Topological aspects of strong correlations and gauge theories (ONLINE)

INTERNATIONAL CENTRE for THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Topology, scaling, and a threshold in hot QCD

8 September 2021

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A. Yu. Kotov, MpL, A. Trunin arXiv:2105.09842

A. Yu. Kotov, MpL, A. Trunin *Eur.Phys.J.A* 56 (2020)

Talks by A. Yu. Kotov and A Trunin at Lattice2021

Reviews

A. Trunin, MpL • *Int.J.Mod.Phys.A* 35 (2020)

A. Yu. Kotov, MpL, A. Trunin Symmetry, 2021, to appear

About QCD plasma

Crossover from strongly coupled around Tc to weakly coupled at high temperatures

Questions for this talk

Does it happen suddenly?
Which is the role of topology, if any?
Which is the role of the critical point and the critical region?

Outline:

Topology
QCD Transition
Critical region

Crossover from strong to weak coupling common?

It is possible to couple QCD to topological charge

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

$$\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a = q(x)$$

Q — topological charge

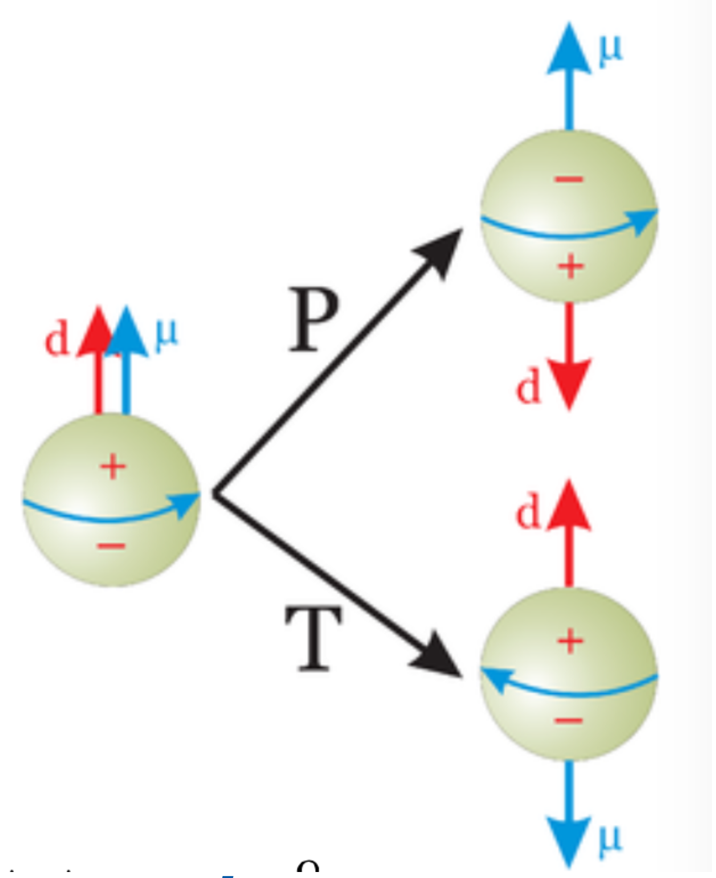
$$Q = \sum q(x)$$

CP-violating term

CP violation in QCD?

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

$$\frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a = q(x)$$



Diagnostic of CP violation: Neutron electric dipole moment d_n

- QCD sum rules: $d_n = 2.4 \times 10^{-16}\theta$ e cm **Pospelov(1999)**
- Chiral perturbation theory: $d_n = 3.6 \times 10^{-16}\theta$ e cm **Pich(1991)**
- Experiments: $d_n = (0.0 \pm 1.1$ (stat) ± 0.2 (sys)$)$ $\times 10^{-26}$ e cm
  $|d_n| < 1.8 \times 10^{-26}$ e cm 90% C.L, **nEDM, Abel et al. (2020)**

$$\theta < 0.5 \times 10^{-10}$$ = 0 here!

The Grand Canonical Partition Function

$$\mathcal{Z}(\theta, T) = \int \mathcal{D}[\Phi]\, e^{-T\sum_t \int d^3x \mathcal{L}(\theta)} = e^{-VF(\theta,T)}.$$

$$P_Q = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} e^{-i\theta Q} e^{-VF(\theta)},\ Q = \int d^4x\, q(x)$$

Taylor expansion..

$$F(\theta, T) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\theta^{2n}}{2n!}\, C_n$$

..and cumulants of the topological charge:

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$

The free energy $F(\theta, T)$

- At zero and low (below or around T_c) temperatures: ChPT
  Villadoro et al. (2015) At leading order

$$F(\theta, T) - F(0, T) = -m_\pi^2 f_\pi^2 \sqrt{(1 - \frac{m_u m_d}{(m_u + m_d)^2} sin^2(\frac{\theta}{2f_a}} \; \theta$$

- At high temperature: DIGA and high temperature perturbation theory Gross et al(1980),Ringwald(1999) At leading order

$$\boxed{F(\theta, T) - F(0, T) \simeq T^{4-\beta_0} \left(\frac{m}{T}\right)^{N_f} (1 - \cos\theta)}$$

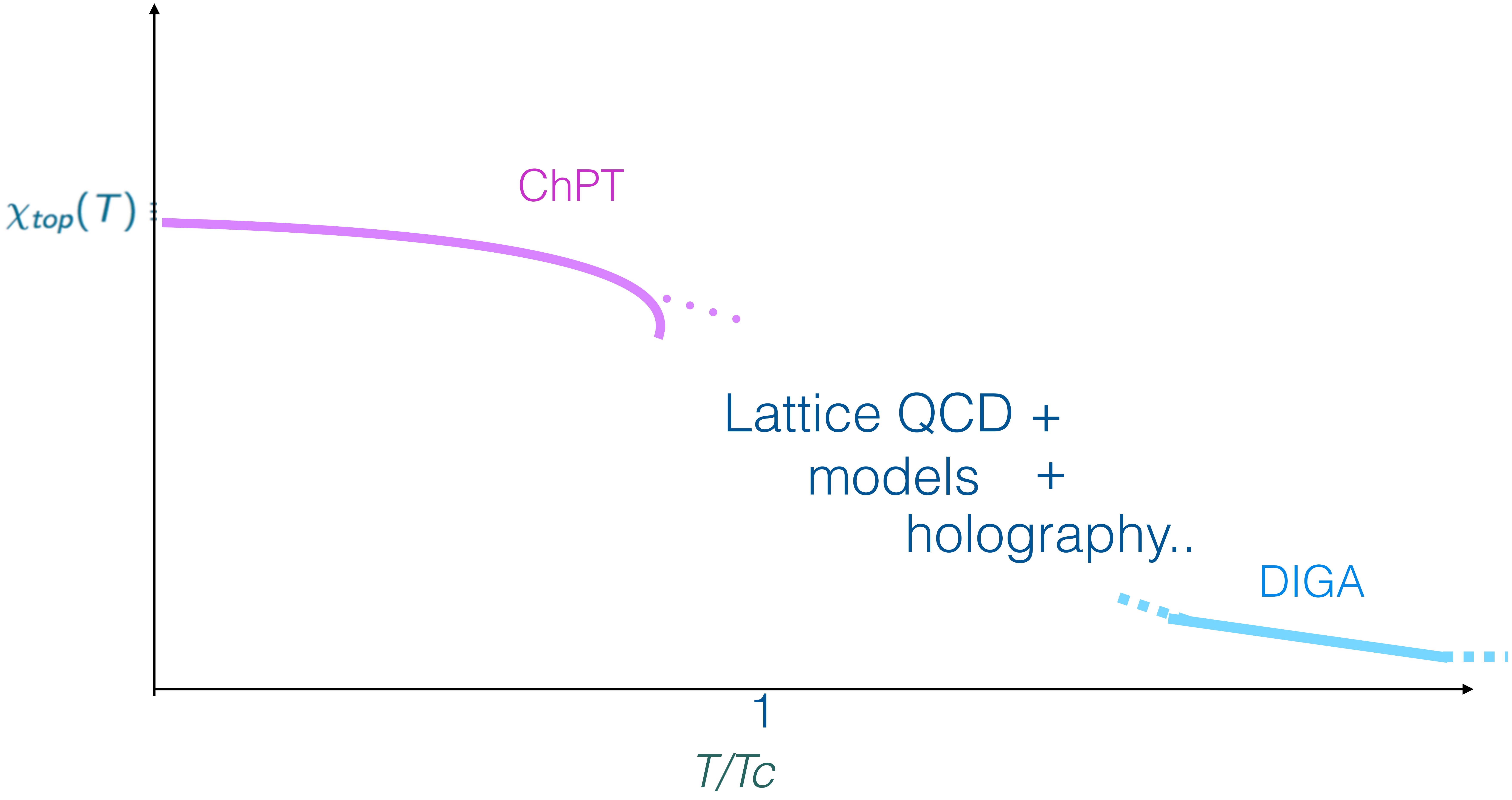
Main diagnostic tool

$\beta_0 = 11N_c/3 - 2N_f/3$

- Around T_c : The interpretation of QCD around T_c in terms of topological structures is of a subject of active research

  Larsen,Sharma,Shuryak et al (2020,–).

What do we know about $\chi_{top}(T) \equiv \left.\frac{\partial^2 F(\theta, T)}{\partial \theta^2}\right|_{\theta=0}$



Lattice QCD and θ term

Lattice simulations rely on a sampling of the phase space weighted with the e^{-S}, where S the Euclidean action.

$$Z = \int \mathcal{D}[U]\, e^{-S_{lat}[U]}$$

- Sign problem: In Euclidean space-time the Minkowskian Lagrangian becomes *complex* for real values of θ.
- Way out: Taylor expansion around $\theta = 0$ or an analytic continuation from imaginary values of θ **Bonati, D'Elia et al. 2019**.

In the following, topological susceptibility and cumulants will be measured at $\theta = 0$:

$$\chi_{top}(T) \equiv \left. \frac{\partial^2 F(\theta, T)}{\partial \theta^2} \right|_{\theta=0}$$

Topology, symmetries and spectrum

$$\mathcal{L} = \sum_{a=1}^{n} \bar{q}_{La} \not{\partial} q_{La} + \bar{q}_{Ra} \not{\partial} q_{Ra} - m(\bar{q}_{La} q_{La} + \bar{q}_{Ra} q_{Ra}) + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a + \mathcal{L}_{gauge}$$

With $m = 0$, invariant under

$q_L \to V_L q_L q_R \to V_R q_R$, with $V \in U(n)$

Global symmetry:

$$U(n)_L \times U(n)_R \cong SU(n) \times SU(n) \times U(1)_V \times U(1)_A$$

.

baryon number

Spontaneously Broken, (n^2- 1) GB

Explicitely broken

Symmetries of QCD

$$\mathcal{L} = \sum_{a=1}^{n} \bar{q}_{La} \partial\!\!\!/ q_{La} + \bar{q}_{Ra} \partial\!\!\!/ q_{Ra} - m(\bar{q}_{La} q_{La} + \bar{q}_{Ra} q_{Ra}) + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a + \mathcal{L}_{gauge}$$

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.

Spontaneously Broken, (n^2 - 1) GB

baryon number

Explicitely broken

Experimental Evidence

Evidences of the explicit $U(1)_A$ breaking.

The $U_A(1)$ symmetry $q \to e^{i\alpha\gamma_5} q$

would be broken by the (spontaneously generated) $\bar{q}q$:

the candidate Goldstone is the η'

too heavy!! (900 MeV)

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c^2)
Pion[6]	π^+	π^-	$u\bar{d}$	139.570 18 ±0.000 35
Pion[7]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a]	134.9766 ±0.0006
Eta meson[8]	η	Self	$\frac{u\bar{u}+d\bar{d}-2s\bar{s}}{\sqrt{6}}$ [a]	547.862 ±0.018
Eta prime meson[9]	$\eta'(958)$	Self	$\frac{u\bar{u}+d\bar{d}+s\bar{s}}{\sqrt{3}}$ [a]	957.78 ±0.06
Kaon[12]	K^+	K^-	$u\bar{s}$	493.677 ±0.016
Kaon[13]	K^0	$\bar{K}^0$	$d\bar{s}$	497.614 ±0.024

BUT:
the divergence of the current $j_5^\mu = \bar{q}\gamma_5\gamma_\mu q,$

contains a mass independent term

$$\partial_\mu j_5^\mu = m\bar{q}\gamma_5 q + \frac{1}{32\pi^2} F\tilde{F}.$$

IF $\frac{1}{32\pi^2}\int d^4x F\tilde{F} \neq 0$

The $U_A(1)$ symmetry is **explicitly** broken

Topology, η' and the $U_A(1)$ problem:

It can be proven that

$$\frac{1}{32\pi^2}\int d^4x F\tilde{F} = Q$$

Gluonic definition

and

$$Q = n_+ - n_-$$

Fermionic definition

The η' mass may now be computed from the decay of the correlation

Q(x) Q(y)

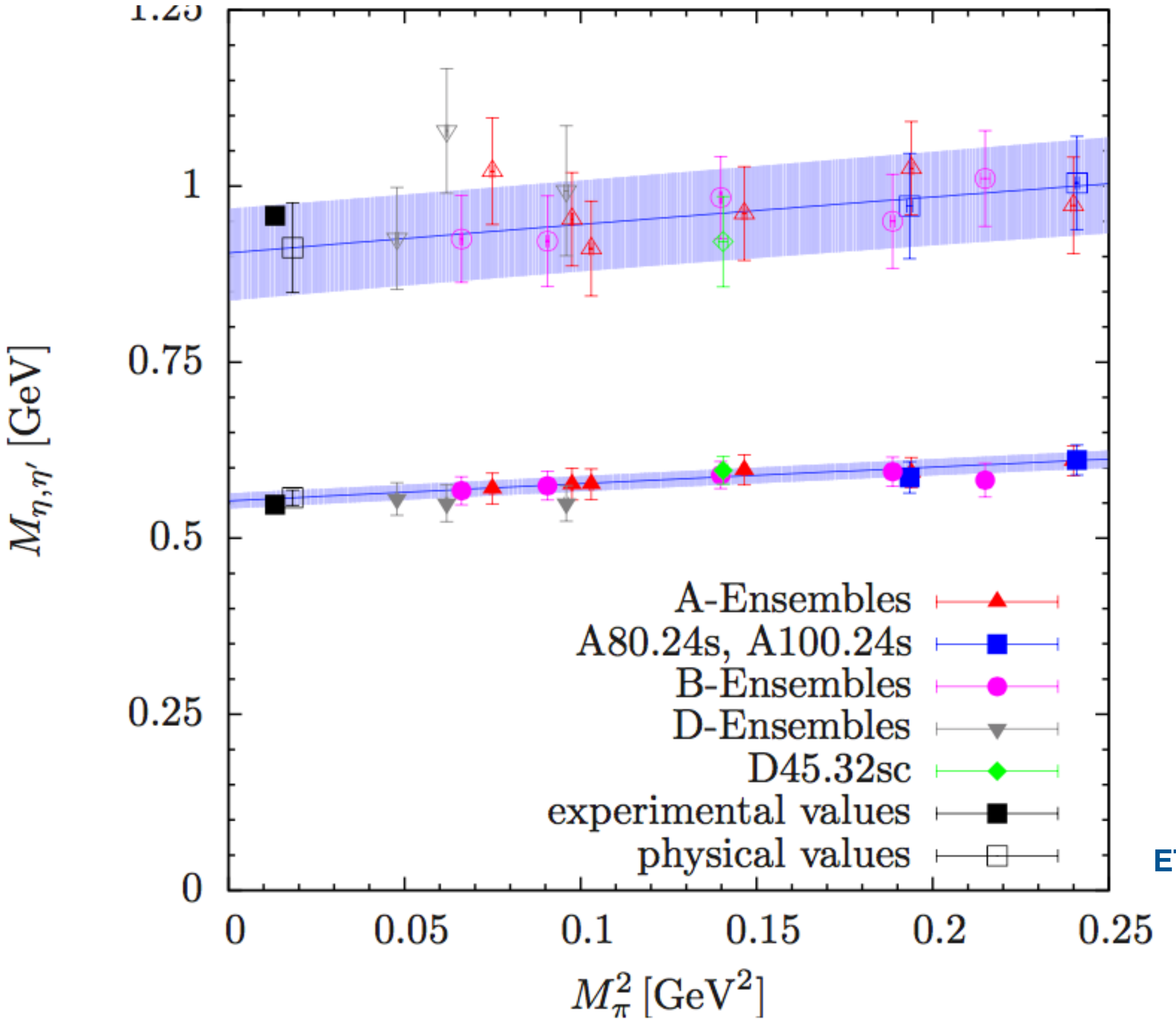
$$\langle \partial_\mu j_5^\mu(x) \partial_\mu j_5^\mu(y) \rangle \propto \frac{1}{N^2} \langle F(x)\tilde{F}(x) F(y)\tilde{F}(y) \rangle$$

which at leading order gives the Witten-Veneziano formula

$$\boxed{m_{\eta'}^2 = \frac{2N_f}{F_\pi^2}\chi_t^{\mathrm{qu}}}$$

Successful at T=0

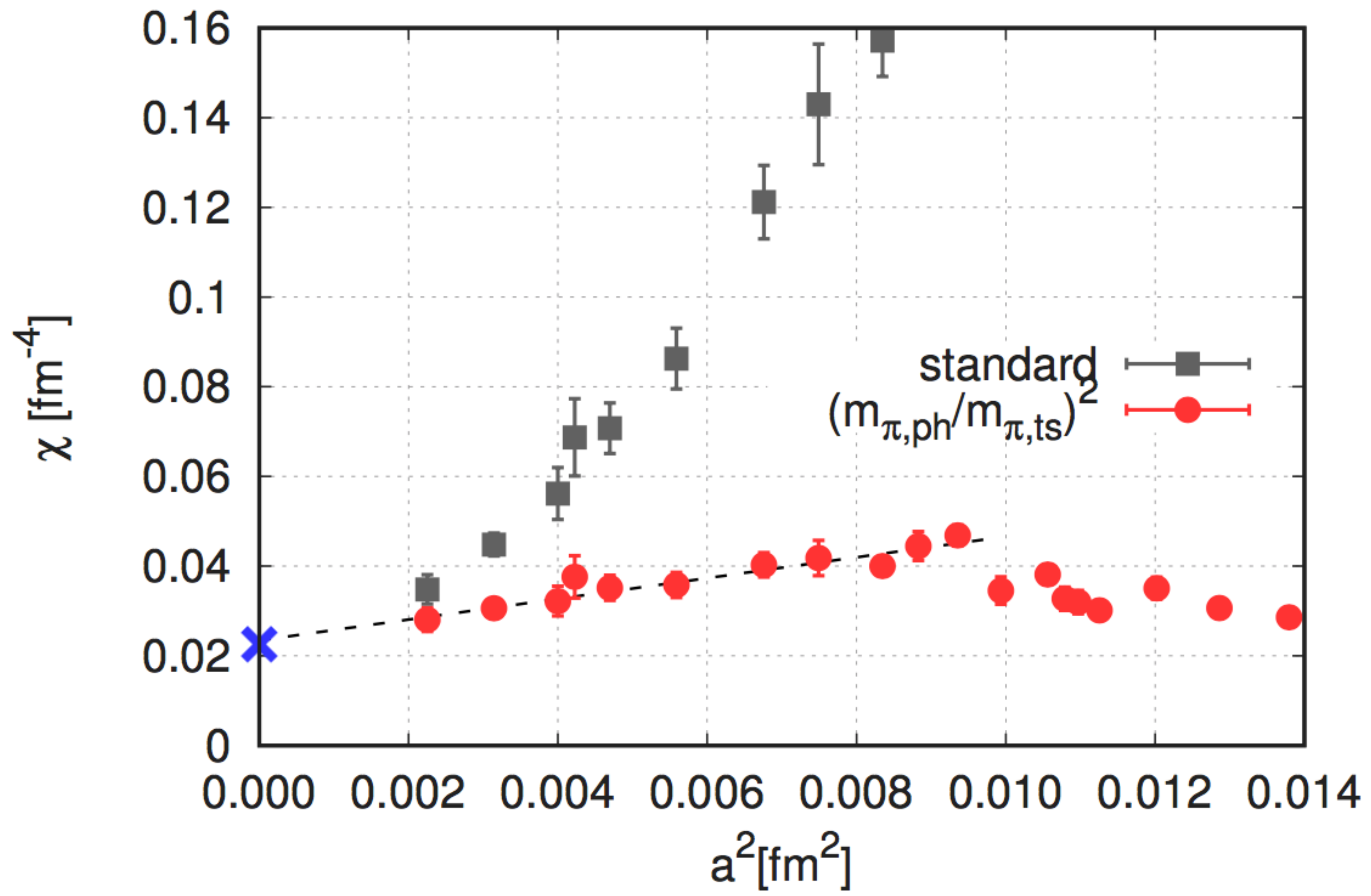
Evidences of the explicit $U(1)_A$ breaking.



