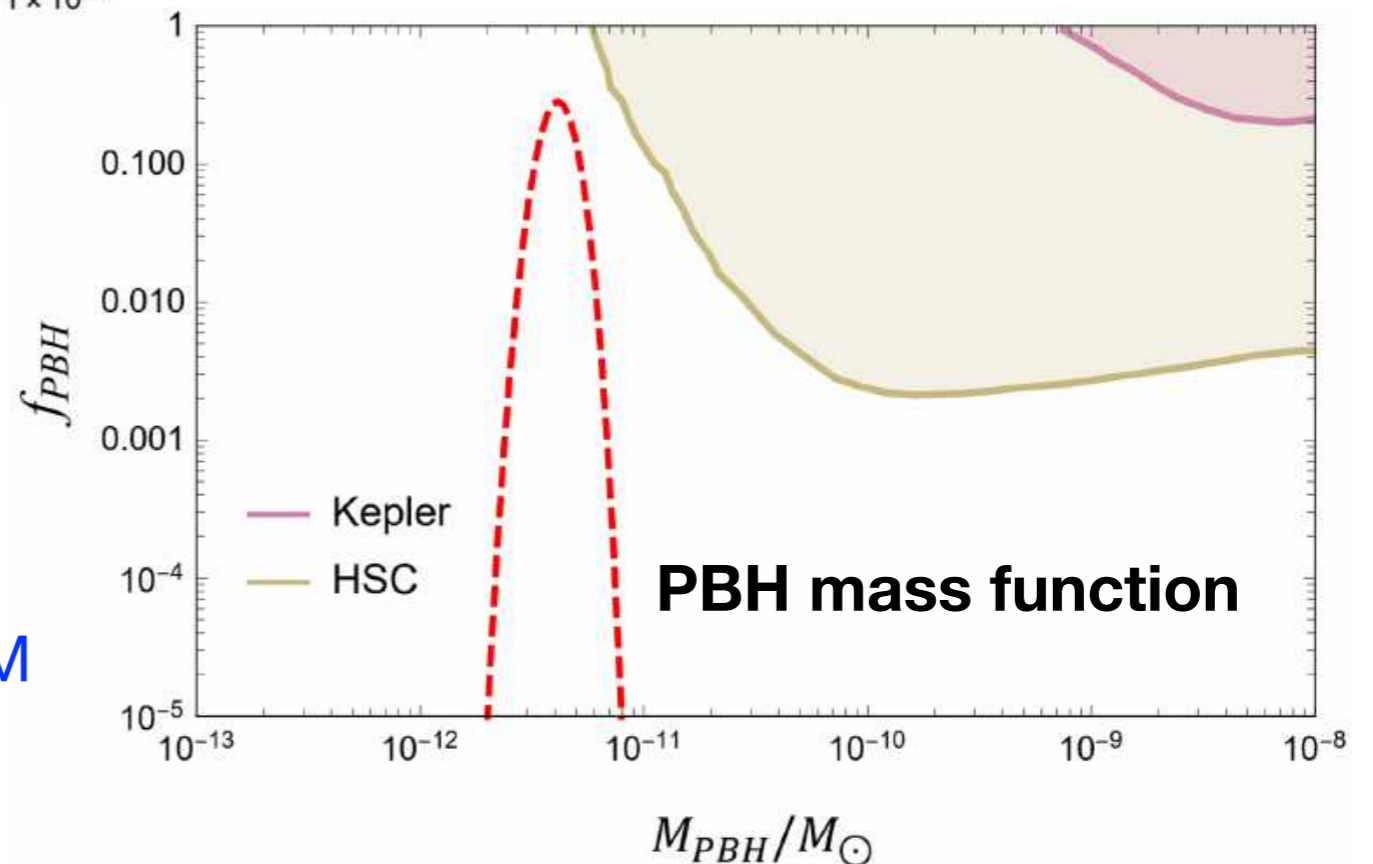
$$\mathcal{A}^2(k) = 1 + \mathcal{A}^2(k_*) \exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2}\right)$$

