Thermal monopole condensation in full QCD

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based on arXiv:2107.02745

Topological aspects of strong correlations and gauge theories - ICTS, September 2021

1 – INTRODUCTION

- The deconfinement transition is a well defined concept in pure gauge theories, where it is associated with the spontaneous breaking of center symmetry.

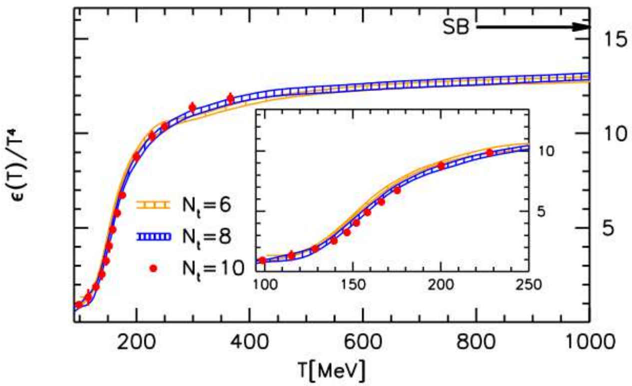
- In full QCD center symmetry is broken explicitly by dynamical fermions.

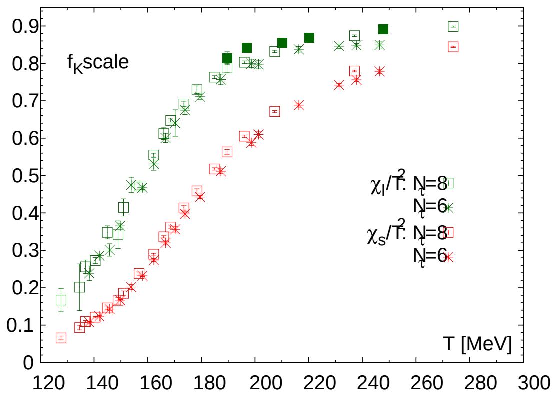
  The dominant (slightly broken) symmetry is the chiral one, with a pseudo-critical restoration temperature $T_c \simeq 155$ MeV

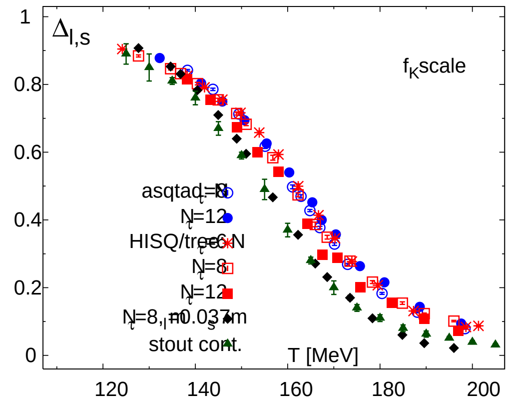
- Around T_c, other thermodynamical observables (pressure, energy density, quark number susceptibilities, ...) provide evidence for a sequential deconfinement.

Results about the QCD crossover

energy density | u/d **and** s **number fluctuations** | **chiral condensate**







arXiv:1007.2580 | **arXiv:1111.1710** | **arXiv:1111.1710**

Temperature and nature of the transition

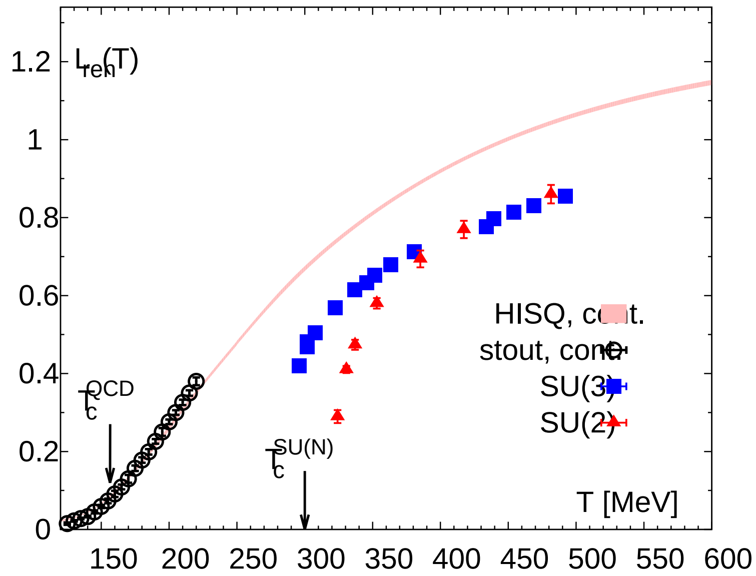
S. Borsanyi *et al.* **JHEP 1009, 073 (2010)** $T_c = 155(6)$ **MeV** (**stout link stag. discretization,** $a_{min} \simeq 0.08$ **fm**)

A. Bazavov *et al.*, **PRD 85, 054503 (2012)** $T_c = 154(9)$ **MeV** (**HISQ/tree stag. discretization,** $a_{min} \simeq 0.1$ **fm**)

- Physical point consistent with a smooth crossover (**Aoki et al., Nature 443, 675 (2006)**)

Pictures are from 10 years ago, there have been refinements in the meanwhile, but the main message is unchanged.

