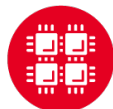


Non-equilibrium attractor in high-temperature QCD plasmas

Michael Strickland
Kent State University

Primary refs: [MS, JHEP2018, 128](#)
[D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 \(2020\).](#)

ICTS Program: Extreme Non-Equilibrium QCD
Oct 5, 2020



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Non-equilibrium attractor in high-temperature QCD plasmas



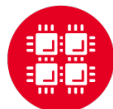
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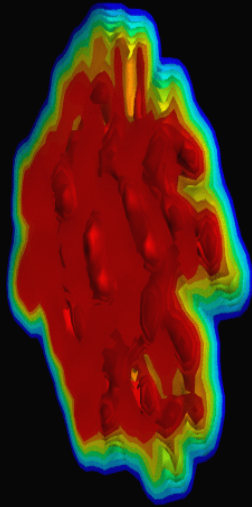
ICTS Program: Extreme Non-Equilibrium QCD
Oct 5, 2020



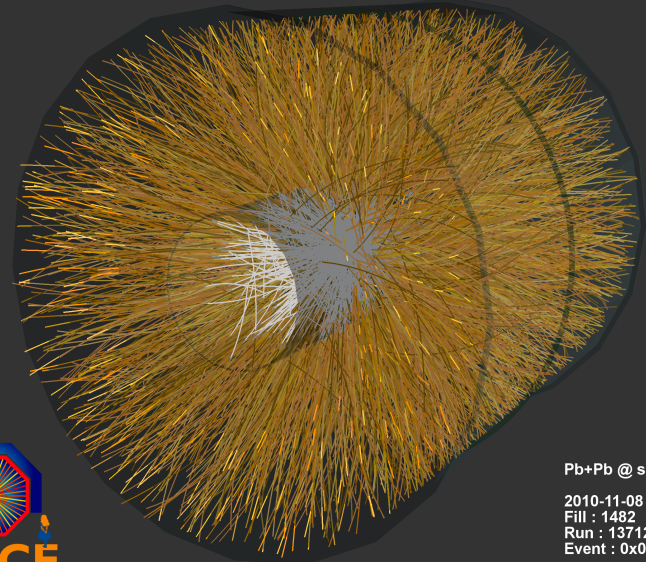
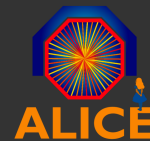
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Ultrarelativistic heavy-ion collisions

- **RHIC**, BNL – Au-Au @ 200 GeV/nucleon (highest energy) $\rightarrow T_0 \sim 400$ MeV
- **LHC**, CERN – Pb-Pb @ 2.76 TeV $\rightarrow T_0 \sim 600$ MeV
- **LHC**, CERN – Pb-Pb @ 5.03 TeV $\rightarrow T_0 \sim 700$ MeV
- **RHIC**, BNL **BES** – Au-Au @ 7.7 - 39 GeV $\rightarrow T_0 \sim 30$ -100 MeV [+finite density]
- **FAIR** (GSI), **NICA** (Dubna) – U-U @ 35 GeV $\rightarrow T_0 \sim 100$ MeV [+finite density]



Animation, B. Schenke et al

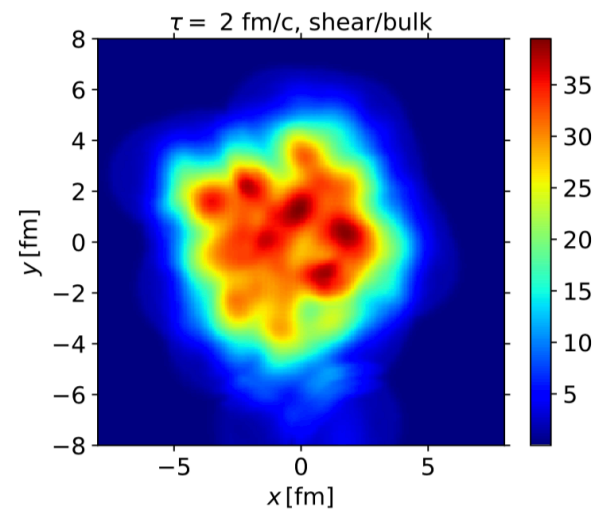
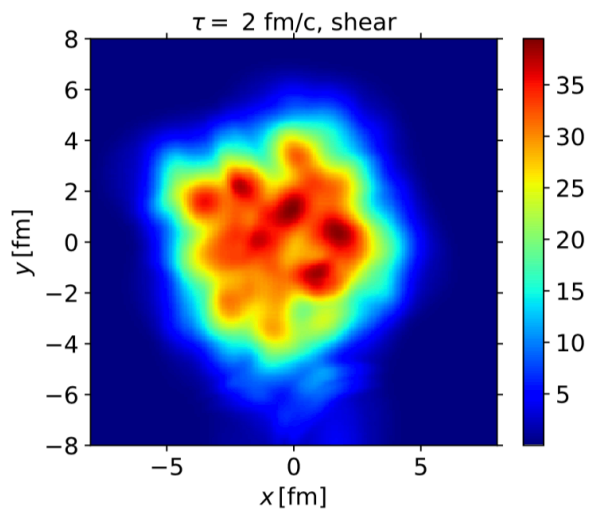
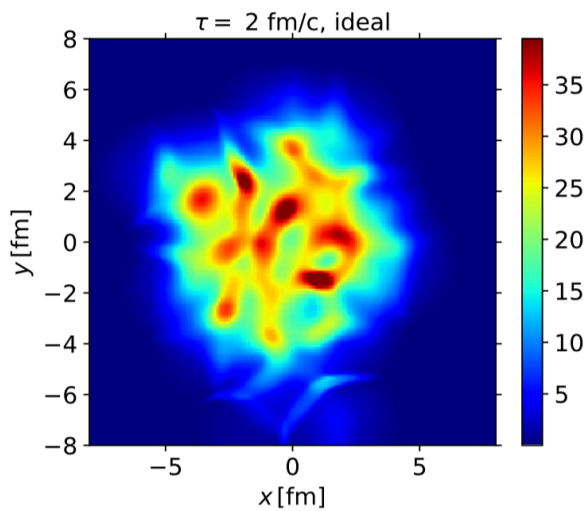
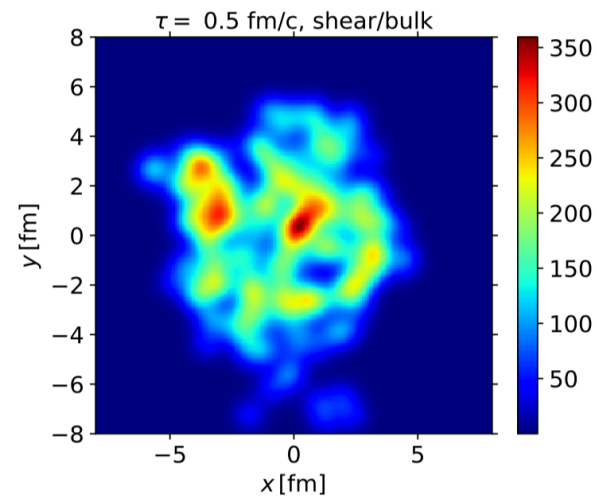
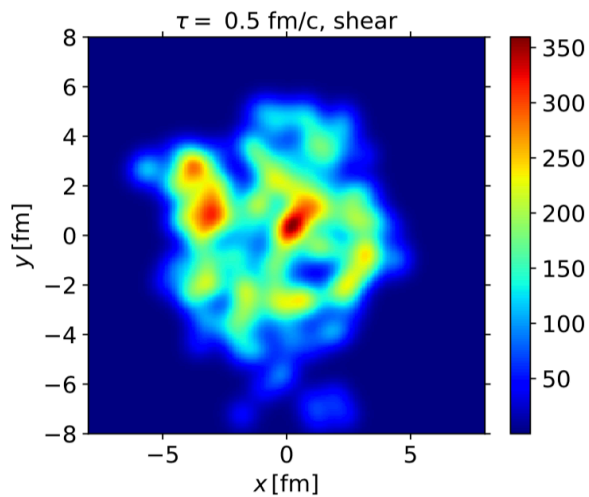
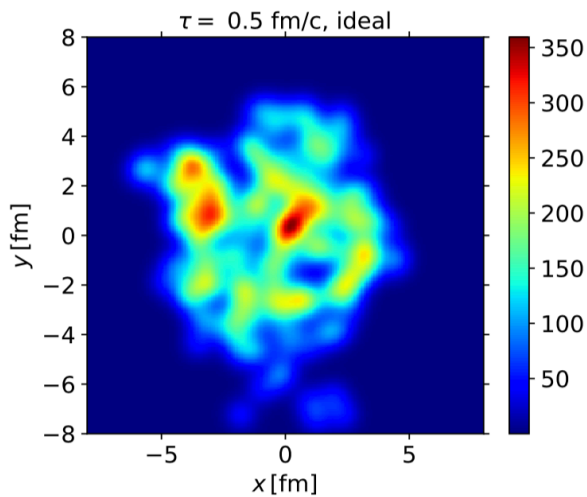


Pb+Pb @ $\sqrt{s} = 2.76$ ATeV
2010-11-08 11:30:46
Fill : 1482
Run : 137124
Event : 0x00000000D3BBE693

Ideal Hydro

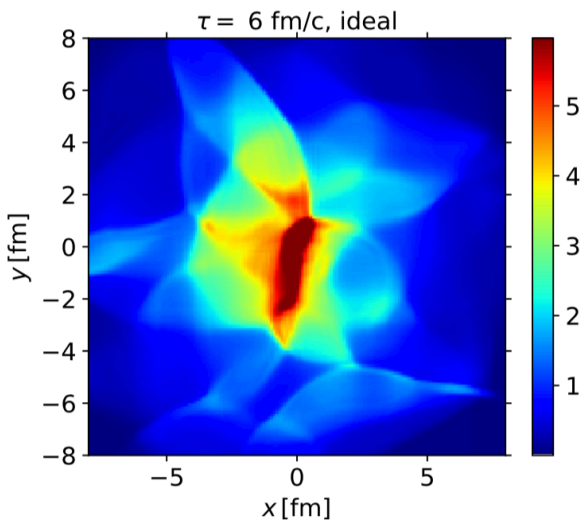
Shear only ($\eta/s = 0.2$)

Shear + Bulk

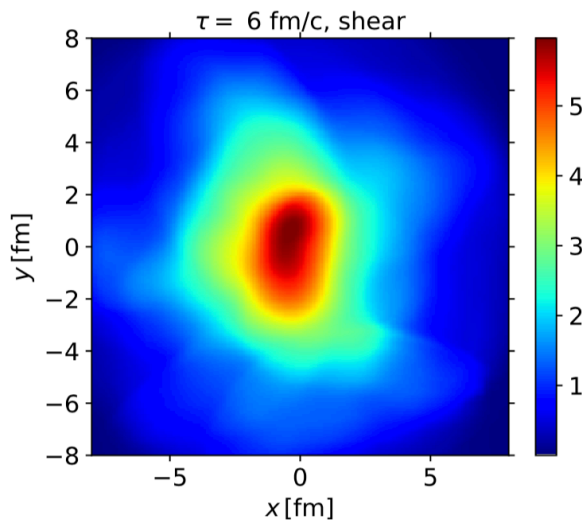


D. Bazow, U. Heinz, and MS, 1608.06577

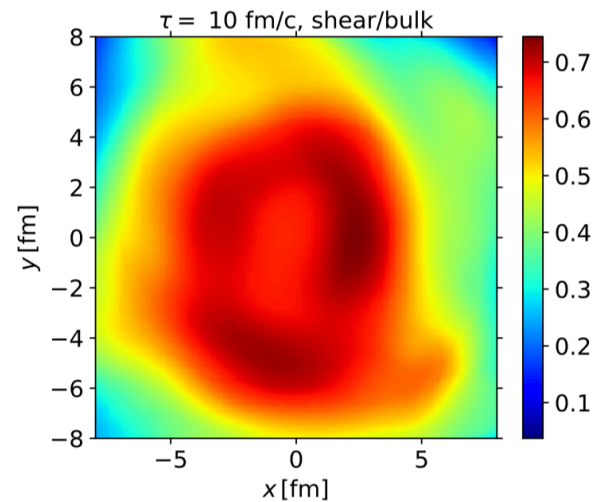
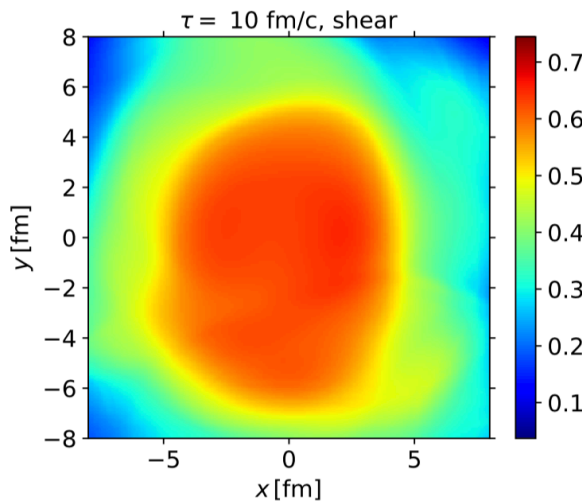
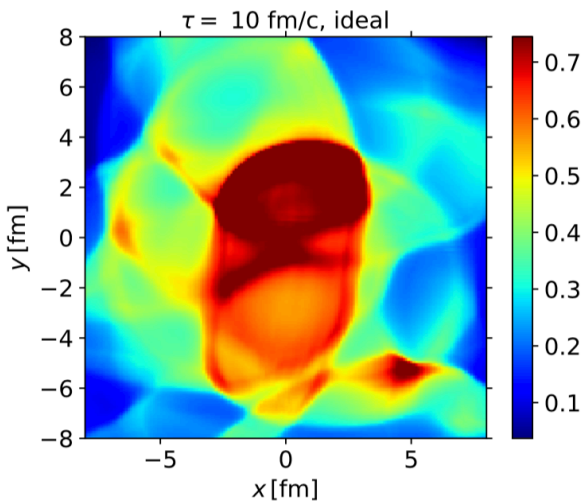
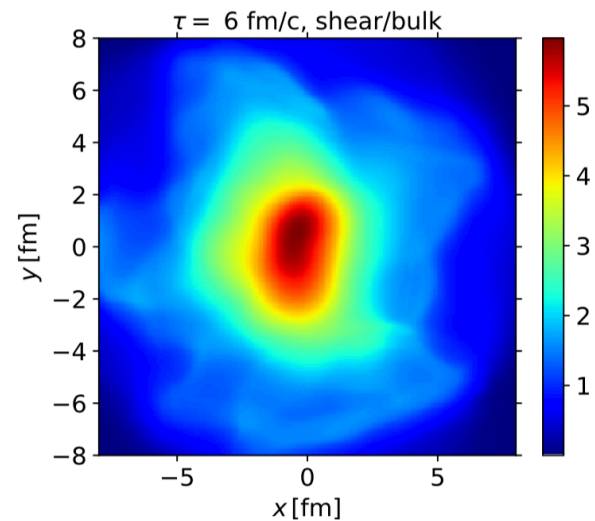
Ideal Hydro



Shear only ($\eta/s = 0.2$)