amplification factor $\sim 10^6$

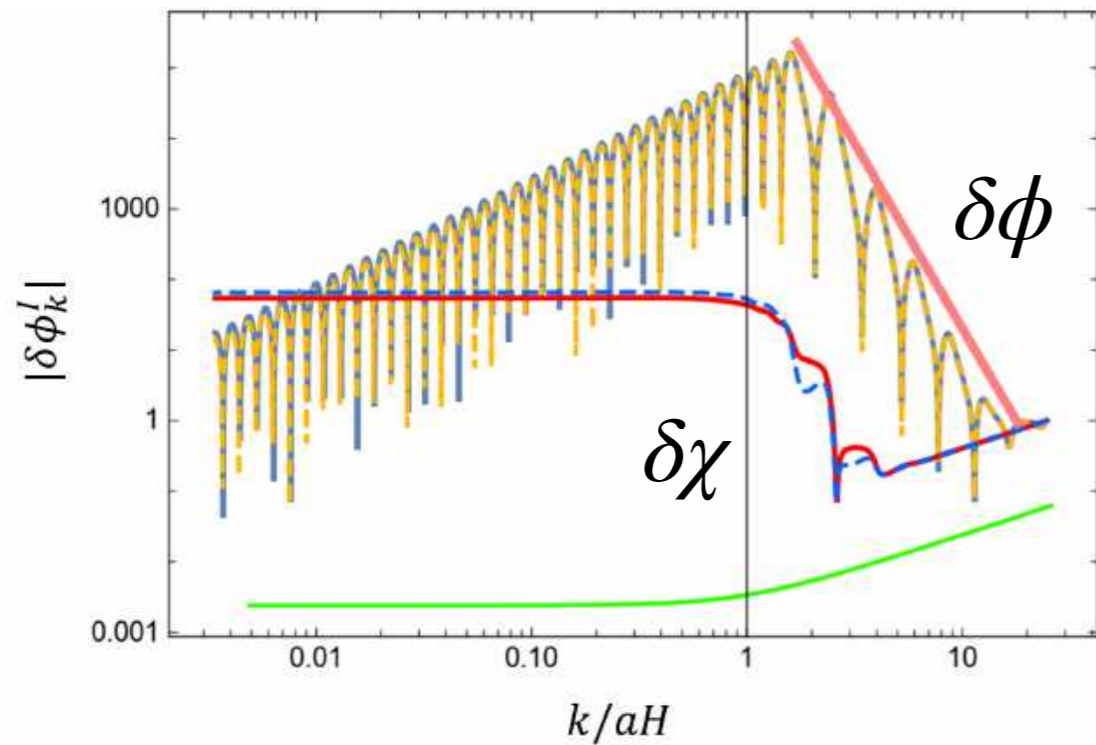
very good fit with log-normal function

$\Delta = O(1)$ for typical values of model parameters

PBHs can account for CDM



GWs Generated during Inflation



This leads to GW generation during inflation

$\delta\phi$: amplified by oscillating potential

$\delta\chi$: amplified through coupling to $\delta\phi$

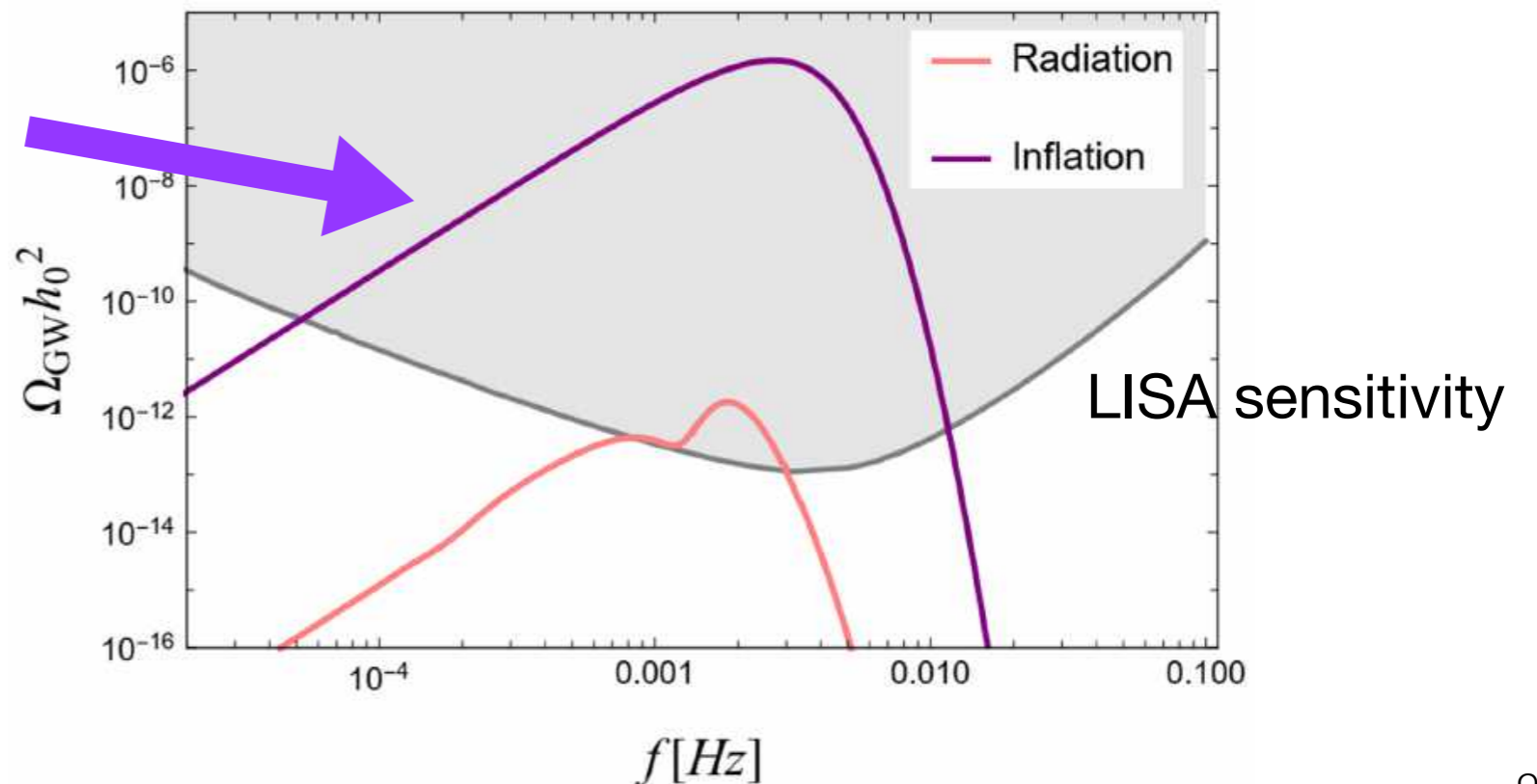
This leads to enhancement of curvature perturbation

