Open Quantum Mechanics for Cosmological Observers

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1 Introduction

The past few decades have heralded a revolution in our understanding of cosmology. The discovery of accelerated expansion, precision analysis of the fluctuations in the cosmic microwave background, galaxy surveys and, more recently, the JWST data have provided us with great avenues to explore the history of our universe. Gravitational wave astronomy has also opened up an entirely unexplored territory by allowing direct observations of electromagnetically dark or opaque regions of the cosmos, as nothing can hide from gravity.

This abundance of experimental data has allowed us to build an increasingly precise timeline for the universe. The standard cosmological model, the Λ-CDM, tells the story [1]: It all started with an era of exponentially rapid expansion that washed out any erstwhile structure, leaving behind only tiny quantum fluctuations. These fluctuations then seeded the early perturbations of the universe, which, over time, triggered a much more dramatic series of events—giving rise to matter-antimatter asymmetry, nuclei, atoms and eventually stars, galaxies and kittens. Two new protagonists orchestrated the dynamics of this subsequent era: dark matter and dark energy. Whereas dark matter provided the additional mass required for structure formation at cosmological scales, dark energy provided an accelerating expansion that stopped this structure from eventually collapsing under its own gravity. Unlike all matter or radiation in the expanding universe, dark energy doesn't dilute away, bearing the role of a sole witness to an ever-emptying cosmos.

A holography for cosmology?

Both the initial rapid expansion of the universe and the dark energy in the later epochs can be effectively described by a positive cosmological constant(Λ) in the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}\, R\, g_{\mu\nu} \, + \Lambda\, g_{\mu\nu} \, = \, \kappa\, T_{\mu\nu} \, . \tag{1}$$

To those of us who would like to embed our current understanding of cosmology into a quantum gravitational theory, the positivity of Λ can play quite the spoilsport and the reason is straightforwardly stated. One of the most promising ideas to come out of quantum gravity is *holography*: the idea that gravitational dynamics is in one-to-one correspondence with a non-gravitational theory that lives on the boundary of the gravitating spacetime[1].

However, when we apply holographic ideas to cosmological spacetimes, a major obstacle arises: these spacetimes do not have the conventional boundaries on which dynamical systems can reside. In fact, they only have boundaries in the far future and the far past, where one is left without the notion of a time. One perspective is to treat the late time boundary as the place where the holographic dual resides. This approach is geared towards understanding the imprint of quantum fluctuations during inflation on the cosmic microwave background [4–7]. Yet, due to the lack of time direction in the late time boundary, we cannot define a non-gravitational 'holographic dual' for cosmology as a dynamical quantum system: in the absence of time, everything is frozen forever.

One possible way out of this conundrum seems promising. Consider a local observer probing such an expanding universe. Their own trajectory through spacetime provides us with a boundary equipped with a notion of time: the parameter in which the local observer moves along the trajectory. This is the boundary on which the non-gravitational holographic dual lives. Since this trajectory has no spatial extensions, it is one-dimensional, and the holographic dual can only be a sufficiently complicated quantum mechanics[2]. This idea is referred to as *solipsistic holography*[3] [8,9].

Such a proposal seems preposterous at first glance: how does a local observer access all the information available in the entire universe? However, we should bear in mind that this is quite similar to how we understand all of cosmology today: we measure things on and around the earth and rebuild our cosmological past based on these local observations. Hence, instead of preemptively dismissing such an idea, we can ask if anything can be gained by embracing this point of view as it fits naturally into our conventional reconstruction of cosmology.

An open system for the observer

So let us adopt this idea of solipsistic holography and ask what an observer sitting in the middle of a cosmological spacetime measures. This observer interacts constantly with its environment:

[1]The AdS/CFT correspondence [2, 3], which maps a gravitational theory in anti-de Sitter(AdS) spacetime to a conformal field theory on the boundary of AdS, is the quintessential example of this idea and has been widely explored for more than two decades. Yet its straightforward application to cosmology is severely hindered by the fact that AdS comes with $\Lambda < 0$.

[2]Unlike in AdS/CFT where the boundary theory is a quantum *field* theory.

