Motivation to Oscillons/I-balls : α-attractor inflation models

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- An online School -

$$c = \hbar = M_G^2 = 1/(8\pi G) = 1$$

Contents

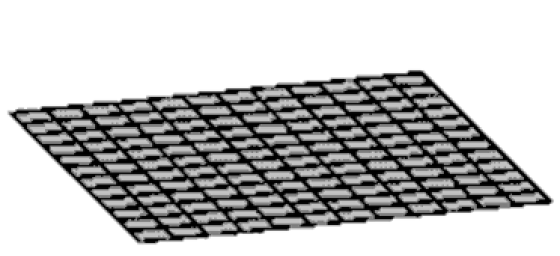
- Introduction
- α-attractor models
- Pole inflation
- After inflation
- Motivation to oscillons/I-balls

c.f. Lectures given by Prof. Wands and Prof. Kinney

Introduction

Generic predictions of inflation, which is an accelerated expansion in the early Universe

- Spatially flat universe



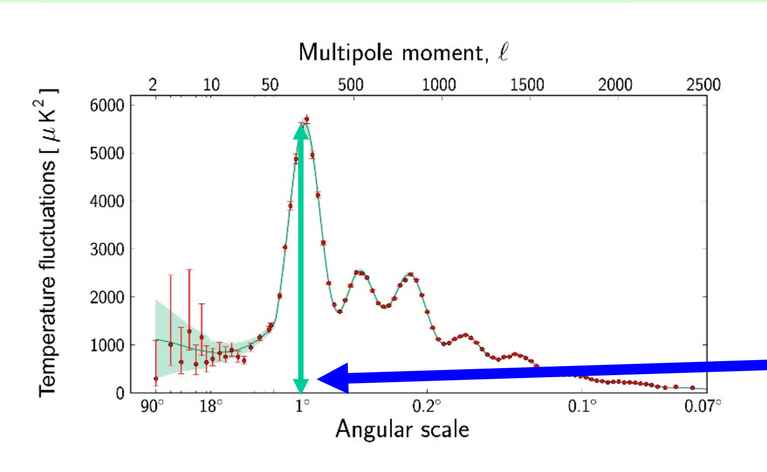
- Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations

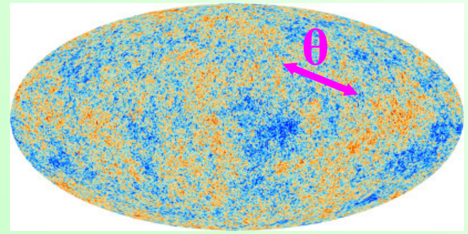
- Almost scale invariant and Gaussian primordial tensor fluctuations

→ Generate observed anisotropies of CMBR.

Inflation is strongly supported by CMB observations

Planck TT correlation :





Green line : prediction by inflation

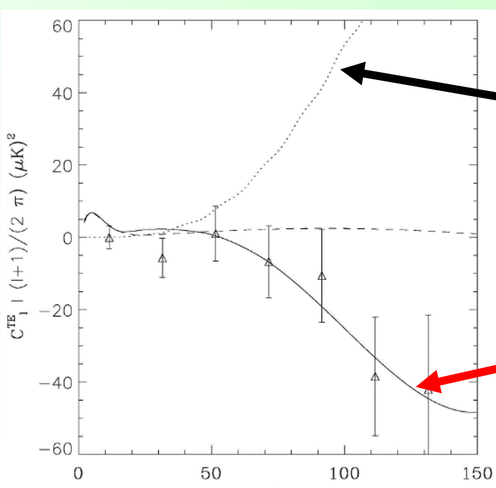
Red points : observation by PLANCK

Angle θ ~ 180° / 1

Total energy density ⇔ Geometry of our Universe

Our Universe is spatially flat as predicted by inflation !!

WMAP TE correlation :