$$\square h_{ij} \sim G\partial_i\delta\phi\partial_j\delta\phi$$

GW probes $\delta\phi$



PBH probes $\delta\chi$



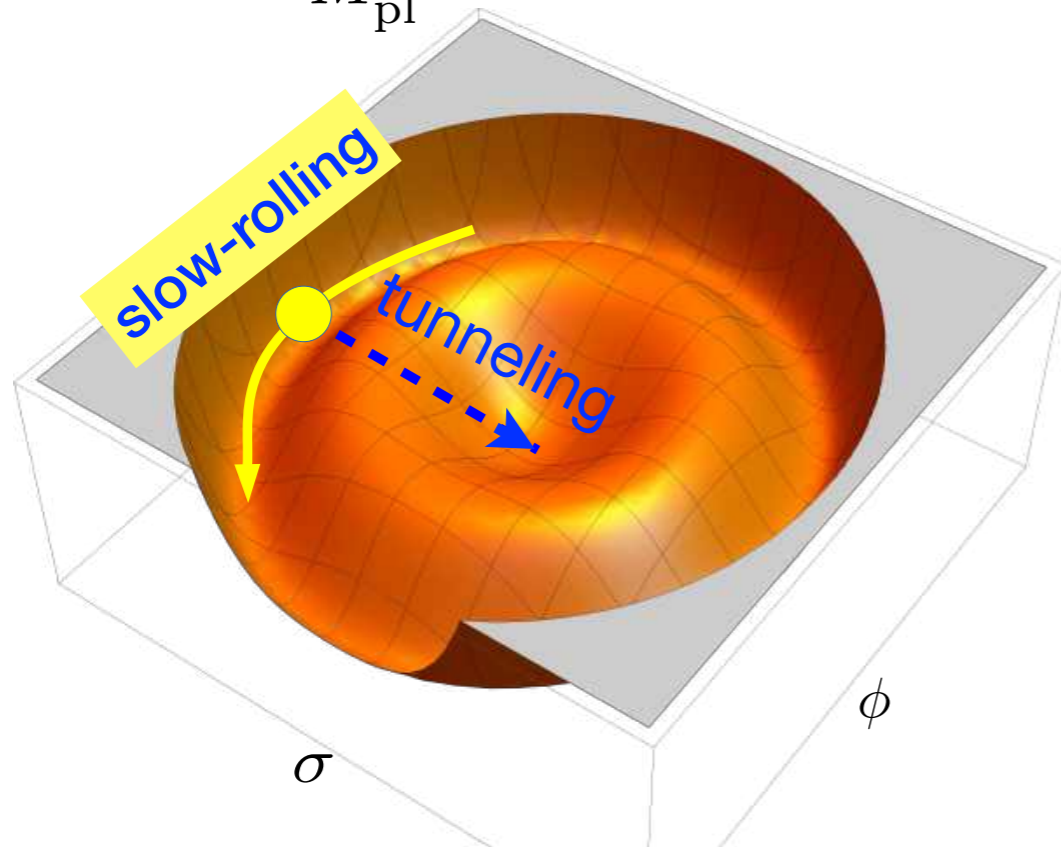
Model 3: PBH-as-MVP scenario

PBH formation during inflation due to vacuum tunneling

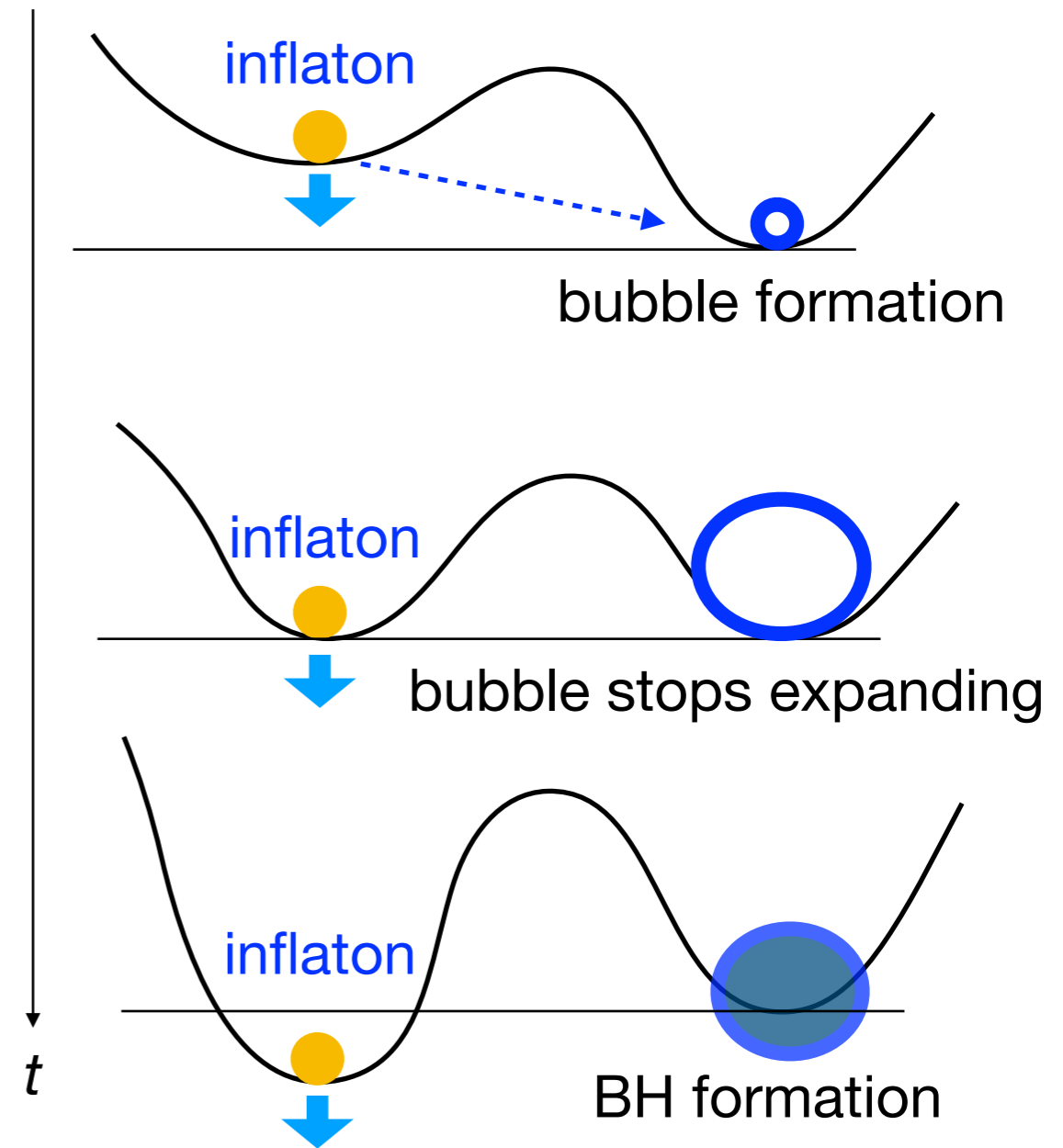
Garriga, Vilenkin & Zhang '15, Deng & Vilenkin '17,...

example:

$$V(\phi, \sigma) = m^2(\phi^2 + \sigma^2) - a(\phi^2 + \sigma^2)^2 + \frac{c}{M_{\text{pl}}^2}(\phi^2 + \sigma^2)^3 + gM_{\text{pl}}^4 \sin\left(\frac{\phi}{fM_{\text{pl}}}\right)$$

