Maths Circle Explorations: Session 1

October 2021

1. Take a standard A4 sized sheet of paper. With a pair of scissors, cut a hole in the paper that is large enough for your entire body to pass through it.

2. Take a sheet of paper, and draw a line passing through the center and parallel to one of the edges. This line divides the sheet into two equal halves. Mark a dot anywhere in one of the halves. Mark another dot anywhere in the other half. Now fold the sheet along the line, such that the dots are on the visible side of the sheet. If this line is made horizontal, stick the two edges that are vertical. The object you get has a cylindrical shape with two open faces. One face has a line along it. Hold the object with this face pointing up. Imagine you are holding a mug or a cup with its rim alongside the line (and the bottom covered). On the outside of this cup is a dot. Imagine that an ant is sitting at that dot. On the inside of the cup is another dot. Imagine that there is a grain of sugar stuck at this dot. So, our ant, call it lazy or efficient, wants to crawl to the grain of sugar with the shortest possible path. How should it proceed?

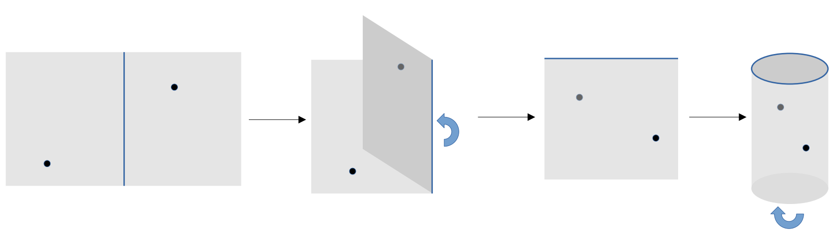

Figure 1: Activity 2

3. Get hold of a toothpick. Alternatively, you can use a matchstick, or any object of a similar shape and size. Measure the length l of the toothpick. Create a ruled paper in which the parallel lines are a distance $2l$ apart. Throw the toothpick on the paper randomly (i.e., without “taking aim”). If it lands so that it intersects a line on the paper, write “1”, otherwise, write “0”. If the toothpick does not land completely on the paper, then

the result is declared invalid. Repeat this 100 times, and then add all the numbers you have written. Compare notes with others. Do they have the same number? Similar numbers?

Can you find an "explanation" for your experimental results based purely on mathematical reasoning? What would you expect if you repeated the experiment 1 million times? What would change if you modified the spacing between lines on the paper?

4. Grab a chessboard, or create one on a piece of paper by drawing an 8×8 grid, and shading alternate squares. Recall that a knight is a chess piece that moves two steps parallel to one edge of the board and one step perpendicular to the same edge in any given move. Can a knight starting out at one corner of the board (square $a1$) reach the diagonally opposite corner (square $h8$) visiting each square on the chessboard *exactly once* along the way? You may want to start by asking the same question for square chessboards of size 3×3, 4×4, and so on (an $m \times n$ chessboard is simply a grid with m rows and n columns).

5. On a piece of paper, draw three rows of boxes. Each row can contain any number of boxes. Each player in turn chooses any number of boxes from one (*and only one!*) of the rows to erase. The winner is the one who erases the last box. Find a partner, and play this game with them.

   Let n_k denote the number of boxes in row k. For what values of (n_1, n_2, n_3) does the first player (the one who makes the first move) have a winning strategy? When does the second player have a winning strategy? Start with a small number of boxes in each row and experiment.