Manipulation and measurement of trapped-ion motional states

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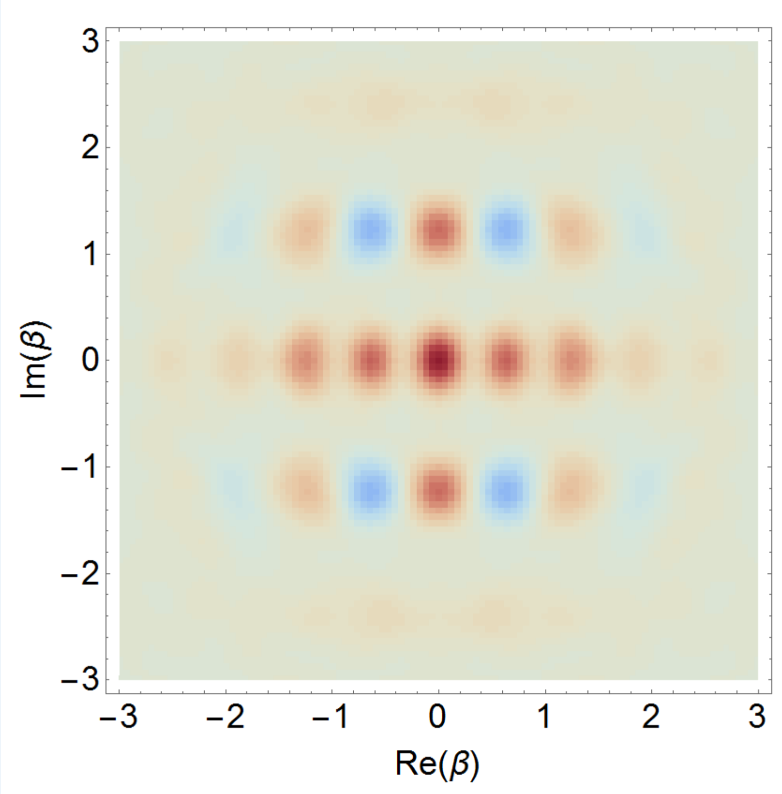


Gottesman, Kitaev, Preskill PRA 64, 012310 (2001)

The trapped-ion mechanical oscillator

Ion is well-isolated from environment: trapped in vacuum by EM fields

Spring constant defined by curvature of trapping potential

$$H = \frac{p^2}{2m} + \frac{\boxed{m\omega^2}}{2}x^2$$

Often cooled to quantum regime: $H = \hbar\omega(a^\dagger a + 1/2)$

Oscillation frequencies, 1 - 10 MHz: zero-point motion $x_0 = \sqrt{\frac{\hbar}{2m\omega}} \sim 10 \text{ nm}$

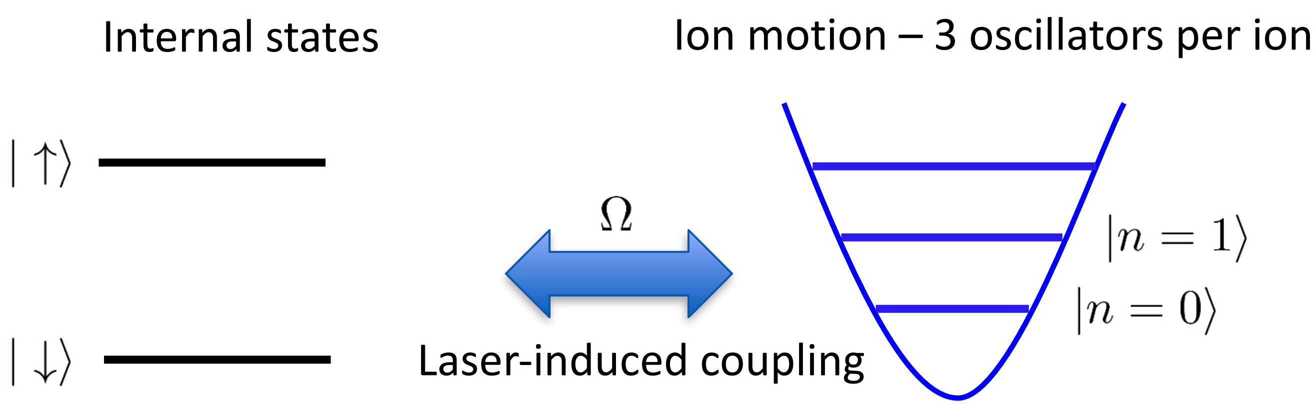
What is the interest in motional states of trapped-ions

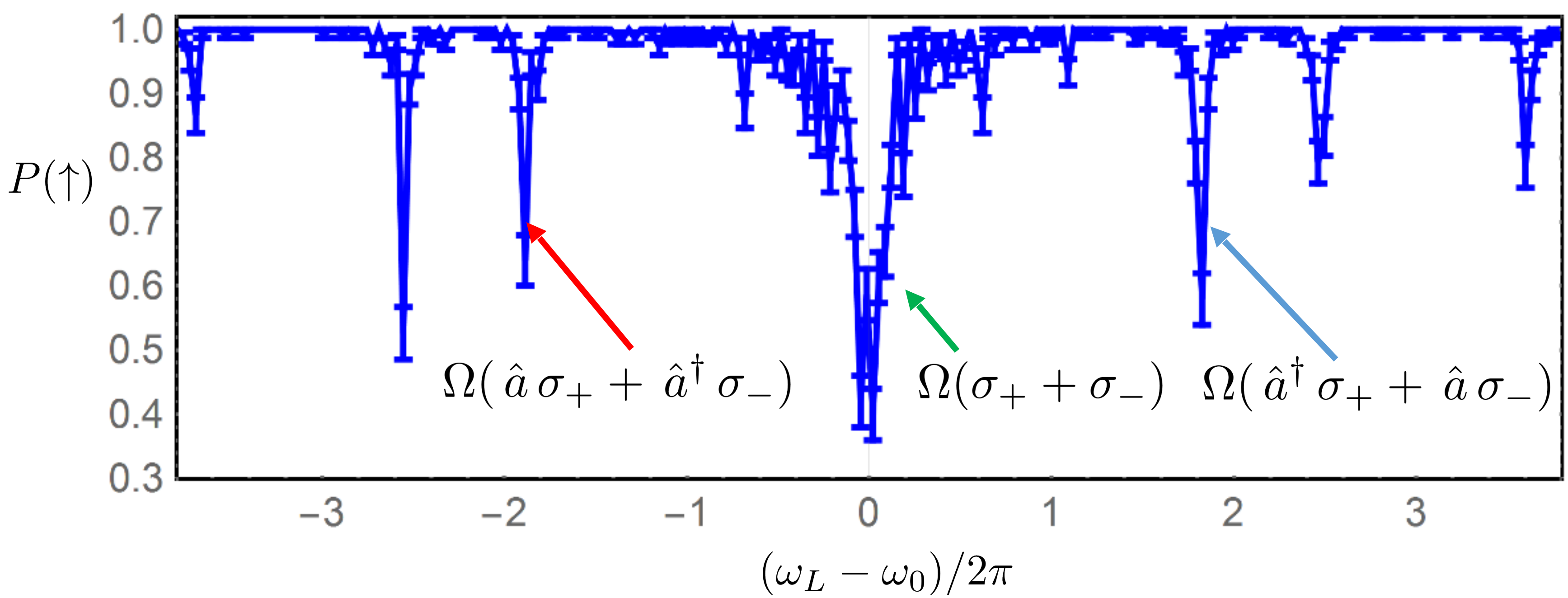
Entry point for quantum control of ions: cooling to the quantum ground state

- Low-decoherence controllable mechanical oscillator: testbed for quantum control
- Position of ion is coupled to other ions via Coulomb interaction (Primary quantum bus for trapped-ion quantum computing)
- Crossover system from quantum to classical (oscillation amplitude tuneable from small to large amplitude)
- Sensitive to environment (electric fields)
- Sensitive to photon recoil - spectroscopy tool (Talk of S. Willitsch)

Quantum control of the trapped-ion oscillator

Laser induced coupling and control

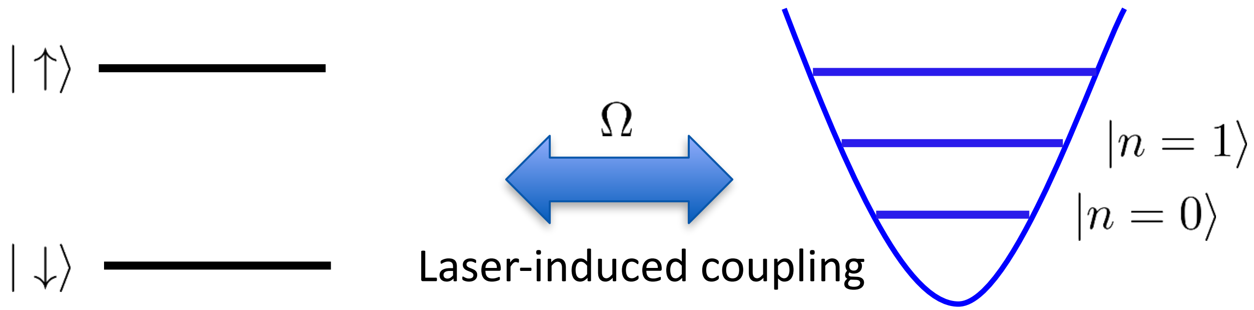




Lamb-Dicke approximation (review, see D. Leibfried)

Internal states

Ion motion – 3 oscillators per ion



$$H_{\text{laser}} = \Omega\left[\sigma_+ e^{i(\omega_{\text{spin}}-\omega_L)t} e^{ikx} + \text{h.c.}\right]$$

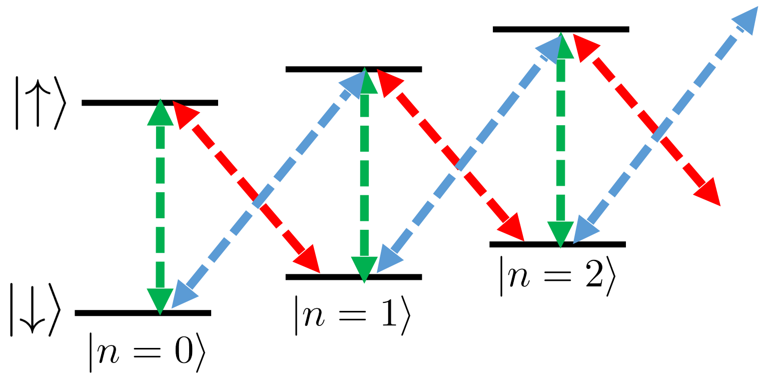
In interaction picture of the oscillator $\quad a \rightarrow a e^{-i\omega t}\ ,\ a^\dagger \rightarrow e^{i\omega t}$

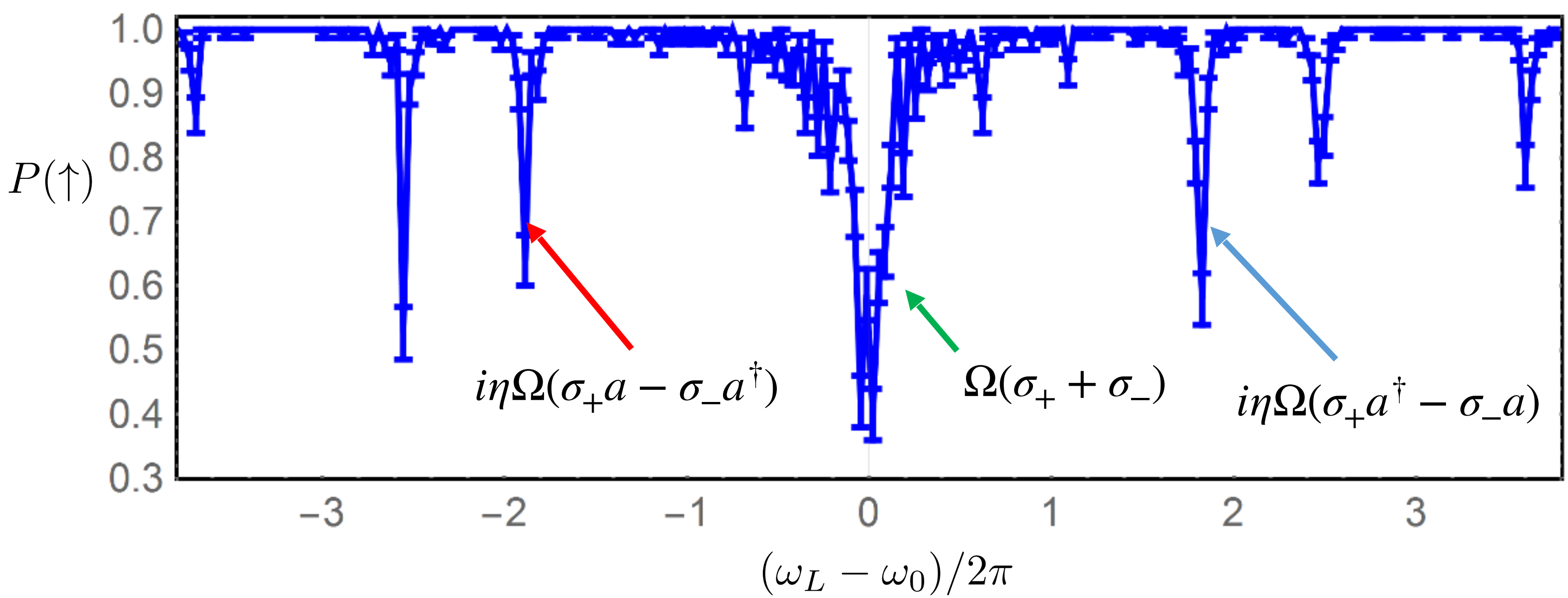
$$e^{ikx} = \simeq 1 + i\eta(a e^{-i\omega t} + a^\dagger e^{i\omega t}) + \text{higher order}$$

Lamb-Dicke parameter $\quad \eta^2 = \left(\dfrac{2\pi x_0}{\lambda}\right)^2 = \dfrac{E_{\text{recoil}}}{\hbar\omega} \ll 1\ (x_0 \sim 10\ \text{nm}, \lambda \sim 700\ \text{nm})$

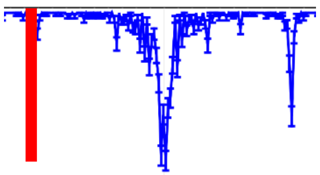
Ion oscillation phase-modulates the laser

Modulation sidebands, with modulation index η, well resolved for $\omega \gg \Omega, \Gamma$