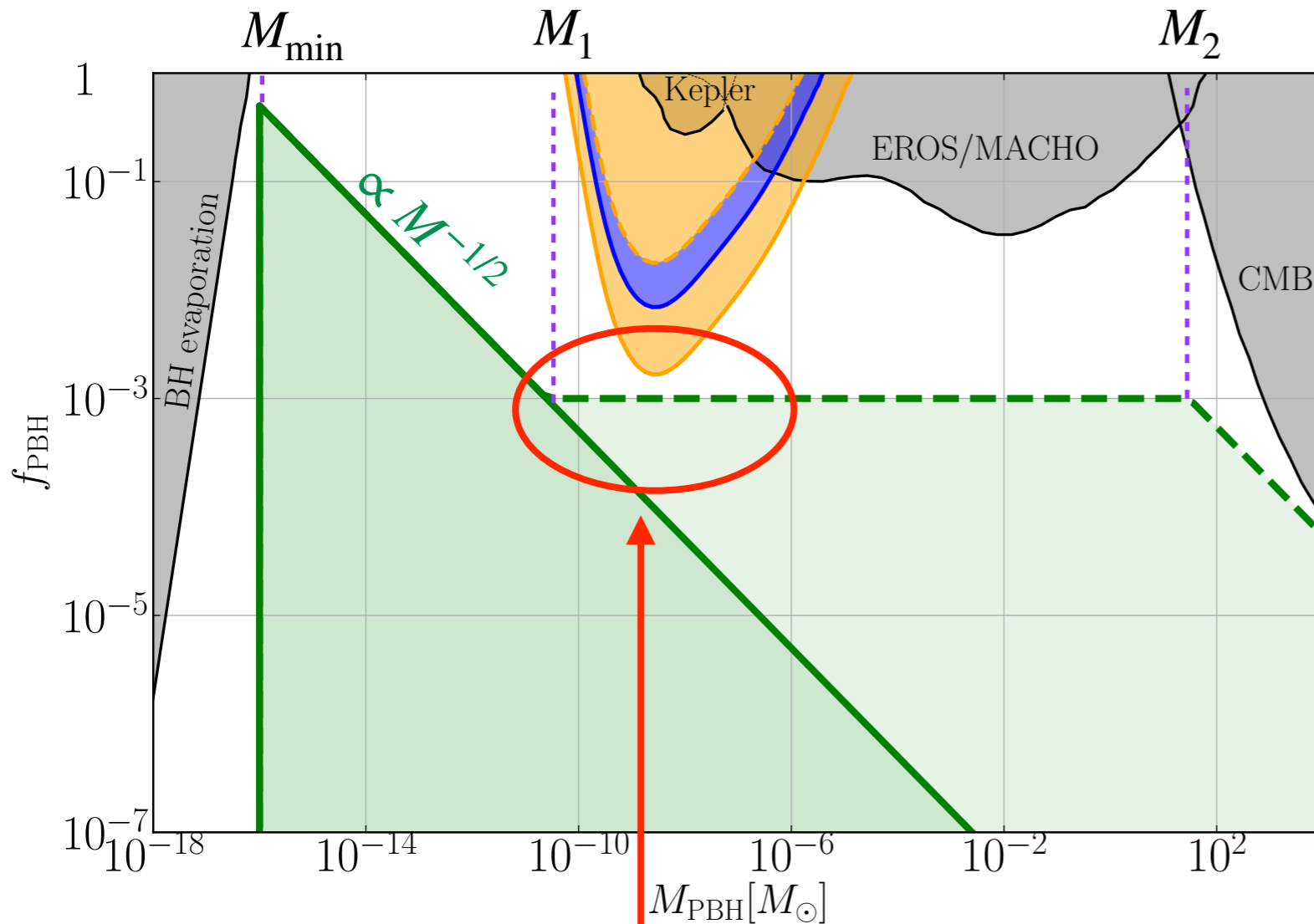


can probe multiverse!



PBH mass function

Kusenko, MS, Sugiyama, Takada, Takhistov & Vitagliano '20



may be tested by Subaru HSC

Subaru accepted our proposal!
obs is ongoing now!

- for scale M re-entering horizon during radiation-dom stage

$$f(M) = \lambda \left(\frac{M}{M_{\min}} \right)^{-1/2} \quad : M_{\min} < M$$

$M \simeq M_{\min}$... CDM

- if there is an intermediate matter-dom stage

$$f(M) = \lambda \left(\frac{M_1}{M_{\min}} \right)^{-1/2} \quad : M_1 < M < M_2$$

