Lattice QCD and EIC: Generalized Parton Distribution (and some more)

February 2024, ICTS,
Bangaluru, India

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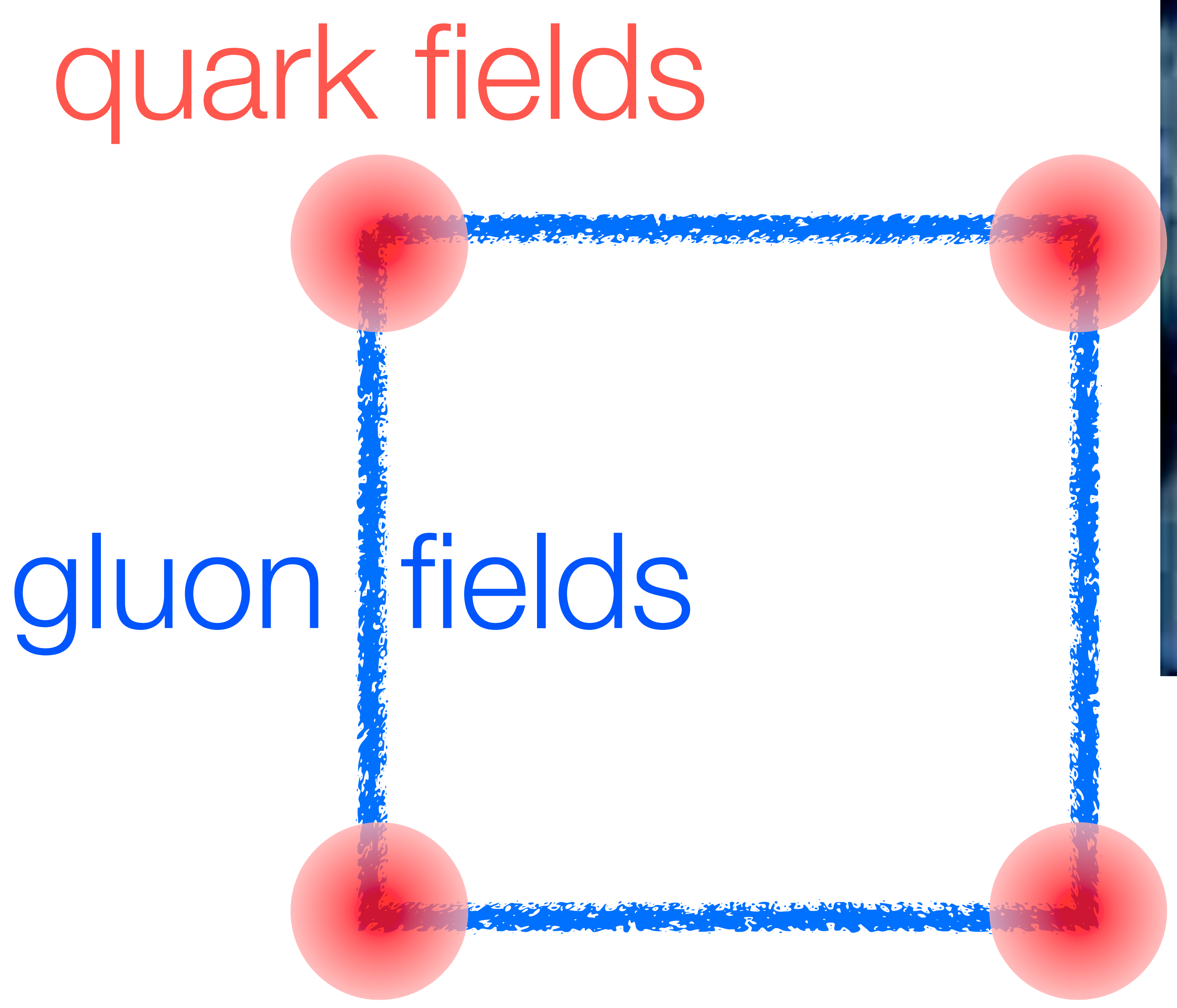
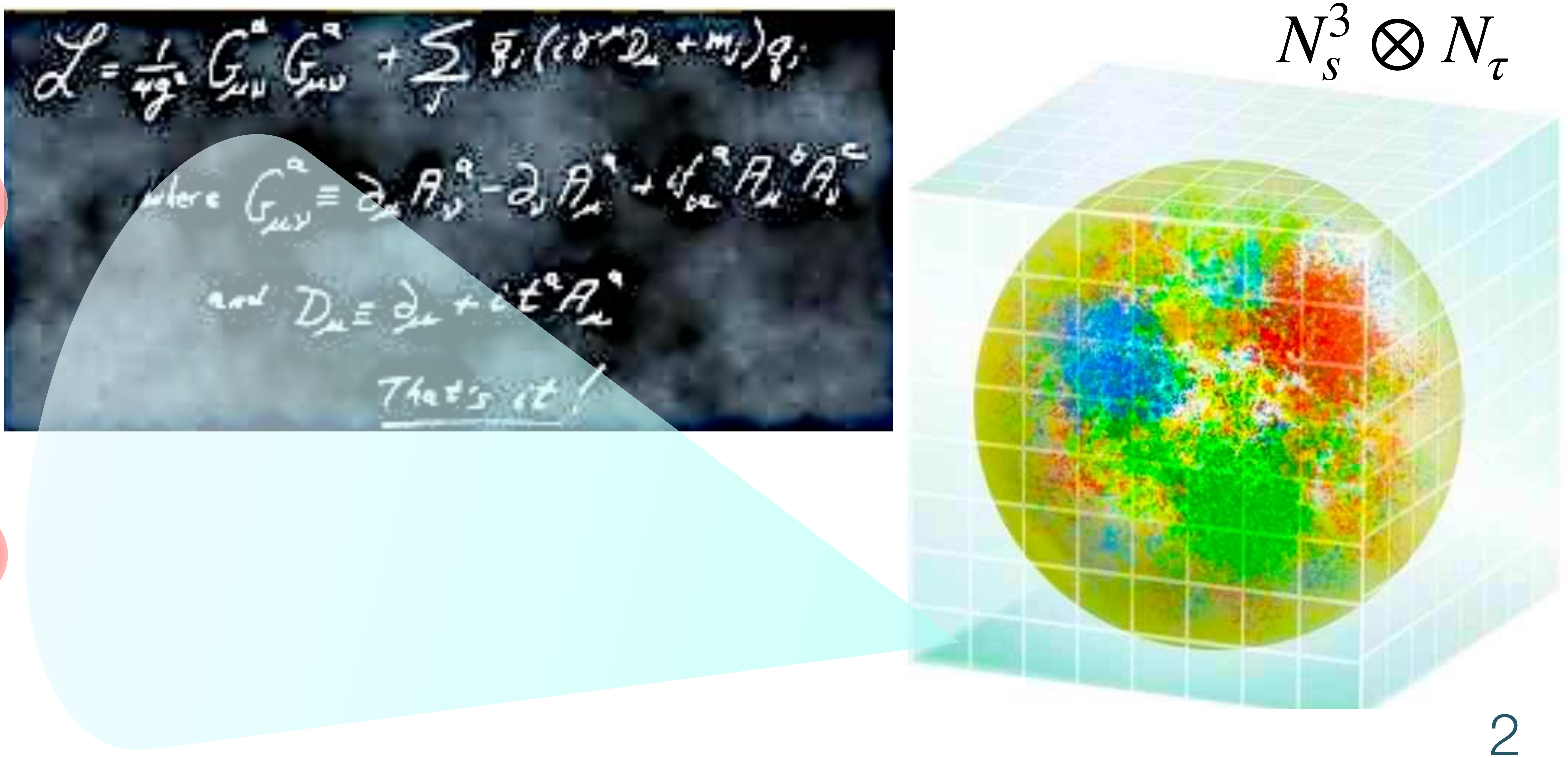
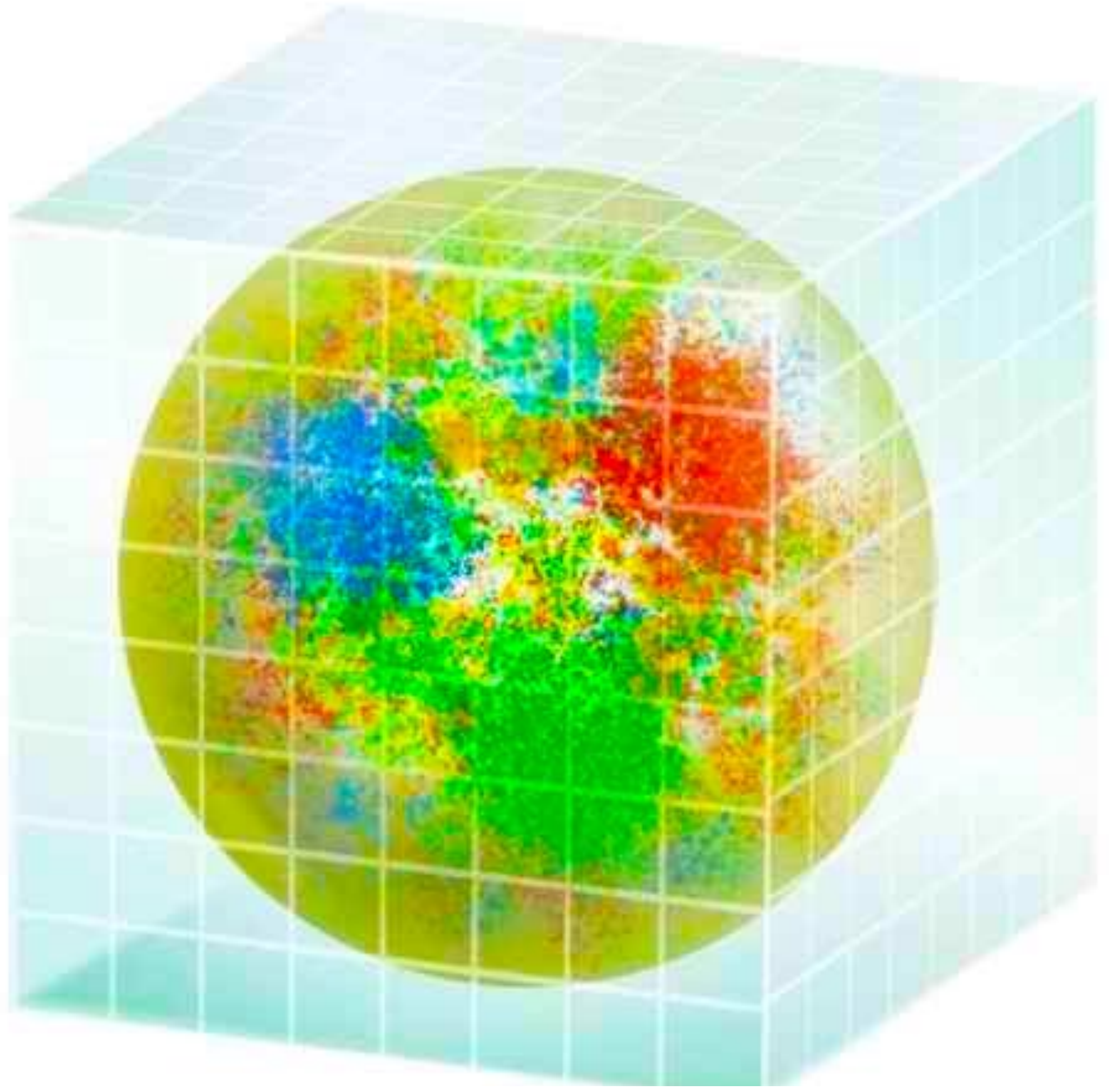
lattice quantum chromodynamics (QCD)

non-perturbative regularization of *field theory*

discretized Euclidean space and *Euclidean* time







QCD path integral $\sim \int \mathscr{D}[U]\mathscr{D}[\psi]\mathscr{D}[\bar{\psi}]e^{-S_{QCD}[U,\psi,\bar{\psi}]}$

$N_s^3 \otimes N_\tau \otimes N_{color} \otimes N_{spin} \otimes N_{flavor}$

> 10 billion degrees of freedom

solve via numerical Monte-Carlo using computers

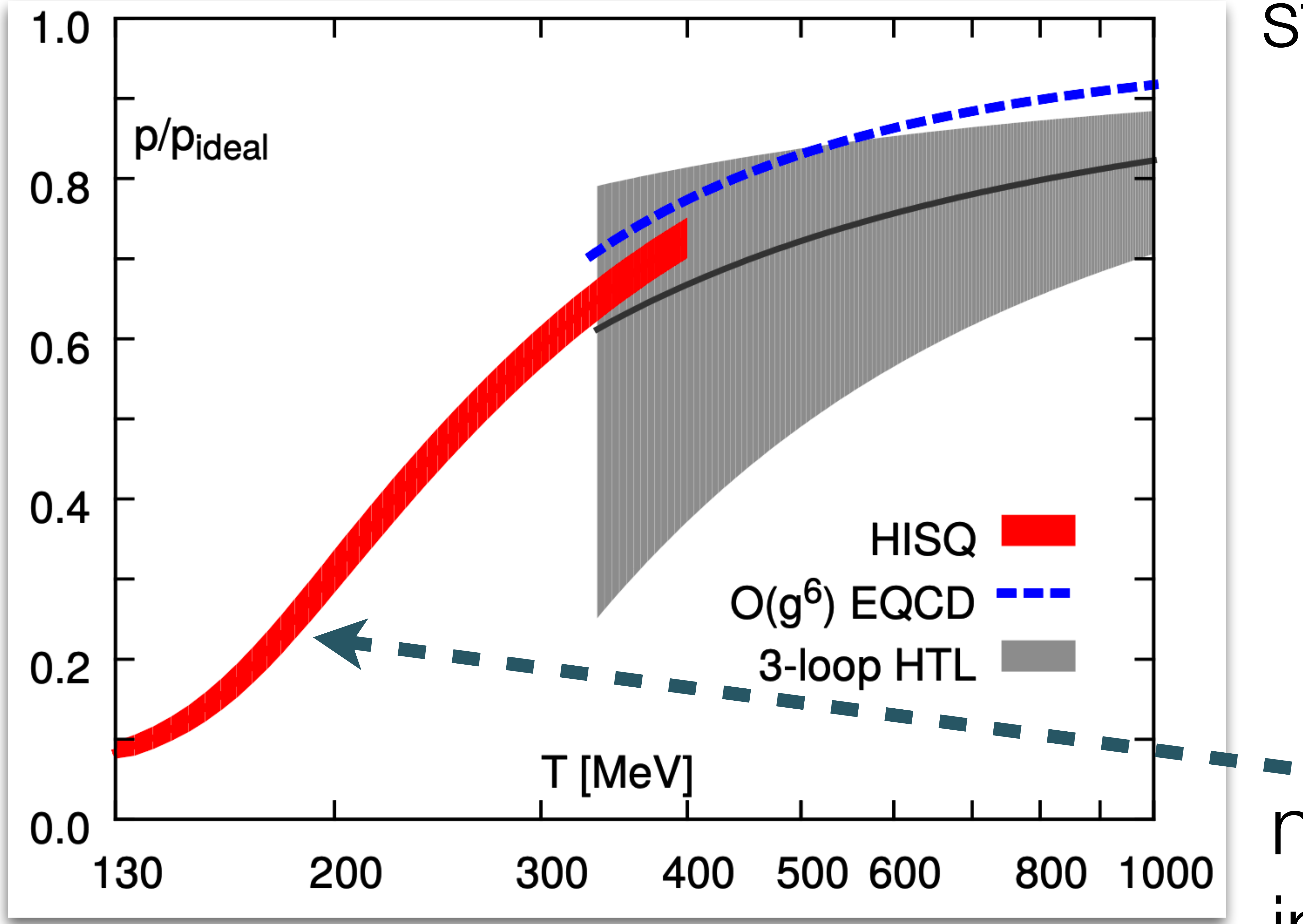
exascale supercomputers

each person,

1 calculation/s,

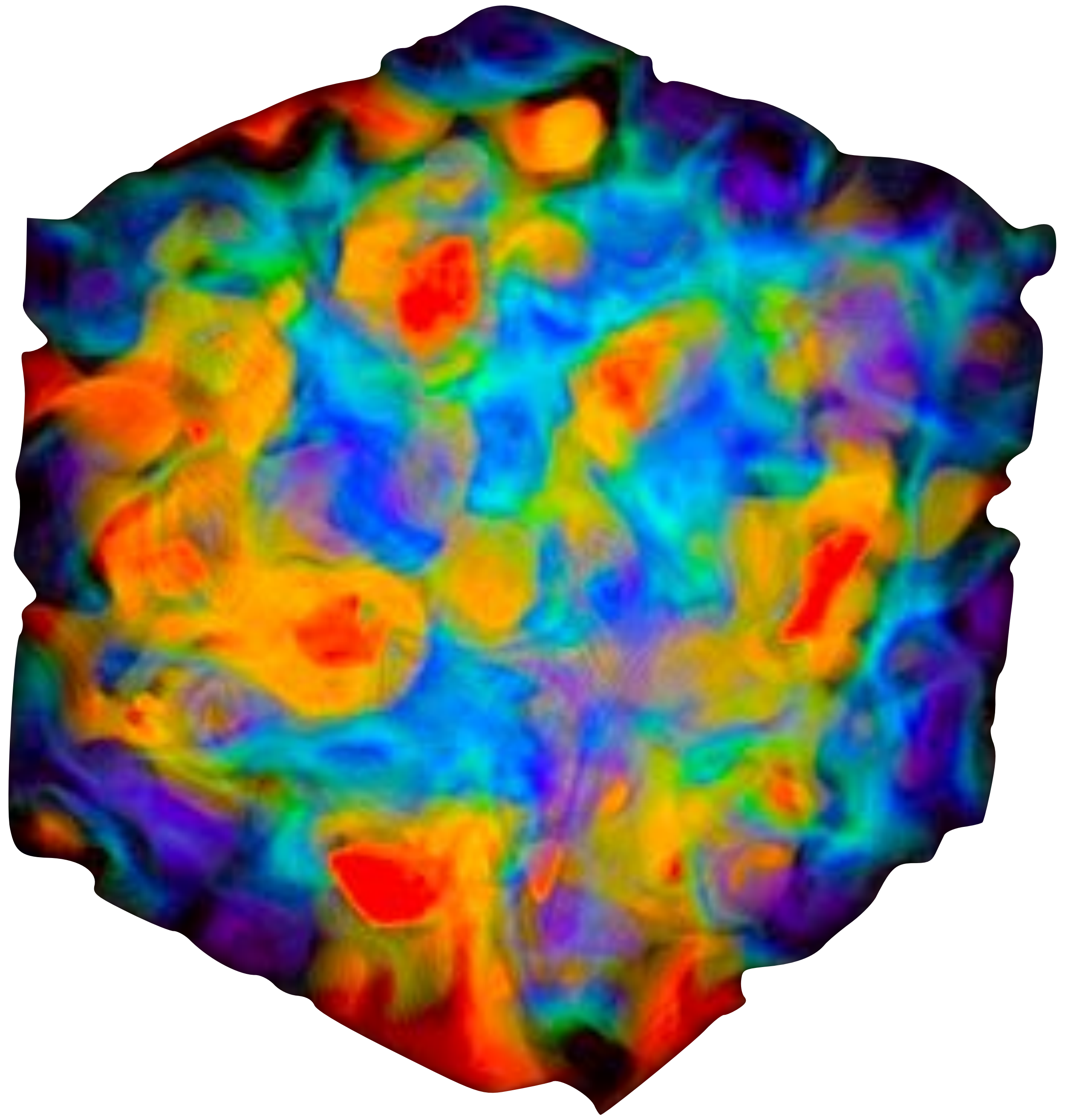
~ 4 years

QCD equation of state



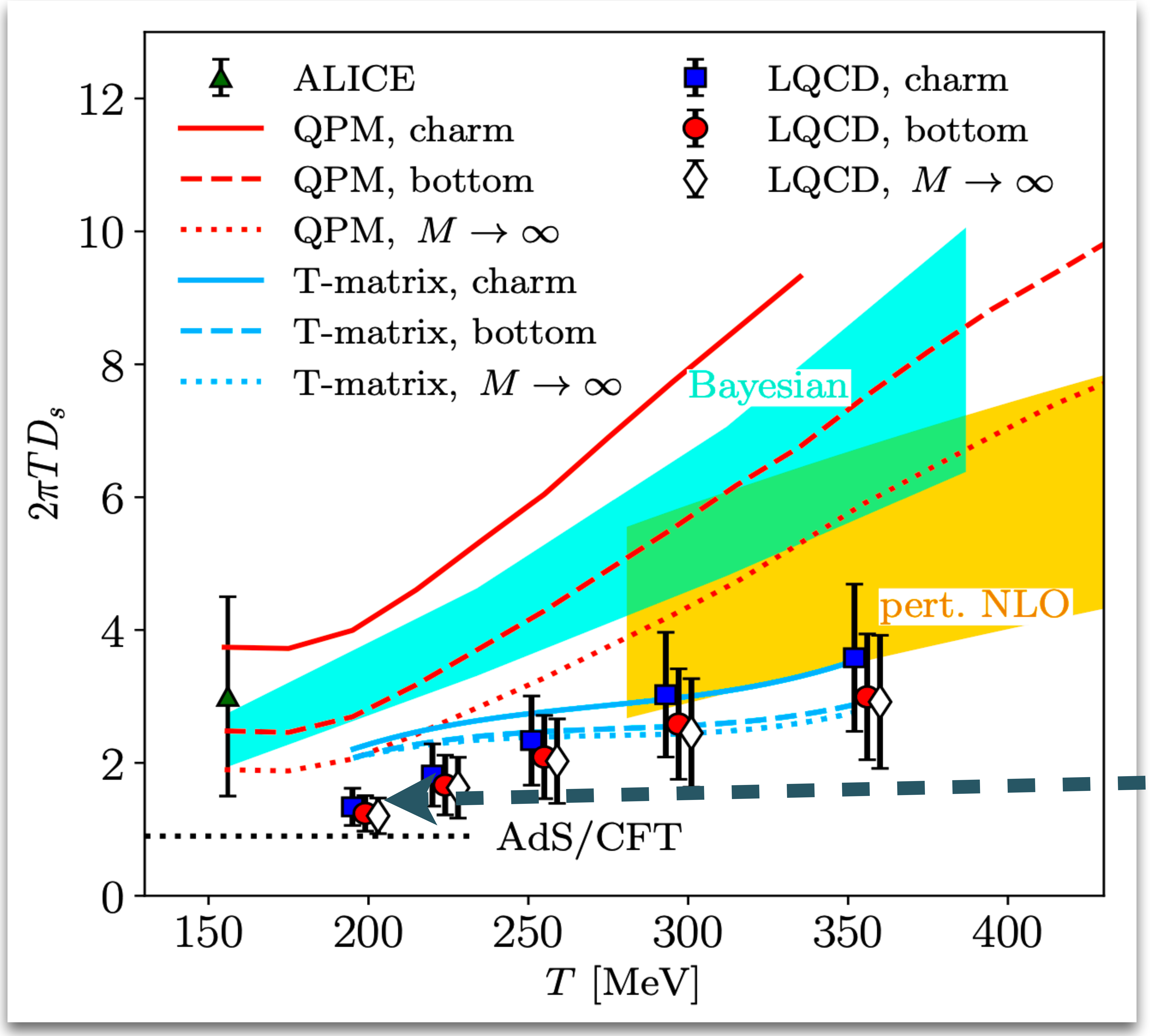
HotQCD: Phys.Rev.D90, 094503 (2014)

