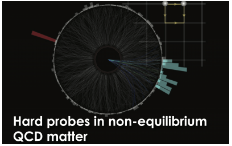

Heavy-quark dynamics in a non(near)-equilibrium hot QCD matter

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Outline

Be prepared for a journey, but don't expect a destination." — Craig F. Bohren

- Heavy quarks (charm c, bottom b) are "hard" probes created in the initial, high-energy stage of heavy-ion collisions.
- Unlike light quarks, their large mass ($M >> T$) results in a longer thermalization time, allowing them to record the entire spacetime history of the Quark-Gluon Plasma (QGP).
- Their evolution is modeled via the Boltzmann equation, which simplifies to the Fokker-Planck framework for small momentum transfers.
- Key Observables: Nuclear Modification Factor (R_{AA}) and flow coefficients (v_2,v_1) to extract the properties of the QGP medium
- These observables depend strongly on: HQ Drag coefficient(s) and the Momentum diffusion coefficient(s)

Anisotropic Medium!

- Momentum anisotropy might be present throughout the space-time evolution of the QGP

- In early (pre-equilibrium) stage of heavy-ion collisions the plasma is not isotropic.

$$f(\mathbf{p}) \neq f(p)$$

- This affects: screening, transport coefficients, heavy quark energy loss and thus R_{AA} and collective flow coefficients

- Momentum anisotropy can be introduced via Anisotropic Distribution Function:

$$f_{\text{aniso}}(\mathbf{p}) = f_{\text{iso}}\left(\sqrt{p^2 + \xi(\mathbf{p}\cdot\mathbf{n})^2}\right)$$

where ξ : anisotropy parameter, $\mathbf{n}$: anisotropy direction

Anisotropy and QGP!

- Spatial-momentum anisotropy and flow: Ollitrault (19192), Volosin and Zhang (1996)
- Early time anisotropy and non-equilibrium QGP: Strickland (2013)
- Anisotropic Hydro: Florkowski and Ryblewski, PRC(2010), Martinez, Strickland (2014)
- Glasma and anisotropic momentum broadening, Ipp et. al, PRD (2020)
- Dileptons from anisotropic QGP: Ryblewski, Strickland (2015)
- Transport models: Bass et al., Bleicher et al. (2000s)
- Heavy quark diffusion in strongly anisotropic plasma: Giataganas et al., JHEP (2014).
- HQ potential in anisotropic QGP: Dumitru, Guo, Strickland (2009–2010)
- Plasma instabilities: Romatschke, Strickland (2003–2004)
- Momentum anisotropy leads to Chromo-Weibel instability and anamalous transport in the hot QCD medium: the transport coefficient are suppressed hugely: one plausible explanation for the small η/S in the hot QCD medium, Asakawa, Bass, Muller, PRL (2006), VC, Ravishankar EPJC (2007), VC, PRD (2012).

The Core Question here is How does this anisotropy drag and diffusion felt by a heavy quark ?

The Question will be discussed/addressed based on:

- *Drag of heavy quarks in an anisotropic QCD medium beyond the static limit*, Kumar, Das, Kurian, VC, Phys. Rev. C **105** (2022) 5, 054903
- *Heavy quark radiation in an anisotropic hot QCD medium*, Prakash, VC, Das, Phys. Rev. **D 108**, 096016 (2023).

- The Heavy quark evolution in the medium is described by the Boltzmann transport equation

$$\frac{\partial f_{HQ}}{\partial t} + \mathbf{v} \cdot \nabla f_{HQ} = C[f_{HQ}]$$

where, $f_{HQ}(\mathbf{p}, t)$ is the heavy quark distribution
and $C[f_{HQ}]$ is the collision integral

- For small momentum transfer, we expand the collision term. In this limit the Boltzmann equation reduces to Fokker–Planck Equation (Svetitsky, 1987)

$$\frac{\partial f_{HQ}}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\mathbf{p}) f_{HQ} + \frac{\partial}{\partial p_j}\Big(B_{ij}(\mathbf{p}) f_{HQ}\Big)\right], \tag{1}$$

where, A_i : drag force and B_{ij} : momentum diffusion tensor (The interactions of HQ with the light quarks and gluons are quantified in terms of drag force A_i and momentum diffusion B_{ij} in the QGP medium).

- The drag force of HQ describes the thermal average of the momentum transfer due to the interaction, whereas the momentum diffusion quantifies the average of the square of the momentum transfer (elastic HQ and light quarks and gluon collision processes)

$$\begin{aligned} A_i =& \frac{1}{2E_p}\int \frac{d^3\mathbf{q}}{(2\pi)^3 2E_q}\int \frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{q'}}\int \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_{HQ}} \\ &\times \sum |\mathcal{M}_{2\rightarrow 2}|^2 (2\pi)^4 \delta^4(P+Q-P'-Q') f_k(\mathbf{q}) \\ &\times \Big(1 + a_k f_k(\mathbf{q}')\Big)\Big[(\mathbf{p}-\mathbf{p}')_i\Big] = \langle\langle(\mathbf{p}-\mathbf{p}')_i\rangle\rangle, \end{aligned} \tag{2}$$

$$B_{ij} = \frac{1}{2}\langle\langle(\mathbf{p}-\mathbf{p}')_i(\mathbf{p}-\mathbf{p}')_j\rangle\rangle, \tag{3}$$

where γ_{HQ} is the statistical degeneracy factor of the HQ, f_k represents the near-equilibrium distribution function of quark/antiquark and gluon.