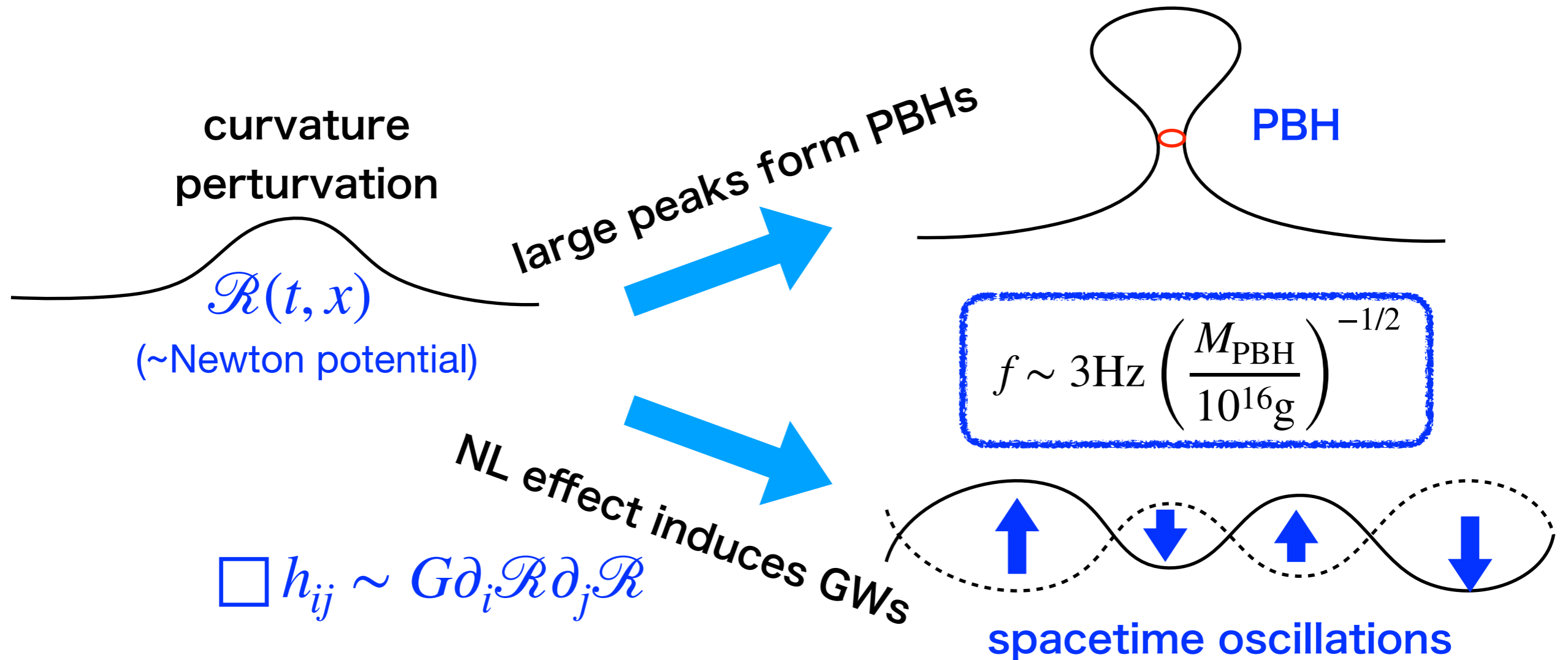
$M \simeq M_2$... LIGO BHs

$$f(M) = \lambda \left(\frac{M_2}{M_1} \right)^{1/2} \left(\frac{M}{M_{\min}} \right)^{-1/2} \quad : M_2 < M$$

$M \gg M_2$... SMBHs

GWs from Large Curvature Perturbation

GWs can capture PBHs!



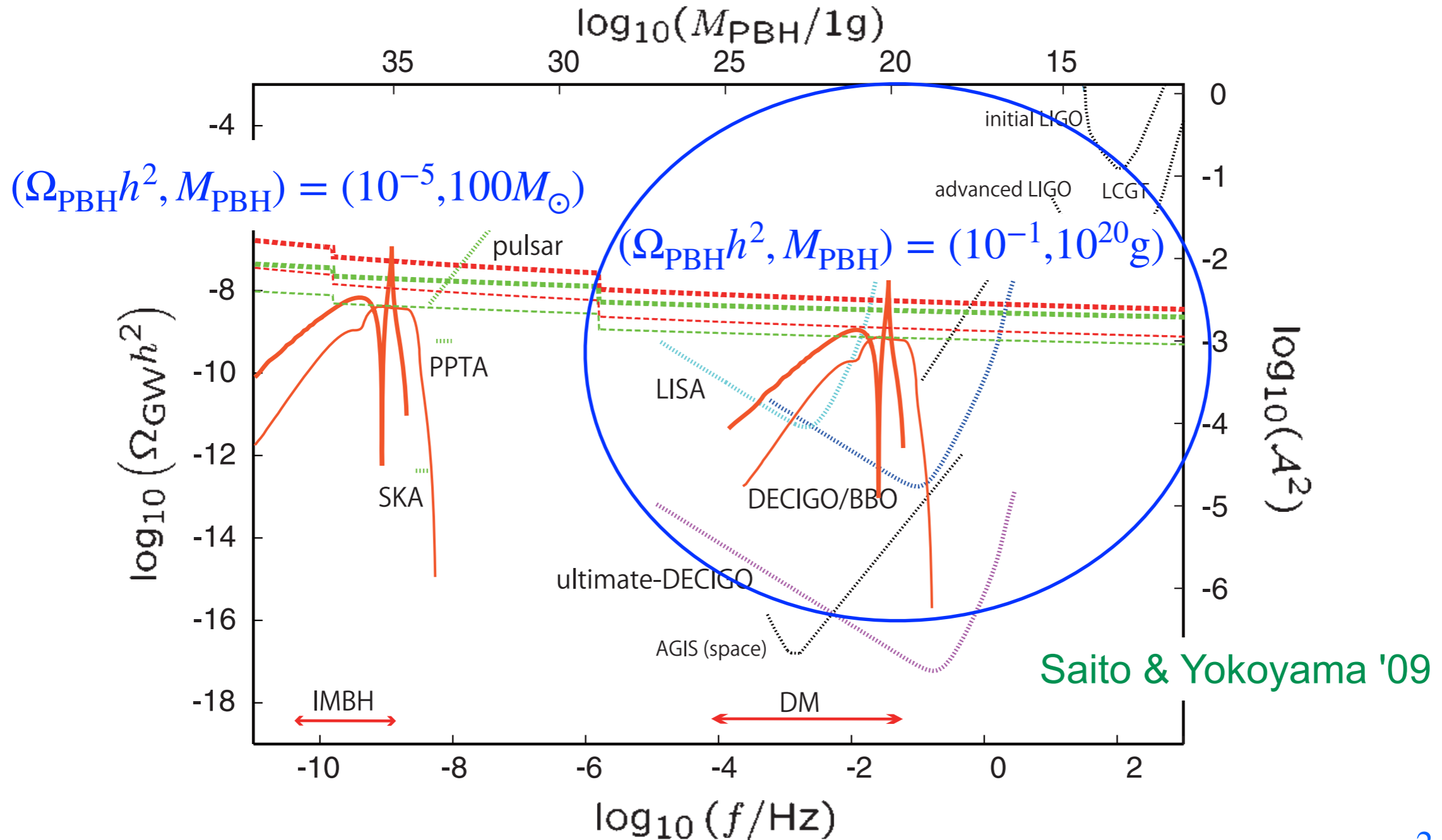
PBHs = CDM with $M_{\text{PBH}} \sim 10^{21}\text{g}$
generates GWs with $f \sim 10^{-3}\text{Hz}$

Background GWs
at LISA band!

cf. $f \sim 10^{-9}\text{Hz}$ for $M_{\text{PBH}} \sim 1-10 M_{\odot}$
~ Pulsar Timing Array band

LIGO-Virgo(-KAGRA): 10-1000 Hz
too high...

