Applications of Chiral Kinetic Theory

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"Extreme nonequilibrium QCD," October 7, 2020

Contents

- Chiral transport phenomena
- Chiral kinetic theory
- Chiral plasma instability
- Applications to core-collapse supernovae

Chiral transport phenomena

Transport phenomena

- Classical and familiar examples:
    - Ohm's law: $\boldsymbol{j}_e = \sigma \boldsymbol{E}$
    - Fourier's law: $\boldsymbol{j}_Q = \kappa(-\boldsymbol{\nabla} T)$

Chiral magnetic effect (CME)

$$\boldsymbol{j}_{\mathrm{V}} = \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{4\pi^2} e\boldsymbol{B} \equiv \frac{\mu_{\mathrm{A}}}{2\pi^2} e\boldsymbol{B}$$

$$\boldsymbol{j}_{\mathrm{A}} = \frac{\mu_{\mathrm{R}} + \mu_{\mathrm{L}}}{4\pi^2} e\boldsymbol{B} \equiv \frac{\mu_{\mathrm{V}}}{2\pi^2} e\boldsymbol{B}$$

Vilenkin (1980); Nielsen, Ninomiya (1983); Fukushima, Kharzeev, Warringa (2008), …

From chiral anomaly to CME

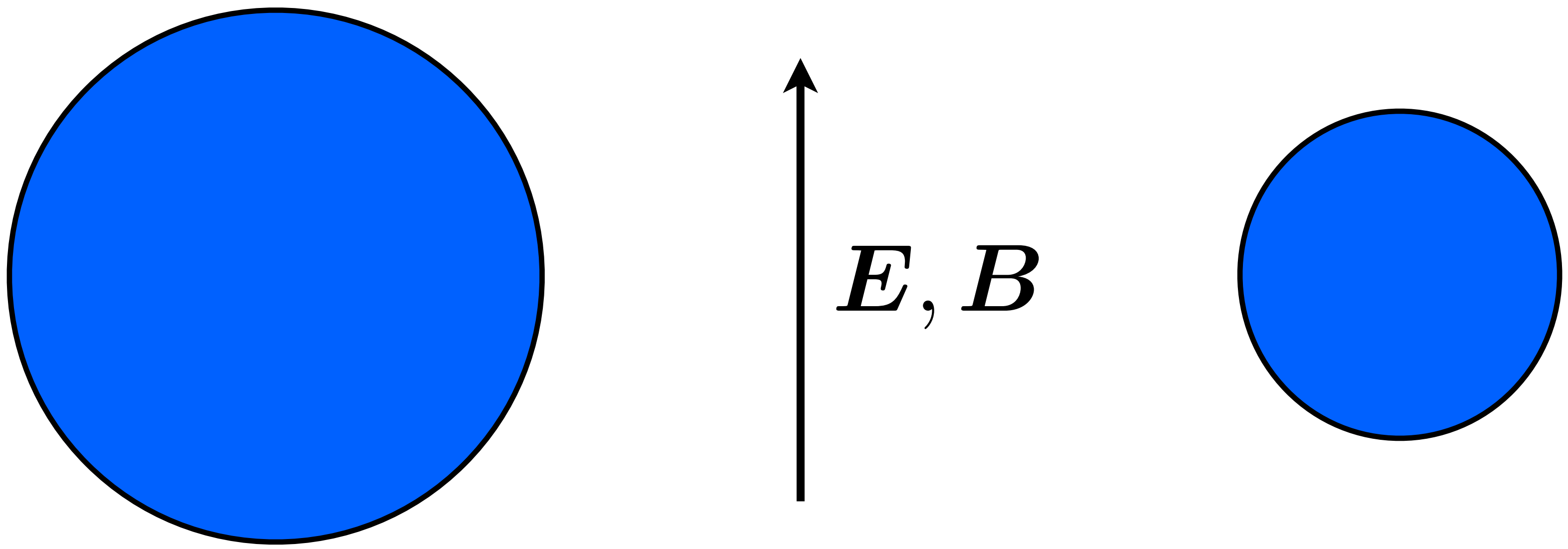
Nielsen, Ninomiya (1983)

Right-handed fermions

Left-handed fermions

From chiral anomaly to CME

Nielsen, Ninomiya (1983)

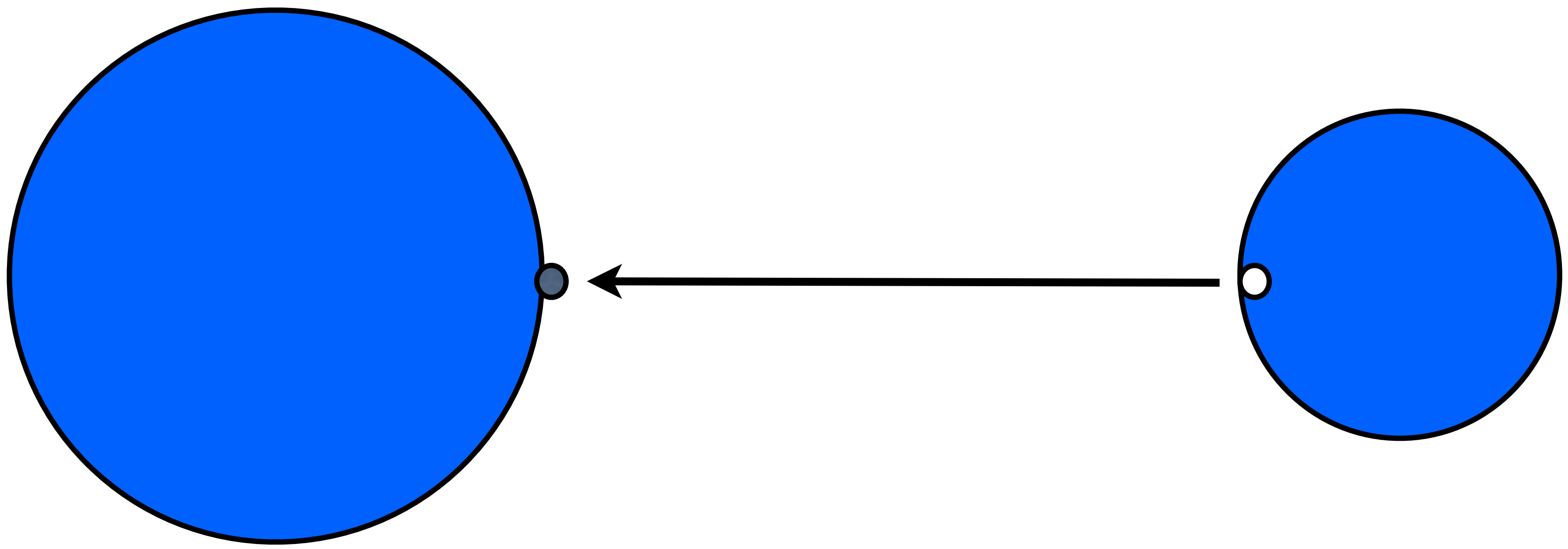


Right-handed fermions Left-handed fermions

$$\Delta Q_{\mathrm{A}} = \frac{e^2}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}$$

From chiral anomaly to CME

Nielsen, Ninomiya (1983)



Right-handed fermions

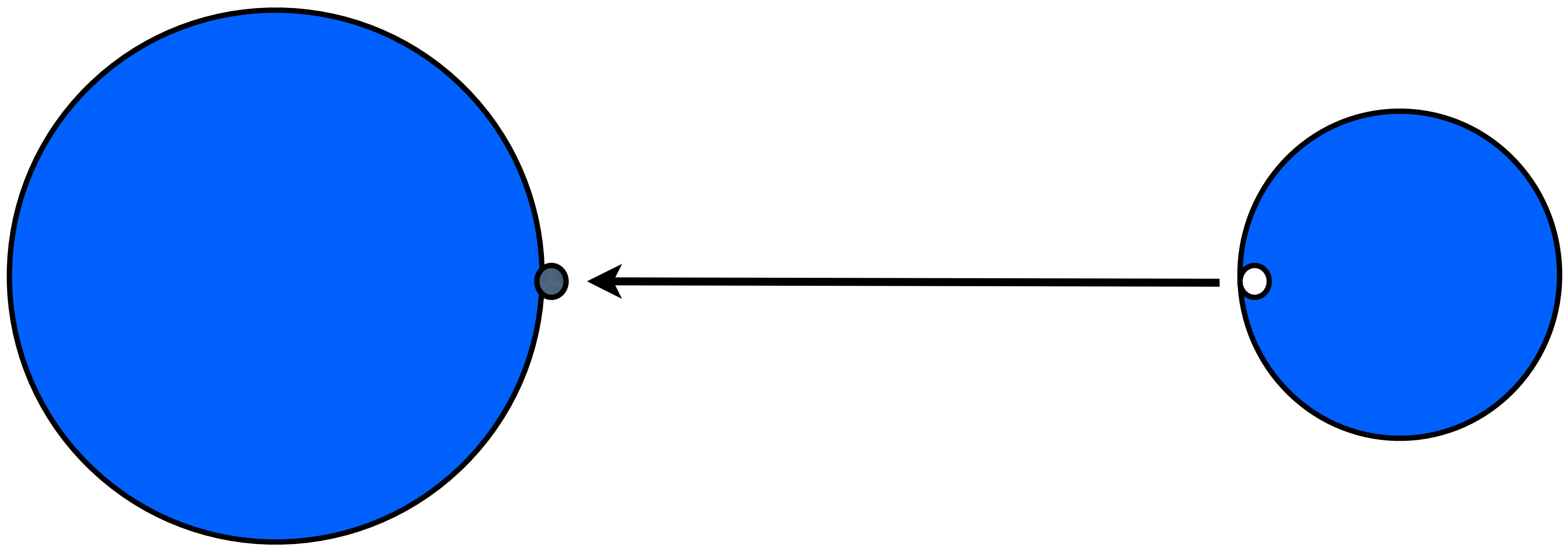
Left-handed fermions

$$\Delta Q_{\mathrm{A}} = \frac{e^2}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}$$

Energy cost: $\mu_{\mathrm{A}} \Delta Q_{\mathrm{A}}$

From chiral anomaly to CME

Nielsen, Ninomiya (1983)



Right-handed fermions

Left-handed fermions

$$\Delta Q_{\mathrm{A}} = \frac{e^2}{2\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}$$

Energy conservation: $\mu_{\mathrm{A}} \Delta Q_{\mathrm{A}} = e \boldsymbol{j} \cdot \boldsymbol{E}$

Chiral magnetic effect: $\boldsymbol{j} = \frac{\mu_{\mathrm{A}}}{2\pi^2} e \boldsymbol{B}$

Anomalies in effective theories

Anomalies are independent of the energy scale

- QCD → Chiral perturbation theory + WZW term

anomalies $\qquad p \ll \Lambda_{\rm chiral} \qquad$ anomalies

Anomalies in effective theories

Anomalies are independent of the energy scale

- QCD → Chiral perturbation theory + WZW term

anomalies $\quad p \ll \Lambda_{\text{chiral}} \quad$ anomalies

- QFT → Kinetic theory → Hydrodynamics

$p \ll T, \mu \qquad p \ll l_{\text{mfp}}^{-1}$

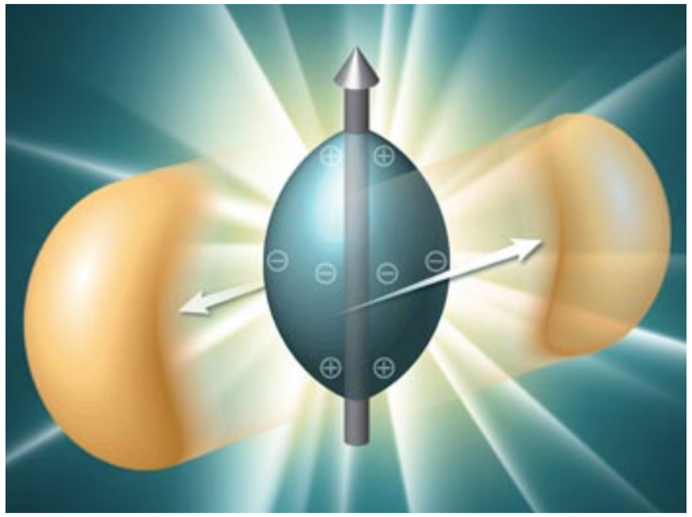
anomalies $\quad$ anomalies? $\quad$ anomalies?

