Tensionless AdS/CFT

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Based mainly on work with
Rajesh Gopakumar and Lorenz Eberhardt

AdS / CFT duality

Much recent progress in string theory has been related to AdS/CFT duality

[Maldacena '97, ...]

superstrings on
$\mathrm{AdS}_5 \times \mathrm{S}^5$

=

SU(N) super Yang-Mills theory in 4 dimensions

4d non-abelian gauge theory similar to that appearing in the standard model of particle physics.

AdS/CFT correspondence

The relation between the parameters of string theory on AdS and the dual CFT is

$$g_s \sim \frac{1}{N} \qquad\qquad \frac{R}{l_s} \sim g_{\mathrm{YM}}^2 N = \lambda$$

string coupling constant

AdS radius in string units

't Hooft parameter

AdS/CFT correspondence

The relation between the parameters of string theory on AdS and the dual CFT is

$$g_s \sim \frac{1}{N} \qquad \frac{R}{l_s} \sim g_{\rm YM}^2 N = \lambda$$

small

large

For example, supergravity regime corresponds to strongly coupled gauge theory —- strong/weak duality.

AdS/CFT correspondence

However, if we want to **prove duality** we should consider weakly coupled (planar) gauge theory: this corresponds to the tensionless regime of string theory

$$g_s \sim \frac{1}{N} \qquad\qquad \frac{R}{l_s} \sim g_{\mathrm{YM}}^2 N = \lambda$$

small

small

$l_{\mathrm{s}} \rightarrow \infty$ `tensionless strings'

[Sundborg '01] [Witten '01]
[Sezgin,Sundell '01]

Tensionless limit

This is the regime where AdS/CFT becomes perturbative:

tensionless strings on AdS	$\longleftrightarrow$	weakly coupled/free SYM theory

- very stringy (far from sugra)
- higher spin symmetry
- maximally symmetric phase of string theory

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Could it have a free worldsheet description?

Lower dimensions

In order to simplify things it is also useful to consider the lower-dimensional case, in particular, string theory on AdS3.

The advantage of going to the 3d case is that

- Solvable world-sheet theory for strings on AdS3 exists [sl(2,R) WZW model]

  [Maldacena, (Son), Ooguri ’00 - ’01]
  [Berkovits, Vafa, Witten ’99]

- Much better control over 2d CFTs

AdS3

It has long been suspected that the CFT dual of string theory on

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

is on the same moduli space of CFTs that also contains the symmetric orbifold theory

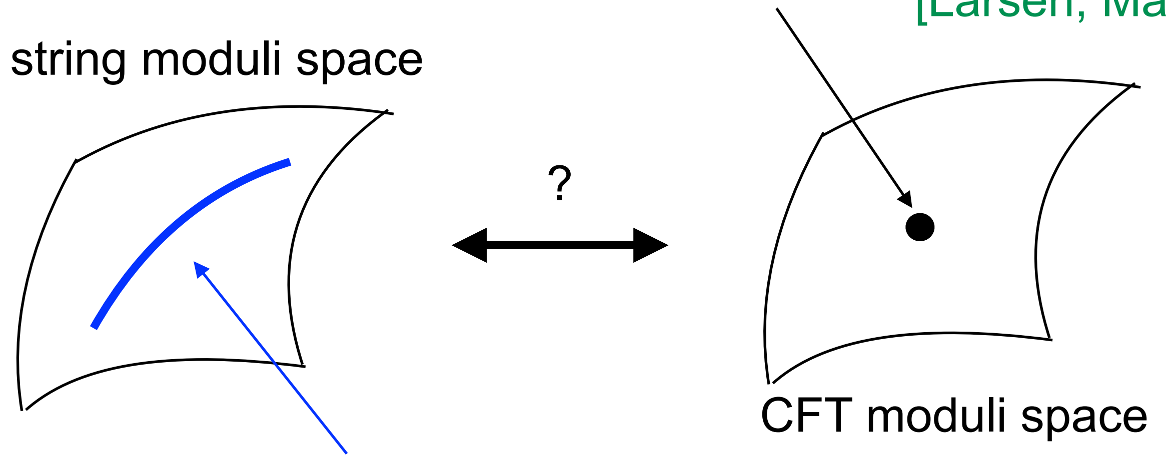
$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^N / S_N$$

[Maldacena '97], …. see e.g. [David et.al. '02]

AdS3

However, it was not known what precise string background is being described by the symmetric orbifold theory itself.