Causal seed models

Superhorizon models

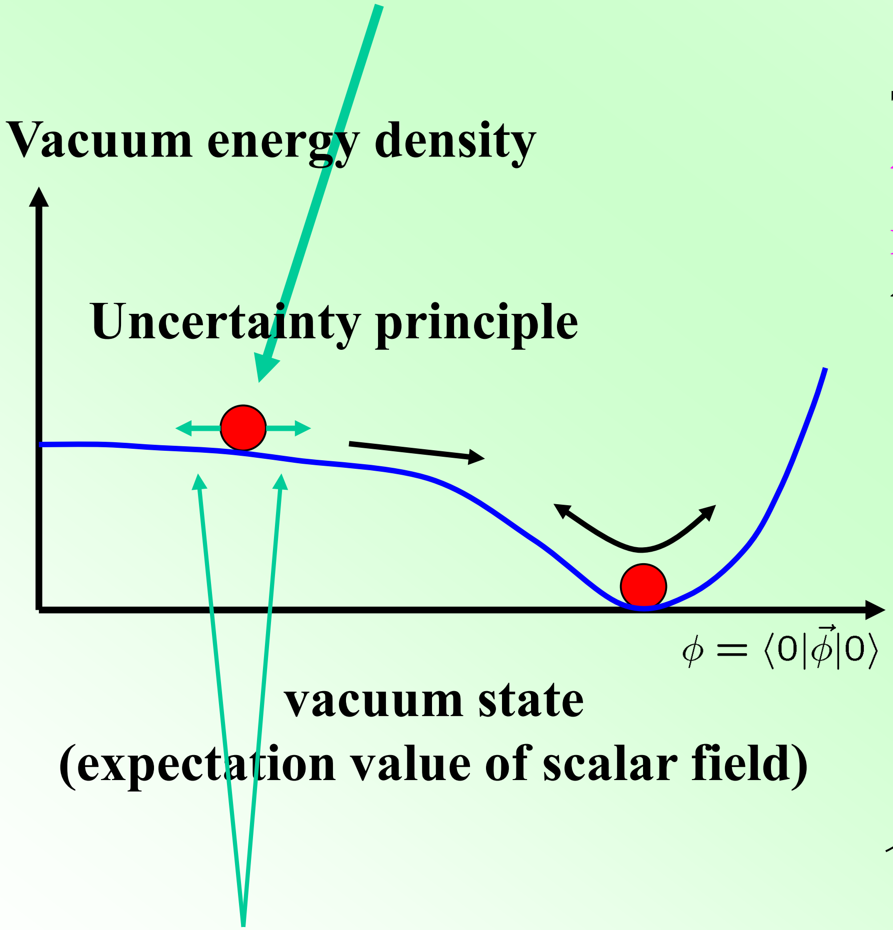
(adiabatic perturbations)

Unfortunately, primordial tensor perturbations have not yet been observed.

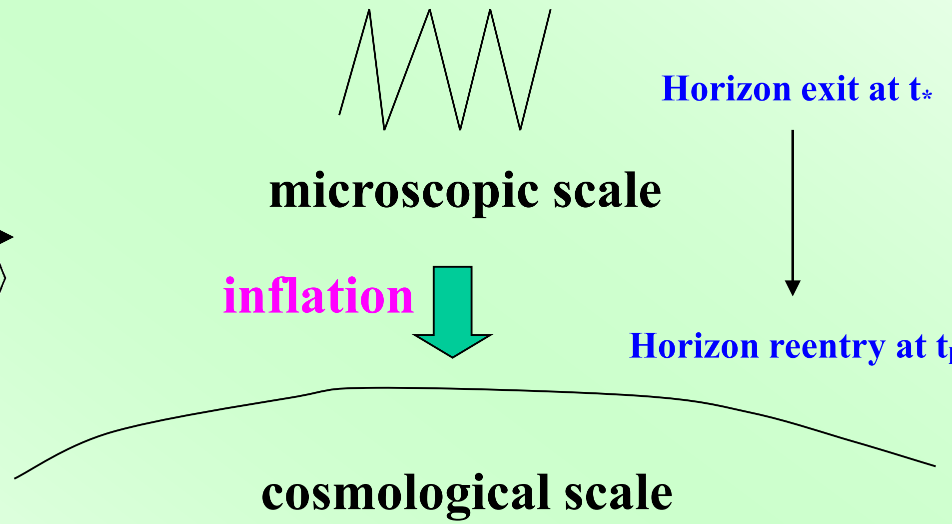
Generation mechanism of primordial perturbations

Primordial density (curvature) fluctuations

The position, φ, fluctuates quantum mechanically.

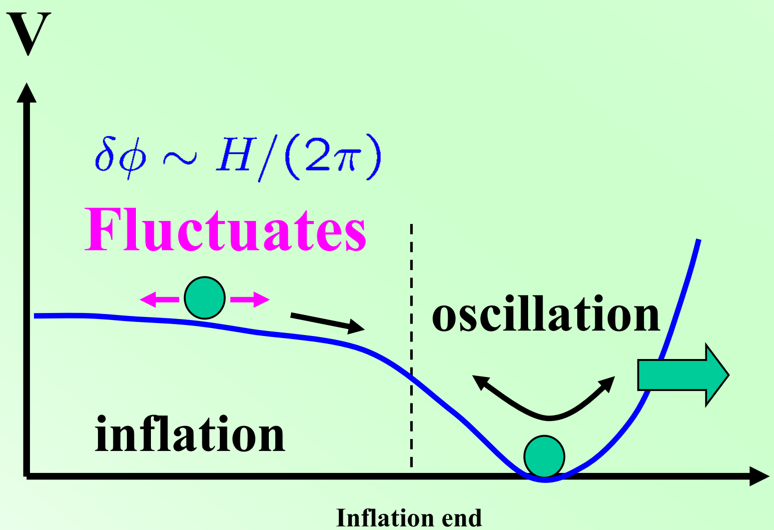


These quantum fluctuations are stretched to cosmological scales thanks to inflationary expansion, and become seeds to produce stars and galaxies.