Son, Yamamoto (2012);
Stephanov, Yin (2012);
Chen et al. (2013), …

Erdmenger et al. (2009);
Banerjee et al. (2009);
Son-Surowka (2009), …

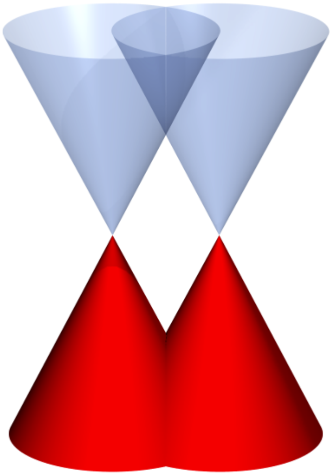
Chiral Matter

- Electroweak plasma in early Universe — Joyce, Shaposhnikov (1997), …
- Quark-gluon plasma in RHIC/LHC — Fukushima, Kharzeev, Warringa (2008), …
- Weyl semimetals (“3D graphene”) — Nielsen, Ninomiya (1983), …
- Neutrino matter in supernovae — Yamamoto (2016), …

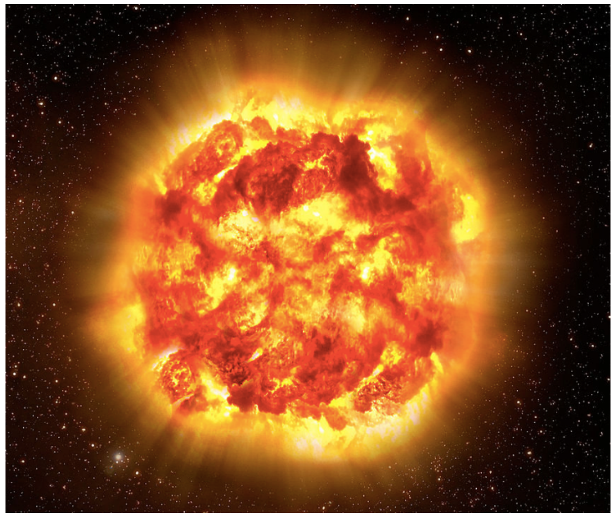


Quark-Gluon Plasma

http://www0.bnl.gov/rhic/news2/



Weyl semimetals



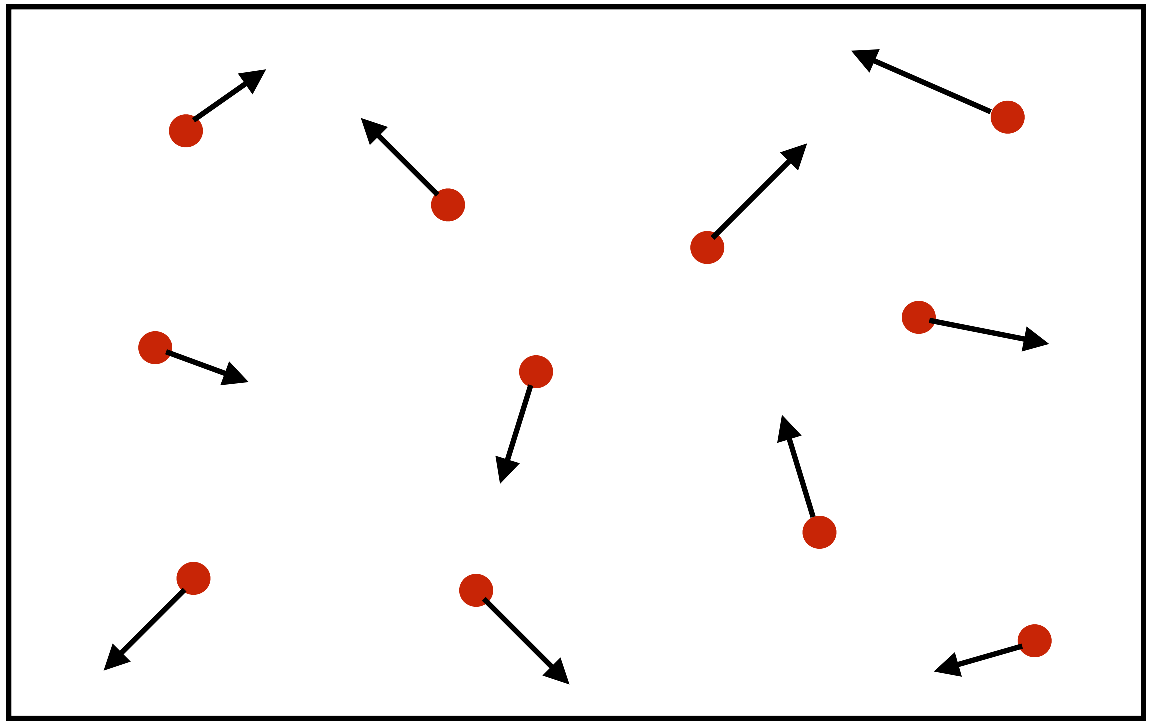
Supernovae

Chiral kinetic theory

Kinetic theory

Description of the statistical behavior in and out of equilibrium

$$\boxed{\frac{\partial n_{\boldsymbol{p}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = C[n_{\boldsymbol{p}}]}$$



Boltzmann equation

- Formulated w.r.t. the distribution function $n_{\boldsymbol{p}}(t, \boldsymbol{x})$
- First ignore collisions; Liouville's theorem dictates

$$\frac{\mathrm{d}n_{\boldsymbol{p}}}{\mathrm{d}t} = \frac{\partial n_{\boldsymbol{p}}}{\partial t} + \underline{\dot{\boldsymbol{x}}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + \underline{\dot{\boldsymbol{p}}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = 0$$

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$$\frac{\mathrm{d}n_{\boldsymbol{p}}}{\mathrm{d}t} = \frac{\partial n_{\boldsymbol{p}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + \underbrace{e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})}_{\text{Lorentz force}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = 0$$

Boltzmann equation

- Formulated w.r.t. the distribution function $n_{\boldsymbol{p}}(t, \boldsymbol{x})$
- Add collisions:

$$\boxed{\frac{\partial n_{\boldsymbol{p}}}{\partial t} + \boldsymbol{v} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = C[n_{\boldsymbol{p}}]}$$

- Ohm's law:

$$\boldsymbol{j}_{\text{noneq}} = \int d\boldsymbol{p}\ \boldsymbol{v} \delta n_{\boldsymbol{p}} = \sigma \boldsymbol{E}$$

Boltzmann equation

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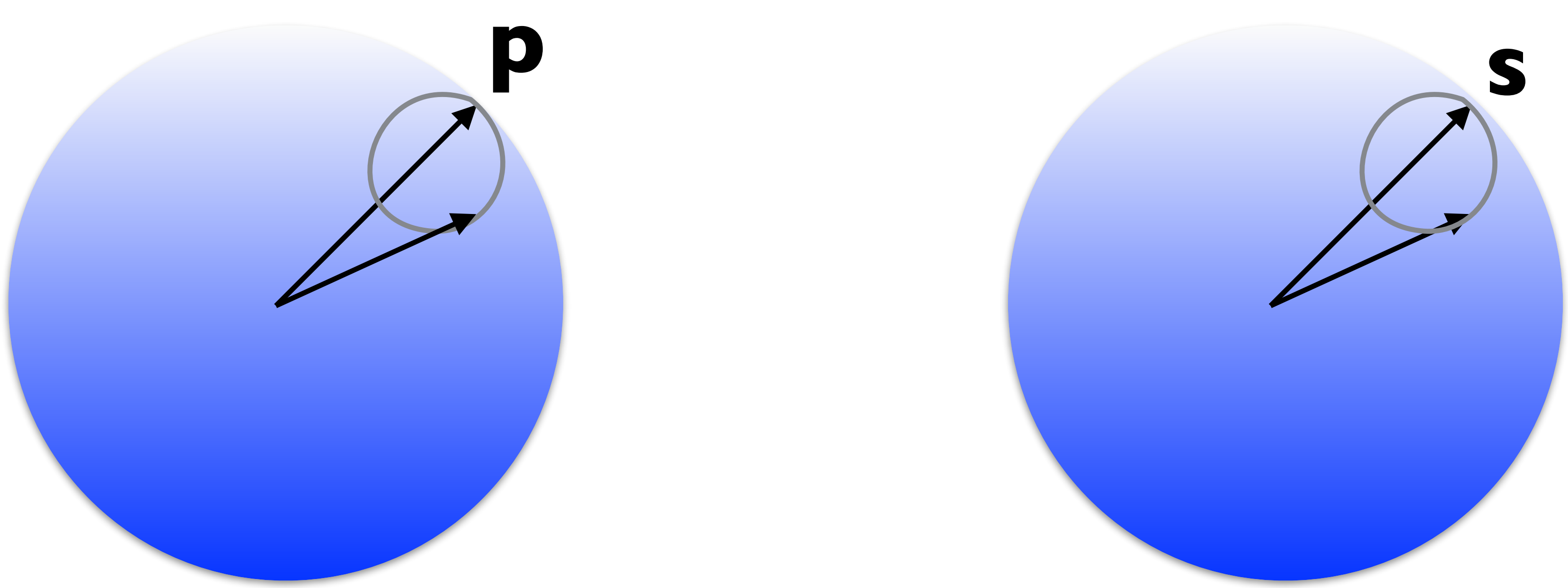
- Ohm's law:

$$\boldsymbol{j}_{\text{noneq}} = \int d\boldsymbol{p}\ \boldsymbol{v} \delta n_{\boldsymbol{p}} = \sigma \boldsymbol{E}$$