- In general, HQ drag and diffusion coefficients can be schematically described as,

$$X_c = \int \text{phase space} \times \text{ interaction} \times \text{transport part}.$$

- In an isotropic medium, the drag force depends on the HQ momentum and A_i can be decomposed as,

$$A_i = p_i A_0(p^2), \tag{4}$$

  where $p^2 = |\mathbf{p}|^2$ and A_0 is the drag coefficient of the HQ in the isotropic QGP medium.

- The drag coefficient can be obtained from Eq. (2) and Eq. (4) as,

$$A_0 = p_i A_i / p^2 = \langle\langle 1 \rangle\rangle - \frac{\langle\langle \mathbf{p} \cdot \mathbf{p}' \rangle\rangle}{p^2}. \tag{5}$$

- Similarly, B_{ij} can be decomposed into longitudinal and transverse components in the isotropic QCD medium as,

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) B_0(p^2) + \frac{p_i p_j}{p^2} B_1(p^2), \tag{6}$$

- where the transverse and longitudinal diffusion coefficients can be defined as follows,

$$B_0 = \frac{1}{4}\left[\langle\langle p'^2\rangle\rangle - \frac{\langle\langle(\mathbf{p}'\cdot\mathbf{p})^2\rangle\rangle}{p^2}\right], \tag{7}$$

$$B_1 = \frac{1}{2}\left[\frac{\langle\langle(\mathbf{p}'\cdot\mathbf{p})^2\rangle\rangle}{p^2} - 2\langle\langle(\mathbf{p}'\cdot\mathbf{p})\rangle\rangle + p^2\langle\langle 1\rangle\rangle\right]. \tag{8}$$

- The kinematics of $2\rightarrow 2$ process can be simplified in the center-of-momentum (COM) frame of the system, and the average of a function $F(\mathbf{p})$ in the COM frame for the isotropic medium can be described as follows,

$$\begin{aligned}\langle\langle F(\mathbf{p})\rangle\rangle = & \frac{1}{(512\,\pi^4)E_p\gamma_{HQ}}\int_0^\infty dq\left(\frac{s-m_{HQ}^2}{s}\right) f_k^0(E_q)\\ & \times\int_0^\pi d\chi\,\sin\chi\int_0^\pi d\theta_{cm}\,\sin\theta_{cm}\sum|\mathcal{M}_{2\rightarrow 2}|^2\\ & \times\int_0^{2\pi} d\phi_{cm}(1+a_k f_k^0(E_{q'}))\,F(\mathbf{p}),\end{aligned} \tag{9}$$

χ quantifies the angle between the incident heavy quarks and the constituent particles of the medium in the lab frame. The quantities θ_{cm} and ϕ_{cm} respectively describe the zenith and azimuthal angle in the COM frame.

- Here, the Mandelstam variables s, t, u are defined as follows,

$$s = (E_p + E_q)^2 - (p^2 + q^2 + 2pq\cos\chi), \tag{10}$$

$$t = 2\,p_{cm}^2(\cos\theta_{cm} - 1), \tag{11}$$

$$u = 2\,m_{HQ}^2 - s - t, \tag{12}$$

- $p_{cm} = |\mathbf{p}_{cm}|$ as the magnitude of initial momentum of HQ in the COM frame.

- Next, we will proceed to incorporate the anisotropy in the whole formalism.

- The momentum anisotropic distribution:

$$f_k^{(\text{aniso})}(\mathbf{q}) = \sqrt{1+\xi}\, f_k^0\left(\sqrt{q^2+\xi(\mathbf{q}\cdot\mathbf{n})^2}\right), \tag{13}$$

in Weakly anisotropic medium, $\xi \ll 1$ reduces to: $f_k^{(\text{aniso})}(\mathbf{q}) = f_k^0 + \delta f_k$ with: (Srivastava et. al, Phys. Rev. C 91, 044903 (2015))

$$\delta f_k = -\frac{\xi}{2E_q T}(\mathbf{q}\cdot\mathbf{n})^2 (f_k^0)^2 \exp\left\{\left(\frac{E_q}{T}\right)\right\}. \tag{14}$$

Define $\tilde{n}^i = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) n^j$ such that $\mathbf{p}\cdot\tilde{\mathbf{n}} = 0$