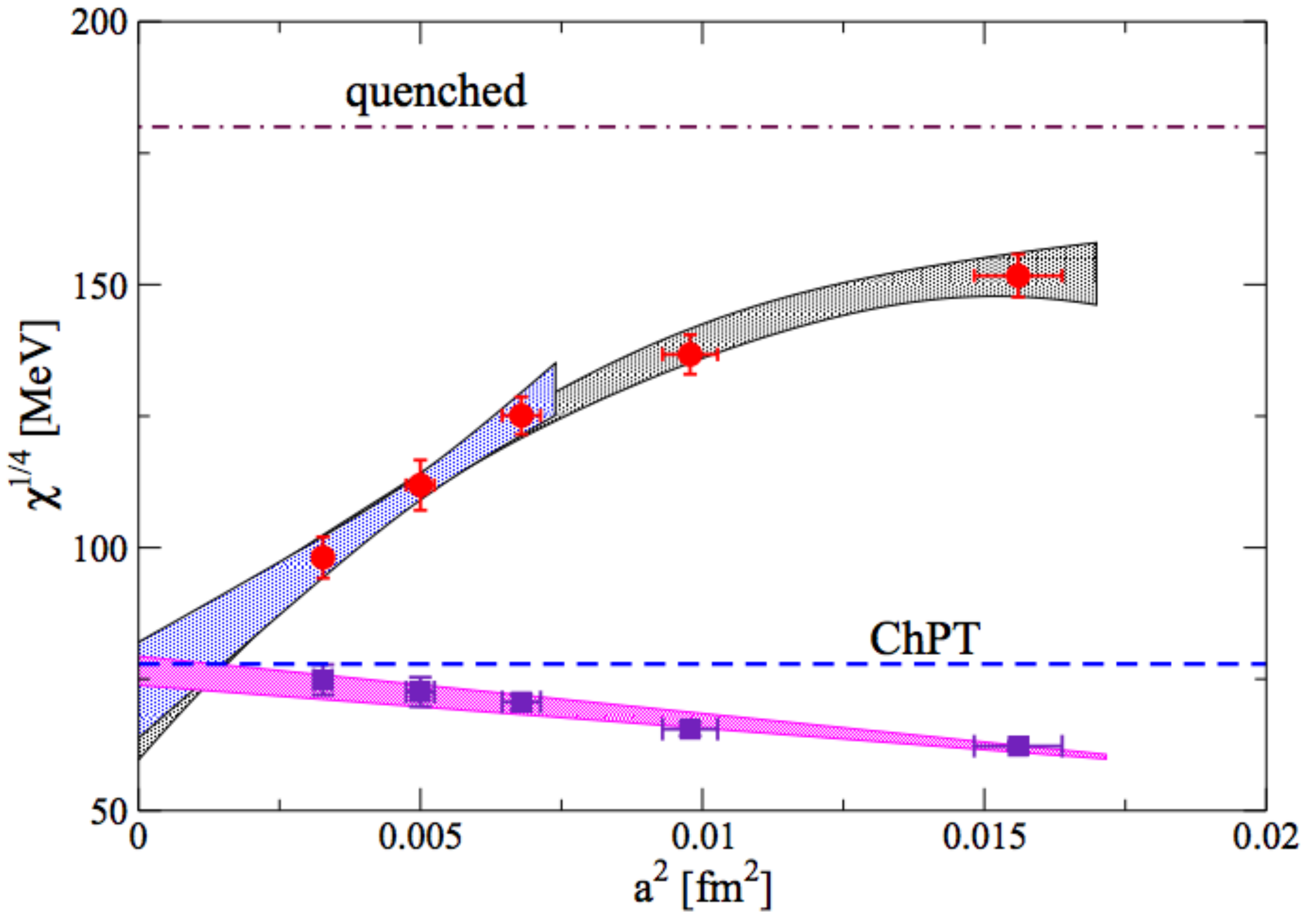
ETMC 2017

T=0

QCD Topological Susceptibility



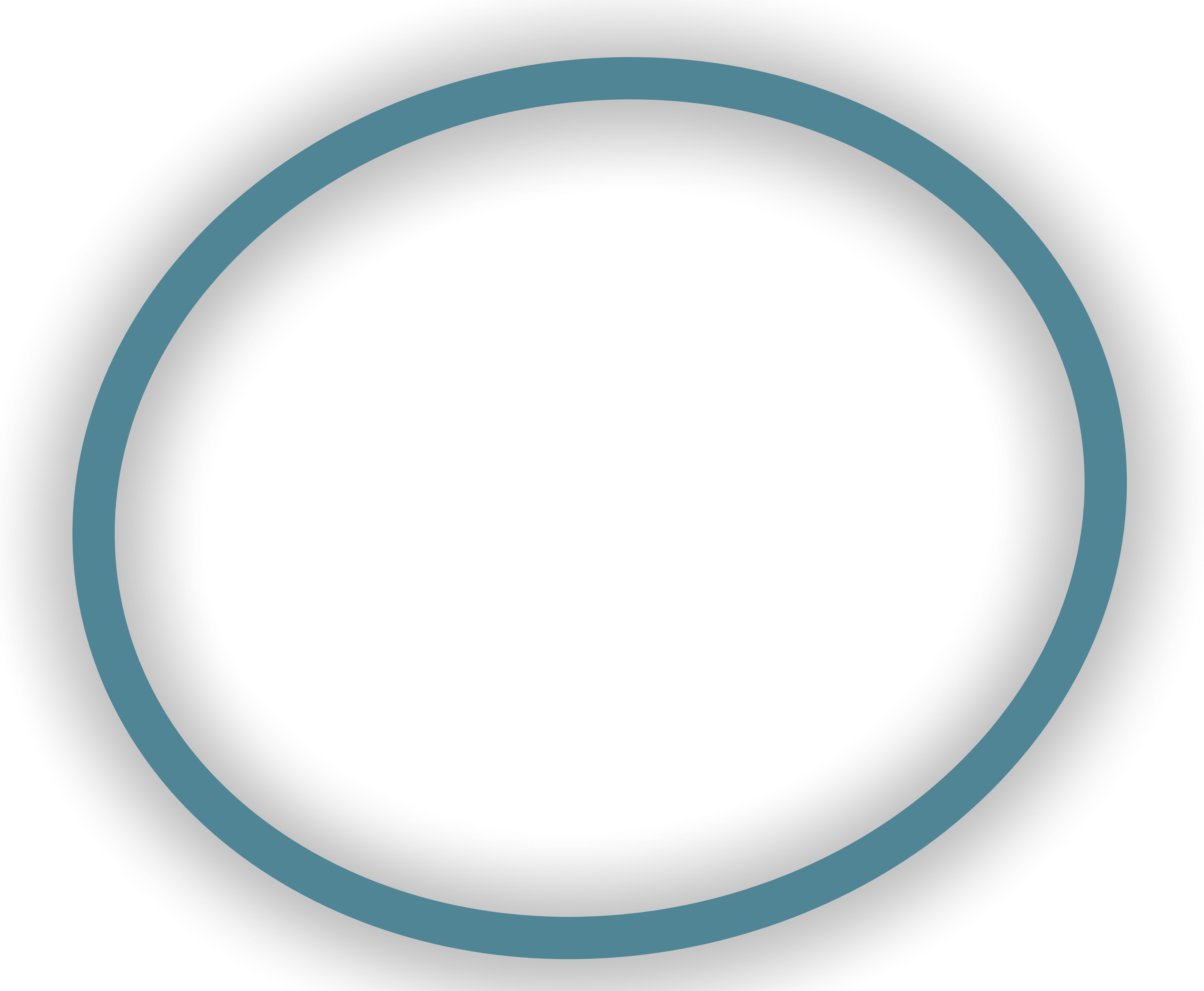
ETMC (2017)



Bonati , D'Elia et al (2016)

Topology from low to high Temperature

What happens to topology in the Quark Gluon Plasma?



Lattice Topology Michael Mueller-Preussker(2015)

- Gluonic:**Luscher(2010), Bonati,d'Elia e al (2014),Alexandrou et al . (2015)**

$$Q = \frac{a^4}{32\pi^2}\varepsilon_{\mu\nu\rho\sigma}\sum_n \mathrm{Tr}[F^{\mu\nu}_{lat}(n)F^{\rho\sigma}_{lat}(n)],$$

Need smooth configurations,using smearing,cooling, gradient flow..

$$\dot{V}_\mu(n,\tau) = -g^2[\partial_{n,\mu}S_G(V(\tau))]V_\mu(n,\tau), \qquad V_\mu(n,0) = U_\mu(n),$$

Pros: Easy
Cons: suffers very much from lattice artifacs

- Fermionic:**Atiyah Singer(1971,1984)**

$$Q = \frac{1}{32\pi^2}\varepsilon_{\mu\nu\rho\sigma}\int \mathrm{Tr}[F^{\mu\nu}(x)F^{\rho\sigma}(x)]\,d^4x = n_+ - n_-$$

Pros: not affected but UV fluctuations
Cons: very high computational cost

- Fermionic - simple but approximate: **Kogut et al.(1996),Petreczky, Sharma(2016)**

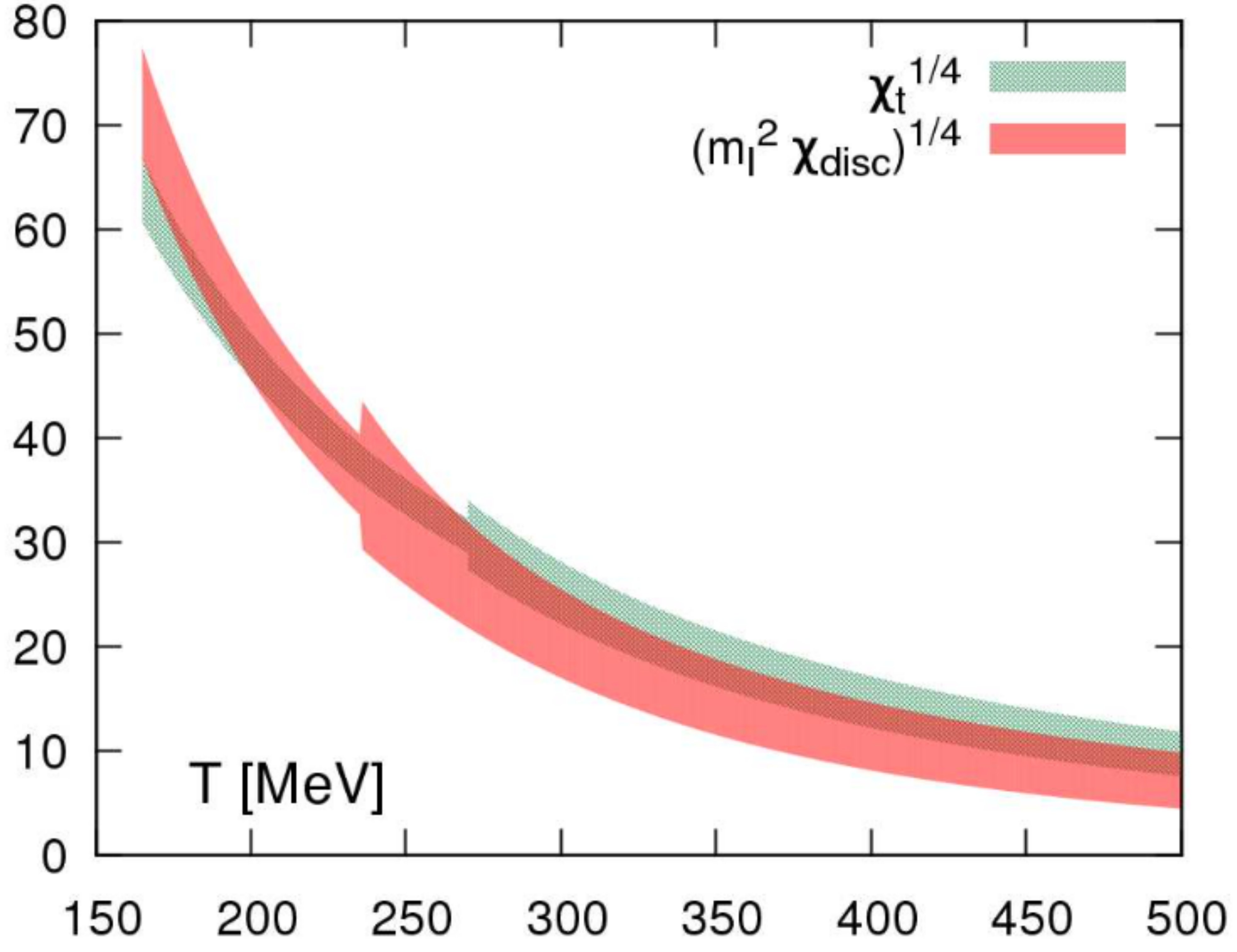
$$\chi_{top} = \frac{\langle Q^2\rangle}{V} = m_l^2\,\chi_{5,disc}$$

$$\chi_{top}(T \gtrsim T_c) = m_l^2\,\chi_{disc} = m_l^2\,\frac{V}{T}\left(\langle(\bar\psi\psi)^2\rangle_l - \langle\bar\psi\psi\rangle_l^2\right).$$

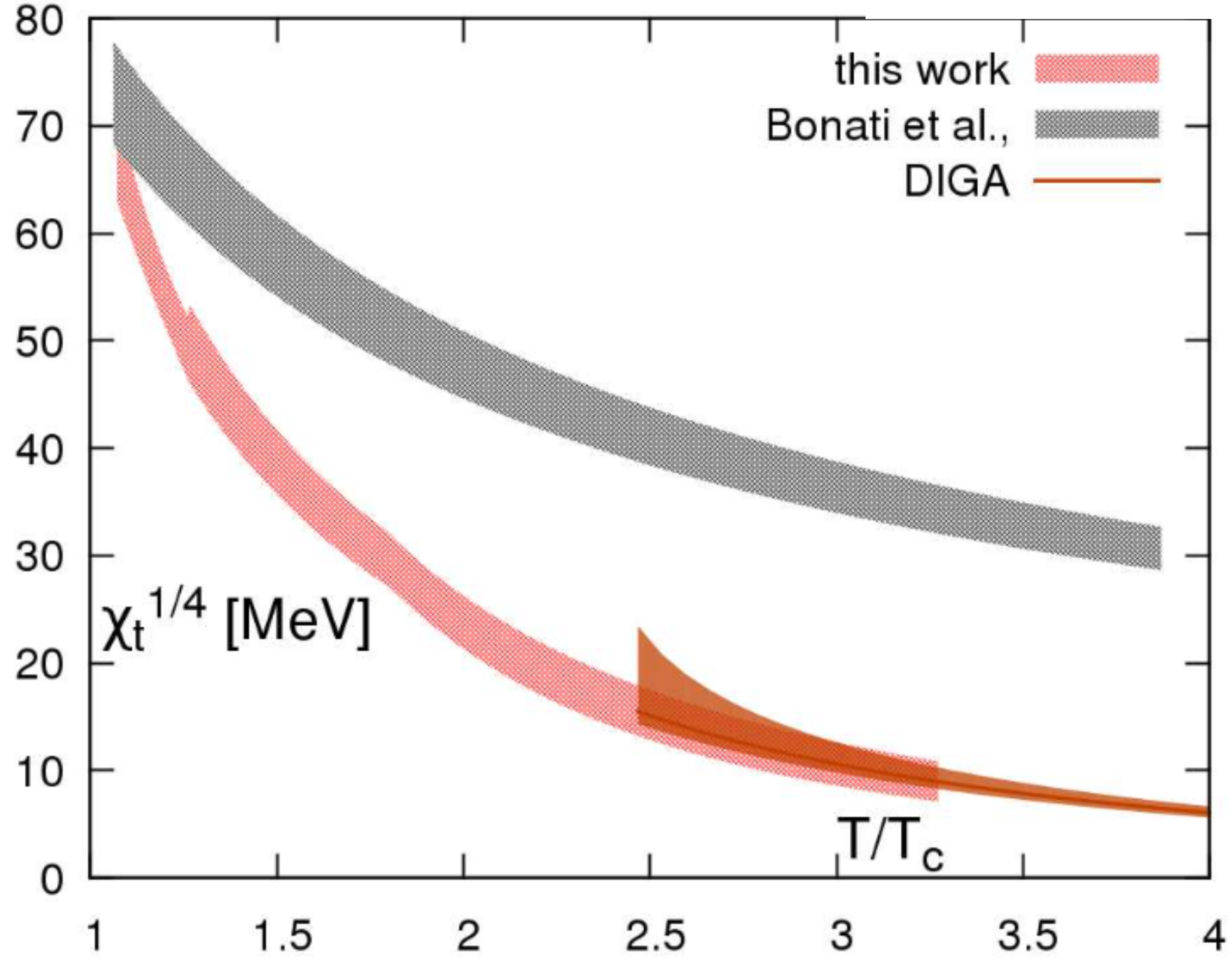
Petreczky, Schlaeder, Scharma (2016)

$$T^{4-\beta_0}\left(\frac{m}{T}\right)^{N_f}$$

Comparison between methods



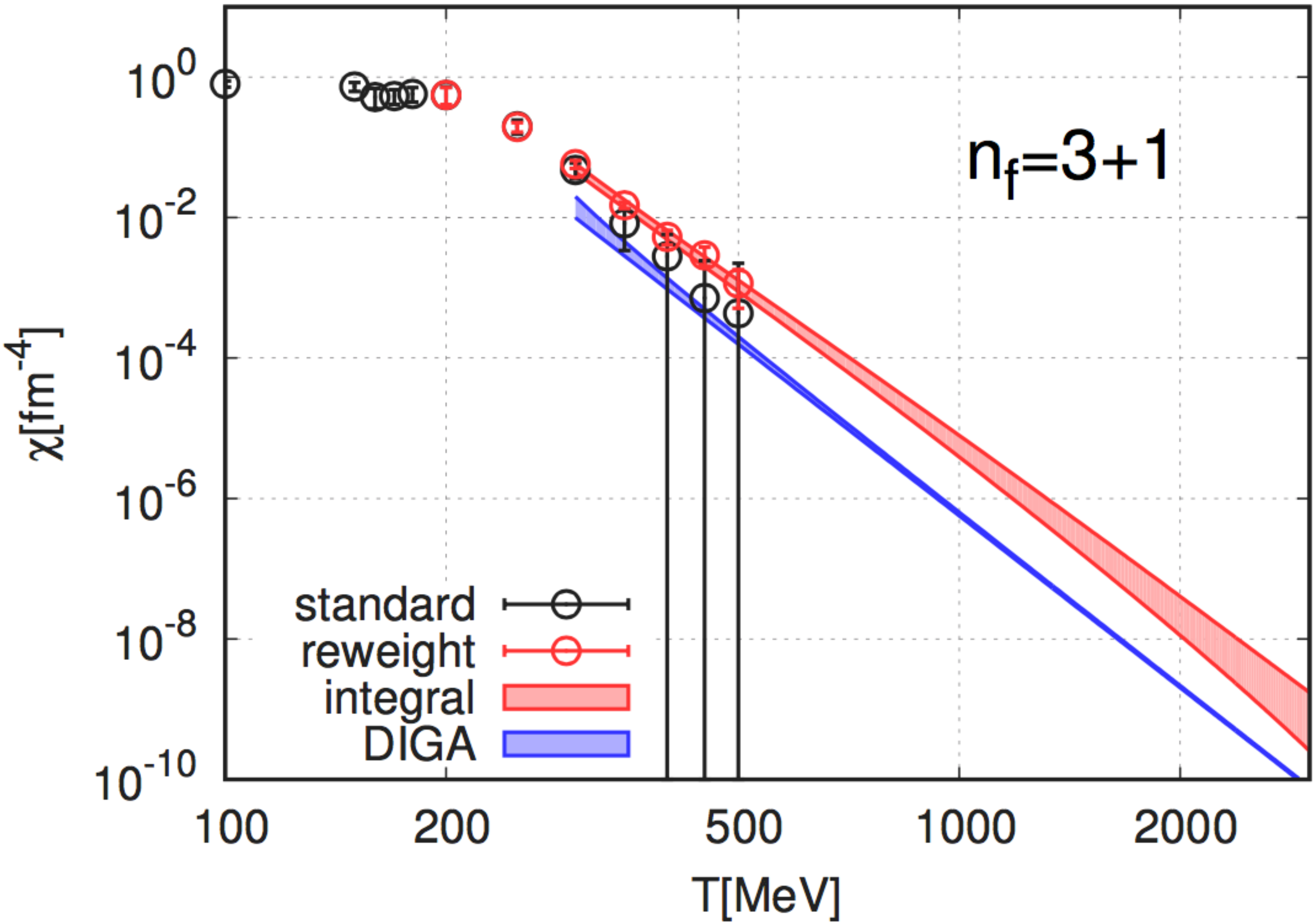
Approach to DIGA



Different fit ranges: T < 240 b approx. 6 ,
T > 240 MeV approach to DIGA

HISQ fermions, 2+1 flavors

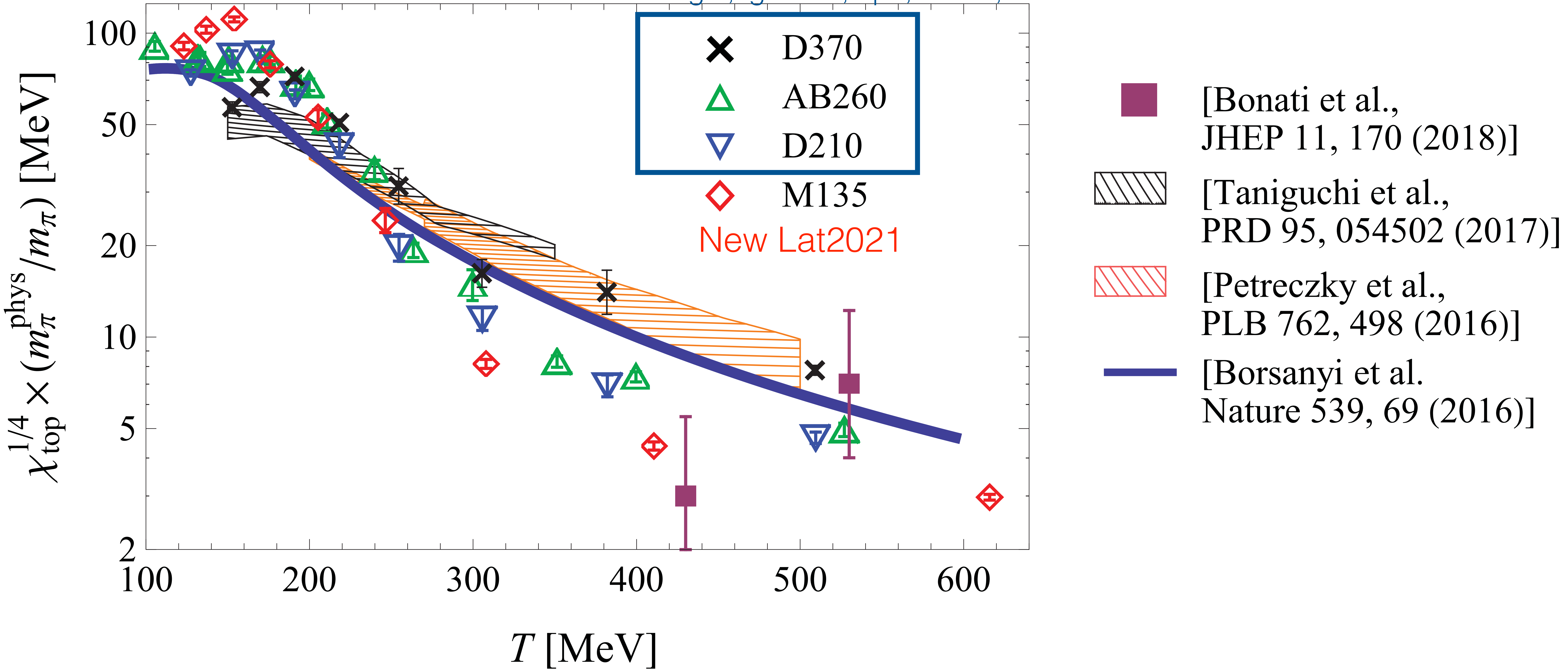
Borsanyi et al. (2016)



Results for physical pion mass from rescaling

$T^{4-\beta_0}\left(\frac{m}{T}\right)^{N_f}$

Burger,Ilgenfritz,MpL, Trunin, PRD2018

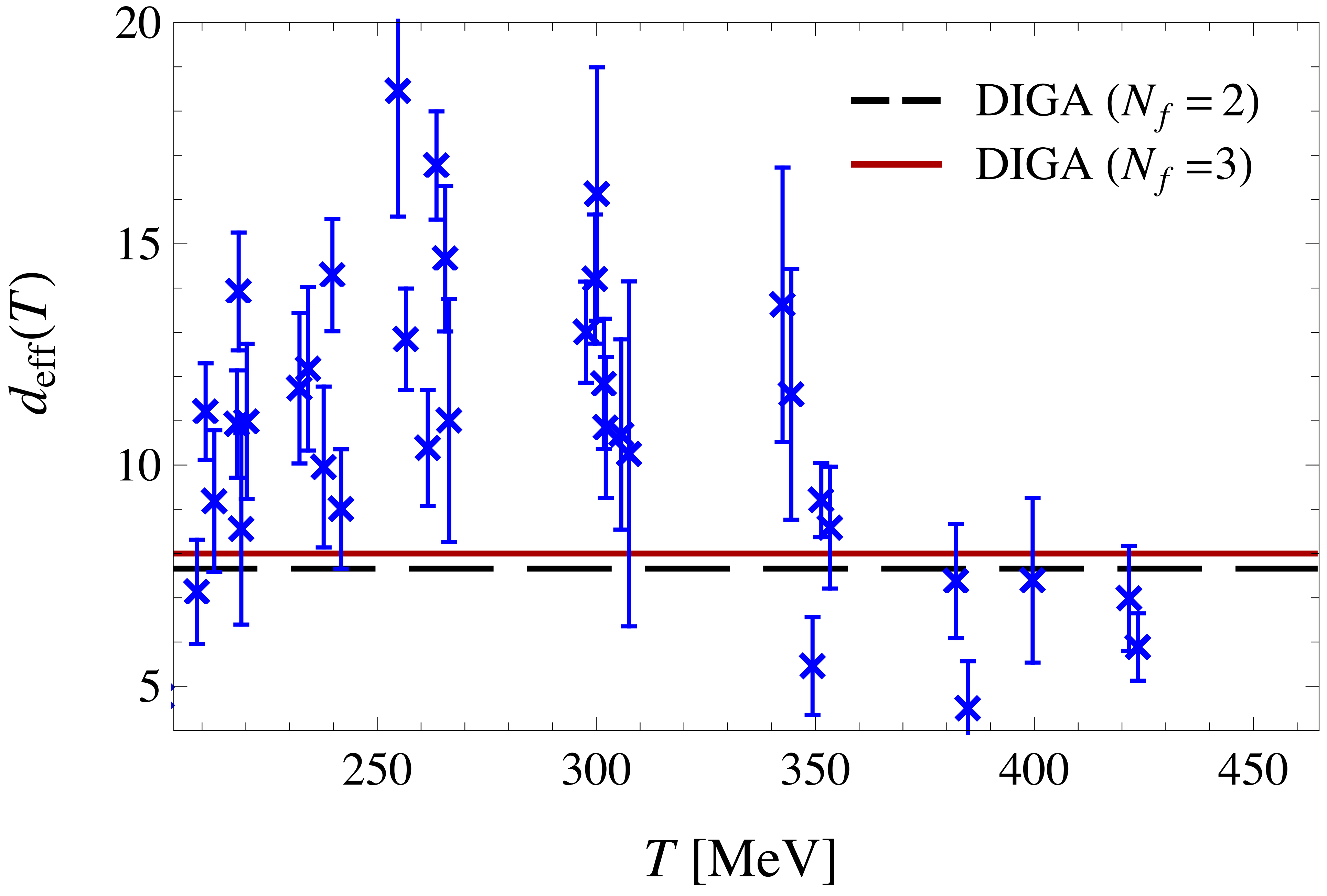


Power-law decay?