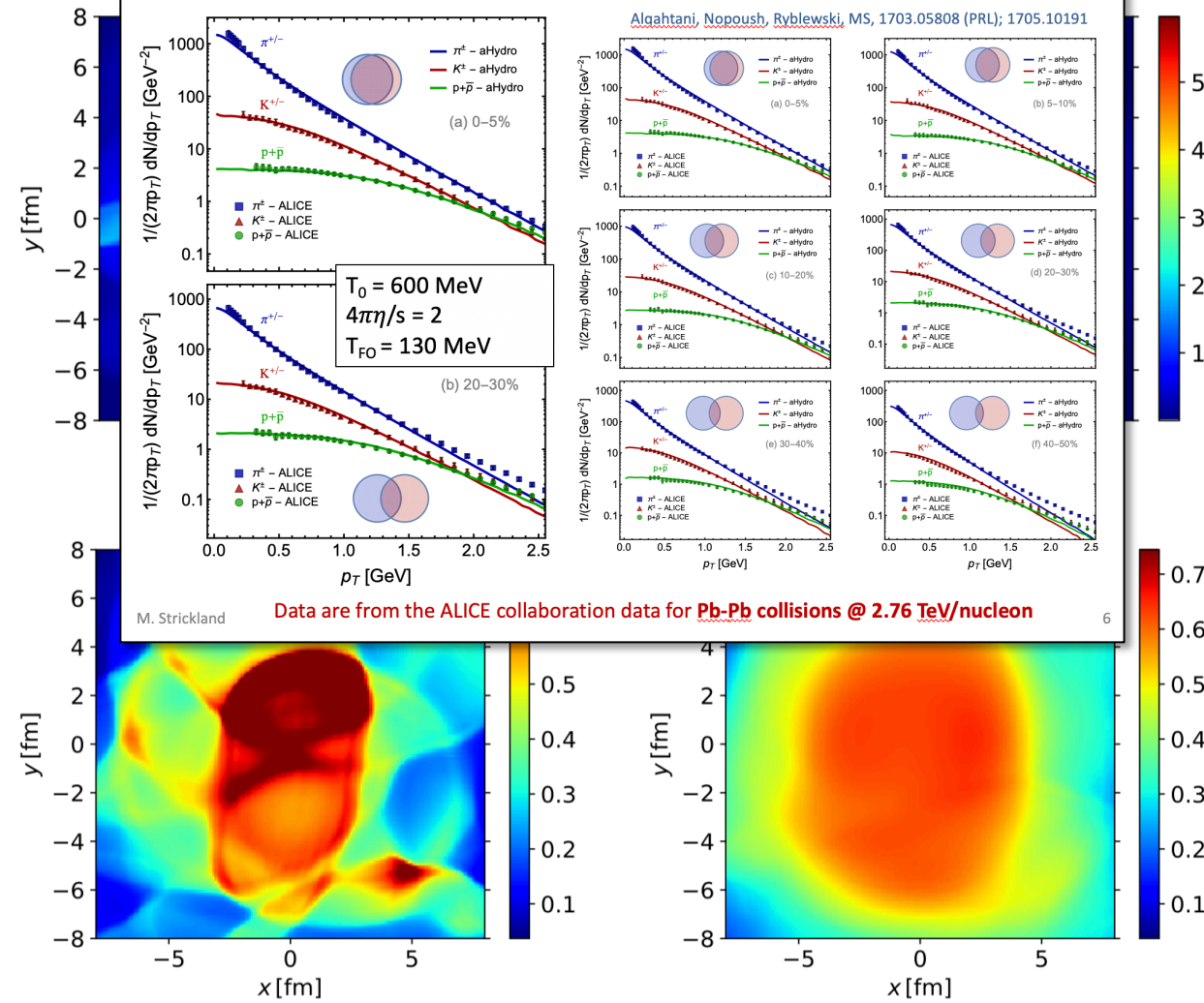


Shear + Bulk

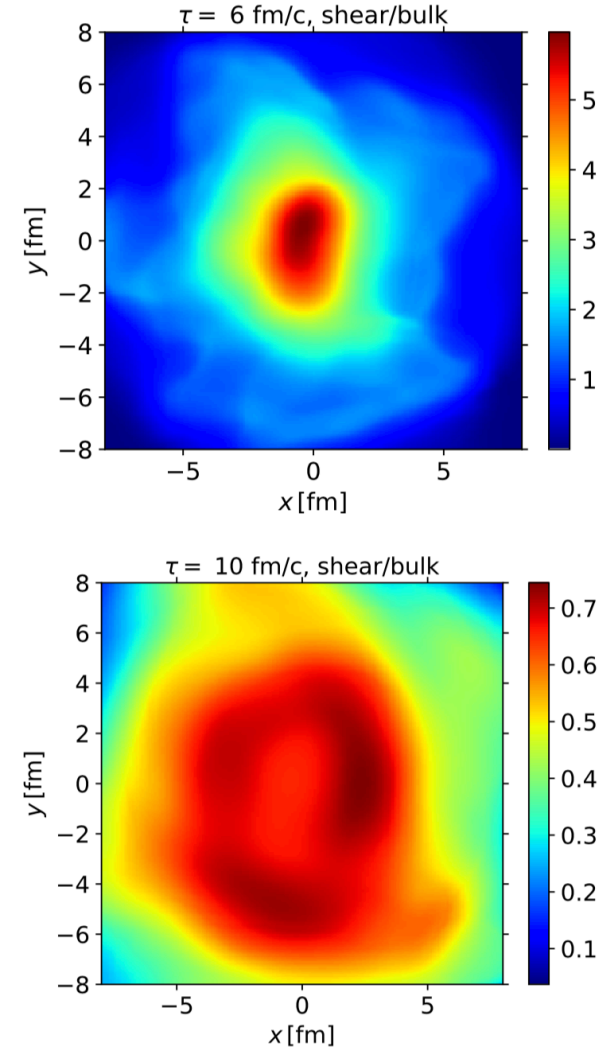


D. Bazow, U. Heinz, and MS, 1608.06577

Identified particle spectra

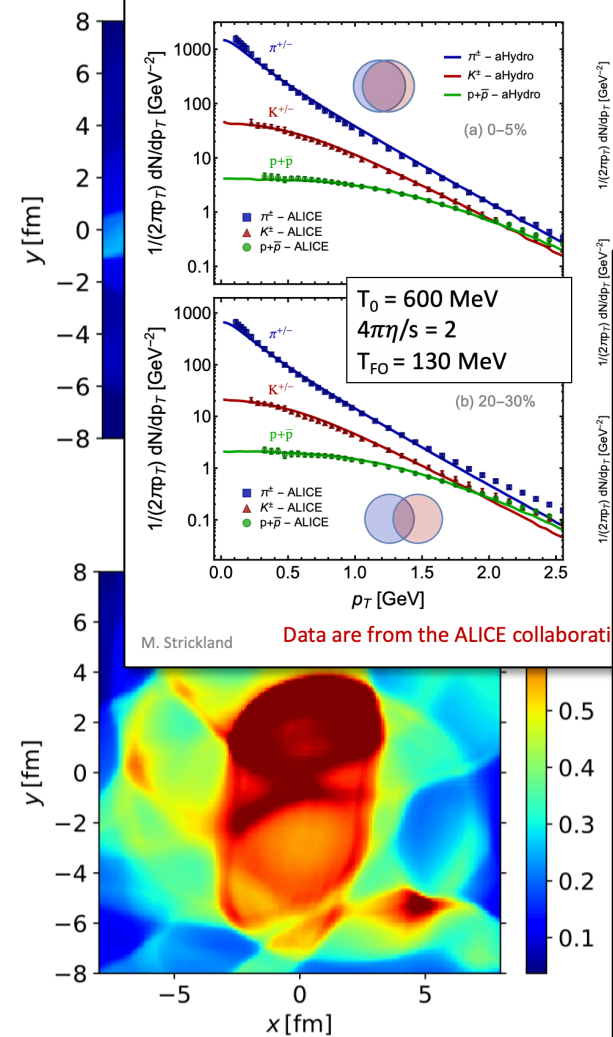


Shear + Bulk

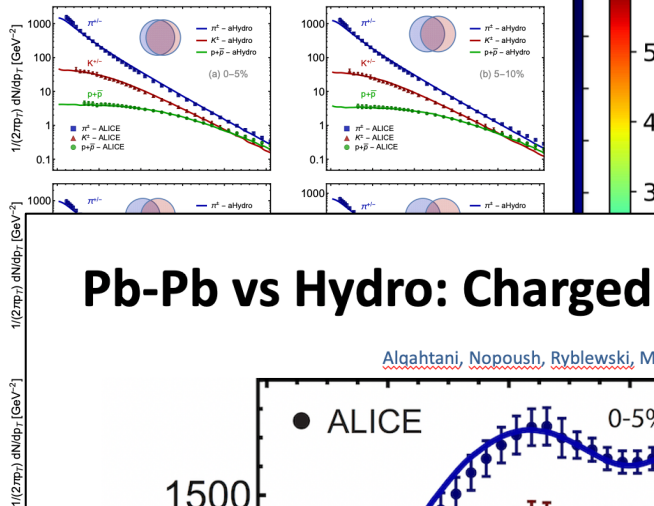


D. Bazow, U. Heinz, and MS, 1608.06577

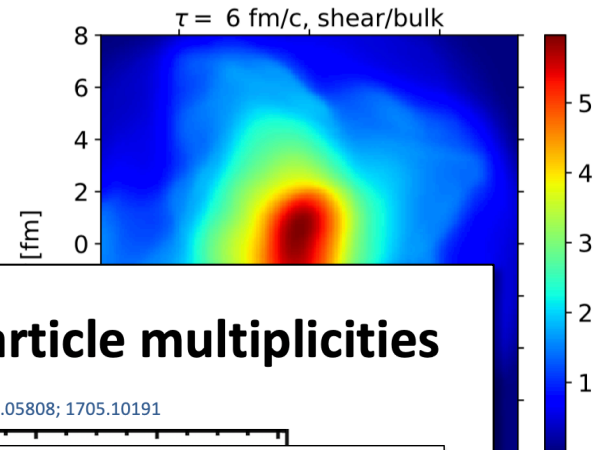
Identified particle spectra



Alqahtani, Nopoush, Ryblewski, MS, 1703.05808 (PRL); 1705.10191

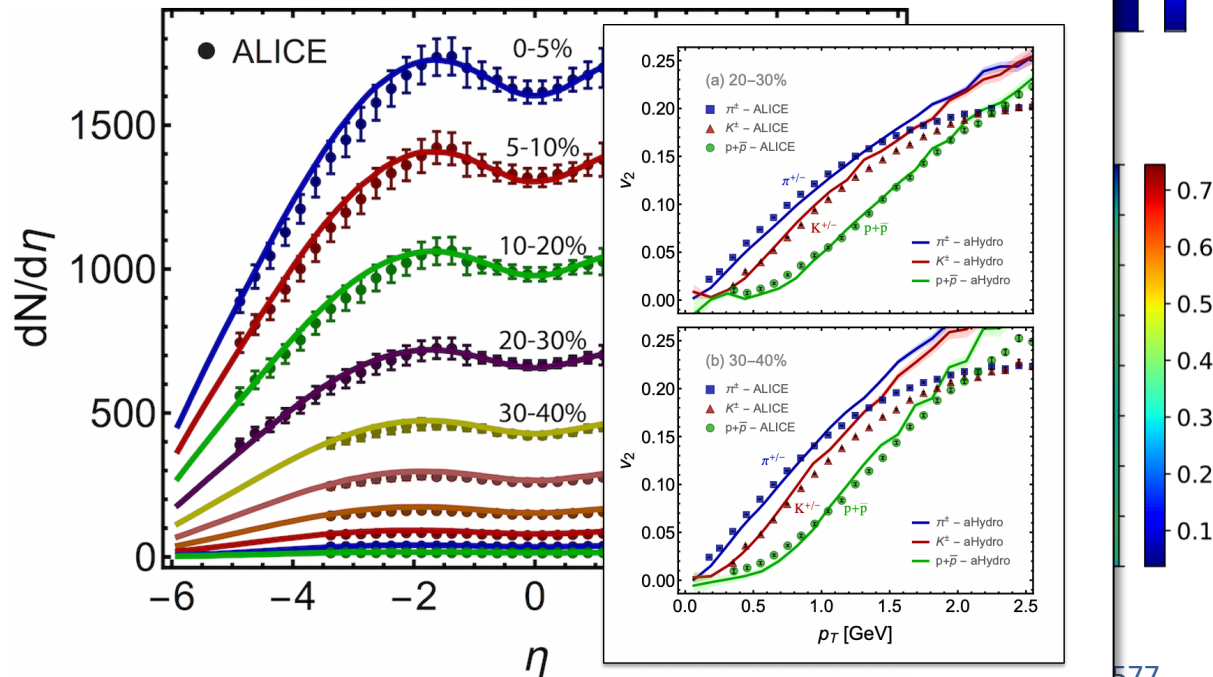


Shear + Bulk



Pb-Pb vs Hydro: Charged particle multiplicities

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191



M. Strickland

8

577

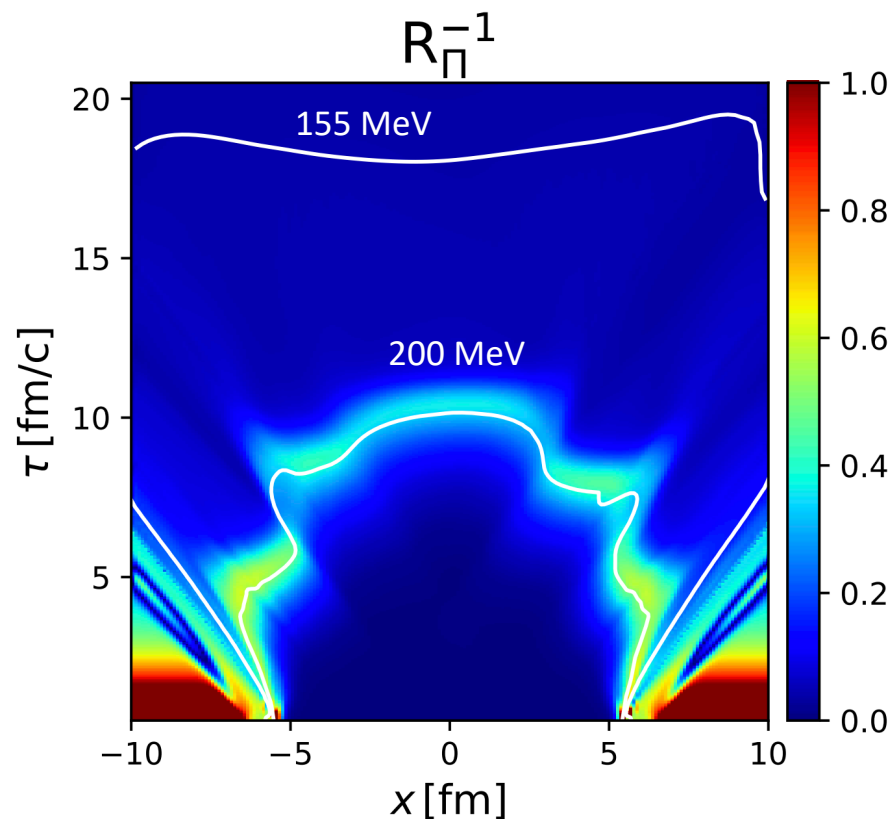
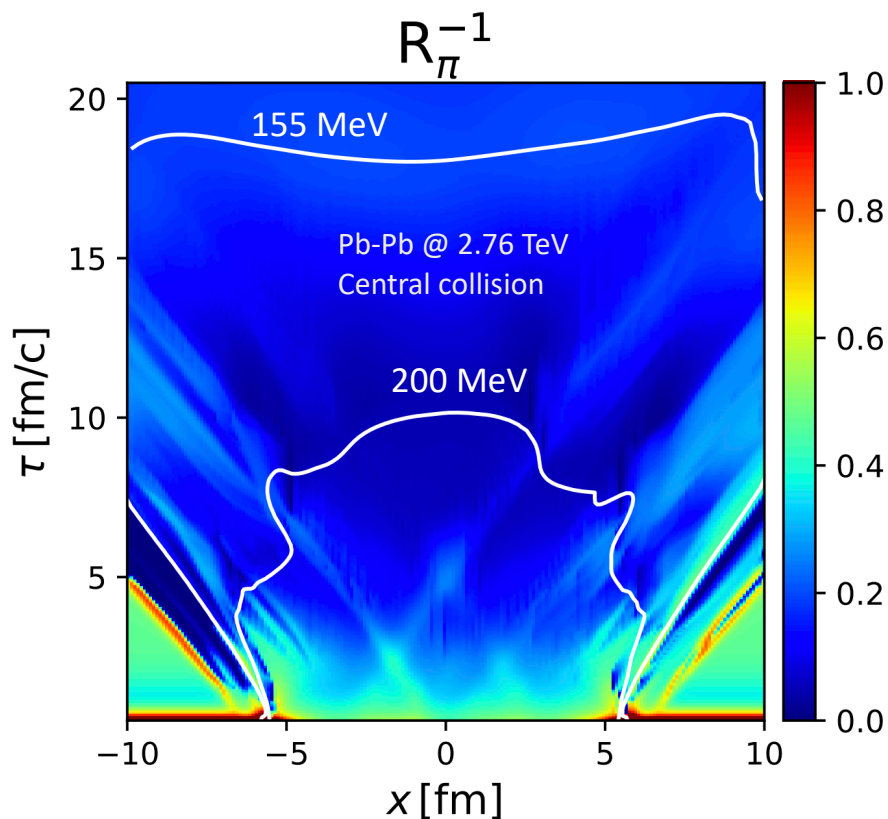
Pb-Pb @ 2.76 TeV - Don't worry, be happy?