The renormalized Polyakov loop, even if not an order parameter any more, starts rising around there, but the rise is spread over a wide range.



Picture taken from P. Petreczky, arXiv:2011.01466

- On the other hand, various mechanisms have been proposed, which tipically interpret confinement in terms of the condensation of effective degrees of freedom of topological nature (monopoles, vortices, ...)

- A univocal view about the mechanism is still lacking, however all descriptions lead to a correct identification of the deconfinement transition for pure gauge theories.

- It is therefore of great interest to investigate such mechanisms in full QCD as well

- In this study we consider the dual superconductor model ('t Hooft, 1975, Mandelstam, 1976) and the associated condensation of Abelian magnetic monopoles

- The mechanism has been investigated on the lattice in various different ways, like looking at the expectation value of magnetically charged operators or at the effective monopole action A. Di Giacomo *et al* 1995-2011, arXiv:2010.04232; P. Cea and L. Cosmai 2001, hep-lat/0103019; M. N. Chernodub, M. I. Polikarpov and A. I. Veselov, 1996, hep-lat/9610007

- The possible role played around and above T_c by thermal monopoles "evaporating" from the zero T condensate attracted lot of attention in the last few years. (Liao-Shuryak 2006, 2008; Chernodub-Zakharov, 2006; D'Alessandro, M. D. 2008; Chernodub, D'Alessandro, Zakharov, 2009; Bornyakov, Braguta, 2011-2012; Bornyakov, Kononenko, 2012) Ratti-Shuryak, 2009).
  They are identified as magnetic currents wrapping non-trivially around the thermal direction, resembling path-integral contributions of thermal quasi-particles (Chernodub, Zakharov, 2006; Bornaykov, Mitrjushkin, Mueller-Preussker, 2002; Ejiri, 2006).

- The distribution of wrapping trajectories shows that thermal monopoles indeed condense at T_c both for $SU(2)$ and $SU(3)$ pure gauge theories A. D'Alessandro, M.D., E. Shuryak, arXiv:1002.4161; C. Bonati, M.D., arXiv:1308.0302

The purpose of this study is to extend the analysis to full QCD

2 – Abelian projection and monopoles in $SU(N)$ gauge theories ('t Hooft, '74, '81)

- in $SU(N)$, one can identify $N-1$ independent Abelian subgroups ($U(1)^{(N-1)}$) by diagonalization of a traceless adjoint Higgs field $X(x)$

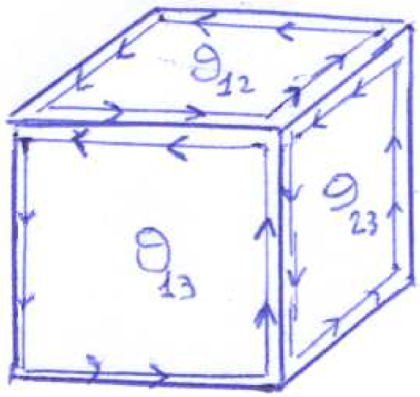
- Fix the gauge where $X(x) = X^D(x) = \mathrm{diag}(X_1(x), X_2(x), \ldots, X_N(x))$ with $X_j(x) \geq X_{j+1}(x)$. That leaves a residual $U(1)^{(N-1)}$ gauge symmetry. An Abelian e.m. 't Hooft tensor $F^{(k)}_{\mu\nu}$ is associated to each residual $U(1)$ group, all tensors are mutually neutral.

- Points where two eigenvalues of X coincide define the location of magnetic monopoles. The residual $U(1)$ is enlarged to a full $SU(2)$ subgroup.

- In a lattice setup, looking for points where two eigenvalues coincide is ill defined. One then works in the diagonal gauge and looks for monopole fields via the De Grand - Toussaint procedure.

3 – Abelian monopoles on the lattice

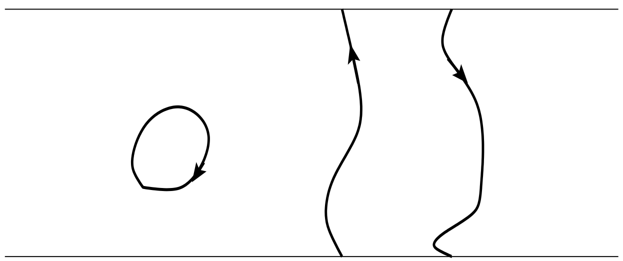
In compact $U(1)$ lattice gauge theory, magnetic monopoles are identified via the De Grand - Toussaint procedure. Let $u_\mu(n) \equiv e^{i\theta_\mu(n)}$ be the $U(1)$ link variables on a cubic 4D lattice, from which Abelian plaquettes are constructed $\theta_{\mu\nu} \equiv \hat{\partial}_\mu \theta_\nu - \hat{\partial}_\nu \theta_\mu$



Monopole currents are then constructed as

$$m_\mu = \frac{1}{2\pi} \varepsilon_{\mu\nu\rho\sigma} \hat{\partial}_\nu \bar{\theta}_{\rho\sigma} \quad ; \qquad \theta_{\mu\nu} = \bar{\theta}_{\mu\nu} + 2\pi n_{\mu\nu}$$

i.e. one measures the net magnetic flux going out of a 3D cube, modulo Dirac string contributions



Monopole currents form closed loops, since $\hat{\partial}_\mu m_\mu = 0$. In a thermal theory, currents which wrap around the periodic time direction are identified with thermal monopoles.