- The drag force in the anisotropic medium can be decomposed on the orthogonal basis as follows,

$$A_i = p_i A_0^{(\text{aniso})} + \tilde{n}_i A_1^{(\text{aniso})}. \tag{15}$$

with

$$A_0^{(\text{aniso})} = p_i A_i / p^2 = \langle\langle 1 \rangle\rangle - \frac{\langle\langle \mathbf{p}\cdot\mathbf{p}' \rangle\rangle}{p^2}, \tag{16}$$

$$A_1^{(\text{aniso})} = \tilde{n}_i A_i / \tilde{n}^2 = -\frac{1}{\tilde{n}^2}\langle\langle \tilde{\mathbf{n}}\cdot\mathbf{p}' \rangle\rangle, \tag{17}$$

where $\tilde{n}^2 = 1 - \frac{(\mathbf{p}\cdot\hat{\mathbf{n}})^2}{p^2} = 1 - \cos^2\theta_n$.

- The average of a function $F(p')$ in the anisotropic medium can be defined as,

$$\langle\langle F(p')\rangle\rangle = \langle\langle F(p')\rangle\rangle_0 + \langle\langle F(p')\rangle\rangle_a, \tag{18}$$

where the isotropic part $\langle\langle F(p')\rangle\rangle$ is defined in Eq. (9).

- Following the same prescription as in the case of isotropic case, we can represent $\langle\langle F(p')\rangle\rangle_a$ in the COM frame as,

$$\begin{aligned}\langle\langle F(\mathbf{p})\rangle\rangle_a = &\frac{1}{(1024\,\pi^5)E_p\gamma_{HQ}}\int_0^\infty dqq\left(\frac{s-m_{HQ}^2}{s}\right)\\ &\times\int_0^\pi d\chi\,\sin\chi\int_0^{2\pi}d\phi\int_0^\pi d\theta_{cm}\,\sin\theta_{cm}\sum|\mathcal{M}_{2\to2}|^2\\ &\times\int_0^{2\pi}d\phi_{cm}\left[\delta f_k(\mathbf{q})\Big(1+a_k f_k^0(\mathbf{q}')\Big)+a_k f_k^0(\mathbf{q})\delta f_k(\mathbf{q}')\right]F(\mathbf{p}).\end{aligned} \tag{19}$$

Employing Eq. (18) in Eq. (16), we obtain the non-equilibrium correction to the HQ drag coefficient described in Eq. (5) as,

$$A_0^{(\text{aniso})} = A_0 + \delta A_0$$

- To decompose the HQ diffusion, one needs to construct the appropriate tensor basis for the symmetric matrix B_{ij} with the momentum vector p^i and anisotropy vector n^i. Following: Romatschke and Strickland, Phys. Rev. D 68, 036004 (2003):

$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) B_0^{(\text{aniso})} + \frac{p_i p_j}{p^2} B_1^{(\text{aniso})} + \frac{\tilde{n}_i \tilde{n}_j}{\tilde{n}^2} B_2^{(\text{aniso})} + (p^i \tilde{n}^j + p^j \tilde{n}^i) B_3^{(\text{aniso})}, \quad (20)$$

- The components:

$$B_0^{(\text{aniso})} = \left[\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) - \frac{\tilde{n}_i \tilde{n}_j}{\tilde{n}^2}\right] B_{ij} = \frac{1}{2}\left[\langle\langle p'^2 \rangle\rangle - \frac{\langle\langle (\mathbf{p}' \cdot \mathbf{p})^2 \rangle\rangle}{p^2} - \frac{\langle\langle (\mathbf{p}' \cdot \tilde{\mathbf{n}})^2 \rangle\rangle}{\tilde{n}^2}\right], \quad (21)$$

$$B_1^{(\text{aniso})} = \frac{p_i p_j}{p^2} B_{ij} = \frac{1}{2}\left[\frac{\langle\langle (\mathbf{p}' \cdot \mathbf{p})^2 \rangle\rangle}{p^2} - 2\langle\langle (\mathbf{p}' \cdot \mathbf{p}) \rangle\rangle + p^2 \langle\langle 1 \rangle\rangle\right], \quad (22)$$

$$B_2^{(\text{aniso})} = \left[\frac{2\tilde{n}_i \tilde{n}_j}{\tilde{n}^2} - \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right)\right] B_{ij} = \frac{1}{2}\left[-\langle\langle p'^2 \rangle\rangle + \frac{\langle\langle (\mathbf{p}' \cdot \mathbf{p})^2 \rangle\rangle}{p^2} + \frac{2\langle\langle (\mathbf{p}' \cdot \tilde{\mathbf{n}})^2 \rangle\rangle}{\tilde{n}^2}\right], \quad (23)$$

$$B_3^{(\text{aniso})} = \frac{1}{2p^2 \tilde{n}^2}(p^i \tilde{n}^j + p^j \tilde{n}^i) B_{ij} = \frac{1}{2p^2 \tilde{n}^2}\left[-p^2 \langle\langle (\mathbf{p}' \cdot \tilde{\mathbf{n}}) \rangle\rangle + \langle\langle (\mathbf{p}' \cdot \mathbf{p})(\mathbf{p}' \cdot \tilde{\mathbf{n}}) \rangle\rangle\right]. \quad (24)$$