strongly coupled quark, gluon fields

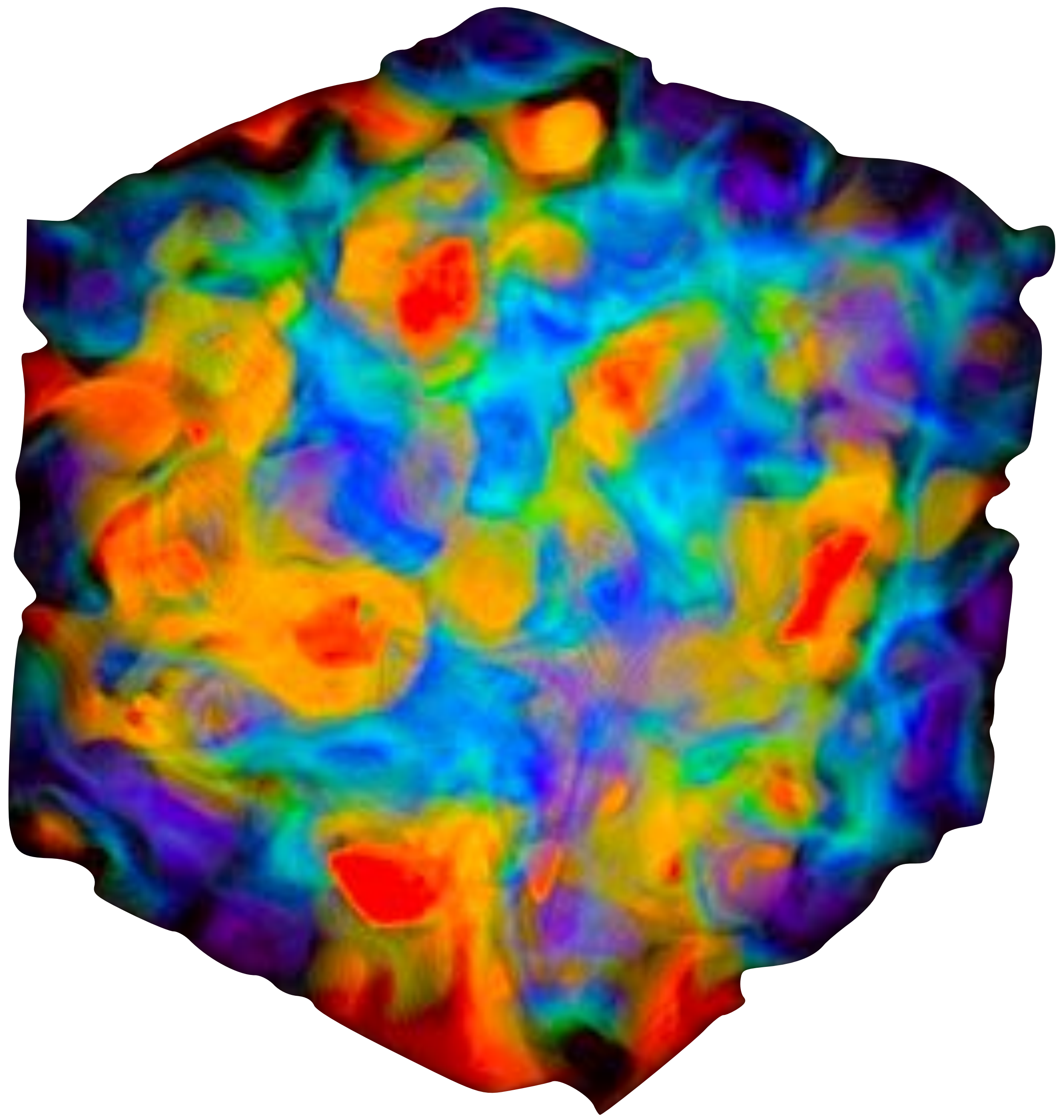


no obvious description in terms of individual partons (quarks & gluons)

heavy quark diffusion constant in quark gluon plasma



strongly coupled quark, gluon fields



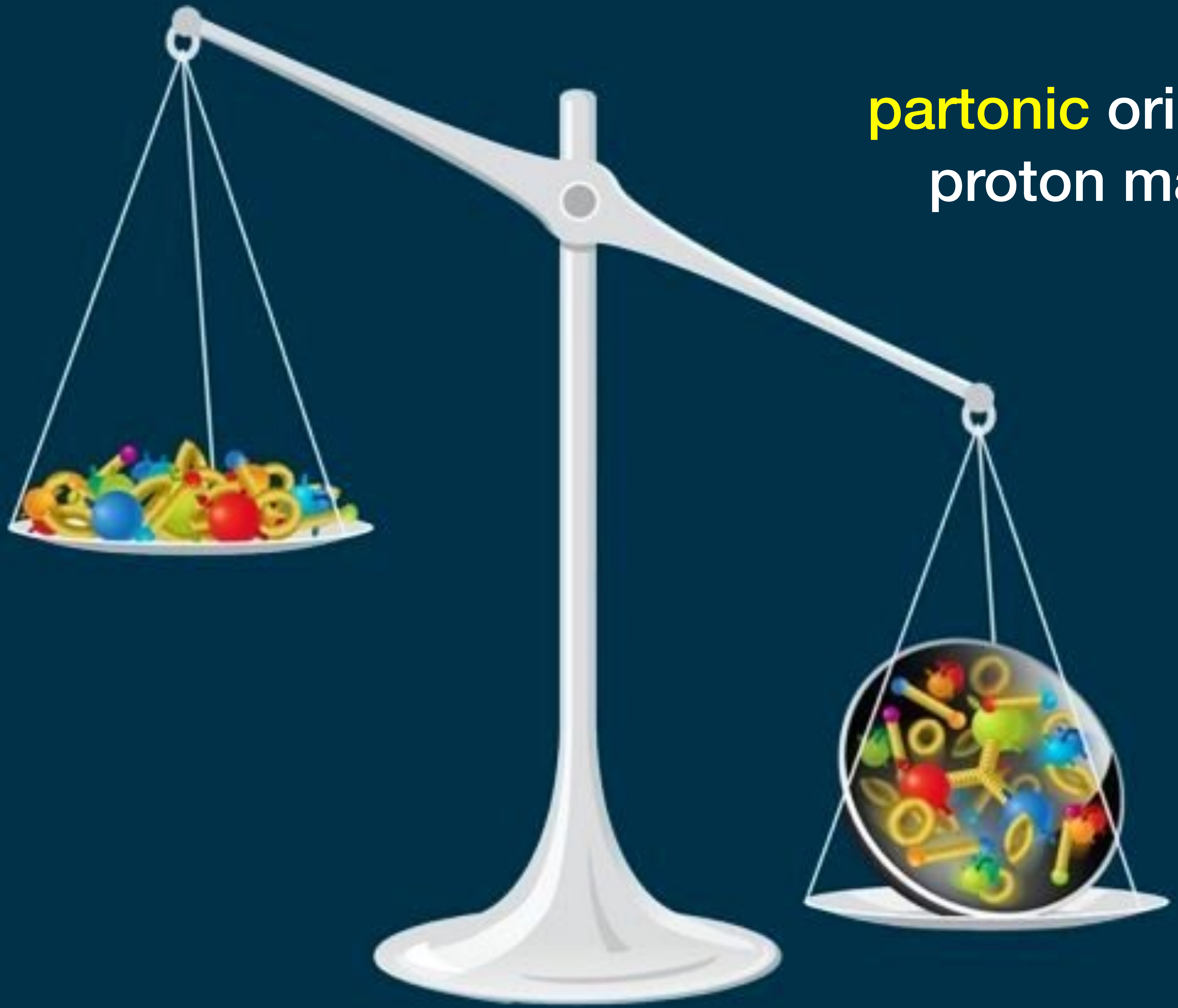
no obvious description in terms of individual partons (quarks & gluons)

H.-T. Shu et. al.: Phys.Rev.Lett. 132, 051902 (2024)