- Chiral anomaly? CME? No distinction between R & L

Chirality and topology

Right-handed fermions



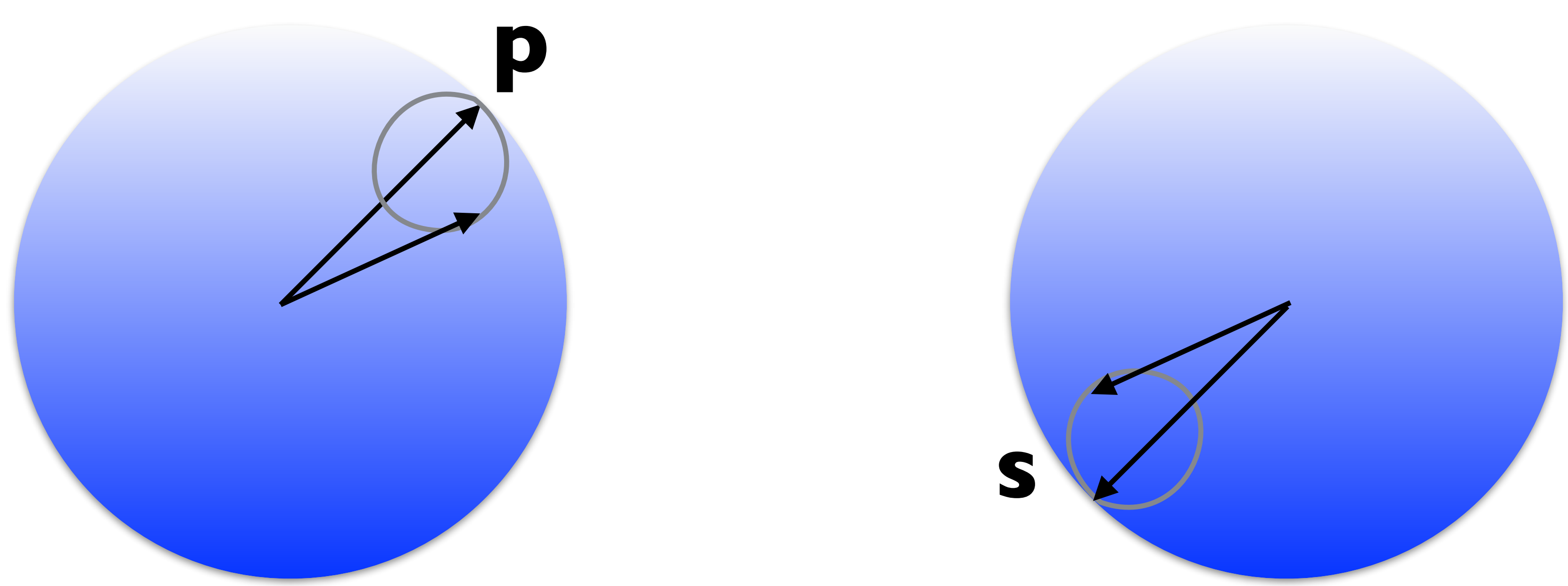
momentum space

spin space

Mapping: S^2 ($\boldsymbol{p}$-space) → S^2 (spin space) winding number +1

Chirality and topology

Left-handed fermions



momentum space　　　　　　spin space

Mapping: S^2 ($\boldsymbol{p}$-space) $\rightarrow$ S^2 (spin space)　winding number -1

Chirality = Topological invariant

Topology and Berry curvature

- More generically, $\pi_2(S^2) = \mathbb{Z}$ → Monopole at $\boldsymbol{p}$**=0**
- "Magnetic field" of monopole = Berry curvature $\boldsymbol{\Omega_p} = K \frac{\hat{\boldsymbol{p}}}{2|\boldsymbol{p}|^2}$

$$\frac{1}{2\pi}\int \boldsymbol{\Omega_p}\cdot \mathrm{d}\boldsymbol{S} = K$$

Berry monopole charge
(±1 for chiral fermions)

- $\boldsymbol{\Omega_p}$ modifies the equation of motion & kinetic theory

Equations of motion

$$\dot{\boldsymbol{x}} = \boldsymbol{v}$$

$$\dot{\boldsymbol{p}} = e(\boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B})$$

Lorentz force in ***x***-space

Equations of motion

"Lorentz force" in ***p***-space

$$\dot{\boldsymbol{x}} = \boldsymbol{v} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}_{\boldsymbol{p}}$$

Sundaram, Niu (1999)

$$\dot{\boldsymbol{p}} = e(\boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B})$$

Lorentz force in ***x***-space

Equations of motion

$$\dot{\boldsymbol{x}} = \boldsymbol{v} + \dot{\boldsymbol{p}} \times \boldsymbol{\Omega}_{\boldsymbol{p}} = \frac{1}{\sqrt{\omega}}[\boldsymbol{v} + e\boldsymbol{E} \times \boldsymbol{\Omega}_{\boldsymbol{p}} + (\boldsymbol{v} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}]$$

$$\dot{\boldsymbol{p}} = e(\boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{B}) = \frac{1}{\sqrt{\omega}}[e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega}_{\boldsymbol{p}}]$$

$$\sqrt{\omega} = 1 + e\boldsymbol{B} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}}$$

$$\longrightarrow \quad \frac{\partial n_{\boldsymbol{p}}}{\partial t} + \underline{\dot{\boldsymbol{x}}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} + \underline{\dot{\boldsymbol{p}}} \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = C[n_{\boldsymbol{p}}]$$

Chiral kinetic theory

$$\sqrt{\omega}\frac{\partial n_{\boldsymbol{p}}}{\partial t} + [\boldsymbol{v} + e\boldsymbol{E} \times \boldsymbol{\Omega}_{\boldsymbol{p}} + (\boldsymbol{v} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}] \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}}$$
$$+ \left[e\boldsymbol{E} + \boldsymbol{v} \times e\boldsymbol{B} + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega}_{\boldsymbol{p}}\right] \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = \sqrt{\omega}C[n_{\boldsymbol{p}}]$$

Son, Yamamoto (2012); Stephanov, Yin (2012); Chen et al. (2013).

Full chiral kinetic theory

$$\boxed{\begin{aligned}\sqrt{\omega}\frac{\partial n_{\boldsymbol{p}}}{\partial t} &+ \left[\tilde{\boldsymbol{v}} + e\tilde{\boldsymbol{E}} \times \boldsymbol{\Omega}_{\boldsymbol{p}} + (\tilde{\boldsymbol{v}} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}\right] \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}} \\ &+ \left[e\tilde{\boldsymbol{E}} + \tilde{\boldsymbol{v}} \times e\boldsymbol{B} + e^2(\tilde{\boldsymbol{E}} \cdot \boldsymbol{B})\boldsymbol{\Omega}_{\boldsymbol{p}}\right] \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = \sqrt{\omega}C[n_{\boldsymbol{p}}]\end{aligned}}$$

$$\epsilon_{\boldsymbol{p}} = |\boldsymbol{p}|(1 - e\boldsymbol{B} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}}), \quad \tilde{\boldsymbol{v}} = \frac{\partial \epsilon_{\boldsymbol{p}}}{\partial \boldsymbol{p}}, \quad e\tilde{\boldsymbol{E}} = e\boldsymbol{E} - \frac{\partial \epsilon_{\boldsymbol{p}}}{\partial \boldsymbol{x}}$$

Corrections due to the magnetic moment

Son, Yamamoto (2013); Chen, Son, Stephanov, Yee, Yin (2014); Hidaka, Pu, Yang (2017)

Chiral kinetic theory

$$\boxed{\begin{aligned}&\sqrt{\omega}\frac{\partial n_{\boldsymbol{p}}}{\partial t}+[\boldsymbol{v}+e\boldsymbol{E}\times\boldsymbol{\Omega}_{\boldsymbol{p}}+(\boldsymbol{v}\cdot\boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}]\cdot\frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}}\\&+\left[e\boldsymbol{E}+\boldsymbol{v}\times e\boldsymbol{B}+e^2(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}_{\boldsymbol{p}}\right]\cdot\frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}}=\sqrt{\omega}C[n_{\boldsymbol{p}}]\end{aligned}}$$

Son, Yamamoto (2012); Stephanov, Yin (2012); Chen et al. (2013).

$$n\equiv\int_{\boldsymbol{p}}\sqrt{\omega}n_{\boldsymbol{p}}$$

$$\boldsymbol{j}\equiv\int_{\boldsymbol{p}}\sqrt{\omega}\dot{\boldsymbol{x}}n_{\boldsymbol{p}}=\int_{\boldsymbol{p}}[\boldsymbol{v}+e\boldsymbol{E}\times\boldsymbol{\Omega}_{\boldsymbol{p}}+(\boldsymbol{v}\cdot\boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}]\,n_{\boldsymbol{p}}$$

Chiral kinetic theory

$$\sqrt{\omega}\frac{\partial n_{\boldsymbol{p}}}{\partial t} + [\boldsymbol{v} + e\boldsymbol{E} \times \boldsymbol{\Omega}_{\boldsymbol{p}} + (\boldsymbol{v} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}] \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}}$$
$$+ [e\boldsymbol{E} + \boldsymbol{v} \times e\boldsymbol{B} + e^2(\boldsymbol{E} \cdot \boldsymbol{B})\boldsymbol{\Omega}_{\boldsymbol{p}}] \cdot \frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}} = \sqrt{\omega}C[n_{\boldsymbol{p}}]$$

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$$\boldsymbol{j} \equiv \int_{\boldsymbol{p}} \sqrt{\omega}\dot{\boldsymbol{x}} n_{\boldsymbol{p}} = \int_{\boldsymbol{p}} [\cancel{\boldsymbol{v} +} e\boldsymbol{E} \cancel{\times \boldsymbol{\Omega}_{\boldsymbol{p}}} + (\boldsymbol{v} \cdot \boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}]\, n_{\boldsymbol{p}}$$

spherical Fermi distribution

Chiral kinetic theory

$$\boxed{\begin{aligned}&\sqrt{\omega}\frac{\partial n_{\boldsymbol{p}}}{\partial t}+[\boldsymbol{v}+e\boldsymbol{E}\times\boldsymbol{\Omega}_{\boldsymbol{p}}+(\boldsymbol{v}\cdot\boldsymbol{\Omega}_{\boldsymbol{p}})e\boldsymbol{B}]\cdot\frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{x}}\\&+\left[e\boldsymbol{E}+\boldsymbol{v}\times e\boldsymbol{B}+e^2(\boldsymbol{E}\cdot\boldsymbol{B})\boldsymbol{\Omega}_{\boldsymbol{p}}\right]\cdot\frac{\partial n_{\boldsymbol{p}}}{\partial \boldsymbol{p}}=\sqrt{\omega}C[n_{\boldsymbol{p}}]\end{aligned}}$$

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spherical Fermi distribution