4 – Maximal Abelian Gauge (MAG) Projection in $SU(2)$ and extension to $SU(N)$

No natural Higgs field exist in QCD, so Abelian projection requires a choice, implying some arbitrariness

For $SU(2)$, MAG is the gauge where the following functional has a maximum

$$F_{\mathrm{MAG}} = \sum_{\mu,n} \mathrm{tr} \left(U_\mu(n) \sigma_3 U_\mu^\dagger(n)\, \sigma_3 \right) = \sum_{\mu,n} 2 \left(|U_\mu(n)_{11}|^2 + |U_\mu(n)_{22}|^2 - 1 \right)$$

On stationary points of F_{MAG}, the diagonal Hermitean, traceless Higgs field is

$$X^{\mathrm{MAG}}(n) = \sum_{\mu} \left[U_\mu(n) \sigma_3 U_\mu^\dagger(n) + U_\mu^\dagger(n-\mu) \sigma_3 U_\mu(n-\mu) \right] \, ,$$

Part of the popuparity of the MAG projection is due to the fact that abelian projected fields retain most of the original dynamics (Abelian Dominance).

The properties of magnetic monopoles defined after MAG projection also show a nice scaling to the continuum limit.

Extension to $SU(N)$

A standard extension adopted for $SU(N)$ still considers maximization of diagonal elements (A. S. Kronfeld, G. Schierholz and U. J. Wiese, 1987) **but has some problems:**

- **No diagonal Higgs field is naturally associated to it**
- **On extremal points, the residual symmetry includes global permutations of group indexes, so that Abelian charges are not well defined.**

Possible alternative: generalized MAG (J. D. Stack, W. W. Tucker 2002; C. Bonati, M.D., arXiv:1308.0302)

$$\tilde{F}_{\rm MAG} = \sum_{\mu,n} \textbf{tr} \left(U_\mu(n) \tilde{\lambda} U_\mu^\dagger(n)\, \tilde{\lambda} \right) \quad ; \quad \tilde{\lambda} = {\rm diag}(\tilde{\lambda}_1, \tilde{\lambda}_2, \dots \tilde{\lambda}_N)\,,$$

where $\tilde{\lambda}$ is a generic element of the Cartan subalgebra. Some properties:

- **A diagonal Higgs field exists, provided $\tilde{\lambda}$ has no pair of coinciding eigenvalues**

$$\tilde{X}(n) = \sum_\mu \left[U_\mu(n) \tilde{\lambda} U_\mu^\dagger(n) + U_\mu^\dagger(n-\mu) \tilde{\lambda} U_\mu(n-\mu) \right] \,.$$

Summary for $SU(3)$

- Gauge is fixed by maximization of the functional

$$\tilde{F}_{\rm MAG} = \sum_{\mu,n} \textbf{tr}\left(U_\mu(n) \tilde{\lambda} U_\mu^\dagger(n)\, \tilde{\lambda} \right) \quad ; \quad \tilde{\lambda} = \frac{1}{3}\mathrm{diag}(1,\ 0,\ -1)\,,$$

That corresponds to $b^k = 1\ \ \forall\ k$, which treats all monopole species symmetrically. A standard local over-relaxed algorithm is adopted, working over $SU(2)$ subgroups.

- On the gauge fixed configuration, the diagonal of gauge links is extracted,

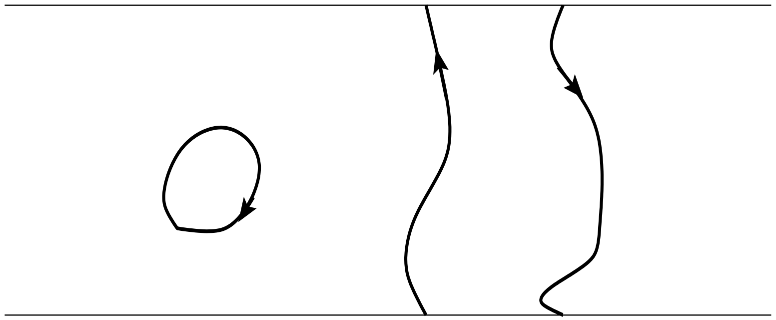
$$U_\mu^D(n) = \mathrm{diag}(e^{i\phi^1_\mu(n)}, e^{i\phi^2_\mu(n)}, e^{i\phi^3_\mu(n)})$$

where $U_\mu^D(n)$ is the diagonal $SU(3)$ matrix maximizing $\mathrm{Re}(\mathrm{tr}(U_\mu^D(n) U_\mu^\dagger(n)))$

- The two Abelian phases are then extracted according to

$$\theta^1_\mu(n) = \phi^1_\mu(n) \qquad \theta^2_\mu(n) = \phi^1_\mu(n) + \phi^2_\mu(n) = -\phi^3_\mu(n)$$

the monopole currents m^1_μ and m^2_μ are then determined following the De Grand-Toussaint method.