EIC @ BNL

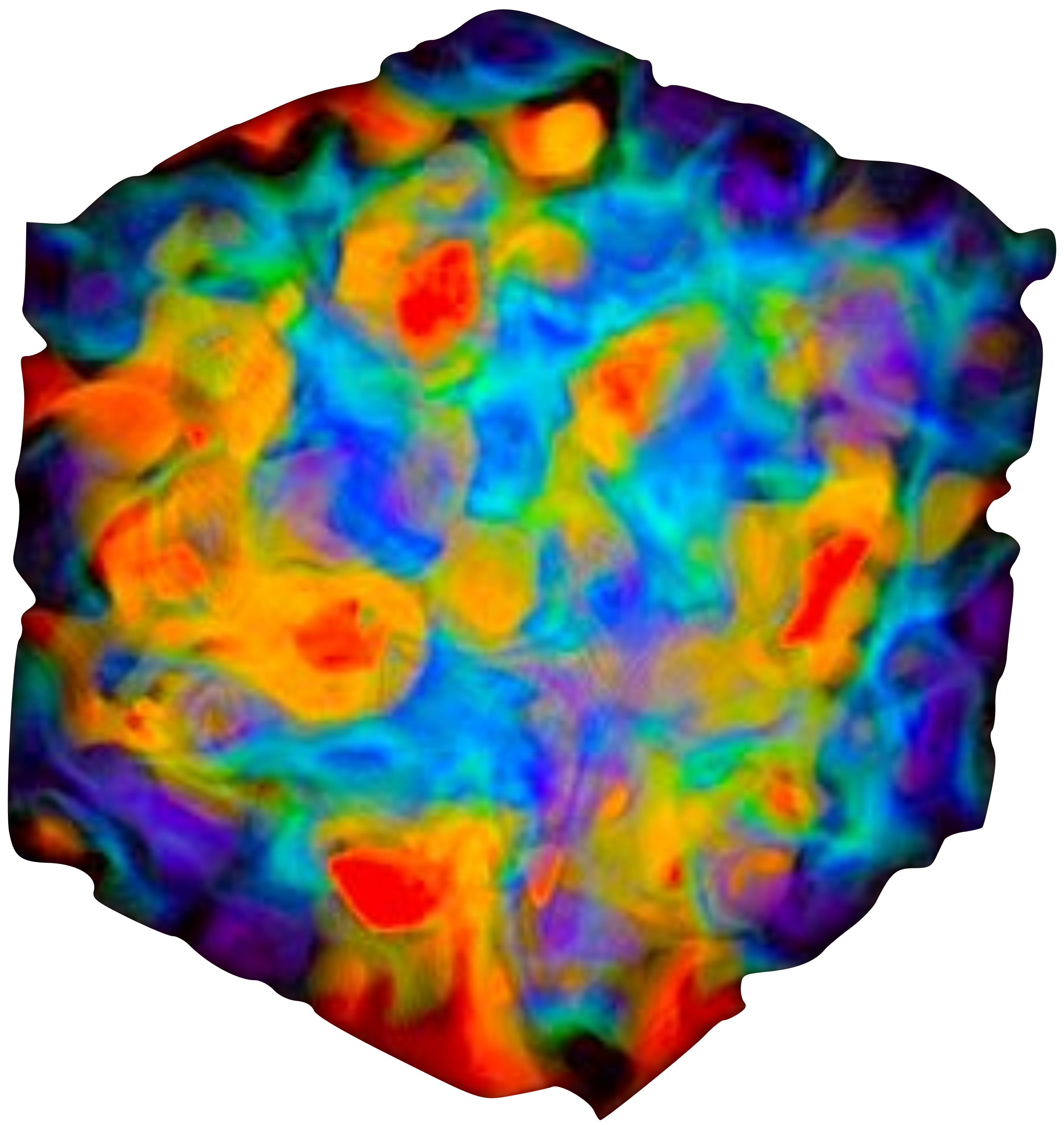
shining light on proton

partonic origin of proton mass

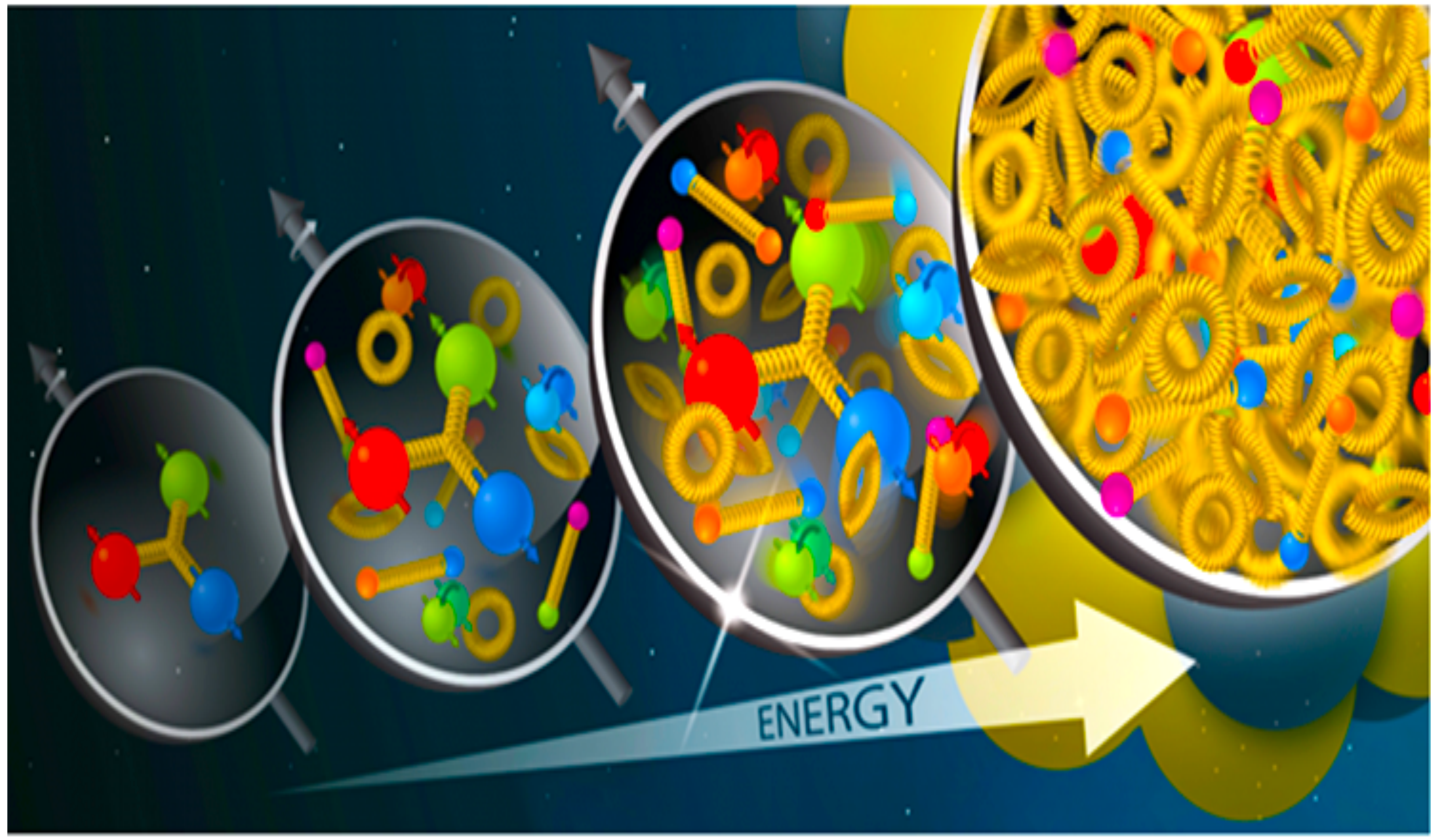


partonic origin of proton spin

challenge for lattice QCD in the EIC era

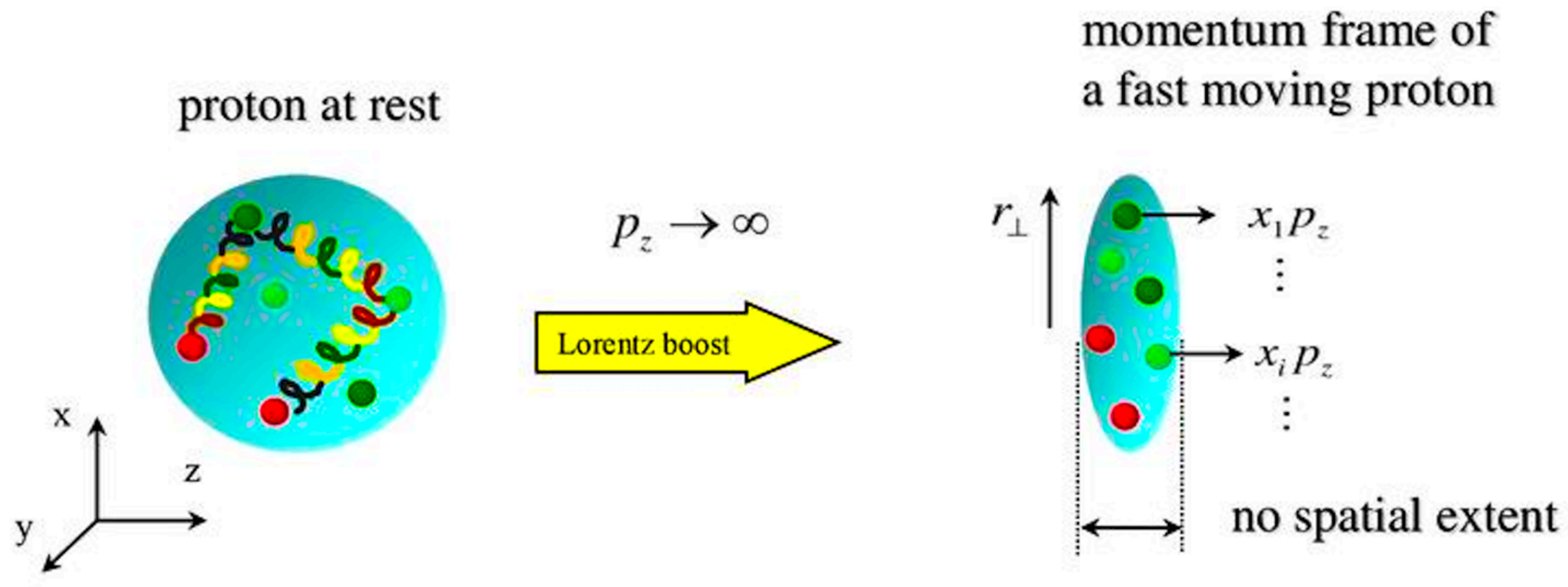


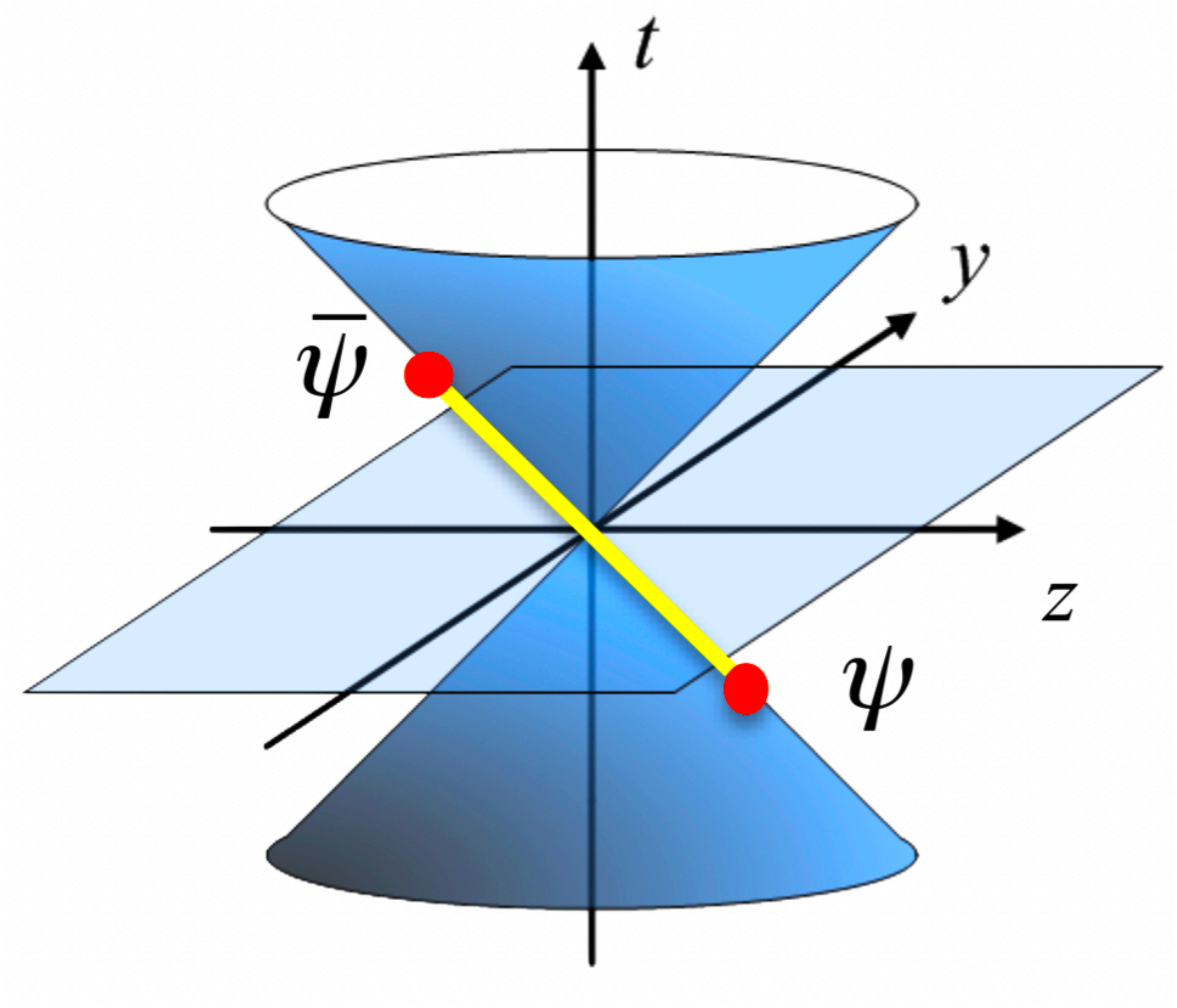
????



how to ‘see’ a parton on the lattice ?

partonic picture

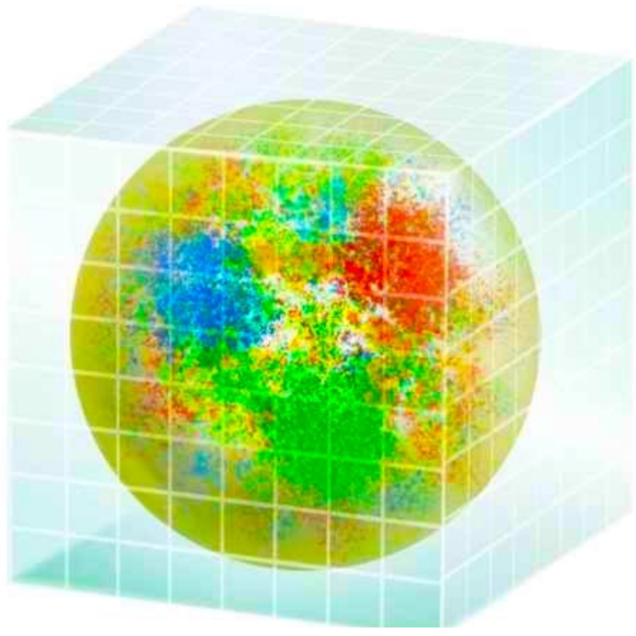


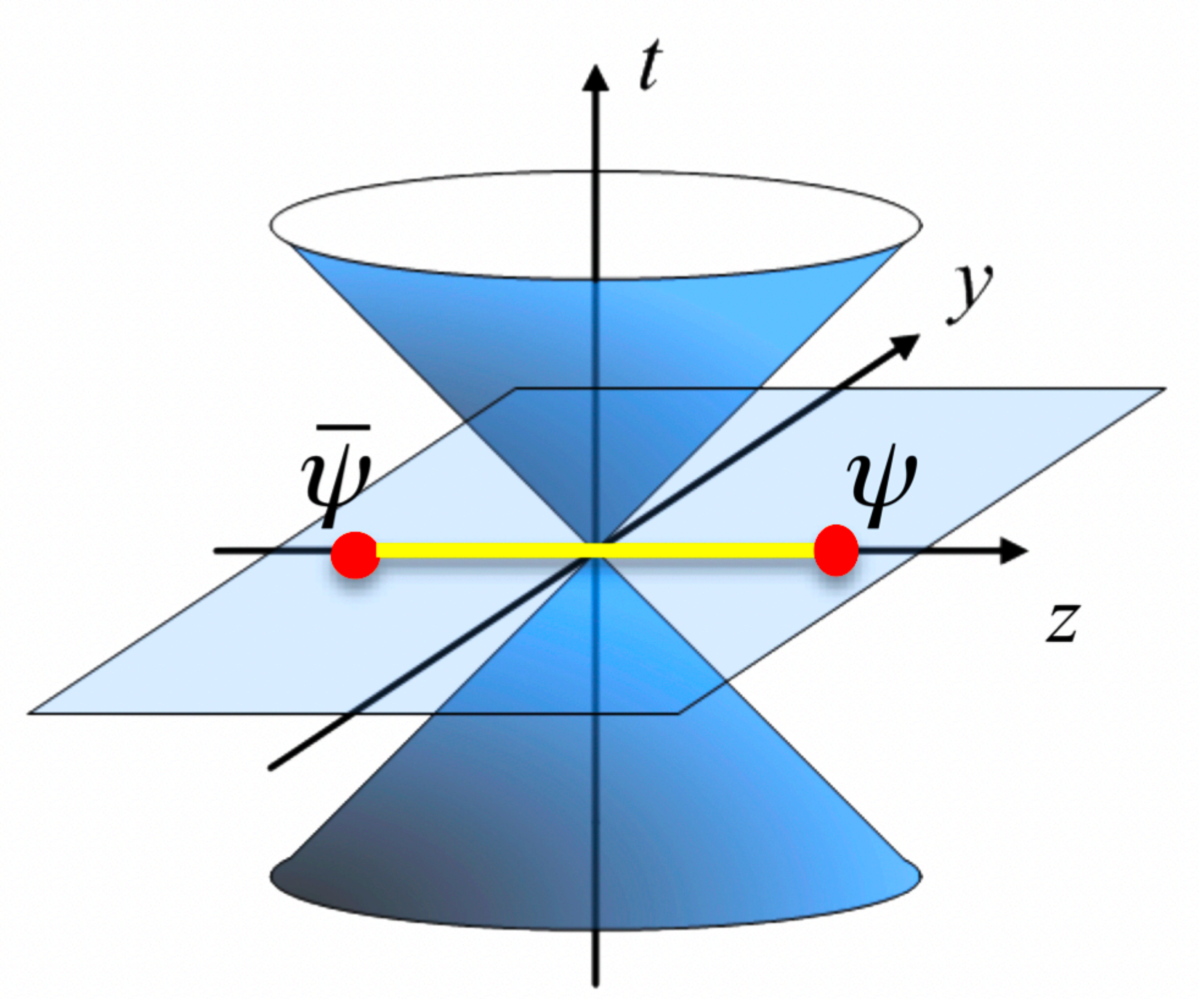


- QCD in infinite-momentum fame / on lightcone
- simplified / effective description of QCD
- $P_z \to \infty$ / $z^2 \to 0$ first, regularize QFT later