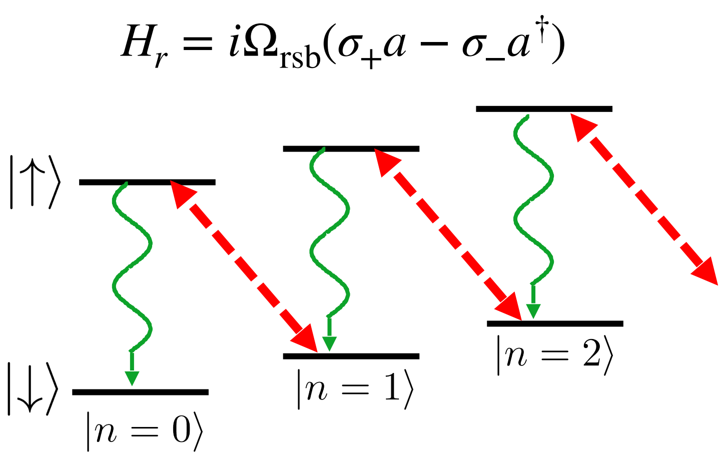


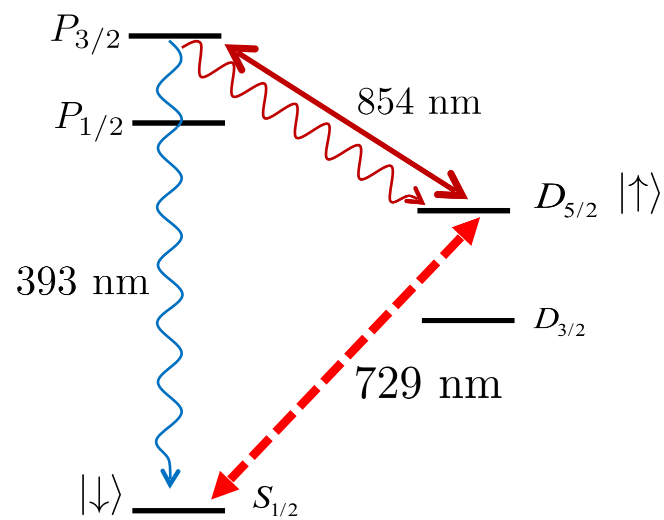


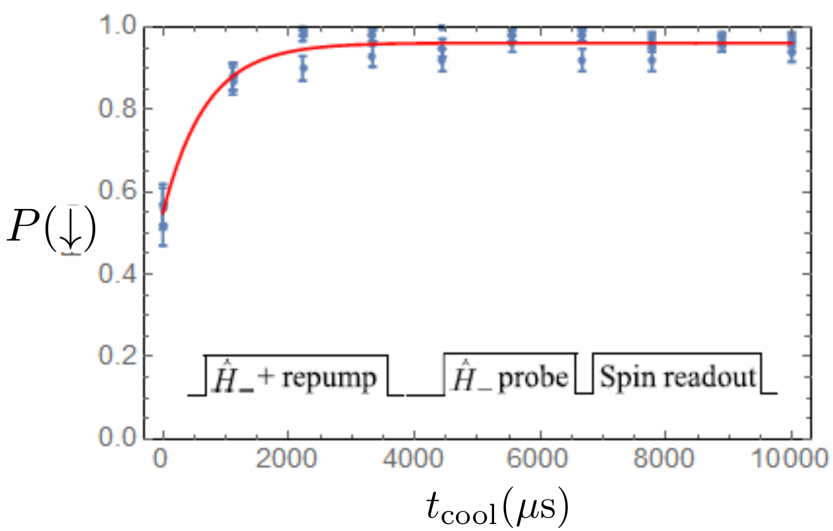
Ground-state laser cooling

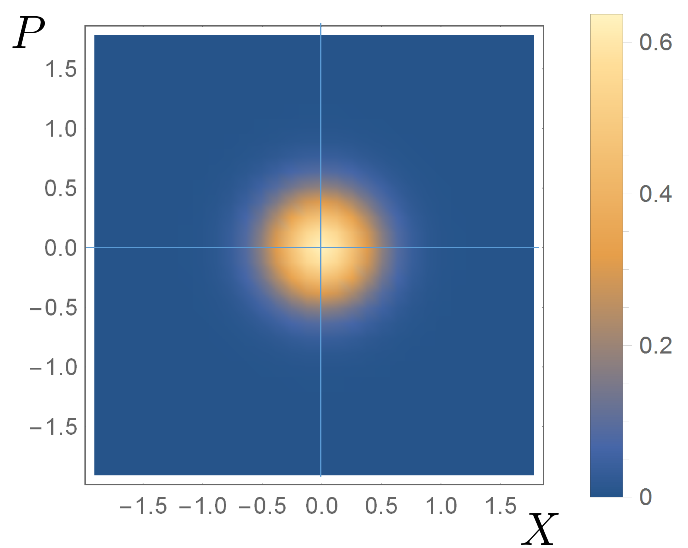


Monroe et al. PRL 75, 4011 (1995), Meekhof et al. PRL 76, 1796 (1996)









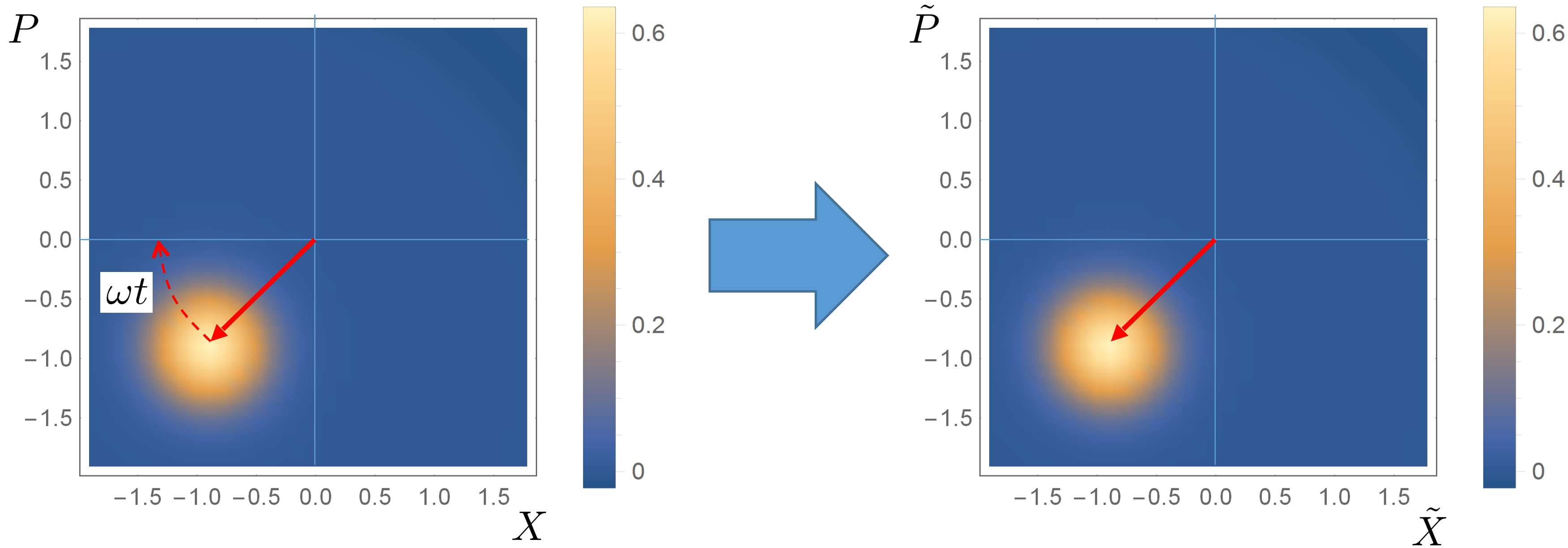
Phase-space picture

Wigner function – quasi-probability distribution

$$W(\beta) = W(\tilde{X}, \tilde{P}) \qquad |\psi(\tilde{X})|^2 = \int W(\tilde{X}, \tilde{P}) d\tilde{P}$$

Laboratory frame

Rotating frame

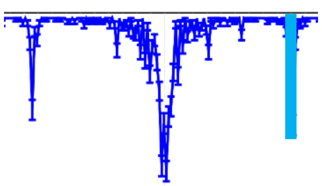


$$|\beta| = \frac{X_{\text{realspace}}}{2x_{\text{r.m.s.}}} \qquad x_{\text{r.m.s.}} \sim 7 \text{ nm}$$

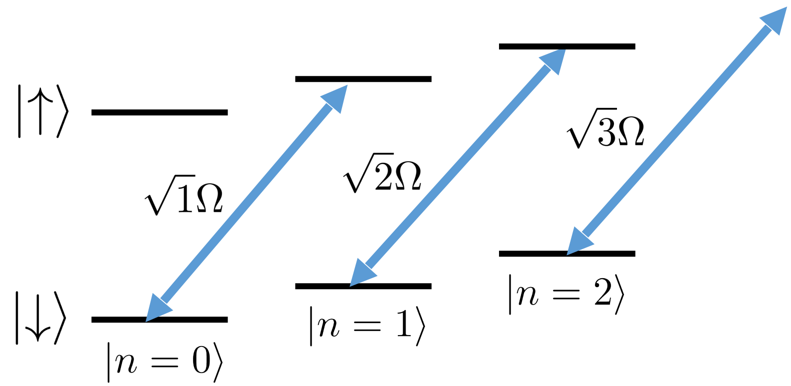
Energy eigenstate distribution measurement

Meekhof et al. PRL 76, 1796 (1996)

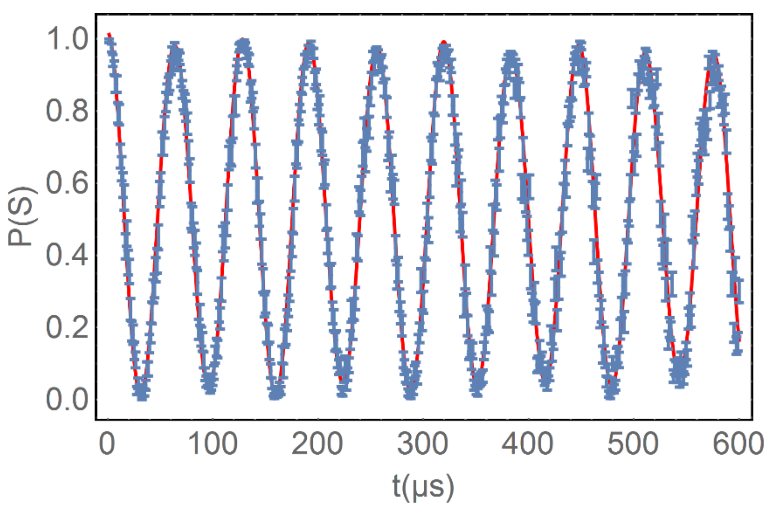
Anti-Jaynes-Cummings Hamiltonian



$$H_r = i\Omega(\sigma_+ a - \sigma_- a^\dagger)$$



Ground state – only one frequency component



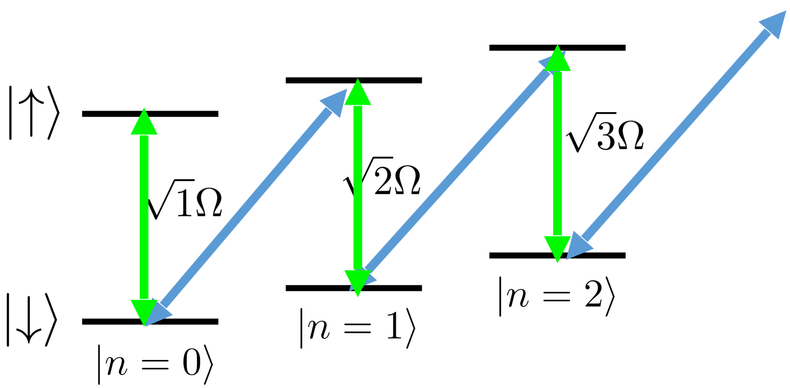
$$\bar{n} = 0.03$$

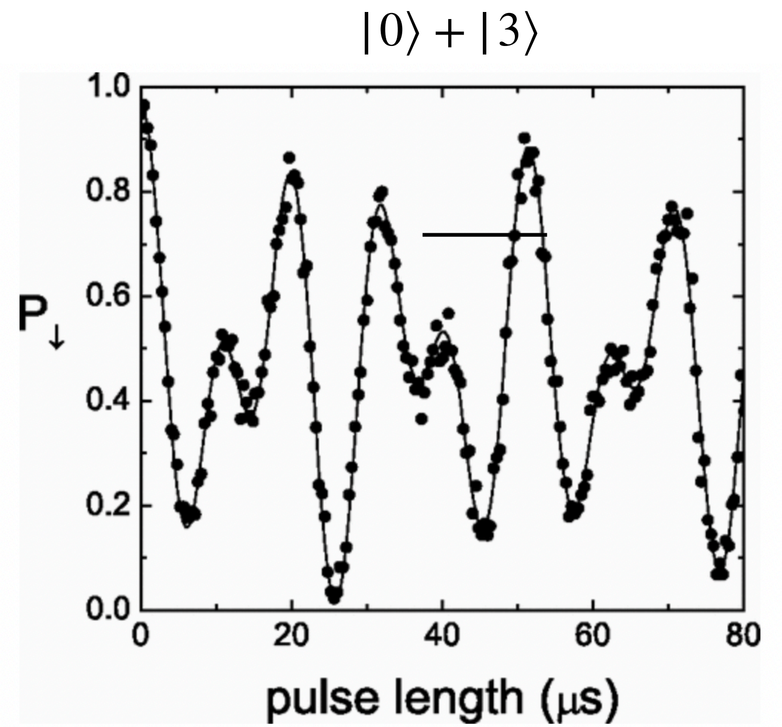
$$p(n=0) = 0.97$$

Superpositions of Fock states

Ben-Kish et. al. PRL **90**, 037902 (2003)

Sequences of appropriate length carrier and bsb (or rsb) can achieve arbitrary superpositions





K. McCormick et al. Nature 572, 86 (2019): Fock states up to $n = 100$, superpositions up to $|0\rangle + |18\rangle$

Electronic production of coherent states by "tickling"

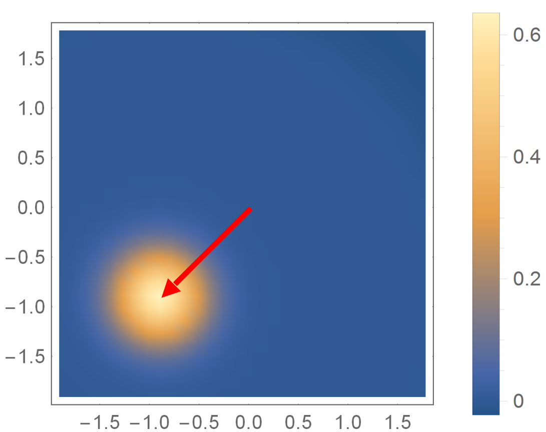
Meekhof et al. PRL 76, 1796 (1996)

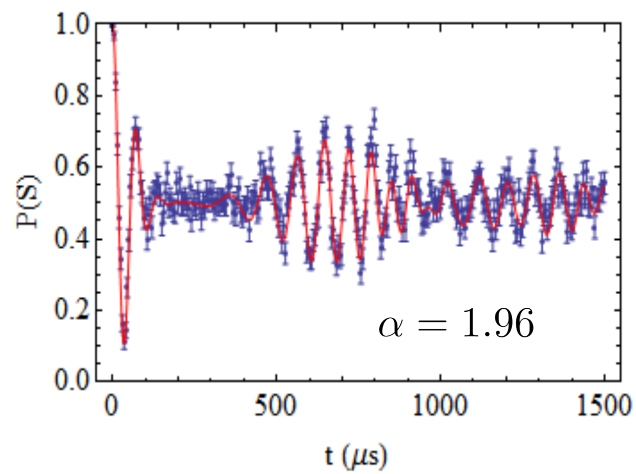
Electric potentials also allow production of interesting states

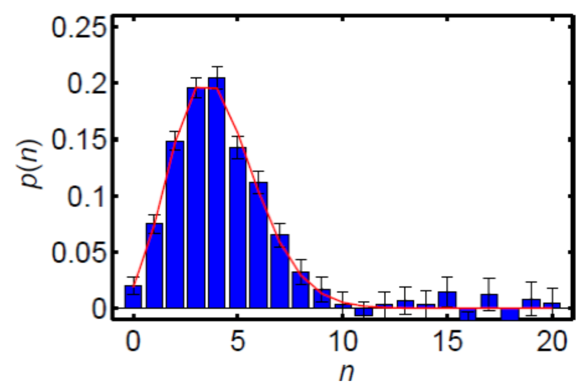
Oscillating electric field $H_{\text{tickle}} = eE\cos(\omega_t t)x$

$\omega_t = \omega$ resonant Hamiltonian is: $H_{\text{disp}} = \dfrac{eE}{2}x$ $\quad U(t) = D(ieEt/2) = e^{-i\frac{eEt}{2}(a+a^\dagger)}$

Displacement operator: generates coherent states from g.s.