[3]Solipsism in philosophy refers to the notion that one's mind is the only thing that surely exists.

it influences everything in the ambient spacetime and is in turn influenced by the spacetime and everything else in it. Even if all other interactions are turned off, gravity will never allow the observer to decouple completely from the rest of the universe. This observer, hence, is naturally modelled by an open system. In the language of quantum mechanics, its dynamics are described by the evolution of a density matrix rather than as a quantum state. The main objective of this thesis is to provide a prescription to describe the dynamics of the cosmological observer's density matrix.

Which scenarios are the most amenable to our foray into such an analysis? The simplest cosmological spacetime in which we could attempt to compute the observer's dynamics is a purely dark energy universe, also known as the *de Sitter* spacetime [10]. Such a spacetime has several advantages over the Friedmann-Lemaitre-Robertson-Walker(FLRW) spacetime that describes a universe much closer to reality. As I had alluded to earlier, dark energy doesn't dilute away. Consequently, a purely dark energy universe has a time translation symmetry: a feature lost once any matter or radiation is added. de Sitter, in fact, has as much symmetry as flat spacetime. This feature assists us in several ways, as will be revealed throughout the rest of this synopsis: not only do the symmetries allow us to obtain exact analytic expressions for physical quantities, but they also constrain the results in ways that provide checks on our computations.

Another fascinating feature of de Sitter is the presence of a cosmological horizon [10]. If you are standing in the middle of an exponentially expanding universe, someone else standing far away enough will never be able to send any signal to you. The space between the two of you stretches too fast for the signal to reach you. The point from beyond which you can't receive any signals forms a spherical horizon around you. Hawking taught us that such horizons radiate thermally at a temperature fixed by the curvature [11]. Placing the observer in such a setting is equivalent to studying an open system coupled to a thermal bath. We will find that this feature assists considerably in our analysis of the open system's dynamics.

Given this highly symmetric de Sitter universe as our toy model, we can now move on to modelling the nature of our observer. We want a localised observer that interacts with the various fields in the universe: an observer that emits and absorbs radiation. Hence, for the moment, we model the observer as a source for several fields living in de Sitter. These sources can be made more explicit when one considers a more specific model of the observer[4], but for

[4]Which could, in principle, be anything from a charged particle, which is a case we will explore further, to

now, let's keep them as abstract sources. For example, in the case of electromagnetism, one can think of the observer as a localised current density distribution.

Now that we have defined our open system and the environment, we come to the question of understanding the evolution of the system's(or equivalently the observer's) density matrix. At a generic level, one starts with the evolution of the density matrix of the system and the environment together and then traces over the degrees of freedom of the environment to obtain the density matrix of the observer.

The procedure we just outlined is phrased in the language of path integrals by Feynman and Vernon [12, 13], which we will now try to understand. One begins with two copies of the system and the environment, such that they encode the physics of the whole density matrix. These doubled set of degrees of freedom can be thought of as ket and bra degrees of freedom: the ket evolves forward in time, whereas the bra evolves backwards. The 'tracing out' of the environment then amounts to integrating out the environment's degrees of freedom in the path integral. The resulting path integral has an extra term in the action along with the action describing the system's degrees of freedom. This additional term encodes the effect of the environment's coupling on the system's dynamics. This term is dubbed *'the influence phase'*.

The core of my thesis is the computation of the influence phase for scalar and electromagnetic fields in de Sitter interacting with the observer's degrees of freedom. This computation can be geometrised using ideas inspired by the AdS/CFT correspondence [14–17]. The section §2 details how one can obtain the influence phase using such a geometric prescription adapted to de Sitter, as we have shown in [18, 19].

Once we have the influence phase, we can extract the physics of the observer from it. The observer is a generic localised source in an otherwise empty universe: it radiates into the fields and loses energy. Hence, it sees the dissipation due to its own radiation. As we saw earlier, de Sitter also provides a thermal bath of Hawking radiation for the observer, and hence, the observer's action also encodes the thermal fluctuations due to such a bath. We will find that the influence phase we compute encodes both the dissipation and the fluctuations correctly to satisfy the fluctuation-dissipation theorem [13].

more generically a binary system or even a liquid drop, etc. We only assume that it is localised, i.e., it doesn't extend all the way to the horizon.