Gaussian Case

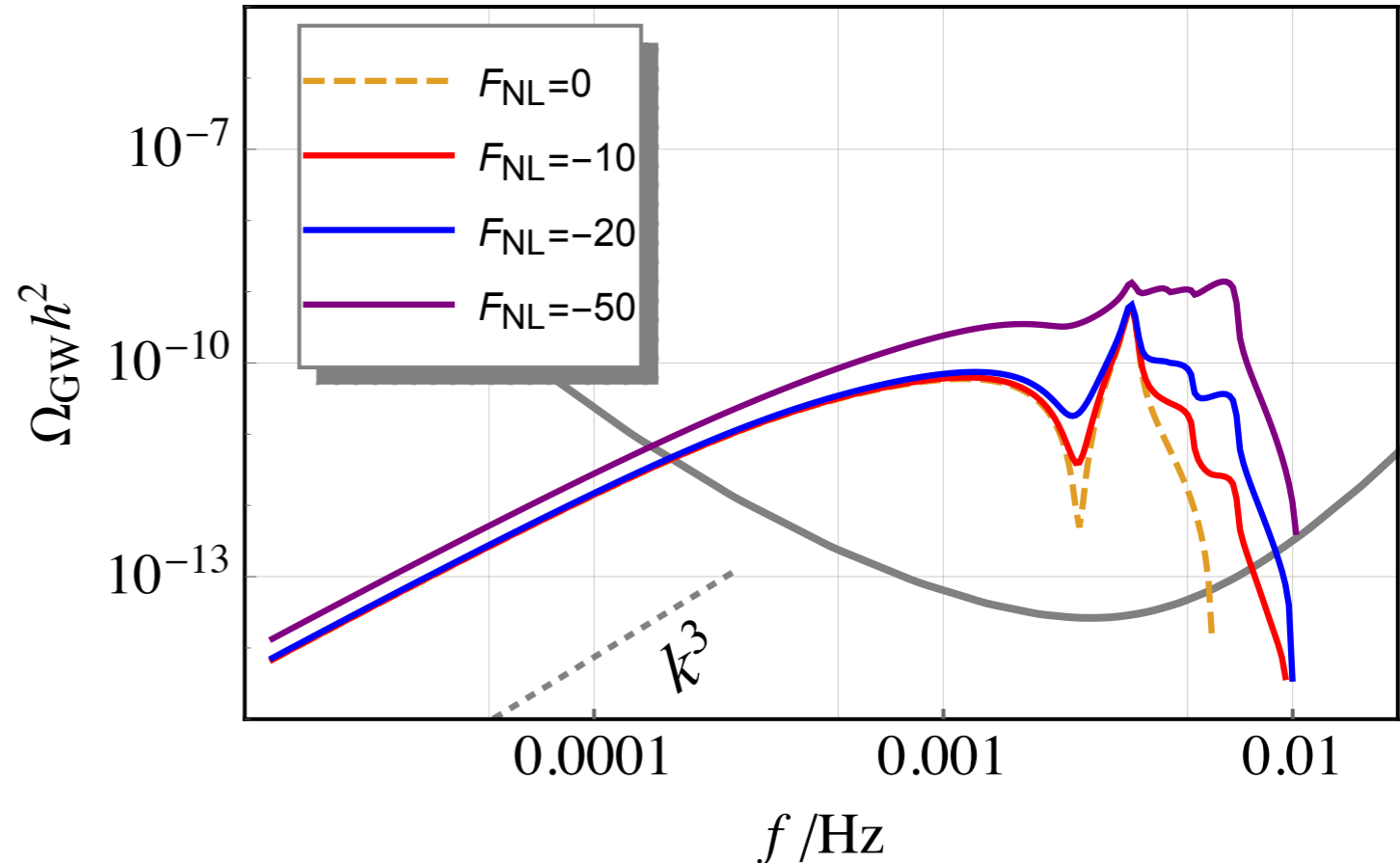
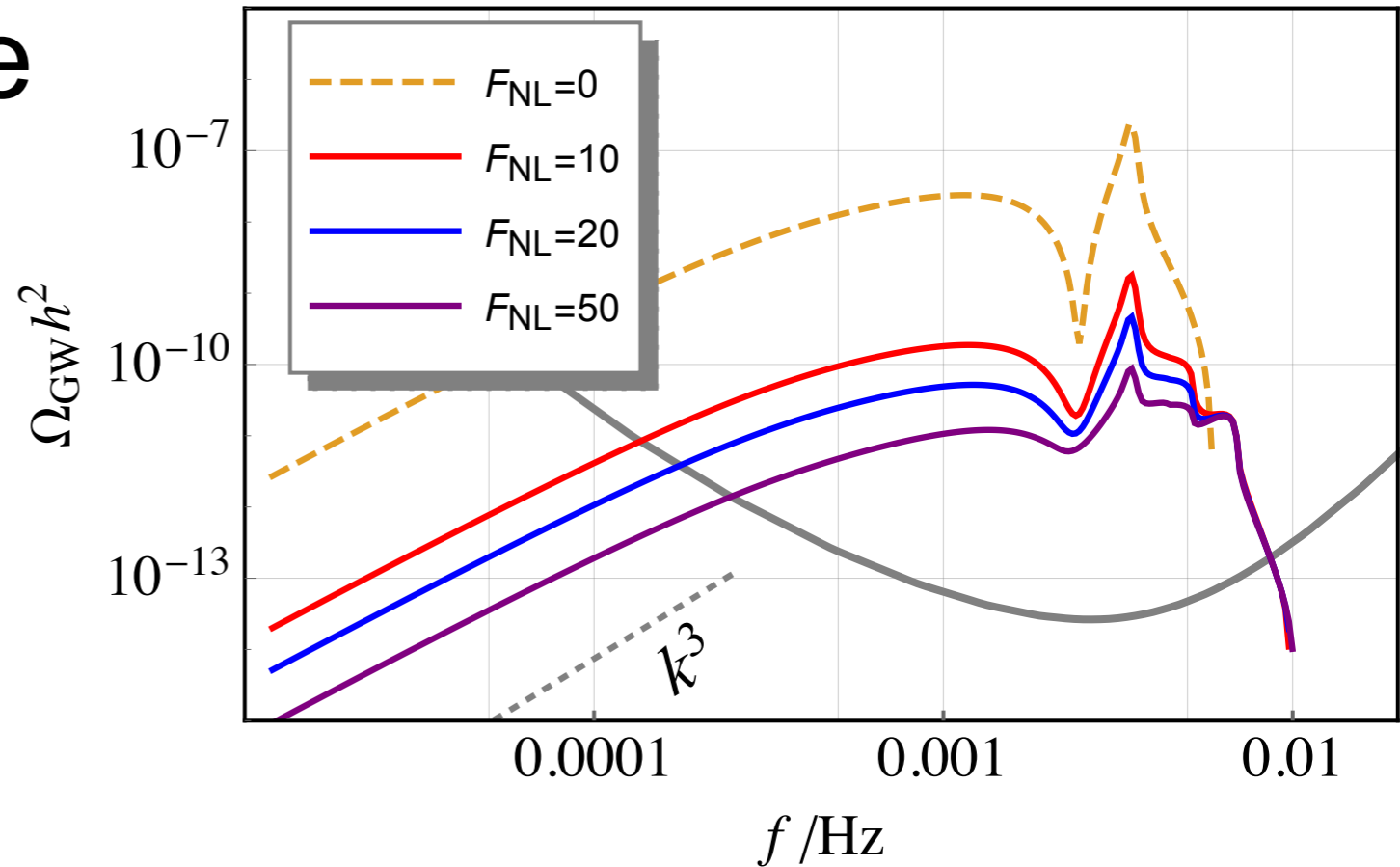