- observe that the term $\langle\langle \tilde{\mathbf{n}} \cdot \mathbf{p}' \rangle\rangle$ is non-zero and modifies the HQ transport

- Estimation of $\langle\langle \tilde{\mathbf{n}} \cdot \mathbf{p}' \rangle\rangle$ in COM:
  We have $\mathbf{n} = (\sin\theta_n, 0, \cos\theta_n)$, where angle θ_n is the angle between anisotropy vector and $\mathbf{n}$. It is important to note that the analysis is also valid for the choice $\mathbf{n} = (0, \sin\theta_n, \cos\theta_n)$.
- Light quark momentum can be decomposed as $\mathbf{q} = (q\sin\chi\cos\phi, q\sin\chi\sin\phi, q\cos\chi)$ and HQ momentum chosen as $\mathbf{p} = (0, 0, p)$ such that we have,

$$\mathbf{p} \cdot \mathbf{q} = pq\cos\chi, \tag{25}$$

$$\mathbf{p} \cdot \mathbf{n} = p\cos\theta_n, \tag{26}$$

$$\mathbf{q} \cdot \mathbf{n} = q\sin\chi\cos\phi\sin\theta_n + q\cos\chi\cos\theta_n, \tag{27}$$

We have $\tilde{n}^i p'^{\,i} = p'^{\,i}\left(\delta^{ij} - \dfrac{p^i p^j}{p^2}\right)n^j$. Hence, we have

$$\langle\langle \tilde{\mathbf{n}} \cdot \mathbf{p}' \rangle\rangle = \langle\langle \mathbf{n} \cdot \mathbf{p}' \rangle\rangle - \langle\langle \mathbf{p} \cdot \mathbf{p}' \rangle\rangle \frac{\cos\theta_n}{p}. \tag{28}$$

$$\mathbf{p}' = \gamma_{cm}\left(\hat{\mathbf{p}}'_{cm} + \mathbf{v}_{cm}\hat{E}'_{cm}\right), \tag{29}$$

where $\gamma_{cm} = \frac{E_p + E_q}{\sqrt{s}}$ and the velocity of the center-of-mass $\mathbf{v}_{cm} = \frac{\mathbf{p}+\mathbf{q}}{E_p+E_q}$. The energy conservation leads to $\hat{p}'^{\,2}_{cm} = \hat{p}^2_{cm}$.

- In the center-of-mass frame, $\hat{\mathbf{p}}^{'}_{cm}$ can be decomposed as follows,

$$\tilde{\mathbf{n}} \cdot \mathbf{p}' = \frac{\gamma_{cm}}{1+\gamma_{cm}^2 v_{cm}^2}\Bigg\{\hat{p}_{cm}\Bigg(\cos\theta_{cm}(\hat{\mathbf{x}}_{cm}\cdot\mathbf{n}) + \sin\theta_{cm}\sin\phi_{cm}(\hat{\mathbf{y}}_{cm}\cdot\mathbf{n}) + \sin\theta_{cm}\cos\phi_{cm}(\hat{\mathbf{z}}_{cm}\cdot\mathbf{n})\Bigg) + \gamma_{cm}E_p^{'}\frac{(p\cos\theta_n + q\cos\chi\cos\theta_n + q\sin\chi\cos\phi\sin\theta_n)}{E_p+E_q}\Bigg\} - \frac{\gamma_{cm}}{1+\gamma_{cm}^2 v_{cm}^2}\frac{\cos\theta_n}{p}\Bigg\{\hat{p}_{cm}\Bigg(\cos\theta_{cm}(\hat{\mathbf{x}}_{cm}\cdot\mathbf{p}) + \sin\theta_{cm}\sin\phi_{cm}(\hat{\mathbf{y}}_{cm}\cdot\mathbf{p})\Bigg) + \gamma_{cm}E_p^{'}\frac{(p^2+pq\cos\chi)}{E_p+E_q}\Bigg\}. \tag{30}$$

$$\mathbf{p}\cdot\mathbf{p}' = \frac{\gamma_{cm}}{1+\gamma_{cm}^2 v_{cm}^2}\Bigg\{\hat{p}_{cm}\Bigg(\cos\theta_{cm}(\hat{\mathbf{x}}_{cm}\cdot\mathbf{p}) + \sin\theta_{cm}\sin\phi_{cm}(\hat{\mathbf{y}}_{cm}\cdot\mathbf{p})\Bigg) + \gamma_{cm}E_p^{'}\frac{(p^2+pq\cos\chi)}{E_p+E_q}\Bigg\}$$
$$= E_pE_p^{'} - \hat{E}_{cm}^2 + \hat{p}_{cm}^2\cos\theta_{cm}.$$