Radiation reaction in de Sitter

Given that our influence phase computes the physics of radiation reaction(RR), our work provides a new way of looking at a century-old problem: How does the electromagnetic radiation from a moving charge backreact on the charge itself? The answer to this question in $3+1$ dimensional flat spacetime is provided by the famous Abraham-Lorentz-Dirac(ALD) force [20], found in standard textbooks on electromagnetism. Our computation will allow us to calculate the ALD force in arbitrary dimensional flat spacetime, along with the effects of the universe's expansion on such a force.

The usual computation of the RR force is plagued with regularisation issues: if one naively computes the electromagnetic fields at the position of the moving charge, they blow up. Dirac addressed this issue of the finiteness of the force. His prescription is to split the field at the point charge into time-reversal odd and even pieces. The even piece is infinite and is thrown away, whereas the odd piece is retained and produces the ALD force. De Witt and Brehme [21] generalised this procedure to curved spacetimes.

Our computation of the influence phase provides a way to understand this regularisation procedure by adding counterterms to an action, which is the standard technique to deal with divergences in quantum field theory. In a sense, the conservative, time-reversal even piece of the field that we drop in the Dirac procedure redefines the multipole moments of the particle, an idea similar to how one absorbs infinities in quantum field theory by redefining the theory's parameters.

Addressing the problem of self-force in de Sitter for scalar [18] and electromagnetic fields [19] meant that we had to generalise several classical field theory results, mostly known in $3+1$ dimensional flat spacetime, to arbitrary dimensional de Sitter. This includes understanding multipole radiation for scalar and electromagnetic fields in de Sitter. The electromagnetic problem in higher dimensional flat spacetime itself suffers from several gaps in the literature, and there isn't a single source of comprehensive analysis of flat space electromagnetism in higher dimensions. Partly, the issue was the lack of understanding of vector spherical harmonics on higher dimensional spheres to the same level as they have been understood on 2-dimensional spheres. On our way to calculating the self-force, we have addressed several of these issues to give a treatment of de Sitter electromagnetism [19], where, at each stage, we contrasted it with the flat space results in the zero curvature limit.

Having outlined the broad motivations for the thesis work, we can move on to summarising our methodology and main results in the following two sections.

2 The Cosmological Influence Phase

In this section, I will illustrate the computation of a de Sitter observer's open system dynamics. As promised, I will describe a simple geometric prescription to obtain the cosmological 'influence phase' on the observer, but a few preliminary remarks are required before we proceed.

To begin with, let us understand the geometry of de Sitter spacetime. de Sitter can be thought of as a sphere that first contracts and then expands in time. This geometry is represented by the 'Penrose diagram' shown in Fig.1. Since the sphere has no boundaries, de Sitter spacetime's only boundaries lie in the future or the past. Consider our observer to be sitting at the south pole of this sphere. The part of the spacetime geometry in causal contact with the observer, i.e., the region where signals can be sent to and received from by the observer, is referred to as the static patch. The static patch is bounded by two horizons, one which marks the extent to which the observer can send signals (also known as the past horizon/particle horizon), whereas the other marks the extent from which the observer may receive signals (otherwise called future horizon/event horizon).

The metric on the $d+1$-dimensional static patch in *outgoing Eddington-Finkelstein* coordinates is given as [10]:

$$ds^2 = -(1 - r^2 H^2)du^2 - 2dudr + r^2 d\Omega_{d-1}^2 \ , \tag{2}$$

where u is the 'retarded time', i.e., outgoing light rays travel along constant u surfaces. The term $d\Omega_{d-1}^2$ is the metric on the unit $d-1$ dimensional sphere. H is defined to be the Hubble constant and is related to the cosmological constant as:

$$H = \sqrt{\frac{2\Lambda}{d(d-1)}} \tag{3}$$

Let us now set this Hubble constant to 1 for convenience. We will restore it later when we compute Hubble corrections to the ALD force. We will also work with the velocity of light c

set to 1.