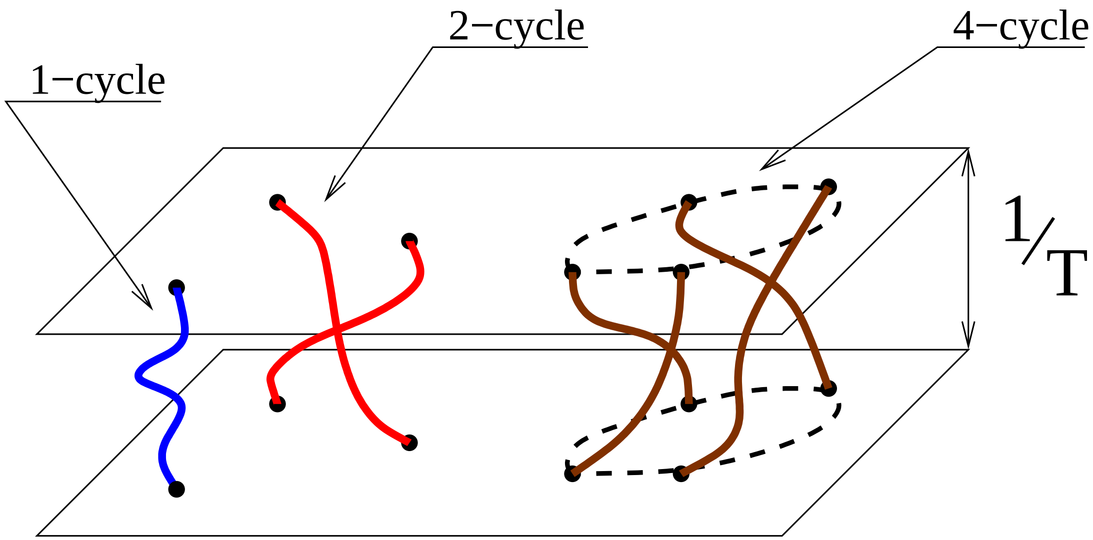
For each configuration, we locate monopole currents with non-trivial winding number in time, and their position at a given reference time slice. After that, we can investigate various quantities.

Density of thermal monopoles

$$\rho = \sum_k k\rho_k \;\; ; \qquad \rho_k \equiv \frac{N_{\mathrm{wrap},k}}{V_s}$$

where $V_s = a^3 L^3$ is the spatial volume and $N_{\mathrm{wrap},k}$ is the number of currents wrapping k times.

Distribution of trajectories with multiple wrappings



Like for a path-integral of bosonic particles, monopole trajectories with multiple windings in the time direction can be associated with two-(or multiple)-particle exchange.

Their T-dependence can be used to investigate thermal monopole condensation.

(M. Cristoforetti and E. Shuryak, arXiv:0906.2019) (A. D'Alessandro, M.D., E. Shuryak, arXiv:1002.4161)

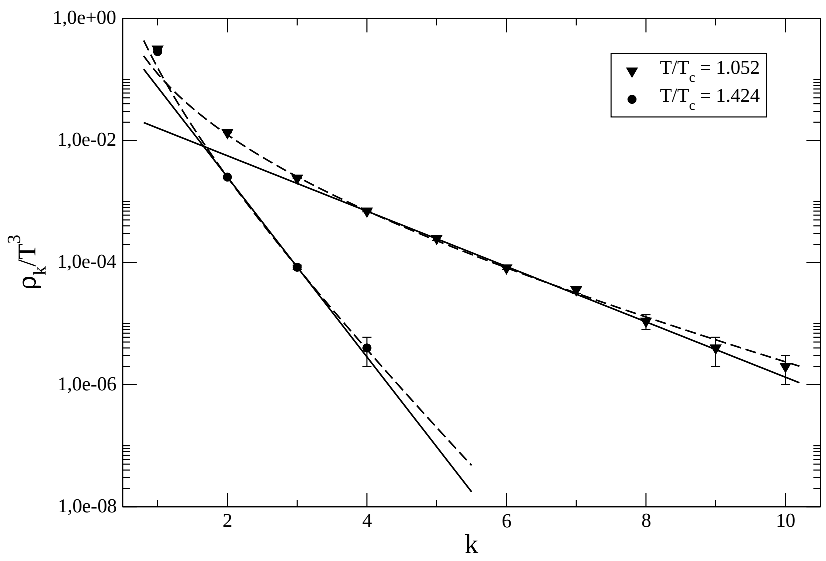
For a system of free bosons, if $\mu = -T\hat{\mu}$ is the chemical potential,