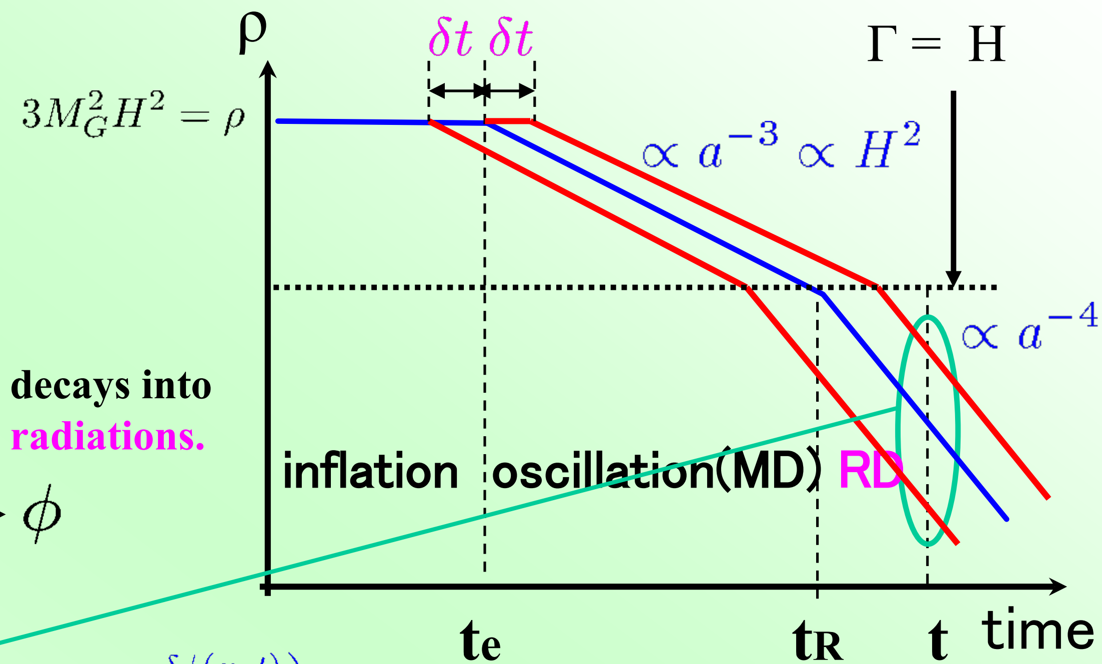


How are these fluctuations transformed into density fluctuations ?

Primordial density fluctuations II



decays into **radiations.**



Curvature perturbation | Density fluctuation | $\left(\delta t(\boldsymbol{x}, t) = \dfrac{\delta\phi(\boldsymbol{x}, t)}{\dot{\phi}_{\rm cl}}\right)$

$$\zeta\Big|_{t_k} \sim \delta\rho/\rho\Big|_{t_k} \sim \delta t/t \sim (\delta\phi/\dot{\phi})/H^{-1} \sim H^2/\dot{\phi}\Big|_{t_*}$$

Almost scale invariant and Gaussian fluctuations are predicted.

(Time translational invariance) (Vacuum fluctuations of a non-interacting field are Gaussian)

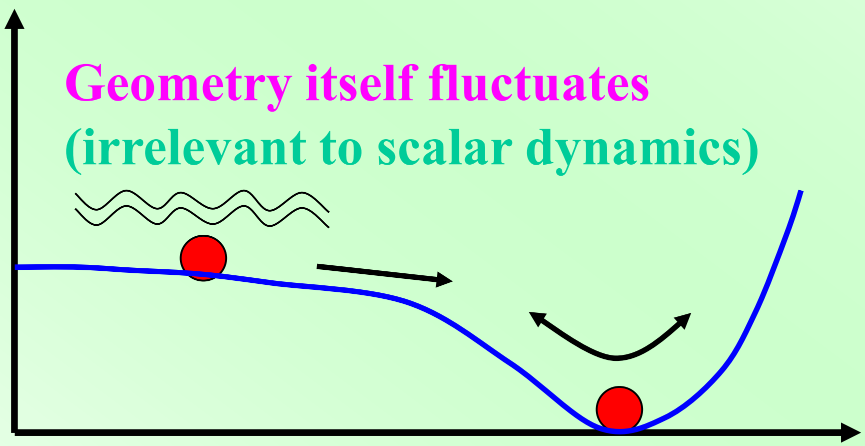
An almost flat potential prohibits
the non-linearity of an interaction (without derivatives).

Primordial tensor fluctuations (gravitational waves)

(Starobinsky)

Vacuum fluctuates quantum mechanically.

Vacuum energy density



vacuum state
(expectation value of scalar field)

$$h \sim \frac{H}{M_G}$$

(directly probes the energy density of the Universe)

$$\left(H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho = \frac{\rho}{3M_G^2} \right)$$

Such vacuum fluctuations generate not only density fluctuations, but also ripples of spacetime, i.e. gravitational waves as quantum gravity effects (quantization of spacetime).