partonic structure from lattice QCD

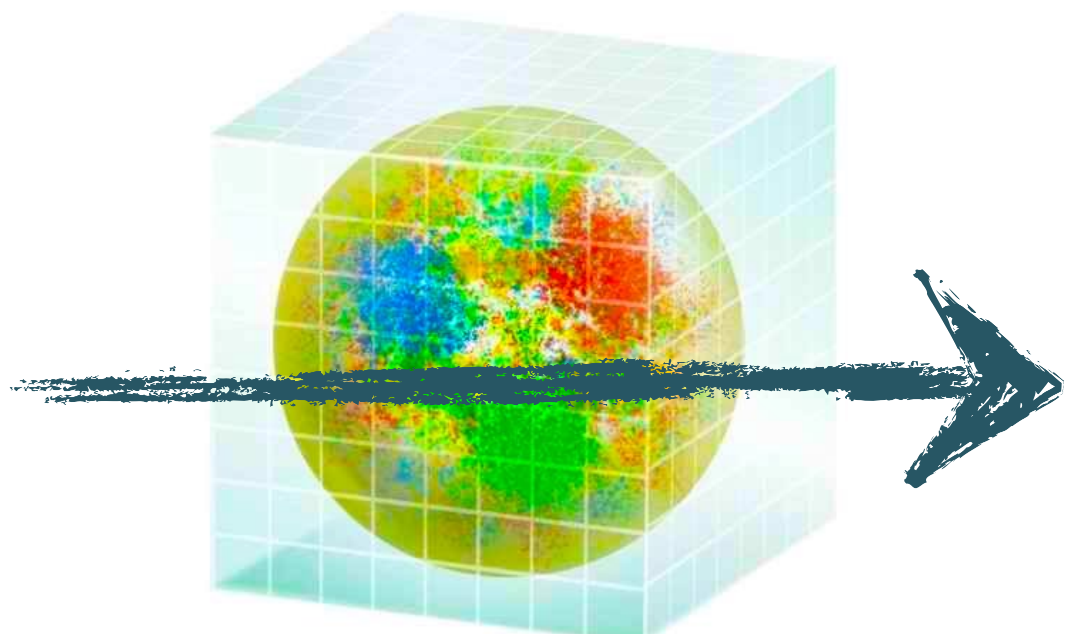
hadron at rest



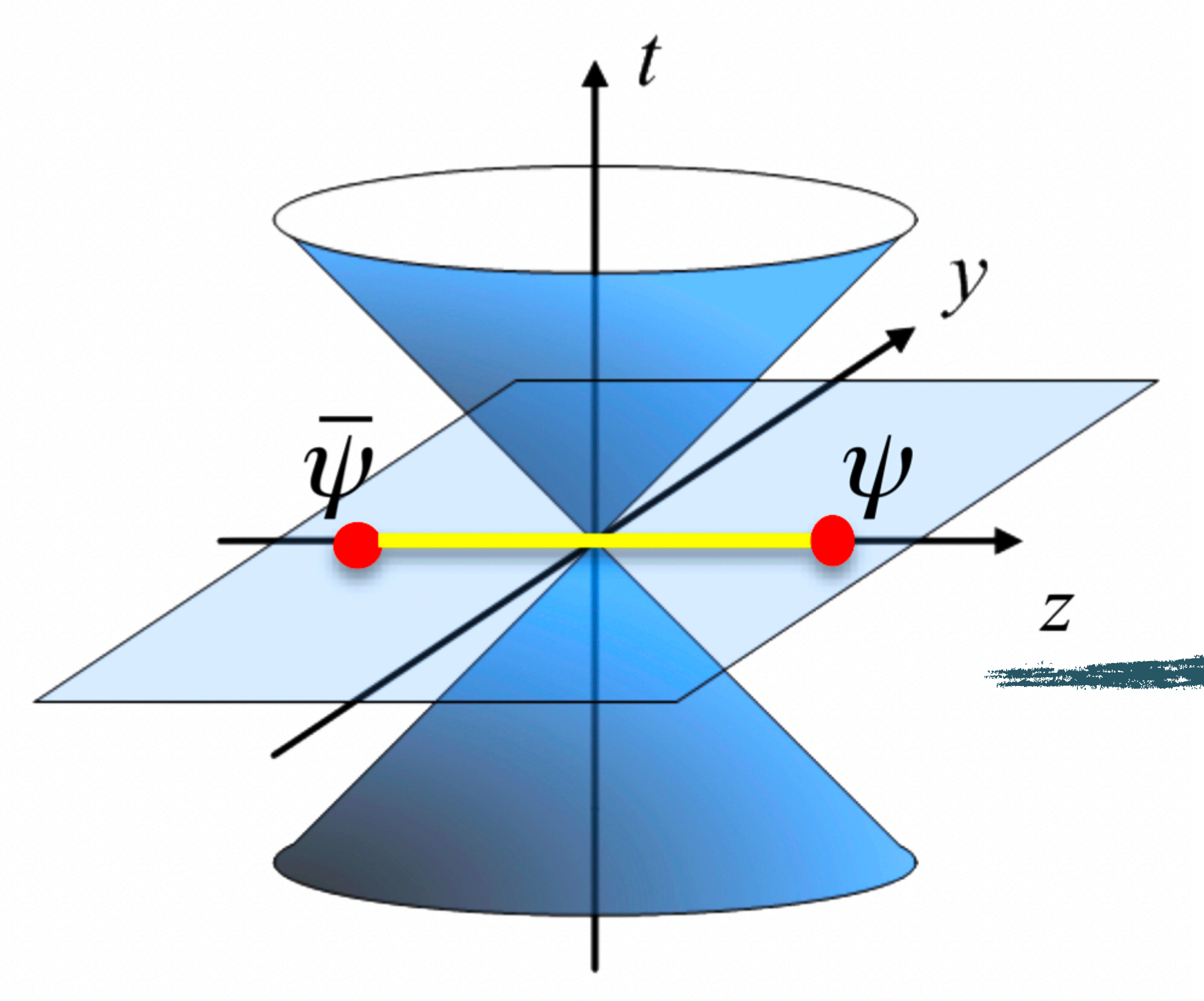


renormalize

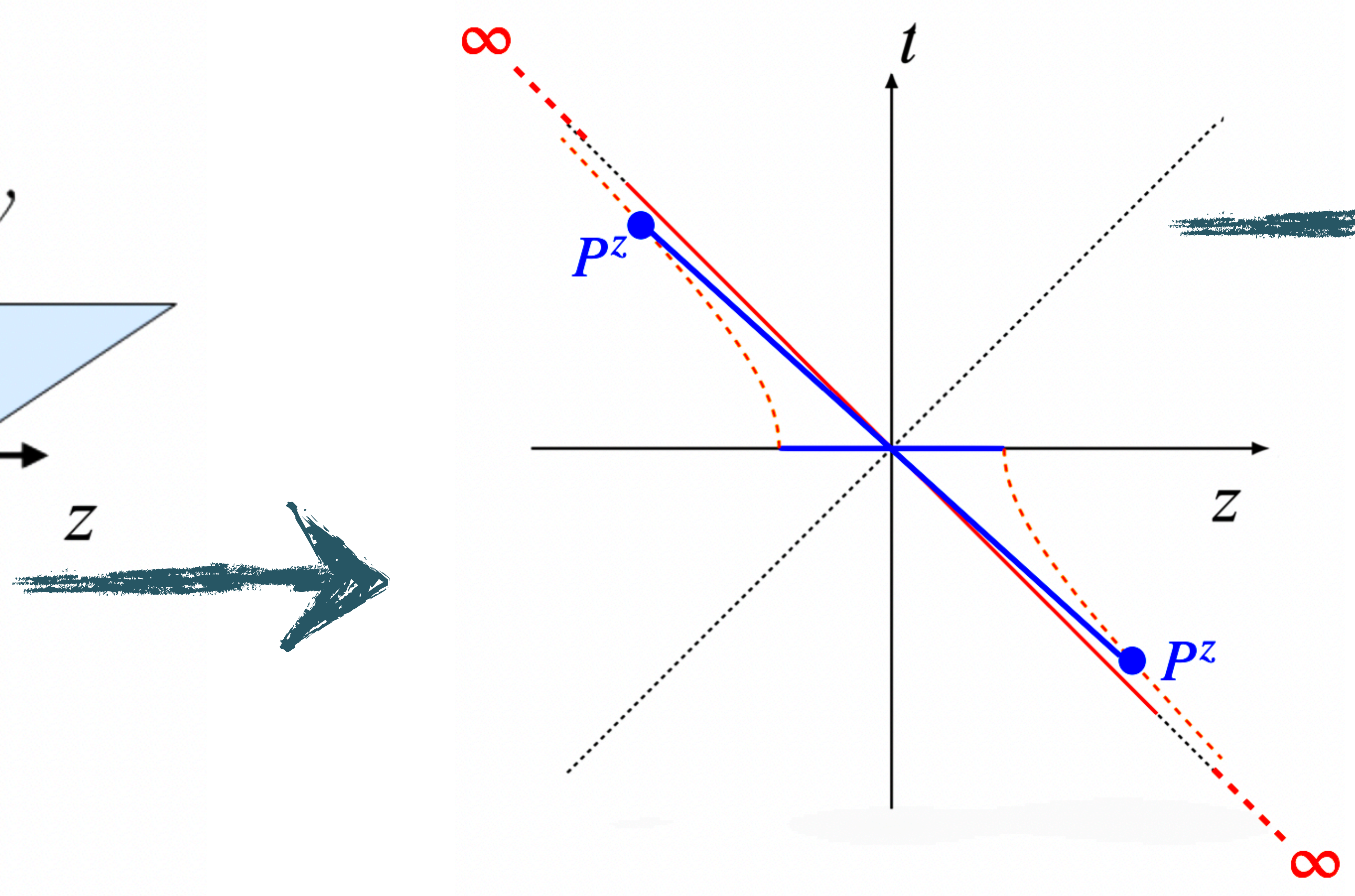
fast-moving hadron

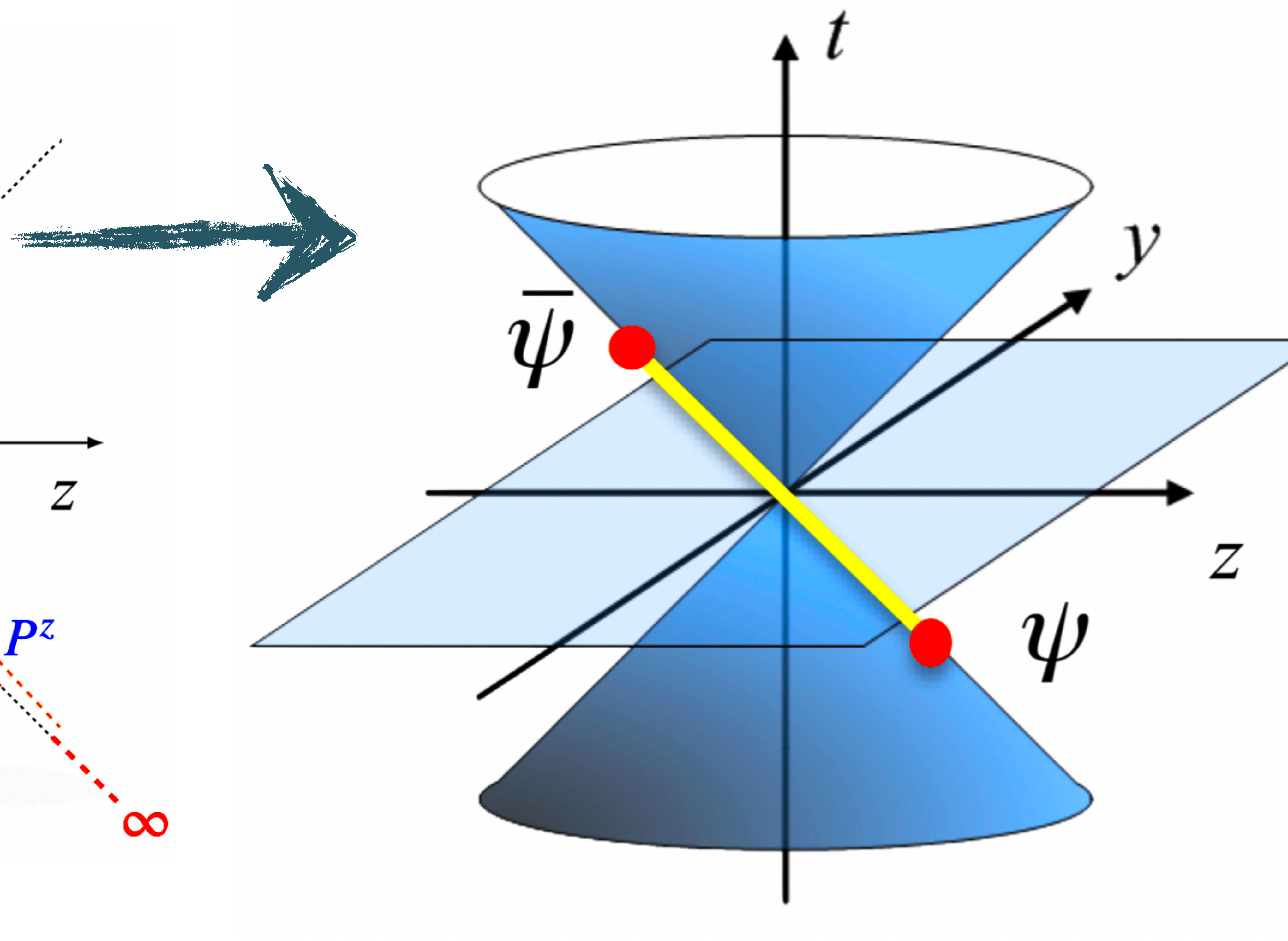


$P_z \approx E$

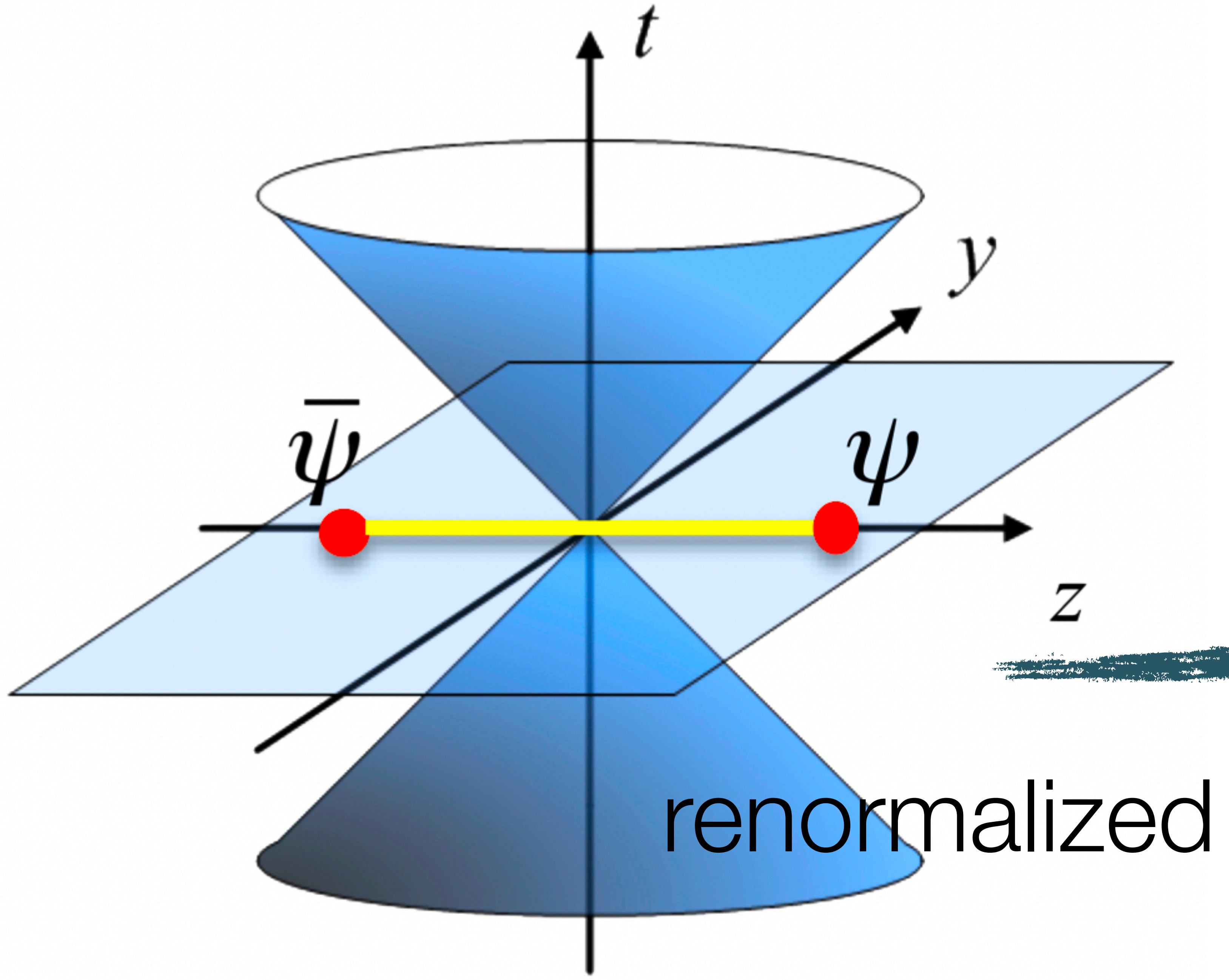


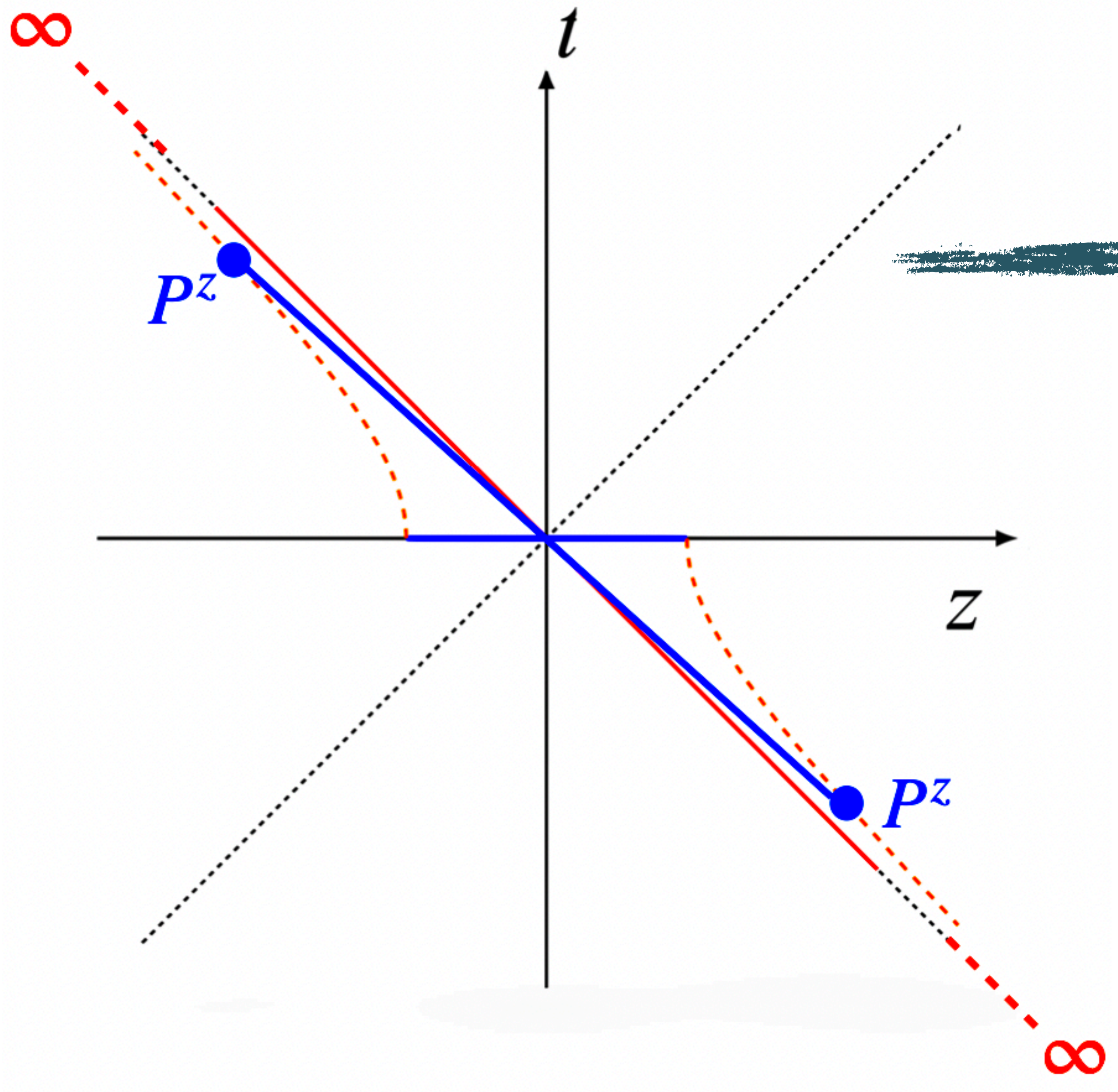
renormalize

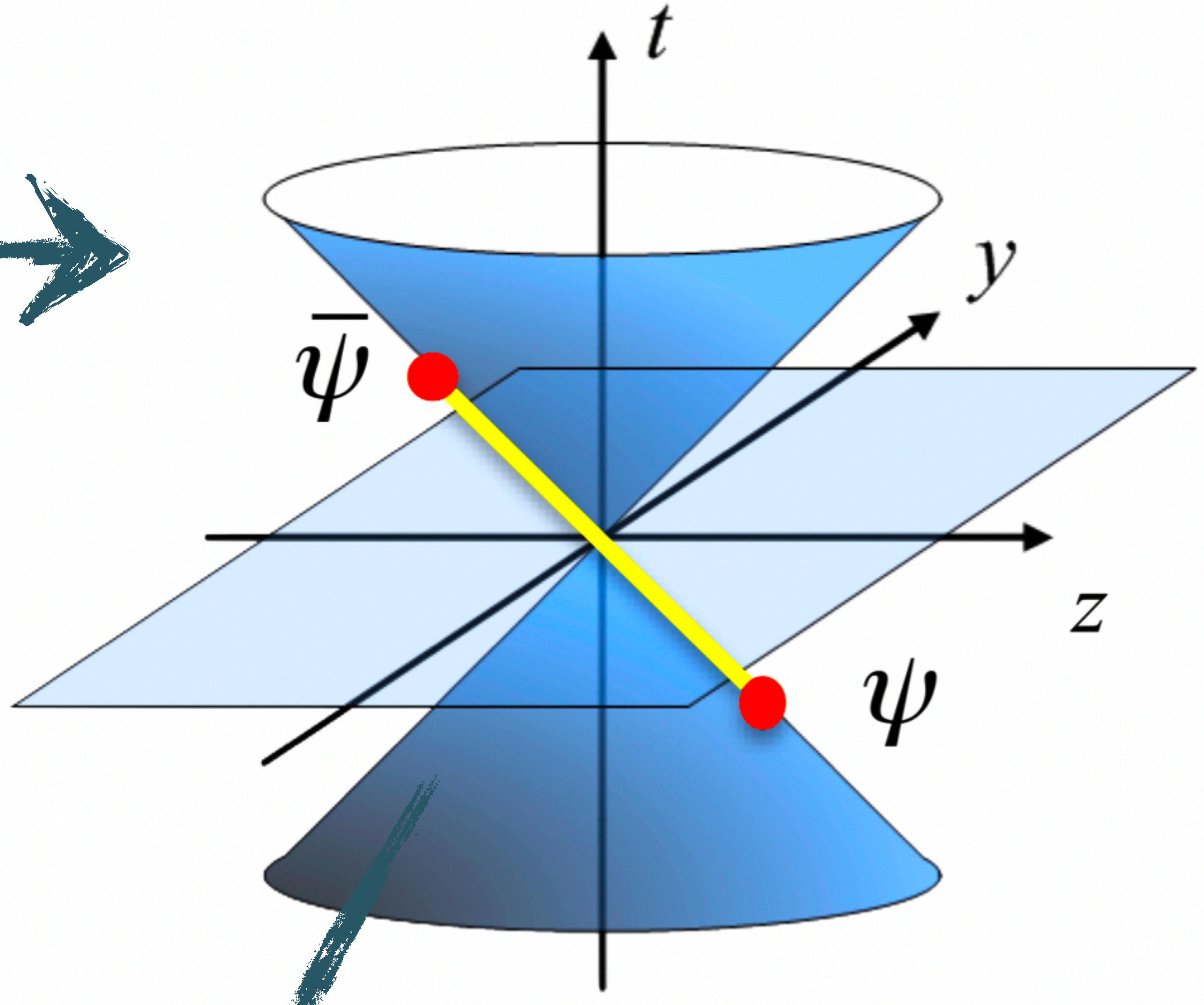




- first regularize QCD on a lattice, then $P_z \to \infty \,/\, z^2 \to 0$
- opposite order of limits; two limits don't commute
- difference is UV physics, can be taken care of through pQCD







parton physics

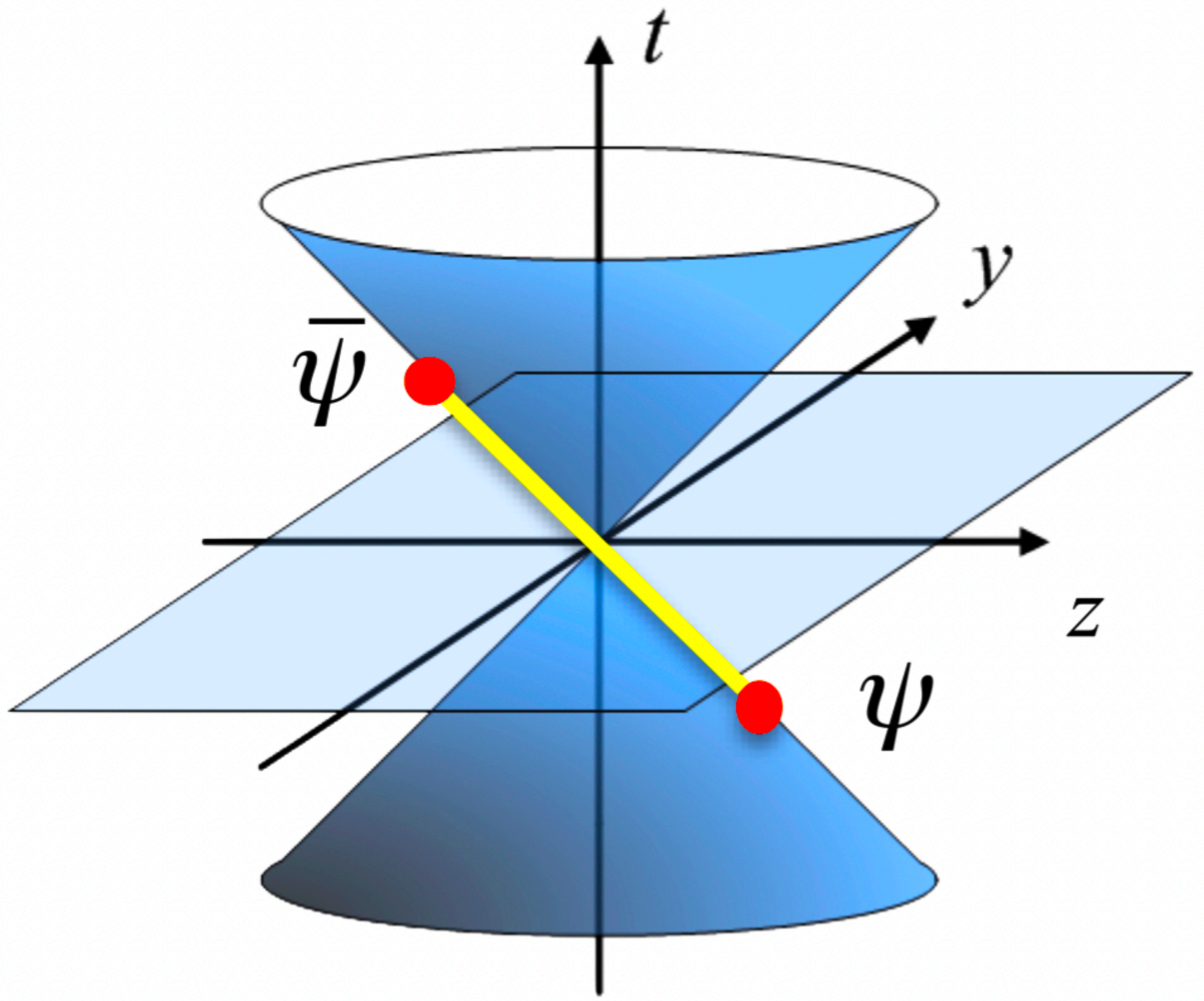
pQCD

$C(x, P_z, \mu) \otimes$ momentum space

$$+\mathcal{O}\left[\frac{\Lambda^2_{\rm QCD}}{x^2P_z^2}, \frac{\Lambda_{\rm QCD}}{(1-x)P_z}, \frac{M_H^2}{P_z^2}, \ldots\right]$$