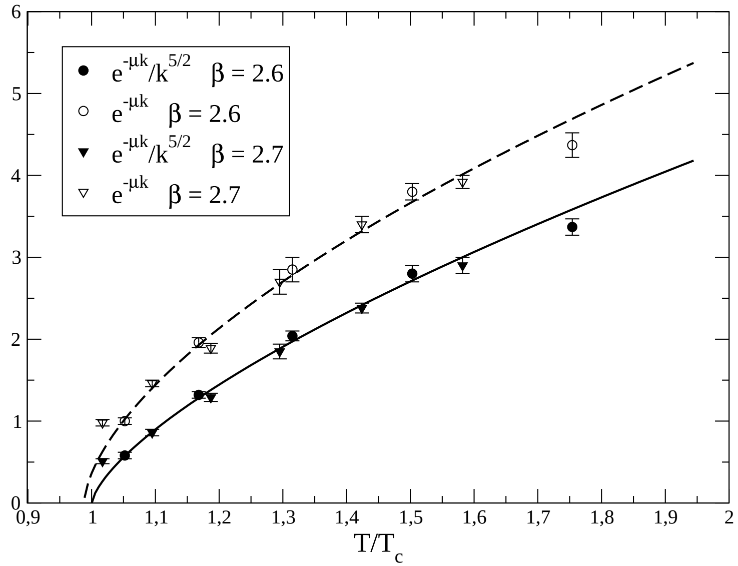
$$\rho_k \;\propto\; e^{-\hat{\mu}k}/k^{5/2} \tag{1}$$

$\mu \to 0$ signals Bose-Einstein condensation (BEC)

Numerical Results for pure gauge $SU(2)$

A. D'Alessandro, M.D., E. Shuryak, arXiv:1002.4161



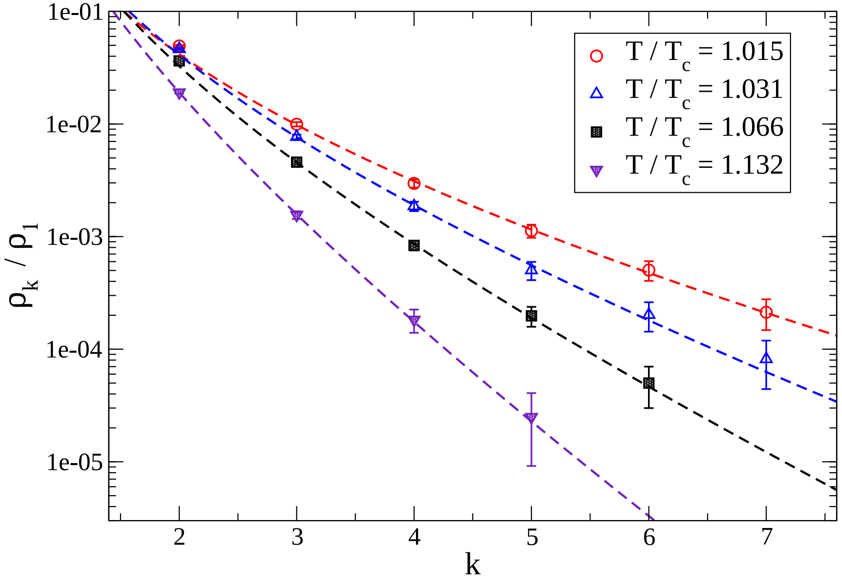


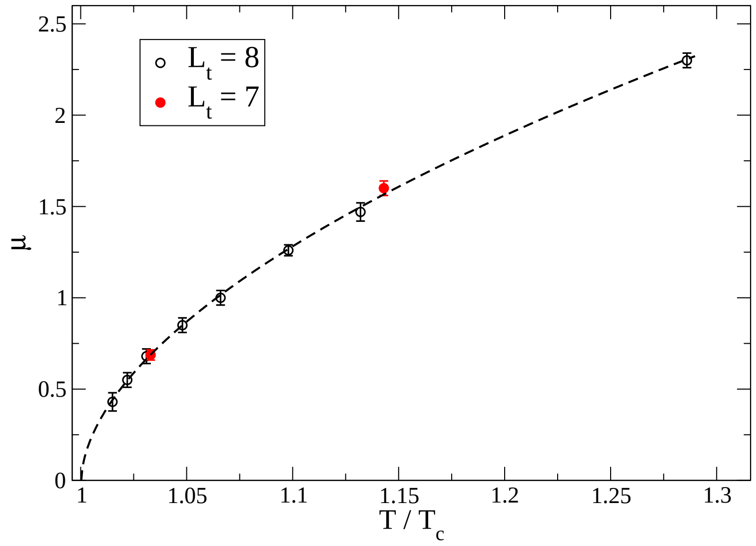
- The density of trajectories winding k-times, ρ_k, is negligible, for $k > 1$, at high T. It becomes significant only as one approaches T_c from above.

- If we assume the simple ansatz in Eq. (1) for the monopole ensemble, we can extract $\hat{\mu}$.
  Then a fit $\hat{\mu} = A\,(T - T_{\rm BEC})^{\nu'}$ returns $T_{\rm BEC} \simeq T_c$ within errors and $\nu' \simeq 0.6 - 0.7$, where $T_{\rm BEC}$ is the Bose-Einstein condensation temperature.