Constraints on scalar and tensor perturbations from the PLANCK satellite

Observational constraints :

$$\begin{cases} \Delta_\zeta(k_0) &= 2.099^{+0.030}_{-0.029} \times 10^{-9}, \\ n_s &= 0.9649 \pm 0.0042, \\ r &< 0.10, \quad \text{(95\% CL TT,TE,EE+lowE+lensing)} \\ & \text{at} \quad k_0 = 0.002\text{Mpc}^{-1}. \end{cases}$$

Theoretical predictions :

$$\begin{cases} \Delta_\zeta(k) &\simeq \dfrac{1}{8\pi^2\epsilon}\left(\dfrac{H}{M_G}\right)^2, \\ n_s - 1 &= \dfrac{d\ln\Delta_\zeta(k)}{d\ln k} \simeq -2\epsilon - 2\eta, \quad \left(\epsilon = -\dfrac{\dot{H}}{H^2}, \quad \eta = \dfrac{\dot{\epsilon}}{\epsilon H}\right) \\ \Delta_h(k) &\simeq \dfrac{2}{\pi^2}\left(\dfrac{H}{M_G}\right)^2, \quad n_T = \dfrac{d\ln\Delta_h(k)}{d\ln k} \simeq -2\epsilon, \\ r &\equiv \dfrac{\Delta_h(k)}{\Delta_\zeta(k)} \simeq 16\epsilon\,(= -8n_T)\,. \end{cases}$$

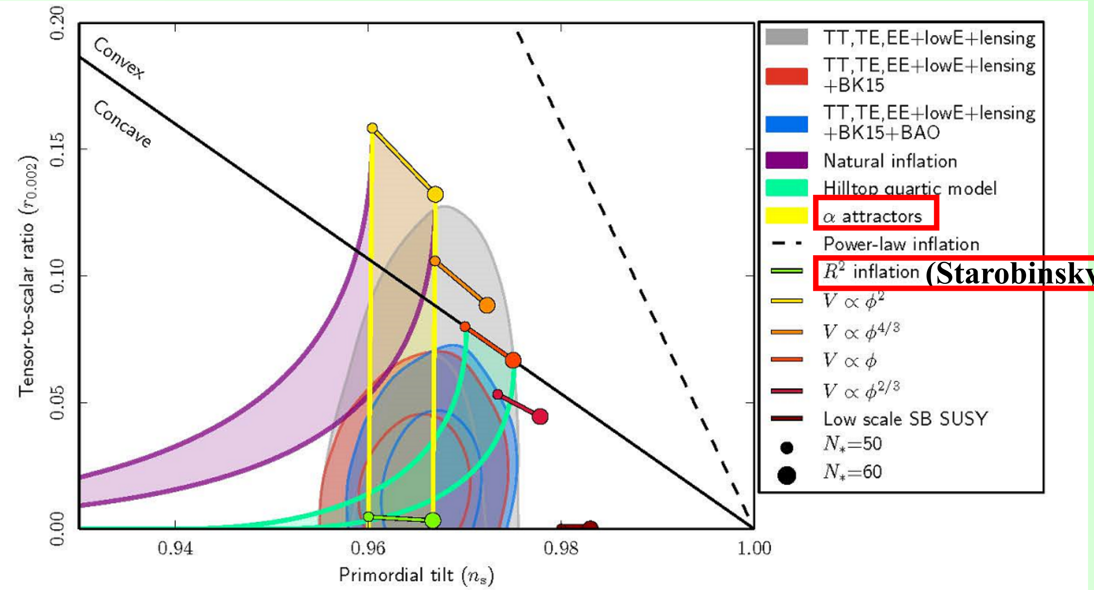


Fig. 8. Marginalized joint 68 % and 95 % CL regions for n_s and r at $k = 0.002\,\text{Mpc}^{-1}$ from *Planck* alone and in combination with BK15 or BK15+BAO data, compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions assume $dn_s/d\ln k = 0$.

Attractor models like Starobinsky model fit the data well.

Planck 2018 results. X 1807.06211

α-attractor models

R^2 (Starobinsky) model

(Starobinsky)

$(M_G = 1)$

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2}R + \frac{R^2}{12M^2}\right)$$

$$\left[\; S = \int d^4x \sqrt{-g}\, f(R) \;\Longleftrightarrow\; S_{\rm eq} = \int d^4x \sqrt{-g}\left(f(\phi) + \frac{df}{d\phi}(R-\phi)\right) \right.$$

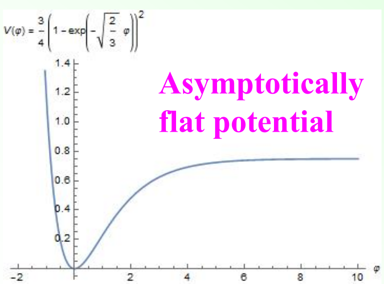
$$\left. \frac{d^2 f}{d\phi^2} \neq 0 \qquad \because\; \frac{\delta S_{\rm eq}}{\delta\phi} = \sqrt{-g}\,\frac{d^2 f}{d^2\phi}(R-\phi) = 0. \;\right]$$

$$S_{\rm eq} = \int d^4x \sqrt{-g}\left[\frac{1}{2}\left(1 + \frac{\phi}{3M^2}\right)R - \frac{\phi^2}{12M^2}\right]$$