Chiral plasma instability

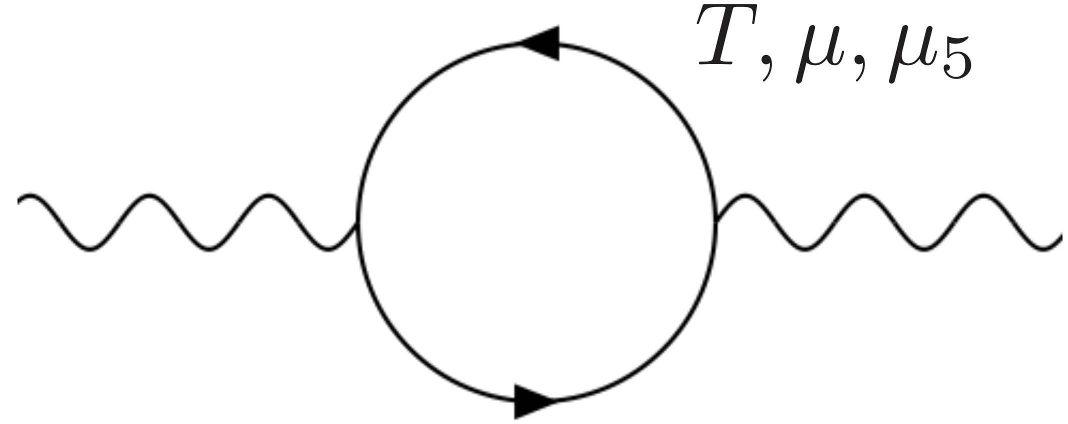
Electroweak theory Redlich, Wijewardhana (1985); Rubakov (1986); Laine (2005)

Early Universe Joyce, Shaposhnikov (1997); Boyarsky et al. (2012); Tashiro et al. (2012)

Quark-gluon plasma Akamatsu, Yamamoto (2013); Manuel, Torres-Rincon (2015), …

Neutron stars Ohnishi, Yamamoto (2014), …

Polarization tensor with μ_5

$\Pi^{\mu\nu}(k) =$ 

- $\Pi^{ij}(k)$ can be decomposed by the tensors ($A_0 = 0$):

$$\hat{k}^i \hat{k}^j, \;\; \delta^{ij} - \hat{k}^i \hat{k}^j, \;\; i\epsilon^{ijk}\hat{k}^k$$

$$\Pi^{ij}_{-}(k) = \frac{e^2 \mu_5}{4\pi^2} i\epsilon^{ijk} \left(1 - \frac{\omega^2}{|\boldsymbol{k}|^2}\right) \left(1 - \frac{\omega}{2|\boldsymbol{k}|} \ln \frac{\omega + |\boldsymbol{k}|}{\omega - |\boldsymbol{k}|}\right) k^k$$

T=0 [Son, Yamamoto (2013)] T≠0 [Manuel, Torres-Rincon (2014)]

Collective modes with μ_5

- Maxwell equation: $\partial_\nu F^{\nu\mu} = j^\mu$
- Linear response: $j^\mu(k) = \Pi^{\mu\nu}(k) A_\nu(k)$

$$\left.\begin{array}{l} \partial_\nu F^{\nu\mu} = j^\mu \\ j^\mu(k) = \Pi^{\mu\nu}(k) A_\nu(k) \end{array}\right\} [k^2\delta^{ij} - k^i k^j + \Pi^{ij}(k)] A^j = 0$$

($A_0 = 0$ gauge)

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- Maxwell equation: $\partial_\nu F^{\nu\mu} = j^\mu$
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($A_0 = 0$ gauge)

Focus on transverse gauge fields:

- For μ_5=0, $\omega = -\frac{4ik^3}{\pi m_D^2}$ → $e^{-i\omega t} \sim e^{-\gamma(k)t}$ Landau damping

Collective modes with μ_5

- Maxwell equation: $\partial_\nu F^{\nu\mu} = j^\mu$
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$$[k^2\delta^{ij} - k^i k^j + \Pi^{ij}(k)]A^j = 0$$

($A_0 = 0$ gauge)

Focus on transverse gauge fields:

- For μ_5=0, $\omega = -\frac{4ik^3}{\pi m_D^2}$ → $e^{-i\omega t} \sim e^{-\gamma(k)t}$ Landau damping
- For μ_5≠0, $\omega = \frac{4ik^2}{\pi m_D^2}\left(\frac{e^2|\mu_5|}{4\pi^2} - k\right)$ → $e^{-i\omega t} \sim e^{\gamma(k)t}$ for small *k*

Chiral plasma instability

Akamatsu, Yamamoto (2013)

Non-Abelian Chiral Instability

- QGP has color charges → non-Abelian chiral instability

$$l_{\rm inst} \sim (g^2\mu_5)^{-1} \ll (g^4 T)^{-1} \sim l_{\rm mfp}$$

$$\tau_{\rm inst} \sim [g^4 T \ln(1/g)]^{-1} \sim \tau_{\rm mft}$$

- For μ_5~T, non-Abelian CPI *cannot* be captured by hydro, but may be described by chiral kinetic theory or chiral Langevin theory

Akamatsu, Yamamoto (2013, 2014)

Core-collapse supernovae explosions

Core-collapse supernova explosions

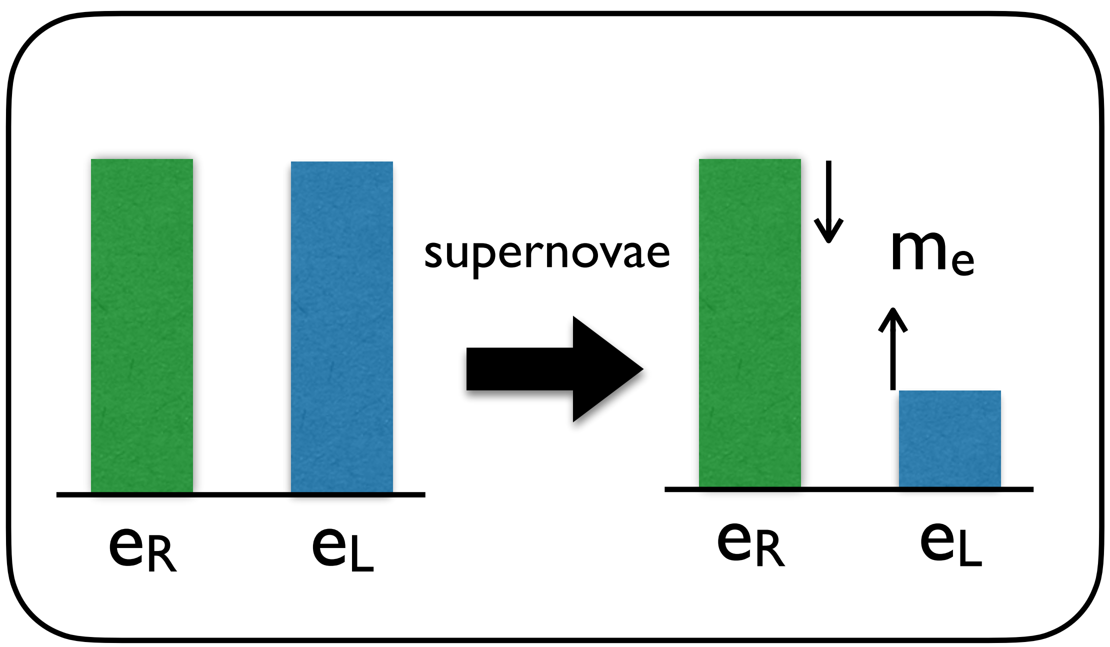
- One of the most energetic phenomena in the Universe
- Transition to neutron stars & origin of heavy elements
- But explosion is difficult in conventional 3D hydrodynamic theory

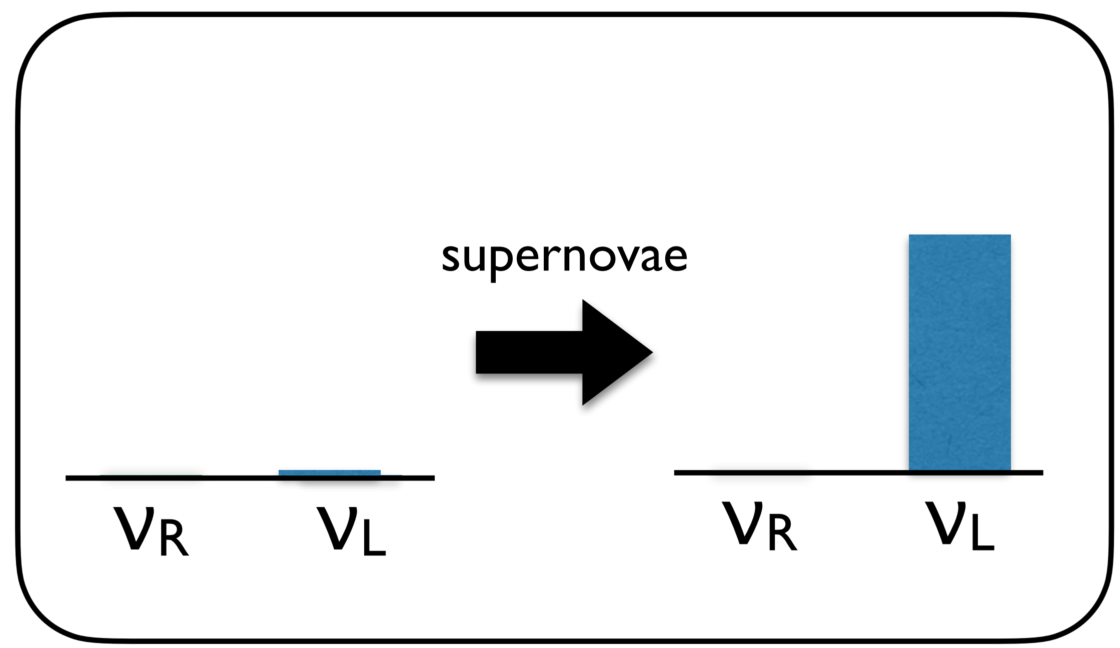
One of the puzzles in astrophysics

http://www.riken.jp/pr/press/2009/20091211/

Supernova = Giant Parity Breaker

$$p + \underline{e^{\mathrm{L}}} \rightarrow n + \underline{\nu_e^{\mathrm{L}}}$$