$$T^{\mu\nu} = T_{\text{ideal}}^{\mu\nu} + \Pi^{\mu\nu}$$

viscous stress tensor

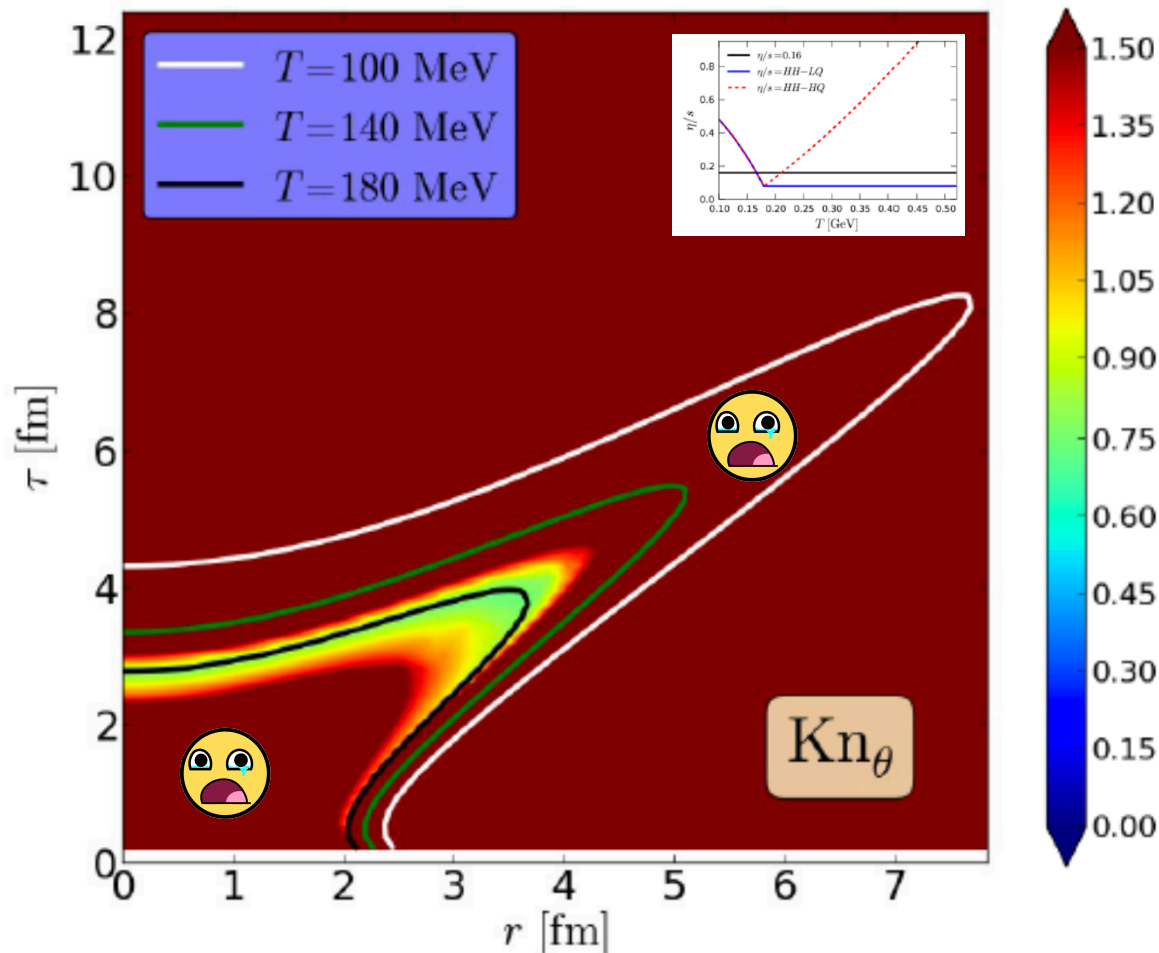
Shear and bulk Inverse Reynolds Numbers

$$R_{\pi}^{-1} \equiv \frac{\sqrt{\pi^{\mu\nu}\pi_{\mu\nu}}}{\mathcal{P}_0}, \quad R_{\Pi}^{-1} \equiv \frac{|\Pi|}{\mathcal{P}_0}.$$



p-A @ 2.76 TeV - Don't be happy, worry!

Figure (sans emoticons): H. Niemi and G. Denicol, 1404.7327



- Large gradients (Knudsen #) induce non-equilibrium deviations (measured by inverse Reynolds #)
- Evolution equations truncated at fixed order in these quantities $\rightarrow$ potential inaccuracy
- System has short lifetime $\rightarrow$ distribution function still far from equilibrium at freeze out

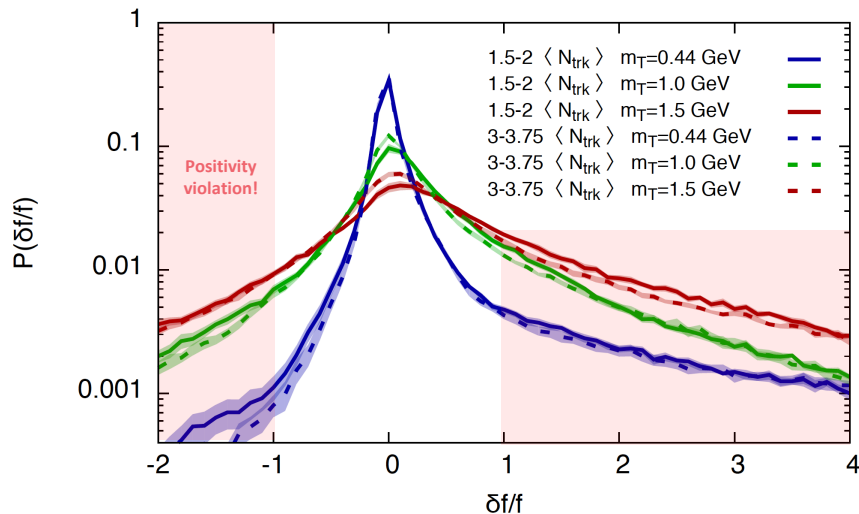
Are the viscous corrections under control?

p-Pb with IP-Glasma + MUSIC + URQMD

H. Mäntysaari, B. Schenke, C. Shen, P. Tribedy, PLB 772, 681 (2017)

$v_n(p_T)$, however, studying the distribution of corrections relative to the thermal distribution from all freeze-out surface cells, we find a significant share (10% for $p_T \sim 0.45$ GeV, 25% for $p_T \sim 1$ GeV, 45% for $p_T \sim 1.5$ GeV for pions) of large (shear) corrections ($|\delta f|/f \gtrsim 100\%$).

This demonstrates that our results are plagued by large viscous corrections, in particular for $p_T \gtrsim 1$ GeV. Nevertheless, p_T -integrated quantities are dominated by low p_T contributions and less sensitive to these problems.



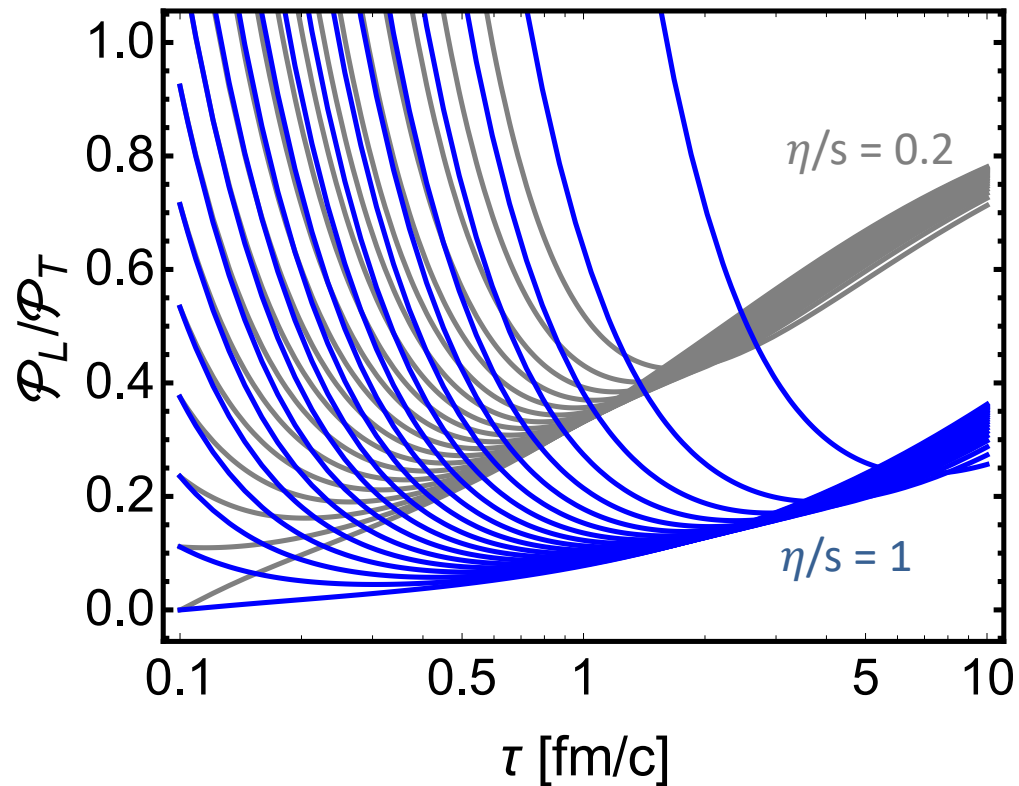
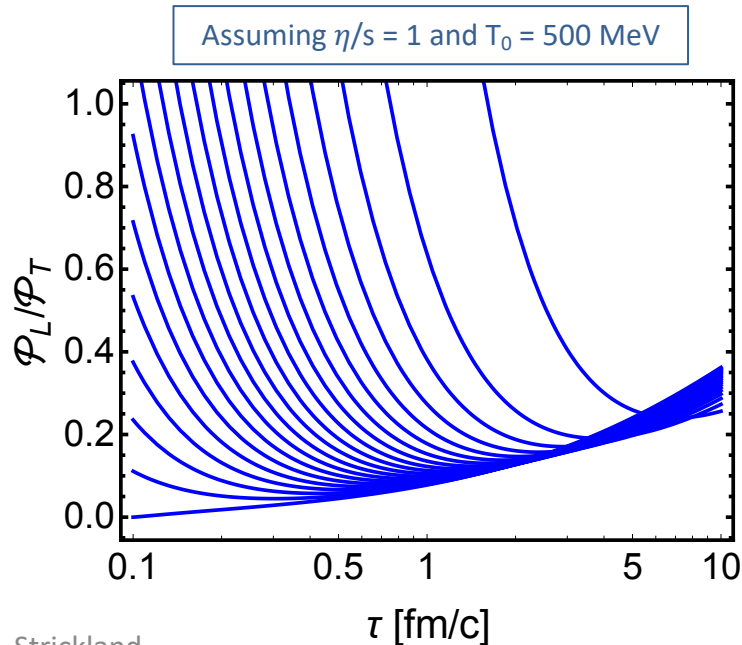
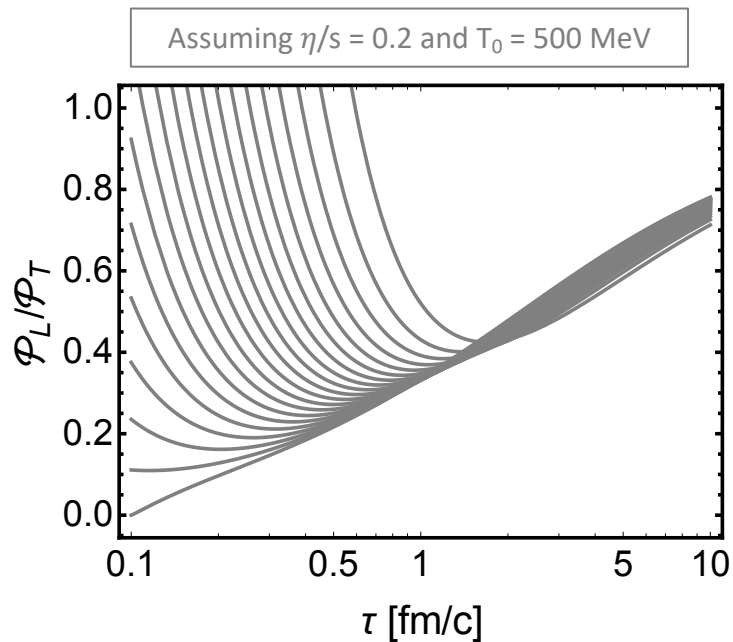
- In small systems, no.
- **Short lifetime + large viscous corrections at freezeout $\rightarrow$ large δf corrections on freeze-out hypersurface**
- As a result, simulations suffer from negative effective pressures and f in a large hypervolume.
- Groups deal with this differently. SONIC, for example, uses an “exponentiation method” introduced by Pratt and Torrieri in PRC 82, 044901 (2010) to prevent $f < 0$ on the switching surface (still large correction).