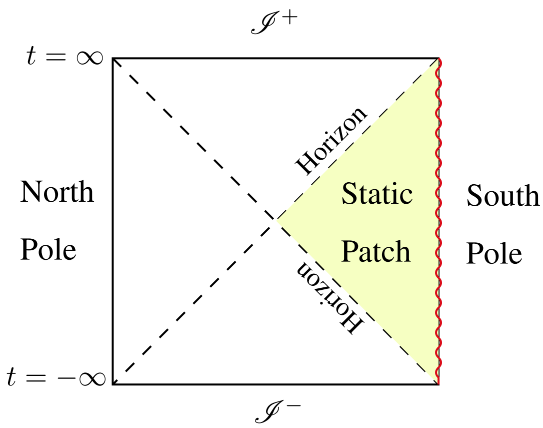

Figure 1: The Penrose diagram represents the de Sitter spacetime in a way that allows us to understand its causal structure with ease. de Sitter is a sphere that first contracts and then expands in time. We move forward in time as we go vertically up in the diagram. The horizontal direction takes us from the sphere's north pole to the south. We place our observer, i.e. the red wavy line, at the south pole. The part of the spacetime that influences and is influenced by the observer is labelled here as the static patch.

The static patch, which forms the main playground for our analysis[5], derives its epithet from the fact that when restricted to the static patch, the dynamics display a time-translation symmetry, which allows us to work in the Fourier expansion of u. This time-translation symmetry also plays a crucial role in the thermality of the Hawking fluctuations. Working in the static patch allows one to exploit all such properties of the problem to obtain greater analytic control.

Now that we have understood the geometry of the setup, let's consider our prescription for computing the *influence phase* of the fields in the static patch on our observer. As we discussed earlier, the Feynman-Vernon formalism requires a doubling of both the environment as well as the system. Our prescription does that in the following manner: we take two copies of the static patch, each equipped with its own observer. Then, we smoothly stitch these two static patches at the future horizon(see Figure 2)[6]. We will call this geometry the de Sitter Schwinger Keldysh geometry or dS-SK in short. In the usual Schwinger-Keldysh notation, the two sides are referred to as R(evolving forward in time, like kets) and L(evolving backwards in time, like bras). Given this setup, we are ready to state our prescription:

The influence phase of the observer

= The on-shell action computed on the dS-SK geometry.

[5]Readers familiar with black holes in AdS should contrast the static patch with the exterior geometry of the eternal black hole.

[6]Such a geometry can be made precise by complexifying the static patch and then considering a hypersurface within the complexified geometry.

Here, 'on-shell' action refers to the action evaluated on solutions to the classical equations of motion.

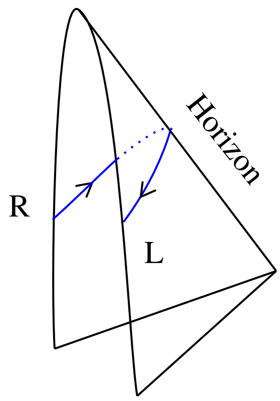

Figure 2: The two sheeted complex dS-SK geometry can be thought of as two static patches smoothly connected at the future horizon. The radial contour along an outgoing Eddington-Finkelstein slice (i.e., a constant u slice) is shown in blue. The radial contour has an outgoing R branch and an incoming L branch.

Naively, this on-shell action diverges and needs to be appropriately countertermed to cancel the divergences. Such counterterming procedures have been understood in the context of AdS/CFT under the moniker of holographic renormalisation. We obtain a corresponding procedure of holographic renormalisation for the solipsistic picture of holography in de Sitter. As we argued in the previous section, the influence phase encodes the physics of radiation reaction(RR), and hence, the regularisation of this on-shell action is equivalent to regulating the self-force. This connects two heretofore disconnected problems—we obtain self-force regularisation through holographic renormalisation.

Let us then understand this influence phase explicitly, starting with the case of scalar fields. We consider a two-parameter family of scalar fields described by the action:

$$S = -\frac{1}{2}\int d^{d+1}x\sqrt{-g}\, r^{\mathcal{N}+1-d}\Bigg\{(\partial\Phi_{\mathcal{N}})^2 + \frac{\Phi_{\mathcal{N}}^2}{4r^2}\left[(d+\mathcal{N}-3)(d-\mathcal{N}-1) - r^2\left(4\mu^2-(\mathcal{N}+1)^2\right)\right]\Bigg\}\,. \tag{4}$$

Along with the kinetic term, the scalar comes with a mass and a centrifugal term in the above action. These terms are defined such that for specific values of the parameters $\mathcal{N}$ and μ, these reproduce the actions useful for a multitude of problems: massive Klein-Gordon scalar, electromagnetism and linearised gravity.