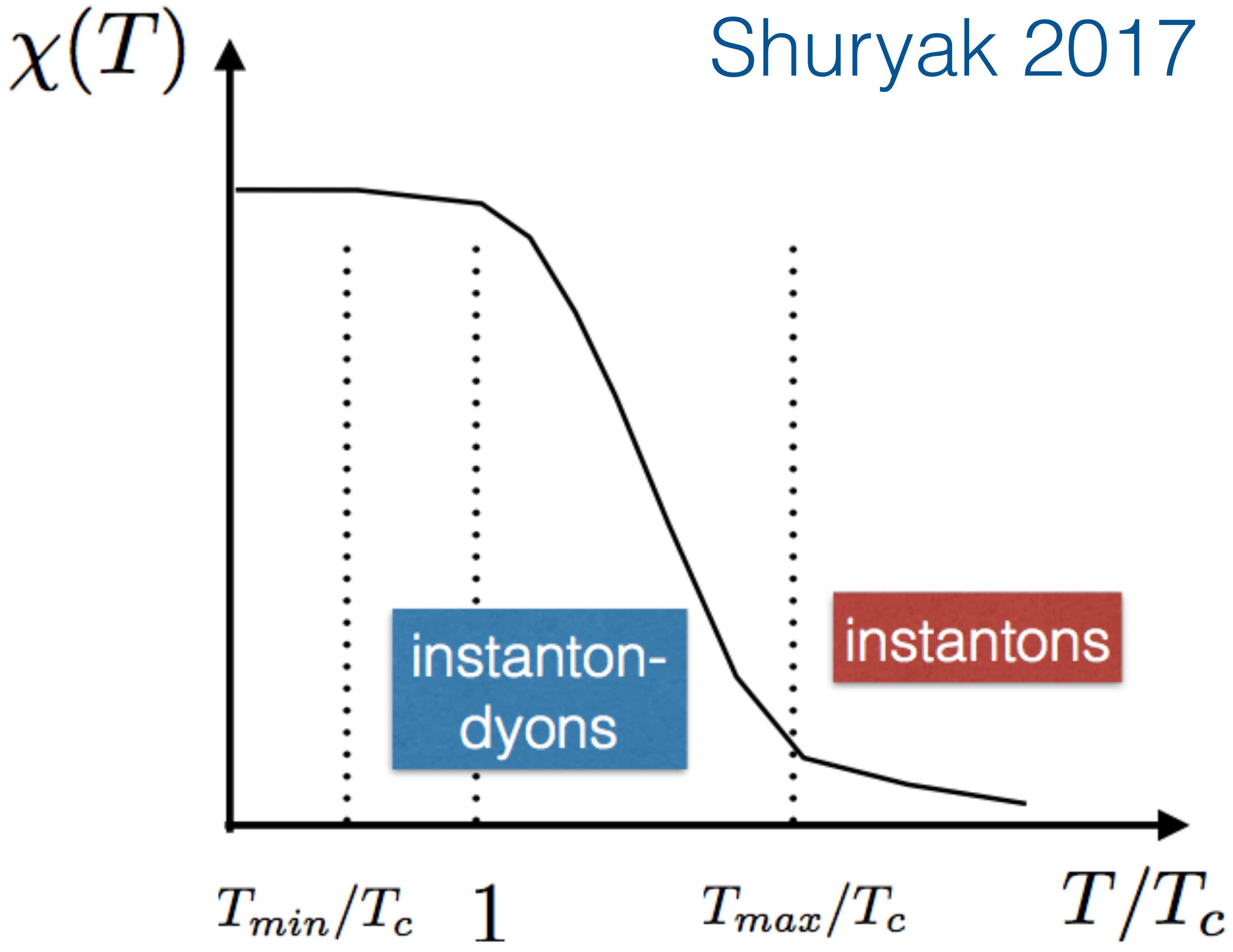
$$\chi^{0.25}(T) = aT^{-d(T)}$$

For instanton gas $\quad d(T) \equiv const \simeq (7 + \frac{N_f}{3})$

$$d(T) = -T\frac{d}{dT}\ln\chi^{0.25}(T)$$



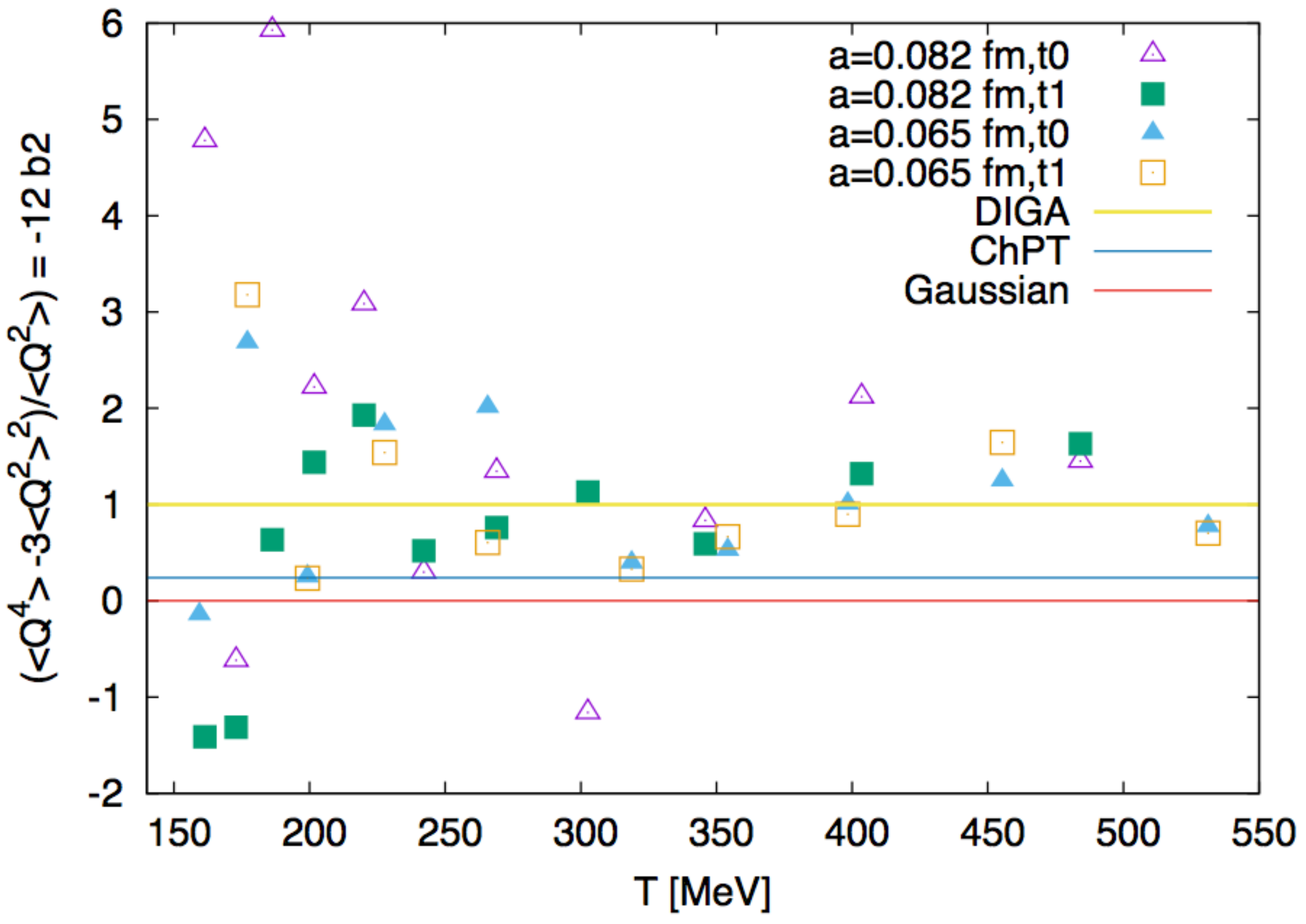
Possibly consistent with instant -dyon? Shuryak 2017



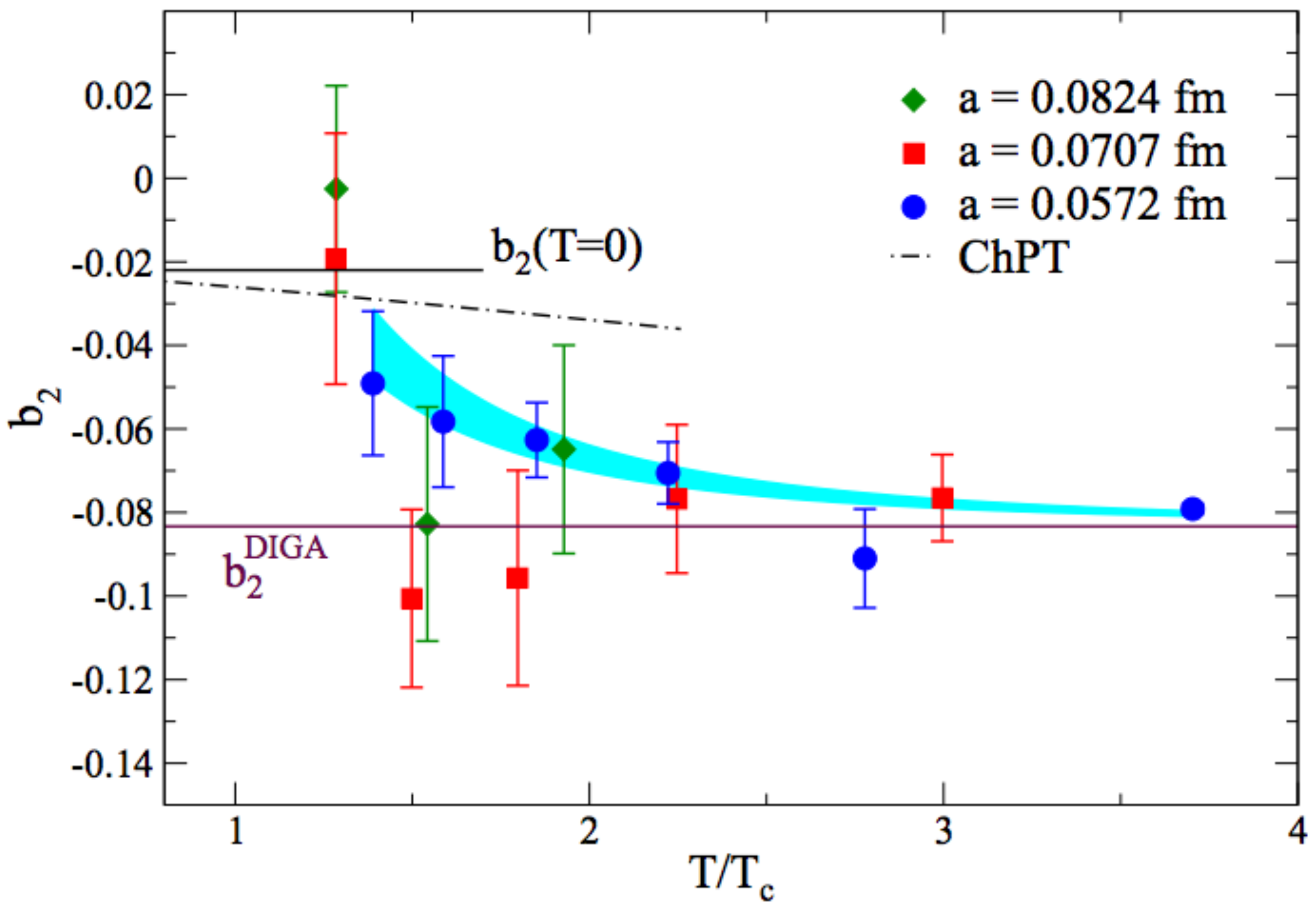
Faster decrease before DIGA sets in

Beyond topological susceptibility   d'Elia, Vicari 1301.7640

$$C_n = (-1)^{n+1} \frac{d^{2n}}{d\theta^{2n}} F(\theta, T)\Big|_{\theta=0} = \langle Q^{2n} \rangle_{conn}.$$



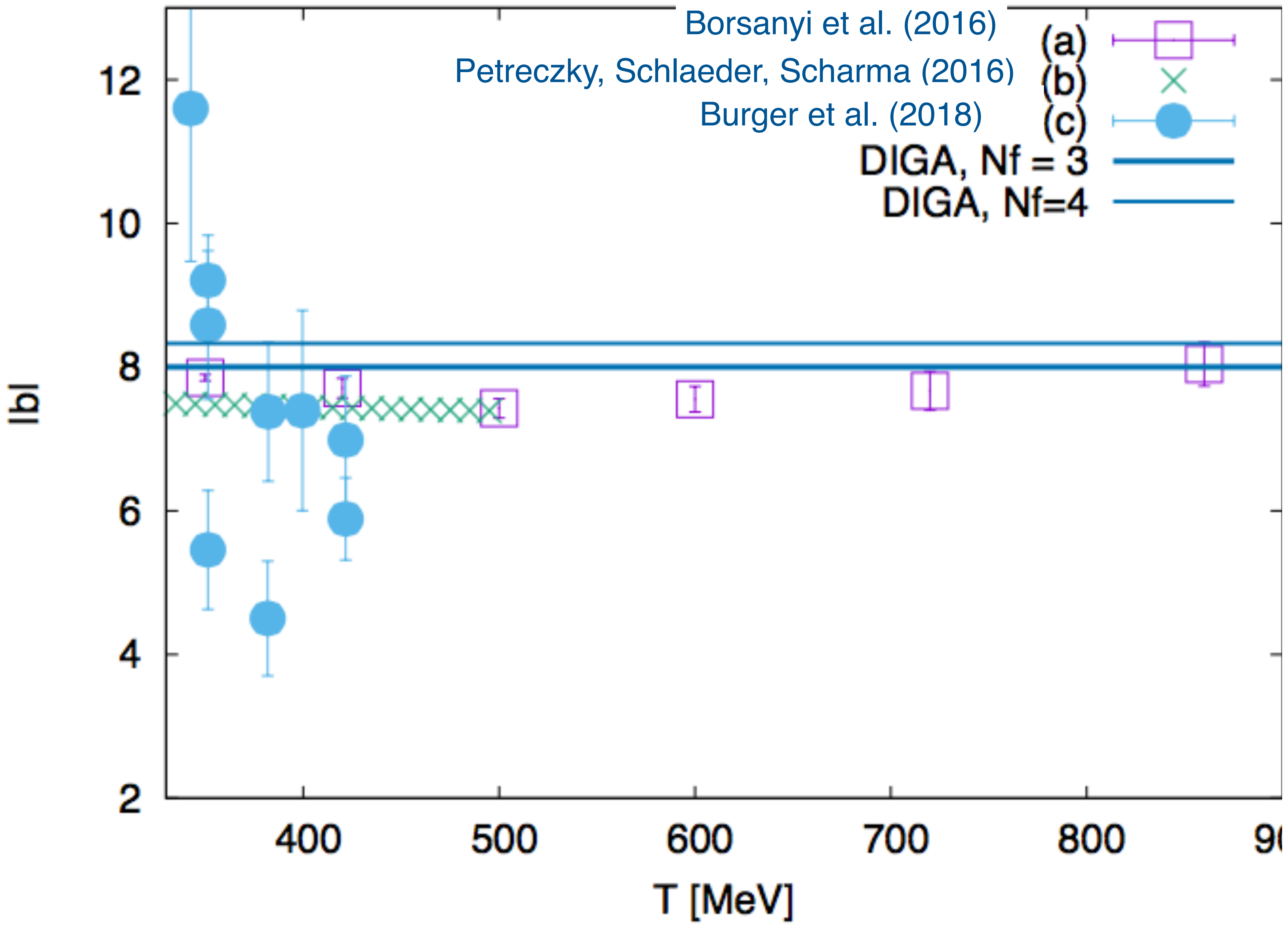
Trunin at al (2018)

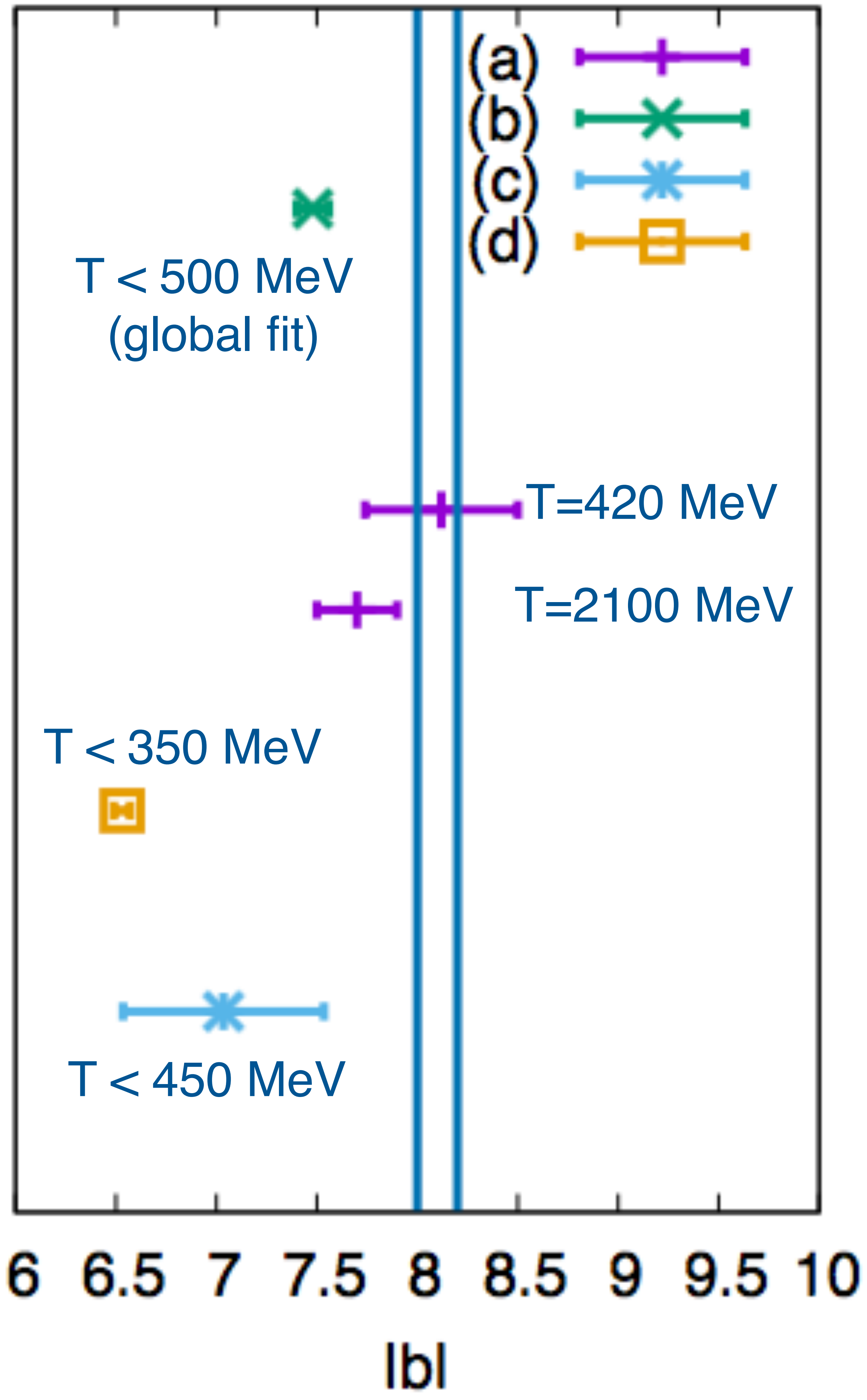


Bonati et al. (2016)

QCD - Summary of b parameter

$$\chi(T) = A\,T^b$$





Topology from low to high Temperature - again

In the hadronic phase topology solves the η' puzzle by explicit breaking $U(1)_A$

What happens to η' in the Quark Gluon Plasma?