[Larsen, Martinec '99]



Conversely, it was not known what the precise CFT dual of the explicitly solvable worldsheet theory for strings in terms of an sl(2,R) WZW model is.

[Maldacena, (Son), Ooguri '00 & '01]

AdS3

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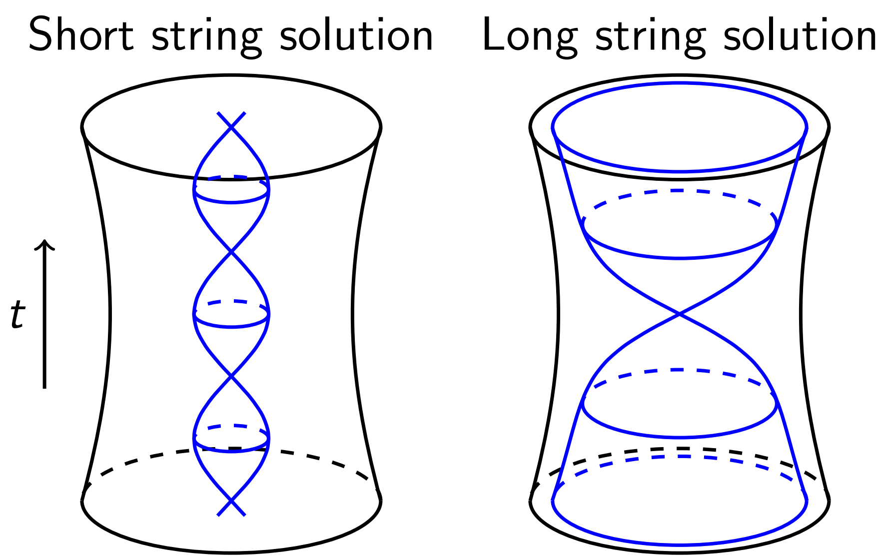

The basic reason for this is that the WZW model describes the background with pure NS-NS flux, which is known to have long string solutions.

[Seiberg, Witten '99], [Maldacena, Ooguri '00]

AdS3

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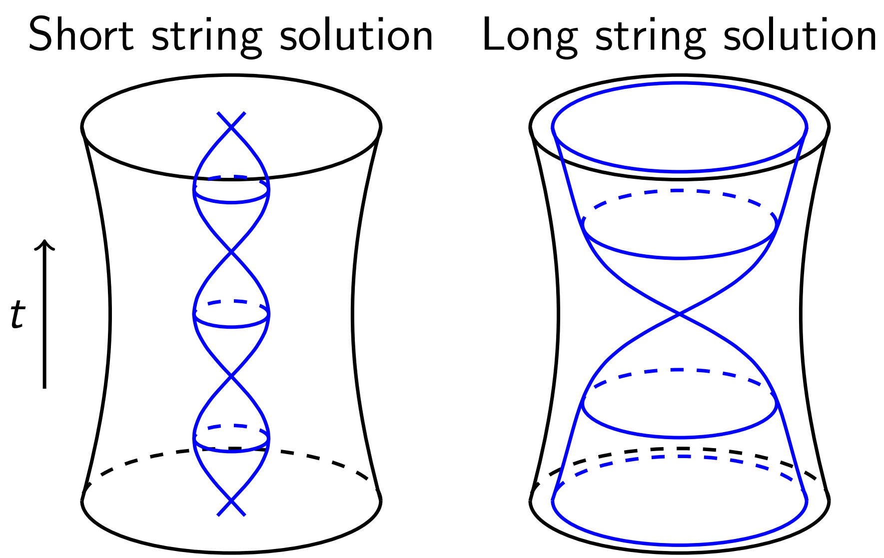


These long strings give rise to a continuum of excitations that are not present in the actual symmetric orbifold theory.