Practical goals

Improved hydrodynamic treatments in far from equilibrium systems:

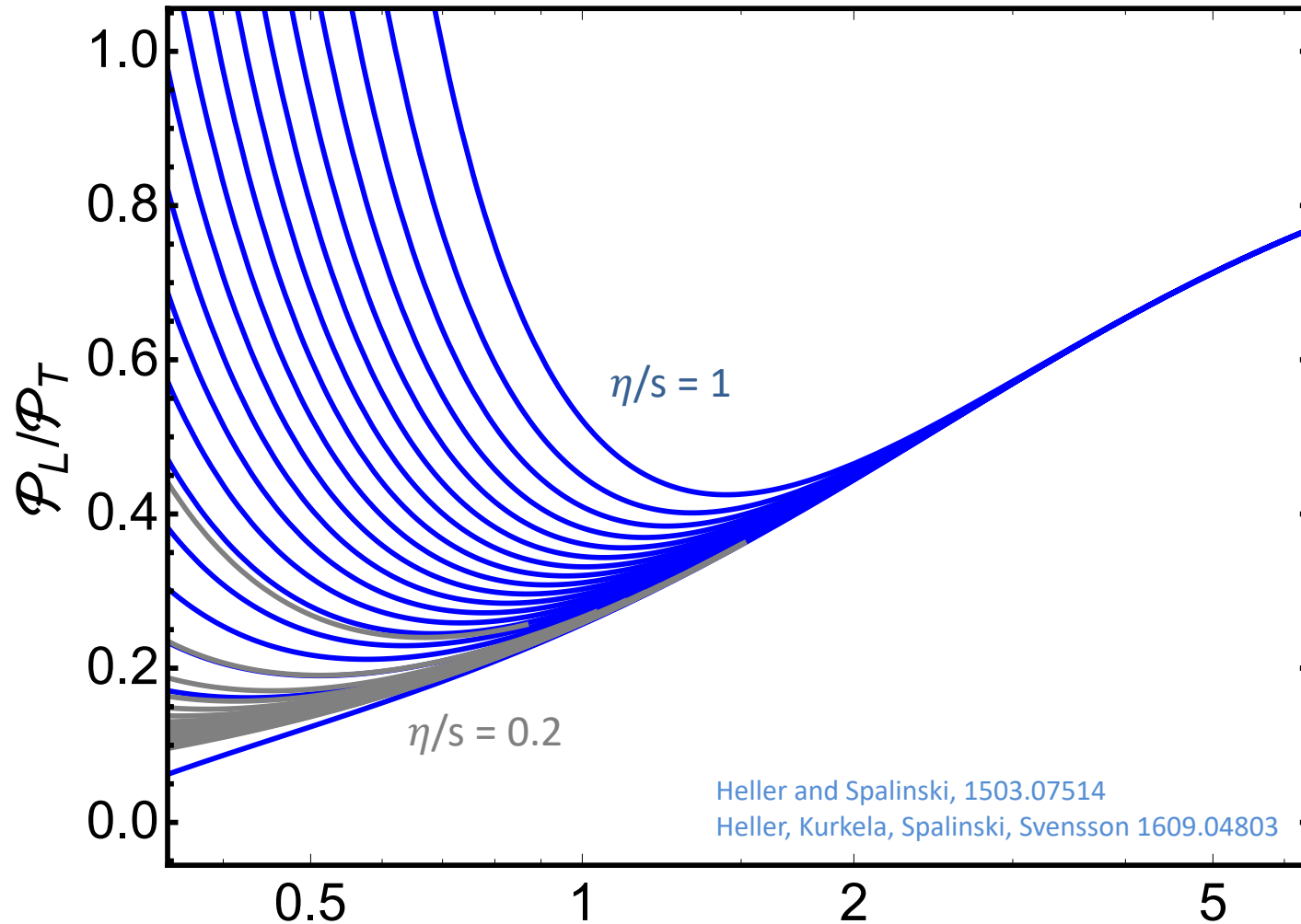
- Can we construct hydrodynamic frameworks that more accurately describe QCD thermalization and apply them to phenomenology? ** more computationally efficient than doing 3+1d kinetic theory simulations and can be extended across the phase boundary using a realistic equation of state
- Is there an attractor for the one-particle distribution function using QCD effective kinetic theory (EKT) → improved description at freezeout?
- Today, I will present progress towards these goals

The non-equilibrium attractor



- Keep it simple: Bjorken 0+1d dynamics.
- Solve dynamical equations for different initial conditions and different values of the shear viscosity (gray vs blue)
- Hints of universal behavior at late times visible (similar levels of momentum anisotropy)

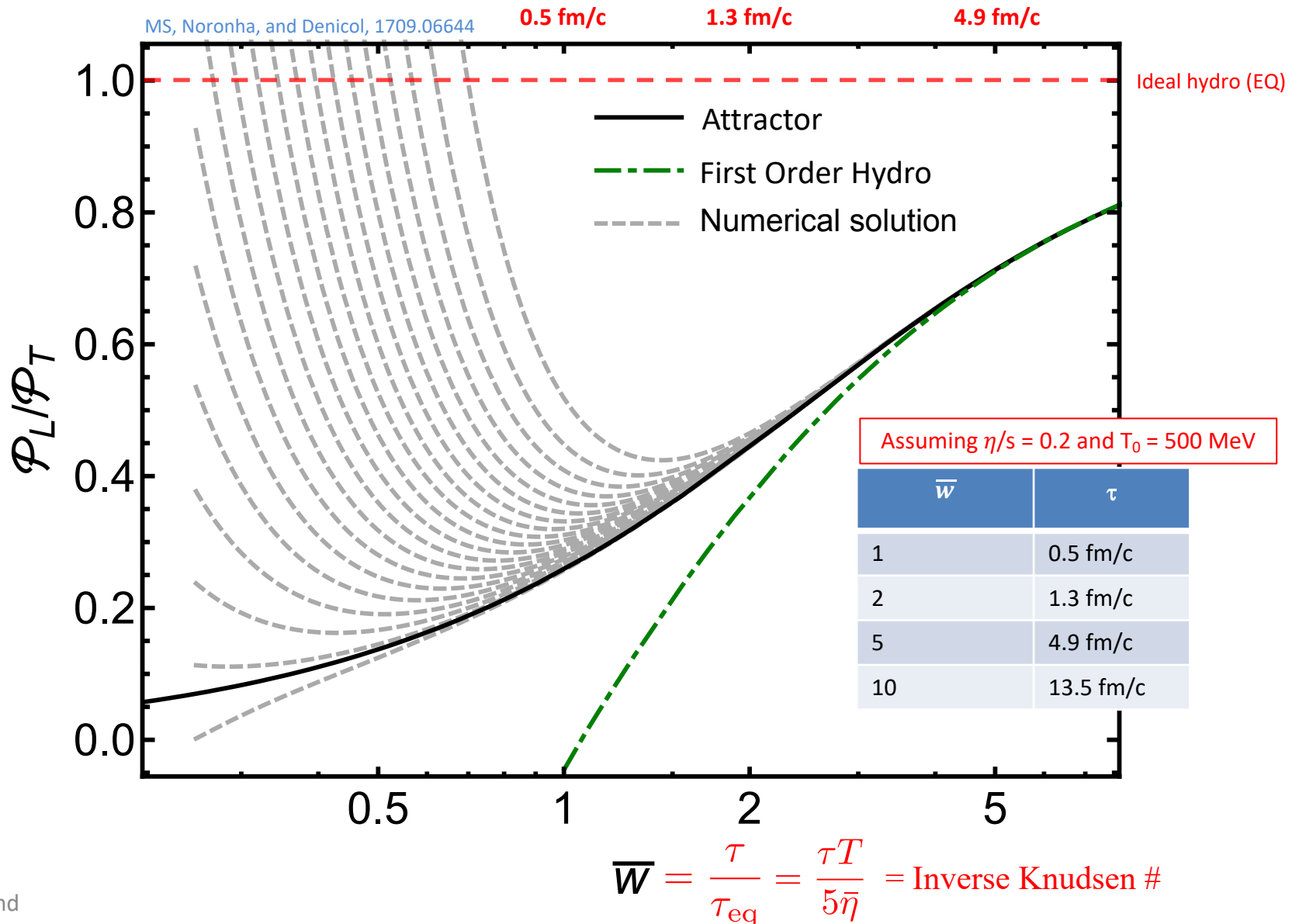
Collapsing the data to the attractor



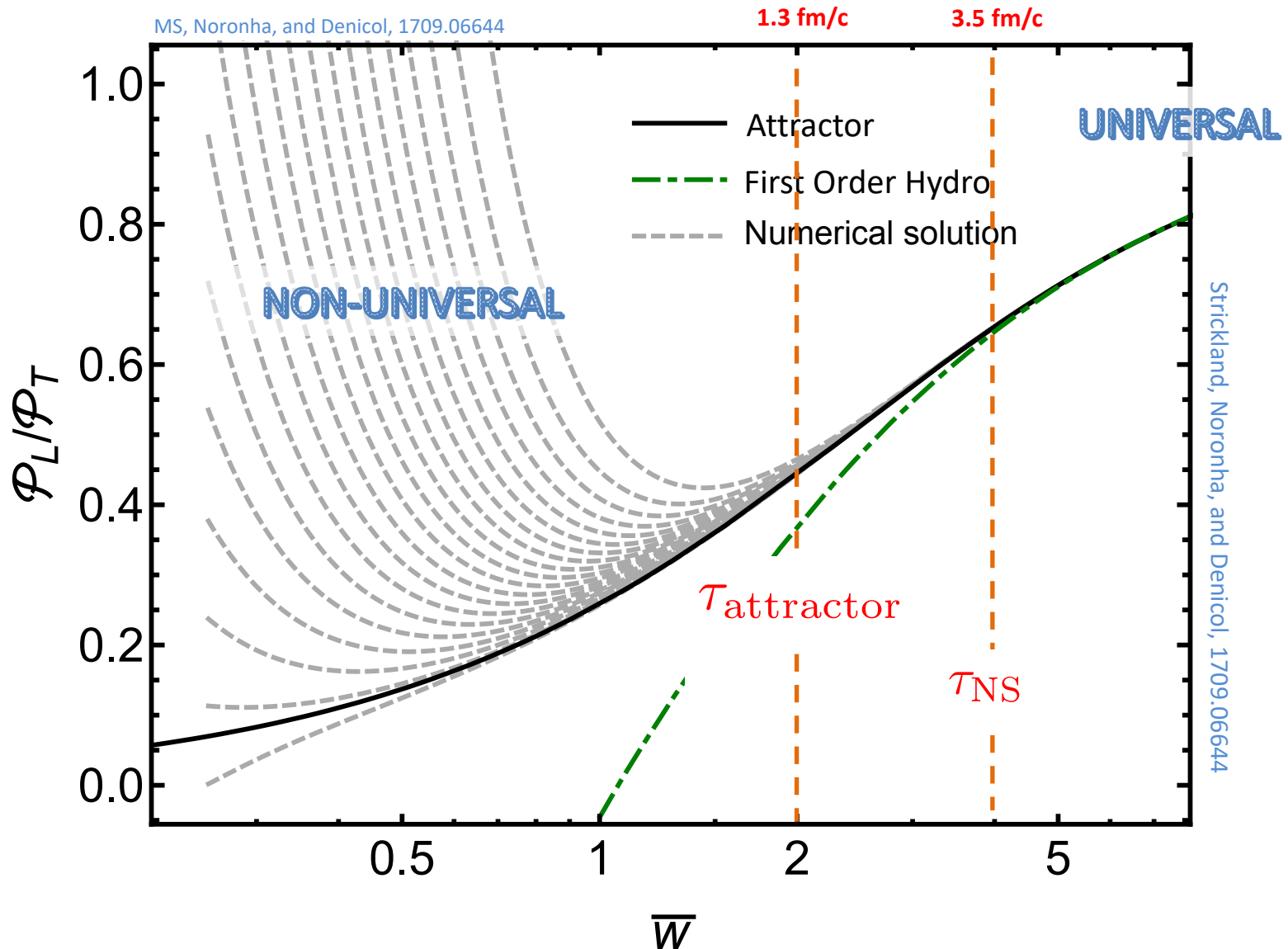
Heller and Spalinski, 1503.07514
Heller, Kurkela, Spalinski, Svensson 1609.04803

$$\bar{W} = \frac{\tau}{\tau_{\text{eq}}(\tau)} = \frac{\tau T(\tau)}{5\bar{\eta}} = \text{Inverse Knudsen \#}$$

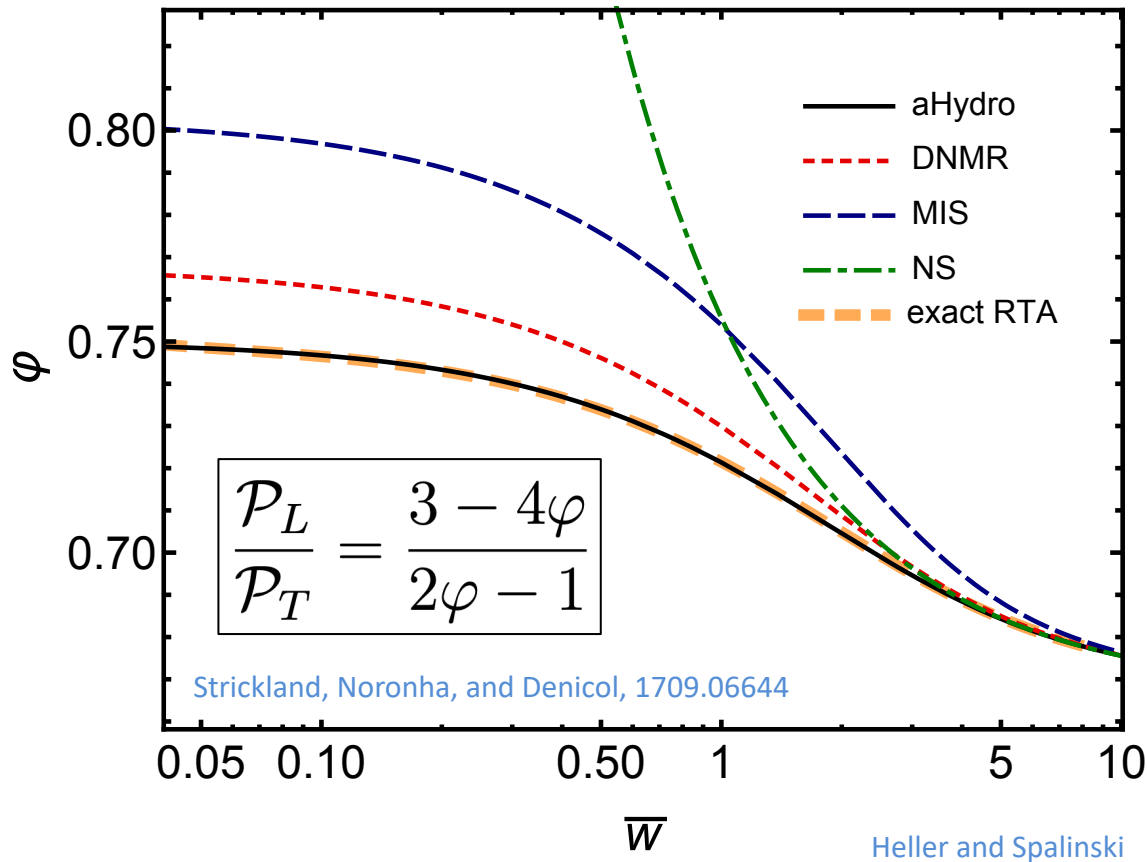
The attractor concept



The attractor concept



Can be used to test hydro approximations!