Table 1: $\mathcal{N}, \mu$ values that show up in the analysis of EM and gravitational perturbations

	KG Scalar	EM Vector	EM Scalar	Grav Tensor	Grav Vector	Grav Scalar
$\mathcal{N}$	$d-1$	$d-3$	$3-d$	$d-1$	$1-d$	$3-d$
μ	$\frac{d}{2}$	$\frac{d}{2}-1$	$\frac{d}{2}-2$	$\frac{d}{2}$	$\frac{d}{2}-1$	$\frac{d}{2}-2$

The observer's degrees of freedom are encoded into the multipole moments on either side of the geometry as $\mathfrak{J}_R(\omega, \ell, \vec{m})$ and $\mathfrak{J}_L(\omega, \ell, \vec{m})$ that source the field $\Phi_{\mathcal{N}}$, where ω is the frequency corresponding to a Fourier transform in the retarded time u. The symbols $\{\ell, \vec{m}\}$ label the spherical harmonics $\mathscr{Y}_{\ell,\vec{m}}$ on $d-1$ dimensional spheres, in which we expand the fields as well as the sources.

We then solve for the $\Phi_{\mathcal{N}}$ on the dS-SK geometry by imposing double Dirichlet boundary conditions on either side as $r \to 0$ in terms of the sources $\mathfrak{J}_R$ and $\mathfrak{J}_L$. Given the solutions on the geometry, we substitute them in the action and regulate it to obtain the cosmological influence phase:

$$S_{\text{CIP}} = -\sum_{\ell,\vec{m}} \int \frac{d\omega}{2\pi} \left[K_{\text{Out}}(\omega, \ell)\, \mathfrak{J}_D^* \mathfrak{J}_A + \left(n_\omega + \frac{1}{2} \right) K_{\text{Out}}(\omega, \ell)\, \mathfrak{J}_D^* \mathfrak{J}_D \right] . \tag{5}$$

Let's understand the different quantities that appear in this formula:

- The average and difference combinations of the observer's multipole moments: $\mathfrak{J}_A = \frac{1}{2}[\mathfrak{J}_R + \mathfrak{J}_L]$ and $\mathfrak{J}_D = \mathfrak{J}_R - \mathfrak{J}_L$ are defined to cleanly separate out the dissipative part of the action (coefficient of $\mathfrak{J}_D^* \mathfrak{J}_A$) from the fluctuations (coefficient of $\mathfrak{J}_D^* \mathfrak{J}_D$).
- The function K_{Out} encodes the physics of the RR: it dictates the time scales for the decay of the multipole moments.
- $n_\omega = \frac{1}{e^{2\pi\omega}-1}$ is the Bose-Einstein distribution, which states that the fluctuations seen by the observer are thermal, at the Hawking temperature of the horizon.

The fact that there are no $\mathfrak{J}_A^* \mathfrak{J}_A$ complies with the unitarity of the dynamics of both system and the environment together [13]. The fact that the fluctuations are proportional to the dissipative part is a consequence of the fluctuation-dissipation theorem.

We obtain such an influence phase for both scalar fields and electromagnetism in [18] and [19], respectively. The corresponding analysis for linearised gravitational perturbations is under preparation. The electromagnetic fields are mapped to two scalars Φ_E and Φ_B with specific values of $\mathcal{N}$ and μ. Then, one finds the on-shell actions also to be proportional to the scalar action in Eq.(5) for specific parameters with additional ℓ and d dependent factors coming from reducing the electromagnetic action to the scalar ones.

Our procedure for counterterming the electromagnetic self-force can be understood in a gauge-invariant fashion by rephrasing it as a prescription on the boundary behaviour of the fields: Consider the observer to be inside a small spherical shell of radius r_c. We fix the behaviour of the tangential electric field on the sphere in terms of the multipole moments of the observer. This also fixes the radial electric field, which blows up as $r_c \to 0$. We can then renormalise the radial electric field and obtain its boundary value to obtain the K^E_{Out} corresponding to electric radiation. On the other hand, for the magnetic field, we fix the radial component and renormalise the tangential field to obtain K^B_{Out} (see Figure 3).