[Seiberg, Witten '99], [Maldacena, Ooguri '00]

Higher spin symmetry

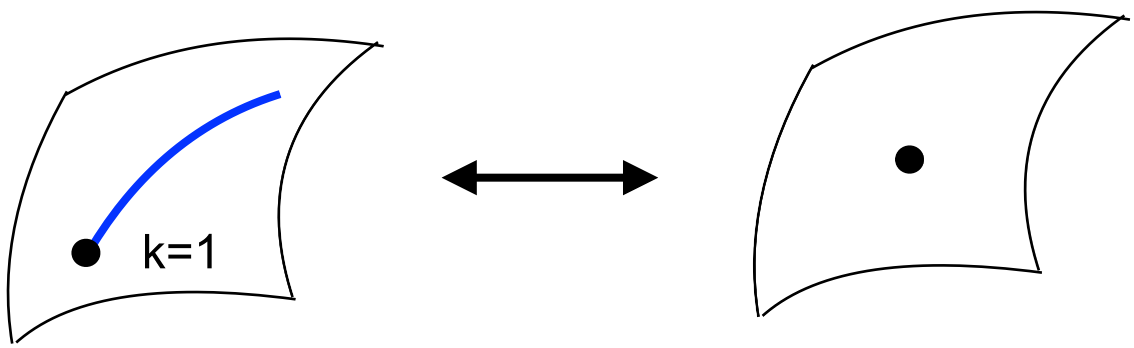
As I will explain below, the symmetric orbifold theory itself has an extended `higher spin' symmetry, and is therefore the analogue of free SYM in 4D.

As such it should be dual to tensionless limit of string theory on AdS3, i.e. to the small radius limit.

In terms of the WZW description, this is the small level limit.

WZW model

This suggests that the precise relation could be:



We have shown that this worldsheet description is indeed exactly dual to the symmetric orbifold.

An exact AdS/CFT duality

[MRG, Gopakumar ’18]
[Eberhardt, MRG, Gopakumar ’18]
[Eberhardt, MRG, Gopakumar ’19]
[Dei, MRG, Gopakumar, Knighton ’20]

AdS5 generalisation

As we will see, the relevant AdS3 worldsheet theory can be formulated in terms of free fields, which seem to describe twistor degrees of freedom.

This suggests a natural generalisation to the case of AdS5, and we have recently made a proposal for a concrete worldsheet theory that should be dual to free N=4 SYM in 4d.

In particular, we have found that this worldsheet theory reproduces exactly the single-trace spectrum of N=4 SYM.

[MRG, Gopakumar '21]

Plan of talk

1. **Introduction and Motivation**
2. Higher Spin symmetry
3. The spectrum for AdS3/CFT2 I: NS-R
4. The spectrum for AdS3/CFT2 II: Hybrid
5. Matching correlators in AdS3/CFT2
6. Generalisation to AdS5