PHYSICAL REVIEW D VOLUME 53, NUMBER 9 1 MAY 1996

Return of the prodigal Goldstone boson

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D. Kharzeev
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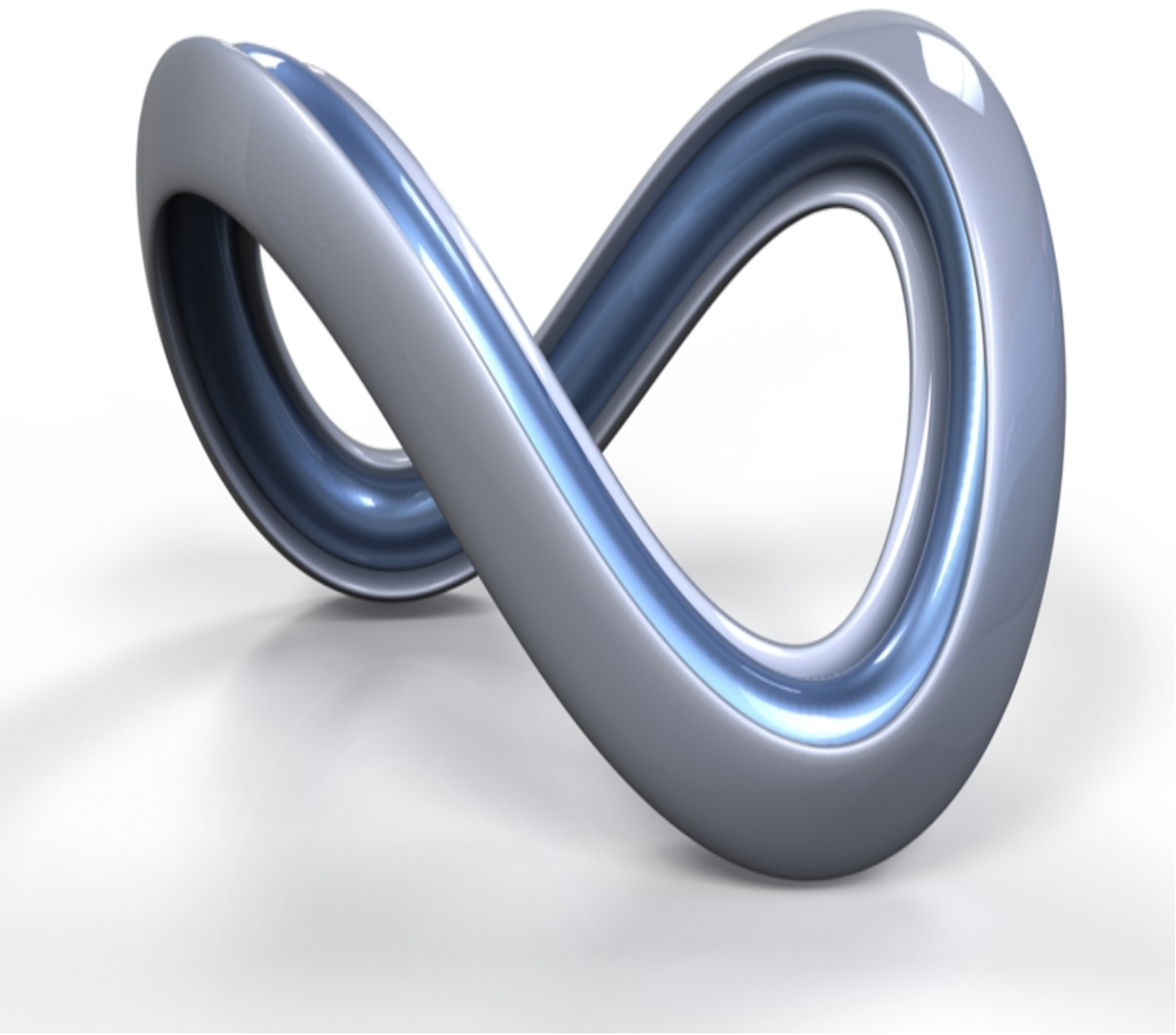
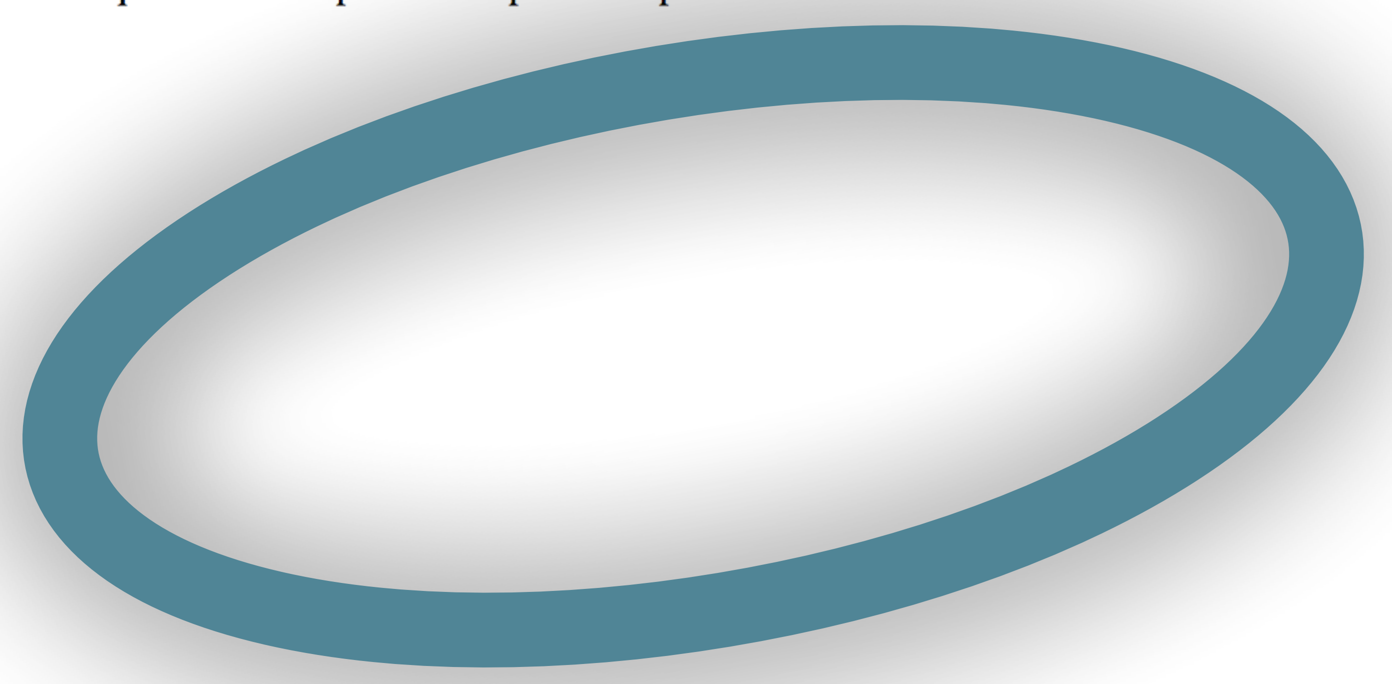
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455
(Received 14 July 1995)

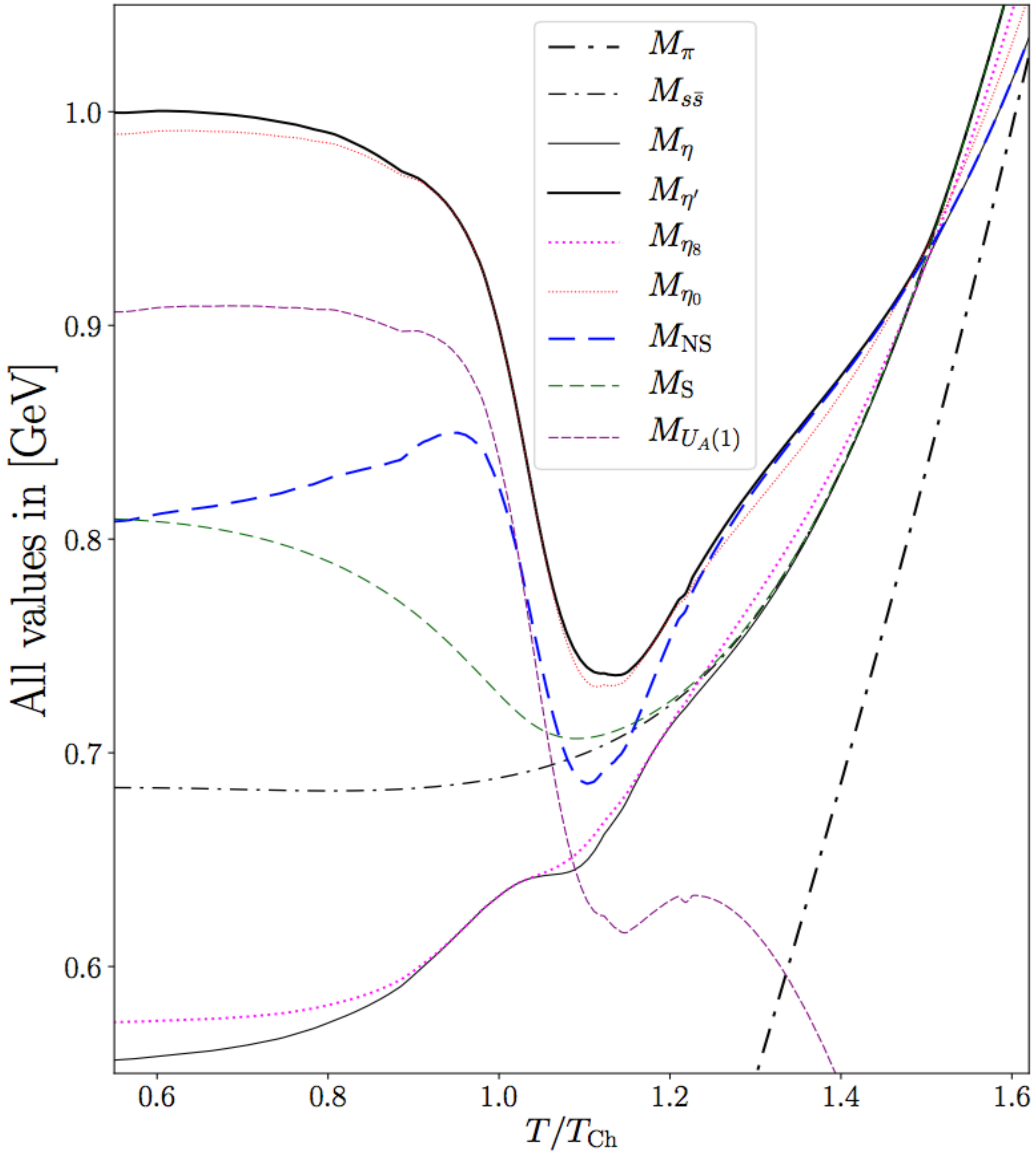
We propose that the mass of the η' meson is a particularly sensitive probe of the properties of finite energy density hadronic matter and quark-gluon plasma. We argue that the mass of the η' excitation in hot and dense matter should be small, and, therefore, that the η' production cross section should be much increased relative to that for pp collisions. This may have observable consequences in dilepton and diphoton experiments.





η' in the QGP

So far, only results
from model's studies



Horvatic et al. 2018

$$|\pi^0\rangle = \left(|u\bar{u}\rangle - |d\bar{d}\rangle\right)/\sqrt{2}$$
$$|\eta_8\rangle = \left(|u\bar{u}\rangle + |d\bar{d}\rangle - 2\,|s\bar{s}\rangle\right)/\sqrt{6}$$
$$|\eta_0\rangle = \left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\right)/\sqrt{3}$$

Different mechanisms leading to

η' (900 MeV) mass reduction

The octet-singlet mass squared matrix in the isospin limit is thus given by

$$M^2 = \begin{pmatrix} M_\pi^2 & 0 & 0 \\ 0 & M_{88}^2 & M_{80}^2 \\ 0 & M_{08}^2 & M_{00}^2 + m_A^2 \end{pmatrix} \qquad m_A^2 = 2\frac{N_f}{f_0^2}\chi^2$$

with matrix elements

$$M_{88}^2 = \tfrac{2}{3}\left(m_{s\bar{s}}^2 + \tfrac{1}{2}m_\pi^2\right) = \tfrac{1}{3}\left(4m_K^2 - m_\pi^2\right)$$
$$M_{80}^2 = M_{08}^2 = \tfrac{\sqrt{2}}{3}\left(m_\pi^2 - m_{s\bar{s}}^2\right) = \tfrac{2\sqrt{2}}{3}\left(m_\pi^2 - m_K^2\right) \quad (3)$$
$$M_{00}^2 = \tfrac{2}{3}\left(\tfrac{1}{2}m_{s\bar{s}}^2 + m_\pi^2\right) = \tfrac{1}{3}\left(2m_K^2 + m_\pi^2\right).$$

$$m_\pm^2 = m_K^2 + \frac{m_A^2}{2} \pm \Delta\,,$$

with m_+, m_- corresponding to the η' and η

$$\Delta = \sqrt{\left(m_K^2 - m_\pi^2\right)^2 - \frac{m_A^2}{3}\left(m_K^2 - m_\pi^2\right) + \frac{m_A^4}{4}}$$

Veneziano, 1981

Non anomalous:

$\eta' \simeq 700$ MeV

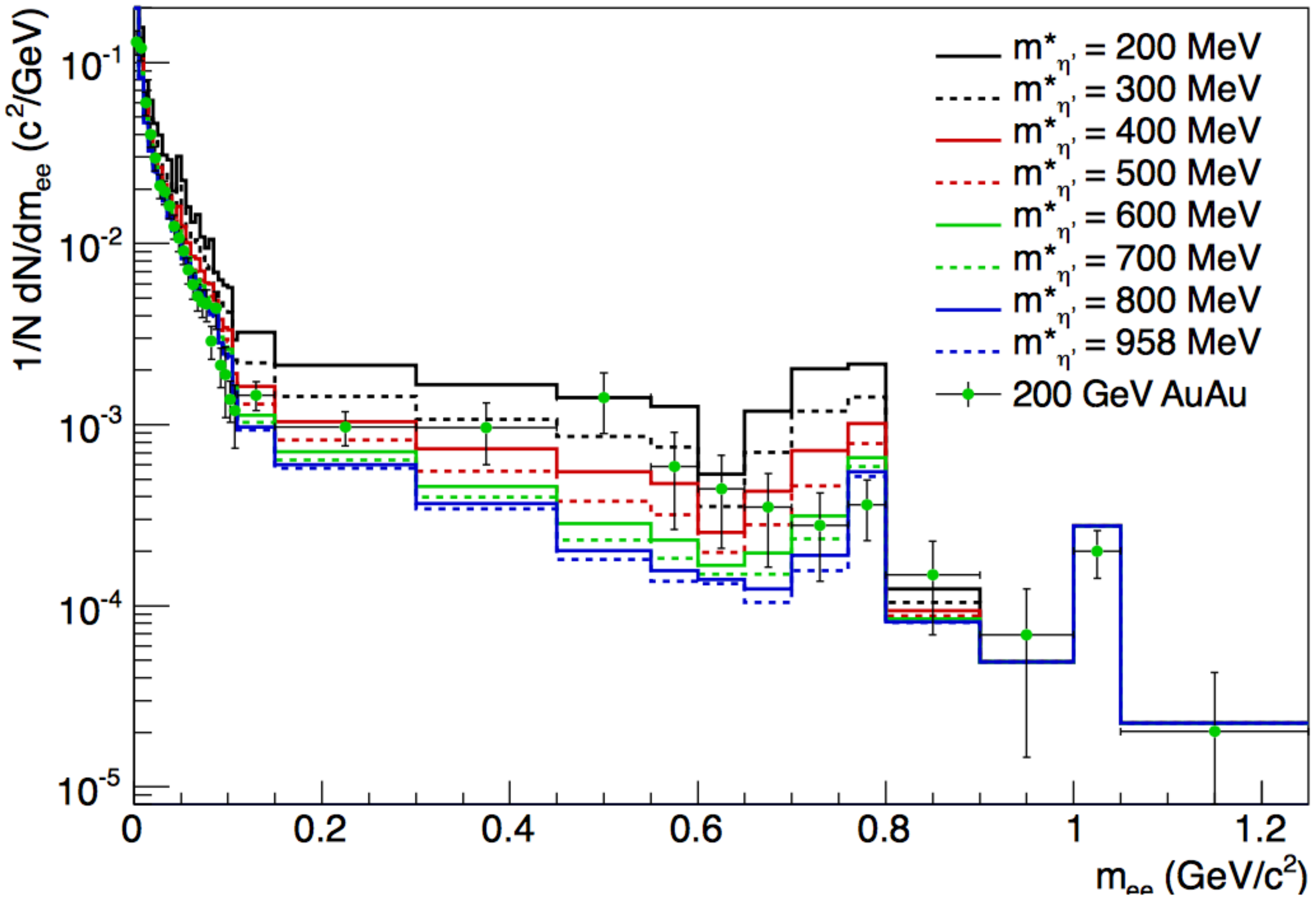
(strange only)

However: also sensitive to SU(2)XSU(2)

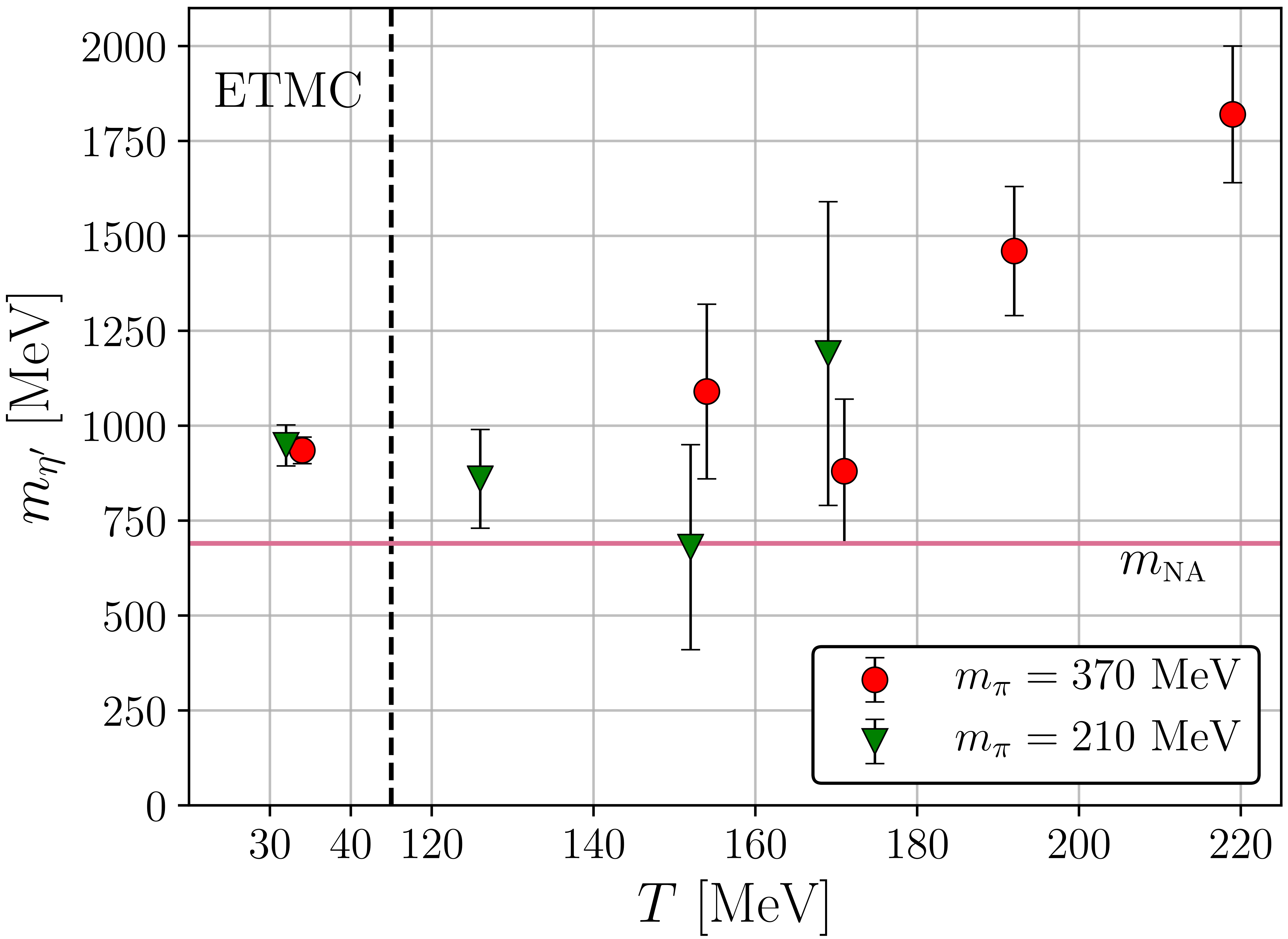
Indication of topology suppression in PHENIX

Effects of chain decays, radial flow and $U_A(1)$ restoration on the low-mass dilepton enhancement in $\sqrt{s_{NN}}$=200 GeV Au+Au reactions

Márton Vargyas[a,b,1], Tamás Csörgő[b,2], Róbert Vértesi[b,c,3]



This is at finite density!



Kotov, Trunin, MpL (2019)

Minimum of the η'

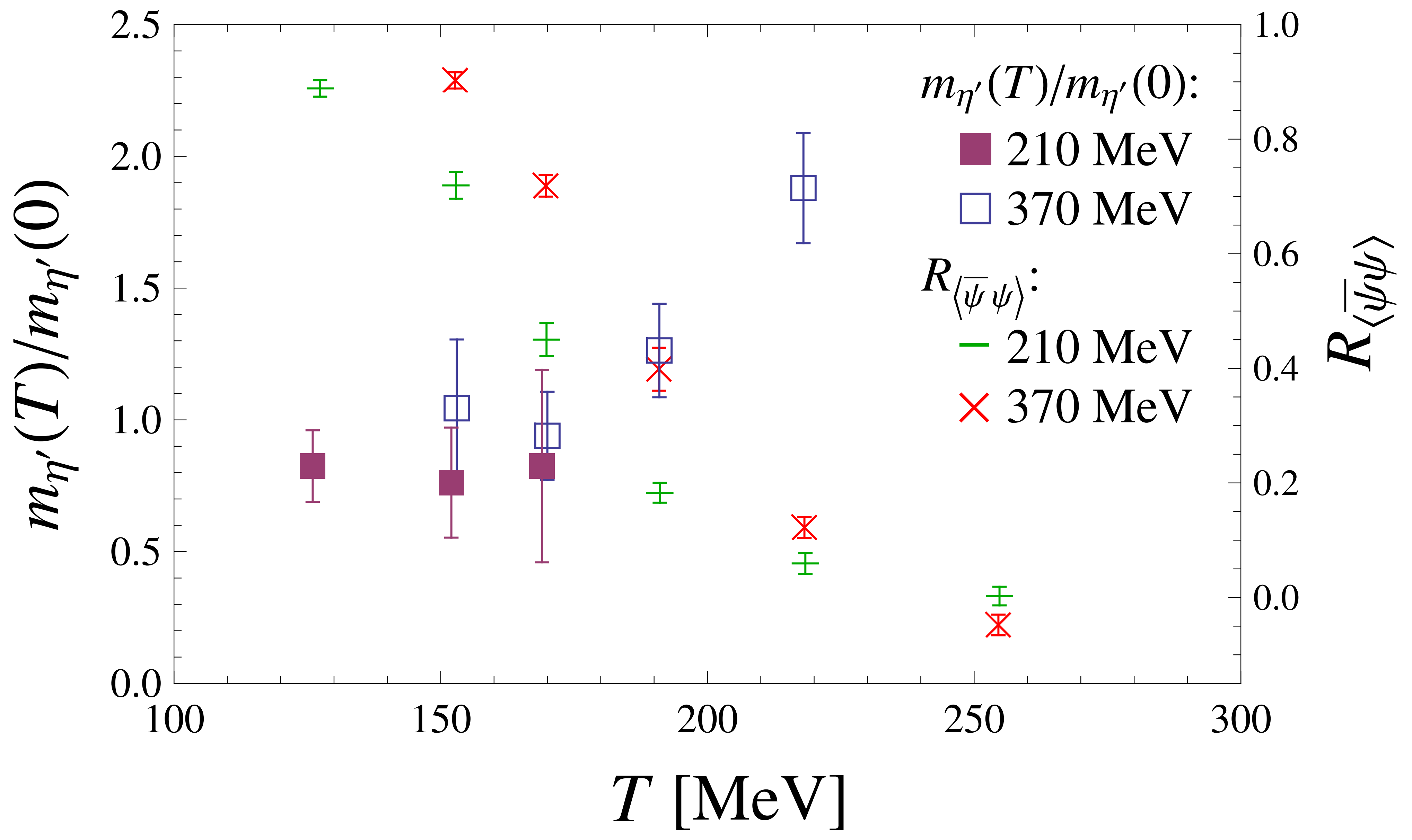
Approx. correlated with T_χ

Ensemble	a [fm]	m_π [MeV]	T_χ [MeV]
D210	0.065	213	158(1)(4)
A260	0.094	261	157(8)(14)
B260	0.082	256	161(13)(2)
A370	0.094	364	185(5)(3)
B370	0.082	372	189(2)(1)
D370	0.065	369	185(1)(3)
A470	0.094	466	200(4)(6)
B470	0.082	465	203(2)(2)