- In the presence of anisotropy: Two independent drag coefficients: parallel to **p**, and along anisotropy direction **n**
- Four diffusion coefficients characterize momentum broadening.
- Transport properties depend on the relative angle

$$\theta = \angle(\mathbf{p},\mathbf{n})$$

Results: drag and diffusion coefficients

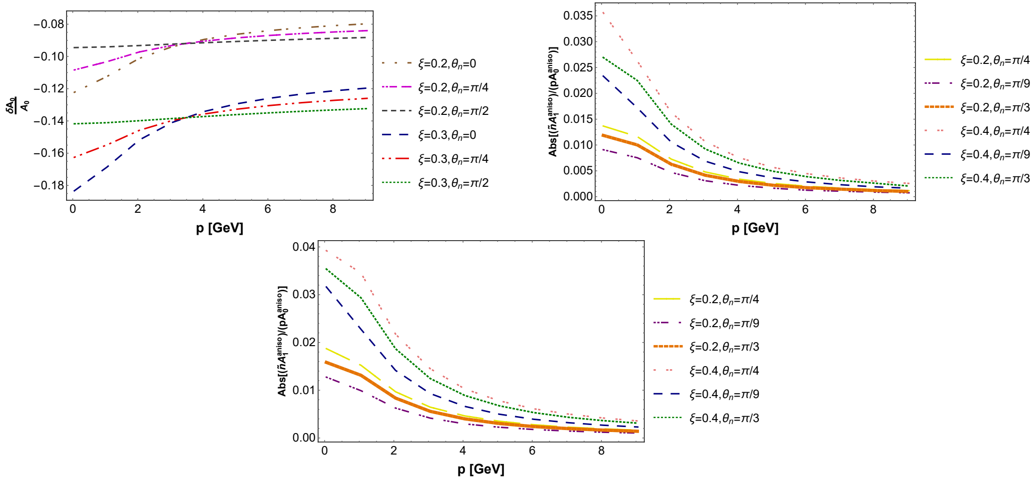


Figure: Anisotropic correction to A_0 as a function of its initial momentum at $T = 360$ MeV (top panel). Relative significance of $A_1^{(\text{aniso})}$ in comparison with $A_0^{(\text{aniso})}$ at $T = 360$ MeV (middle panel) and $T = 480$ MeV (bottom panel).

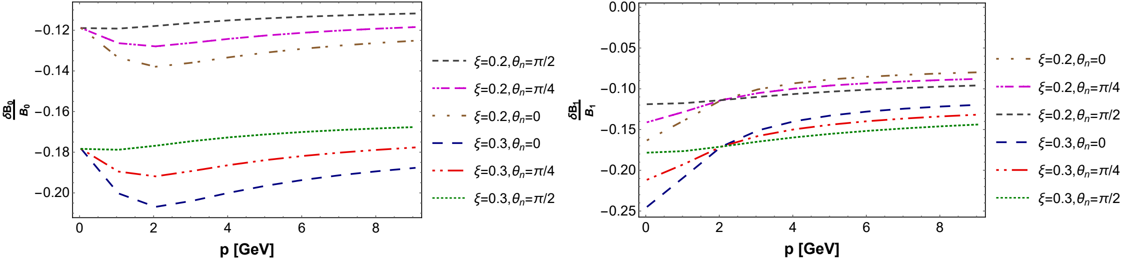


FIGURE: Momentum dependence of anisotropic corrections to B_0 (top panel) and B_1 (bottom panel) at $T = 360$ MeV.

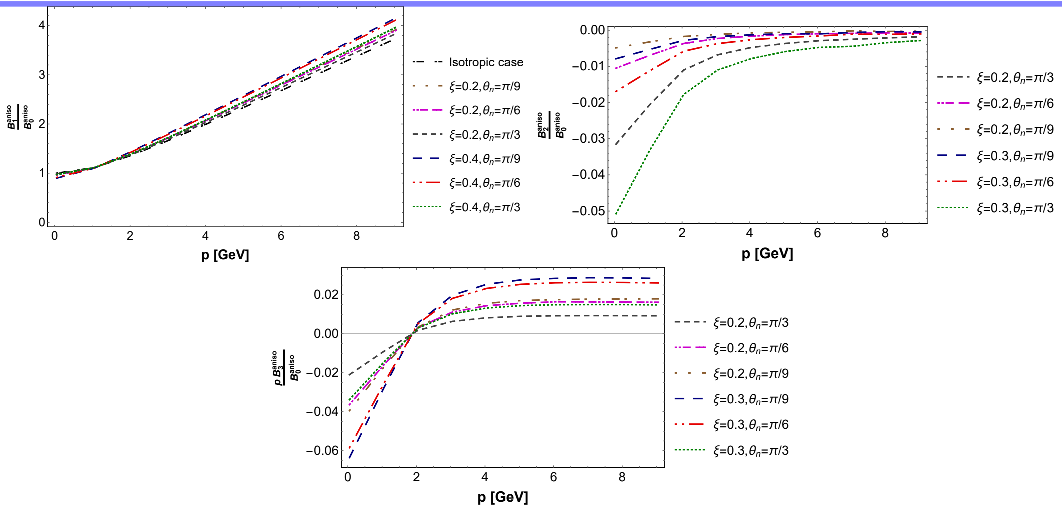


FIGURE: Relative significance of HQ diffusion coefficients in an anisotropic medium: $B_1^{(\text{aniso})}/B_0^{(\text{aniso})}$ (top panel), $B_2^{(\text{aniso})}/B_0^{(\text{aniso})}$ (middle panel), $(pB_3^{(\text{aniso})})/B_0^{(\text{aniso})}$ (bottom panel) at $T = 360$ MeV.