- Can compare exact RTA result to different hydro frameworks
- In each case one has to solve a 1d ODE subject to a self-consistent boundary condition
- **aHydro performs the best because it “resums” an infinite # of terms in Re^{-1}**

2nd order vHydro

$$\overline{w}\varphi\varphi' + 4\varphi^2 + \left[\overline{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\overline{w}}{3} = 0$$

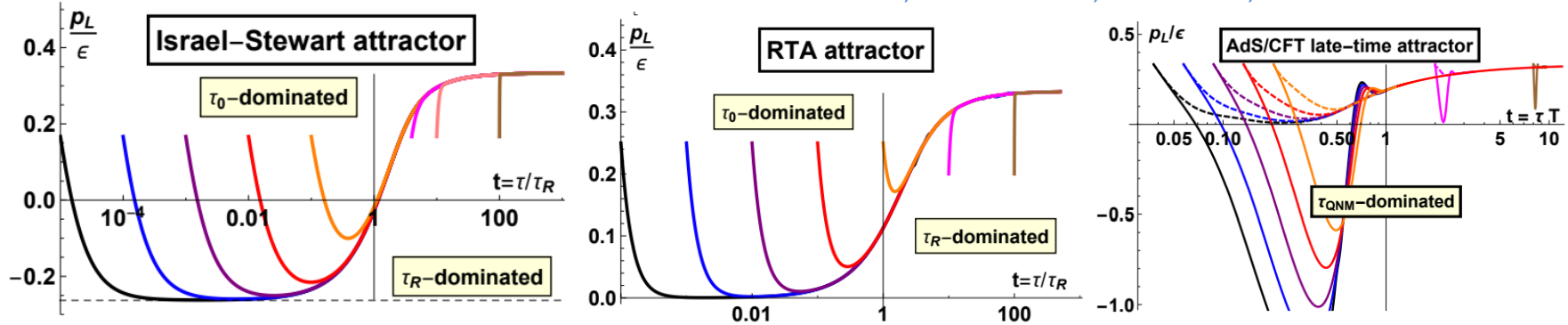
Strickland, Noronha, and Denicol

LO order aHydro

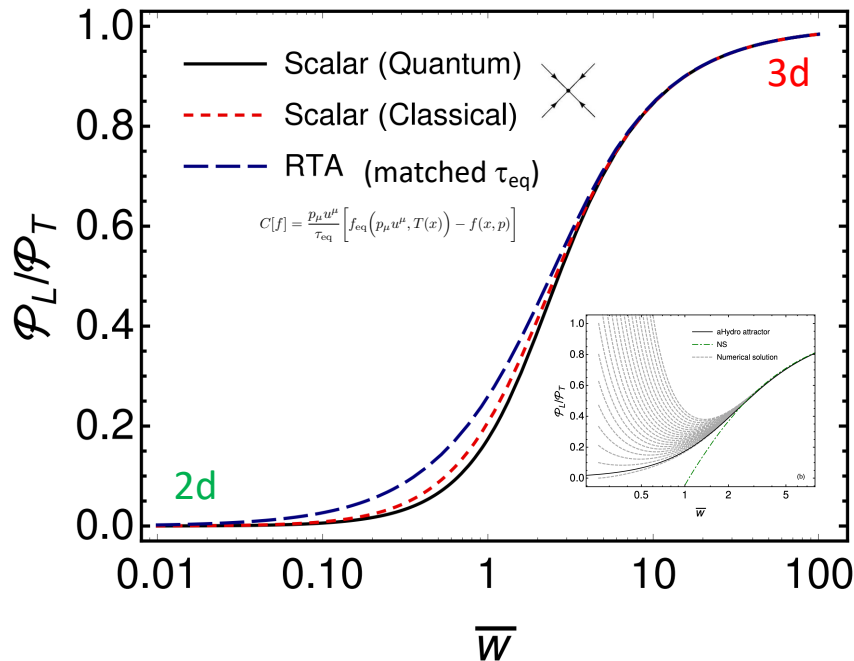
$$\overline{w}\varphi\frac{\partial\varphi}{\partial\overline{w}} = \left[\frac{1}{2}(1 + \xi) - \frac{\overline{w}}{4}\mathcal{H}\right]\overline{\Pi}'$$

Early time behavior sensitive to the model

Kurkela, van der Schee, Wiedemann, and Wu 1907.08101



D. Almaalol and MS, 1801.10173



- Top three panels shown are Mueller-Israel-Stewart (MIS), RTA, and AdS/CFT evolution
- RTA has positive P_L at all times; MIS and AdS/CFT have negative P_L at early times.
- Early time AdS/CFT attractor sensitive to details of the initial conditions (two-body correlations in particular)
- Left panel shows comparison of the attractor for RTA and a scalar QFT with/without quantum statistics

Beyond hydrodynamics?

Beyond hydrodynamics?

MS, JHEP2018, 128; 1809.01200

- Can the concept of a non-equilibrium attractor be extended beyond the 14 degrees of freedom described using the energy-momentum tensor, number density, and diffusion current?
- In kinetic theory we describe things in terms of a one-particle distribution function $\mathbf{f}(\mathbf{x}, \mathbf{p})$ and the energy-momentum tensor is obtained from low-order moments:

$$T^{\mu\nu} = \int dP p^\mu p^\nu f(x, p) \quad dP = \int \frac{d^3p}{(2\pi)^3 E}$$

- What about more general moments of f ? Particularly ones that are sensitive to higher momenta?

Beyond hydrodynamics?

MS, JHEP2018, 128; 1809.01200

- For a conformal system it suffices to consider

$$\mathcal{M}^{nm}[f] \equiv \int dP (p \cdot u)^n (p \cdot z)^{2m} f(x, p)$$

- This encompasses the moments necessary to construct the energy momentum tensor, e.g. below, and more

$$\varepsilon = \mathcal{M}^{20} = \int dP (p \cdot u)^2 f(\tau, w, p_T) = T_{\text{LRF}}^{00}$$

$$P_L = \mathcal{M}^{01} = \int dP (p \cdot z)^2 f(\tau, w, p_T) = T_{\text{LRF}}^{zz}$$

Bjorken Expansion: Exact RTA Solution

- Simple model: Boost-invariant, transversally homogeneous Boltzmann equation in relaxation time approximation (RTA).
- Many results in this model, so we can compare with the literature.

Boltzmann EQ: $p^\mu \partial_\mu f(x, p) = C[f(x, p)]$

RTA: $C[f] = \frac{p_\mu u^\mu}{\tau_{\text{eq}}} \left[f_{\text{eq}}(p_\mu u^\mu, T(x)) - f(x, p) \right]$

Massless Particles

W. Florkowski, R. Ryblewski, and MS,
1304.0665 and 1305.7234

Massive Particles

W. Florkowski, E. Maksymiuk,
R. Ryblewski, and MS, 1402.7348

Solution:

$$f(\tau, w, p_T) = D(\tau, \tau_0) f_0(w, p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\text{eq}}(\tau')} D(\tau, \tau') f_{\text{eq}}(\tau', w, p_T)$$

Time-
dependent
relaxation time

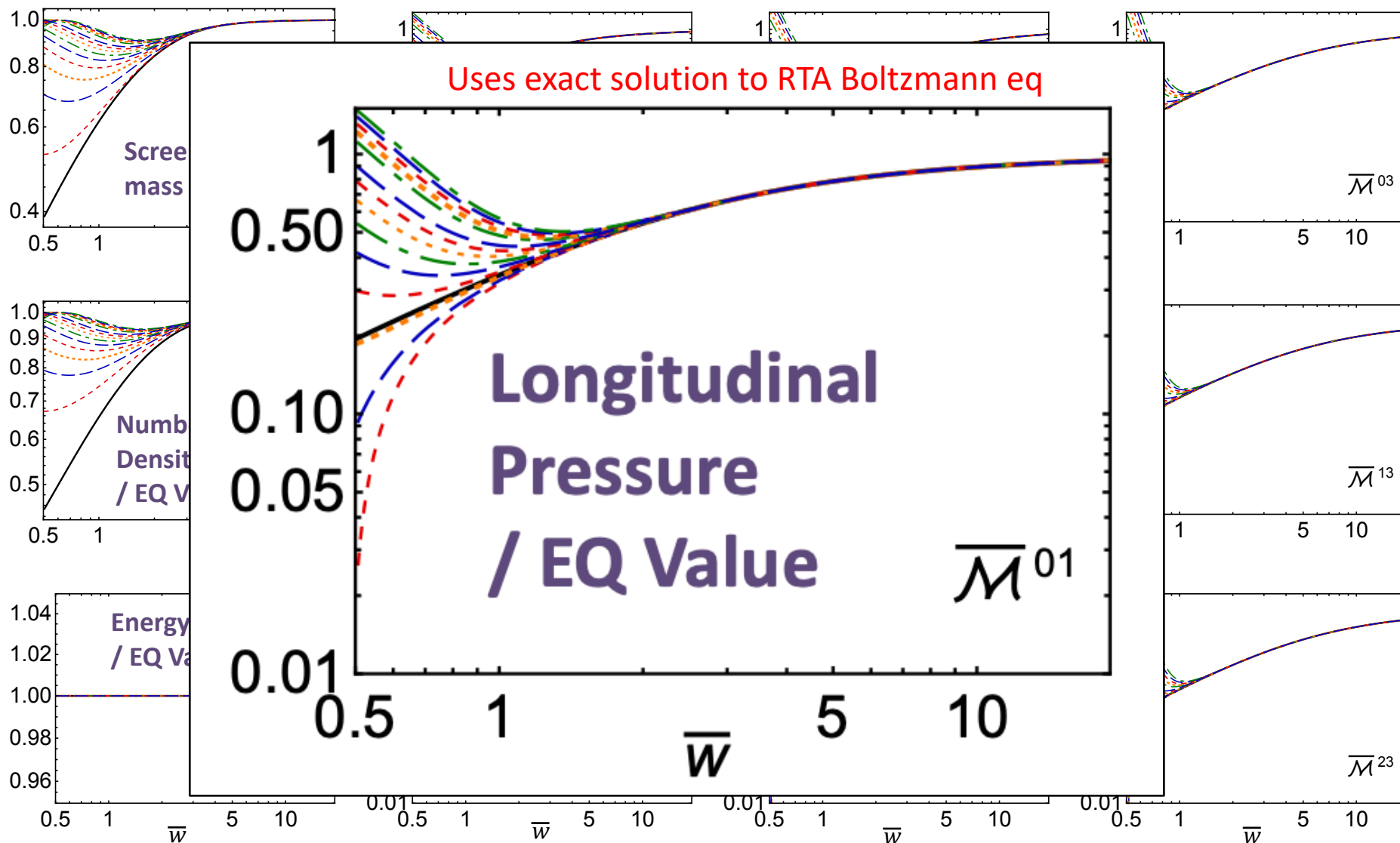
$$\tau_{\text{eq}}(\tau) = \frac{5\bar{\eta}}{T(\tau)}$$

Damping
Function

$$D(\tau_2, \tau_1) = \exp \left[- \int_{\tau_1}^{\tau_2} d\tau \tau_{\text{eq}}^{-1}(\tau) \right]$$