Numerical Results for pure gauge $SU(3)$

C. Bonati, M.D., arXiv:1308.0302





- **Similar results are found for $SU(3)$**
- **notwithstanding the ambiguities related to the Abelian projection procedure, there is no doubt that for pure gauge theories thermal monopoles catch many non-perturbative properties related to confinement/deconfinement.**

5 – Results for $N_f = 2 + 1$ QCD

LATTICE SETUP

$$Z(T) = \int \mathcal{D}U \, e^{-\mathcal{S}_{YM}} \prod_{f=u,\,d,\,s} \det\,(D^f_{\text{st}})^{1/4}\ .$$

- pure gauge: Symanzik tree level improved gauge action

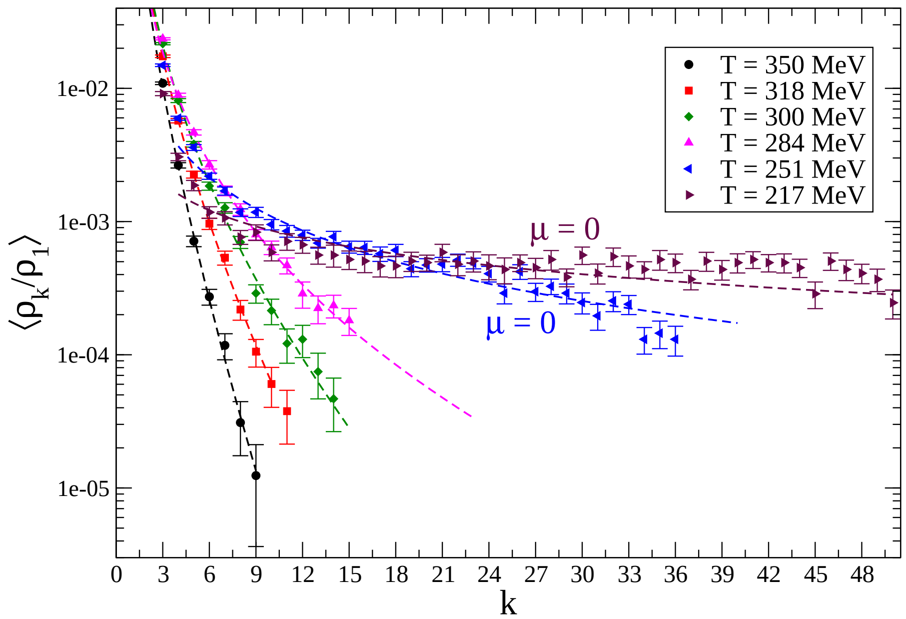
- fermion sector: 2-level stout improved rooted staggered fermions

- bare parameters tuned to stay on a line of constant physics at the physical point

Y. Aoki *et al*, arXiv:0903.4155, S. Borsanyi *et al*, arXiv:1007.2580

- different lattice sizes explored to check finite cut-off and finite size effects :

$24^3 \times 6$, $32^3 \times 8$, $48^3 \times 6$, $\quad T = 1/(N_t a)$.



Results for ρ_k/ρ_1 from simulations on the $48^3 \times 6$ lattice. The dashed lines correspond to best fits to Eq. (1), respectively fixing $\alpha = 5/2$ (for $T > 280$ MeV) or $\hat{\mu} = 0$

- **a striking aspect of our results emerges already looking at the ratio ρ_k/ρ_1 for various T**

- **for $T \gtrsim 280$ MeV the exponential decay is clearly visible, leading to a non-zero $\hat{\mu}$.**

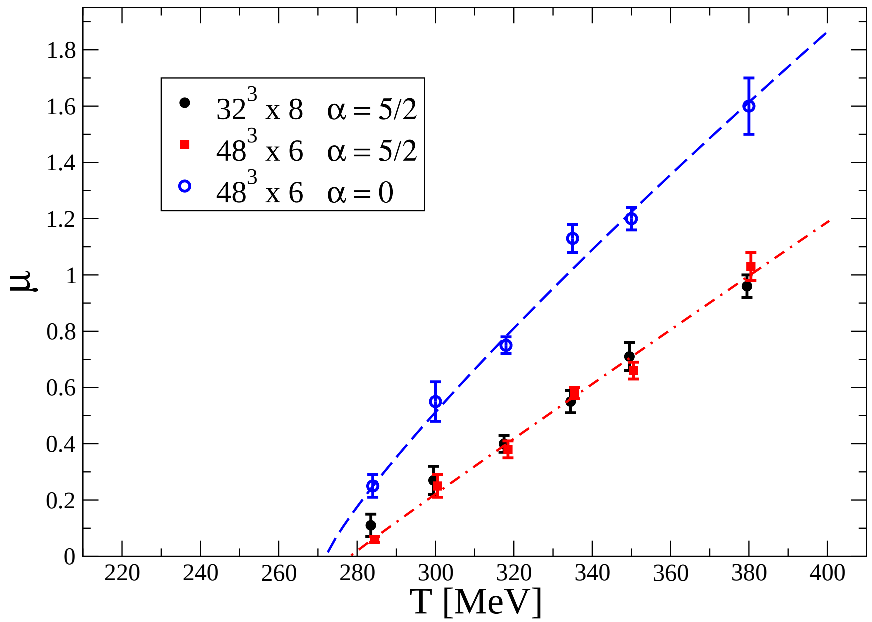
- **for lower temperatures the dependence on k is much flatter and compatible with $\hat{\mu} = 0$.**

Results for $\hat{\mu}$ at $T > 280$ MeV.