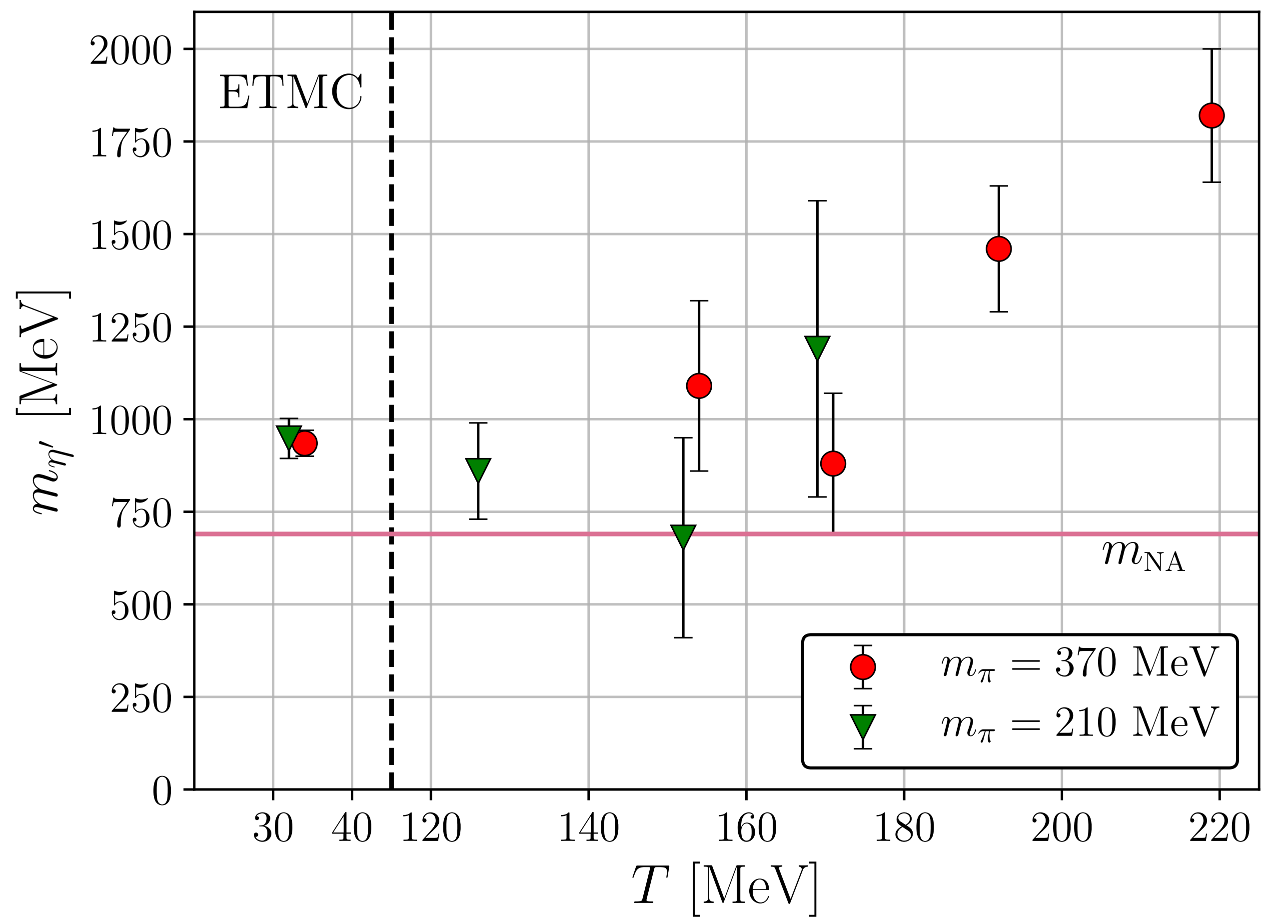
$T_{\eta'}$ [MeV]

$\simeq 150$

$\simeq 170$



Consistent with suppression of the anomalous contribution

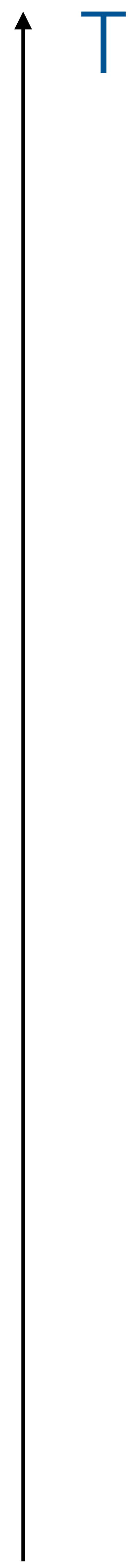


Analysis with
a physical pion
mass in progress

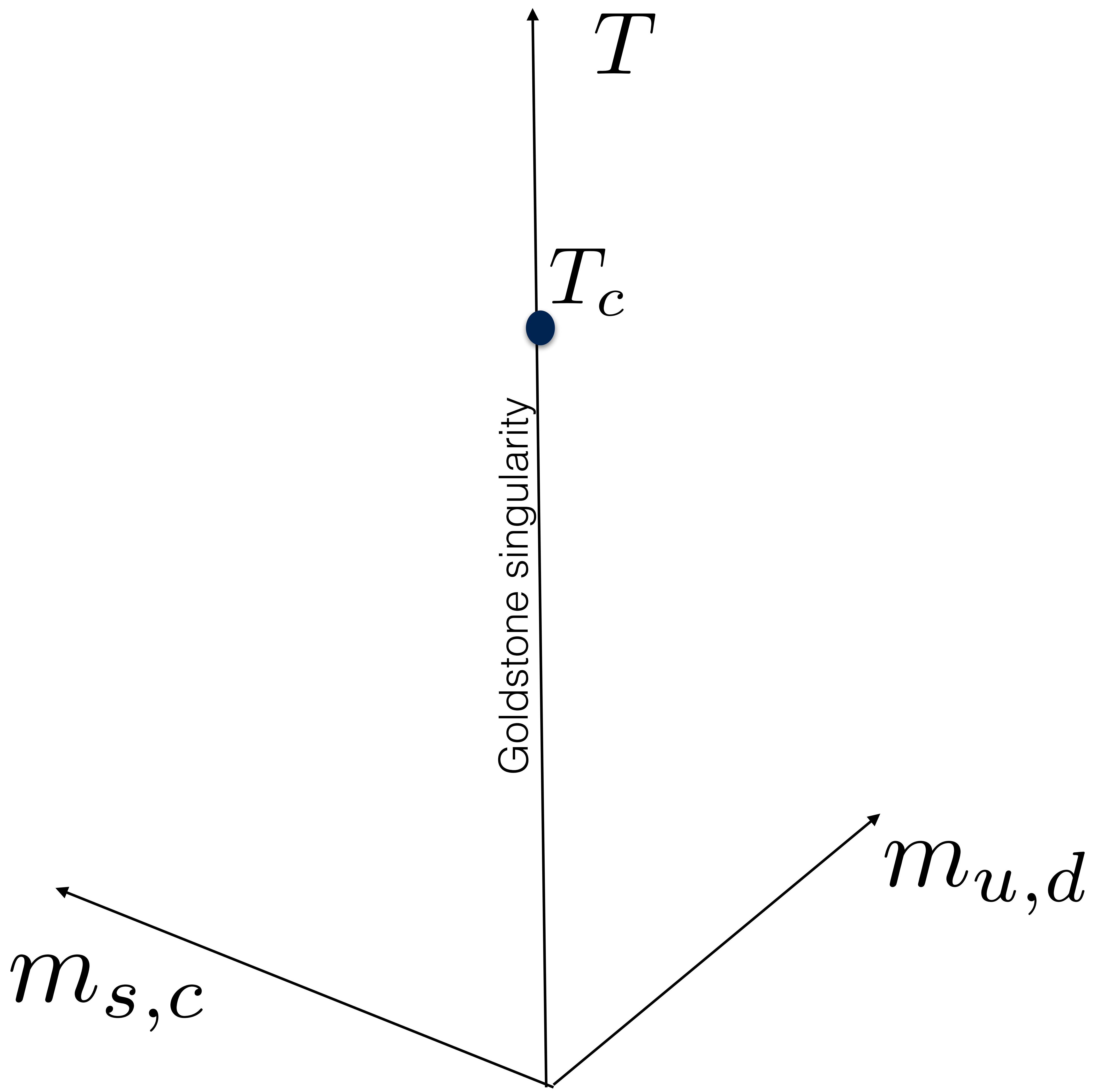
Quick summary topology:

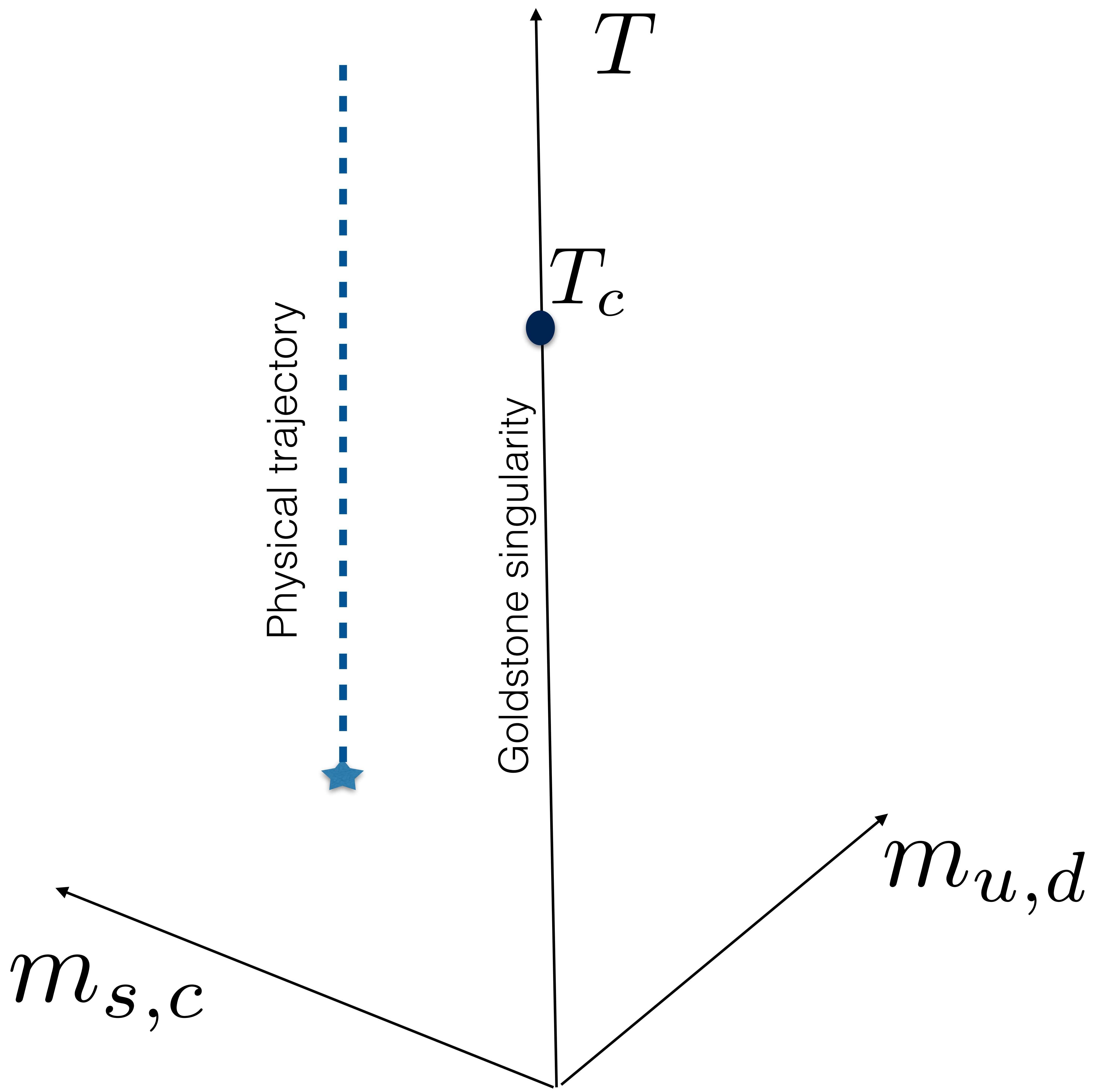
Topological susceptibility drops beyond Tc

Consistence with DIGA (exponent, b2, mass scaling) T > 300 MeV



Focus on critical behaviour:
And on the critical region


T
T_c
Goldstone singularity
$m_{u,d}$
$m_{s,c}$


T
T_c
Physical trajectory
Goldstone singularity
$m_{u,d}$
$m_{s,c}$

WHICH SYMMETRY IS RESTORED IN HOT QCD?

Shuryak(1993)

$$\boxed{SU(N_f)_L \times SU(N_f)_R \times U(1)_A \times U(1)_B.}$$

Breaking/restoration
at Tc
studied a lot
on the lattice

Always broken if topological charge
fluctuates!

BUT:
the 'amount' of breaking
depends on temperature

$\boxed{\text{HOW ARE THESE RELATED??}}$

$\boxed{\text{IMPLICATIONS?}}$

$N_f = ?$

T=0, no difference, just different #Goldstones

$m_{u,d} = 0$

$N_f = 3$

$N_f = 2$

m_s

0 ∞

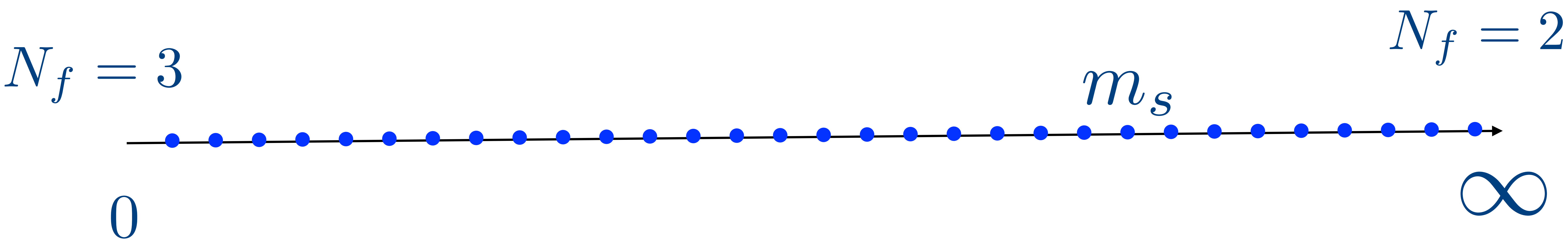
High T restoration: 1st order

2nd or 1st

(or maybe 2nd?? - Cuteri,Philipsen@2021)

Fate of 1st order for Nf=3 : persistence till Nf=2 ?

$m_{u,d} = 0$



1st

High T restoration: 1st order

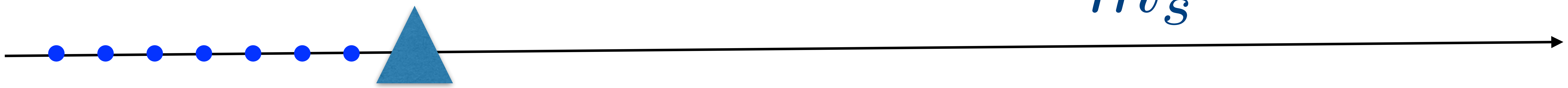
Fate of 1st order for Nf=3 : critical endpoint ?

$m_{u,d} = 0$

$N_f = 3$

$N_f = 2$

m_s



0

∞

High T restoration: 1st order

2nd O(4) 3D

Nature of the singularity for a physical strange mass?

[open parentheses: axial symmetry restoration?

- Consensus: axial symmetry is effectively restored $T \simeq 1.2T_c$

  Kaczmarek(2020),Mazur(2018),Buchoff(2013),Suzuki(2020),Kanazawa(2015),Aoki(2012),
  Tomiya(2016),Brandt(2019),Cossu(2013),Chiu(2013),Tomiya(2016)

- Controversy: how close to T_c the effective restoration may happen.

  same refs as above

- More controversy: **axial symmetry still broken at** $T \simeq 1.6T_c$

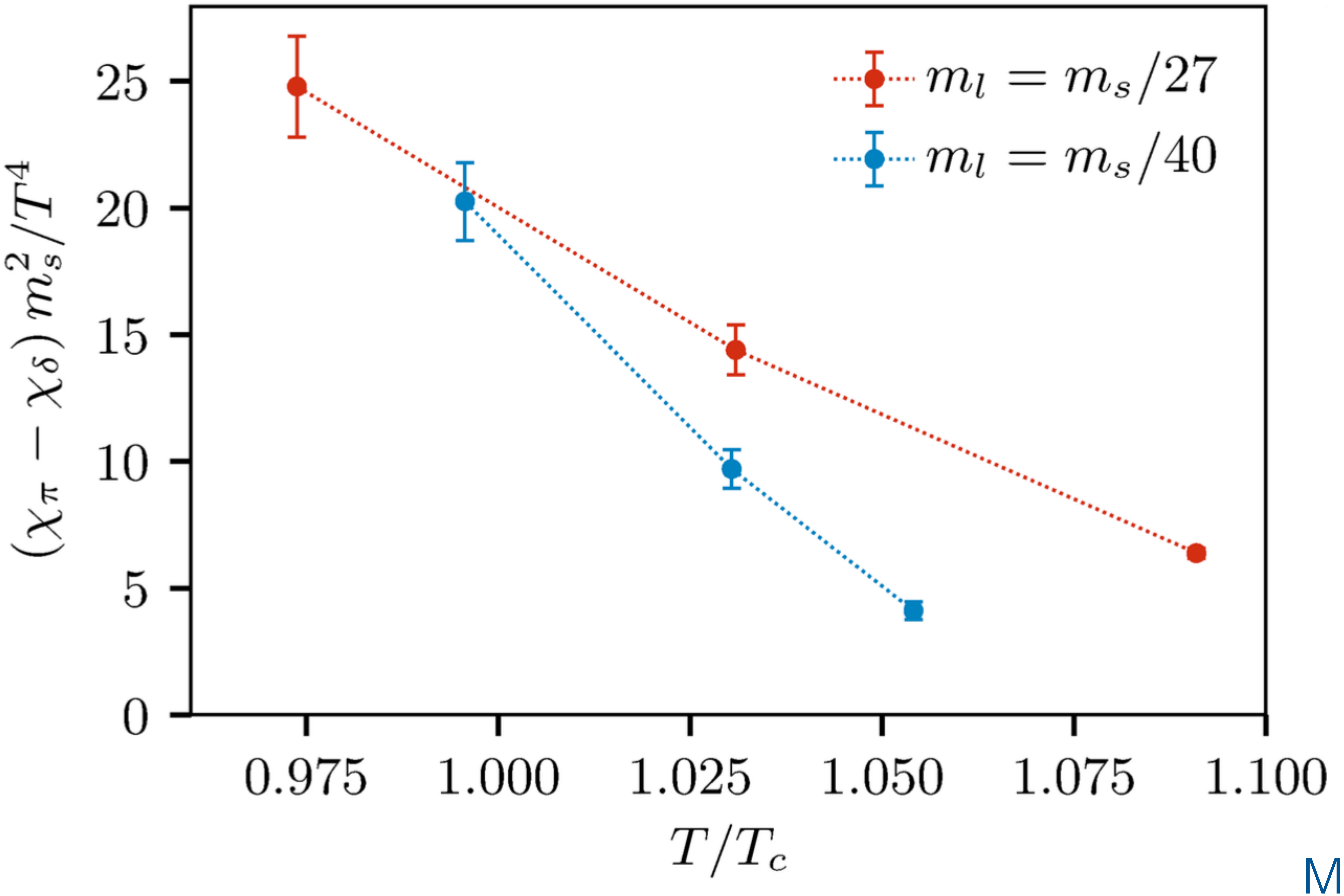
Ding et al. (2020)

A quick look at the axial symmetry as seen in 2pt functions

Correlation functions of local operators **Shuryak(1993),Buchoff(2013)**

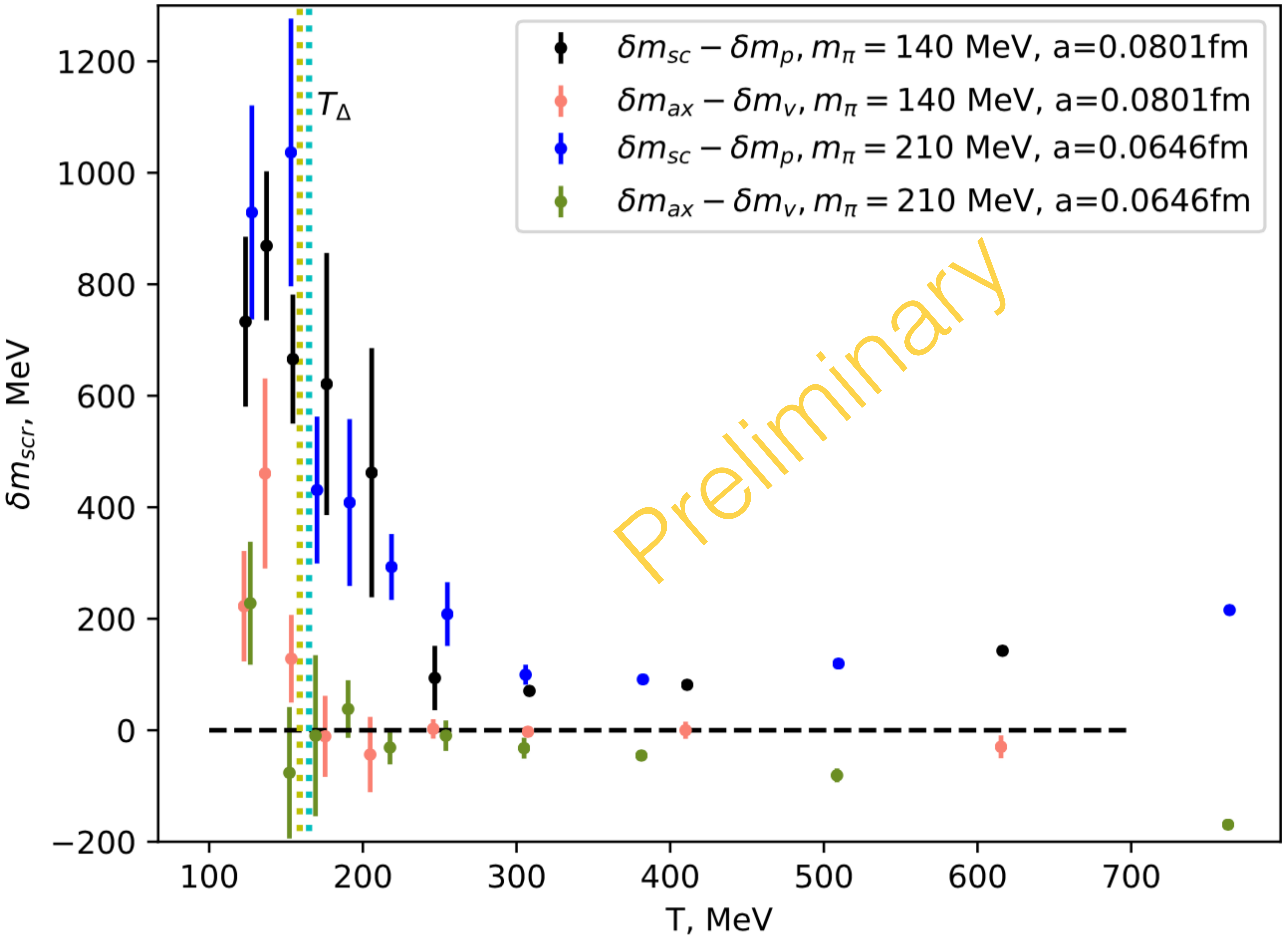
$$
\begin{aligned}
\sigma &= \bar{\psi}_l \psi_l \\
\delta^i &= \bar{\psi}_l \tau^i \psi_l \\
\eta &= i\bar{\psi}_l \gamma^5 \psi_l \\
\pi^i &= i\bar{\psi}_l \tau^i \gamma^5 \psi_l.
\end{aligned}
$$

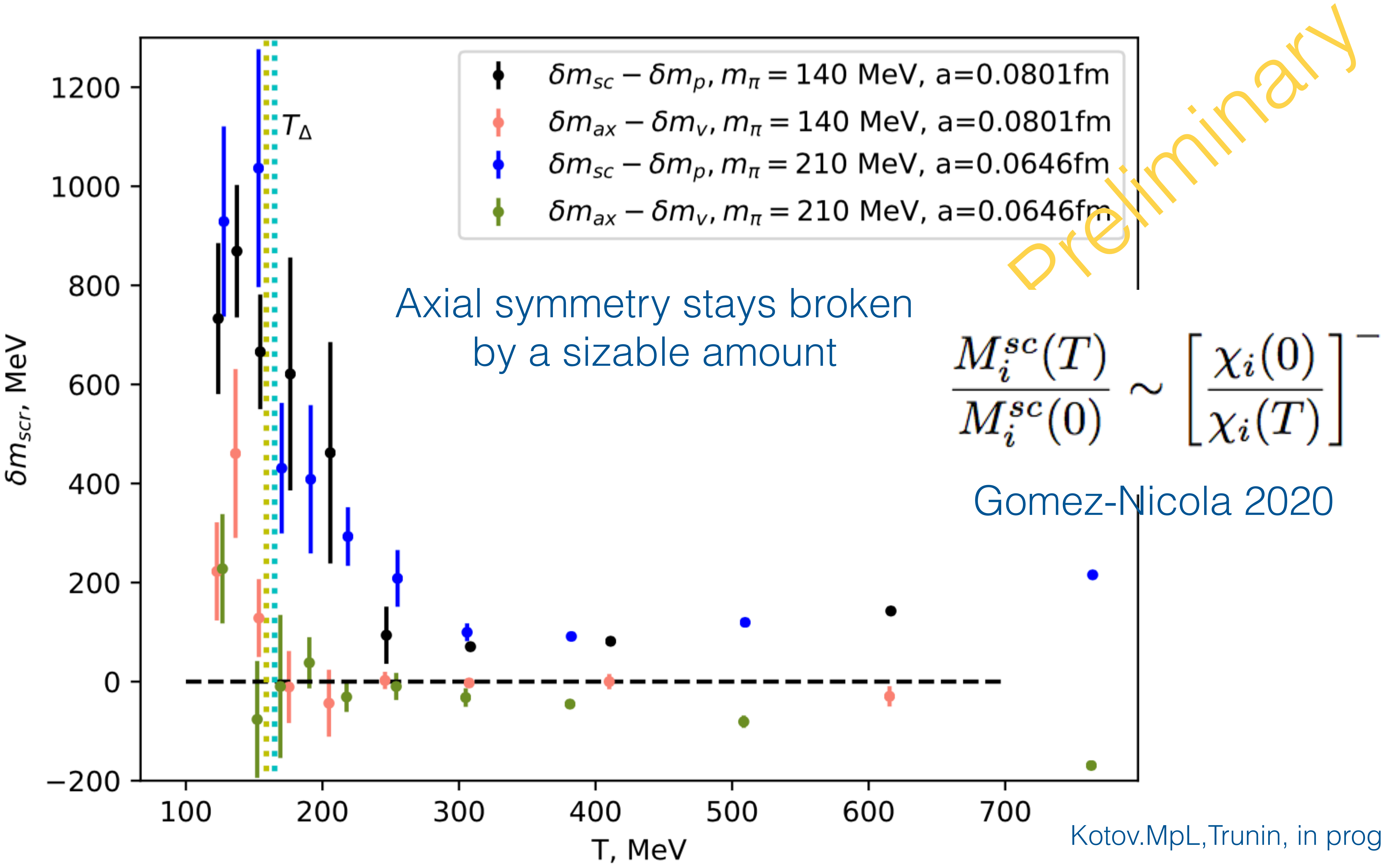
(σ, π^i) and (η, δ^i) are related by $SU(2)_L \times SU(2)_R$ transformations
(σ, η), $(\delta^i, \pi^i)_{1 \leq i \leq 3}$ by $U(1)_A$.
Several lattice studies in mesonic channels.