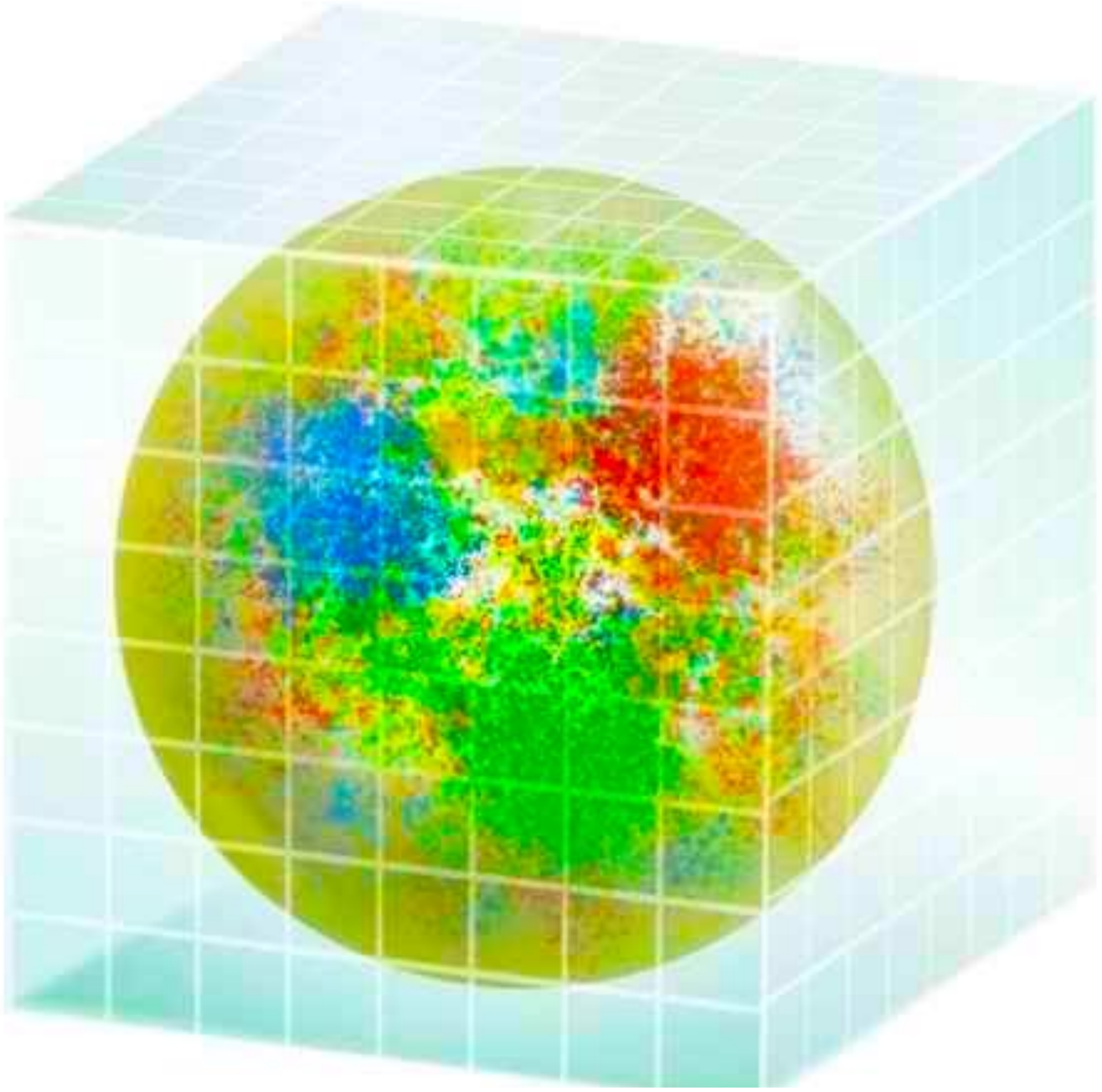
$C(\alpha, z^2, \mu) \otimes$ position space

$$+\mathcal{O}\left[z^2\Lambda^2_{\rm QCD}, z^2M_H^2, \ldots\right]$$



$C(x, P_z, \mu) \otimes$

$C(\alpha, z^2, \mu) \otimes$

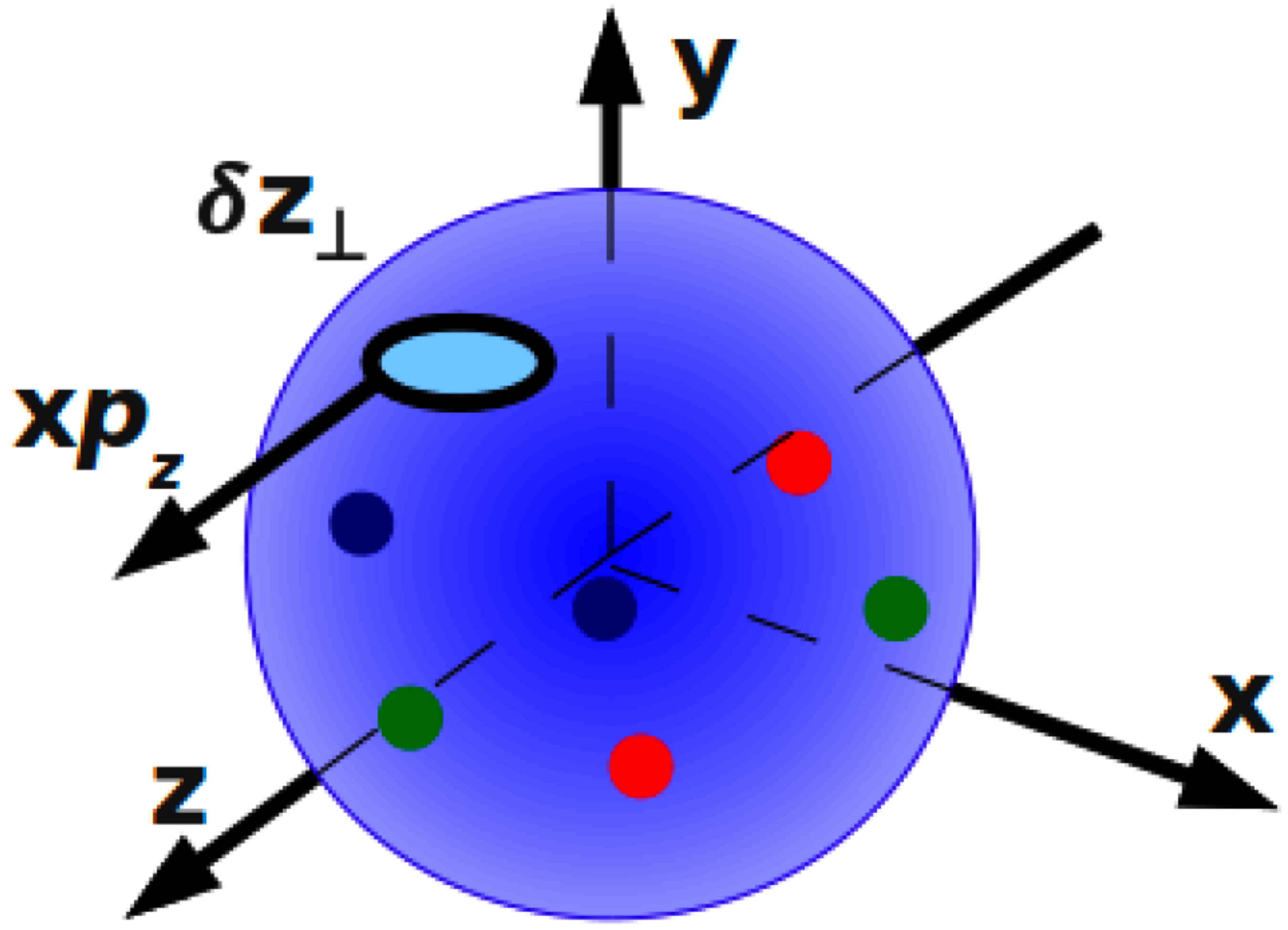


LO NLO NNLO

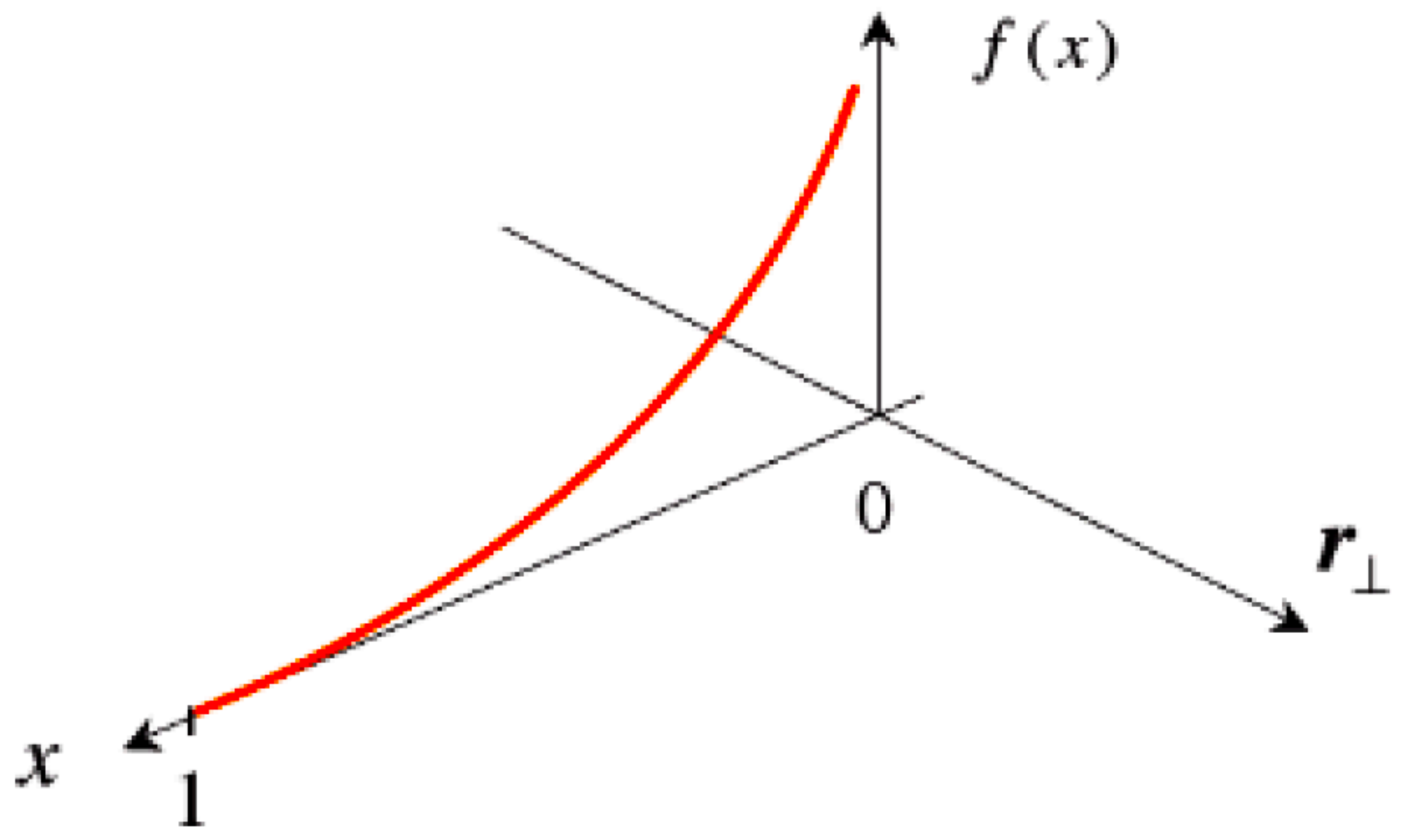
$$C(\mathcal{S}, \mu) \ \sim \ \alpha_s^0(\mu) \ + \ \alpha_s(\mu) f\left(\ln[\mathcal{S}\mu]\right) \ + \ \alpha_s^2(\mu) f\left(\ln[\mathcal{S}\mu]\right) \ + \ \cdots$$

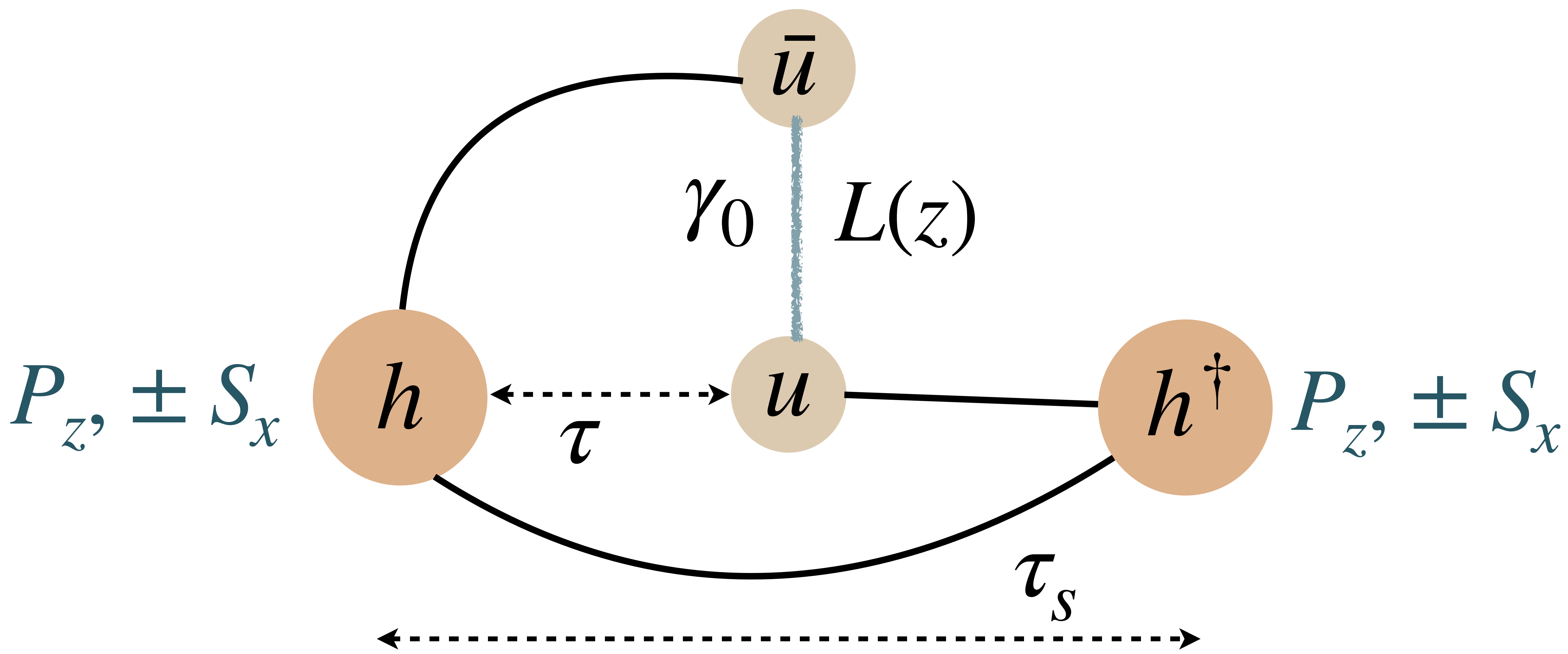
$$\mathcal{S} \ = \ 2xP_z \ , \ z^2$$

parton distribution function (PDF): 1d snapshot of hadron

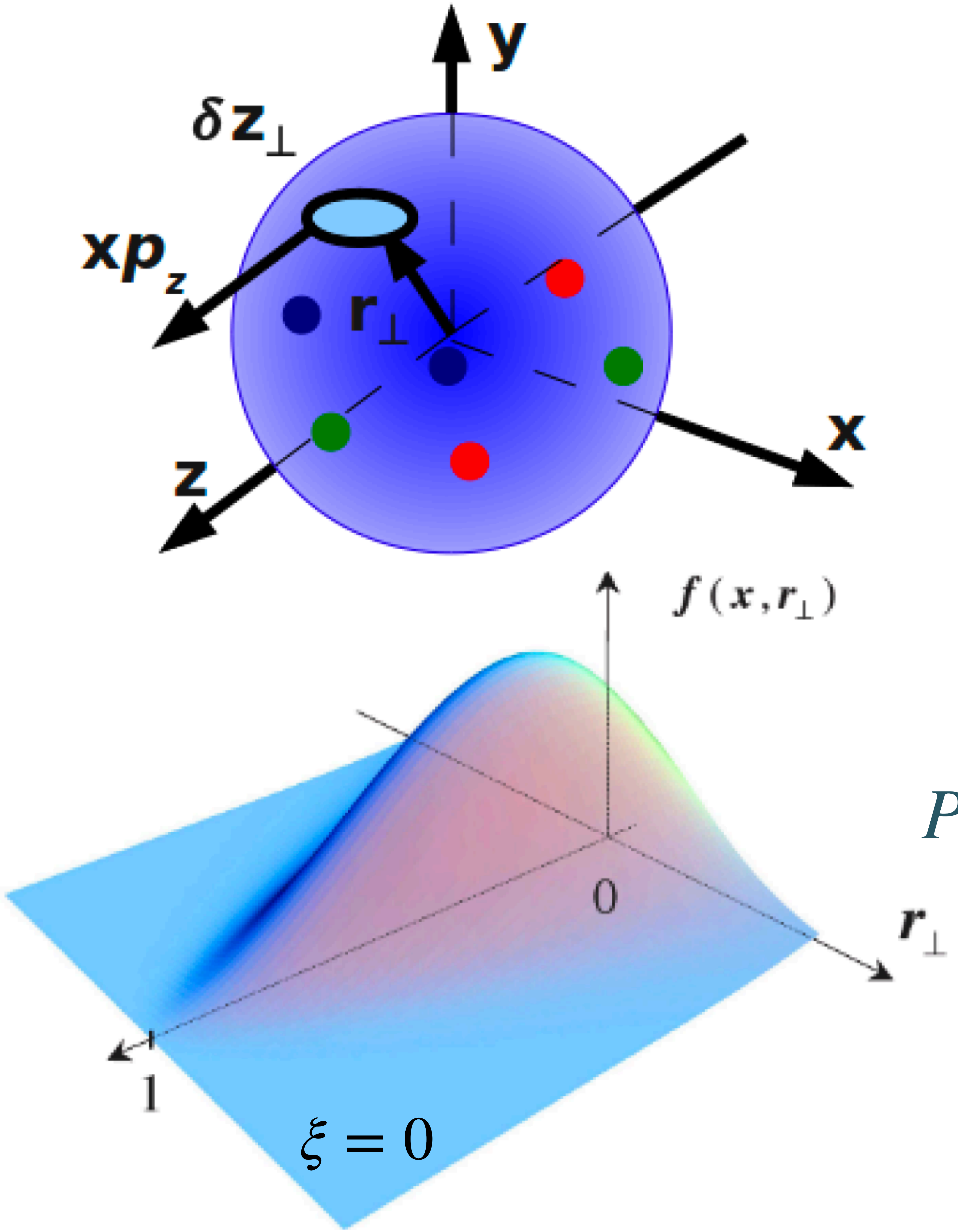


distribution of longitudinal momentum fractions of partons inside a hadron

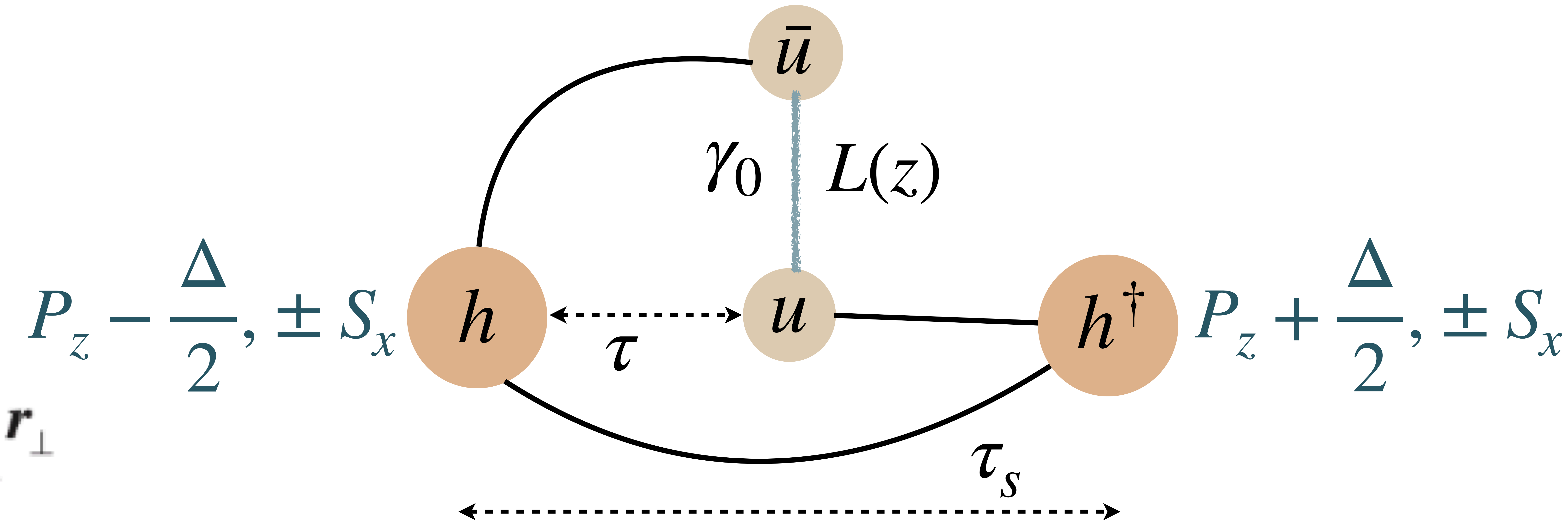




generalized parton distribution (GPD): (1+2)d snapshot of hadron



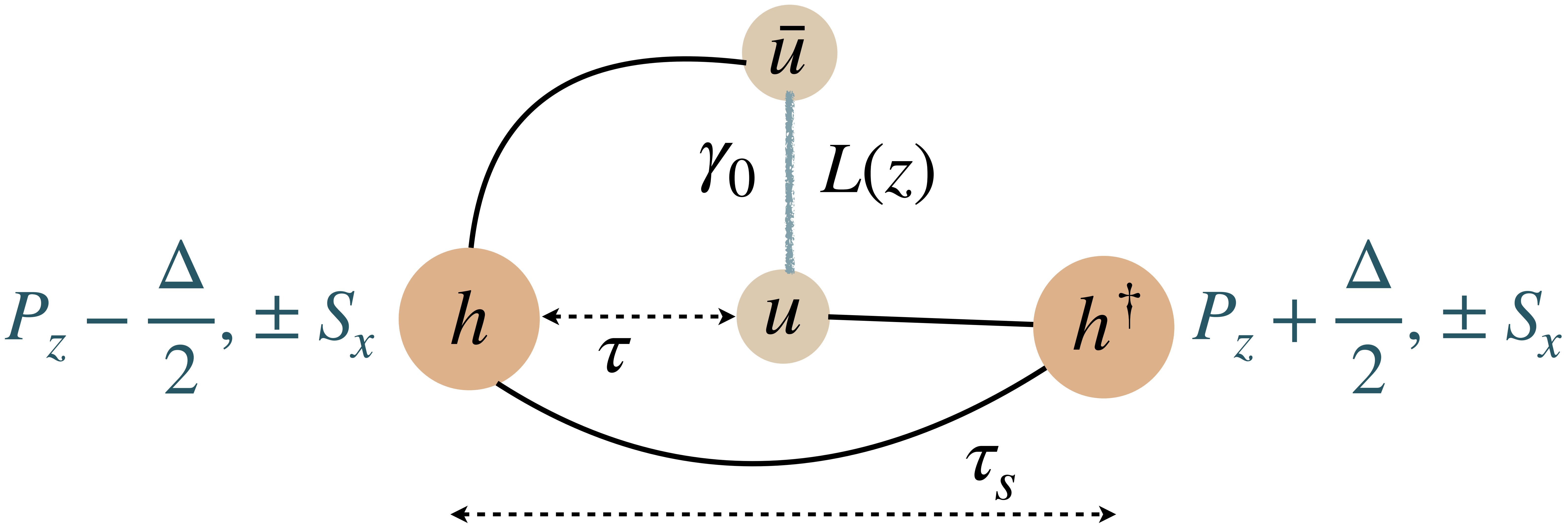
distribution of the longitudinal momentum fractions of partons in the transverse plane the hadron



generalized parton distribution (GPD): (1+2)d snapshot of hadron

spin-1/2 hadron

N / q	U	L	T
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	H_T $\tilde{H}_T$