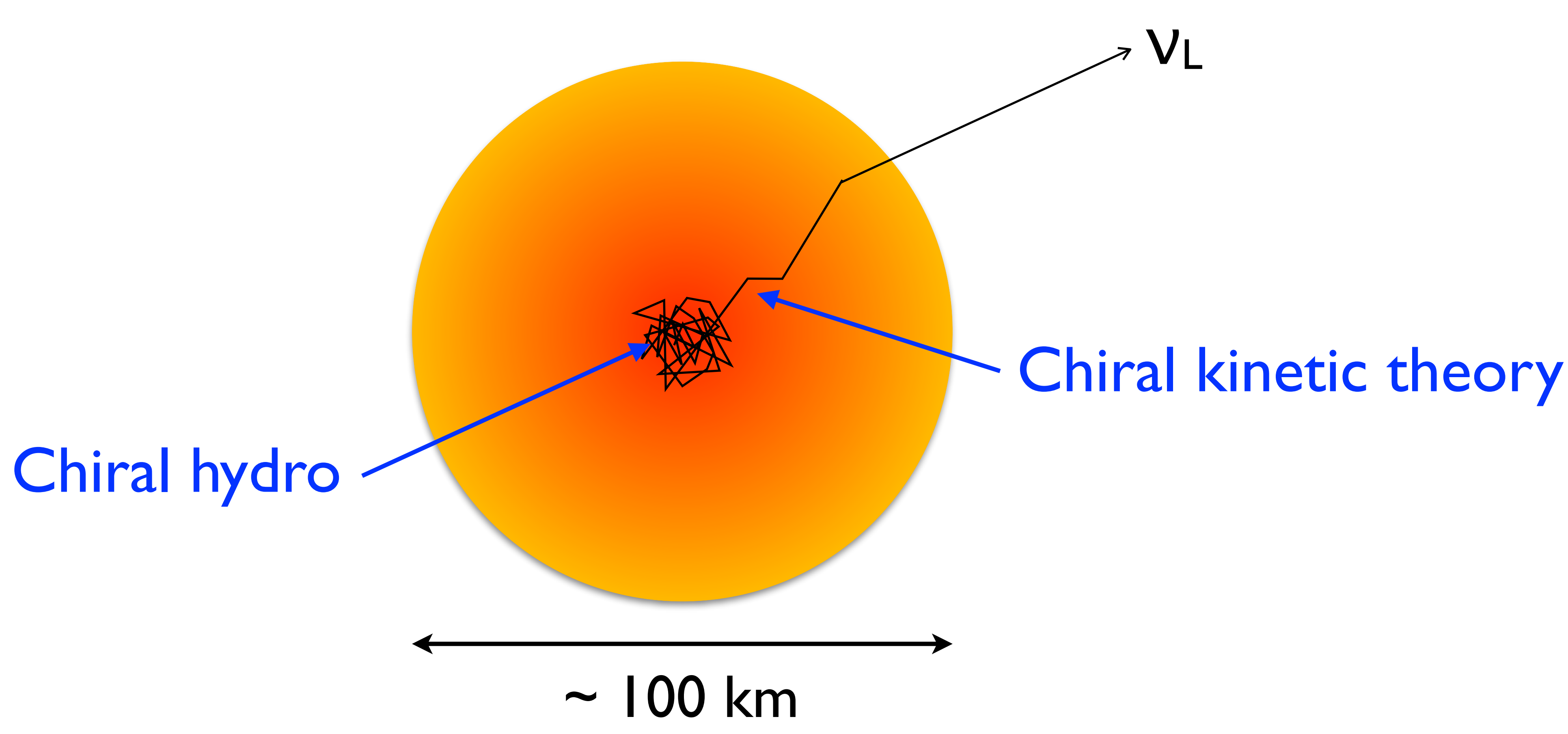


Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015); Sigl, Leite (2016), …

Neutrino matter in supernovae

- At the core ($\rho_N \sim 10^{15}$ g/cm^3), neutrino mean free path ~ 1cm
- Neutrino matter = Chiral liquid ($\mu_\nu \sim 200$ MeV $\gg$ T~10 MeV at core)

  = 3D topological matter   Yamamoto (2016)



Chiral radiation transfer

- Hydrodynamics for ν is not applicable outside the core.
- Generally,

$$\nabla_\alpha (T^{\alpha\beta}_{\mathrm{hyd}} + T^{\alpha\beta}_{\nu}) = 0$$

Stress tensor for matter
(Hydrodynamics)

Stress tensor for ν
(Chiral kinetic theory)

- T_ν and collisions between ν and matter are corrected by chirality

Wigner function formalism

Hidaka, Pu, Yang (2017); A. Huang et al. (2018); Liu et al. (2019)

- Wigner function: $S^{<}(q,x) = \int_y \mathrm{e}^{-\mathrm{i}q\cdot y}\langle \psi^\dagger(x+y/2)\psi(x-y/2)\rangle \equiv \sigma^\mu \mathcal{L}^{<}_\mu$

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- Kadanoff-Baym eqs: $\mathcal{D}\cdot\mathcal{L}^< = 0,$

$$q\cdot\mathcal{L}^< = 0, \quad \mathcal{D}_\mu\mathcal{L}^<_\nu - \mathcal{D}_\nu\mathcal{L}^<_\mu = -2\epsilon_{\mu\nu\rho\sigma}q^\rho\mathcal{L}^{<\sigma}$$

where $\mathcal{D}_\mu\mathcal{L}^<_\nu \equiv \partial_\mu\mathcal{L}^<_\nu - \Sigma^<_\mu\mathcal{L}^>_\nu + \Sigma^>_\mu\mathcal{L}^<_\nu$

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  where $\mathcal{D}_\mu\mathcal{L}^<_\nu \equiv \partial_\mu\mathcal{L}^<_\nu - \Sigma^<_\mu\mathcal{L}^>_\nu + \Sigma^>_\mu\mathcal{L}^<_\nu$
- Solution: $\mathcal{L}^{<\mu} = 2\pi\delta(q^2)\big(q^\mu - S^{\mu\nu}_{(n)}\mathcal{D}_\nu\big)f^<$ where $S^{\mu\nu}_{(n)} = \dfrac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}$

  side-jump

  frame vector

Chen, Son, Stephanov (2015)

ν chiral radiation transfer eq.

Yamamoto, Yang, arXiv:2020.11348

$$\left[q^\mu(\partial_\mu - \Gamma^\lambda_{\mu\rho} q^\rho \partial_{q\lambda}) - (D_\mu S^{\mu\nu}_{(n)})\partial_\nu + S^{\mu\nu}_{(n)} q^\rho R^\lambda_{\rho\mu\nu}\partial_{q\lambda}\right] f = (1-f)\Gamma^<_{(n)} - f\Gamma^>_{(n)}$$

emission absorption

where $D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu} q^\nu \partial_{q\lambda}$

$$\Gamma^{\lessgtr}_{(n)} = \left(q^\nu - D_\mu S^{\mu\nu}_{(n)}\right)\Sigma^{\lessgtr}_\nu$$

side-jump

ν chiral radiation transfer eq.

Yamamoto, Yang, arXiv:2020.11348

$$\left[q^\mu(\partial_\mu - \Gamma^\lambda_{\mu\rho} q^\rho \partial_{q\lambda}) \cancel{- (D_\mu S^{\mu\nu}_{(n)})\partial_\nu} + \cancel{S^{\mu\nu}_{(n)} q^\rho R^\lambda_{\rho\mu\nu} \partial_{q\lambda}}\right] f = \underbrace{(1-f)\Gamma^<_{(n)}}_{\text{emission}} - \underbrace{f\Gamma^>_{(n)}}_{\text{absorption}}$$

where $D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu} q^\nu \partial_{q\lambda}$

$$\Gamma^{\lessgtr}_{(n)} = \Big(q^\nu \underbrace{- D_\mu S^{\mu\nu}_{(n)}}_{\text{side-jump}}\Big) \Sigma^{\lessgtr}_\nu = \Big(q^\nu - S^{\mu\nu}_{(n)} D_\mu\Big) \Sigma^{\lessgtr}_\nu$$

- We take $R^\lambda_{\rho\mu\nu} = 0$ and $n^\mu = (1, \mathbf{0})$.

Collision term

Yamamoto, Yang, arXiv:2020.11348

- Assume that matter (except for ν) is in local thermal equilibrium.
- By symmetry, $\Gamma_q^{\lessgtr} \approx \Gamma_q^{(0)\lessgtr} + \Gamma_q^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma_q^{(B)\lessgtr}(q\cdot B)$

  where $\omega^\mu = \dfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta\,, \quad B^\mu = \dfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$
- We will be particularly interested in the process:

$$\nu_{\mathrm{L}}^{\mathrm{e}}(q) + \mathrm{n}(k) \rightleftharpoons \mathrm{e_L}(q') + \mathrm{p}(k')$$

Absorption and emission rates

Yamamoto, Yang, arXiv:2020.11348

$$\Gamma_q^{\lessgtr} \approx \Gamma_q^{(0)\lessgtr} + \Gamma_q^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma_q^{(B)\lessgtr}(q\cdot B)$$

- Based on the 4-Fermi theory of weak interaction,

$$\Gamma_q^{(0)>} \approx \frac{G_\mathrm{F}^2}{\pi}\left(g_\mathrm{V}^2+3g_\mathrm{A}^2\right)E_\nu^3\left(1-f_q^{(\mathrm{e})}\right)\left(1-\frac{3E_\nu}{M_\mathrm{N}}\right)\frac{n_\mathrm{p}-n_\mathrm{n}}{1-\mathrm{e}^{\beta(\mu_\mathrm{n}-\mu_\mathrm{p})}}$$

$$\Gamma_q^{(B)>} \approx \frac{G_\mathrm{F}^2}{2\pi M_\mathrm{N}}\left(g_\mathrm{V}^2+3g_\mathrm{A}^2\right)E_\nu\left(1-f_q^{(\mathrm{e})}\right)\left(1-\frac{8E_\nu}{3M_\mathrm{N}}\right)\frac{n_\mathrm{p}-n_\mathrm{n}}{1-\mathrm{e}^{\beta(\mu_\mathrm{n}-\mu_\mathrm{p})}}$$

$$\Gamma_q^{(\omega)>} \approx \frac{G_\mathrm{F}^2}{2\pi}\left(g_\mathrm{V}^2+3g_\mathrm{A}^2\right)E_\nu\left(1-f_q^{(\mathrm{e})}\right)\left(2+\beta E_\nu f_q^{(\mathrm{e})}\right)\frac{n_\mathrm{p}-n_\mathrm{n}}{1-\mathrm{e}^{\beta(\mu_\mathrm{n}-\mu_\mathrm{p})}}$$

$\Gamma^{(0)}$ was previously computed in Reddy, Prakash, Lattimer (1998)