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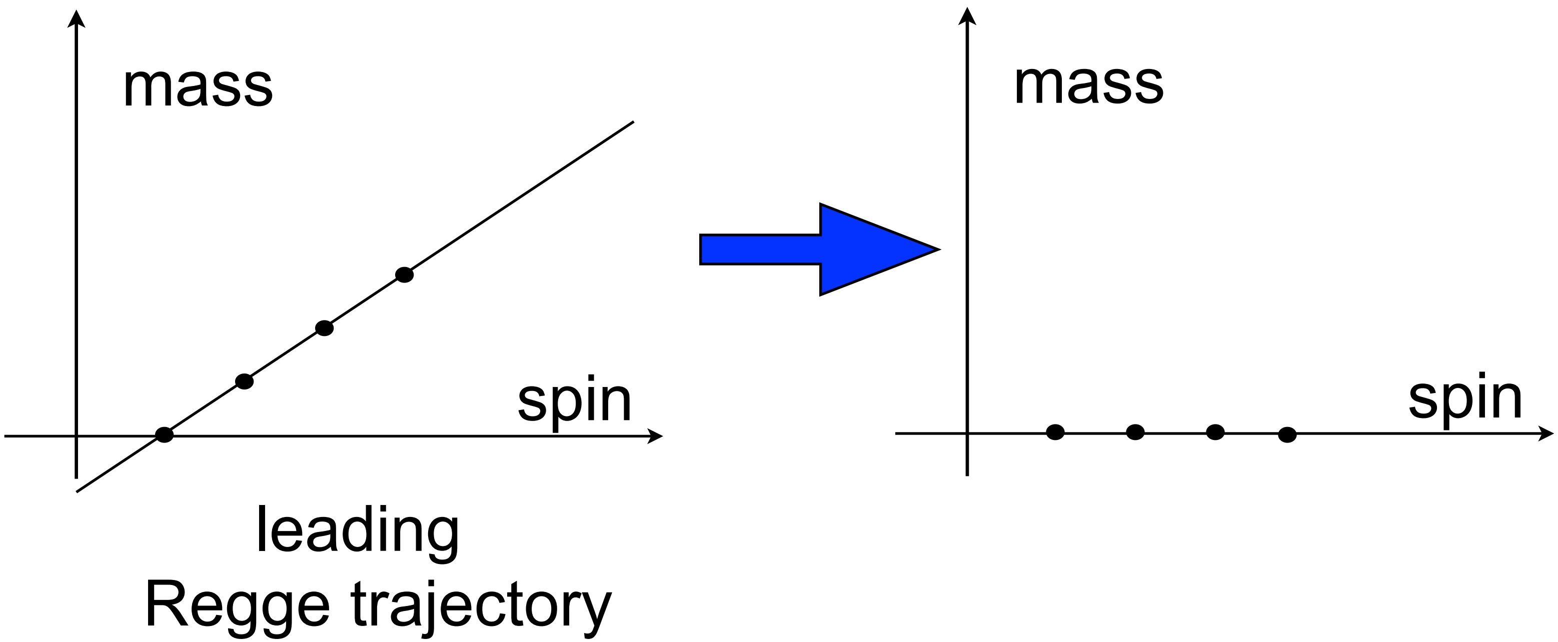
Tensionless limit

In tensionless limit all string excitations become massless:



Higher spin theory

Resulting theory has an infinite number of massless higher spin fields, which generate a very large gauge symmetry.

$\rightarrow$ effective description in terms of Vasiliev Higher Spin Theory.

maximally symmetric/unbroken phase of string theory

Leading Regge trajectory

On the dual CFT side, the traces of bilinears of elementary Yang-Mills fields form closed subsector in free theory.

This subsector is believed to correspond to the leading Regge trajectory from the string point of view:

vector-like HS -- CFT duality

Higher spin CFT duality

First concrete proposal for HS - CFT duality

[Klebanov-Polyakov ’02]
[Sezgin-Sundell ’02]

higher spin theory on AdS4 $\longleftrightarrow$ 3d O(N) vector model in large N limit

Different versions: vector model fields bosons or fermions;
free or interacting fixed point.

Subsequently: generalisation to family of parity-violating theories.

[Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin ’11]
[Aharony, Gur-Ari, Yacoby ’11], …

Checks of the proposal

Various impressive checks of the proposal have been performed, in particular

3-point functions of HS fields on AdS4

have been matched to

3-point functions of HS currents in O(N) model to leading order in 1/N.

[Giombi, Yin ’09-’10]

Furthermore, the symmetries have been identified.

[Giombi, Prakash, Yin ’11], [Giombi, Yin ’11]
[Maldacena, Zhiboedov ’11-’12]

3d proposal

The lower dimensional version of this duality is

[MRG,Gopakumar '10]

AdS3:
higher spin theory
with a complex
scalar of mass M

$\longleftrightarrow$

2d CFT:
$\mathcal{W}_{N,k}$ minimal models
in large N 't Hooft limit
with coupling λ

where $\lambda = \dfrac{N}{N+k}$ and $M^2 = -(1-\lambda^2)$

3d version

The advantage of going to the 3d case (that is dual to a 2d CFT) is that

- AdS3 HS theories are much simpler
- Much better control over 2d CFTs

As a consequence, many precision tests (quantum symmetry, spectrum) were possible.