Conformal transformation with $\tilde{g}_{\mu\nu} = \Omega(\phi) g_{\mu\nu}, \quad \Omega(\phi) = 1 + \dfrac{\phi}{3M^2}$

$$S_{\rm eq} = \int d^4x \sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{12M^4}\frac{1}{\left(1+\frac{\phi}{3M^2}\right)^2}\,\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\phi^2}{12M^2\Omega^2}\right]$$

$$\left(\frac{d\varphi}{d\phi} = \frac{1}{\sqrt{6}M^2}\frac{1}{1+\phi/(3M^2)} \quad\Longleftrightarrow\quad \phi = 3M^2\left(\exp^{\sqrt{2/3}\,\varphi} - 1\right)\right)$$



$$S_{\rm eq} = \int d^4x \sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{3M^2}{4}\left(1 - \exp^{-\sqrt{2/3}\,\varphi}\right)^2\right]$$

Conformal attractors

(Kallosh & Linde, Ferrara et al., Kallosh et al.)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{1}{12}\left(\chi^2-\phi^2\right)R-\frac{1}{36}\left(\chi^2-\phi^2\right)^2F\left(\frac{\phi}{\chi}\right)\right]$$

Local (gauge) conformal symmetry :

$$\tilde{g}_{\mu\nu}=e^{-2\sigma(x)}g_{\mu\nu},\quad \tilde{\chi}=e^{\sigma(x)}\chi,\quad \tilde{\phi}=e^{\sigma(x)}\phi$$

(global SO(1,1) symmetry for constant F(φ/χ))

N.B. χ has wrong sign of kinetic term : compensator field

Gauge fixing with $\chi^2-\phi^2=6$ **:** $\chi=\sqrt{6}\cosh\left(\frac{\varphi}{\sqrt{6}}\right),\quad \phi=\sqrt{6}\sinh\left(\frac{\varphi}{\sqrt{6}}\right)$

(Einstein frame)

⇒ $$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{6}F\left(\tanh\frac{\varphi}{\sqrt{6}}\right)\right]$$

- **If F is constant, the potential is simply C.C.**
- **If F is smooth, the potential is stretched for large** φ
- **Starobinsky model** ⇔ $F\left(\frac{\phi}{\chi}\right)=\frac{3M^2}{(1+\chi/\phi)^2}$

Conformal attractors II

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{1}{12}\left(\chi^2-\phi^2\right)R-\frac{1}{36}F\left(\frac{\phi}{\chi}\right)\left(\chi^2-\phi^2\right)^2\right]$$

- **Gauge fixing with $\chi=\sqrt{6}$:**

(Jordan frame)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\left(1-\frac{\phi^2}{6}\right)R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{36}F\left(\frac{\phi}{\sqrt{6}}\right)\left(6-\phi^2\right)^2\right]$$

Conformal transformation with $\tilde{g}_{\mu\nu}=\Omega(\phi)g_{\mu\nu},\quad \Omega(\phi)=1-\frac{\phi^2}{6}$

(Einstein frame)

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R}-\frac{1}{2}\frac{1}{\left(1-\frac{\phi^2}{6}\right)^2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-F\left(\frac{\phi}{\sqrt{6}}\right)\right]$$

$$\left(\frac{d\varphi}{d\phi}=\frac{1}{1-\phi^2/6}\iff\frac{\phi}{\sqrt{6}}=\tanh\left(\frac{\varphi}{\sqrt{6}}\right)\right)\qquad\left(\phi\to\sqrt{6}\iff\varphi\to\infty\right)$$

Same action

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2}\tilde{R}-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-\frac{1}{6}F\left(\tanh\frac{\varphi}{\sqrt{6}}\right)\right]$$

The pole structure of the kinetic term stretch the potential effectively !!

- **α attractors :**

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}\frac{\alpha}{\left(1-\frac{\phi^2}{6}\right)^2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-F\left(\frac{\phi}{\sqrt{6}}\right)\right]$$

(Conformal attractors including Strobinsky model correspond to α= 1.)

Pole inflation

Pole inflation

(Galante et al., Broy et al.)