Mazur et al 2018

Kotov, Trunin, MpL



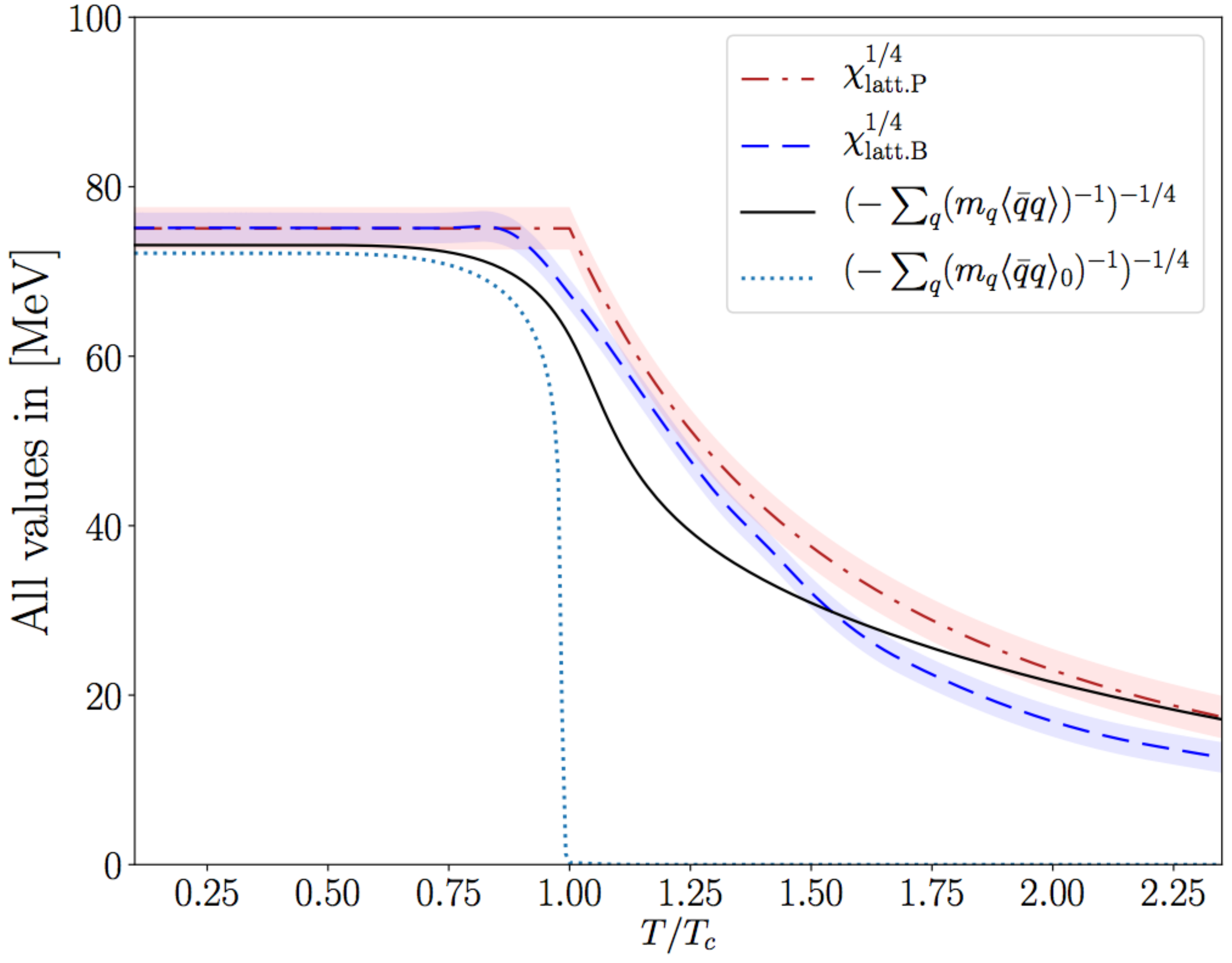


Axial symmetry stays broken by a sizable amount

$$\frac{M_i^{sc}(T)}{M_i^{sc}(0)} \sim \left[\frac{\chi_i(0)}{\chi_i(T)}\right]^{-1/2}$$

Gomez-Nicola 2020

Kotov.MpL,Trunin, in progress



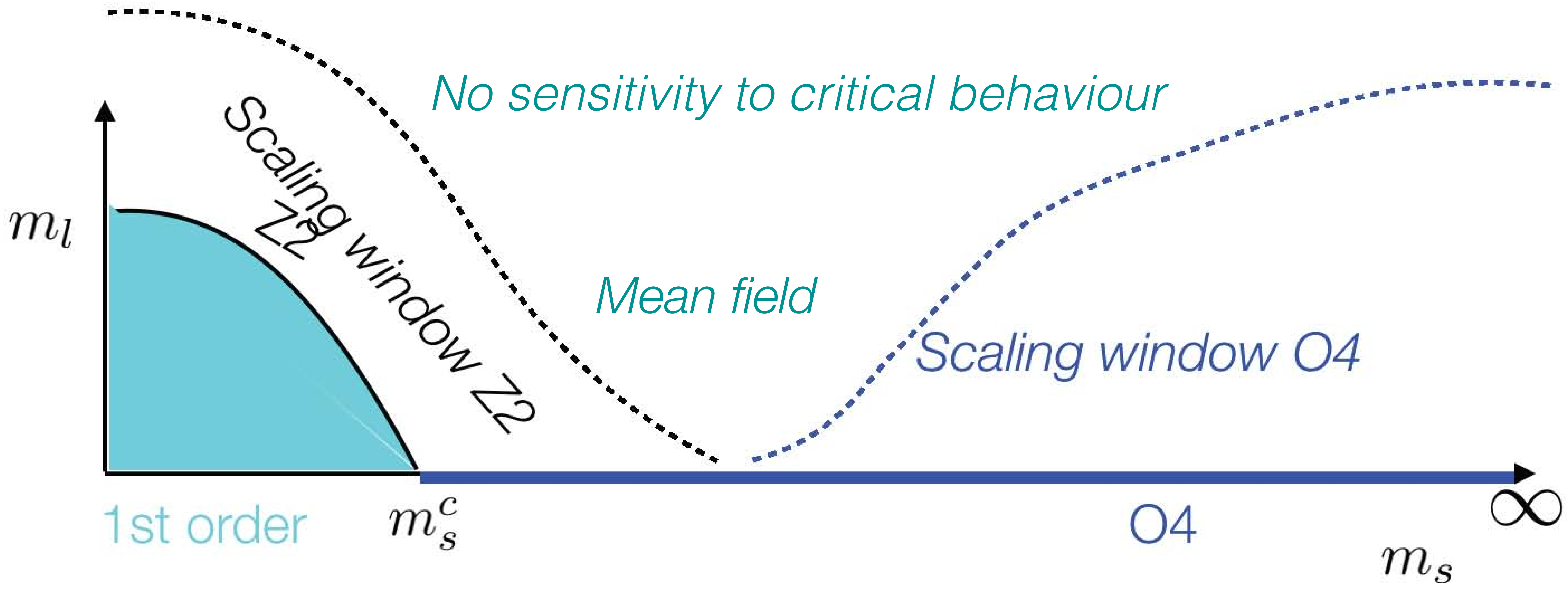
Is chiral symmetry driving axial symmetry?

Horvatic et al (2020)

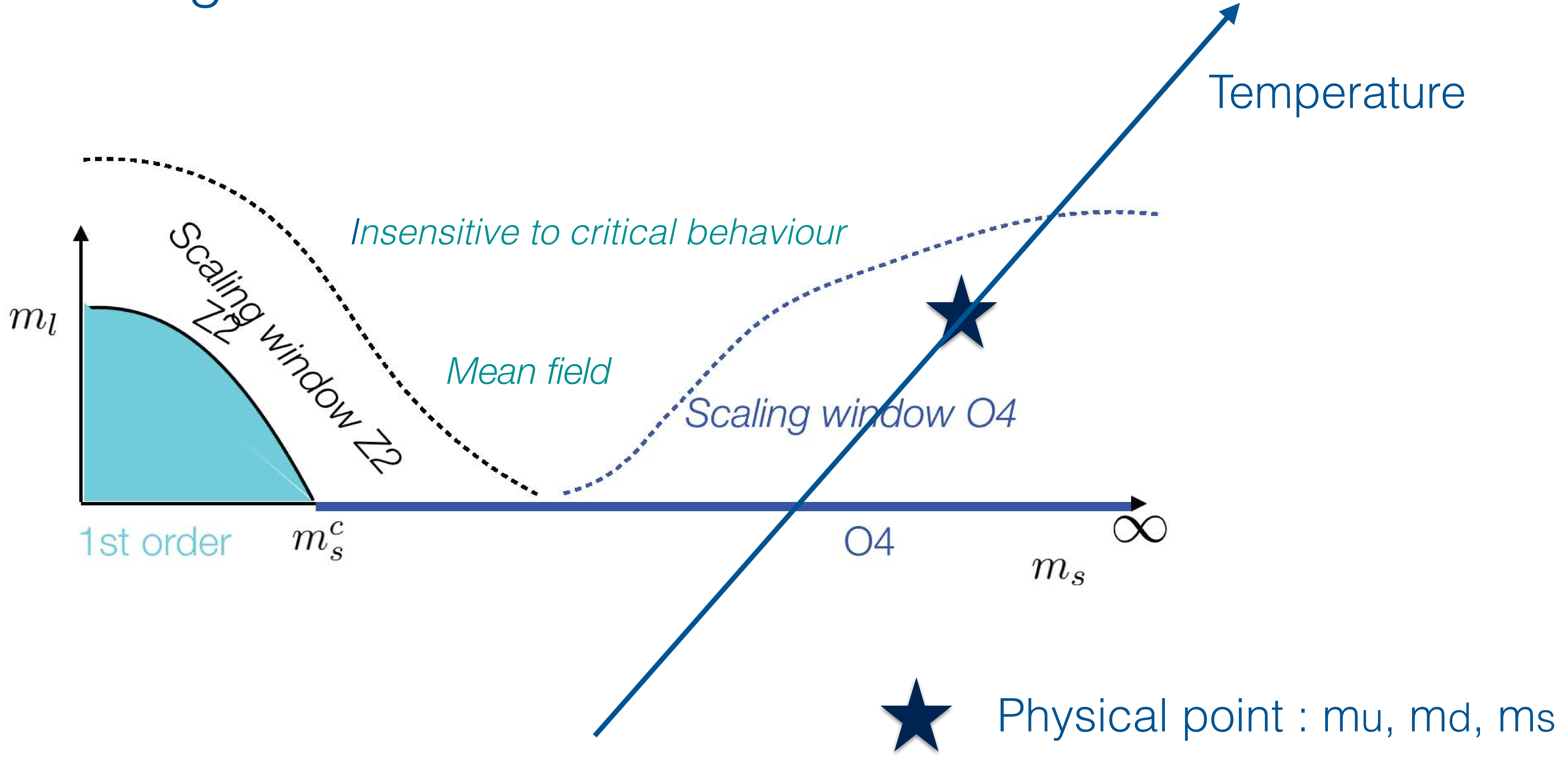
$$\chi_{QCD} = \frac{\chi_{YM}}{1 + a\sum_{i=1}^{N_f} \frac{1}{\mu_i^2(\theta)}} = \chi_{YM}\left(1 - \frac{\chi_{YM}}{\sum_{k=1}^{N_f}(m_i\langle\bar{\psi}\psi\rangle)}\right)^{-1}$$

Di Vecchia, Rossi, Veneziano,Yankielowicz
Gomez-Nicola, ..

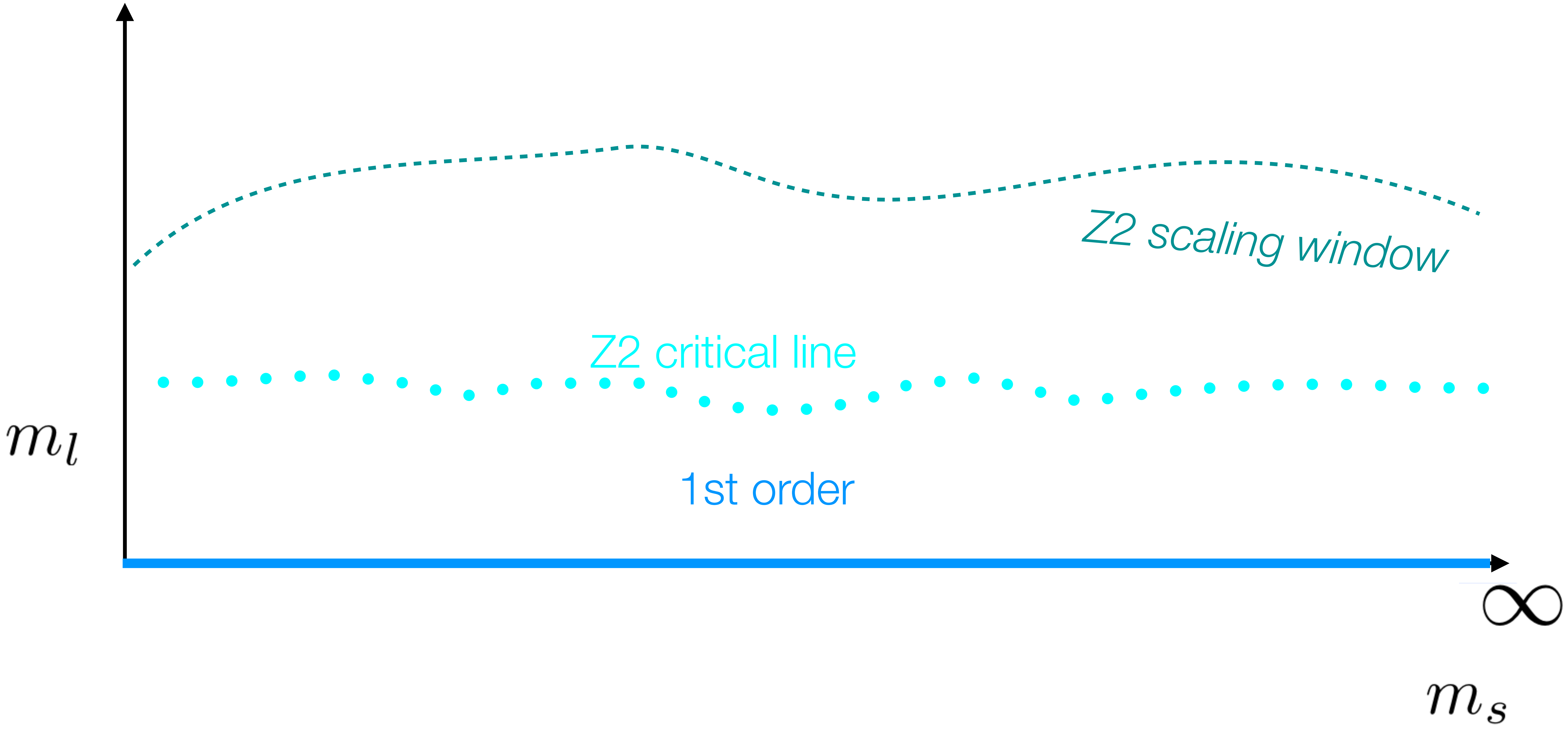
Including the light quark masses



Where is the scaling window in QCD in mass and T?



Alternative scenario, similar question



Scaling window and universal behaviour

Magnetic equation of State

$$h = M^{\delta} f(t/M^{1/\beta\delta})$$

$M \equiv \bar{\psi}\psi$, $h \equiv m_q$, $t \equiv T - T_c$, m_q is the quark mass and T_c is the critical temperature

Pseudocritical (mass dependent) temperatures follow:

$$T_c = T_c(0) + k_s m_{\pi}^{2/\beta\delta}$$

Observable	χ_L	$\bar{\psi}\psi$
k_s	1.35(3)	0.74(4)

$$\chi_L = \frac{\partial \bar{\psi}\psi}{\partial m}.$$

Ratios of k's universal

A ‘new’ order parameter

also mentioned in the PhD thesis by Wolfgang Unger

’Beating’ the regular terms/additive renormalization
for more stringent universality checks

$$\Delta_3 \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m}) \equiv m(\chi_T - \chi_L)$$

Transverse and longitudinal susceptibilities

$$\chi_T = \frac{\bar{\psi}\psi}{m}$$

$$\chi_L = \frac{\partial\bar{\psi}\psi}{\partial m}.$$

$$R_\pi \equiv \chi_T^{-1}/\chi_L^{-1}$$

$$\frac{1}{R_\pi(t,m)} = \delta - \frac{x}{\beta}\frac{f'(x)}{f(x)},$$

$$R_\pi(0,m) = \frac{1}{\delta}$$

Kocic, Kogut, MpL;
Karsch, Laermann

- linear terms in m drop in $\Delta_3 \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m})$

- Δ_3 order parameter, no leading order additive renormalization

Use: $M = h^{1/\delta} f_G(t/h^{1/\beta\delta})$ (parametrization in:

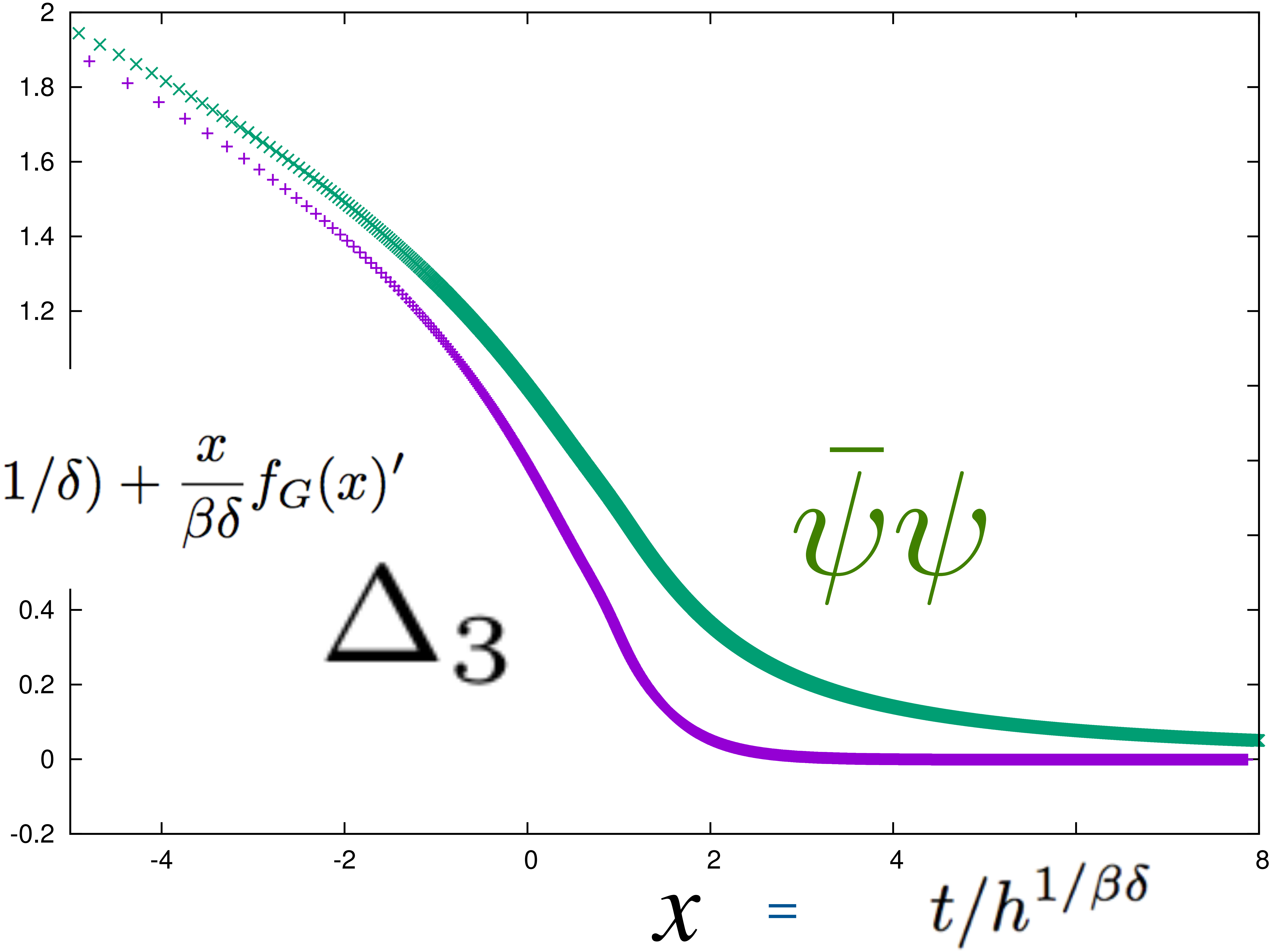
J.Engels and F.Karsch, Phys. Rev. D 85, (2012)