[MRG,Gopakumar, Hartman, Raju '11], [MRG,Gopakumar '12]

Susy version

In the susy version of this HS-CFT duality, the 2d CFT is the so-called Wolf space coset [MRG, Gopakumar '13]

$$\frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2} \oplus \mathfrak{u}(1)_\kappa} \oplus \mathfrak{u}(1)_\kappa \ ,$$

which is dual to an N=4 susy HS theory on AdS3 based on the Lie algebra $\mathrm{shs}_2[\lambda]$ and

$$\lambda = \frac{N}{N+k+2}$$

hs theory in string theory

In the large k (small λ limit), the Wolf space coset becomes a subsector of [MRG, Gopakumar '14]

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathbb{T}^4$$

hs theory based on

$$\mathrm{shs}_2[\lambda]$$

string theory

$$\updownarrow \qquad\qquad \updownarrow$$

Wolf space cosets $\xrightarrow{\lambda \to 0} \subset$ symmetric orbifold

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv \left(\mathbb{T}^{4 \otimes (N)}\right)/S_N$$

Strategy

This viewpoint therefore suggests that the symmetric orbifold

$$\mathrm{Sym}_N(\mathbb{T}^4) \equiv (\mathbb{T}^4)^N / S_N$$

is dual to string theory at the tensionless point.

In order to find direct description of this string theory need to use world-sheet methods — tensionless limit is far from supergravity regime!

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NS-R WZW model

The perturbatively solvable world-sheet theory for AdS3 is formulated in terms of a WZW model based on the Lie algebra sl(2,R). For the case with supersymmetry the relevant algebra is

[Maldacena, (Son), Ooguri ’00 & ‘01]

$$\mathfrak{sl}(2,\mathbb{R})_k^{(1)} \cong \mathfrak{sl}(2,\mathbb{R})_{k+2} \oplus 3 \text{ free fermions}$$

bosonic: $J_n^3, J_n^{\pm}$

decoupled

The free fermions sit in the usual NS/R representations.

Representations

The highest weight representations of the $\mathfrak{sl}(2,\mathbb{R})_k$ affine algebra are of the form

$$J^{a_1}_{-n_1} \cdots J^{a_l}_{-n_l} |j, m\rangle$$

characterised by rep of the $\mathfrak{sl}(2,\mathbb{R})$ zero mode algebra

Geometric considerations (large level) suggest that the relevant representations should be of two kinds:

NS-R WZW model

Discrete series lowest weight reps:

$$\mathcal{D}_j^+ : \qquad C = -j(j-1)\ , \quad J_0^- |j,j\rangle = 0$$

Continuous series reps:

$$C_\alpha^j : \quad C = -j(j-1) = \tfrac{1}{4} + p^2\ , \quad |j,m\rangle \text{ with } m \in \alpha + \mathbb{Z}$$

$$(j = \tfrac{1}{2} + ip)$$

[Maldacena, Ooguri ’00]

No-ghost theorem

Because of the Maldacena-Ooguri (unitarity) bound,

$$\text{MO-bound}: \qquad \frac{1}{2} < j < \frac{k+1}{2}$$

[Petropoulos '90]
[Hwang '91]
[Evans, MRG, Perry '98]
[Maldacena, Ooguri '00]

the (discrete) spectrum is bounded from above. Additional states are spectrally flowed images of these two classes of representations

They are not Virasoro highest weight, and are therefore best described in terms of the spectral flow w.

[Maldacena, Ooguri '00]
see also [Henningson et.al. '91]

Spectral flow automorphism

Basic idea: work with original highest weight rep. space, but define on it a new action (by automorphism):

$$J^3_m = \tilde{J}^3_m + \frac{kw}{2}\,\delta_{m,0}$$

$$J^\pm_m = \tilde{J}^\pm_{m\mp w}$$

$$L_m = \tilde{L}_m - w\tilde{J}^3_m - \frac{k}{4}w^2\delta_{m,0}$$

Here the tilde modes act as in the original highest weight representation, but we think of action in terms of new un-tilde modes.