Electronic production of squeezed states by parametric drive

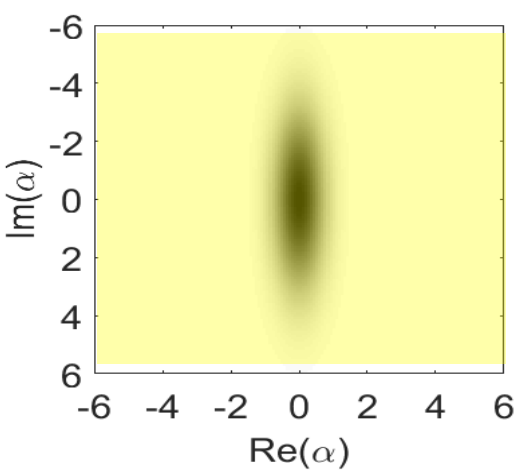
Meekhof et al. PRL 76, 1796 (1996), Verified + used in sensing S. Burd et al. Science 364, 6446 (2019)

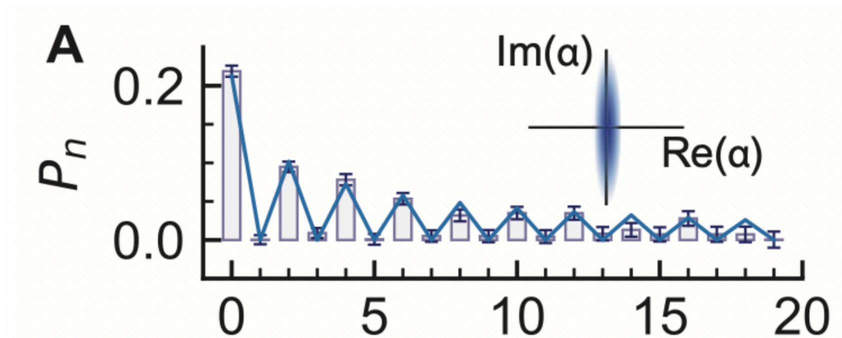
Modulate trap frequency $H_{\text{para}} = \frac{\hbar \eta_s \cos(\omega_s t)}{2} x^2$

$\omega_s = 2\omega$ resonant Hamiltonian is: $H_{\text{sq}} = \frac{\hbar \eta_s}{4} \left(a^2 + (a^\dagger)^2 \right)$

$$U(t) = S(i\eta_m t/4) = e^{-i\frac{\eta_s t}{4}(a^2 + (a^\dagger)^2)}$$

Squeezing operator: generates squeezed states from g.s.

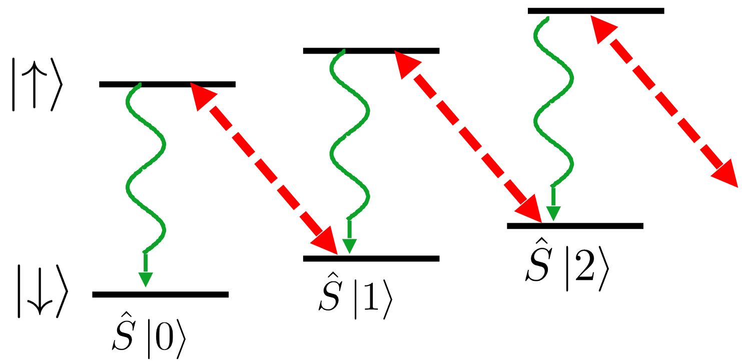




Burd et al: 20 dB reduction in quadrature variance

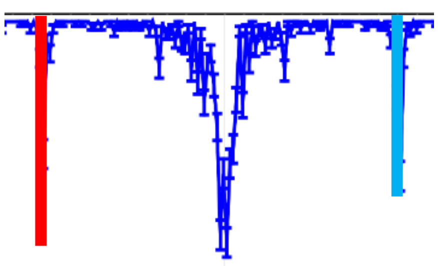
Production of squeezed states by Reservoir engineering

Proposal: Cirac et al. PRL 70, 556 (1993), Expt: D. Kienzler et al. Science 347, 6217 (2015)

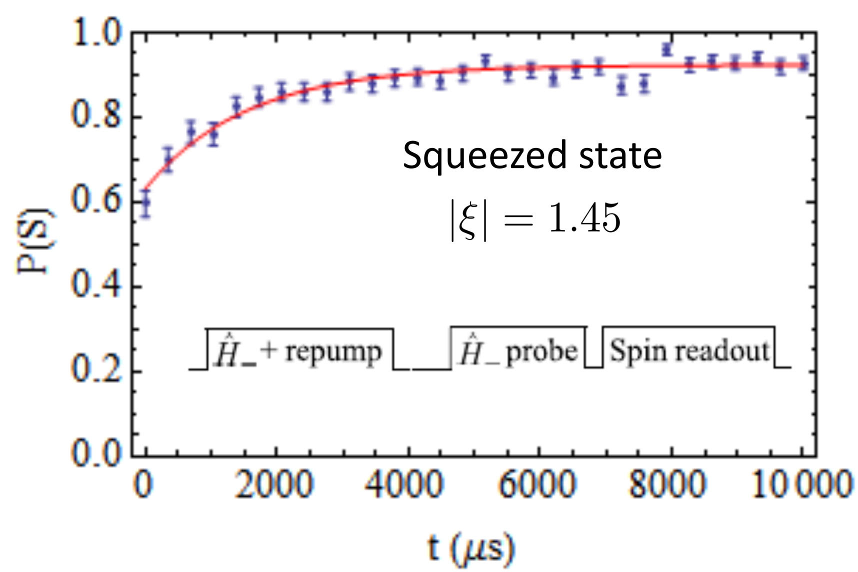


$$\hat{H}_- = \Omega\left(\hat{K}\sigma_+ + \hat{K}^\dagger\sigma_-\right)$$

$$\hat{K} = \hat{S}\,\hat{a}\,\hat{S}^\dagger = \mu\,\hat{a} + e^{i\phi}\nu\,\hat{a}^\dagger$$

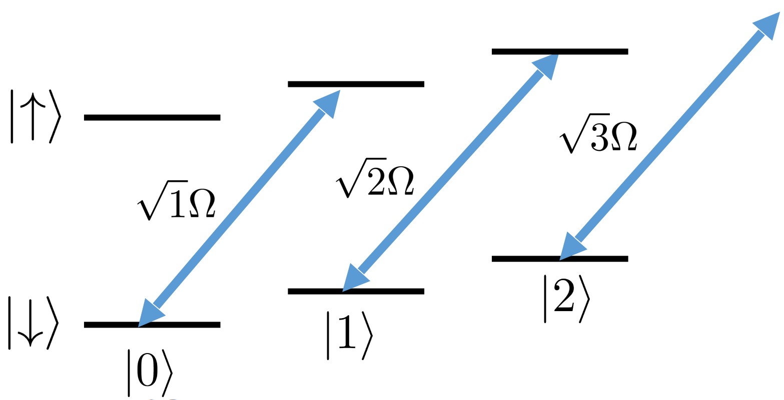


$$K\sigma_+ = \underbrace{\mu a\sigma_+}_{\text{rsb term}} + \underbrace{e^{i\phi}\nu a^\dagger\sigma_+}_{\text{bsb term}}$$

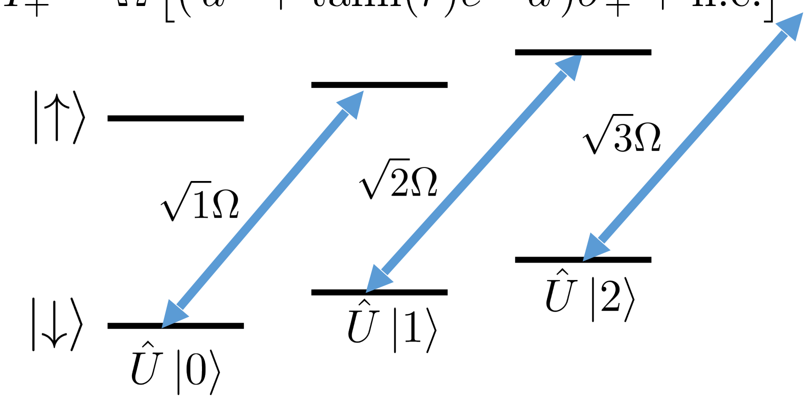


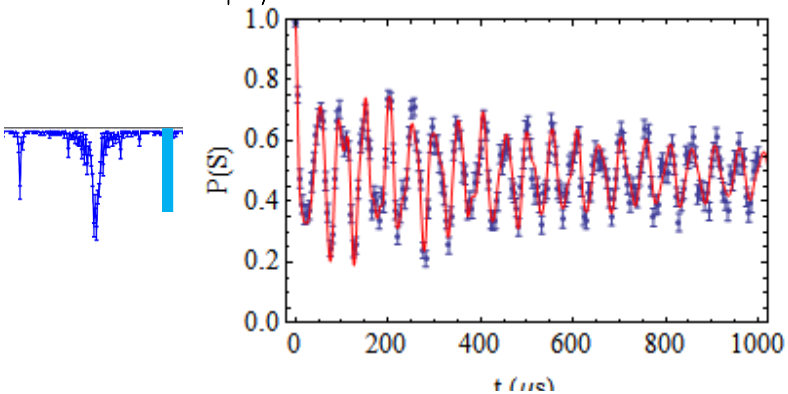
Analysis of quantum states in a squeezed Fock basis

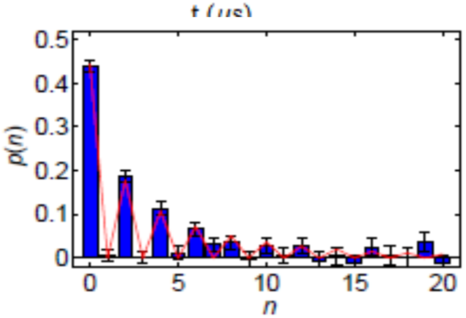
$$\hat{H}_+ = \Omega \left[\hat{a}^\dagger \sigma_+ + \mathrm{h.c.} \right]$$

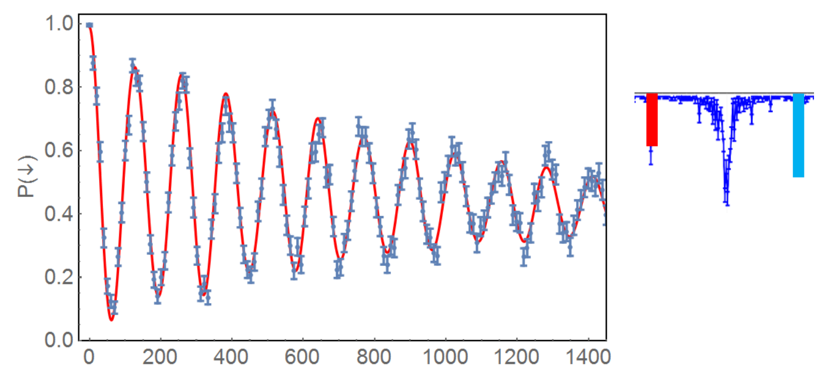


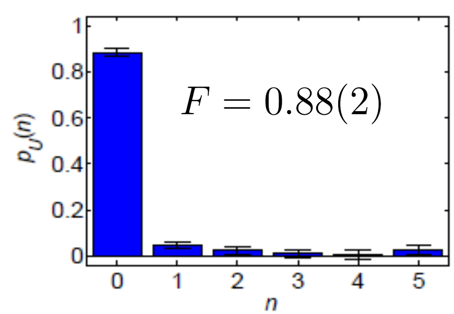
$$\hat{H}_+ = \Omega \left[(\hat{a}^\dagger + \tanh(r) e^{i\phi} \hat{a}) \sigma_+ + \mathrm{h.c.} \right]$$







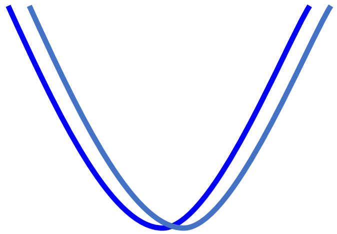




Noise: motional heating

Noisy electric field $E(t)$, with short correlation time

Noisy displacement of the Potential



Fermi golden rule $$\Gamma_{0\to 1} = \frac{e^2}{4m\hbar\omega} S_E(\omega)$$

Noise spectral density

Possible sources

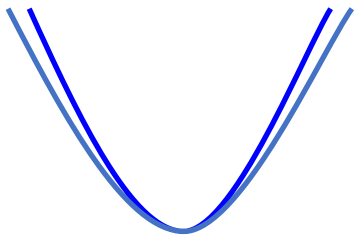
1) Technical: can be suppressed to negligible levels with some work
2) Johnson noise: resistances (much too small to explain observed heating)
3) Surface noise (fluctuating charges or dipoles).

   Surface treatments observed to help but still poorly understood

Noise: dephasing

Trap frequency fluctuations

Noise in the curvature of the potential



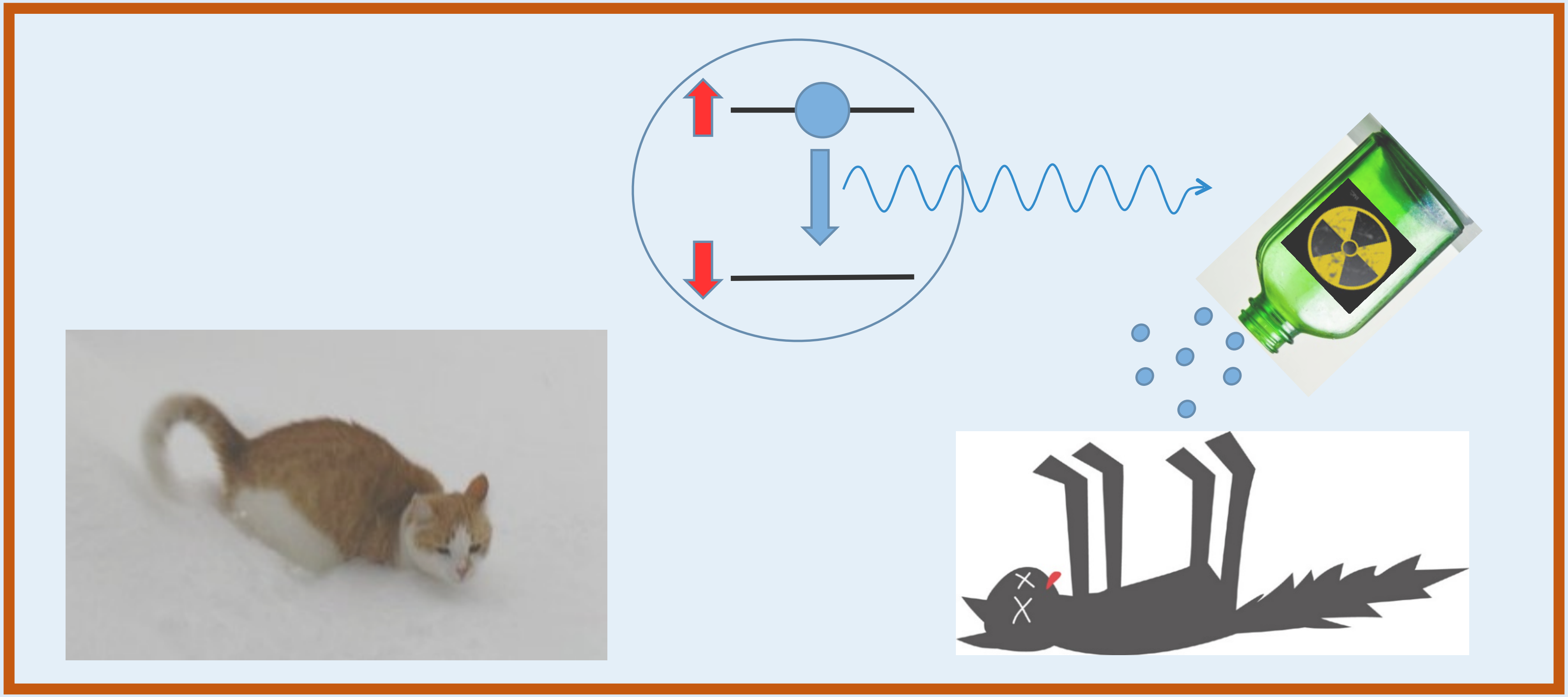
Frequency components: at 2ω - parametric heating: usually insignificant