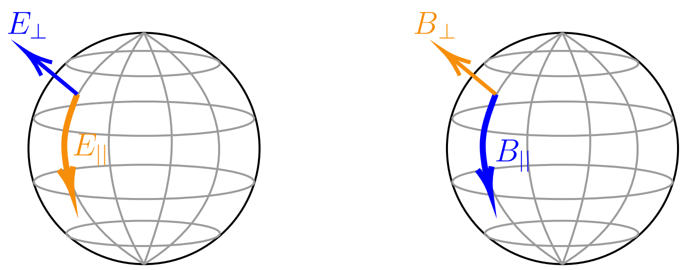

Figure 3: We fix the tangential component of the electric field, but the radial component of the magnetic field on the sphere(shown in orange) in terms of the multipole moments. Then, we renormalise the electric field's radial component and the magnetic field's tangential component (shown in blue) to obtain the RR as the sphere radius goes to zero. See section §3 of [19] for more details about the counterterms.

Now that we have the influence phase that encodes the radiation reaction, we can test this by computing the scalar and electromagnetic ALD force for the observer modelled as a particle moving along an arbitrary trajectory, which is described in the next section.

3 Abraham-Lorentz-Dirac force in de Sitter

The influence phase computation provides us with the two-point function K_{Out} that controls the effect of one multipole moment on the other. Given K_{Out}, one only needs to compute the correct radiative multipole moments of any given model of the observer and feed it into the action to obtain the influence phase in the explicit degrees of freedom of the observer. e.g. for a charged particle, one starts with the standard current density given by:

$$J^\mu(x') = \int_{\text{worldline}} \frac{dx^\mu}{d\tau} \delta^{d+1}(x(\tau) - x') d\tau \tag{6}$$

where $x(\tau)$ describes the trajectory fo the particle. This current density then needs to be appropriately smeared with a Gauss hypergeometric function, henceforth referred to as the 'smearing function'. This smearing of an extended source accounts for the difference in time delays that different parts of the source will radiate.

Once we have the multipole moments of the particle, one can obtain the ALD force [20] by using a post-newtonian expansion adapted to de Sitter. One needs to work with cartesian symmetric tracefree(STF) multipole moments of the source to facilitate this PN-like expansion. But since the de Sitter static patch doesn't admit a description in terms of cartesian coordinates, one performs the multipole expansion of the fields in spherical polar coordinates and then uses certain identities quadratic in spherical harmonics to cast the influence phase in terms of STF multipole moments. These identities are analogues of the spherical harmonic addition theorem:

$$\sum_{m=-\ell}^{\ell} \mathscr{Y}^*_{\ell m}(\hat{r}_0) \mathscr{Y}_{\ell m}(\hat{r}) = P_\ell(\hat{r}_0 \cdot \hat{r}) \tag{7}$$

where P_ℓ is the Legendre Polynomial of degree ℓ. We generalise this identity to scalar and vector spherical harmonics on arbitrary dimensional spheres, which helps us in converting our de Sitter influence phases for scalar and electromagnetic fields into cartesian STF harmonics. The vector spherical harmonic addition(VSH) theorem, which we find in [19] can be stated as follows:

$$\mathcal{N}_{d,\ell} |\mathbb{S}^{d-1}| \sum_{\alpha \vec{m}} \mathbb{V}^{*\alpha}_{I \ell \vec{m}}(\hat{r}_0) \mathbb{V}^{\alpha}_{J \ell \vec{m}}(\hat{r}) = \Pi^V_{IJ}(\hat{r}_0|\hat{r})_{d,\ell} \; . \tag{8}$$

Let us understand the different components in this formula:

- The $\mathbb{V}^{\alpha}_{J\ell\vec{m}}$ are vector spherical harmonics, i.e. they are eigenvectors of the vector Laplacian on the sphere, with eigenvalue fixed by ℓ. The other labels $\vec{m}$ are higher dimensional analogues of the m label of scalar spherical harmonics, whereas the α labels are specific to VSH in higher dimensions. I, J are vector indices on the sphere.

- $\mathcal{N}_{d,\ell} \equiv \frac{(d-2)!!}{(d+2\ell-2)!!}$ is an inverse integer associated with the inner-product of STF harmonics and $|\mathbb{S}^{d-1}|$ is the volume of the $d-1$ dimensional sphere.