Physical consequences

Yamamoto (2016); Yamamoto, Yang, arXiv:2020.11348

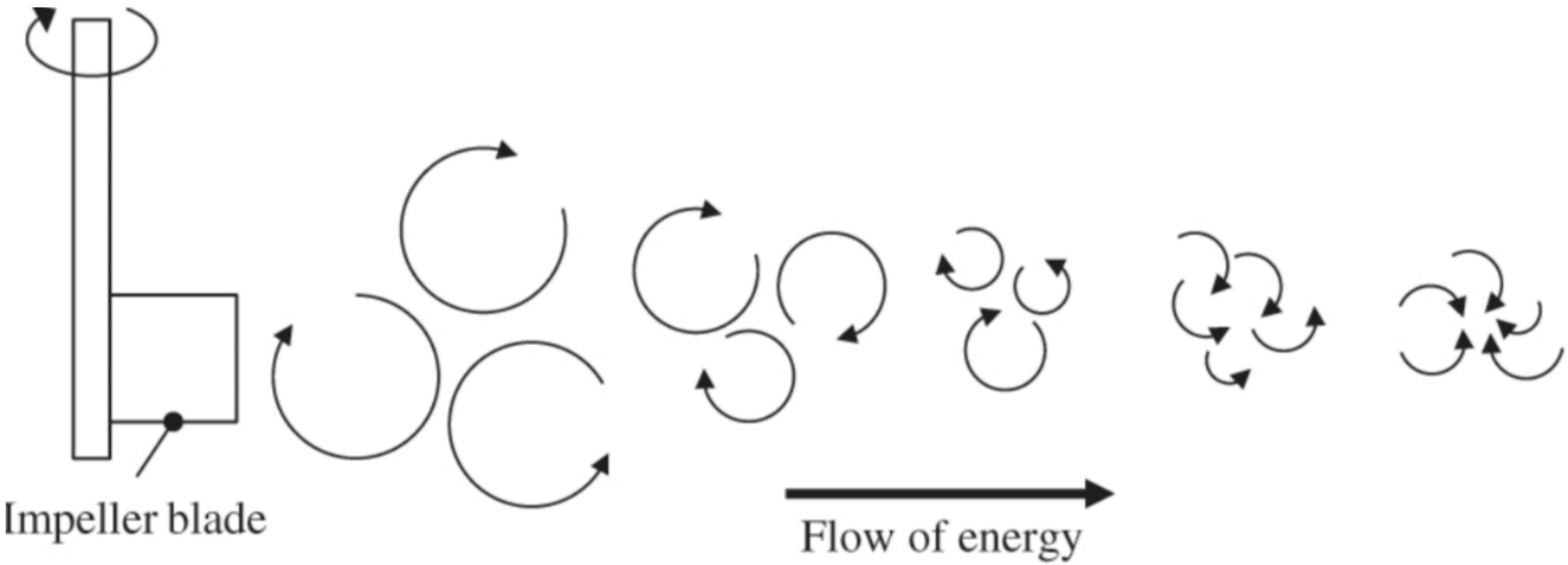
$$\Gamma_q^{\lessgtr} \approx \Gamma_q^{(0)\lessgtr} + \Gamma_q^{(\omega)\lessgtr}(q \cdot \omega) + \Gamma_q^{(B)\lessgtr}(q \cdot B)$$

- ν propagating collinear to the flow generates fluid helicity $\boldsymbol{v} \cdot \boldsymbol{\omega}$ and cross helicity $\boldsymbol{v} \cdot \boldsymbol{B}$.
- Fluid helicity gives an analogue of CME (helical magnetic effect):

$$\boldsymbol{j} \sim (\boldsymbol{v} \cdot \boldsymbol{\omega})\boldsymbol{B}$$

Chiral MHD turbulence in supernovae

Turbulence and cascade



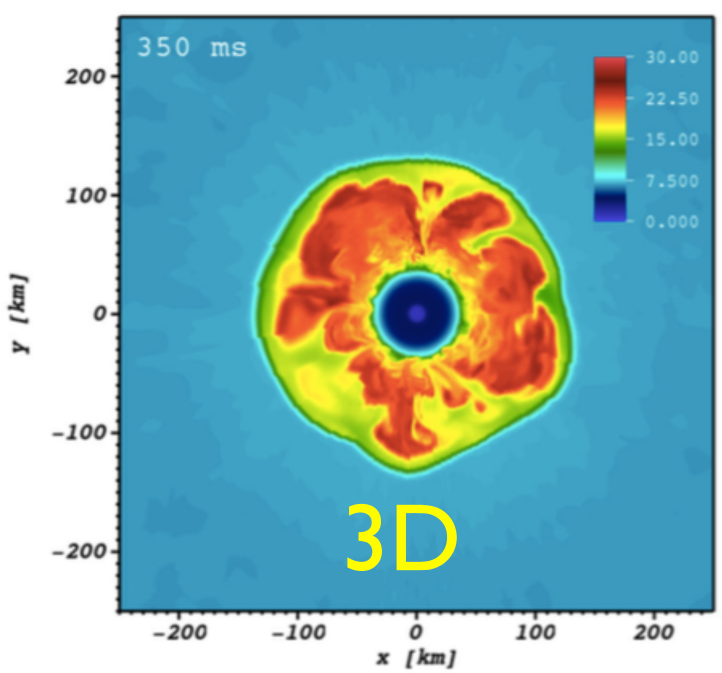
https://doi.org/10.1515/htmp-2016-0043

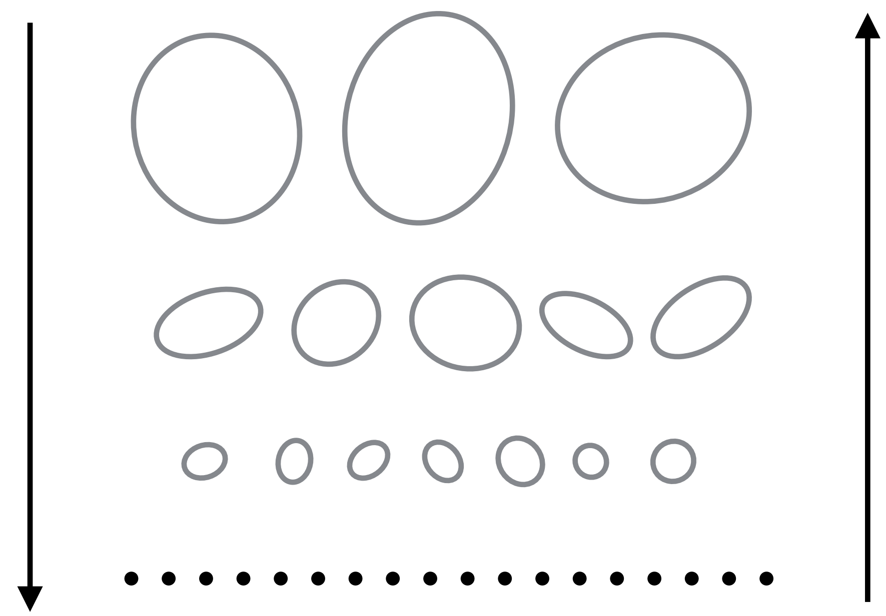
- The structure becomes smaller, and eventually dissipates (direct cascade)
- Similar in magneto-hydrodynamics (MHD)

Cascade and explosion

Direct cascade
(3D usual matter)

explosion difficult

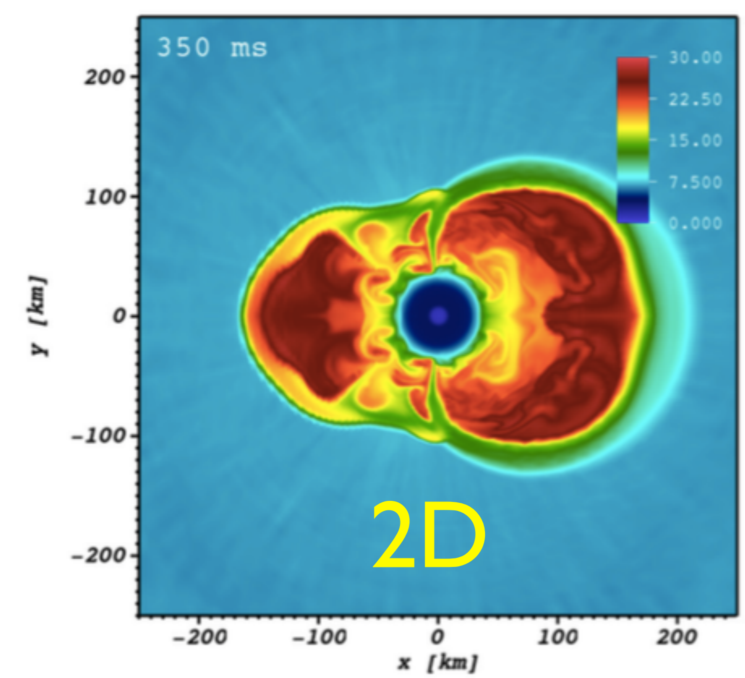




F. Hanke (2014)

Inverse cascade
(2D usual matter)

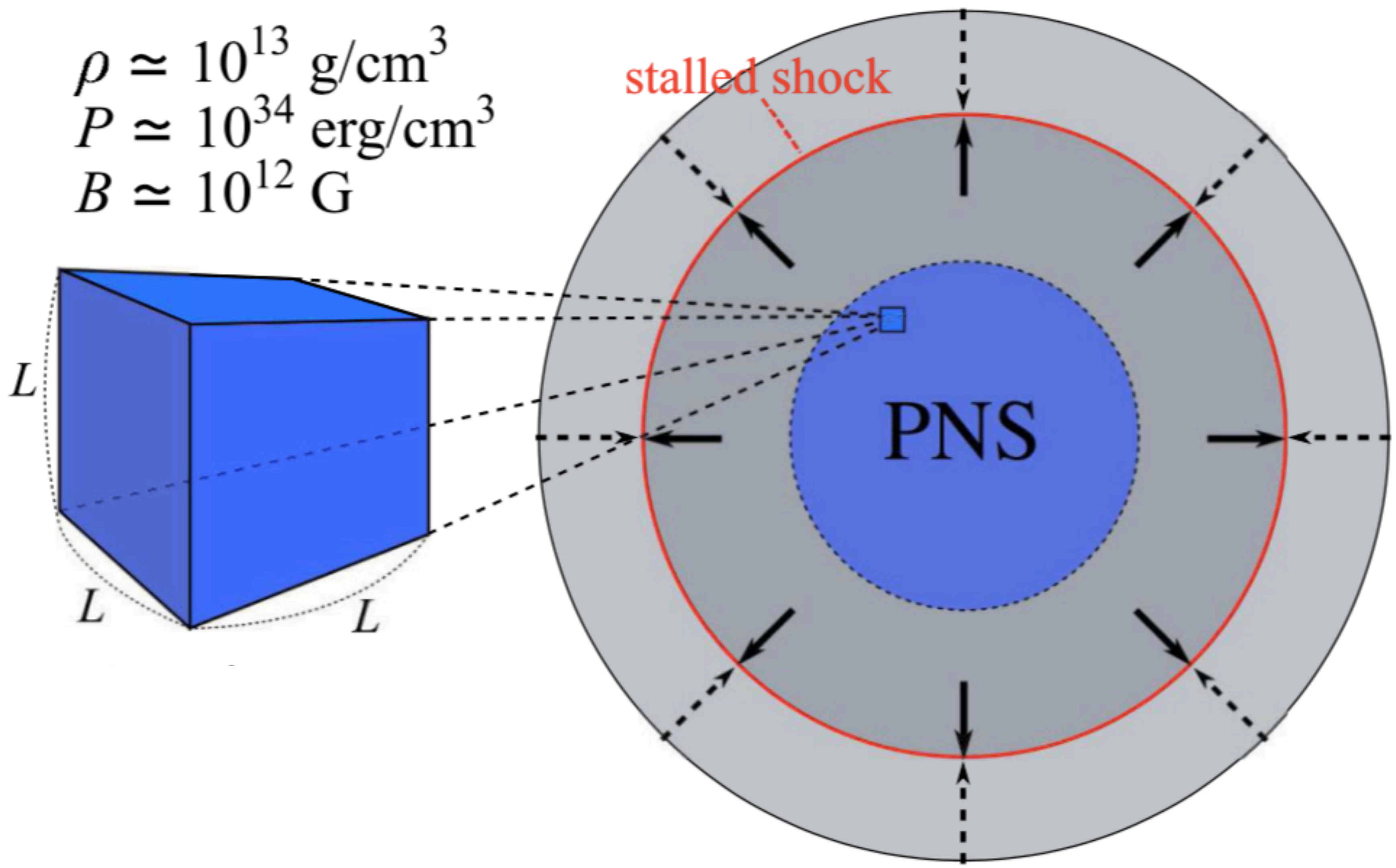
explosion easier



What about 3D chiral matter?

Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419



Proto-neutron star (PNS)

Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419

- Chiral MHD w/o vorticity at the core (proton and electrons):

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$

$$\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \boldsymbol{v}) = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \text{(dissipation)}$$

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \boldsymbol{\nabla} \times \underbrace{(\xi_B \boldsymbol{B})}_{\text{CME or helical magnetic effect}}$$

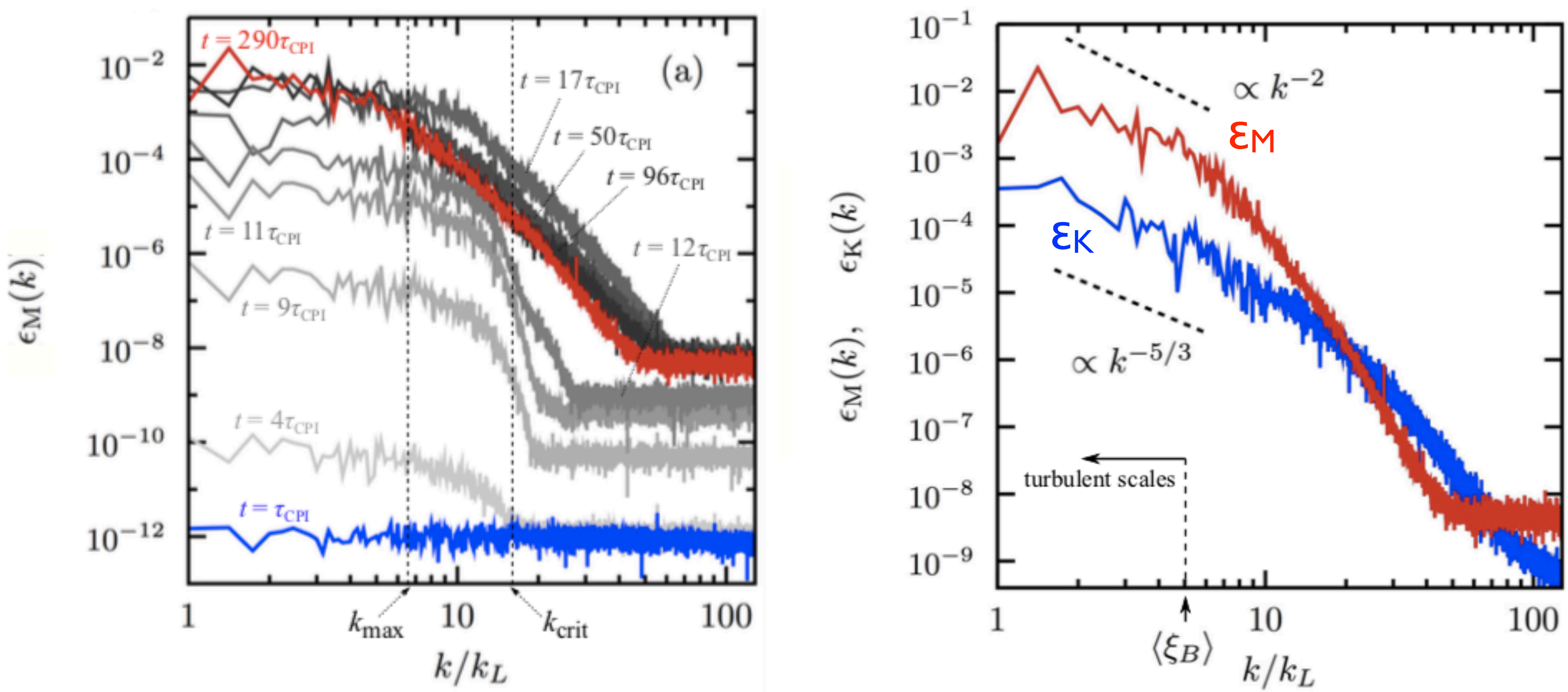
$$\partial_t n_5 = \underbrace{\frac{\eta}{2\pi^2} (\boldsymbol{\nabla} \times \boldsymbol{B} - \xi_B \boldsymbol{B}) \cdot \boldsymbol{B}}_{\text{chiral anomaly}}$$

- Setup for proto-neutron stars (100 MeV = 1):

$$\rho_0 = 5.0,\ P_0 = 1.0,\ \xi_{B0} = 4.2 \times 10^{-3},\ \eta = 100.0$$

Masada et al., arXiv:1805.10419; movie available at
http://www.kusastro.kyoto-u.ac.jp/~masada/movie.mp4

Energy spectra



Chiral effects lead to the inverse cascade, which may be favorable for explosions

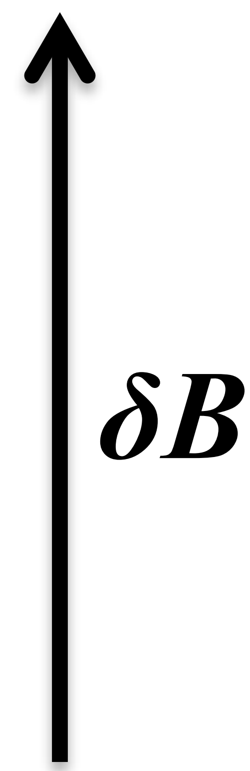
Masada et al., arXiv:1805.10419; see also Brandenburg et al., arXiv:1707.03385; Mace et al., arXiv:1910.01654

Summary and outlook

- Chirality = topology in relativistic many-body systems.
- Non-Abelian chiral plasma instability in QGP.
- Relevance of chiral transport of neutrinos in supernovae.
- Chiral kinetic theory can be generalized to photons.
  Yamamoto (2017); Huang, Mitkin, Sadofyev, Speranza, arXiv:2006.03591;
  Hattori, Hidaka, Yamamoto, Yang, in preparation
- Further applications of chiral kinetic theory to cond-mat, nuclear, and astro physics, and cosmology.

Backup slides

Chiral plasma instability



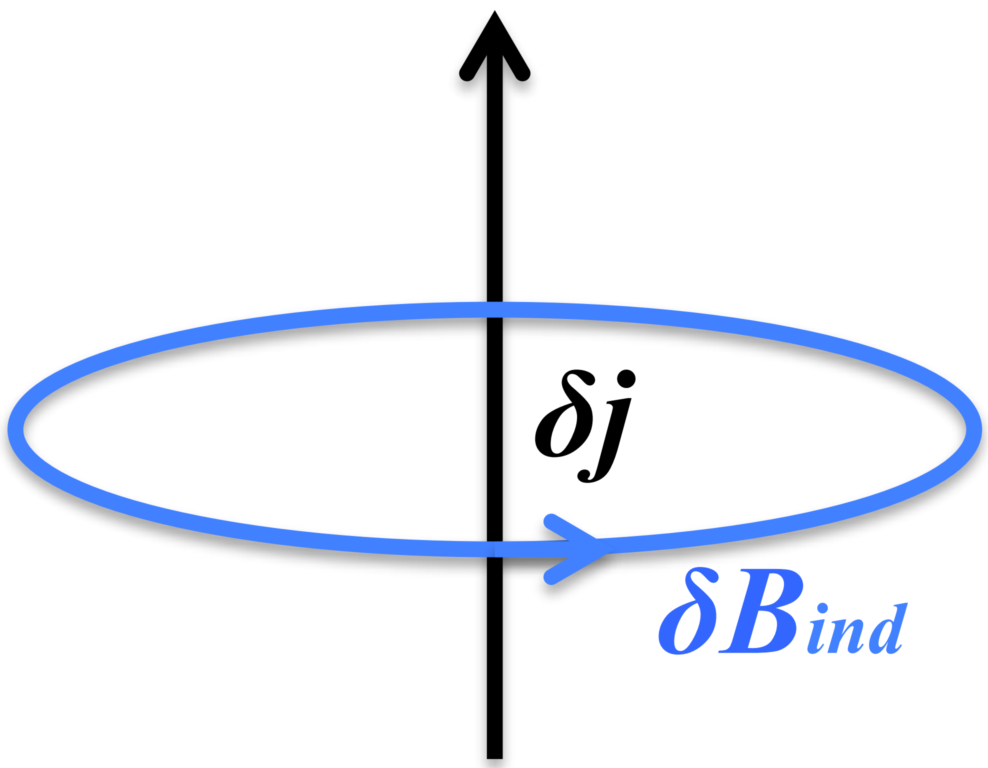
Assume nonvanishing $\mu_5 \equiv \mu_{\mathrm{R}} - \mu_{\mathrm{L}}$ or fluid helicity