Behavior of higher-order moments in RTA

MS, JHEP2018, 128; 1809.01200

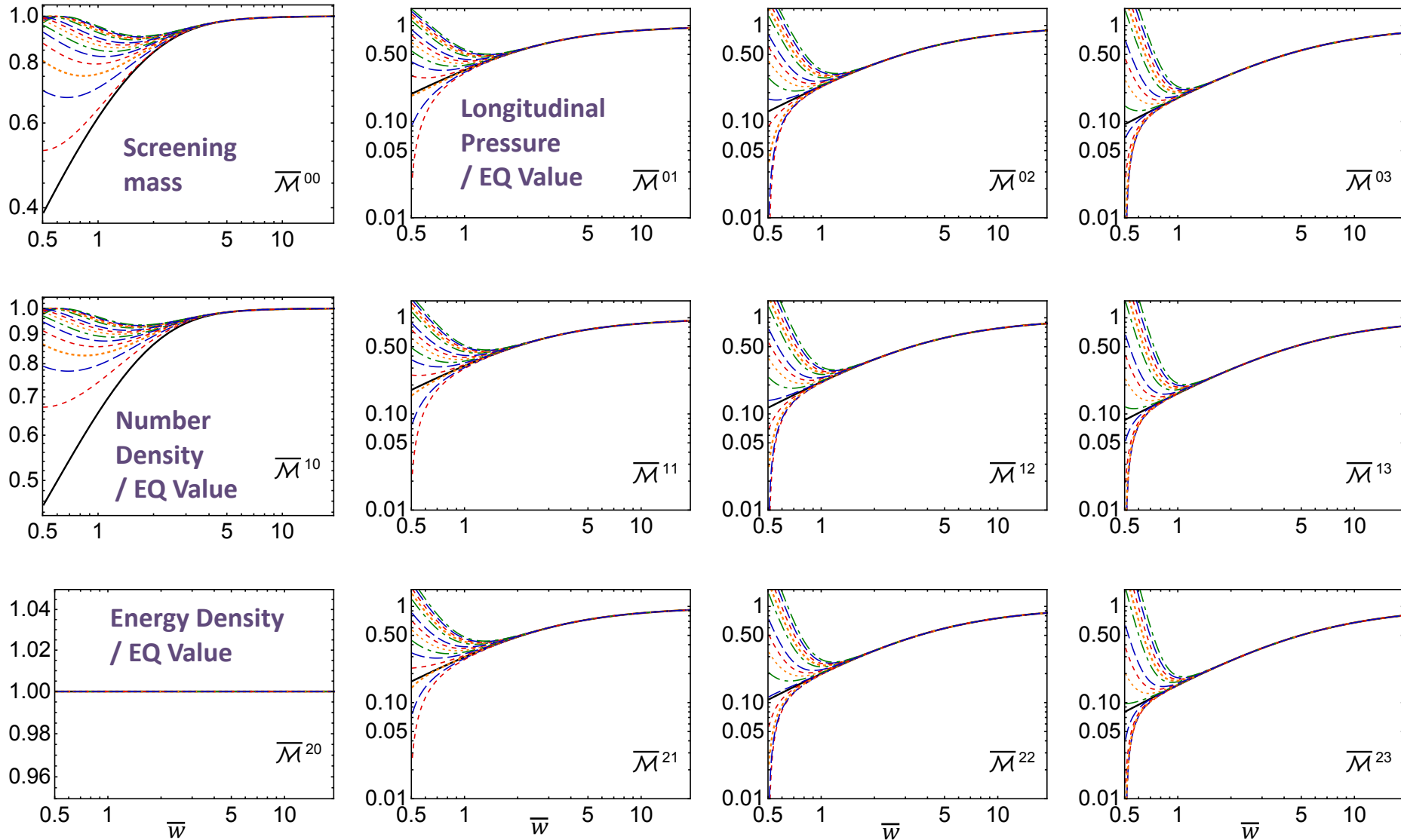


Black Line = Attractor Solution

Dashed colored lines = scan of initial conditions

Behavior of higher-order moments in RTA

MS, JHEP2018, 128; 1809.01200



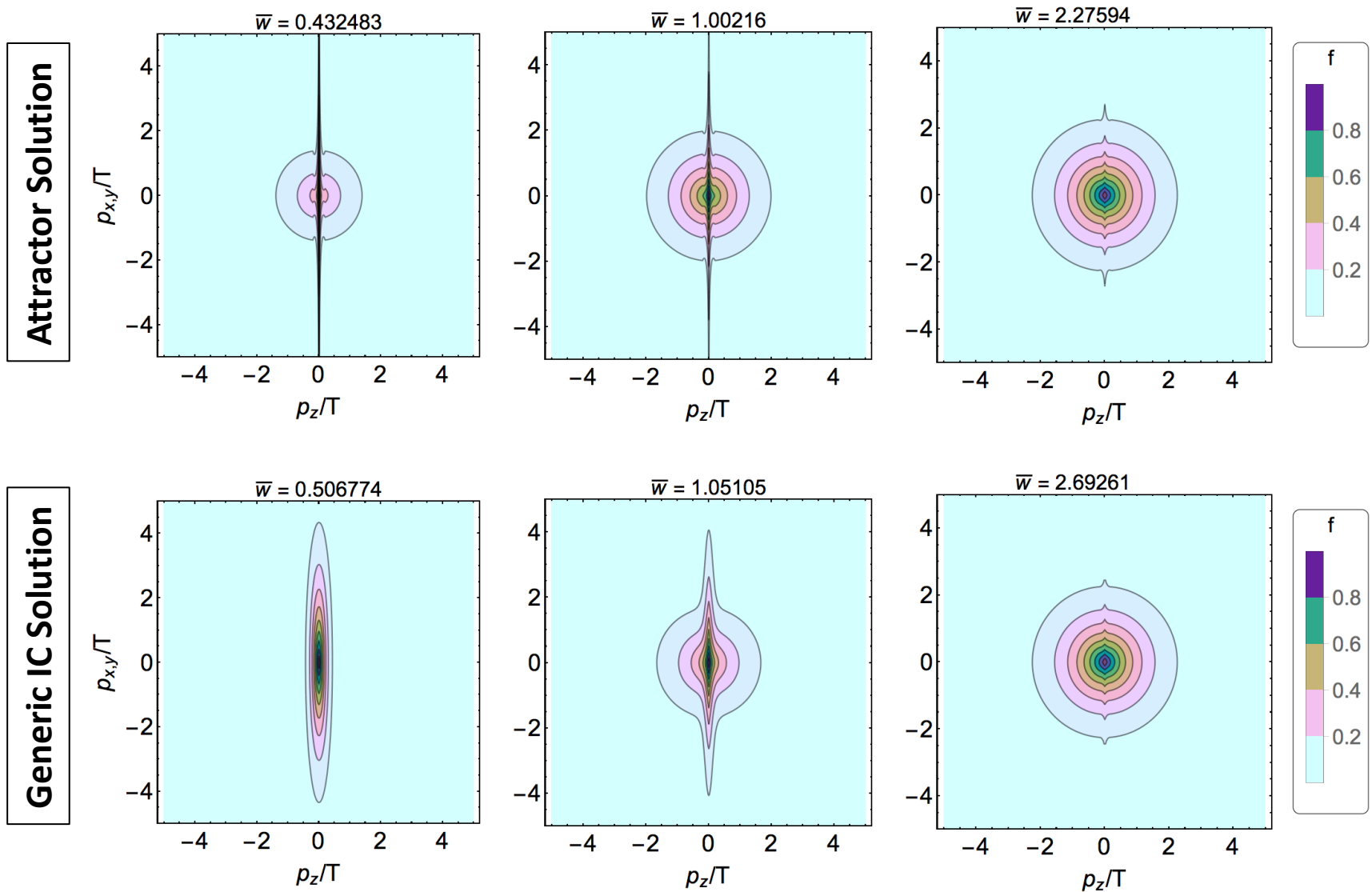
Black Line = Attractor Solution

Dashed colored lines = scan of initial conditions

The attractor for the distribution function itself

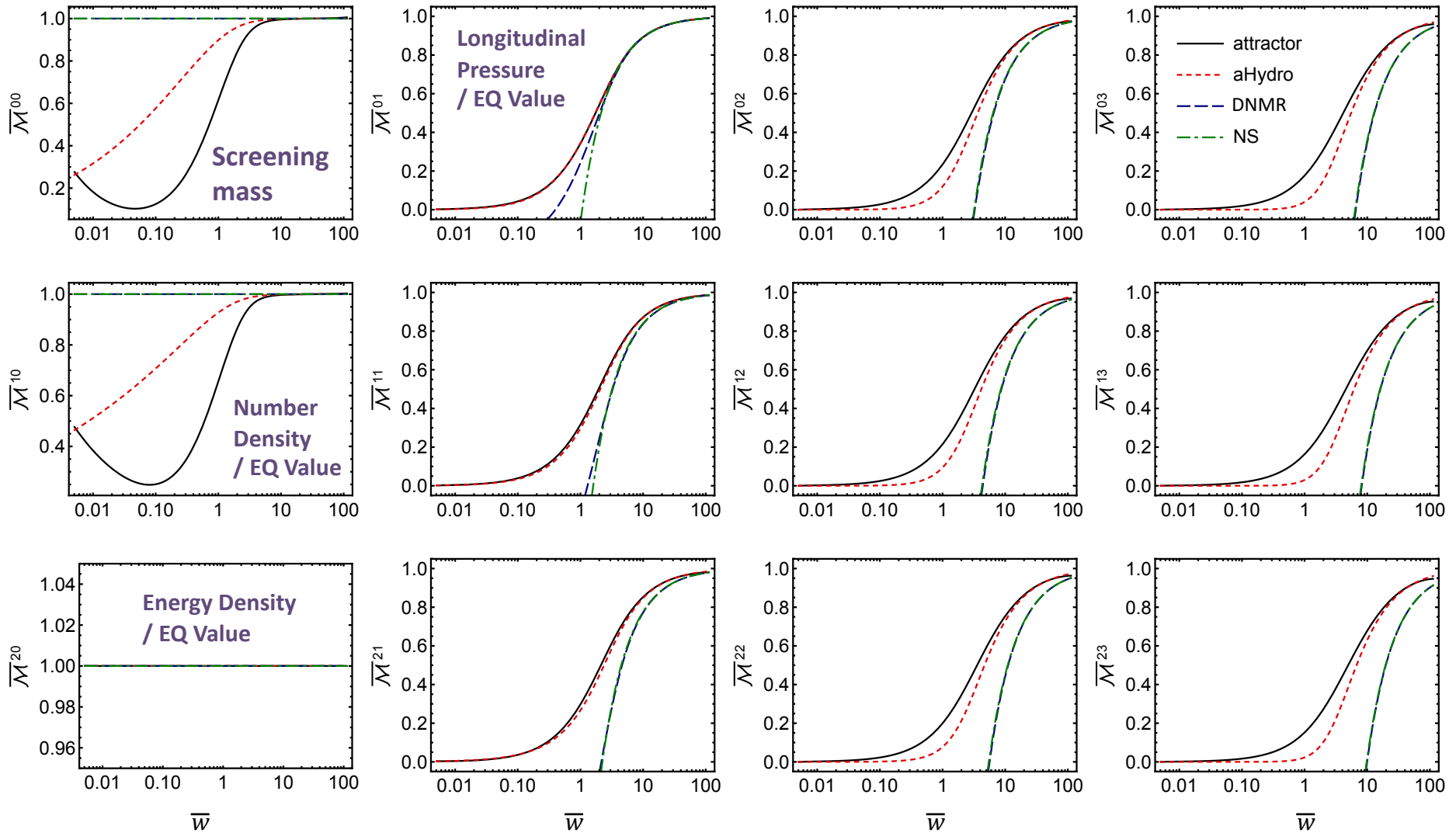
RTA attractor for the distribution function

MS, JHEP2018, 128; 1809.01200



Comparison of exact attractor for moments with different hydrodynamics approximations

Hydrodynamic comparisons



QCD?

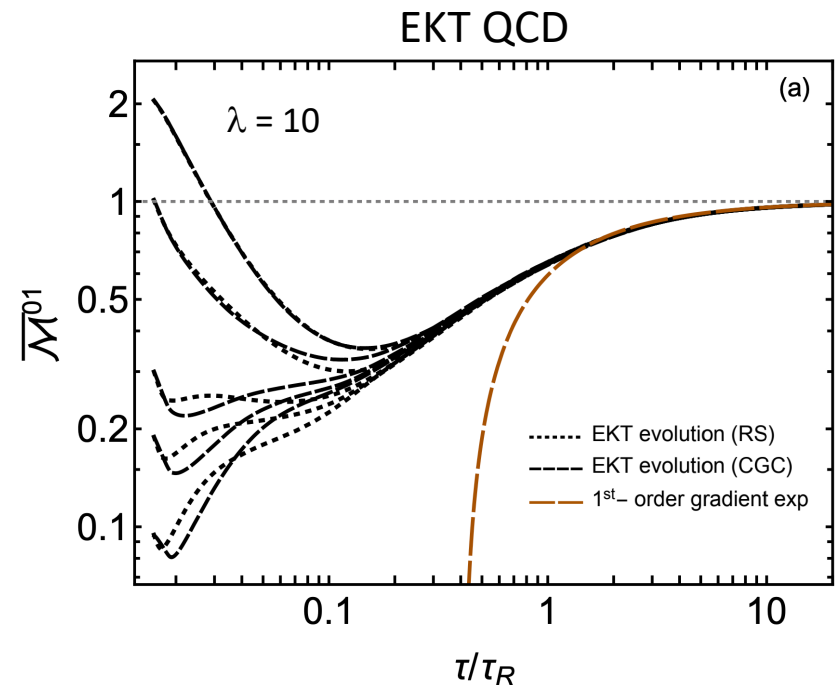
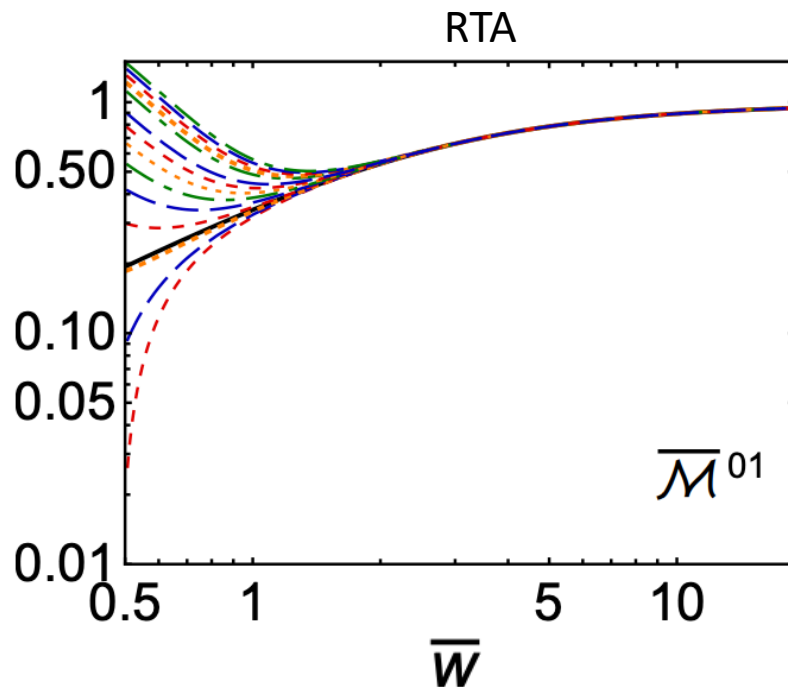
Evidence for a QCD EKT attractor

D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).

- Numerical implementation of pure glue AMY effective kinetic theory (EKT)
- Includes **elastic gluon scattering** and **inelastic gluon splitting** with LPM suppression and detailed balance.
- We use the “pure glue” EKT code of Kurkela and Zhu PRL 115, 182301 (2015).
- 250 x 2000 x 1 grid in momentum space ($N_p \times N_\theta \times N_\phi$)