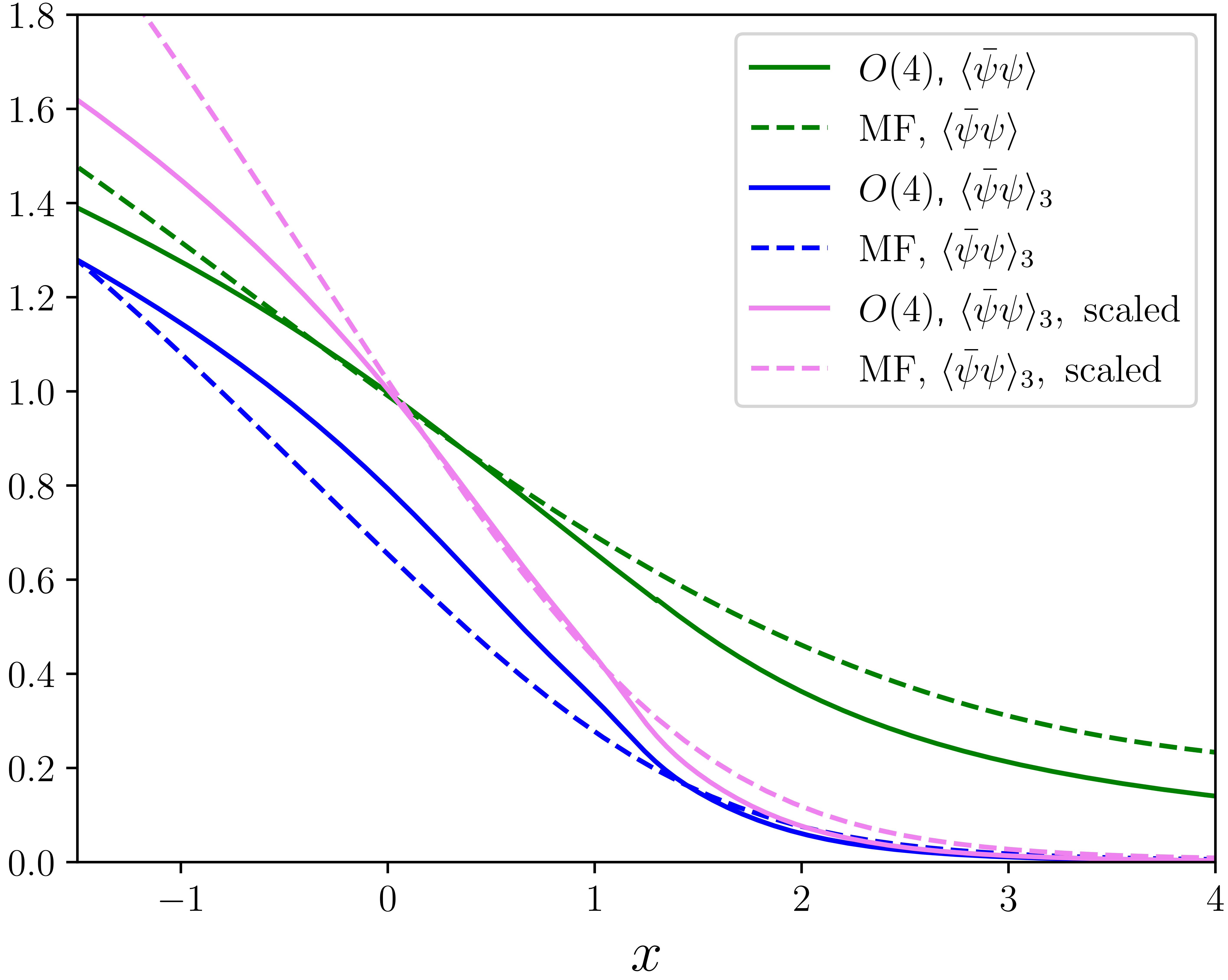
To get EoS for Δ_3

$$\Delta_3 = m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) - 1/\delta m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) + m^{1/\beta\delta+1} f'_G((t/m^{1/\beta\delta})$$

$$\boxed{\frac{\Delta_3}{m^{1/\delta}} = f_G(x)(1-1/\delta) + \frac{x}{\beta\delta} f_G(x)'}$$

$$\frac{\Delta_3}{m^{1/\delta}} = f_G(x)(1 - 1/\delta) + \frac{x}{\beta\delta} f_G(x)'$$



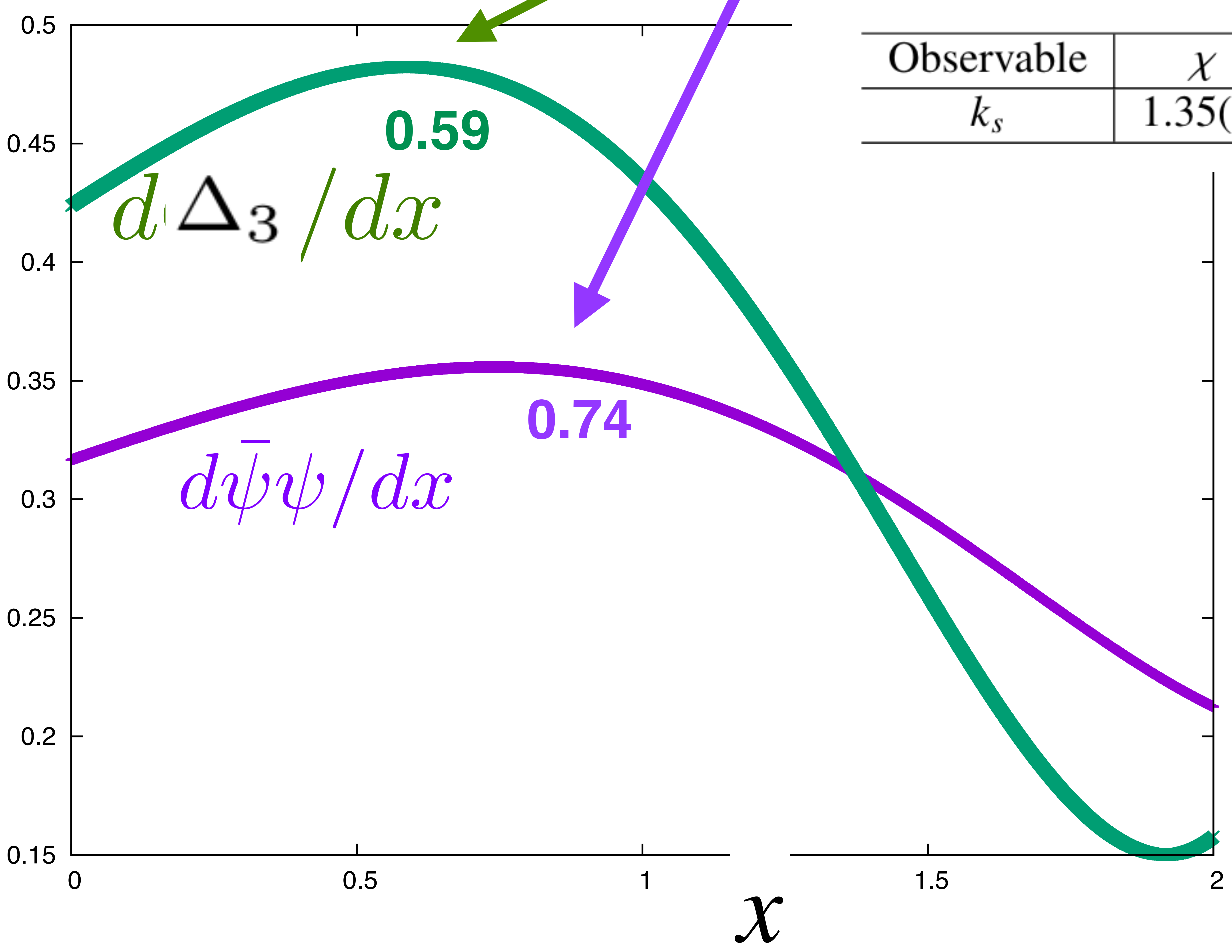


1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1
0
1
2
3
4
x
$O(4)$, $\langle\bar{\psi}\psi\rangle$
MF, $\langle\bar{\psi}\psi\rangle$
$O(4)$, $\langle\bar{\psi}\psi\rangle_3$
MF, $\langle\bar{\psi}\psi\rangle_3$
$O(4)$, $\langle\bar{\psi}\psi\rangle_3$, scaled
MF, $\langle\bar{\psi}\psi\rangle_3$, scaled

Derivatives: give scaling of pseudo critical temperature Tc with mass

$$T_c = T_c(0) + k_s m_\pi^{2/\beta\delta}$$

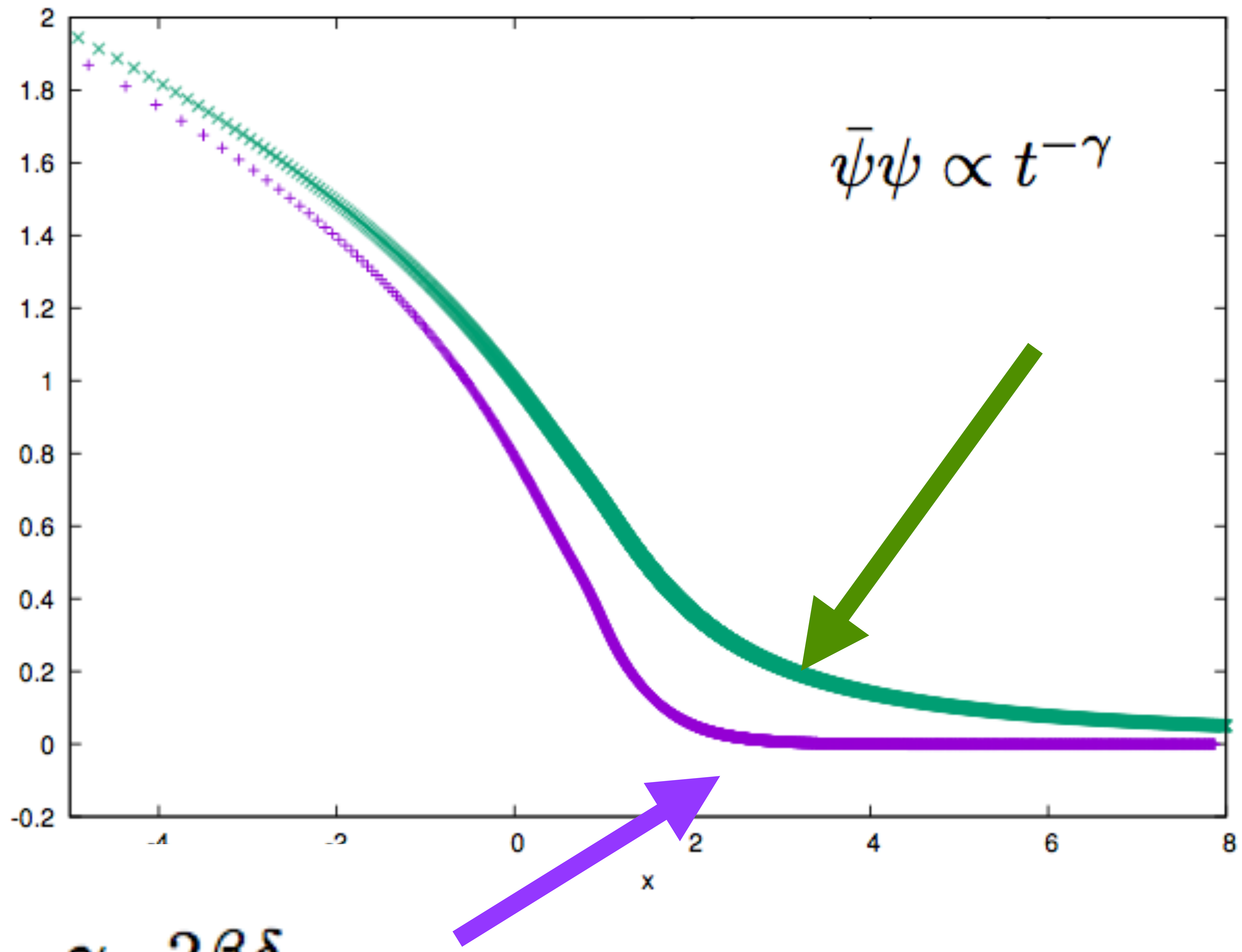
Observable	χ	$\bar\psi\psi$	Δ_3
k_s	1.35(3)	0.74(4)	0.59(1)



Asymptotic behavior - high T expansion

$$f_G(x) = x^{-\gamma} \sum_{n=0}^{\infty} d_n x^{-2n\Delta}$$

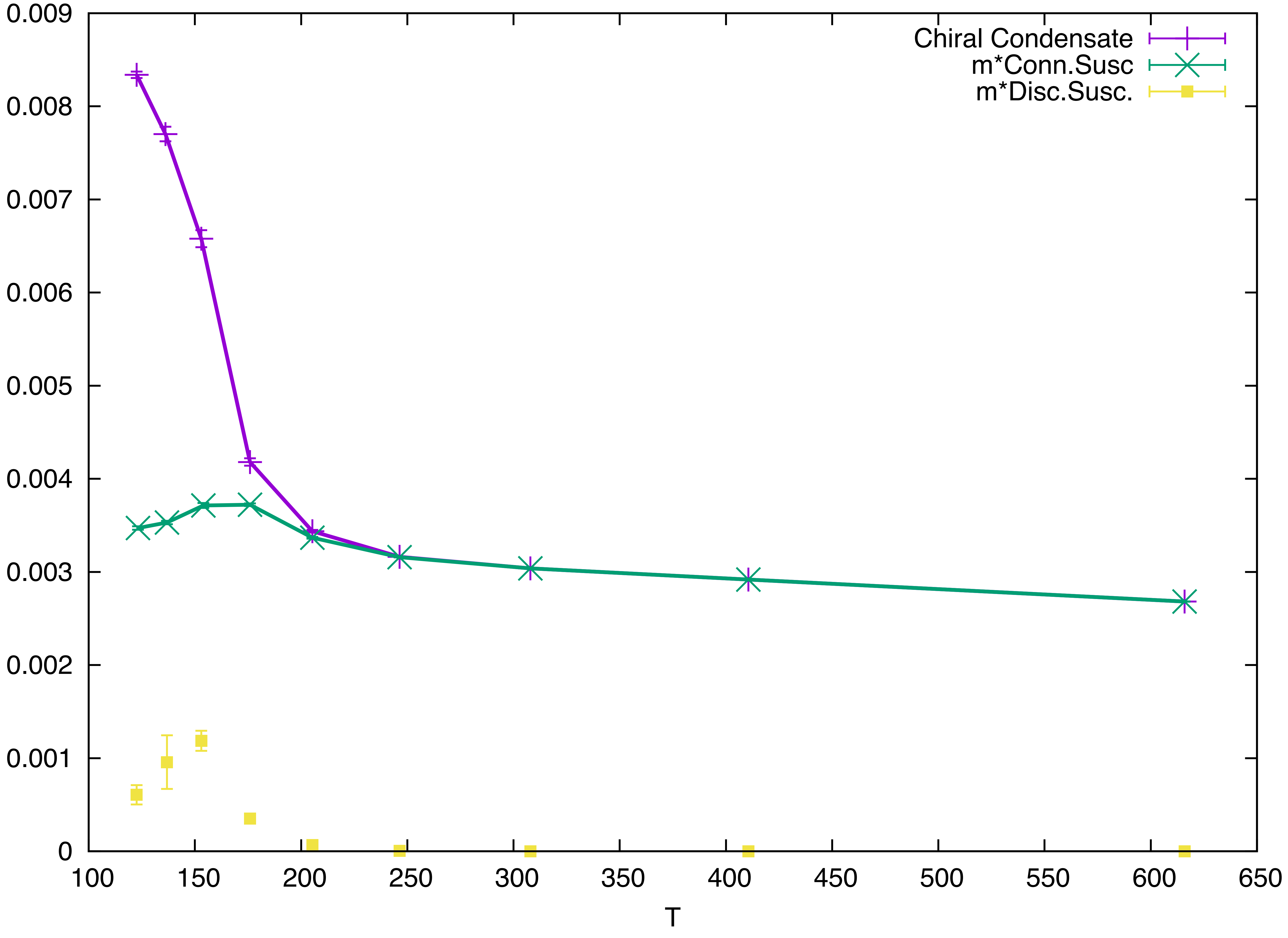
again, linear term
drops in Δ_3



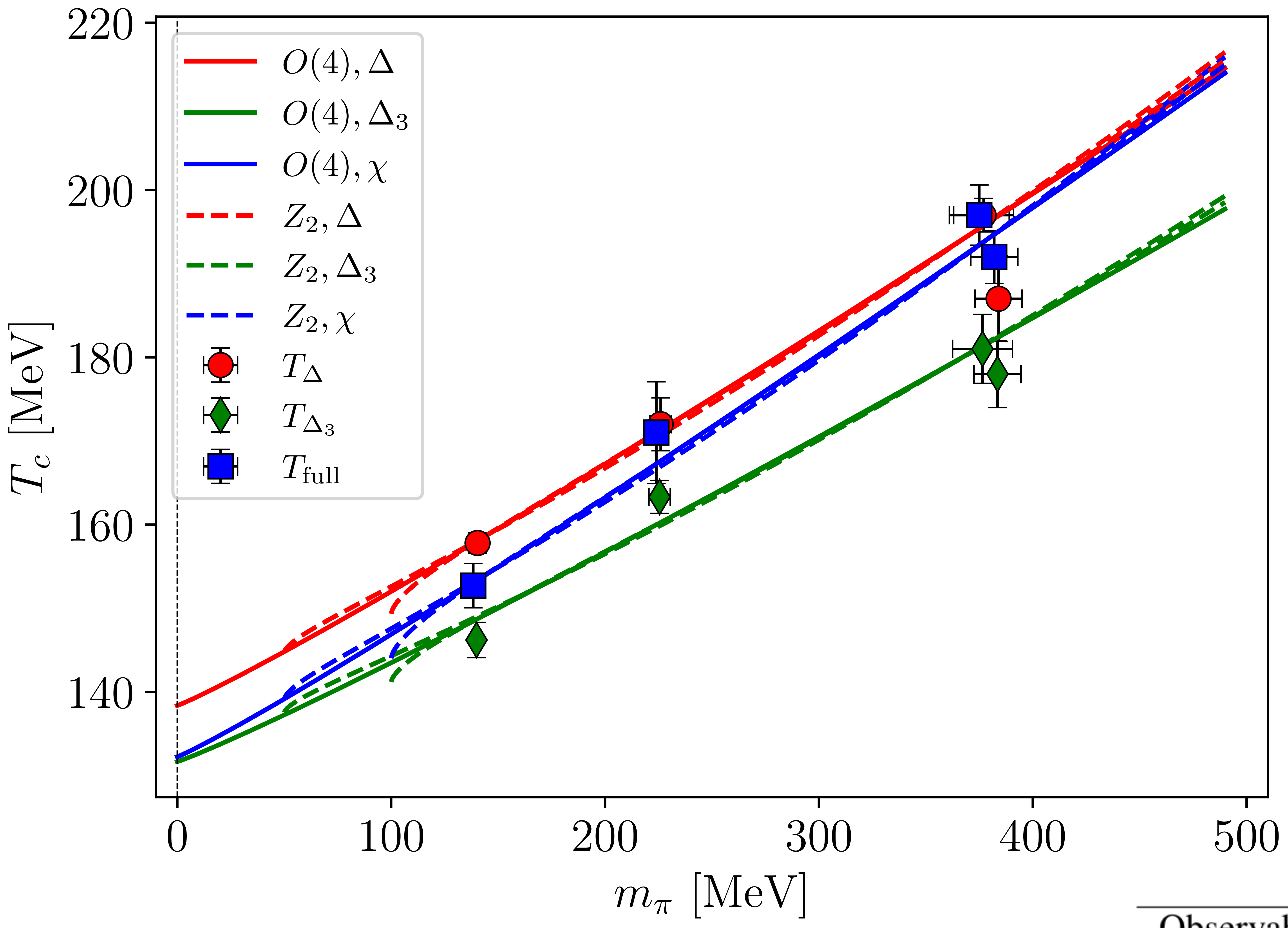
$$\Delta_3 \propto t^{-\gamma-2\beta\delta}$$