$$M_{\text{PBH}} \sim 0.1 M_{\odot} \left(\frac{1 \text{GeV}}{T} \right)^2 \sim 10 M_{\odot} \left(\frac{1 \text{pc}^{-1}}{k} \right)^2$$

Non-Gaussian Case

$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) + F_{\text{NL}} \left[\mathcal{R}_g^2(\mathbf{x}) - \langle \mathcal{R}_g^2(\mathbf{x}) \rangle \right].$$

- Up: $F_{\text{NL}} > 0$, and we fix the PBH abundance to be 1.
- Down: $F_{\text{NL}} < 0$, and we fix the peak amplitude to be $\mathcal{A}_{\mathcal{R}} = 10^{-2}$
- Frequency: PBH window \leftrightarrow LISA band
- GWs will be detected if BHs=CDM
- Conversely, if LISA doesn't see GWs, PBHs \neq CDM



Summary

- 2-field inflation models can produce abundant PBHs as well as GWs.
- If $\text{PBHs} = \text{CDM}$, $M_{\text{PBH}} \sim 10^{19-22} \text{g}$, induced GWs must be detectable by LISA, indep of non-Gaussianity.
- Conversely if LISA doesn't detect the induced GWs, it **constrains the PBH abundances of $M_{\text{PBH}} \sim 10^{19-22} \text{g}$** , where no other experiment can explore.
- If **resonant amplification** occurs, GWs generated during inflation can dominate GW background: **PBHs and GWs** give **complimentary info** of the 2-fields.
- PBHs from **vacuum tunneling** during inflation may **explain everything!**