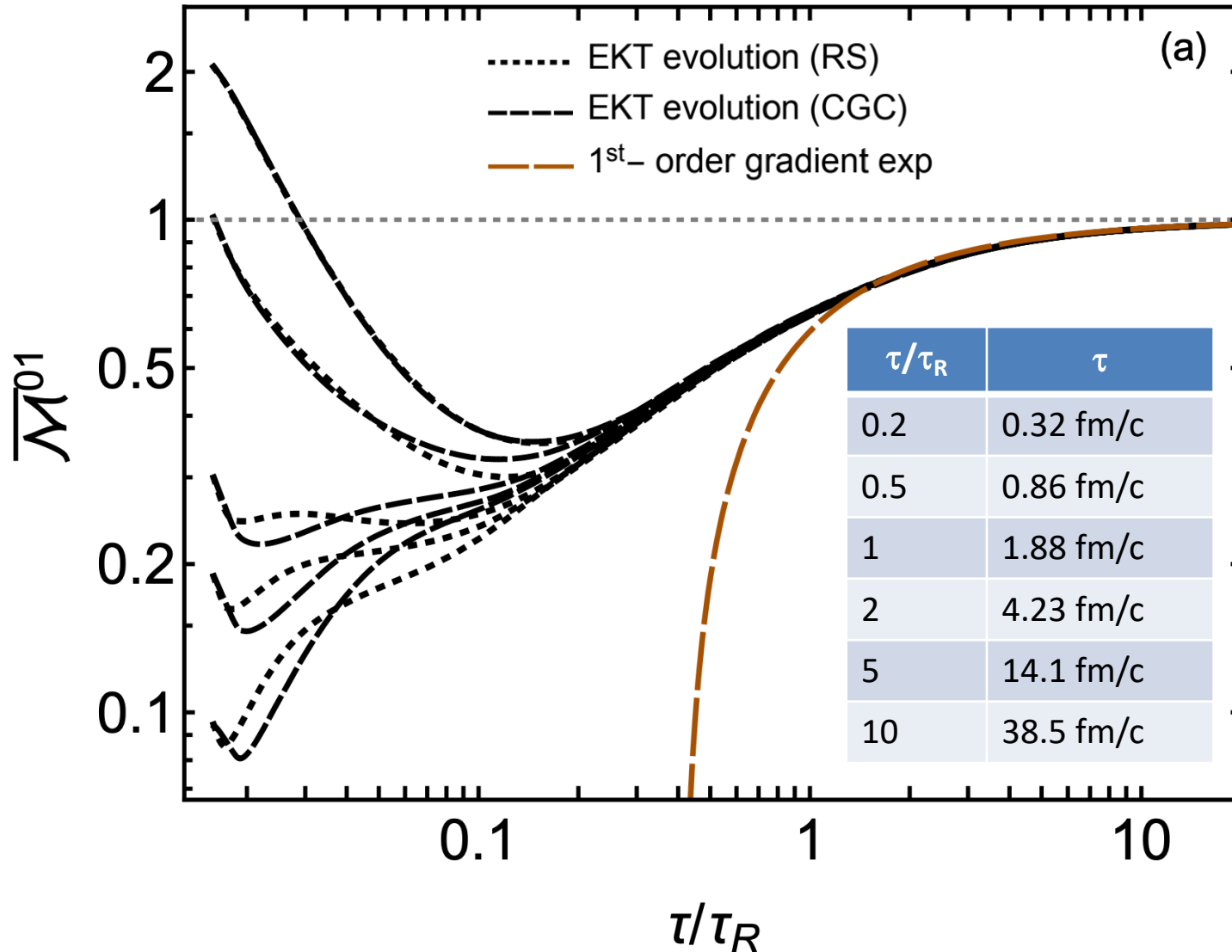


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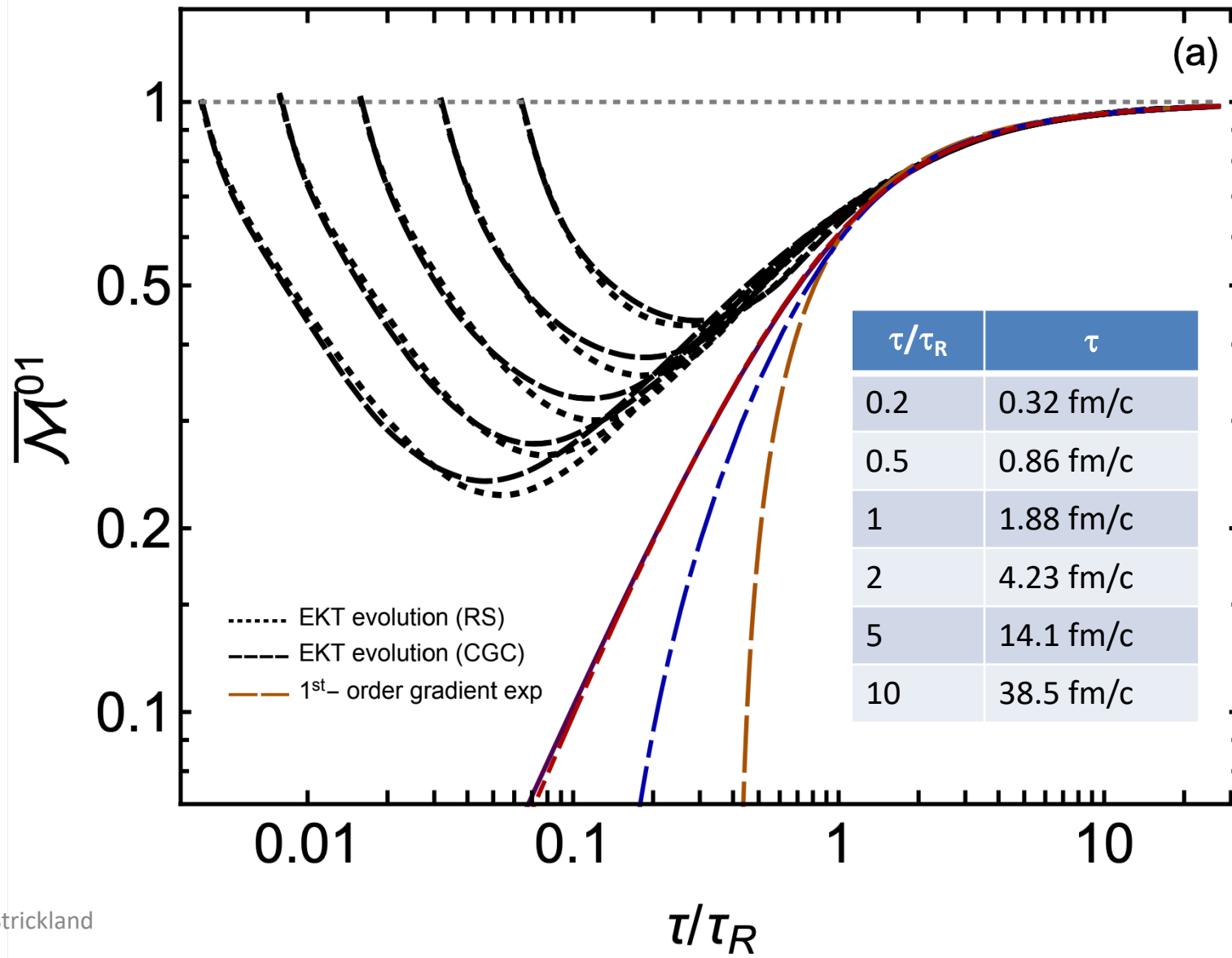
Evidence for a QCD EKT forward attractor

D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).



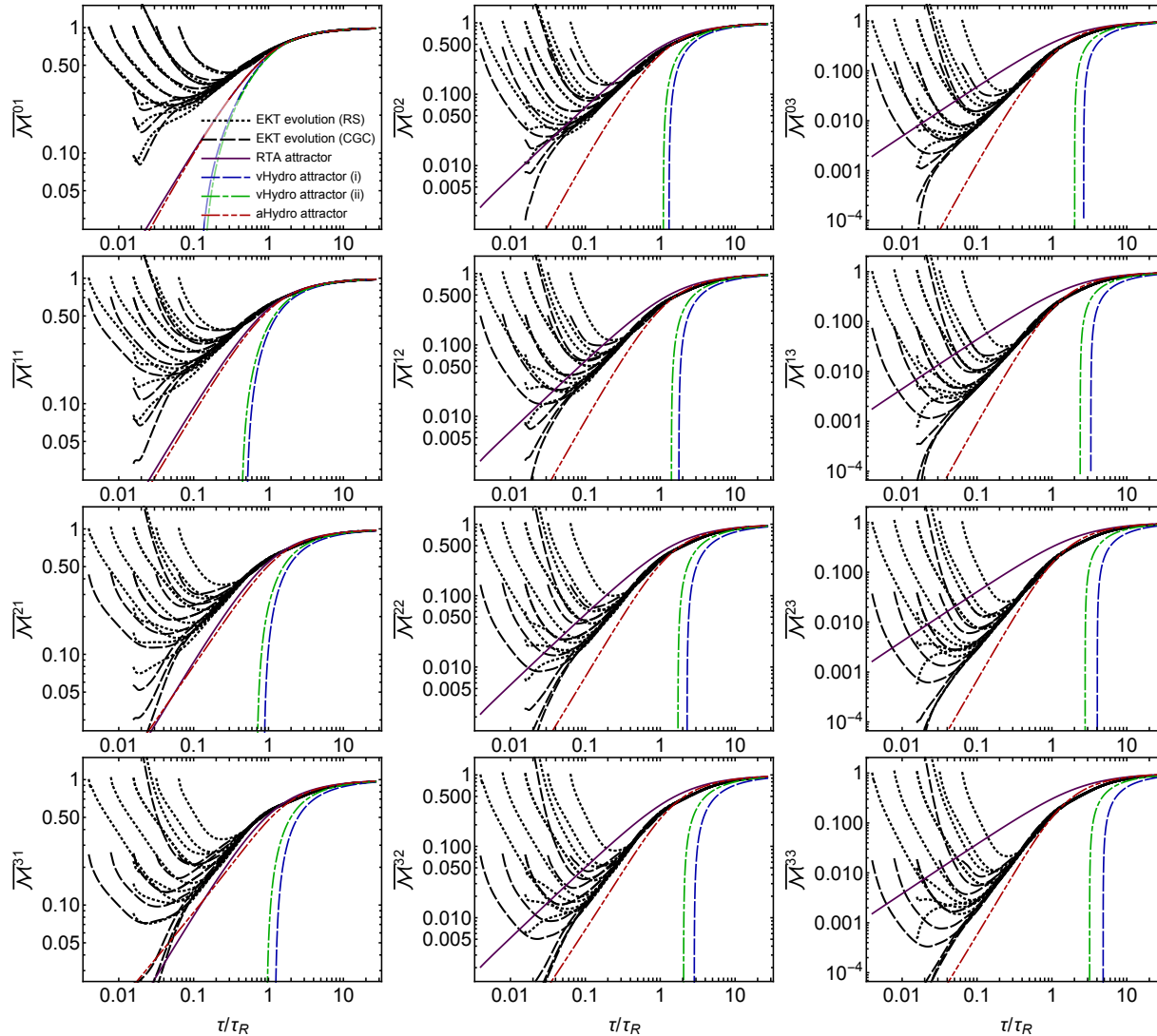
Evidence for a QCD EKT pullback attractor

D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).



Varying initial anisotropy and t_0

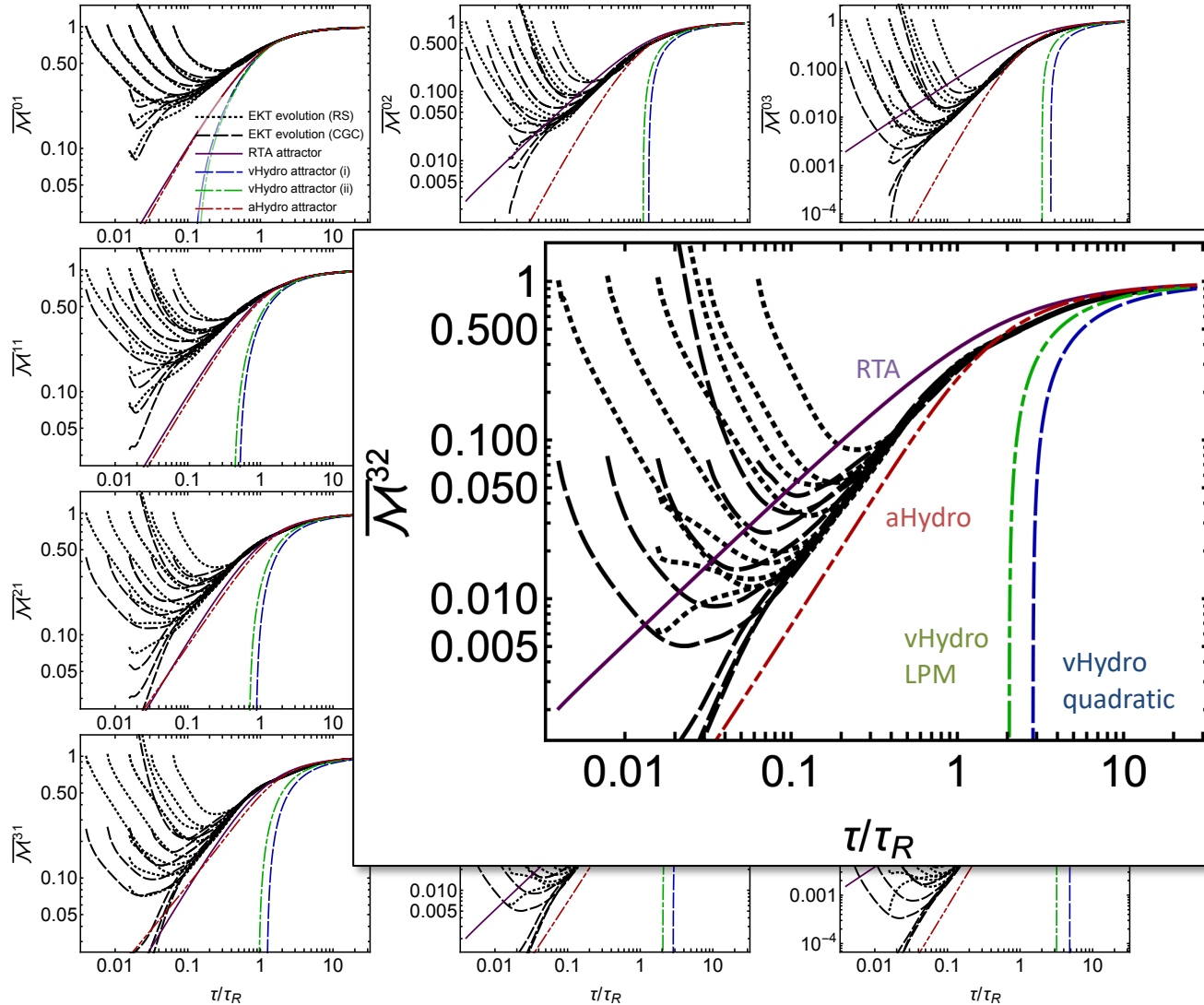
D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).



- Attractor seen in all moments and is the same for both types of initial conditions.
- For low order moments EKT QCD is closer to EQ than RTA and hydro predictions.
- For high order moments the opposite is true.
- **Hydrodynamization is only one corner of a much bigger picture**

Varying initial anisotropy and t_0

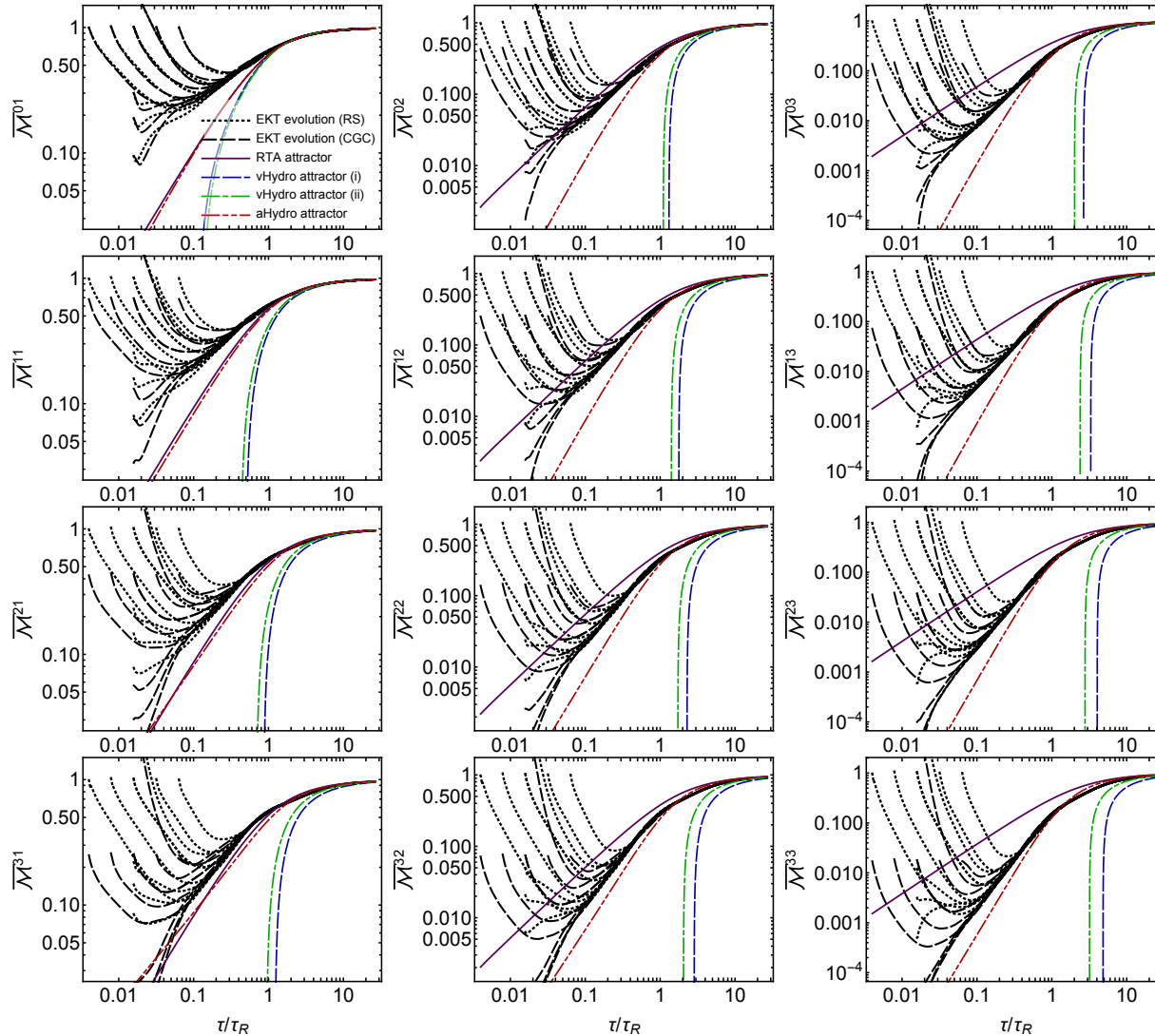
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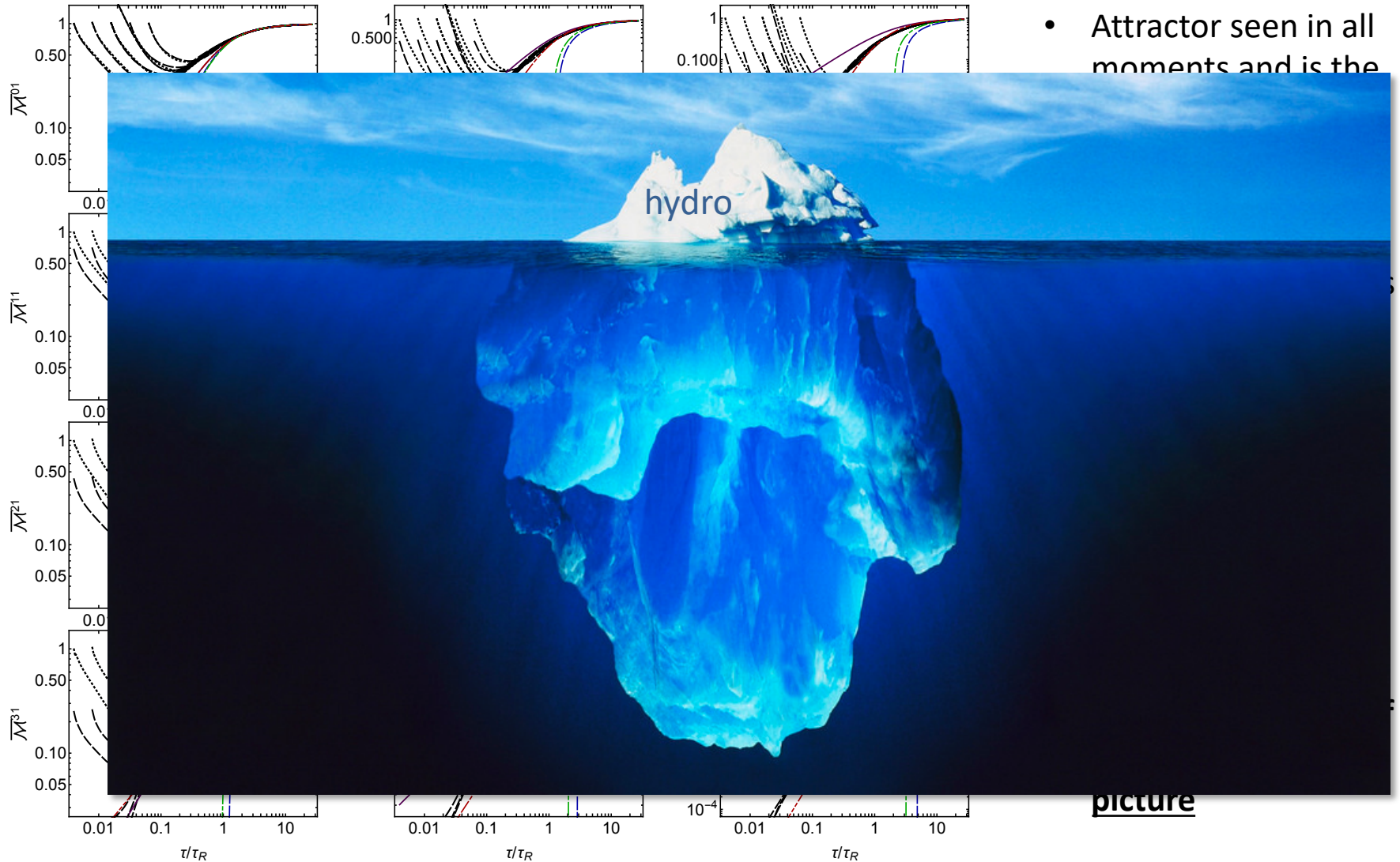
D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).