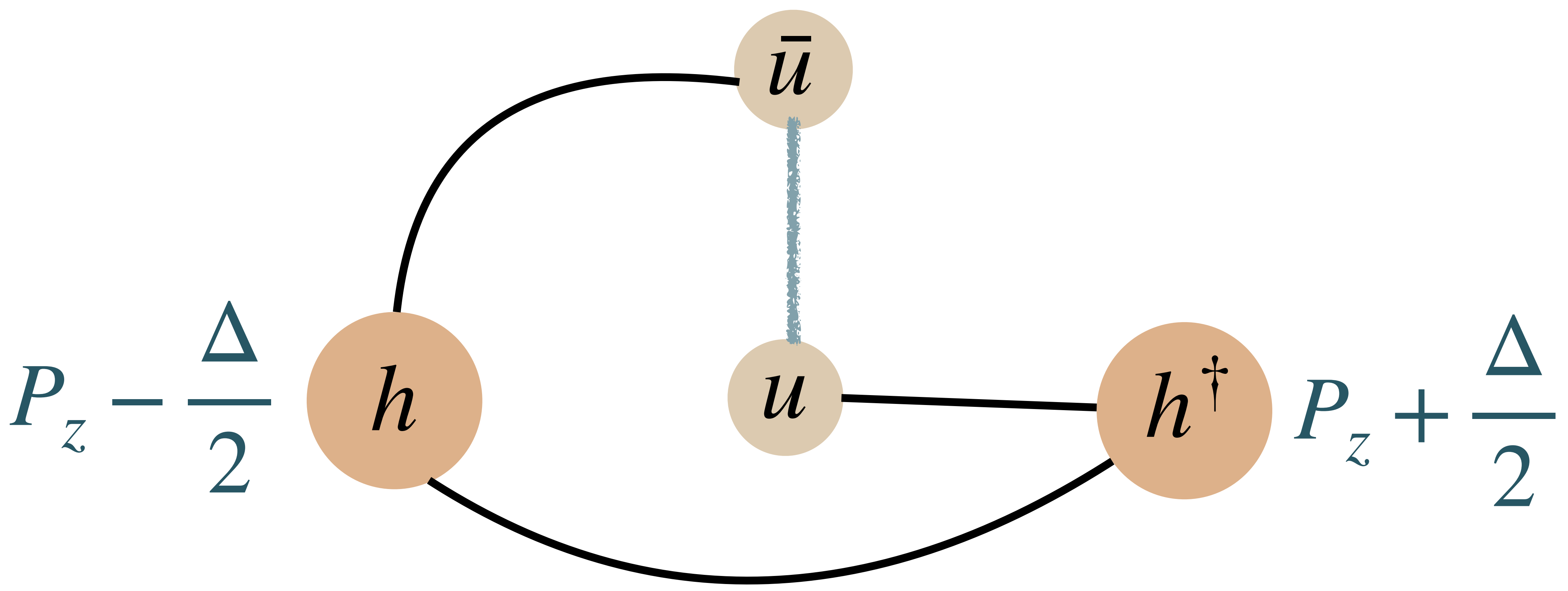


GPD from lattice QCD: a recent breakthrough

traditional method:

symmetric momentum transfer

each Δ needs a separate calculations

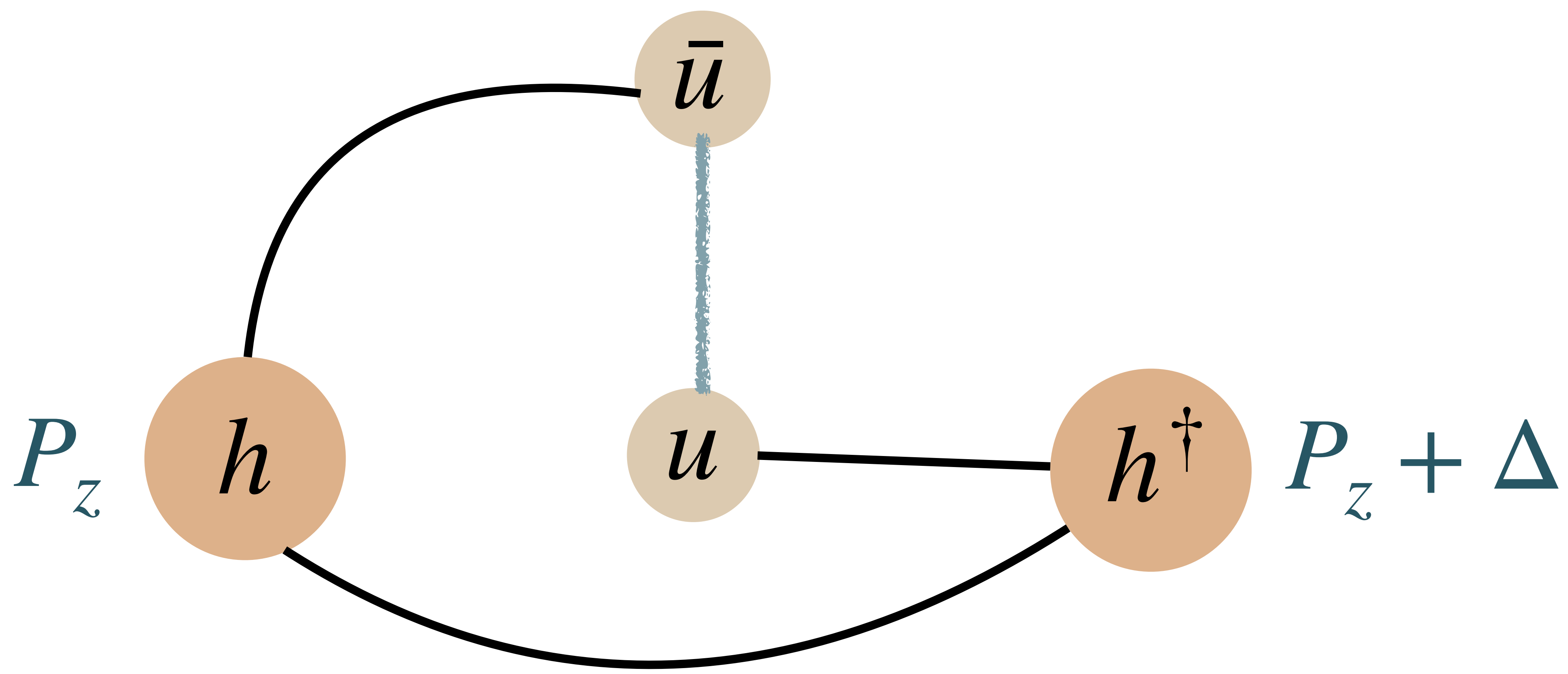


new Lorentz covariant formalism:

asymmetric momentum transfer

single calculations for multiple Δ

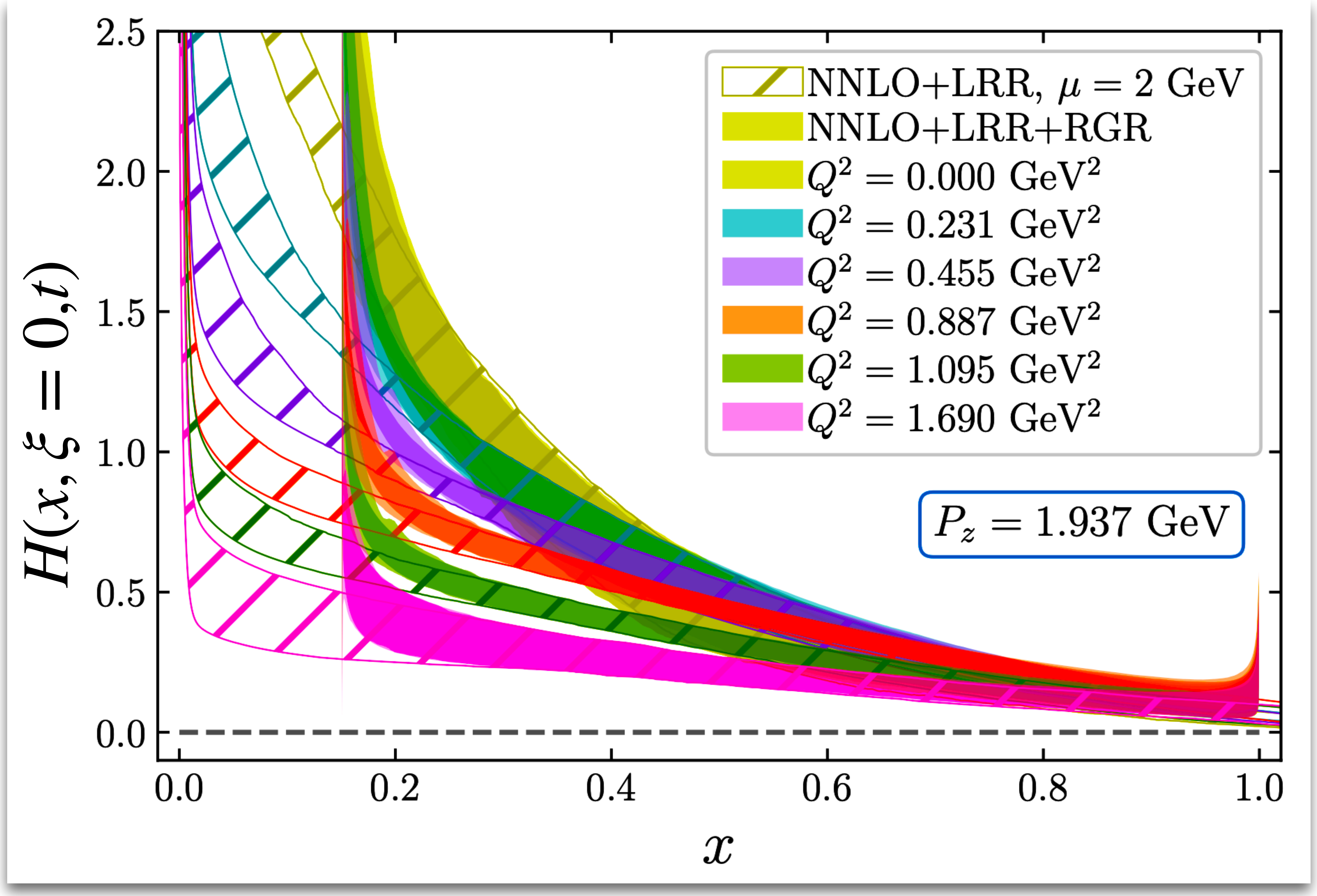
~ 20x faster calculations



S. Bhattacharya et al., Phys. Rev. D 106, 1, 114512 (2022)

pion GPD from lattice QCD

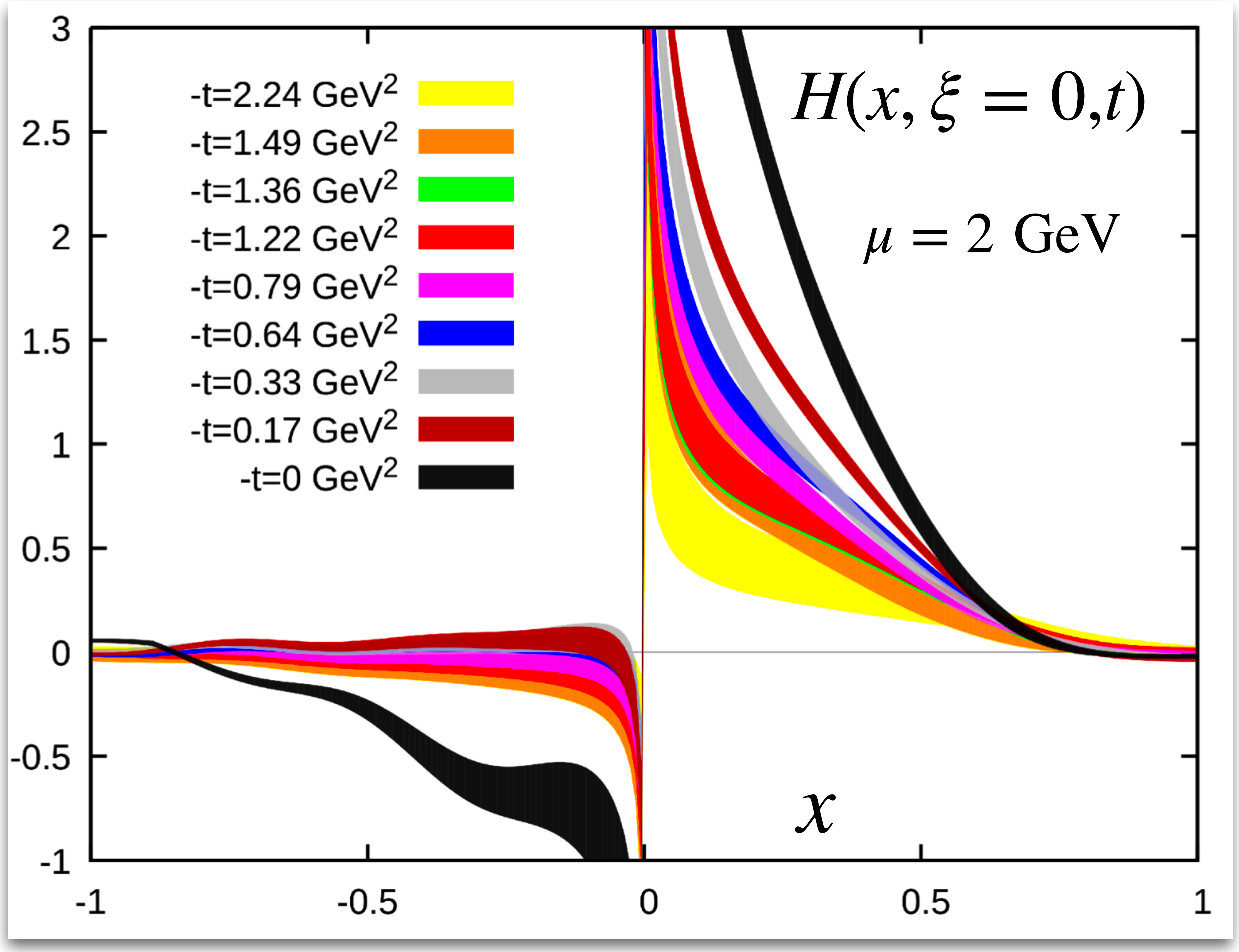
$$t = -Q^2$$



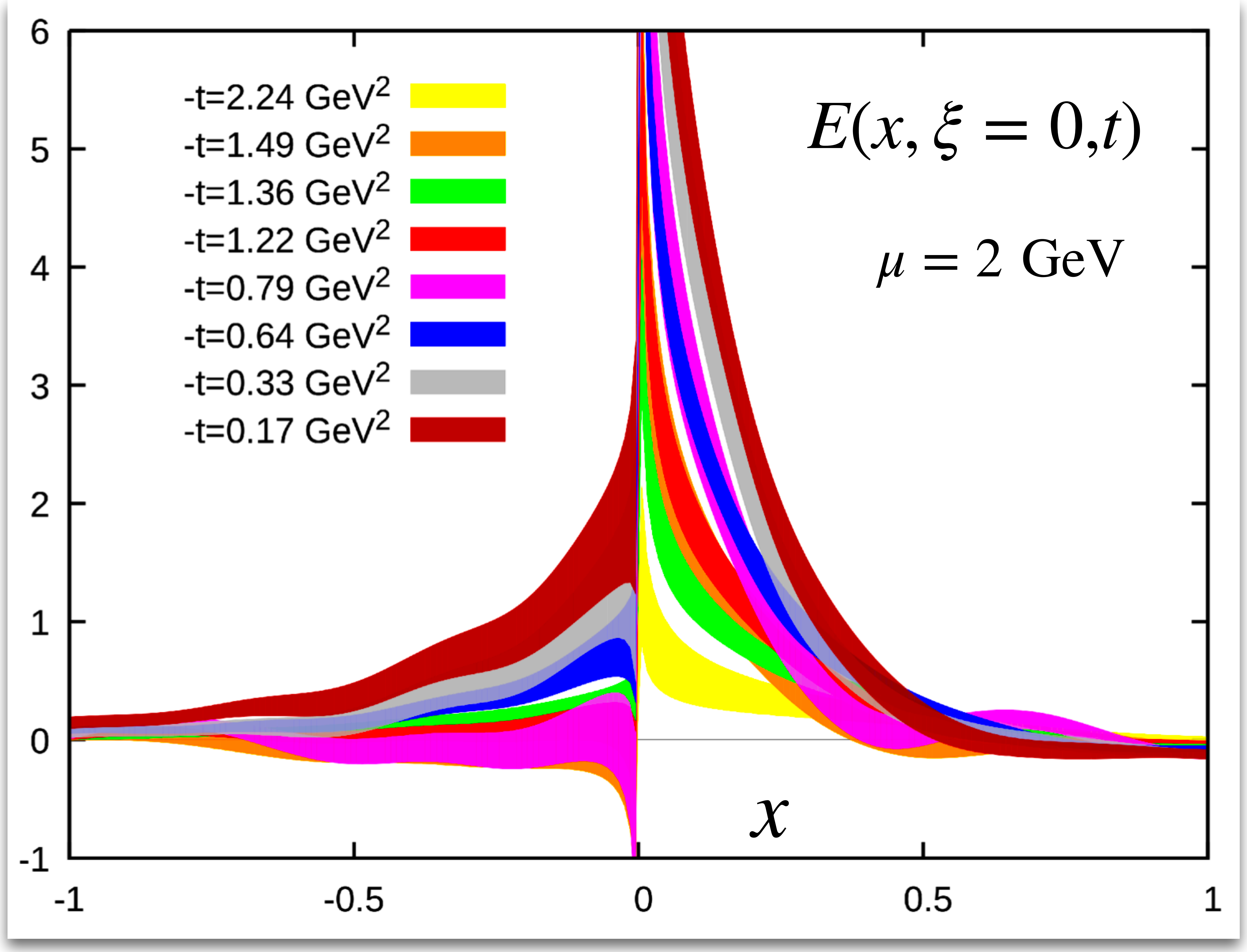
Q. Shi et. al., coming very soon

proton GPD: unpolarized quarks inside …

unpolarized proton



polarized proton



J. Miller et. al., Acta Phys. Polon. Supp. 167, 7-A6 (2023); Phys. Rev. D 106, 1, 114512 (2022)

proton GPD to proton spin

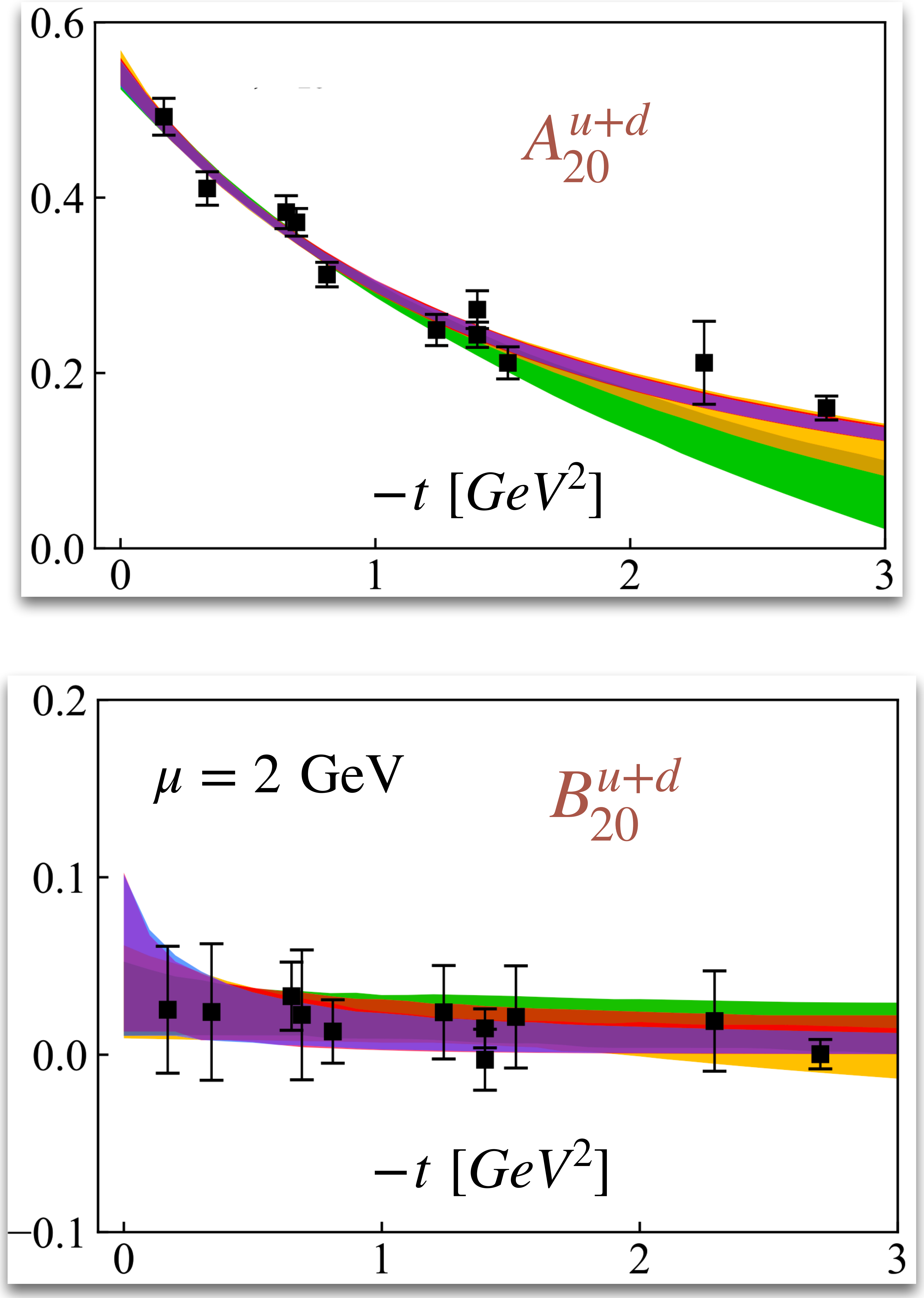
$$A_{2,0}(t) = \int_{-1}^{1} xH^q(x, \xi = 0, t)dx \qquad B_{2,0}(t) = \int_{-1}^{1} xE^q(x, \xi = 0, t)dx$$