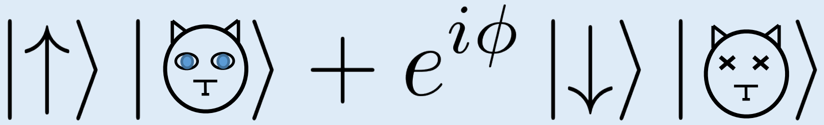
More problematic: slow timescale (technical) noise eg. 50 Hz

$$\phi_{\mathrm{osc}}(t) = \int_0^t \omega(t')dt'$$

Dominates in many experiments with large oscillator states

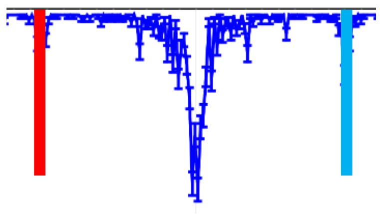
State-dependent forces and Schroedinger's cat





Entangled state of a spin and a "macroscopic" system

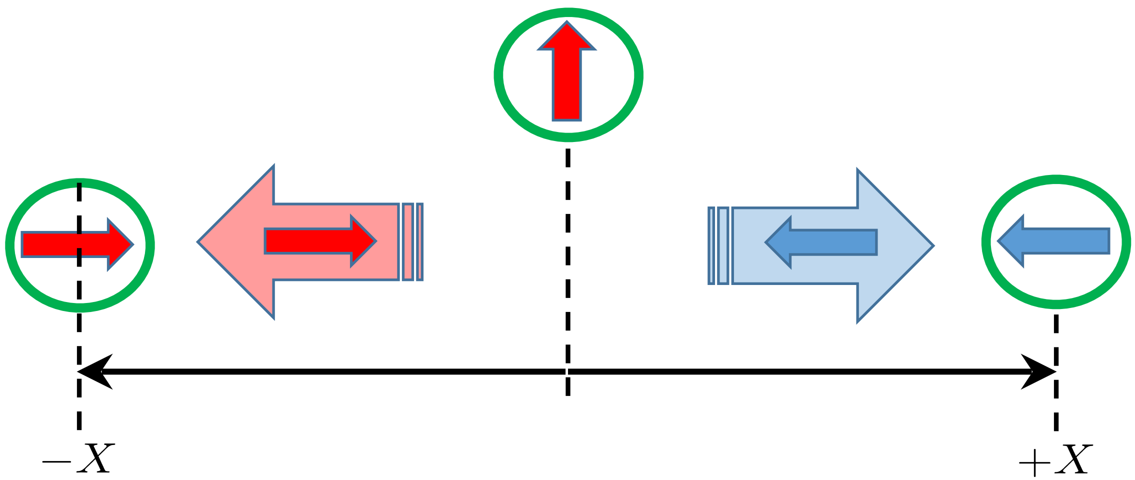
Optical state-dependent force



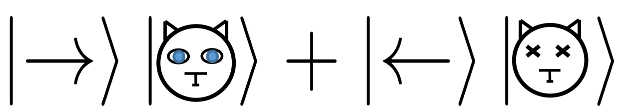
Equally driven sidebands $\hat{H}_I = F_0 X \sigma_x$

$$U(t) = e^{-i\frac{F_0 t}{\hbar} X \sigma_x} = D(\alpha_X(t)\sigma_x)$$

Before $|\uparrow\rangle = |\rightarrow_x\rangle - |\leftarrow_x\rangle$

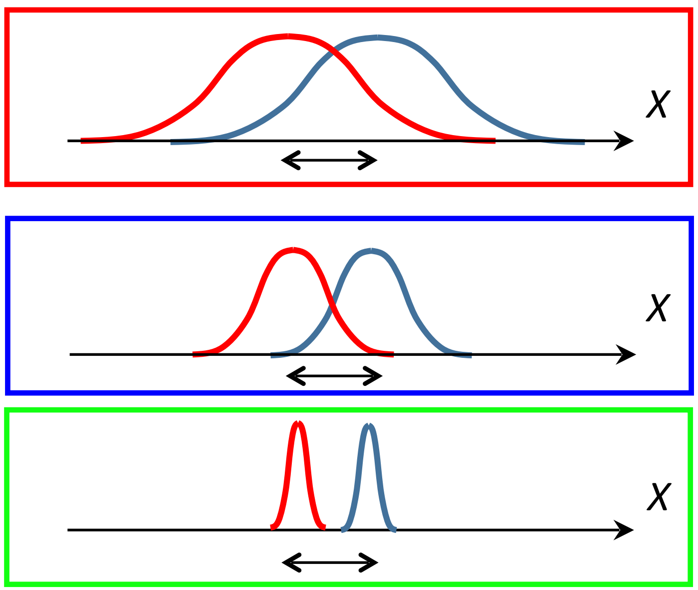


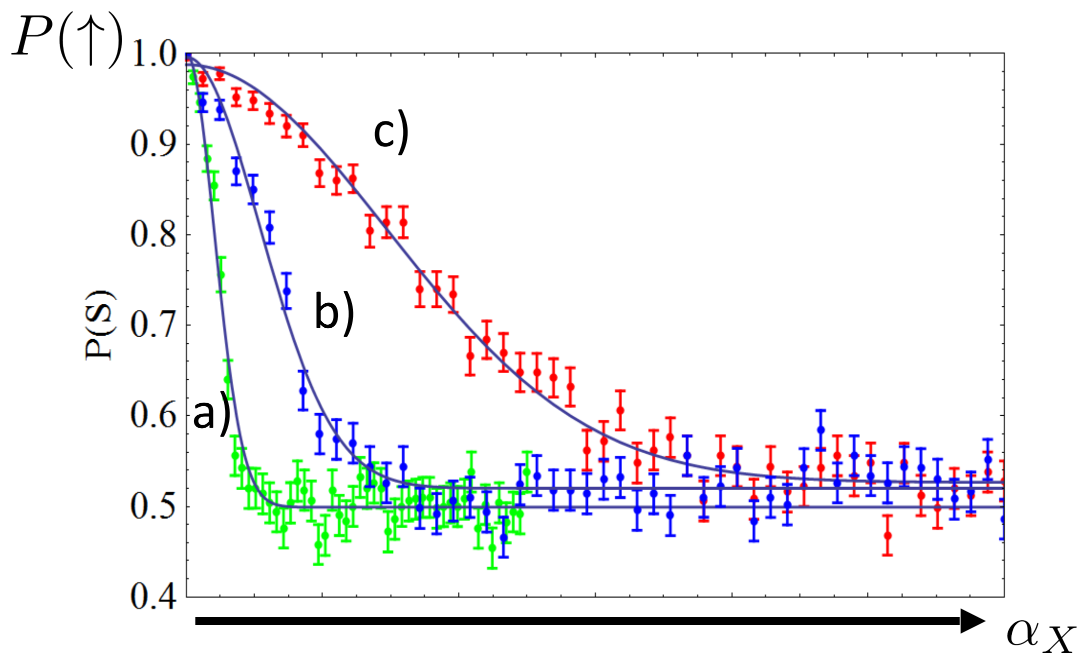
After $|\rightarrow\rangle\,|-\alpha_X\rangle + |\leftarrow\rangle\,|+\alpha_X\rangle$



Spin measurements of the oscillator

$$|\uparrow\rangle = |\rightarrow\rangle - |\leftarrow\rangle \quad \Rightarrow \quad |\rightarrow\rangle\,|{-\alpha_X}\rangle + e^{i\phi}\,|\leftarrow\rangle\,|{+\alpha_X}\rangle$$

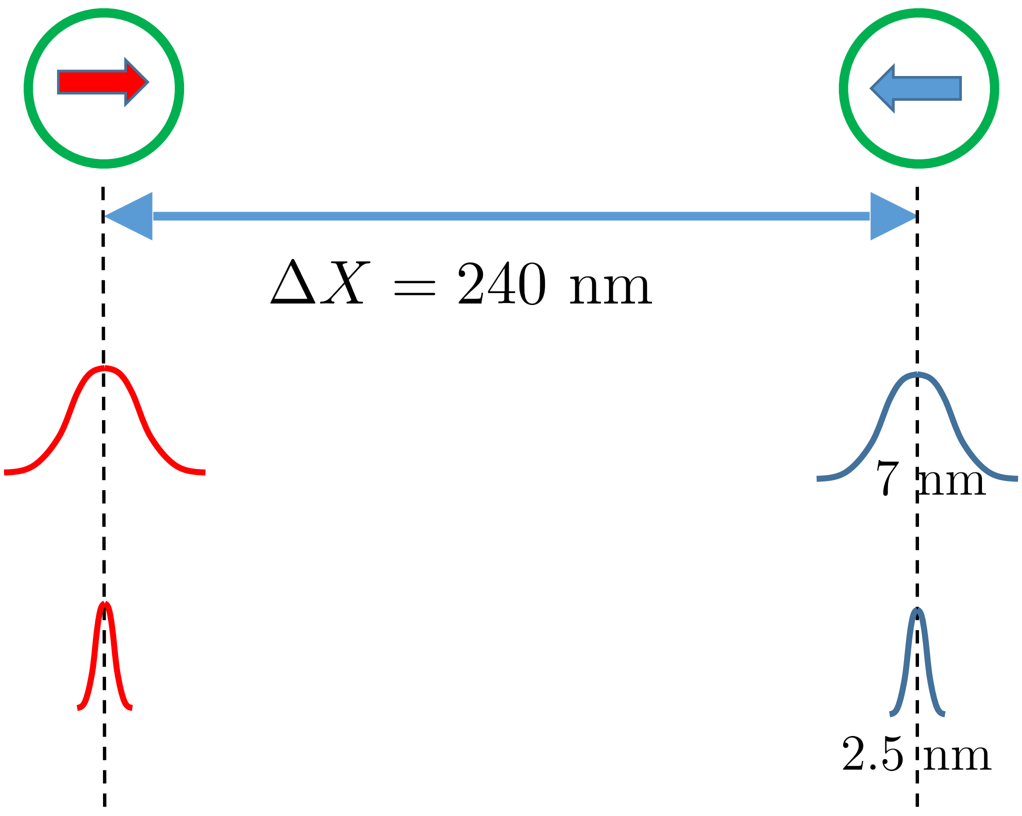




"Spin-motion entanglement and state diagnosis with squeezed oscillator wavepackets"
H-Y Lo. et al., **Nature 521**, 336 (2015)

How big is the "cat"?

$$|\rightarrow\rangle\,|-\alpha_X\rangle + |\leftarrow\rangle\,|+\alpha_X\rangle$$



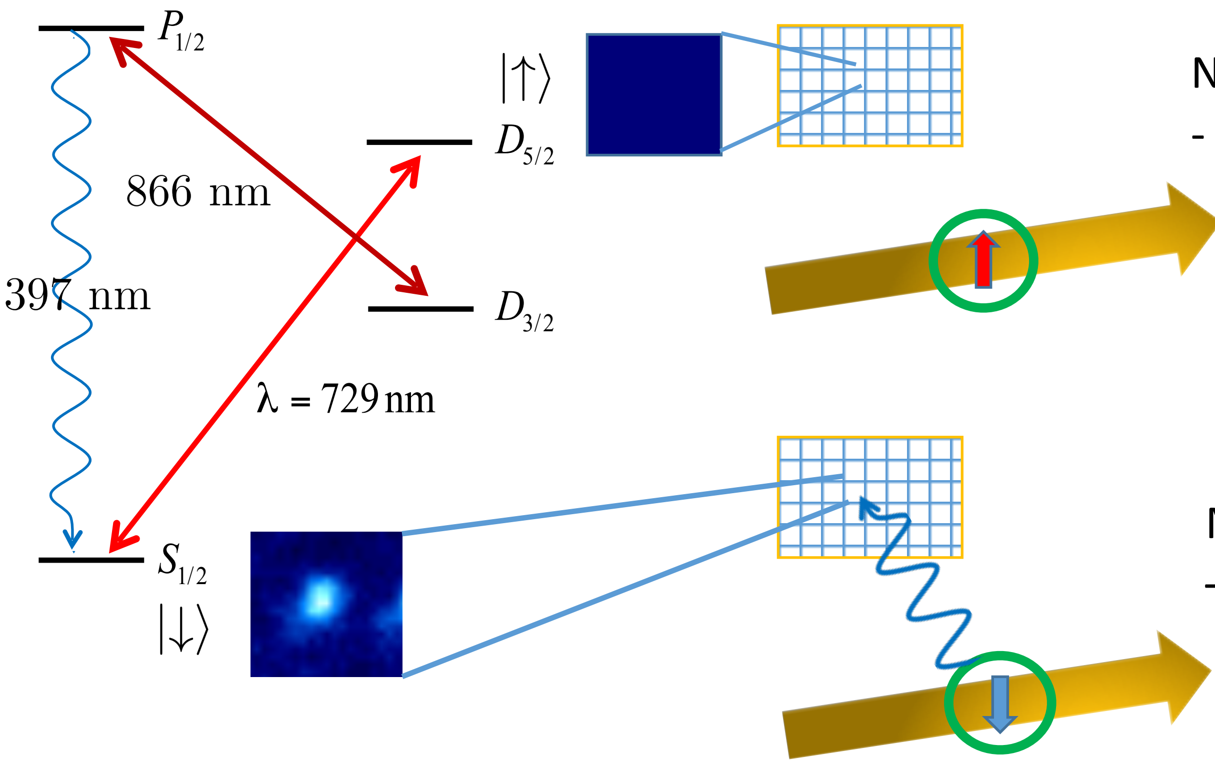
$$P(-\alpha_X \& \leftarrow) \sim 10^{-52}$$

$$P(-\alpha_X \& \leftarrow) \sim 10^{-500}$$

Distinguishable – but how do we verify "quantum"?