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}K(\rho)g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho - V(\rho)\right]$$

- **K(ρ) has a pole at ρ= 0 in Laurent series :**

$$K(\rho) = \frac{a_p}{\rho^p} + \cdots ,$$

canonical field φ

$$\rho \simeq \begin{cases} \rho_0 e^{-\frac{\varphi}{\sqrt{a_2}}} & \text{for } p=2 \\ \left(\frac{(p-2)}{2\sqrt{a_p}}\varphi\right)^{-\frac{2}{p-2}} & \text{for } p>2 \end{cases}$$

- **V(ρ) is regular at ρ= 0 :**

$$V(\rho) = V_0\,(1 - \rho + \cdots)$$

$$V(\varphi) \simeq \begin{cases} V_0\left[1 - \rho_0 e^{-\frac{\varphi}{\sqrt{a_2}}}\right] & \text{for } p=2 \\ V_0\left[1 - \left(\frac{(p-2)}{2\sqrt{a_p}}\varphi\right)^{-\frac{2}{p-2}}\right] & \text{for } p>2 \end{cases}$$

(asymptotically flat)

Primordial perturbations :

$$n_s - 1 \simeq -\frac{p}{p-1}\frac{1}{N},$$

$$r \simeq \frac{8}{a_p}\left[\frac{a_p}{(p-1)N}\right]^{\frac{p}{p-1}} .$$

α attractors ⟺ p=2, a_2 = 3α/2

a_p dependence appears only in r.

Subleading terms yield higher order corrections.

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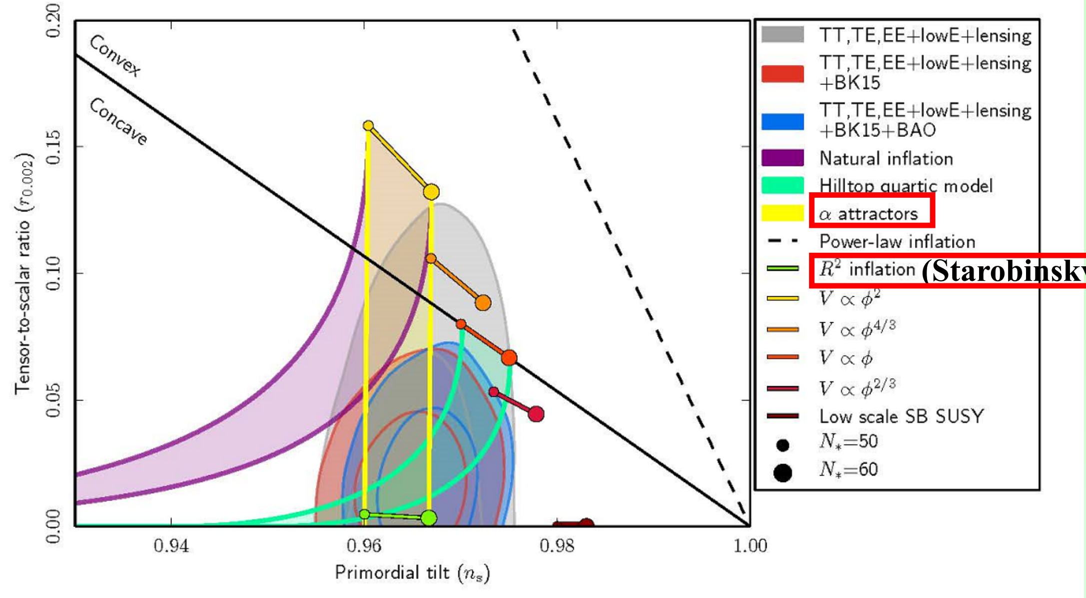

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Planck 2018 results. X 1807.06211

Higgs inflation

Higgs potential (tree-level) :

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2,$$
$$\simeq \frac{\lambda}{4}h^4 \quad \text{for} \quad h \gg v.$$

$H = h / \sqrt{2}$ in Unitary gauge
λ: self-coupling constant
v : expectation value

Chaotic inflation with quartic term

$\lambda \sim 10^{-13}$, too large tensor-to scalar ratio

But, these constraints are obtained
in the minimal coupling to Gravity.

Non-minimal coupling

Model of Higgs inflation

(Futamase & Maeda, Cervantes-Cota & Dehnen, Bezrukov & Shaposhnikov)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\left(1+\xi h^2\right)R-\frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h-\frac{\lambda}{4}\left(h^2-v^2\right)^2\right].$$

(ξ> 0)

$$\tilde{g}_{\mu\nu}=\Omega(h)g_{\mu\nu},\quad \Omega(h)=1+\xi h^2.$$

$$\frac{d\varphi}{dh}=\frac{\sqrt{1+(\xi+6\xi^2)h^2}}{1+\xi h^2}\simeq\frac{\sqrt{6}}{h},\quad h\simeq\frac{1}{\sqrt{\xi}}\exp\left(\frac{\varphi}{\sqrt{6}}\right).$$

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{\tilde{R}}{2}-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-U(\varphi)\right].$$

$$U(\varphi)\simeq\frac{\lambda}{4\xi^2}\left[1-2\exp\left(-\frac{2\varphi}{\sqrt{6}}\right)\right].$$

asymptotically flat potential !!