- The HQs may radiate gluon while propagating through the medium, such processes contribute to the HQ transport coefficients along with the collision processes. For the radiative process $(2 \to 3)$:$HQs(P) + l(Q) \to HQs(P^{'}) + l(Q^{'}) + g(K_5)$, where $K_5 = (E_5, k_\perp, k_z)$.
- The contribution of the radiative processes to the HQ transport coefficient can be written in terms of collisional processes. The transport coefficient for the radiative processes in isotropic medium: Mazumder wt al. Phys. Rev. D 89, 014002 (2014), Liu and R. Rapp, JHEP 08, 168 (2020):

$$X_{r0} = X_{c0} \times \int \frac{d^3\mathbf{k}_5}{(2\pi)^3 2E_5} 12g^2 \frac{1}{k_\perp^2} \left(1 + \frac{M_{HQ}^2}{s} e^{2\eta}\right)^{-2} \times \left(1 + \hat{f}(E_5)\right) \Theta_1(\tau - \tau_F) \Theta_2(E_p - E_5), \tag{31}$$

- Similarly for the anisotropic medium (Prakash, Kurian, Das, and VC, Rev. D 103, 094009 (2021)), written as follows,

$$X_{ra} = X_{ca} \times \int \frac{d^3\mathbf{k}_5}{(2\pi)^3 2E_5} 12g^2 \frac{1}{k_\perp^2} \left(1 + \frac{M_{HQ}^2}{s} e^{2\eta}\right)^{-2} \times \left(1 + \hat{f}(E_5)\right) \Theta_1(\tau - \tau_F) \Theta_2(E_p - E_5). \tag{32}$$

- In the presence of anisotropy in the medium, the transport coefficient for the radiation processes can be decomposed as (inline with collisional processes):

$$X_r = X_{r0} + X_{ra}. \tag{33}$$

- Our analysis is carried under the approximation of soft gluon emission *i.e.*, $K_5 \to 0$. The radiated gluons follow the Bose-Einstein phase space distribution, $\hat{f}(E_5) = \frac{1}{\exp\{(\beta E_5)\}-1}$.
- The emission of gluons from the HQs occurs under the two constraints; (a.)the theta function, $\Theta_2(E_p - E_5)$, ensures that the energy of emitted gluon (E_5) should not be greater than the HQs energy (E_p), (b) In the dense medium, if the mean free time, the average time between two successive collisions, is of the order of formation time or larger, then the emission of gluon will be suppressed due to destructive interference, a phenomenon known as the Landau-Pomeranchuk-Migdal (LPM) suppression: Gyulassy and Wang, Nucl. Phys. B 420, 583 (1994)
- This has been taken care of in a gross way through the theta function ,$\Theta_1(\tau - \tau_F)$ (Zhe and Greiner,Phys. Rev. C 71, 064901 (2005), Das, Alam, Mohanty, Phys. Rev. C 82, 014908 (2010))
- The invariant amplitude corresponding to radiative processes, $|\mathcal{M}|^2_{2\to3}$ can be defined in terms of elastic invariant amplitude $|\mathcal{M}|^2_{2\to2}$ Abir et. al, Phys. Rev. D 85, 054012 (2012):

$$|\mathcal{M}|^2_{2\to3} = |\mathcal{M}|^2_{2\to2} \times 12g^2 \frac{1}{k_\perp^2}\left(1 + \frac{M_{HQ}^2}{s} e^{2\eta}\right)^{-2}, \tag{34}$$

where η is the rapidity of emitted gluons.

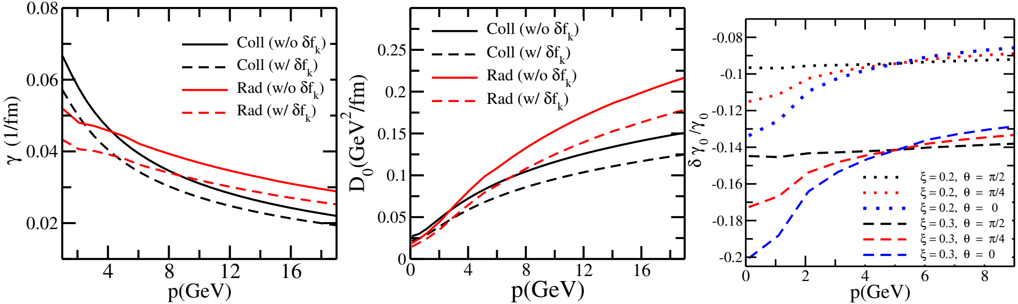


FIGURE: Momentum dependence of the HQs drag coefficient (left panel), diffusion coefficient D_0 (middle) and anisotropic correction to isotropic drag coefficients as a function of momentum at a fixed temperature, $T = 360$ MeV (right panel). We choose $\xi = 0.3$ and $\theta_n = \pi/4$