Chiral plasma instability

Chiral magnetic effect

$$\delta \boldsymbol{j} \sim \mu_5 \delta \boldsymbol{B}$$

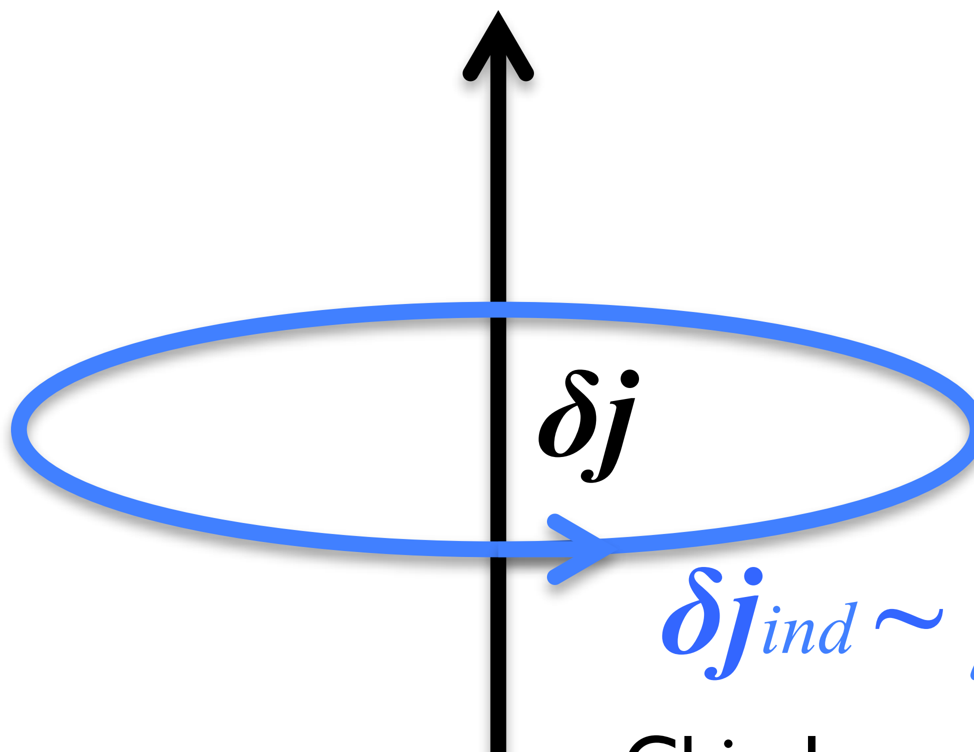
Chiral plasma instability



Ampere's law

$$\nabla \times B = j$$

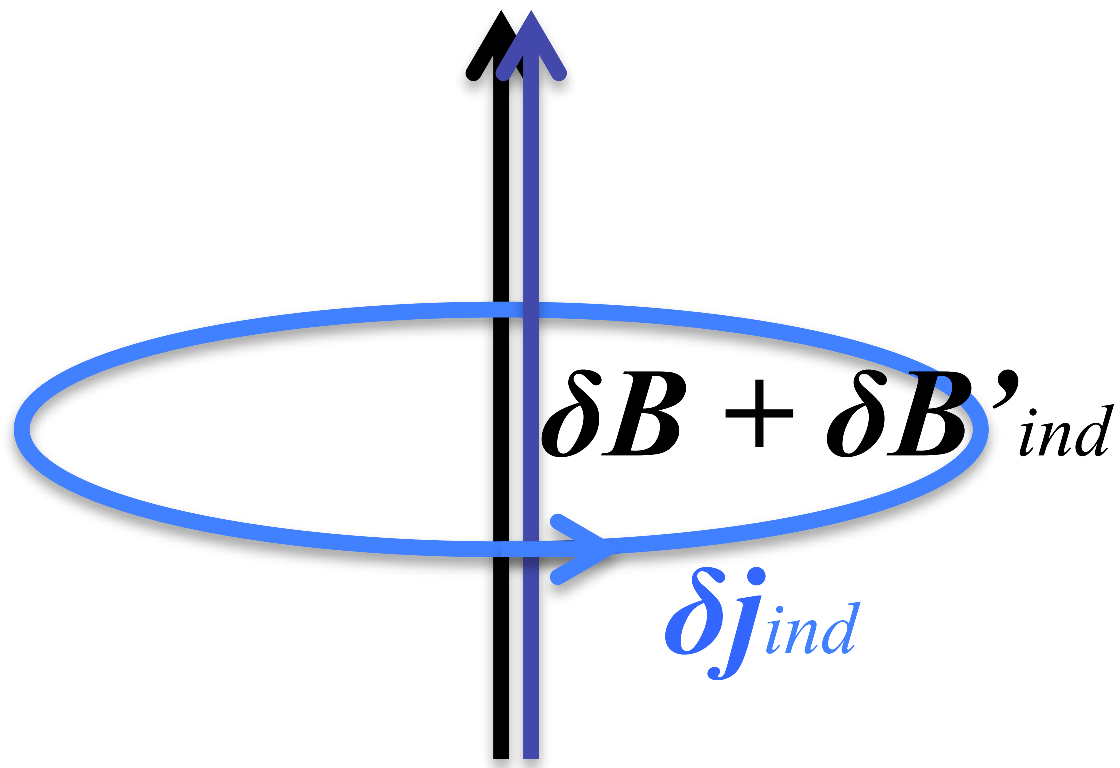
Chiral plasma instability



$\delta \boldsymbol{j}_{ind} \sim \mu_5 \delta \boldsymbol{B}_{ind}$

Chiral magnetic effect

Chiral plasma instability

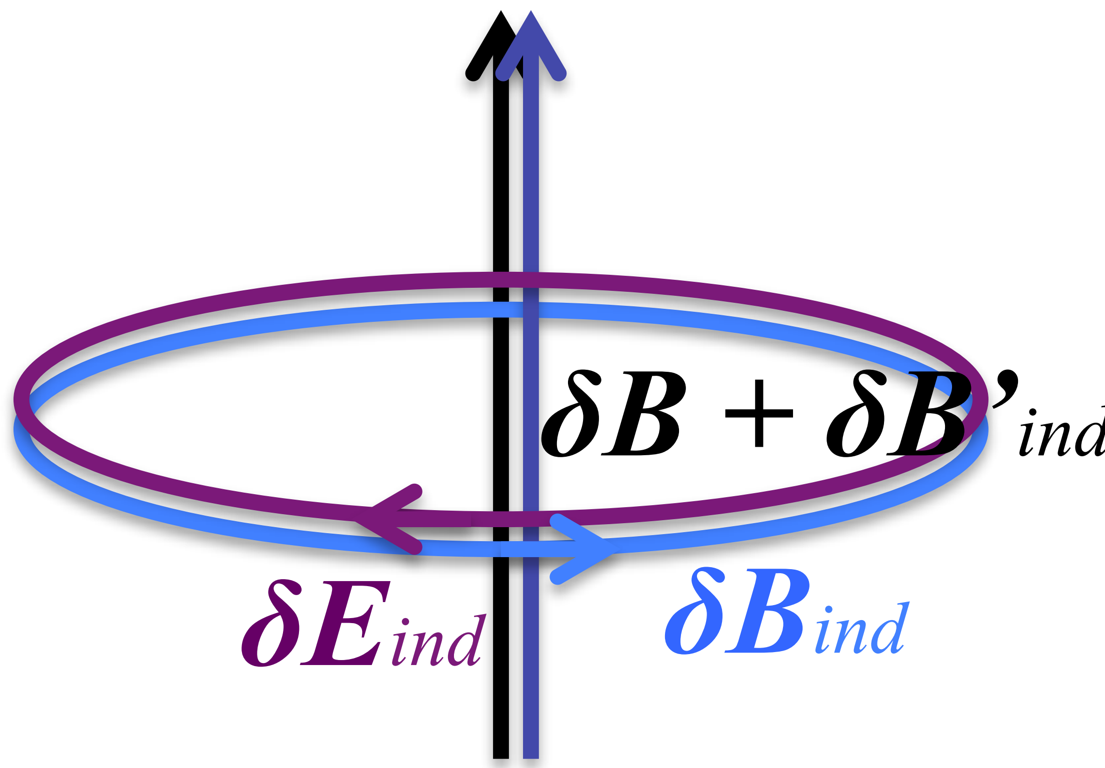


Ampere's law

$$\nabla \times B = j$$

Positive feedback: instability

Chiral plasma instability



Faraday's law

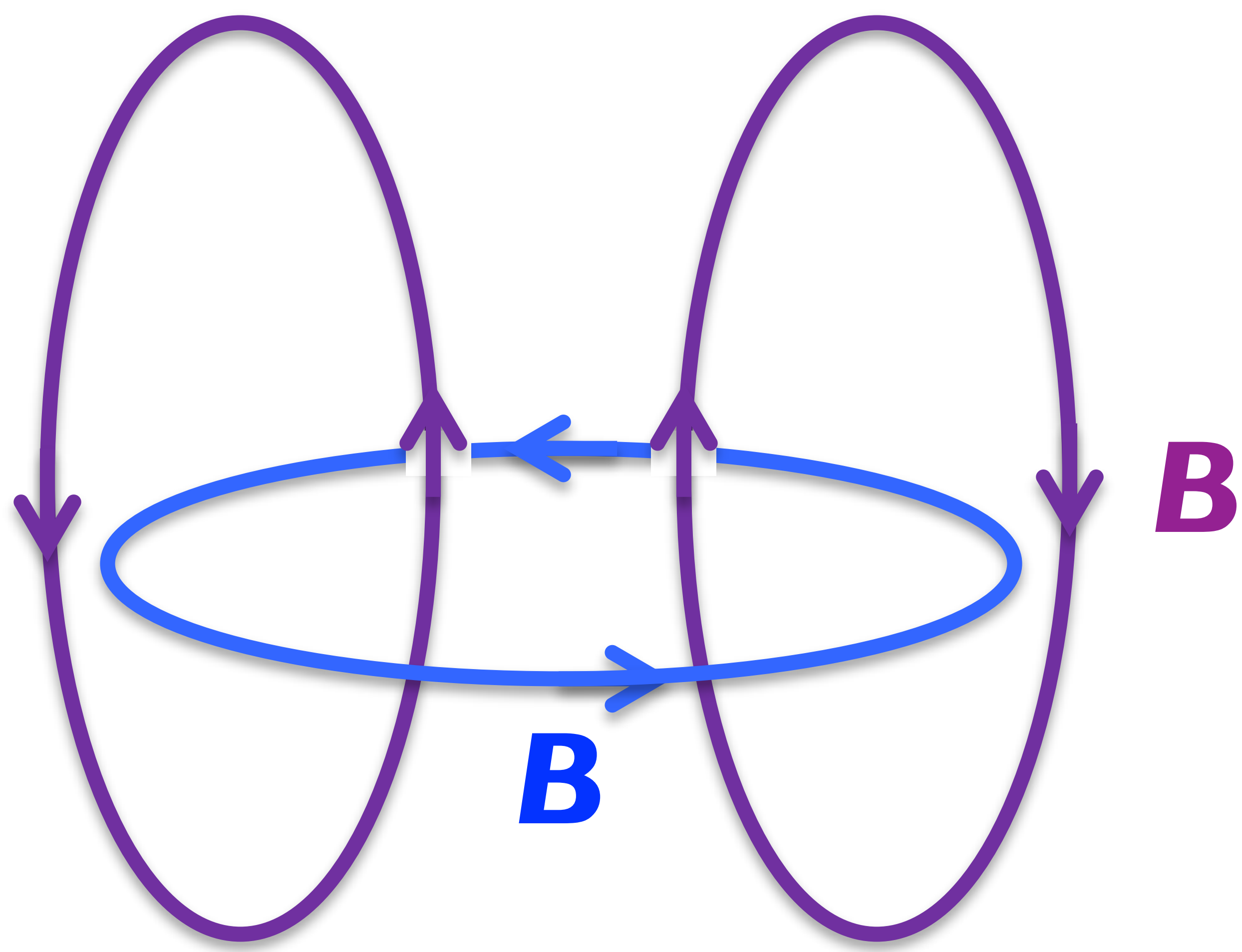
$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$

Chiral anomaly

$$\Delta n_5 = C\boldsymbol{E} \cdot \boldsymbol{B} < 0$$

Chiral anomaly tends to reduce μ_5

Chiral plasma instability



Chirality imbalance → Helical magnetic field