Internal state projection

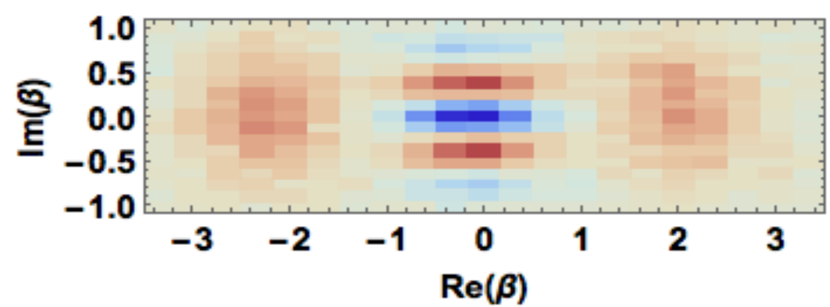
D. Kienzler et al. **Phys. Rev. Lett. 116**, 080502 (2016)



No recoil
- Ideal quantum measurement

Many recoils
– destroys oscillator state

Post-select on “dark” measurement
- previously used in Schrödinger’s cat experiments



Even and odd "cats"

Daniel Kienzler, C. Flühmann, V. Negnevitsky,

$$|+\rangle\,|\alpha\rangle + |-\rangle\,|-\alpha\rangle = \boxed{|\uparrow\rangle\,(|\alpha\rangle + |-\alpha\rangle)} + \boxed{|\downarrow\rangle\,(|\alpha\rangle - |-\alpha\rangle)}$$

No post-selection – repump spin

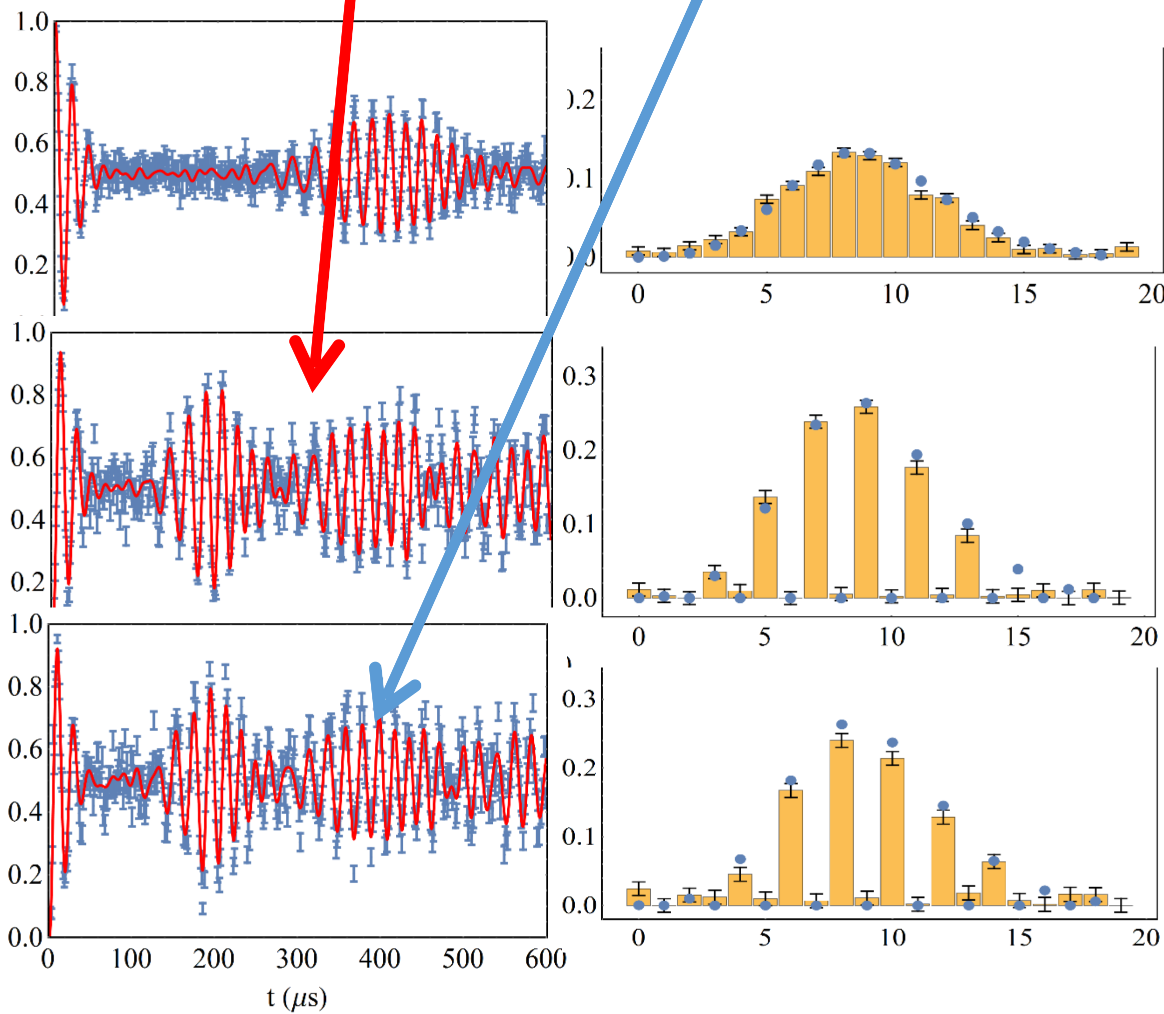
$$\hat{\rho} = \frac{1}{2}(|\alpha\rangle\,\langle\alpha| + |-\alpha\rangle\,\langle-\alpha|)$$

Dark detection

$$|\psi_{\mathrm{even}}\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle)$$

Flip spin then dark detection

$$|\psi_{\mathrm{odd}}\rangle = \frac{1}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle)$$

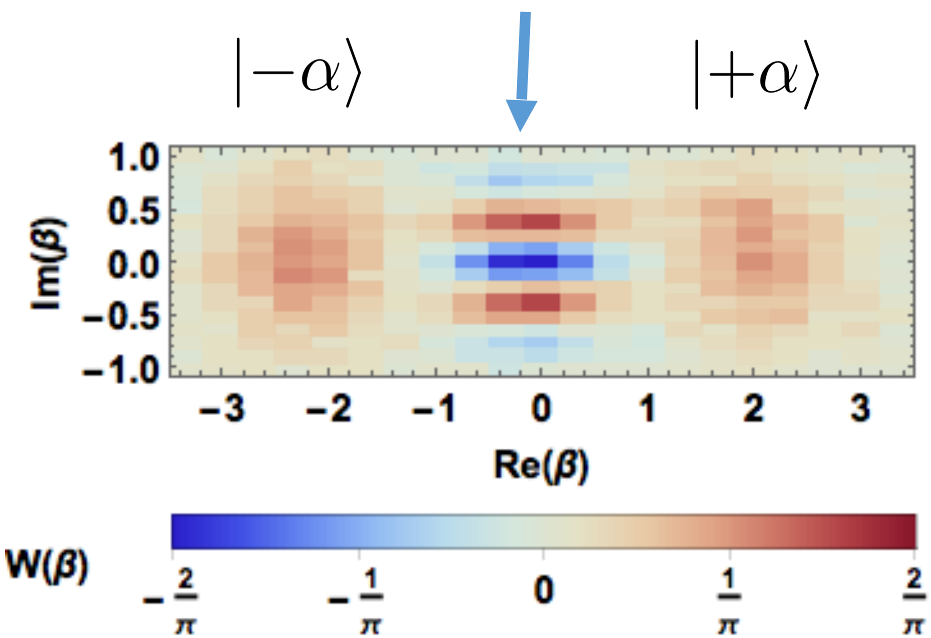


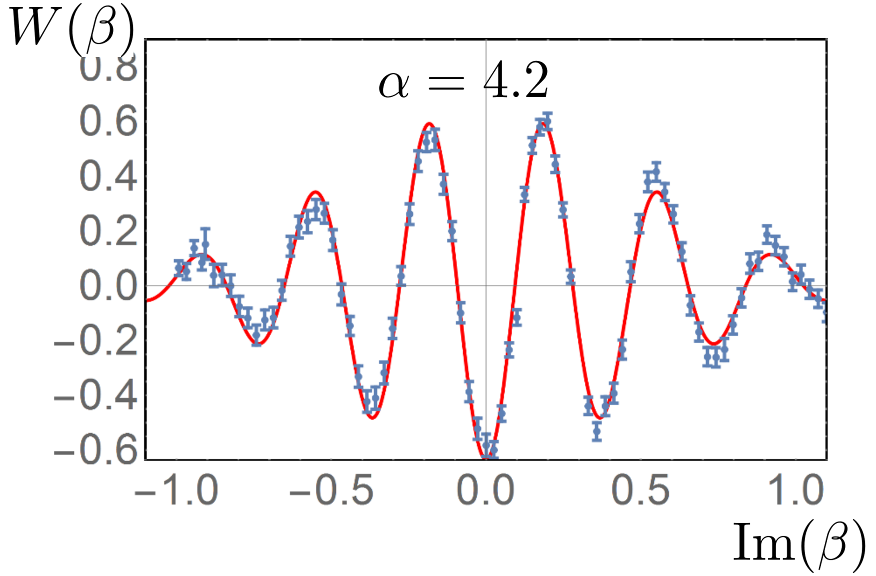
Wigner function reconstruction

Daniel Kienzler, Christa Flühmann

$$W(\beta) = \frac{2}{\pi}\langle \hat{P}(\beta)\rangle = \frac{2}{\pi}\sum_n (-1)^n p_n(\beta)$$

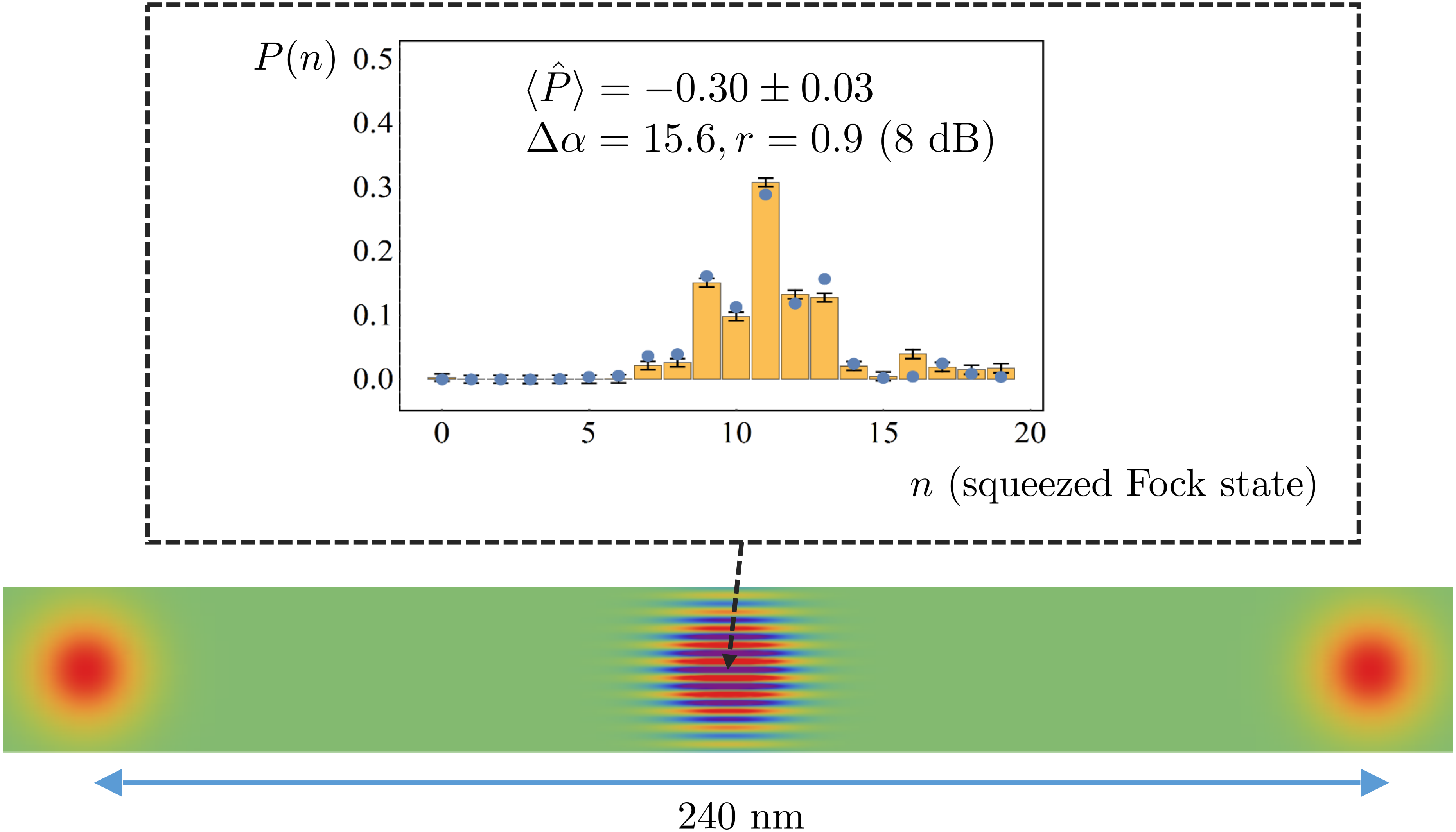
Interference
– quantum phase, negative values





“Largest” cat interference

D. Kienzler et al. **Phys. Rev. Lett. 116**, 080502 (2016)

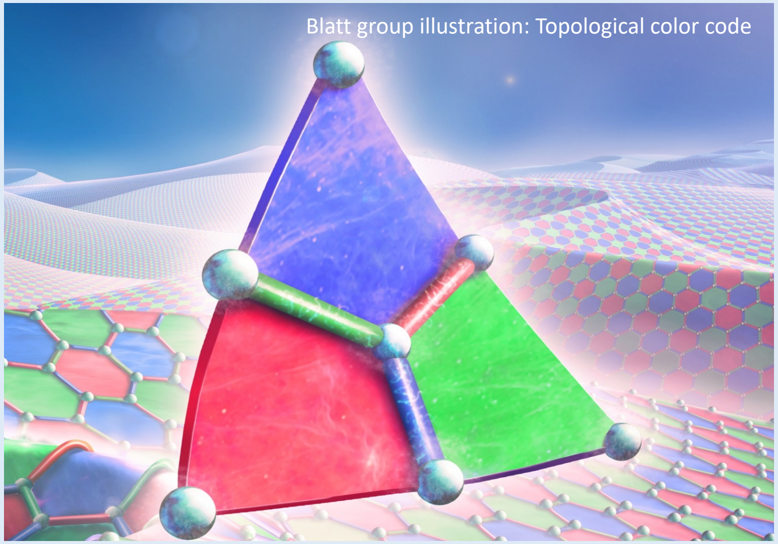


Approaches to quantum error-correction

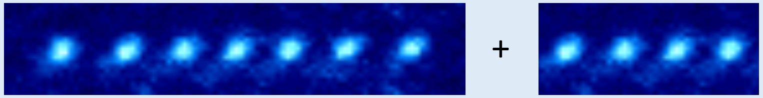
Give information a different footprint to errors

Qubit codes:

- Delocalised entangled states of many qubits
- Errors low prob. + local



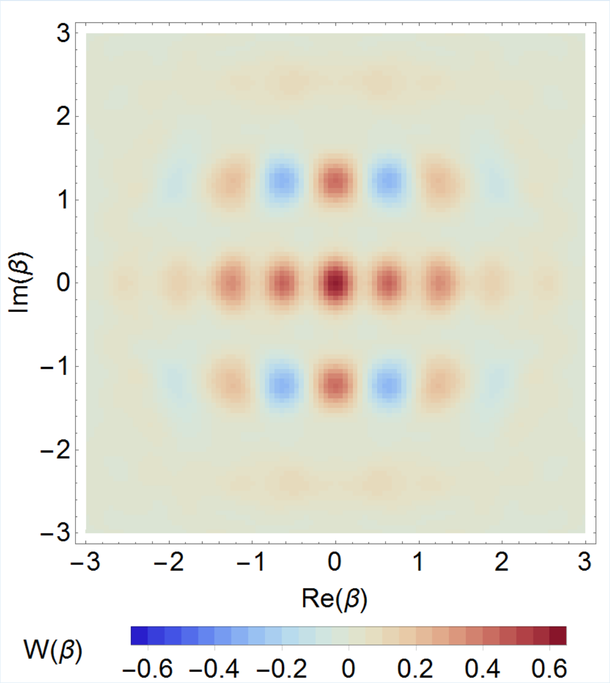
Steane code: 7 data + 4 ancilla @ high fidelity



"End game" for quantum computing
- gates using integrated photonics: end of talk

Bosonic codes:

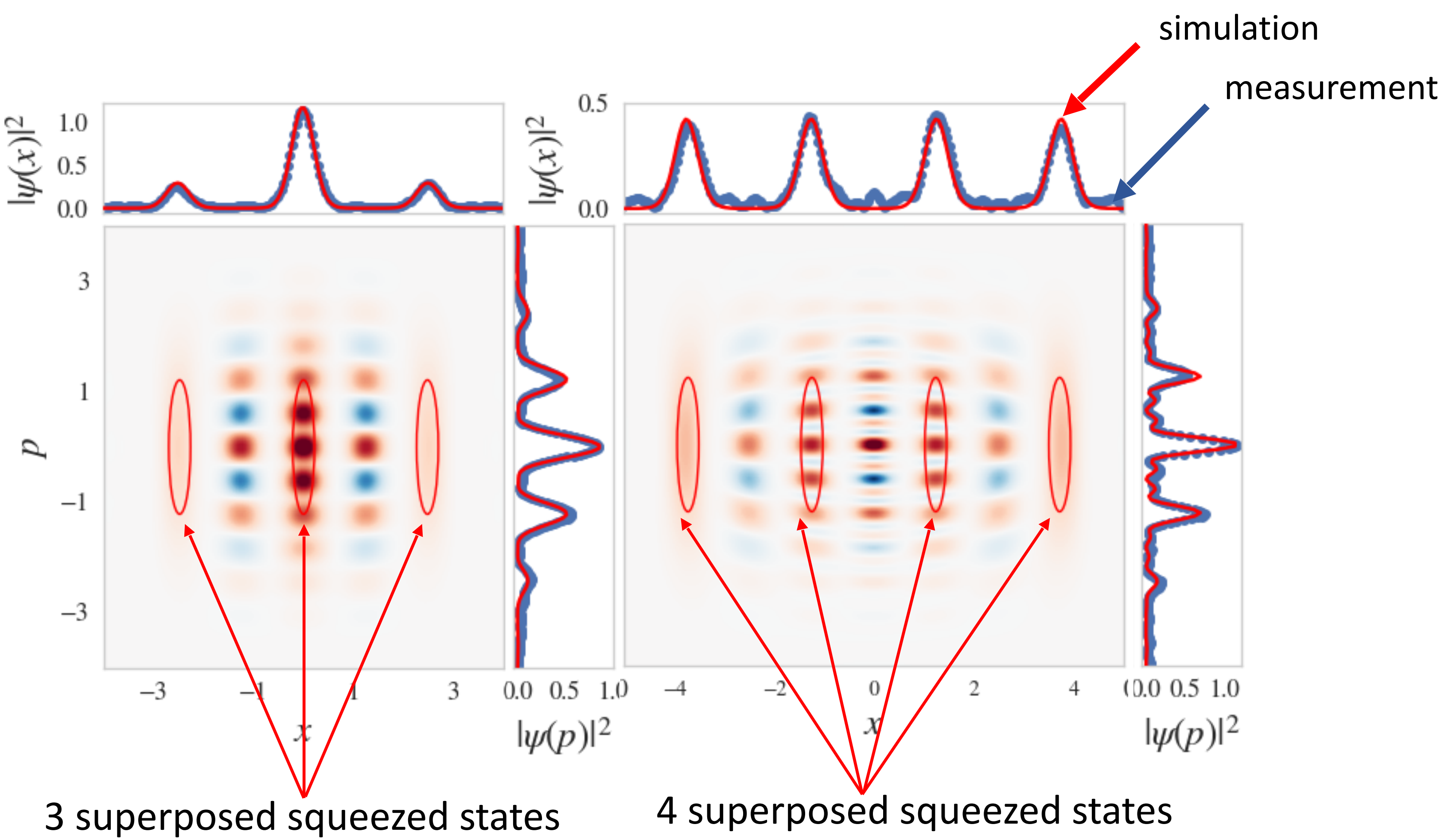
- Delocalized states of single oscillators
- Errors deterministic, small compared to delocalisation



"Base layer" for a quantum computer
- implementation in single ion: main topic

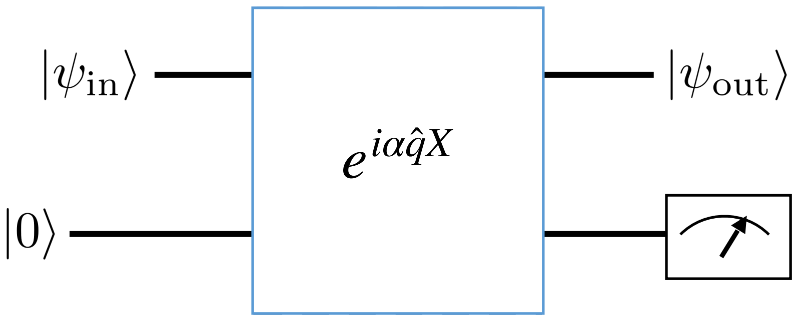
Experimental approximate GKP states

Squeezed state generation (8-10 dB): D. Kienzler et al. Science 347, 6217 (2015)
Repeated SDF + measurement: C. Flühmann et al. PRX 8, 021001, (2018)
Preparation method: B. Terhal, D. Weigand, PRA 93, 012315 (2016)



Measurement of displacements + modular variables

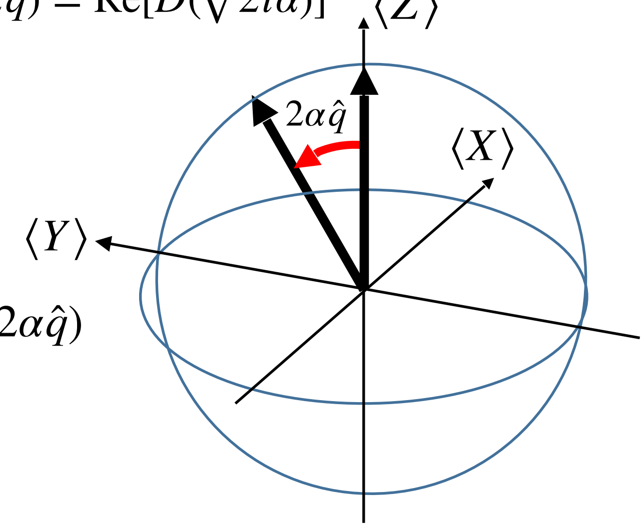
Expt: C. Flühmann et al. PRX 8, 021001, (2018),
Theory: A. Asadian et al. PRL 112, 190402 (2014)



$$\langle\sigma\rangle = \langle\psi_{\rm in}|\hat{Q}|\psi_{\rm in}\rangle$$

$$\hat{Q}_Z = \cos(2\alpha\hat{q}) = \mathrm{Re}[D(\sqrt{2}i\alpha)]$$

$$\hat{Q}_Y = \sin(2\alpha\hat{q})$$

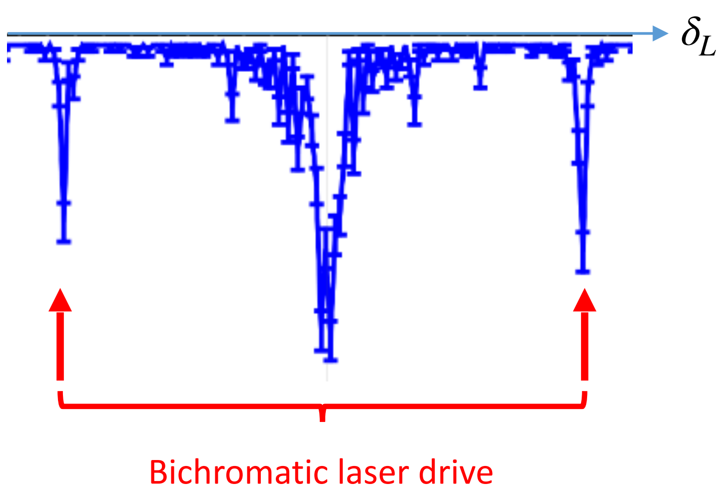


Coherent rotation only if motional wavepackets overlap!

GKP: $2\alpha\hat{q} \to 2\pi m + \delta q$

State-dependent force

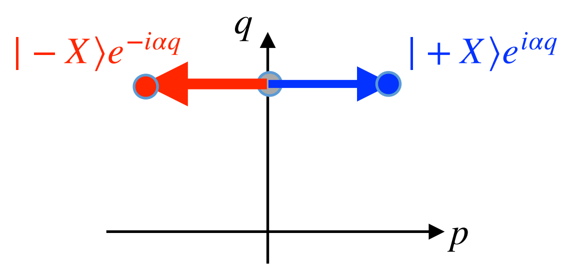
$$\hat{H}_{\rm SDF} = \hbar\Omega(\hat{a}^\dagger + \hat{a})X \Rightarrow \hat{U} = D(i\alpha X/\sqrt{2}) = e^{i\alpha\hat{q}X}$$

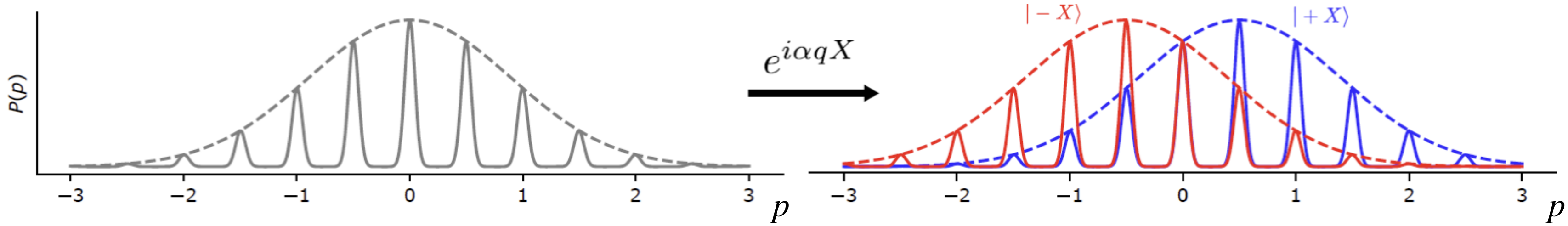


Infinite operator readout, finite GKP code

A measurement of q involves displacements along p

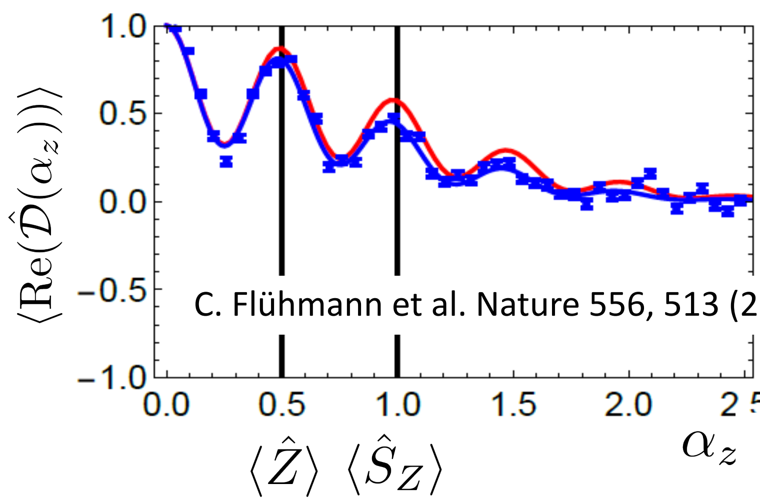
$e^{i\alpha\hat{q}X}$ in phase space





Loss of overlap

1. Degrades readout result
2. Enlarges envelope of finite state
   (diffusion on grid increases energy)

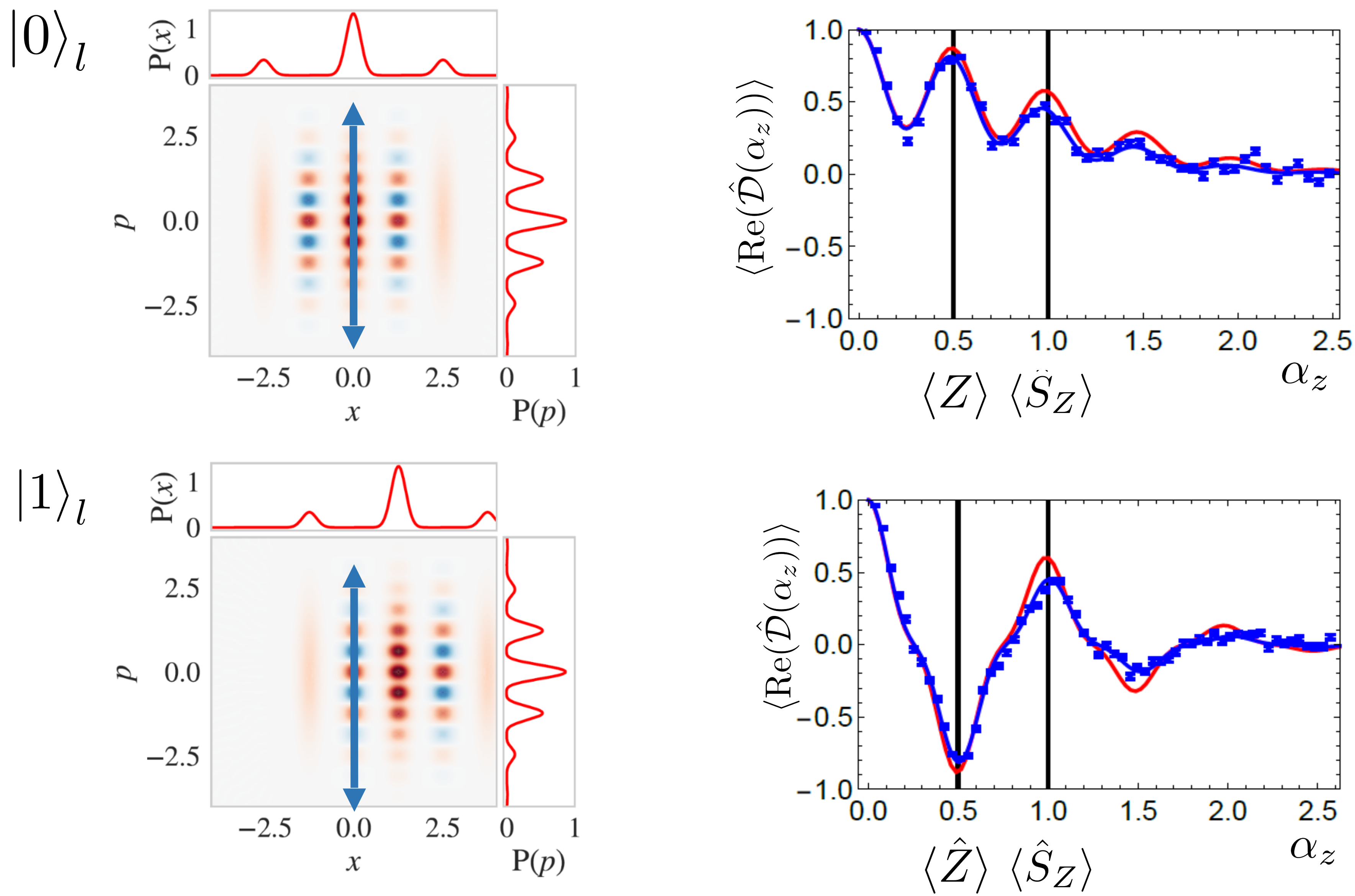


C. Flühmann et al. Nature 556, 513 (2019)

Code states (finite) are not eigenstates of observables (infinite)

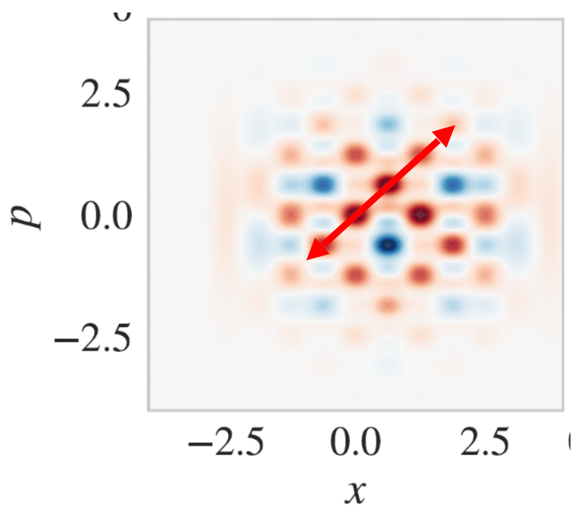
Modular variable measurement: GKP states

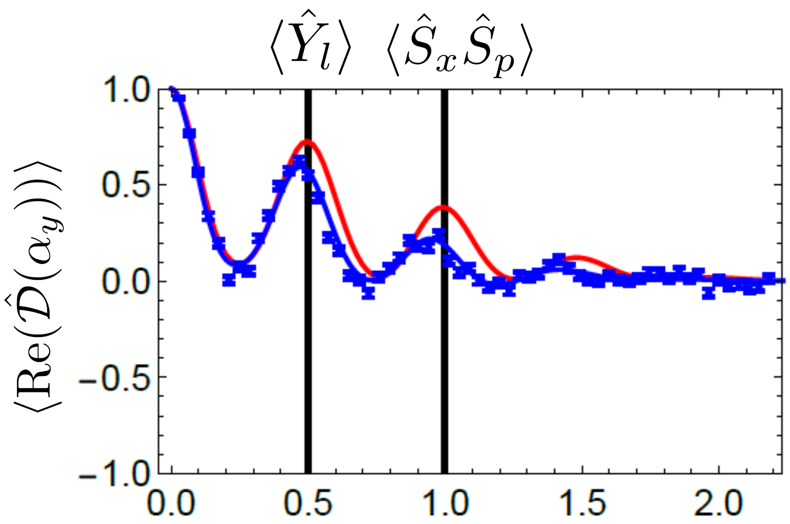
C. Flühmann et al. Nature 556, 513 (2019)