Ji sum rule: $$J^q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$$

contributions of quarks' total angular momentum to proton spin:

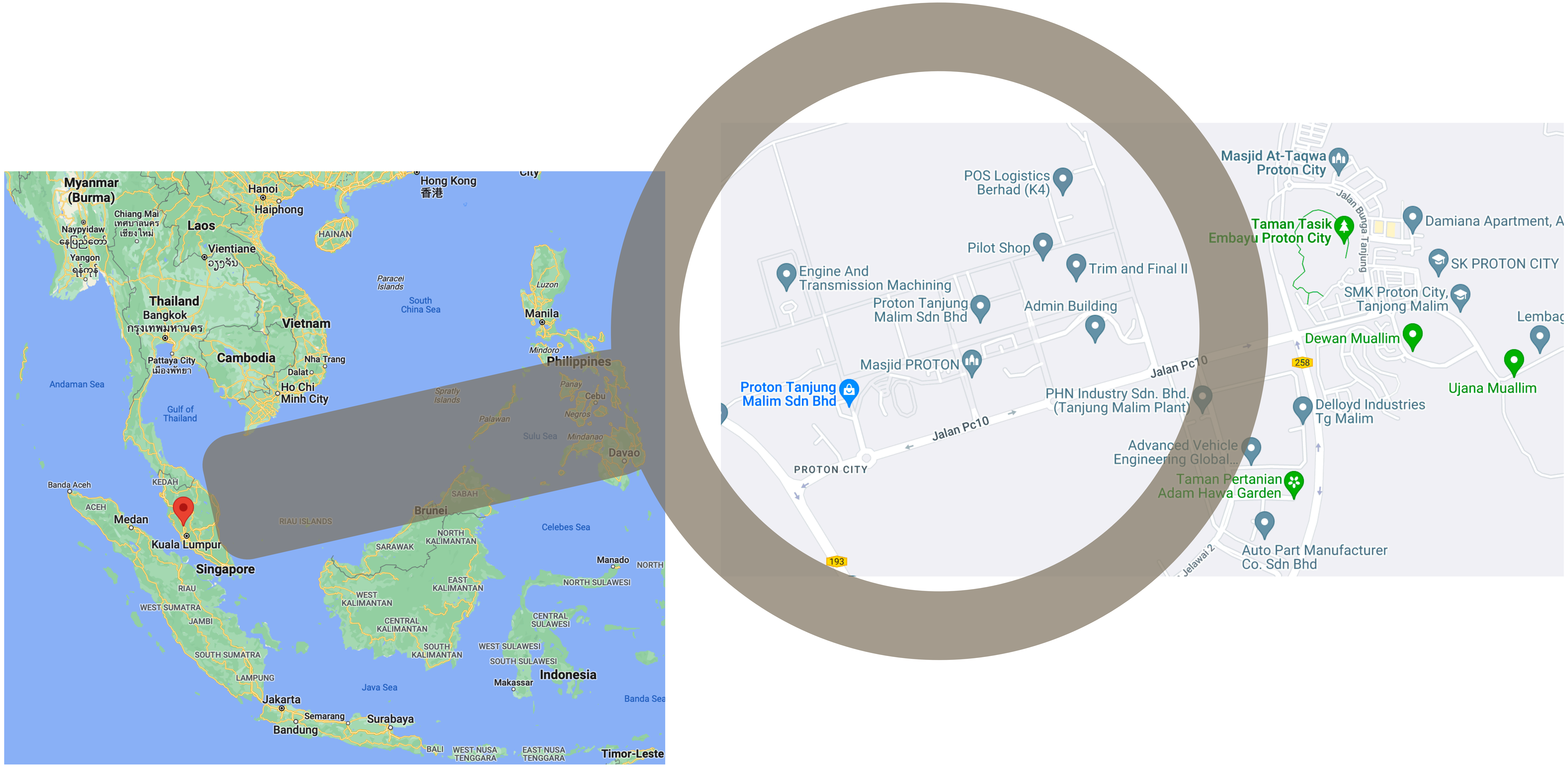
$$J^{u+d} = 0.296(22)(33)$$

X. Gao et. al., Phys. Rev. D108, 1, 014507 (2023)

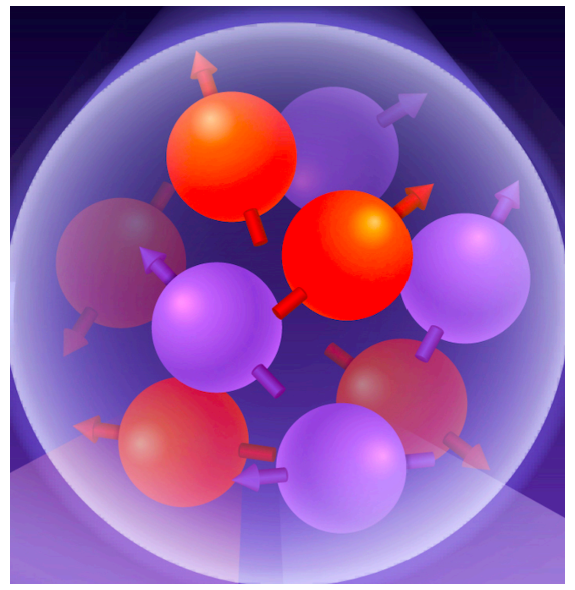


mapping proton

Proton City, Perak, Malaysia



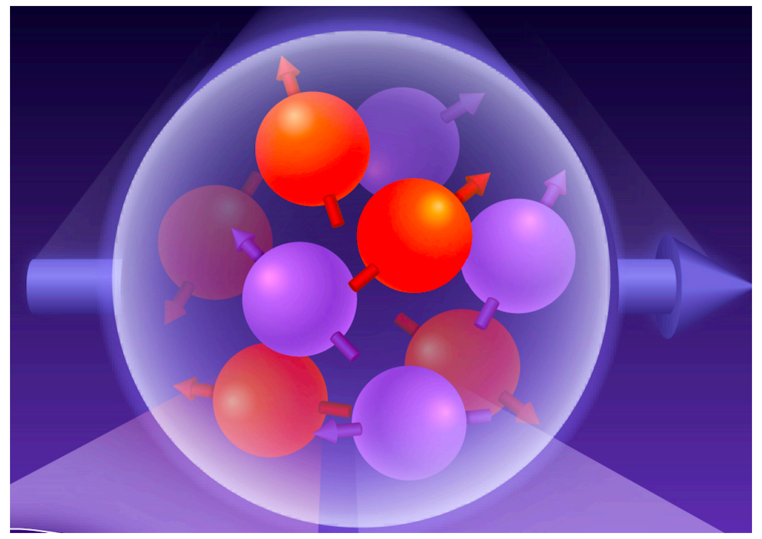
quark's enthalpy density inside proton



$$\rho_2(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} A_{2,0}(-\overrightarrow{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

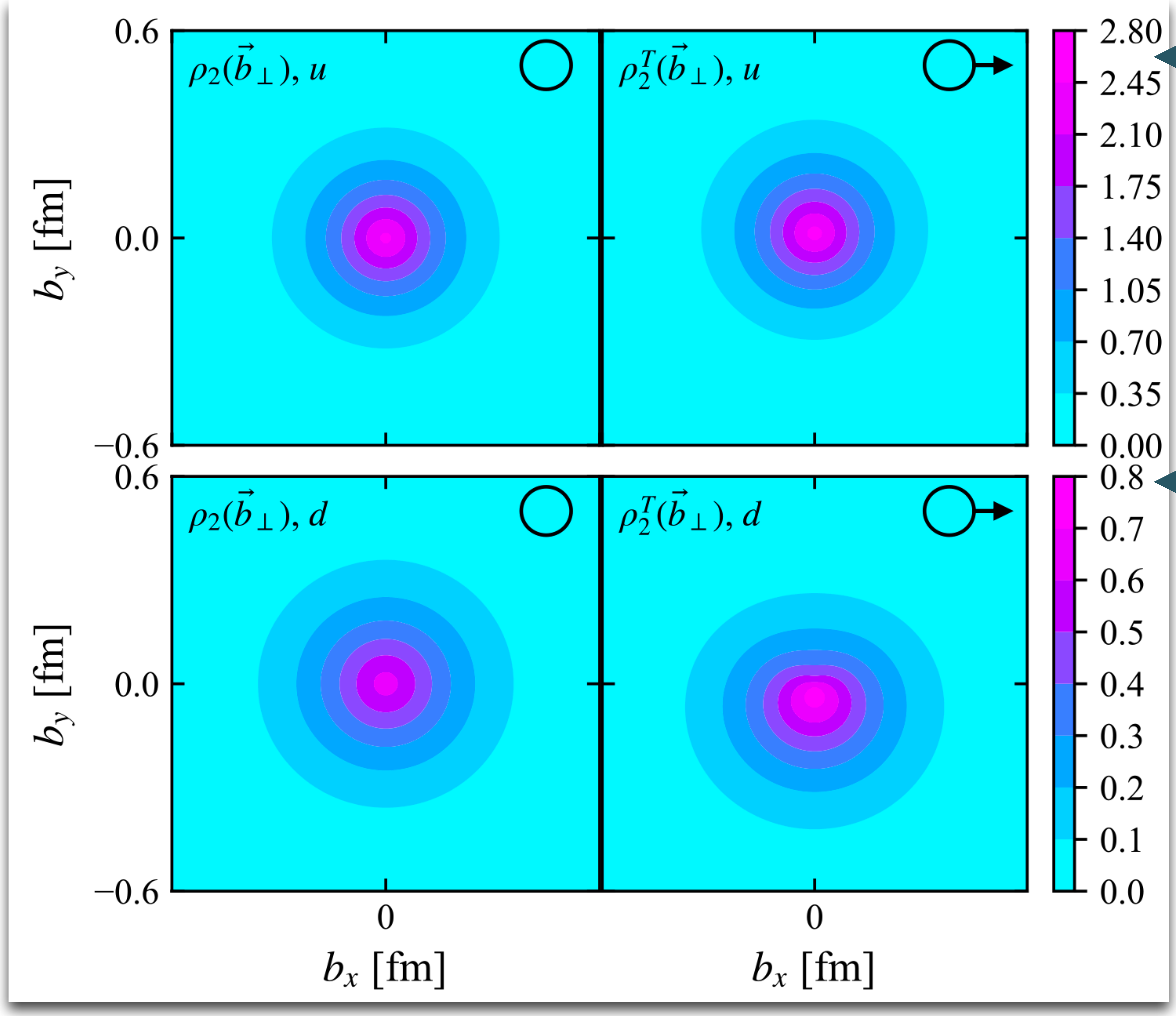
C. Lorcé: Eur. Phys. J. C78, 2, 120 (2018)

quark's angular momentum inside transversely polarized proton

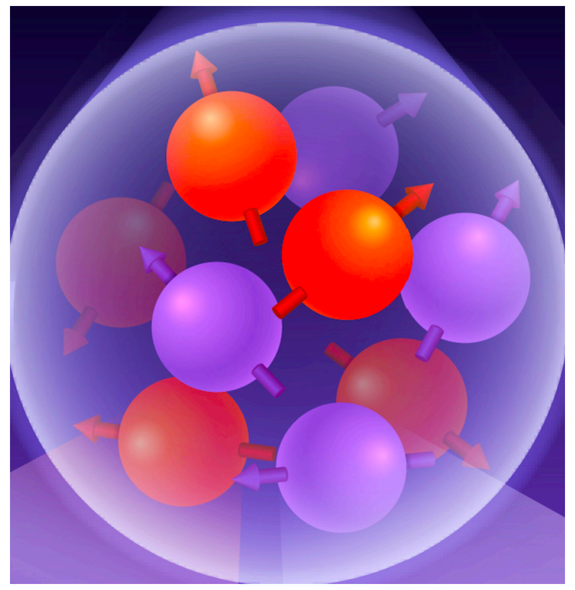


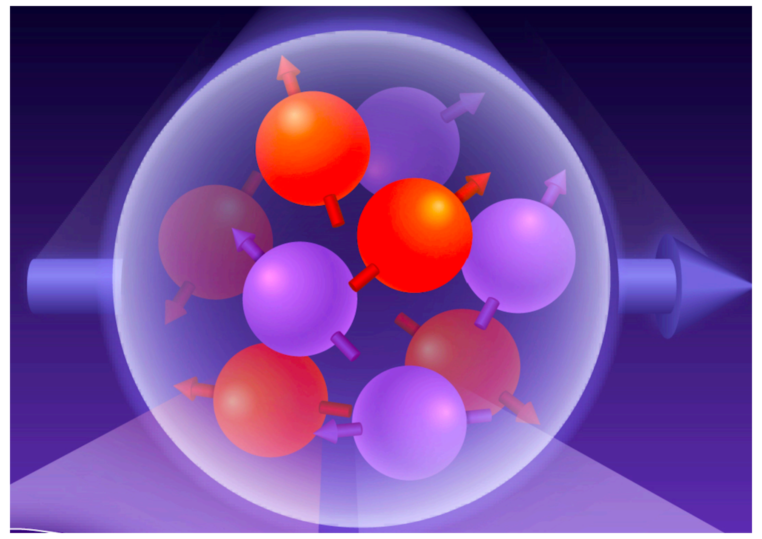
$$\rho_2^T(\vec{b}_\perp) = \int \frac{d^2\overrightarrow{\Delta}_\perp}{(2\pi)^2} \left[A_{2,0}(-\overrightarrow{\Delta}_\perp^2) + \frac{i\Delta_y}{2m_n} B_{2,0}(-\overrightarrow{\Delta}_\perp^2) \right] e^{-i\vec{b}_\perp\cdot\overrightarrow{\Delta}_\perp}$$

M. Burkardt, Int. J. Mod. Phys. A 18, 173 (2003) up quarks



$\mu = 2$ GeV





down quarks

X. Gao et. al., Phys. Rev. D108, 1, 014507 (2023)

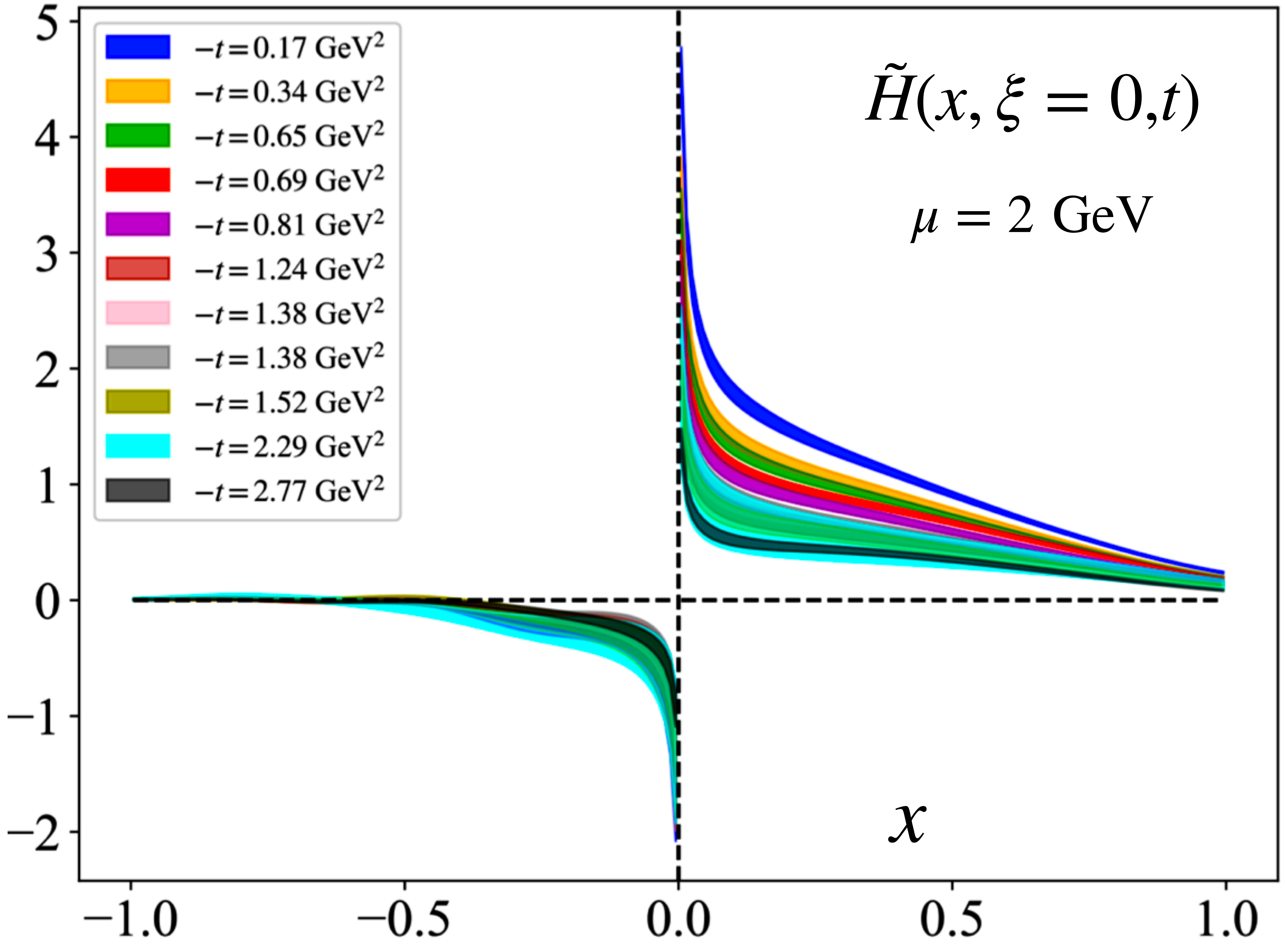
GPD of longitudinally polarized proton

spin-orbit correlation of longitudinally polarized quark

$$4L_l^q S_l^q = \int_{-1}^{1} x\tilde{H}^q(x, \xi = 0, t)dx - 1$$

$$+\mathcal{O}(m_q/m_p)$$

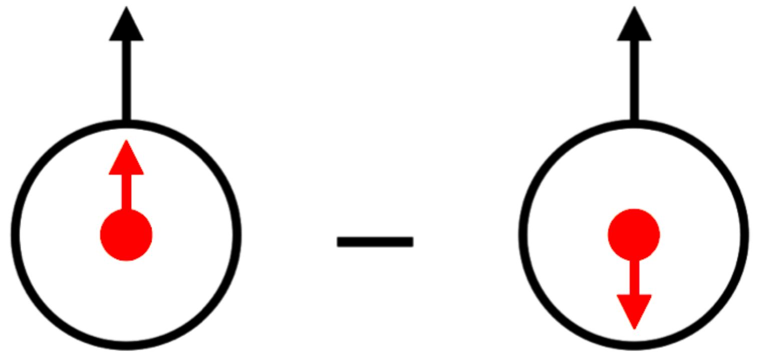
C. Lorcé: Phys. Lett. B 735, 344 (2014)