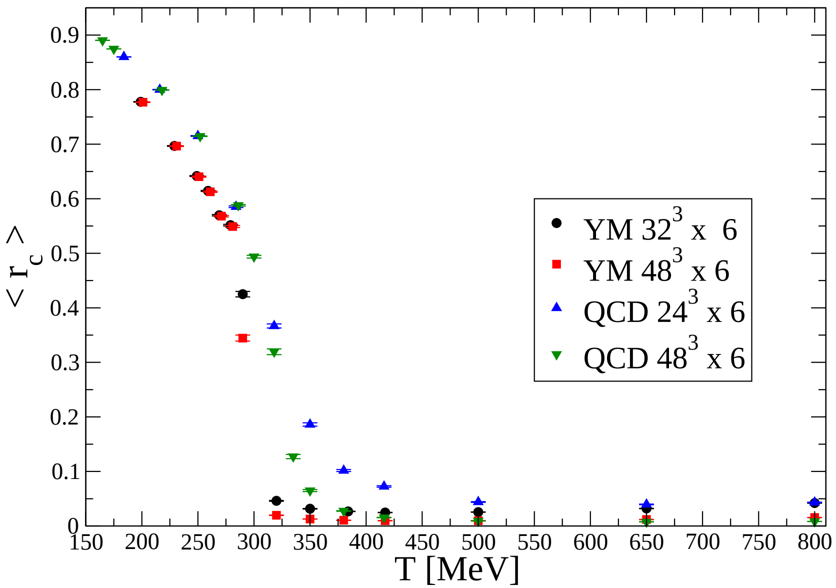
The dashed lines represent best fits of the $48^3 \times 6$ data to a critical behavior

$$\hat{\mu}(T) = a(T - T_{BEC})^{\nu'}$$

returning $T_{BEC} = 272(2)$ ($\nu' \sim 0.9$) and $278(6)$ ($\nu' \sim 1$) respectively for $\alpha = 0$ and $\alpha = 5/2$.



- **as for pure gauge, the outcome is independent of the assumption on α: in both cases $\hat{\mu}$ approaches zero at $T_{BEC} \sim 275$ MeV.**
- **the dependence on the lattice spacing is also negligible**
- **T_{BEC} is almost twice the well established pseudocritical temperature of QCD, $T_c \simeq 155$ MeV!**
- **Can that be confirmed by alternative methods to look for monopole condensation?**



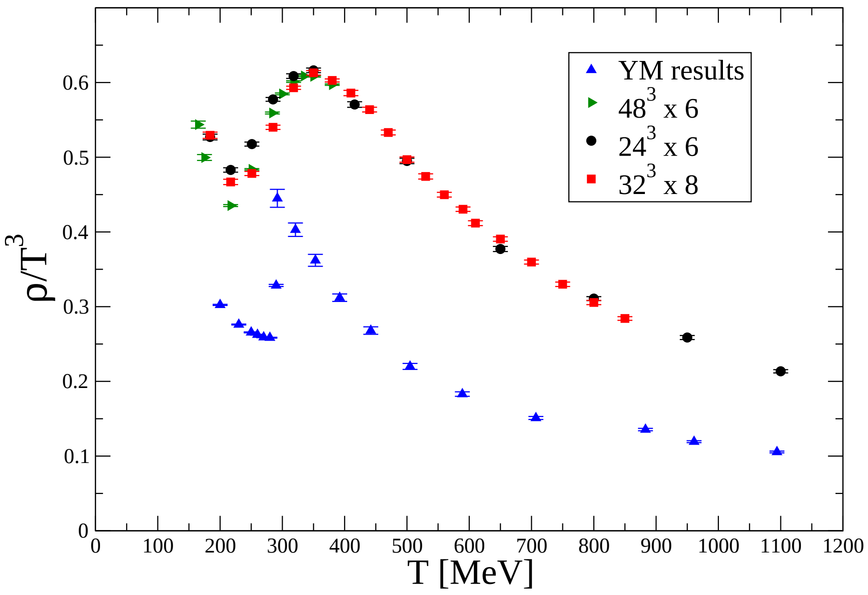
A popular method in the past has been to look for the formation of a percolating cluster of monopole clusters

ratio r_c of the current length of the biggest connected cluster to the total length of monopole currents.

$\langle r_c \rangle \to 0$ in the thermodynamical limit if no dominating cluster forms.

- full QCD results and pure gauge $SU(3)$ results show a quite similar behavior
- the thermodynamical limit of $\langle r_c \rangle$ is non-zero in both cases for $T \lesssim 300$ MeV
- the behavior is just a bit sharper for pure gauge $SU(3)$, where a weak first order transition is at work

Similarities between full QCD and pure gauge also looking at the total thermal monopole density ρ normalized by T^3



- At high T, ρ/T^3 in full QCD is about twice than the quenched value, but similar behavior, consistent with perturbative predictions

  (Giovannangeli, Korthals Altes, hep-ph/0102022; Liao, Shuryak, hep-ph/0611131)

$$\rho/T^3 \propto (\log(T/\Lambda_{eff}))^{-3}$$

  with $\Lambda_{eff} = 47(5)$ MeV ($48(1)$ MeV for pure gauge).

- the drop around T_{BEC} can be interpreted as disappearance of part of the thermal component due to the condensation.

How should we interpret $T_{BEC} \simeq 275$ MeV $> \ T_c \simeq 155$ MeV?