Pole structure of Higgs inflation

(Futamase & Maeda, Cervantes-Cota & Dehnen, Bezrukov & Shaposhnikov)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\left(1+\xi h^2\right)R-\frac{1}{2}g^{\mu\nu}\partial_\mu h\partial_\nu h-\frac{\lambda}{4}\left(h^2-v^2\right)^2\right].$$

($\xi > 0$)

$$\tilde{g}_{\mu\nu}=\Omega(h)g_{\mu\nu},\quad \Omega(h)=1+\xi h^2.$$

$$\Rightarrow\quad \mathcal{L}_K^E=\left(\frac{3\Omega'^2}{2\Omega^2}+\frac{1}{\Omega}\right)(\partial h)^2=\left(\frac{3}{2\Omega^2}+\frac{1}{\Omega\Omega'^2}\right)(\partial\Omega)^2=\left(\frac{3}{2\rho^2}+\frac{\rho}{\rho'^2}\right)(\partial\rho)^2$$

$$\left(\rho=\frac{1}{\Omega}=\frac{1}{1+\xi h^2}\right)$$

$$\rho\to 0\quad\Longleftrightarrow\quad \Omega(h)\to\infty$$

$$\Rightarrow\quad K_E(\rho)=\left(\frac{3}{2\rho^2}+\frac{\rho}{\rho'^2}\right)=\frac{3}{2}\frac{1}{\rho^2}+\frac{1}{4\xi}\frac{1}{\rho^2(1-\rho)}=\frac{3\alpha}{2}\frac{1}{\rho^2}+\frac{1}{4\xi}\frac{1}{\rho}+\cdots.$$

Leading term coincides with α attractors !! $\left(\alpha=1+\frac{1}{6\xi}\right)$

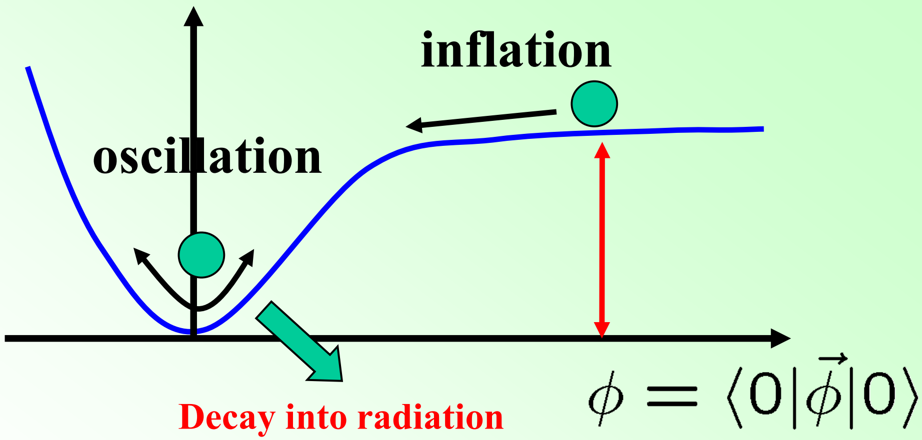
(Density perturbations ➔ ξ~ 10^4 ➔ α ~ 1)

After inflation

From inflation to big-bang

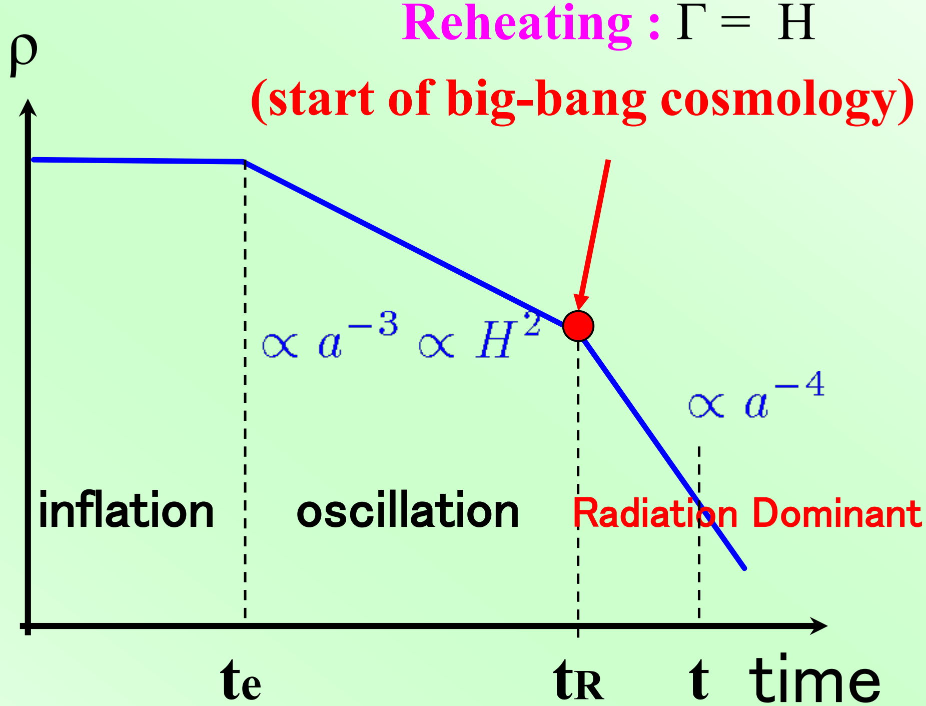
After the rapid expansion (inflation) ends, the vacuum energy is released as the latent heat (called "re"heating) so that the hot and dense Universe (Big-bang Universe) is realized.