Spectral flow

In order to get a sense of what this means, let's concentrate on a continuous series rep for the case k=w=1:

$$J^3_m = \tilde{J}^3_m + \tfrac{1}{2}\,\delta_{m,0}$$
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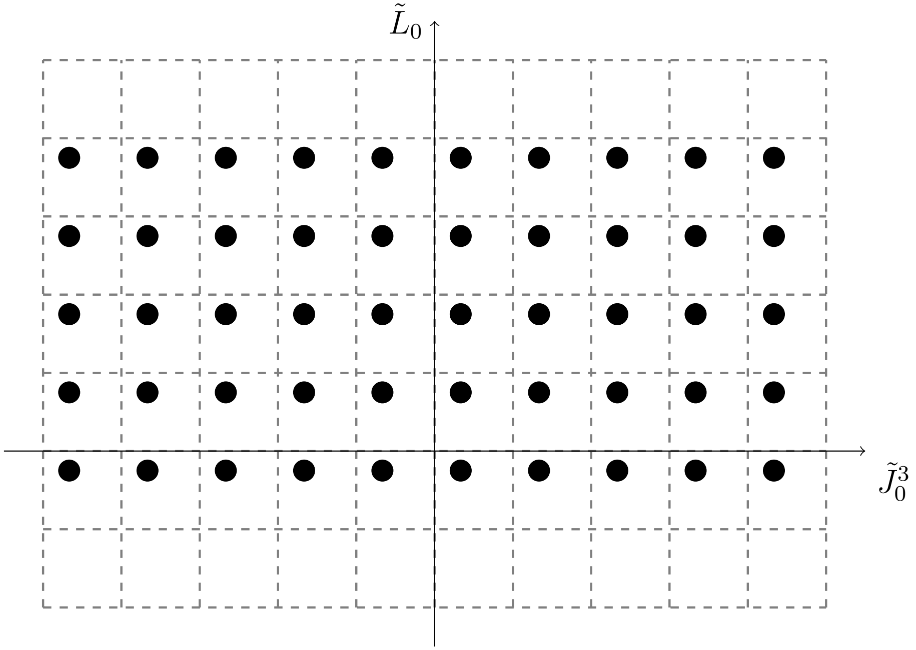
$$L_m = \tilde{L}_m - \tilde{J}^3_m - \tfrac{1}{4}\delta_{m,0}$$



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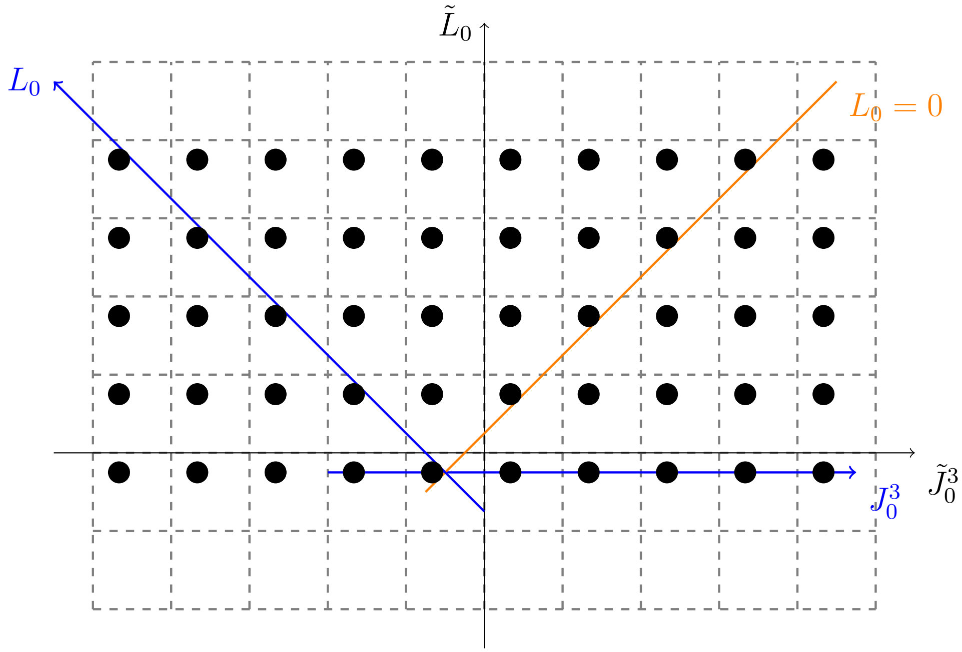