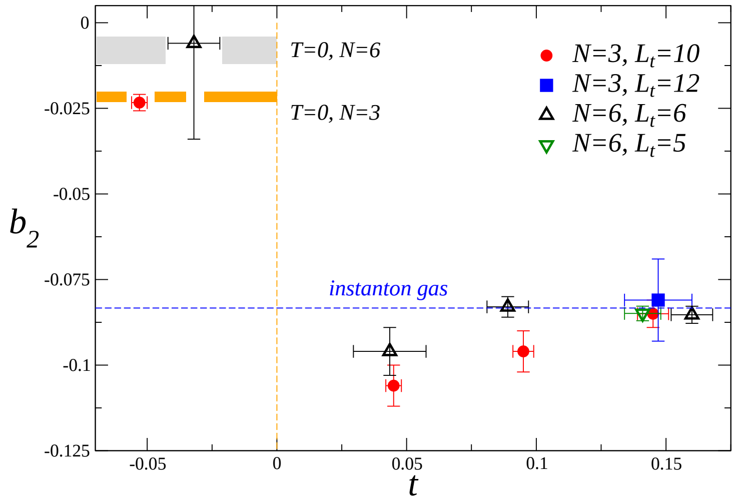
- given that monopole condensation is still not firmly assessed as the correct mechanism for color confinement, an interpretation in terms of a new confined, chiral symmetry restored phase of QCD is not straightforward
- nevertheless, pure gauge theories show that magnetic monopoles correctly catch many interesting aspects of non-perturbative physics related to confinement

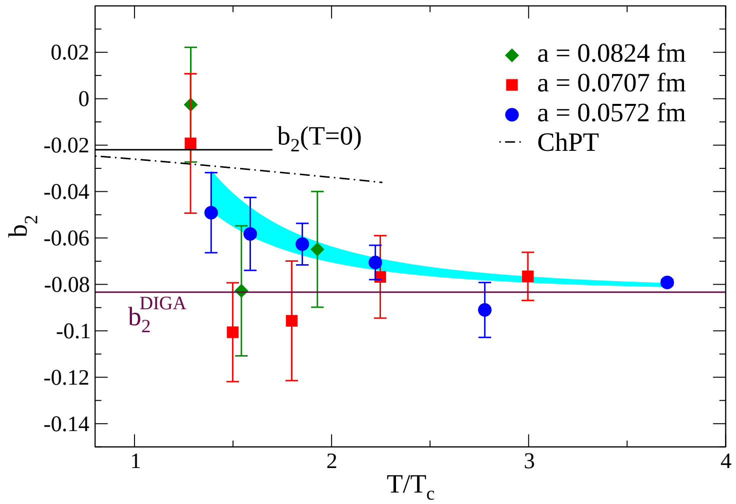
- are there any hints of an intermediate phase above T_c, dominated by non-perturbative effects, from other different observables?

- Generally speaking, the non-perturbative region above T_c is quite large. Quark number susceptibilities and thermodynamical quantities reach values compatible with those of a non-interacting Quark-Gluon Plasma only for $T \gtrsim 300$ MeV
- At the same time, color screening properties show that in-medium quark-antiquark systems behave consistenty with a weak-coupling picture only for $T \gtrsim 300$ MeV

  for a review, see Bazavov, Weber, arXiv:2010.01873

Hints from θ-dependence





- In pure gauge, θ-dependence compatible with instanton gas (DIGA) soon after T_c
- this is visible especially from the kurtosis coefficient b_2, compatible with DIGA, $b_2 = -1/12$, already for $T \gtrsim 1.1\, T_c$ Left figure, Bonati, MD, Panagopoulos, Vicari, arXiv:1301.7640
- in full QCD, instead, b_2 approaches the DIGA value quite slowly, showing appreciable deviations still for $T \sim 2\, T_c$ Right figure, Bonati *et al*, arXiv:1512.06746
- this has been interpreted in terms of the existence of an intermediate phase dominated by an instanton-dyon ensemble

E.Shuryak, arXiv:1701.08089; DeMartini, Shuryak arXiv:2102.11321

A few other non-trivial phenomena observed above T_c

- **Recently, an emergent enhanced symmetry observed in spatial meson correlators** (C. Rohrhofer *et al*, arXiv:1902.03191) **has been interpreted in terms of a possible new phase.**
  The candidate new phase would be chiral symmetric but still confined, a so-called *stringy fluid phase*

  L. Glozman, arXiv:1907.01820

- **Possible evidence for an intermediate phase has been reported also based on the properties of the lowest-lying part of the Dirac spectrum**

  A. Alexandru and I. Horváth, arXiv:1906.08047, arXiv:2103.05607

6 – Conclusions and Perspectives

- there is plenty of evidence that the phase right above the crossover temperature T_c is dominated by non-perturbative effects
- the analysis of thermal monopoles may permit a precise identification of the temperature $T_{BEC} \simeq 275$ MeV where such non-perturbative effects disappear
- Whether T_{BEC} corresponds to some real (percolation-like?) transition or not should be investigated by a careful finite size scaling analysis
- Other order parameters for dual superconductivity or other confinement mechanisms should be investigated to see if they return a similar temperature
- why T_{BEC} so close to the pure gauge critical temperature? Investigations away from the physical point (more or less chiral), or with a different number of flavours, should clarify if this is just accidental