Numerical results for physical pion mass

$$(\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m})$$

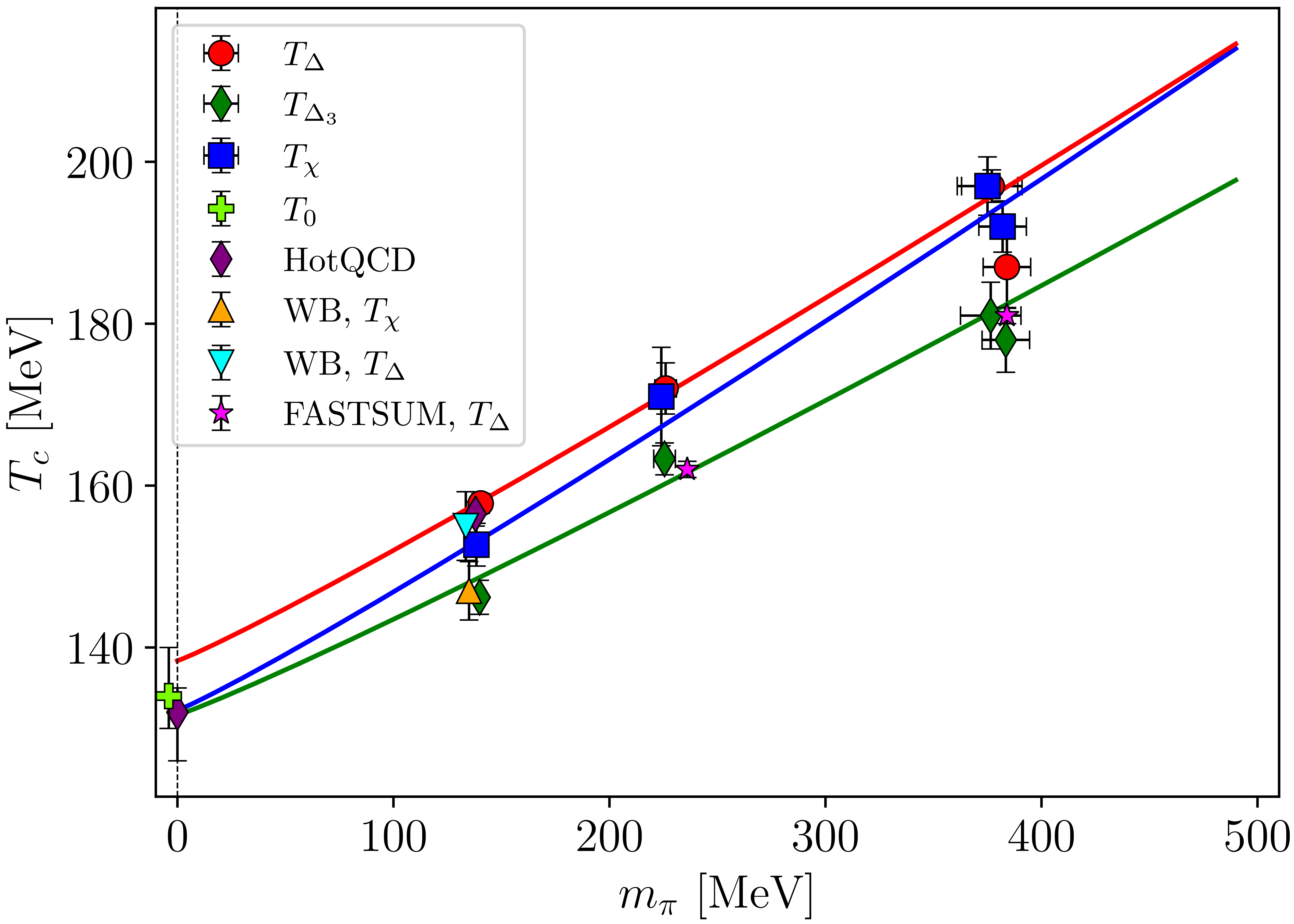


Scaling of the pseudo critical temperatures



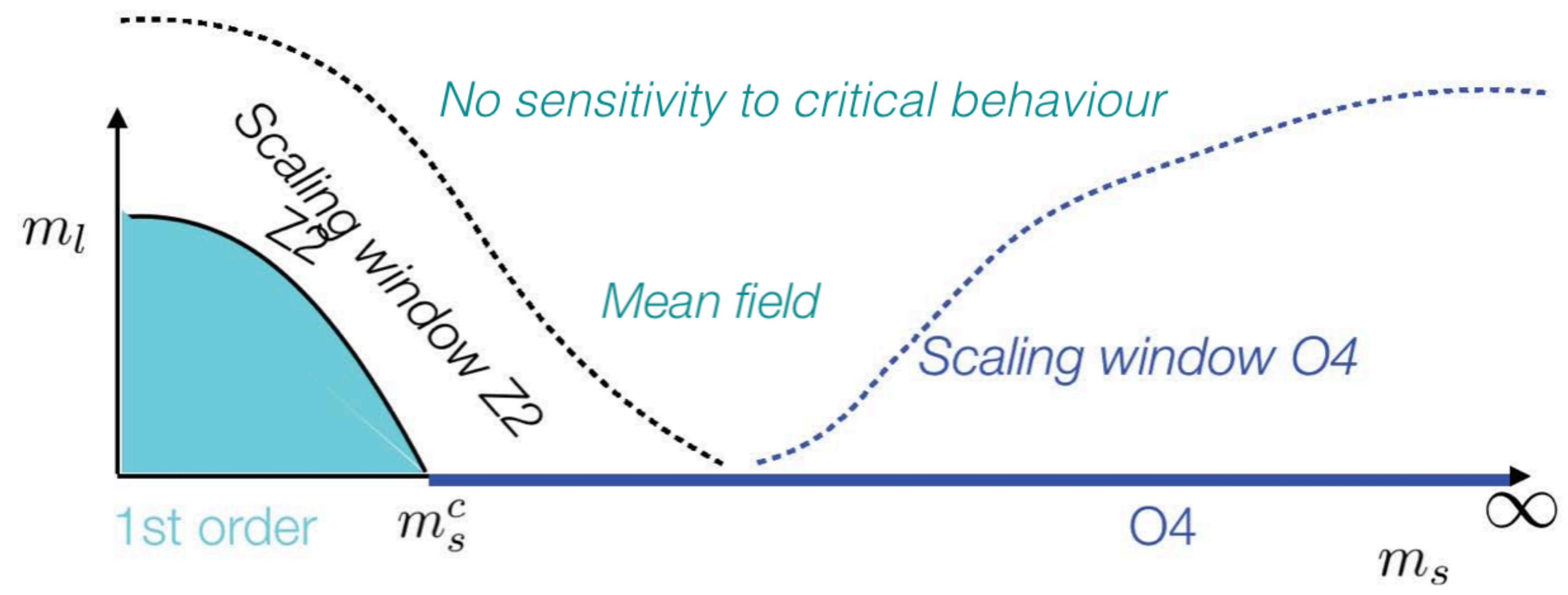
O(4)

Observable	χ	$\bar{\psi}\psi$?	Δ_3
k_s	1.35(3)	0.74(4)	0.59(1)



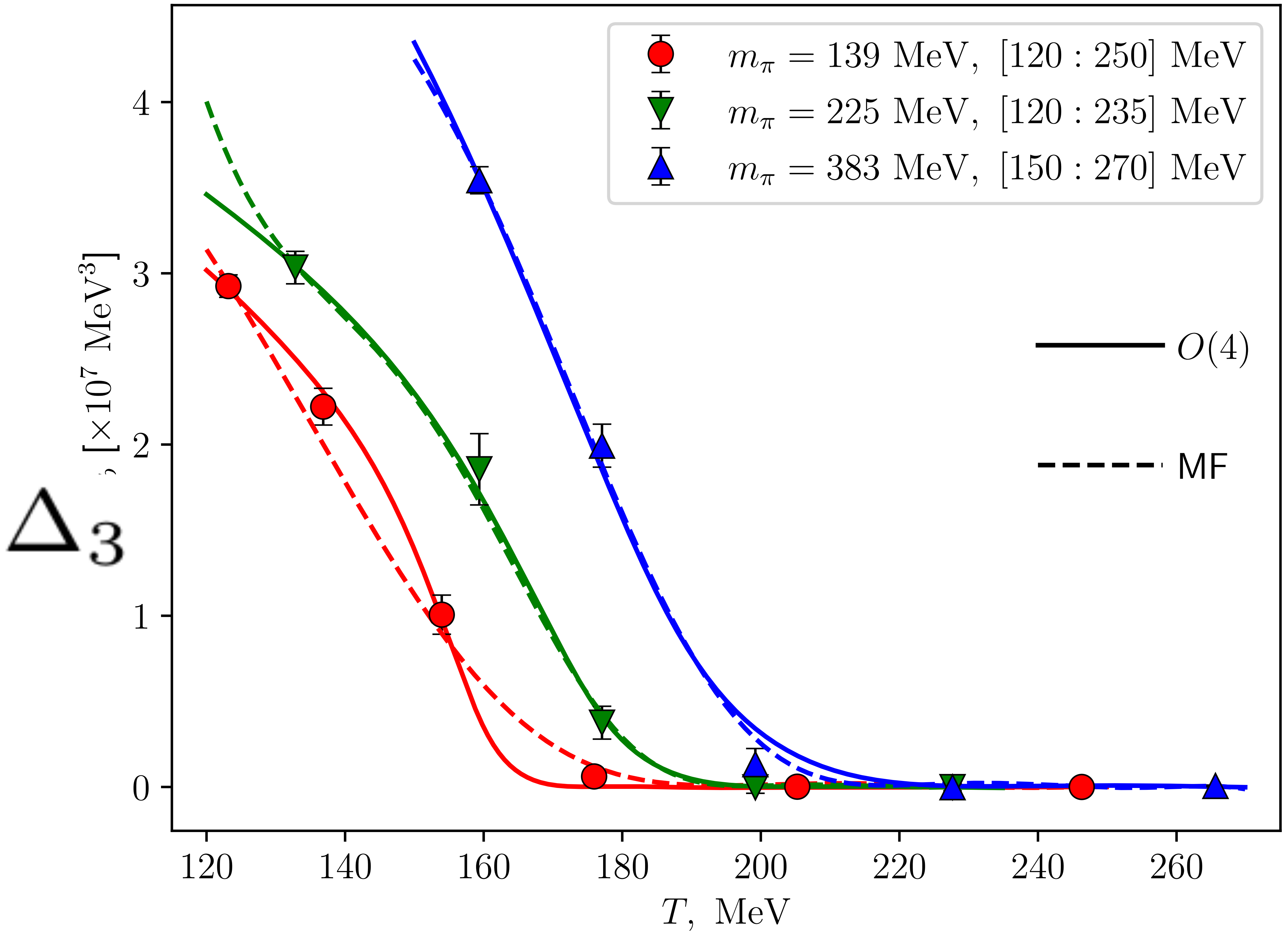
T_Δ
T_{Δ_3}
T_χ
T_0
HotQCD
WB, T_χ
WB, T_Δ
FASTSUM, T_Δ
T_c [MeV]
140
160
180
200
0
100
200
300
400
500
m_π [MeV]

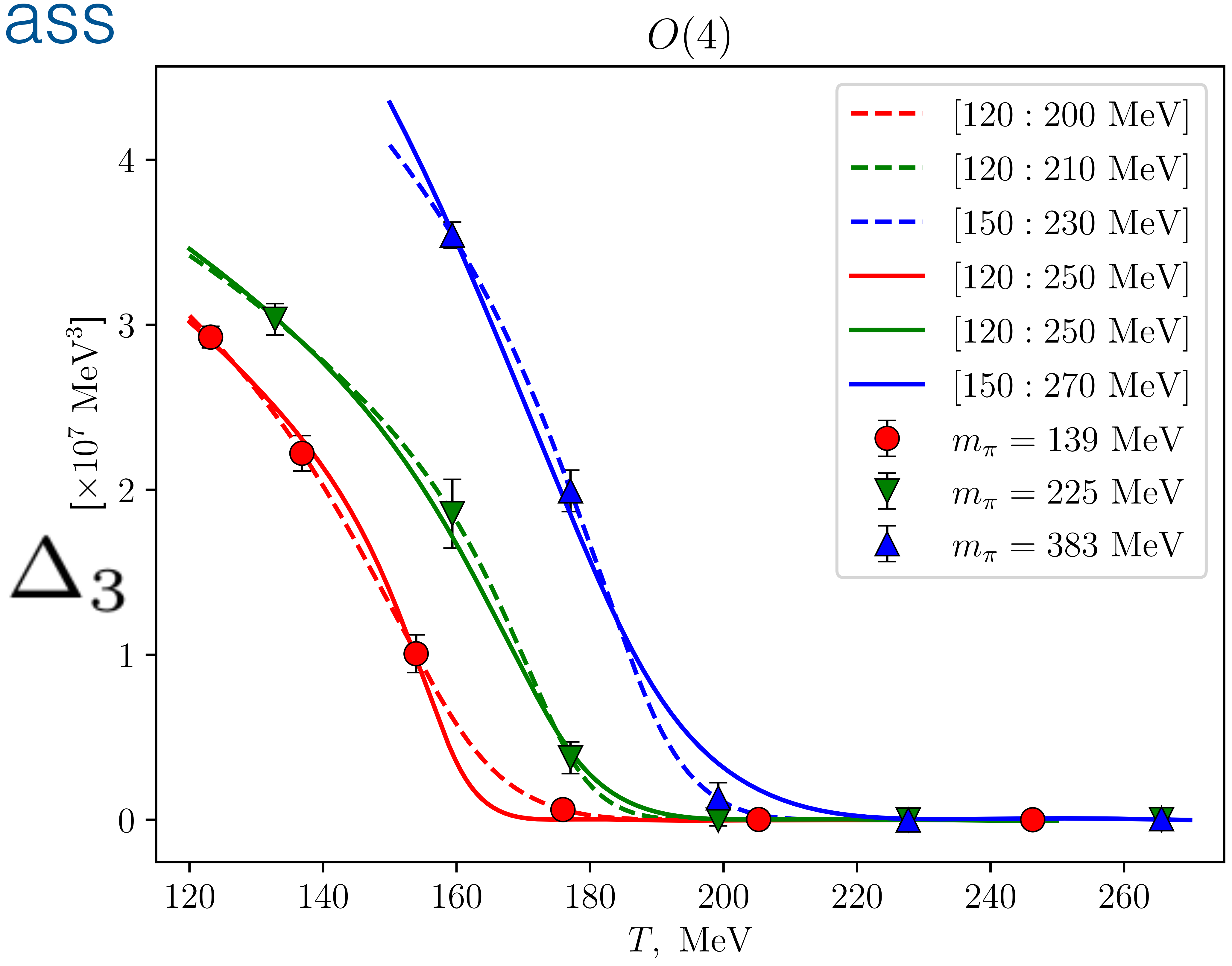
Searching for the scaling window in mass



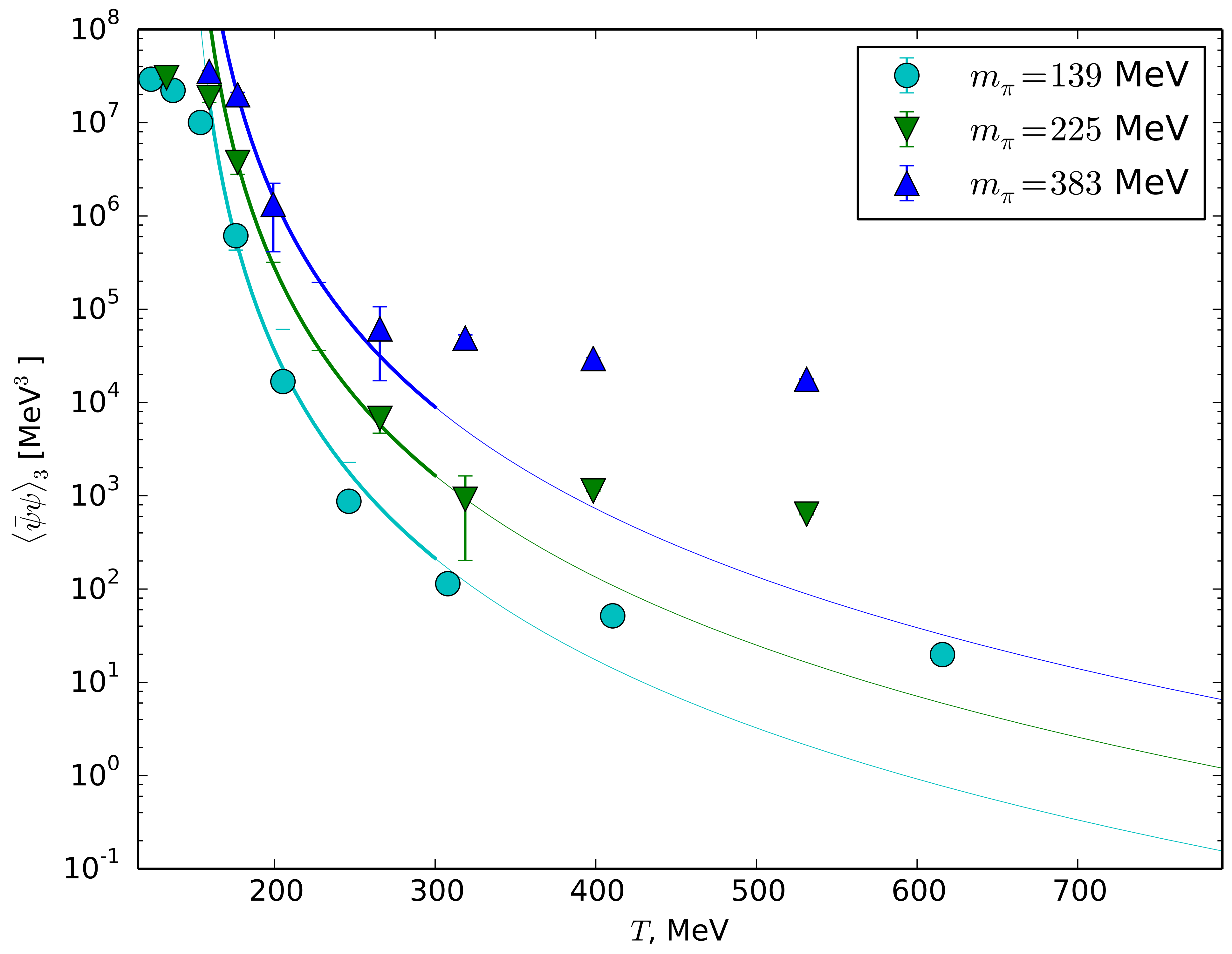
O(4) or mean field?

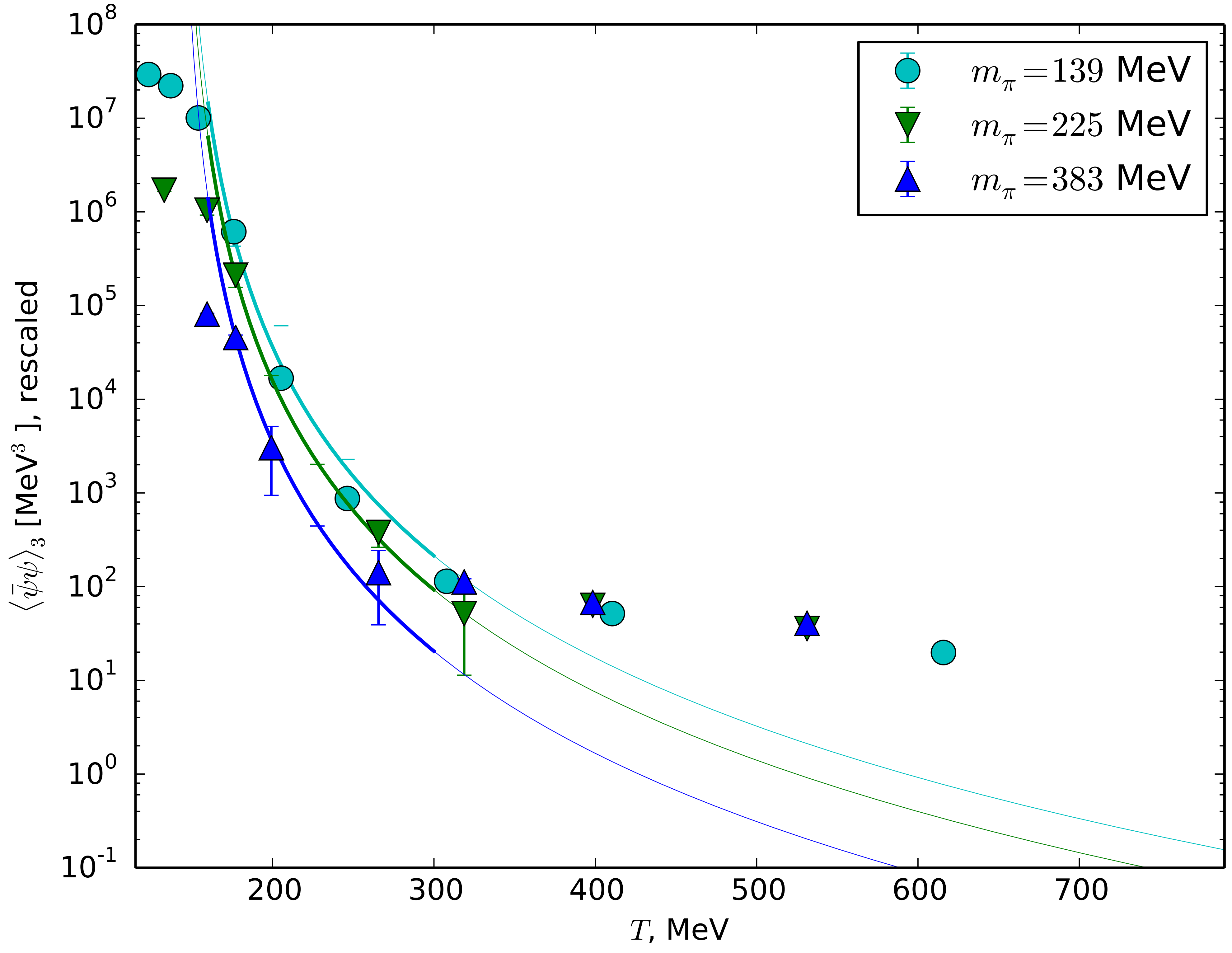
Unrealistic Tc from O4 at high mass





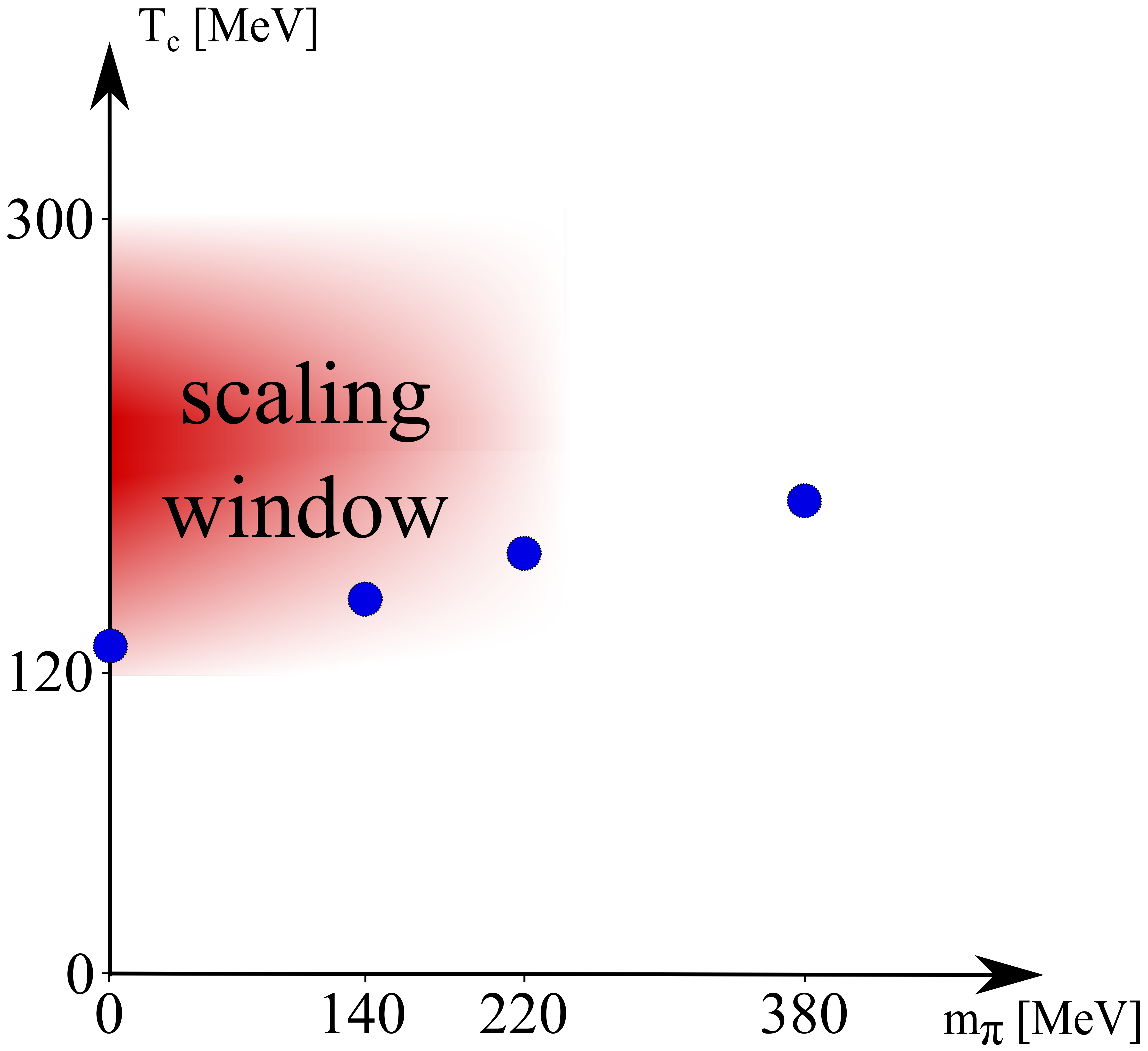
Searching for the scaling window in temperature





$\Delta_3 \propto t^{-\gamma-2\beta\delta}$ T < 300 MeV

$\Delta_3 \propto m_\pi^6$ T > 300 MeV



T_c [MeV]
300
120
0
scaling
window
0
140
220
380
m_π [MeV]

Topology, scaling, and a threshold in hot QCD

Topology: We observe a behaviour consistent with DIGA above T = 300 MeV

Scaling: We observe O(4) scaling for T < 300 MeV, and leading mass scaling T > 300 MeV

Is this a coincidence or does it point to a threshold in hot QCD ?