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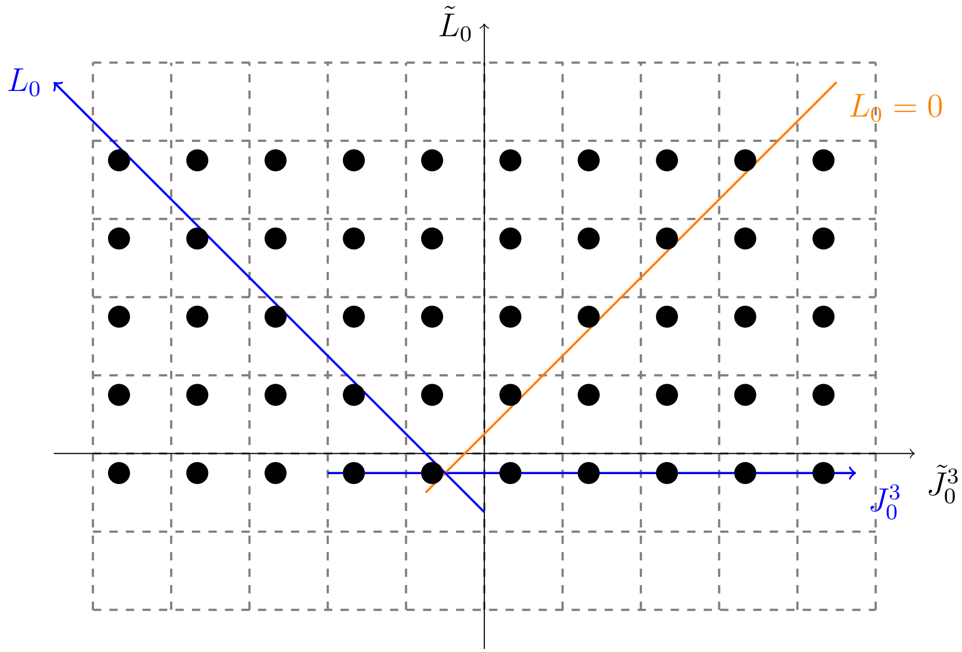

Thus the spectrally flowed representation is not Virasoro highest weight, i.e. the L_0 spectrum is unbounded from below — analogous to string theory on flat Minkowski space.

Physical states

This description is covariant, i.e. we need to impose the physical state condition, e.g. in NS sector

$$G_r^{\mathrm{tot}}\Phi = 0 \quad (r > 0)$$

$$(L_0^{\mathrm{tot}} - \tfrac{1}{2})\Phi = 0 \ .$$

The second condition (mass-shell condition) implies that e.g. in sector without spectral flow

$$\frac{C}{k} + h_0 + N = \frac{1}{2} \qquad (\text{NS-sector})$$

Casimir of sl(2,R)

World-sheet conformal dim. of internal CFT

Dual CFT

The dual (`spacetime') CFT lives on the boundary of AdS3, and we have the identifications

$$L_0^{\mathrm{CFT}} = J_0^3 \ , \qquad L_1^{\mathrm{CFT}} = J_0^- \ , \qquad L_{-1}^{\mathrm{CFT}} = J_0^+ \ ,$$

with a similar relation for the right-movers.

We are interested in the `tensionless' regime of this theory. Since the level k is proportional to the size of the AdS3 space in string units, this should correspond to smallest (non-trivial) value of k: k=1.