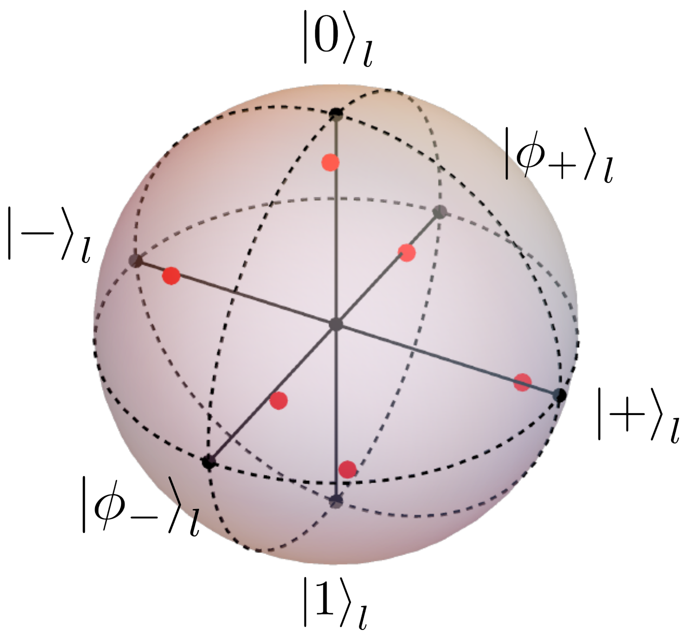


State tomography: X, Y, Z eigenstates

$|Y+\rangle$







$$\langle F \rangle = 87.8 \pm 0.3\%$$

$$F \equiv \langle \phi | \hat{\sigma} | \phi \rangle$$

Pauli gates – displacements in phase space

C. Flühmann et al. Nature 556, 513 (2019)

Oscillating drive to a trap electrode $\hat{H}_D = eE_0 \cos(\omega t)\left(\hat{a}^\dagger e^{i\phi} + \hat{a}\, e^{-i\phi}\right)$

$$S(\rho) = \sum_{i,j} \chi_{ij} \hat{P}_i \rho \hat{P}_j$$

Pauli matrix

	I	X	Y	Z
Z	0.0066	−0.0237	0.0061	0.0064
Y	0.0065	0.0024	0.0067	0.0061
X	−0.0007	0.9747	0.0024	−0.0237
I	0.0121	−0.0007	0.0065	0.0066

$\mathrm{F}_{\hat{X}_l} = 97.47\%$

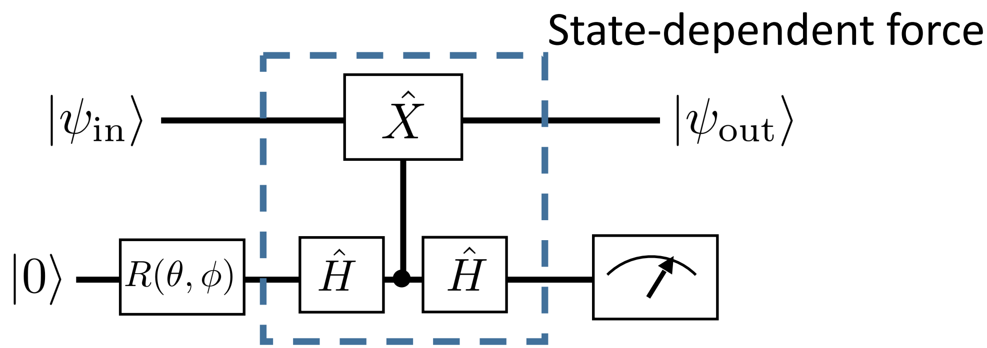
	I	X	Y	Z
Z	0.0112	0.0020	0.0114	0.0102
Y	−0.0073	0.0064	0.9713	0.0114
X	0.0022	0.0057	0.0064	0.0020
I	0.0128	0.0022	−0.0073	0.0112

$\mathrm{F}_{\hat{Y}_l} = 97.13\%$

	I	X	Y	Z
Z	−0.0030	0.0078	−0.0190	0.9659
Y	0.0040	−0.0050	0.0059	−0.0190
X	0.0023	0.0060	−0.0050	0.0078
I	0.0222	0.0023	0.0040	−0.0030

$\mathrm{F}_{\hat{Z}_l} = 96.59\%$

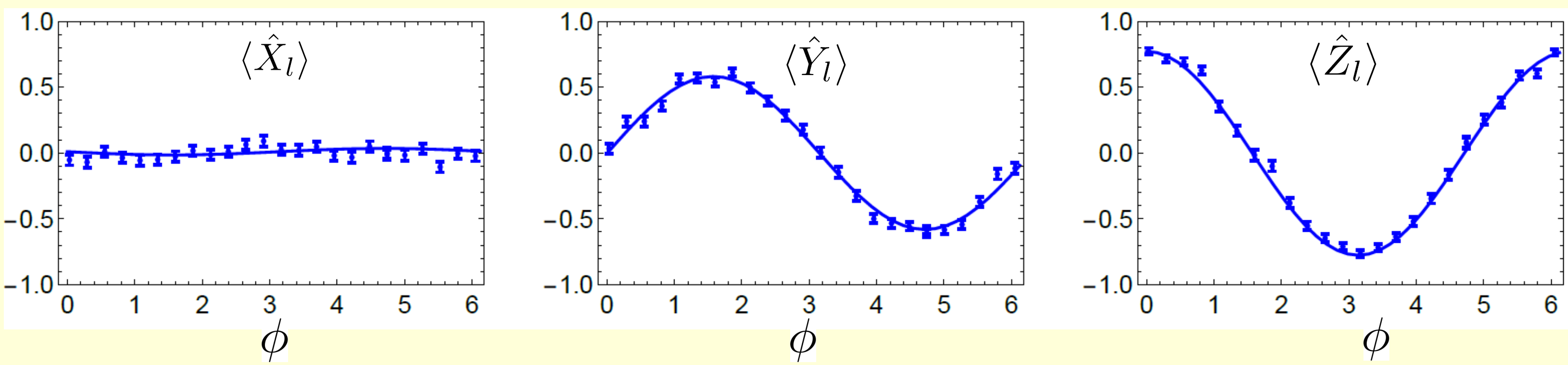
Non-Clifford (universal) – gate teleportation



$$\hat{U}_L = \cos(\theta/2)\,|+X\rangle\langle+X| + \sin(\theta/2)e^{i\phi}\,|-X\rangle\langle-X|$$

$\theta = \pi/2$ - Unitary operation (Rabi oscillation)

C. Flühmann et al. Nature 556, 513 (2019)



$$\hat{T} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

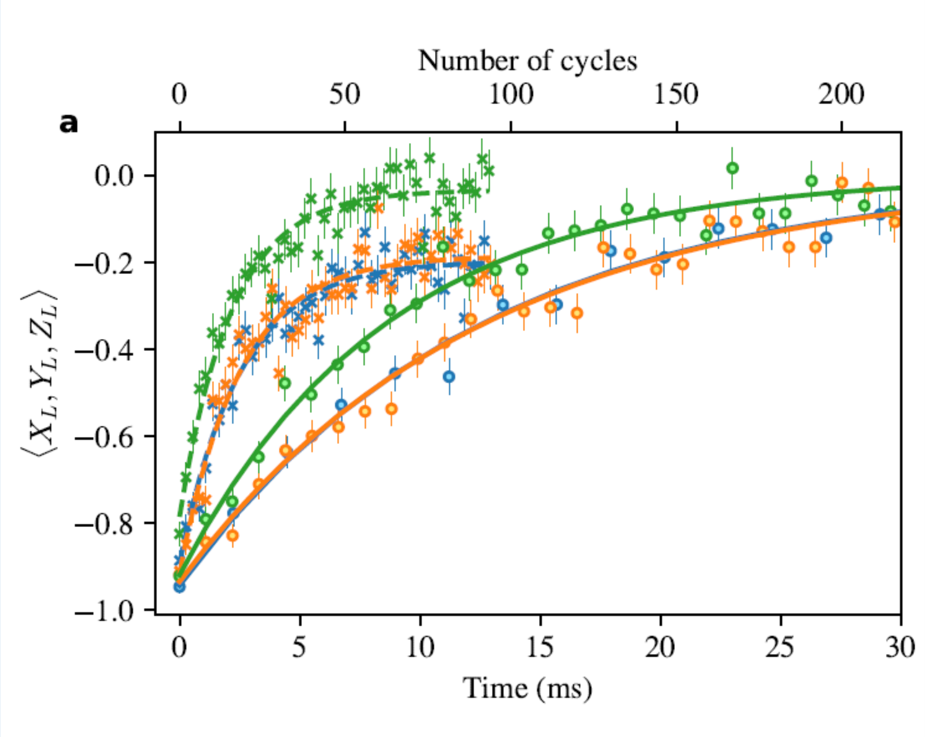
Process Fidelity $F = 92\%$.

Simulations including dephasing, finite size etc. 94%

Quantum error-correction: extension of logical coherence

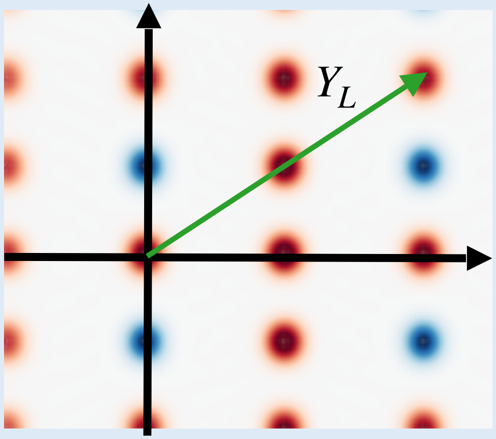
B. deNeeve, T-L. Nguyen, et al. arXiv:2010.09681 (2020)

Autonomous quantum error correction + finite GKP code operators



No correction $\tau = (2.5,\ 2.2,\ 2.5)$ ms

With correction $\tau = (12.6,\ 8.6,\ 12.3)$ ms



- sensitive to finite extent
- sensitive to dephasing (primary error source)

Factor 3.4 increase in logical coherence over uncorrected encoded qubit

Multiple trapped-ion oscillators

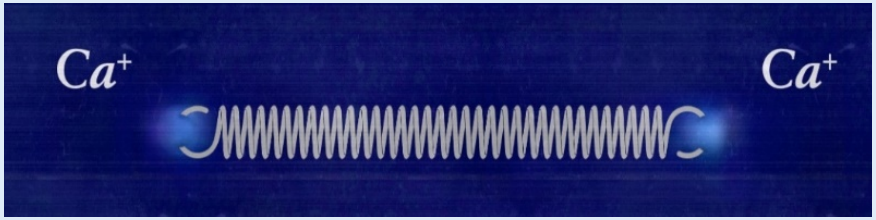
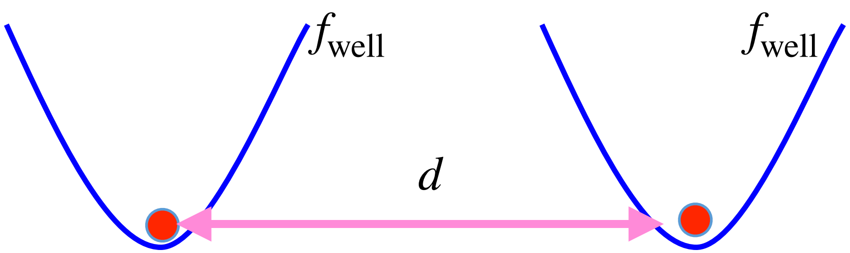


Image: T. Monz, R. Blatt, U. Innsbruck

Multiple oscillators

Two relevant pictures: both correct descriptions

Local oscillators with dynamical exchange



These will exchange energy with frequency

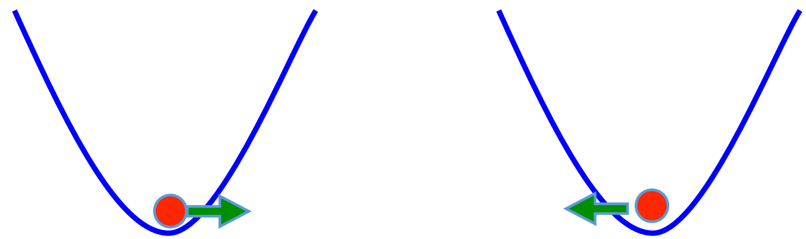
$$f_{\mathrm{exchange}} = \frac{1}{2\pi}\frac{e^2}{(2\pi)^2\epsilon_0 f_{\mathrm{well}} m d^3}$$

Can be considered as coupling Hamiltonian

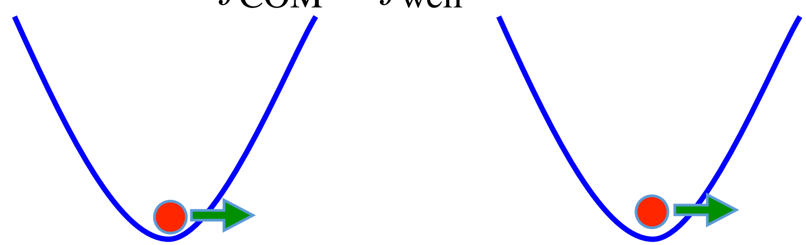
$$H = \frac{h f_{\mathrm{exchange}}}{2}(ab^\dagger + a^\dagger b)$$

Static normal modes

Stretch: $f_{\mathrm{STR}} = f_{\mathrm{COM}} + f_{\mathrm{exchange}}$



COM: $f_{\mathrm{COM}} = f_{\mathrm{well}}$

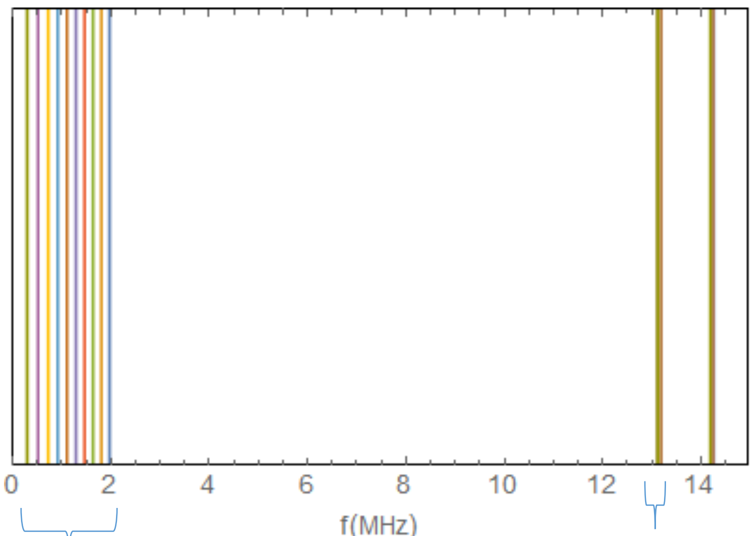


Linear chains + multiple oscillator modes

Mode frequencies: Be ions $f_x = 13.2\ \mathrm{MHz}, f_y = 14.2\ \mathrm{MHz}$

Two different axial trap frequencies: set the **distance** between the 10 ions

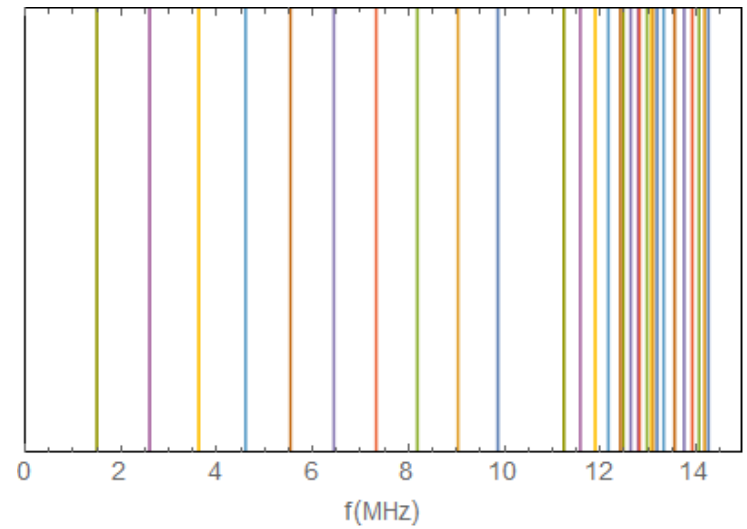
$f_z = 300\ \mathrm{kHz}$



$\sim N_{\mathrm{ions}} f_z$

$\sim f_{\mathrm{exchange}}$

$f_z = 1.5\ \mathrm{MHz}$



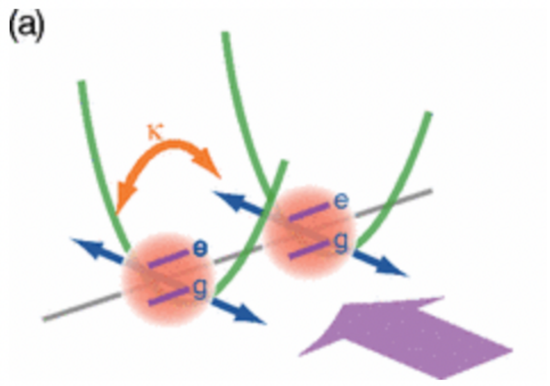
$$f_{\mathrm{exchange}} = \frac{1}{2\pi} \frac{e^2}{2\pi\epsilon_0 \omega_\alpha m_{\mathrm{ion}} d^3}$$