- $\Pi^{V}_{IJ}(\hat{r}_0|\hat{r})_{d,\ell}$ is a vector STF projector that acts on a ℓ degree polynomial in the cartesian coordinates and gives a vector STF harmonic. Several explicit expressions for this projector using distinct methods are obtained in [19].

With these addition theorems, the action can be converted into STF multipole moments, and we can proceed with our modified PN expansion scheme which we explain next.

PN-like expansion in de Sitter

Given ω as the frequency of the radiation and r defining the average deviation of the particle's trajectory from the origin, the usual post-newtonian scheme in flat space involves the following steps:

- One works with small velocities compared to the speed of light ($v \ll 1$).

- We require the particle to not stray too far away from the origin compared to the wavelength of the radiation ($\omega r \ll 1$).

- The multipole expansion has to be consistently truncated to lower ℓ effects since higher ones contribute to more time derivatives.

In de Sitter, we will add two more assumptions to obtain a near-flat expansion(in H):

- We also consider the particle to not stray far away compared to cosmological scales ($rH \ll 1$).

- We will consider the wavelength of the radiation to be much smaller compared to cosmological scales ($\omega \gg H$): our particle should move slowly but not 'cosmologically' slowly.

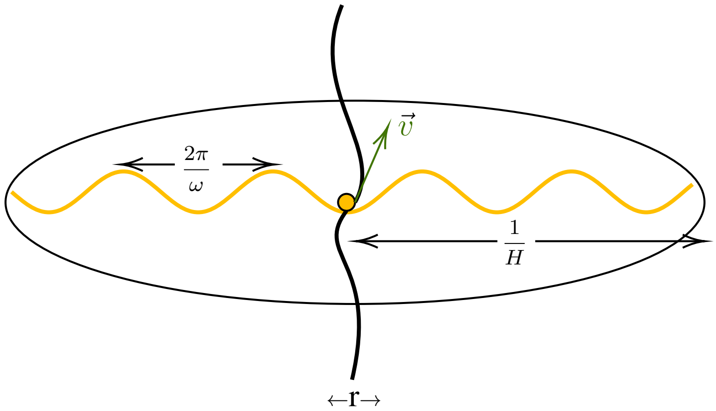

Figure 4: The RR is given in a post-Newtonian expansion: the velocity is taken to be small ($v \ll 1$) and the trajectory is centred about the south pole ($\omega r \ll 1$) while the near-flat expansion requires that the curvature effects are small(i.e. $\omega \gg H$ and $rH \ll 1$).

This set of rules gives a consistent way to define a PN expansion in de Sitter that reproduces flat space RR force along with curvature corrections. Figure 4 visualises these approximations.

As an illustration, consider the first Hubble correction to the electromagnetic ALD force in $d+1$ dimensional de Sitter for odd values of d:

$$F^i_{\text{ALD}} = \frac{(-1)^{\frac{d+1}{2}}(d-1)}{|\mathbb{S}^{d-1}|d!!(d-2)!!}\left\{\partial_t^d x^i - H^2\frac{d}{6}\left[d^2-6d+11\right]\partial_t^{d-2}x^i + O(H^4)\right\}. \tag{9}$$

Here $|\mathbb{S}^{d-1}|$ is the volume of the $d-1$ dimensional sphere. Each cosmological correction comes with 2 fewer time derivatives than the flat space counterpart for dimensional consistency. This RR force, in the $H \to 0$ limit, matches previous calculations in flat spacetimes [22].

The ALD force written in a PN expansion in flat spacetime breaks manifest poincare covariance. Nevertheless, given the symmetries of the spacetime, one expects that the post-newtonian series expansion of the self-force sums up to a manifest poincare covariant force, and indeed, it is found to be so. A similar story holds for the modified PN scheme we used to compute the self-force in de Sitter. The de Sitter self-force should resum to de Sitter covariant results, which we show by explicit resummation. For example, in 6-dimensional de Sitter, the EM force can be written as:

$$f^\mu = \frac{P^{\mu\nu}}{5!!}\left\{-4a_\nu^{(3)} + 10\,(a\cdot a)\,a_\nu^{(1)} + 30\,(a\cdot a^{(1)})\,a_\nu\right\} - H^2\frac{P^{\mu\nu}}{5!!}\left\{16a_\nu^{(1)}\right\} \tag{10}$$

where $P^{\mu\nu} = g^{\mu\nu} + v^\mu v^\nu$ such that $v^\mu = \frac{dx^\mu}{d\tau}$ is the proper velocity computed in the de Sitter background and $a^\mu = \frac{dv^\mu}{d\tau}$ is the corresponding dS covariant acceleration. We compare the flat space limits of these covariant results for electromagnetism with previous literature [23,24] and find agreement.