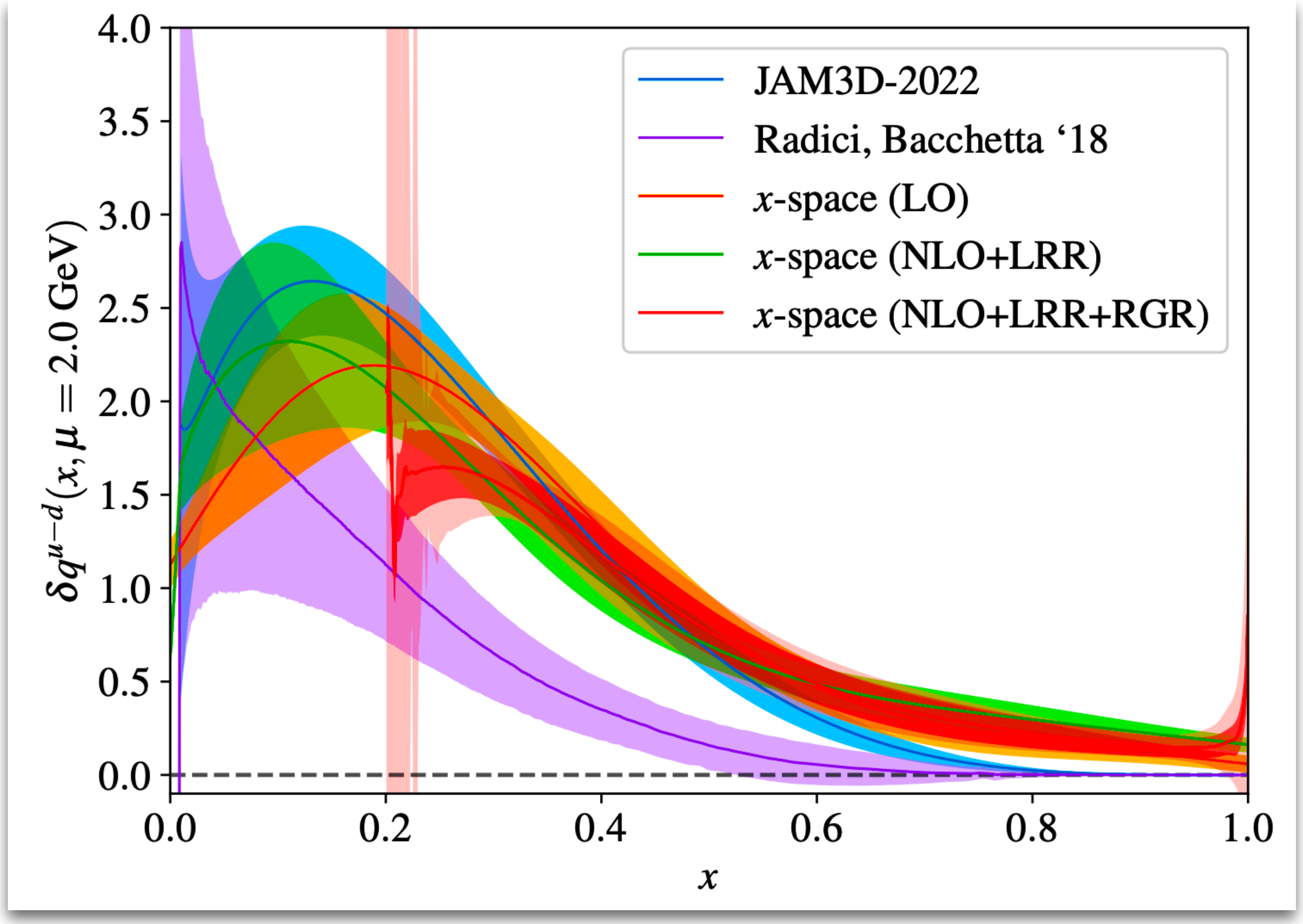


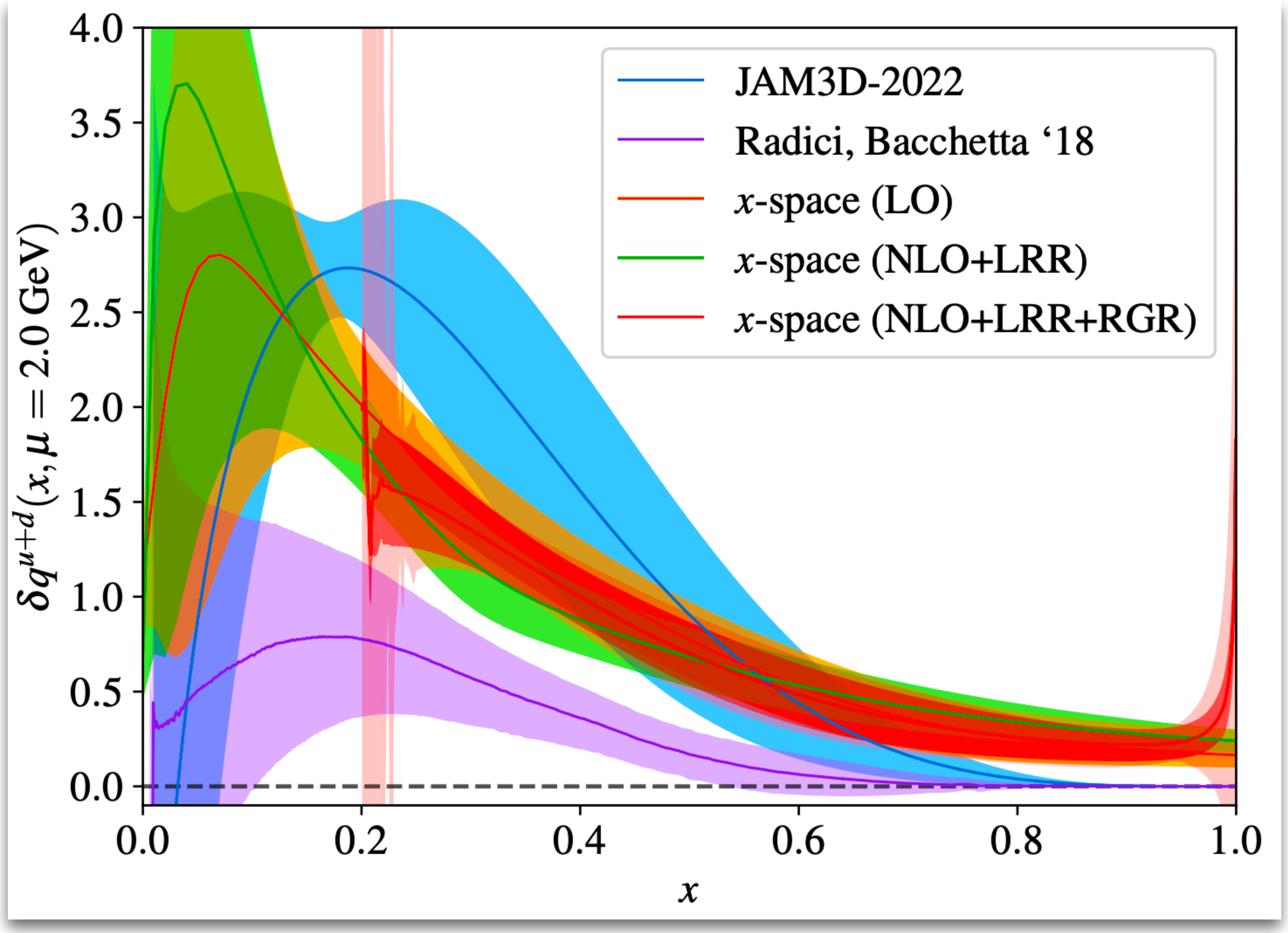
J. Miller et. al., Phys. Rev. D ??? (2024) [2310.13114]

… and some more …

proton transversity PDF





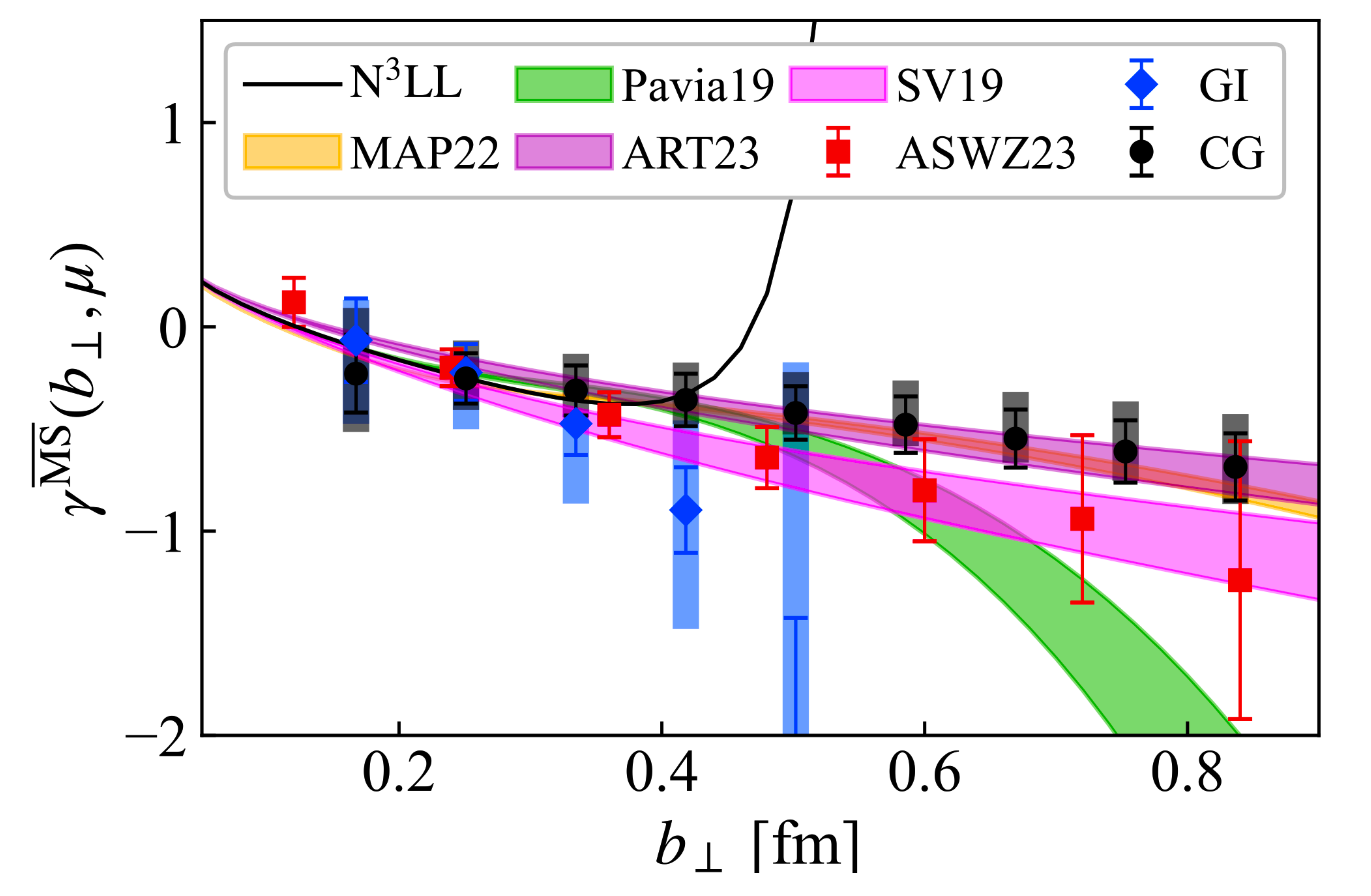


$$g_T^{u-d} = 1.05(2), \quad \overline{\text{MS}}(\mu = 2\ \text{GeV})$$
$$g_T^{u+d} = 0.64(2), \quad \overline{\text{MS}}(\mu = 2\ \text{GeV})$$

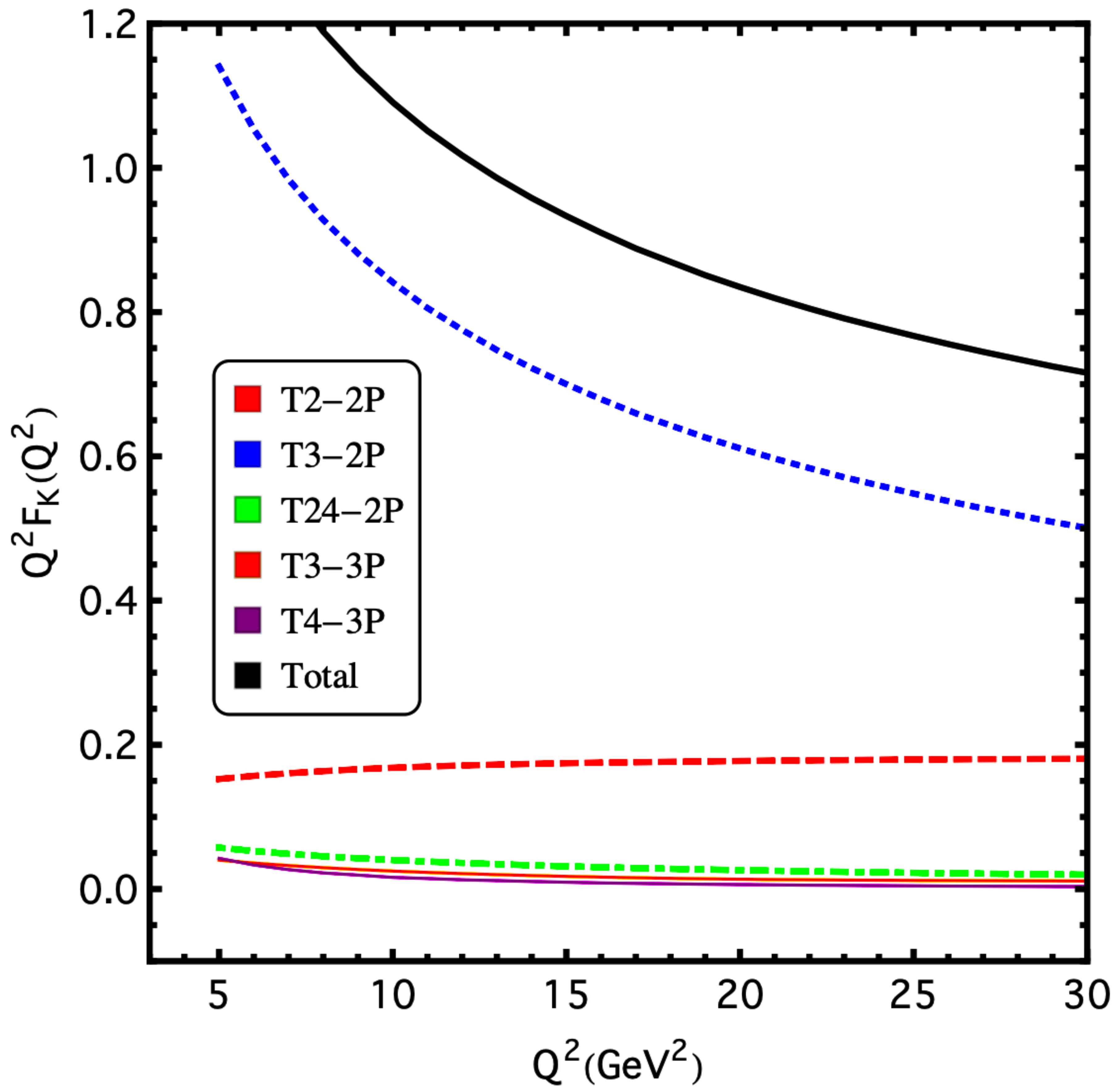
A. Hanlon et. al., Phys. Rev. D ??? (2024) [2310.19047]

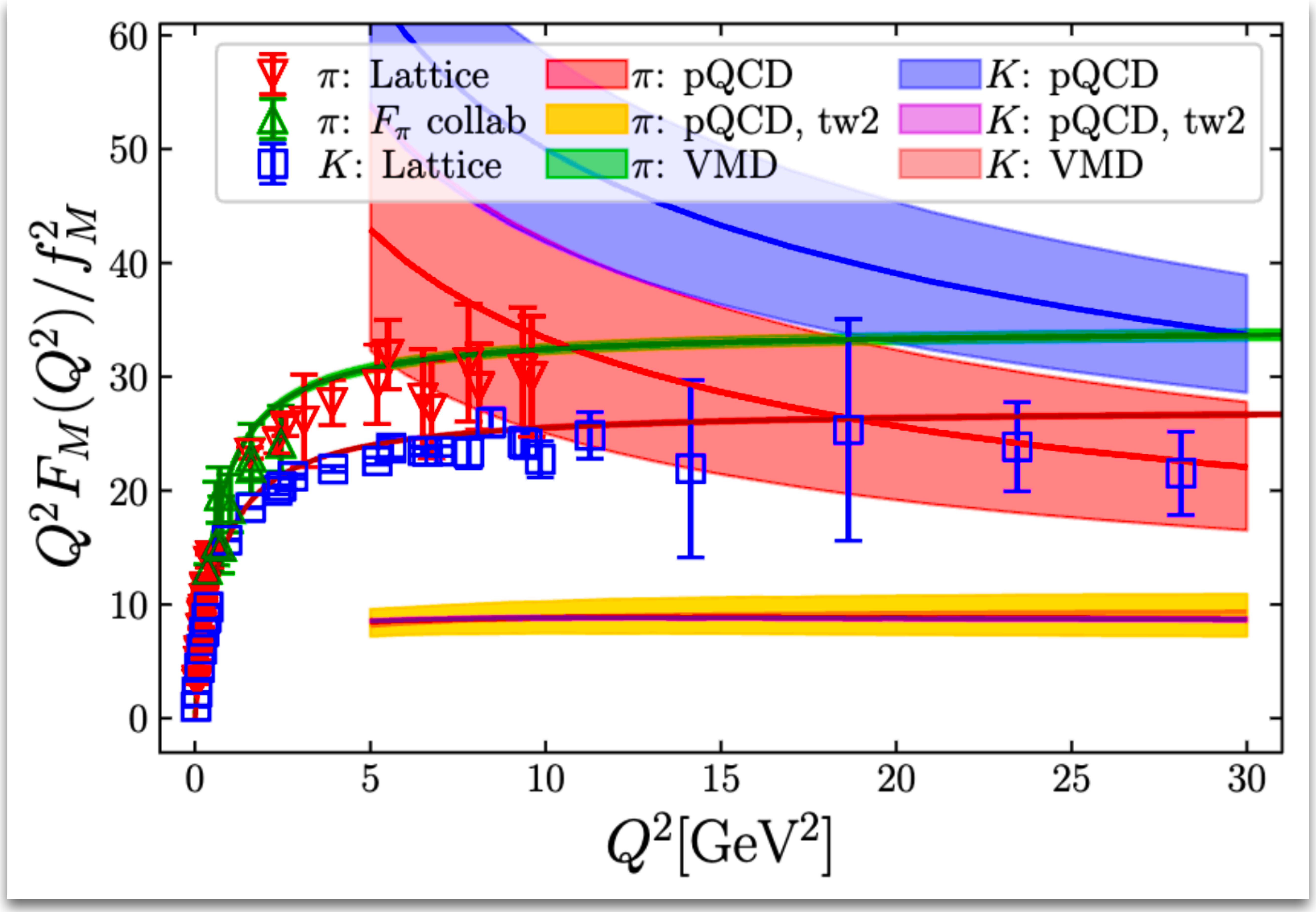
Collins-Soper kernel

X. Gao et. al., coming very soon



multi-patron correlations: pion, kaon form factors at large momenta





S. Cheng, Phys. Rev. D100, 1, 013007, (2019)

Q. Shi et. al., coming very soon

beginning of a new journey ...