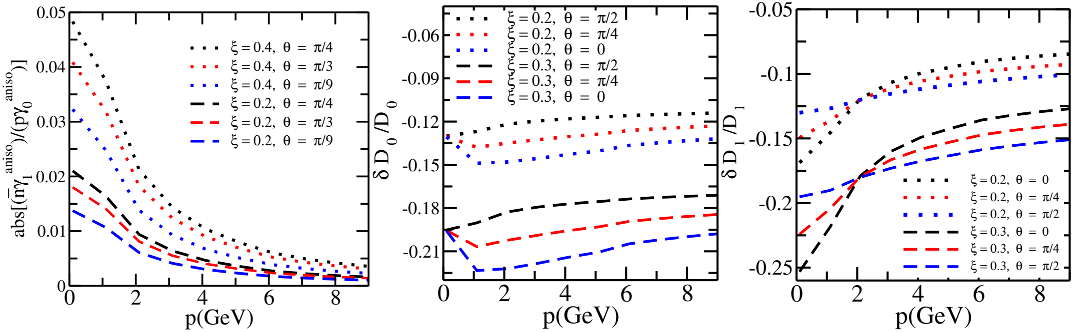


FIGURE: Relative significance of γ_1 in comparison with γ_0 with a momentum (p) at a fixed temperature, $T = 360$ MeV (left panel). Anisotropic corrections to D_0 (middle panel) and the anisotropic corrections to D_1 (right panel).

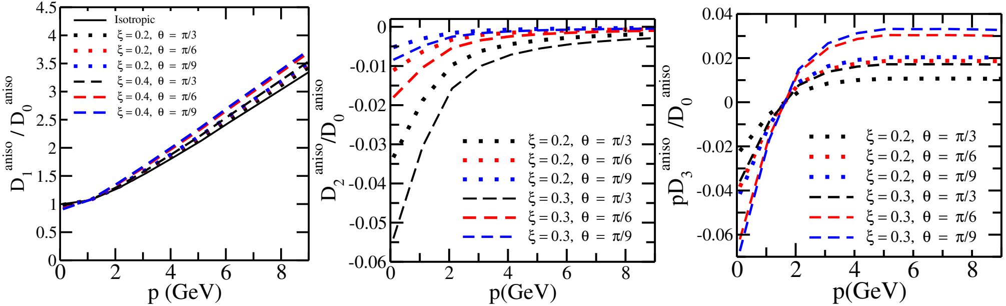


FIGURE: Relative Correction to the HQs diffusion coefficients for radiative processes: $D_1^{(aniso)}/D_0^{(aniso)}$ (left panel), $D_2^{(aniso)}/D_0^{(aniso)}$ (middle panel), and $pD_3^{(aniso)}/D_0^{(aniso)}$ (right panel) at $T = 360$ MeV.

- HQs may lose its energy while traveling through the anisotropic QCD medium due to the collisional processes with the in-medium particles.
- The differential collisional energy loss can be quantified in terms of the HQ drag coefficient due to the elastic collisions in the medium as(Mustafa, Pal, and Srivastava, Phys. Rev. C 57, 889 (1998)):

$$\left(-\frac{dE}{dx}\right)_{\text{aniso}} = A_0^{(\text{aniso})}(p^2, T)p. \tag{35}$$

- Current focus is on the energy loss in the direction of initial HQ momentum. Hence, the contribution from $A_1^{(\text{aniso})}$ will vanish as $\mathbf{p} \cdot \tilde{\mathbf{n}} = 0$. However, the energy loss will have an anisotropic contribution through the δA_0.

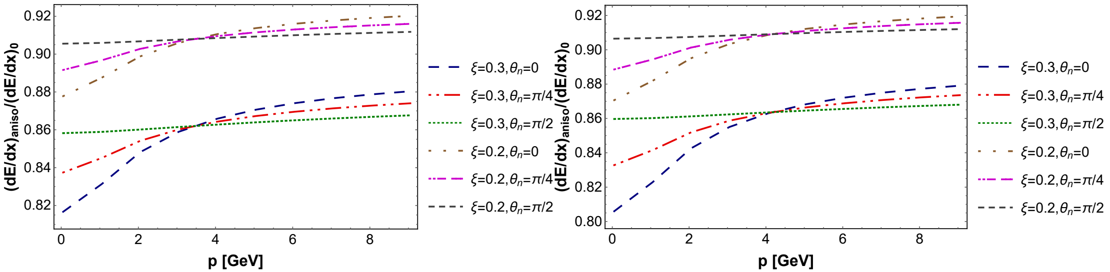


FIGURE: Impact of anisotropy on the momentum behaviour of collisional energy loss of charm quark for the RHIC energy at $T = 360$ MeV (top panel) and for the LHC energy at $T = 480$ MeV.