The above results are meant for even-dimensional spacetimes. The nature of radiation reaction in odd-dimensional spacetimes is qualitatively different: there is no energy loss asymptotically far away from the source and hence there is no true dissipation, only a redefinition of the multipole moments. These features of odd-dimensional radiation reaction in flat space were explored in [20, 24] and we study the analogous problem in de Sitter.

4 Discussion and Outlook

Our analysis of the cosmological influence phase leads us to many interesting results having a wide range of applicability:

- At the most basic level, we understood the multipole radiation of cosmological sources in de Sitter for scalar and electromagnetic fields. The smearing of the sources, required to account for the correct time-delay differences, is accomplished by a Gauss hypergeometric function that includes appropriate Hubble corrections to the flat space smearing—performed by Bessel functions. The main takeaway is that for cosmologically extended sources, the Hubble expansion plays a role in the smearing of the multipole moments. These multipole moments are also defined to reproduce the flat space multipole moments in the zero curvature limit which we compare with [25–28].

- We generically find that the Hubble expansion enhances the RR effects felt by the sources. One can ask if there are any real-world scenarios where one could expect to see such effects. Given the smallness of the Hubble constant in our universe, one can rule out several astrophysical and galactic phenomena. Nevertheless, such enhanced RR would be relevant to extragalactic and large-scale structure evolution.

- We focused only on the dissipative parts of the action and ignored the conservative pieces in the influence phase. In principle, they can be retained and, in fact, they should be retained if one wants to solve for the conservative correction to particle trajectories due to the field interactions. Such a conservative influence phase would be useful in understanding two-body dynamics in the presence of a cosmological constant. One could apply such results in linearised GR to understand the co-evolution of galactic clusters/superclusters.

- Returning to the original motivation, we can ask if our results shed any light upon the hypothesized solipsistic holography. Since the holographic dual theory is unknown, we can only speculate on the boundary implications of the influence phase. The holographic features manifest in this computation of the RR signal that, for any proposed duals, one should look for the characteristic dissipation found in the de Sitter RR.

Although we have only covered scalar fields and electromagnetism in [18, 19], we are in the process of finishing the analogous computations for linearised gravitational fields. Our analysis, till now, has been restricted to linearised fields, but we hope it can be extended perturbatively to include non-linearities. We initiated the study of non-linearities in [18], where we analysed certain constraints on the leading correction in the perturbation series for scalar interactions. Further extensions to include higher-order terms in the perturbation theory and analyse the corresponding non-linear corrections to the ALD force are the next few goals in this program. Bootstrap techniques have been used to understand interactions in the late time holography [6, 7] and we hope that the interacting radiation reaction can be bootstrapped in a similar way using de Sitter isometries.

A similar analysis for gravitational non-linearities may seem daunting. But one could, in principle, formulate a cosmological corrected version of the multipolar-post-minkowskian formalism that works for de Sitter.

The more real-world cosmological scenario, that of FLRW cosmology, disrupts the many nice properties of de Sitter, such as time translation symmetry and the thermality of Hawking radiation. Applying our ideas to FLRW is like studying an open system that, instead of interacting with a thermal bath, interacts with a non-thermal one. Nevertheless, one can proceed by analysing simpler time dependences, as done in textbook quantum mechanics: the adiabatic and sudden approximations. Whereas the adiabatic approximation is a reasonable approximation for the current universe with a slowly varying Hubble constant, the sudden approximation can capture the physics of cosmological phase transitions. One would like to proceed towards the problem of generic FLRW through such simpler steps.

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List of Publications

1. R. Loganayagam and **Omkar Shetye**. *Influence phase of a dS observer. Part I. Scalar exchange*, *JHEP* **01** (2024) 138. [2309.07290]

2. R. Loganayagam and **Omkar Shetye**. *Influence phase of a dS observer II: Electromagnetism,* [2503.00135]