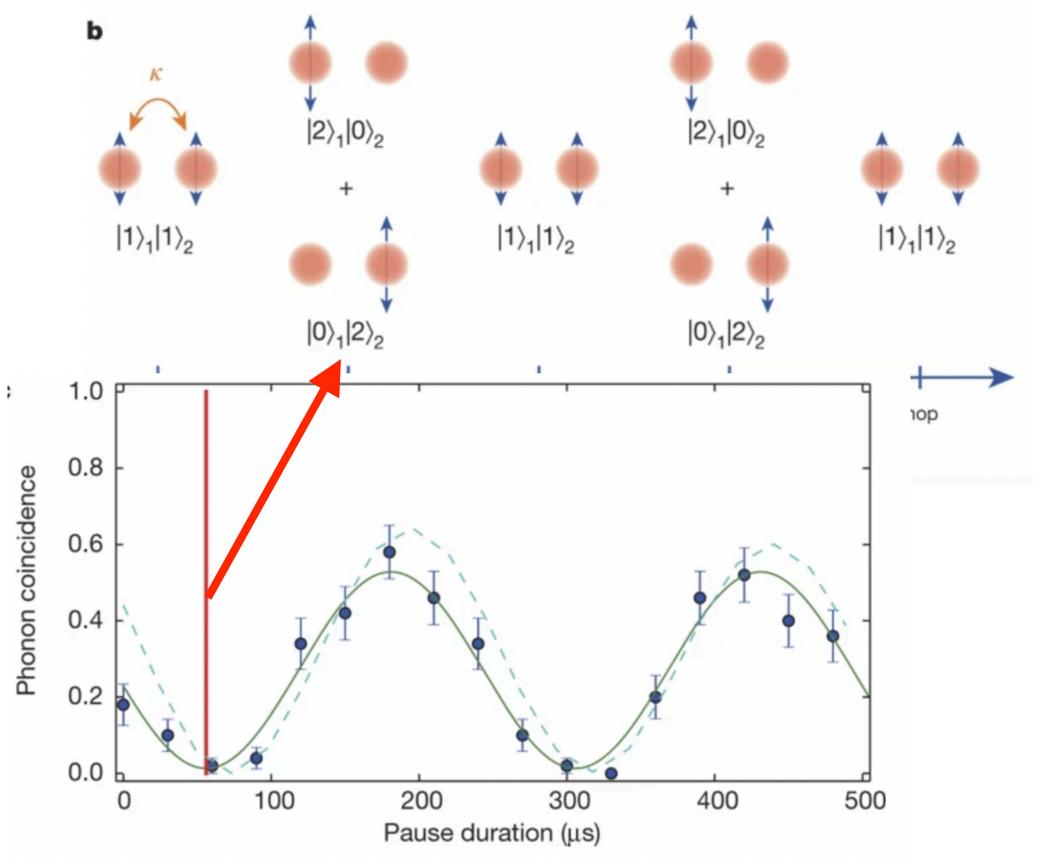
$$\hat{H}_{\mathrm{ex}} = h f_{\mathrm{exchange}} \left(\hat{a}^\dagger \hat{b} + \hat{a} \hat{b}^\dagger \right)$$

Distance between neighboring ions

Multiple oscillators in an ion string

Toyoda et al. Nature 527, 74–77(2015)
Beamsplitter action for bosons produces bunching
- two identical bosons come out on the same output port



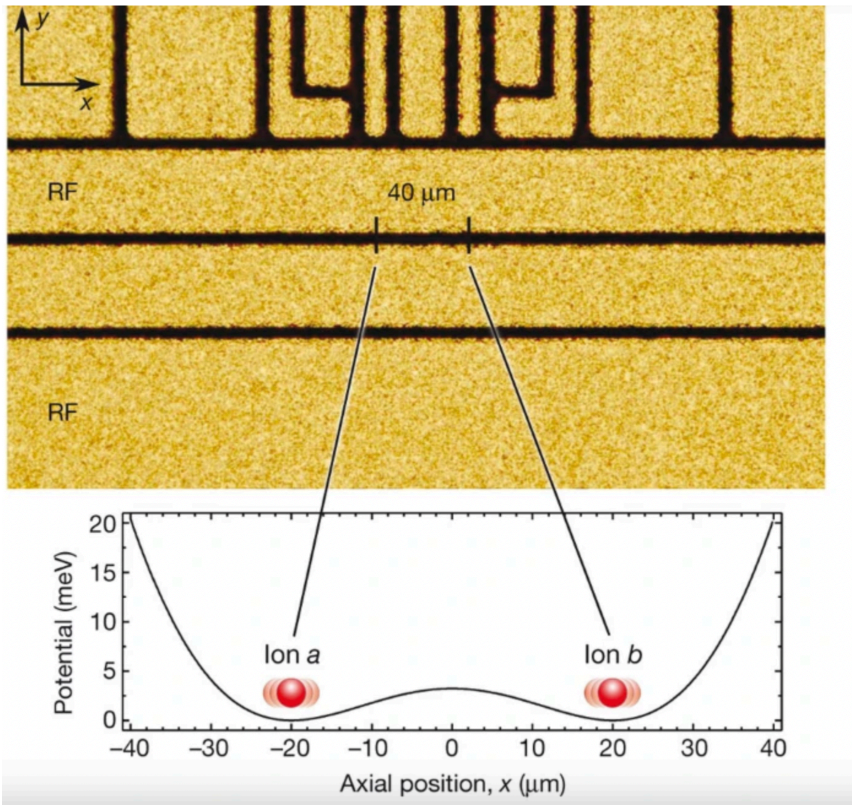


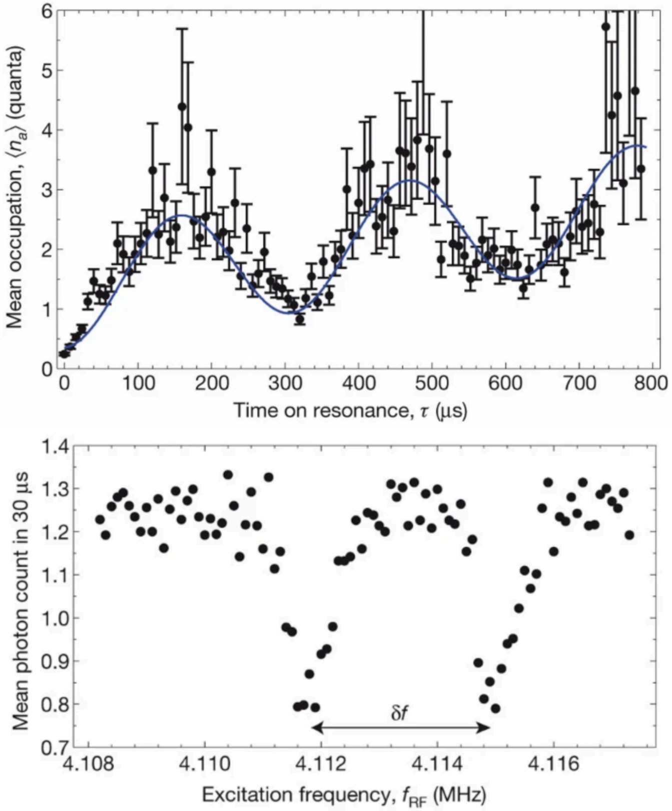
Toyoda et al. PRL 111, 160501 (2013) Combined with RSB drive: Jaynes-Cummings-Hubbard model

Multiple oscillators in separate potentials

Use a small trap - create local wells

K. Brown et al. Nature 471, 196 (2011)

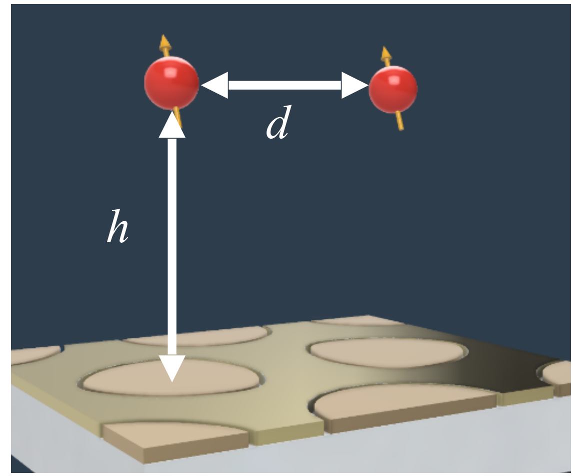


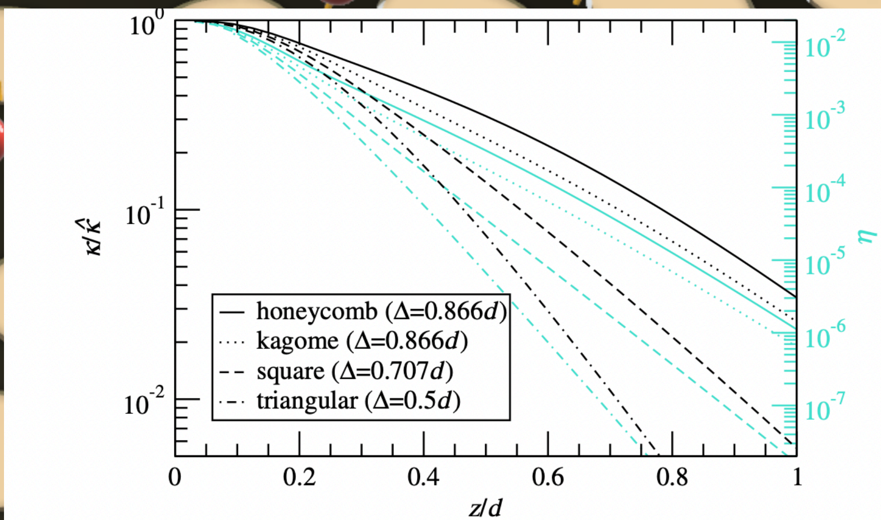


Challenge: motional heating increases rapidly with ion-surface distance

2-D arrays of micro-fabricated traps

$$V_{i,\text{static}}(\mathbf{r}_i) + \Phi_{\text{RF},i}(\mathbf{r}_i)$$



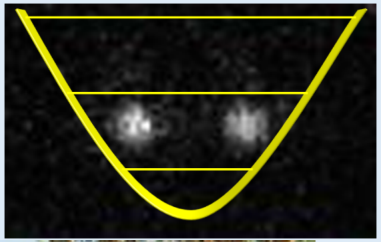


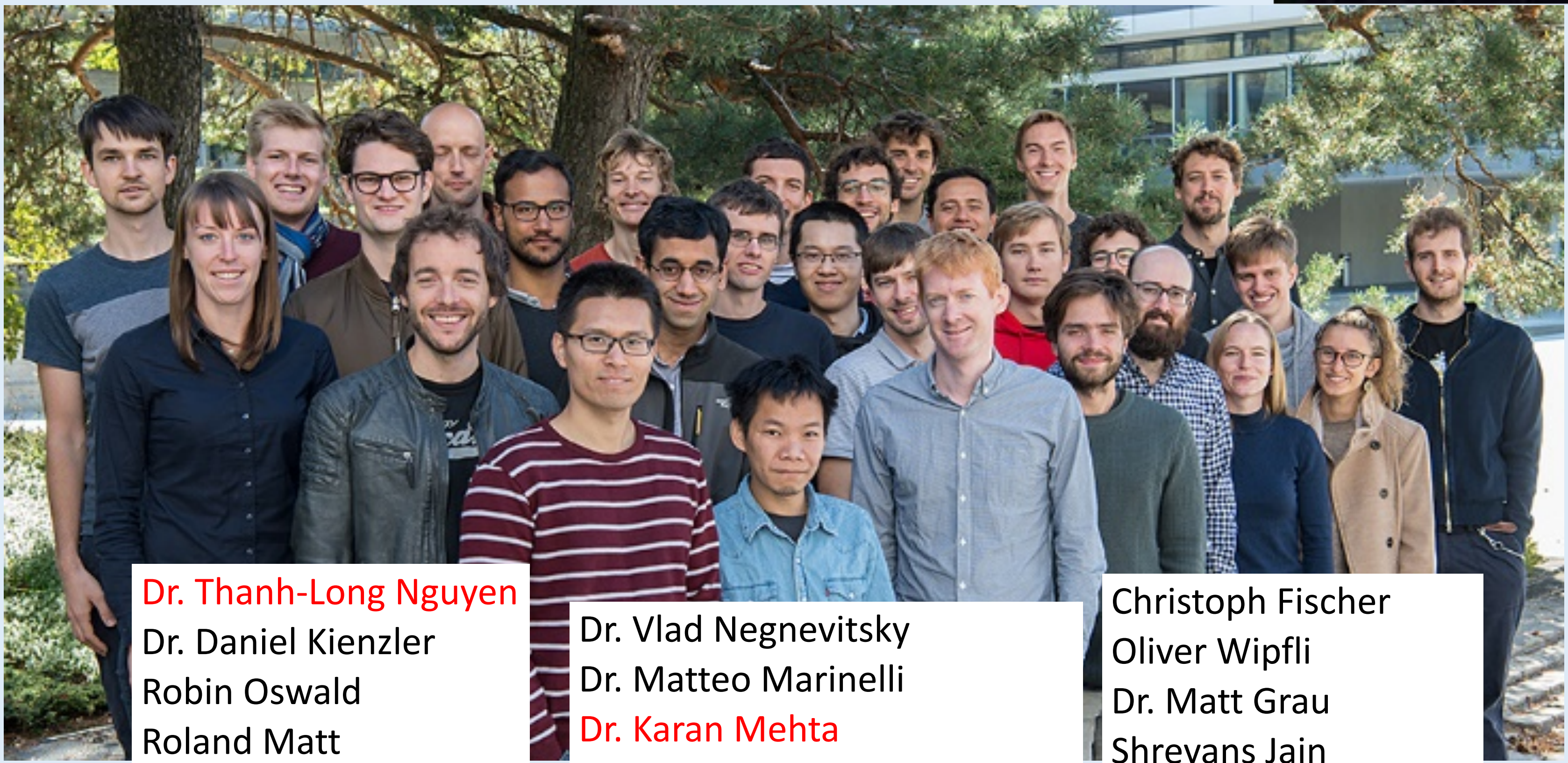
Studied theoretically

Schmied et al. PRL 102, 233002 (2009)

Heating problem critical to realization

Trapped Ion Quantum Information Group
ETH Zürich
www.tiqi.ethz.ch





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