- The nuclear suppression factor, $R_{AA}(p_T)$ for the HQs is defined as follows,

$$R_{AA}(p_T) = \frac{f_{\tau_f}(p_T)}{f_{\tau_0}(p_T)}. \tag{36}$$

- The final momenta spectrum of charm quarks is $f_{\tau_f}(p)$, at the end of time evolution τ_f, which is assumed 6 fm/c in our calculation. The initial momenta spectra, f_{τ_0}, is taken according to the fixed order + next-to-leading log (FONLL) calculations: Cacciari, Nason, and Vogt, Phys. Rev. Lett. 95, 122001 (2005), Cacciari et. al, JHEP 10, 137 (2012).
- We employ stochastic Langevin dynamics (Moore and D. Teaney, Phys. Rev. C 71, 064904 (2005)) as

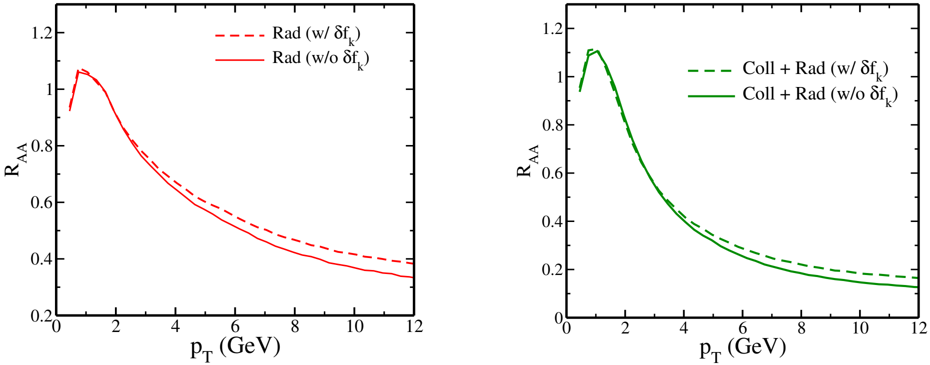
$$dx_i = \frac{p_i}{E} dt, \tag{37}$$

$$dp_i = -\gamma p_i \, dt + C_{ij} \rho_j \sqrt{dt}, \tag{38}$$

where C_{ij} is the covariance matrix that describes stochastic force in terms of independent Gaussian- normal distributed random variable, ρ_j, known as the white noise with $\langle \rho_i \rho_j \rangle = \delta_{ij}$ and $\langle \rho_j \rangle = 0$.
- The covariance matrix is written as follows,

$$C_{ij} = \sqrt{2D_0}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) + \sqrt{2D_1}\frac{p_i p_j}{p^2}. \tag{39}$$

Thus, at $p \to 0$, the coefficient, $D_0 = D_1 = D$, further $C_{ij} = \sqrt{2D}\delta_{ij}$, where D is the diffusion coefficient of the HQ.



- The $R_{AA}(p_T)$ of charm quarks for radiative processes (left panel) and summing both collisional and radiative processes (right panel) as a function of transverse momentum (p_T) at $T = 360$ MeV. We choose, $\xi = 0.4$ and $\theta_n = \pi/9$.

- We studied heavy quark drag and diffusion coefficients in an anisotropic (momentum) medium

- Momentum anisotropy in the medium leads to two independent drag coefficients and four diffusion coefficients, all of which depend on the angle relative to the anisotropy direction.

- We analyzed radiative contribution to drag and diffusion coefficients in contrast to collisional ones
- The R_{AA} for the charm quarks have been estimated for RHIC and LHC energies with the inclusion of both collisional and radiative processes.

- The strength of anisotropy and the angle between HQ momentum and the direction of anisotropy dependence have been analyzed.

Conclusions and outlook

- Momentum anisotropy modifies the transport coefficients of heavy quarks, which in turn affects their energy loss and nuclear suppression factor
- Future work will focus on:
  - Coupling the framework with hydro evolution and computation of v_2 and R_{AA} Kurian et. al, Phys.Rev.C 102 (2020) 4, 044907 Also hydro derived from kinetic theory within momentum dependent relaxation time: Kumar, Kurian, VC, Phys.Rev. D **110** (2024), Kumar, Bhadury, Kurian, VC, Phys.Rev.D **111** (2025)
  - Comparison with the experimental predictions on R_{AA} and flow coefficients
  - Inclusion of electromagnetic fields (space-time dependent) and heavy quark transport (radiative contribution): Some preliminary (initial) works: Kurian, Das, VC, Phys.Rev.D 101 (2020) 9, 094024; Phys.Rev.D 100 (2019) 7, 074003

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