Vacuum energy density : V(φ)

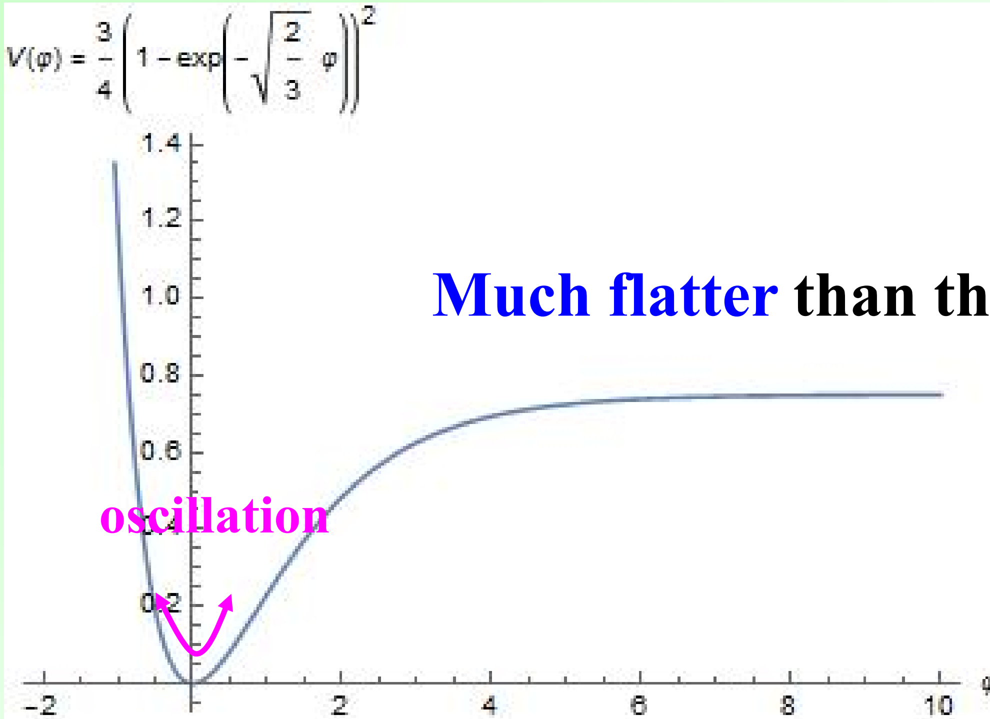


State of vacuum (vacuum expectation value of a scalar field)

Reheating : Γ = H
(start of big-bang cosmology)



Very flat potential



Much flatter than the mass term, φ^2

- **During the oscillation period, can a non-topological soliton like Q-ball be formed ???**
- **But, apparently, there is no conserved charge for a real scalar field !!**

Oscillon/I-ball

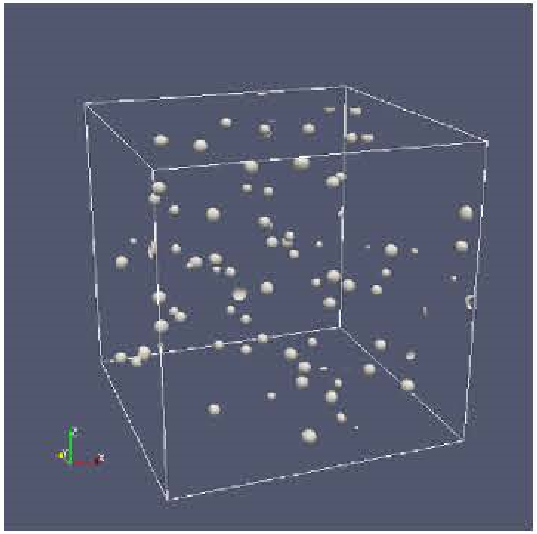


Figure 3. Oscillons in the simulations for $V_A(\phi; \alpha_1 = 1/2)$ with $\epsilon_G = 10^{-2}$. The isosurface with $\rho = 1 m^2 M^2$ at $\eta = 400 m^{-1}$ is shown.

Figure taken from Hiramatsu, Sfakianakis, and MY 2021

Oscillons/I-balls can be formed after inflation.

Summary

- The observational data can be fitted well by
  α-attractor inflation models like R^2 (Starobinsky) model.

- α-attractor models can be understood as a part of pole inflation,
  in which the pole structure of a kinetic term effectively stretches
  a potential after the canonical normalization.

- Such a potential is flatter than the mass term.
  Then, during the oscillation period, can a non-topological soliton
  like Q-ball be formed for such a potential ???

- Yes.
  A non-topological soliton called oscillon/I-ball is formed.

➡ Shall we discuss more detailed properties of oscillon/I-ball
through whiteboard.