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Varying initial anisotropy and t_0

D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).



- Attractor seen in all moments and is the

What can we learn from this for heavy-ion phenomenology?

- Can use the EKT QCD attractor to assess how well different freeze-out prescriptions fare in trying to reproduce the entire distribution function.

$$\frac{\delta f_{(i)}}{f_{\text{eq}}(1 + f_{\text{eq}})} = \frac{3\bar{\Pi}}{16T^2}(p^2 - 3p_z^2)$$

$$\frac{\delta f_{(ii)}}{f_{\text{eq}}(1 + f_{\text{eq}})} = \frac{16\bar{\Pi}}{21\sqrt{\pi} T^{3/2}} \left(p^{3/2} - \frac{3p_z^2}{\sqrt{p}} \right)$$

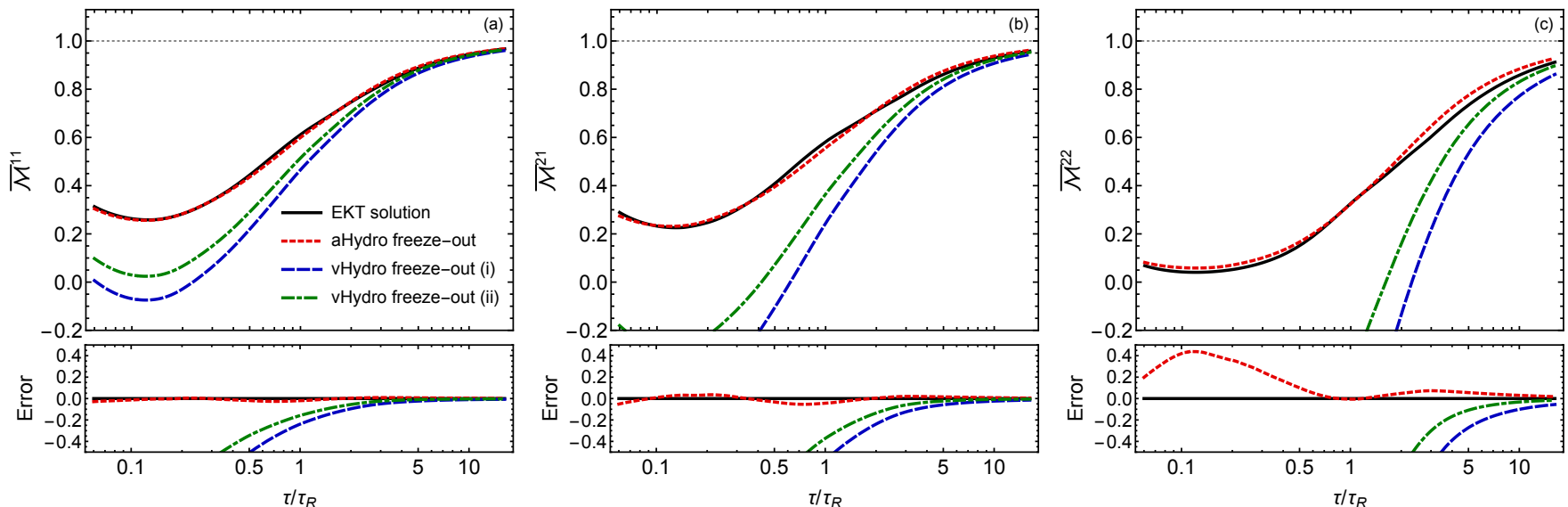
$$f(p) = f_{\text{Bose}}(\sqrt{\mathbf{p}^2 + \xi p_z^2}/\Lambda)$$

- Quadratic approx. which results from a wide set of models including RTA with momentum-independent relaxation time, momentum diffusion approximation, scalar field theory, etc.
- LPM-improved version from Dusling, Moore, and Teaney [nucl-th/0909.0754](#).
- The aHydro “non-perturbative” form.

What can we learn from this for heavy-ion phenomenology?

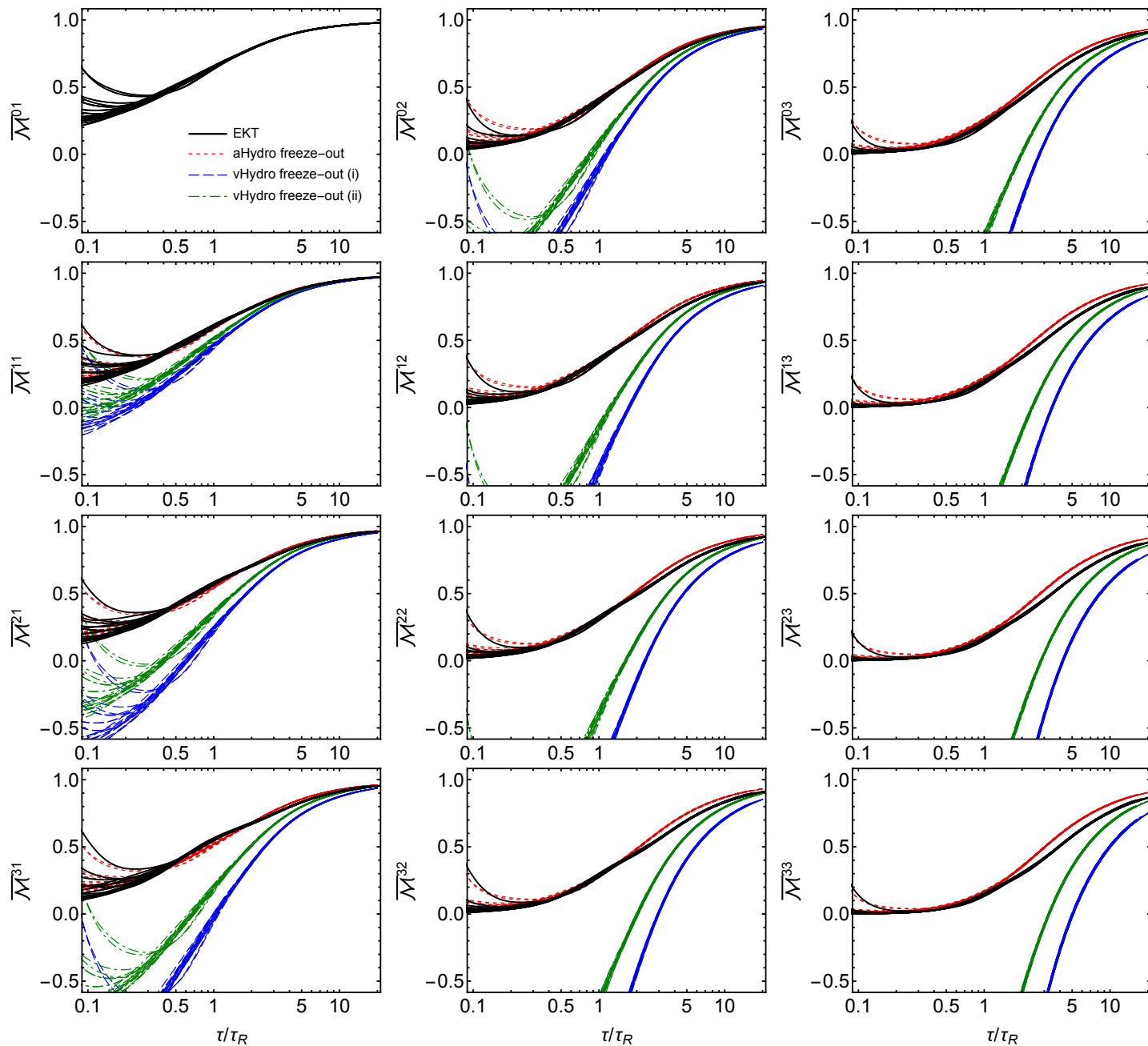
- In all three cases, we fix the independent parameter, Π or ξ , by matching to the scaled longitudinal pressure ($\bar{M}^{01}$) obtained along the EKT attractor.
- Once we match the pressure at every proper time, we then use the ansatz to make predicts for the higher order moments.
- Three examples are shown below. In the paper we presented similar results for all moments considered!

D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).



Another big table of plots!

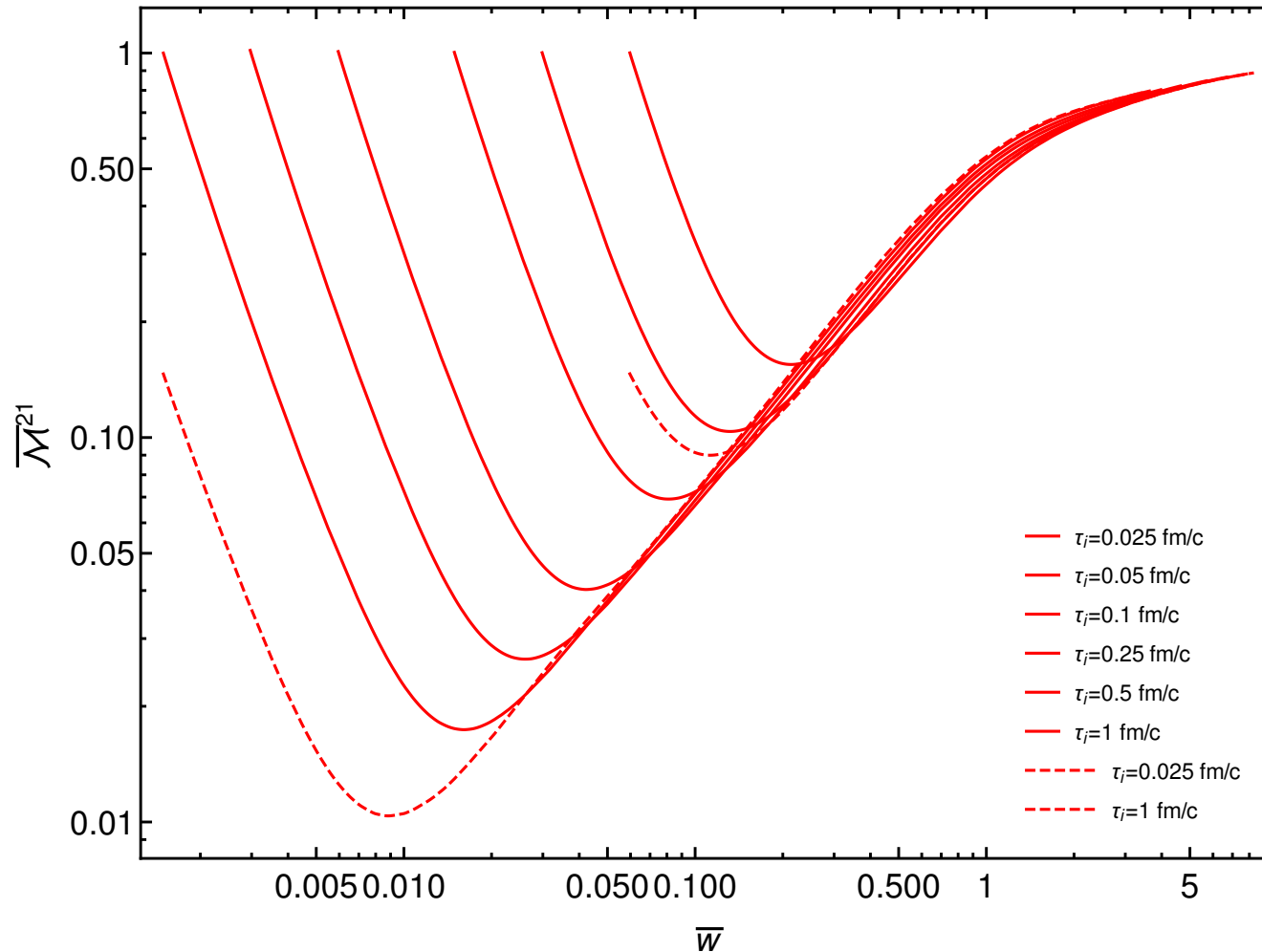
D. Almaalol, A. Kurkela, and MS, Phys. Rev. Lett. 125, 122302 (2020).



Non-equilibrium attractor with quarks

D. Almaalol, A. Mazeliauskas, and MS, forthcoming

Preliminary results obtained using a “small” lattice



Conclusions I

- Attractor for low-order moments well-approximated by hydro but system is not in equilibrium → hydrodynamization instead of thermalization
- RTA and EKT higher-order moments poorly described by standard viscous hydrodynamics
- There is, however, a fast convergence to a non-equilibrium attractor for higher moments → **“pseudo-thermalization” instead of hydrodynamization**

Conclusions II

- Like RTA, EKT QCD has a “beyond hydrodynamics” attractor
- For EKT, we considered two types of initial conditions
- Evidence presented for existence of early-time (“pullback”) attractor for EKT QCD.
- **Collapse occurs in all moments measured**
- Can we use properties of EKT attractor to improve hydro and freeze-out?
- Yes, collaboration with Chun Shen (Wayne State) and Qianqian Du (Kent State/Wuhan) to use aHydro type freezeout in MUSIC. Results coming soon.

Backup Slides

Attractor exists in many theories

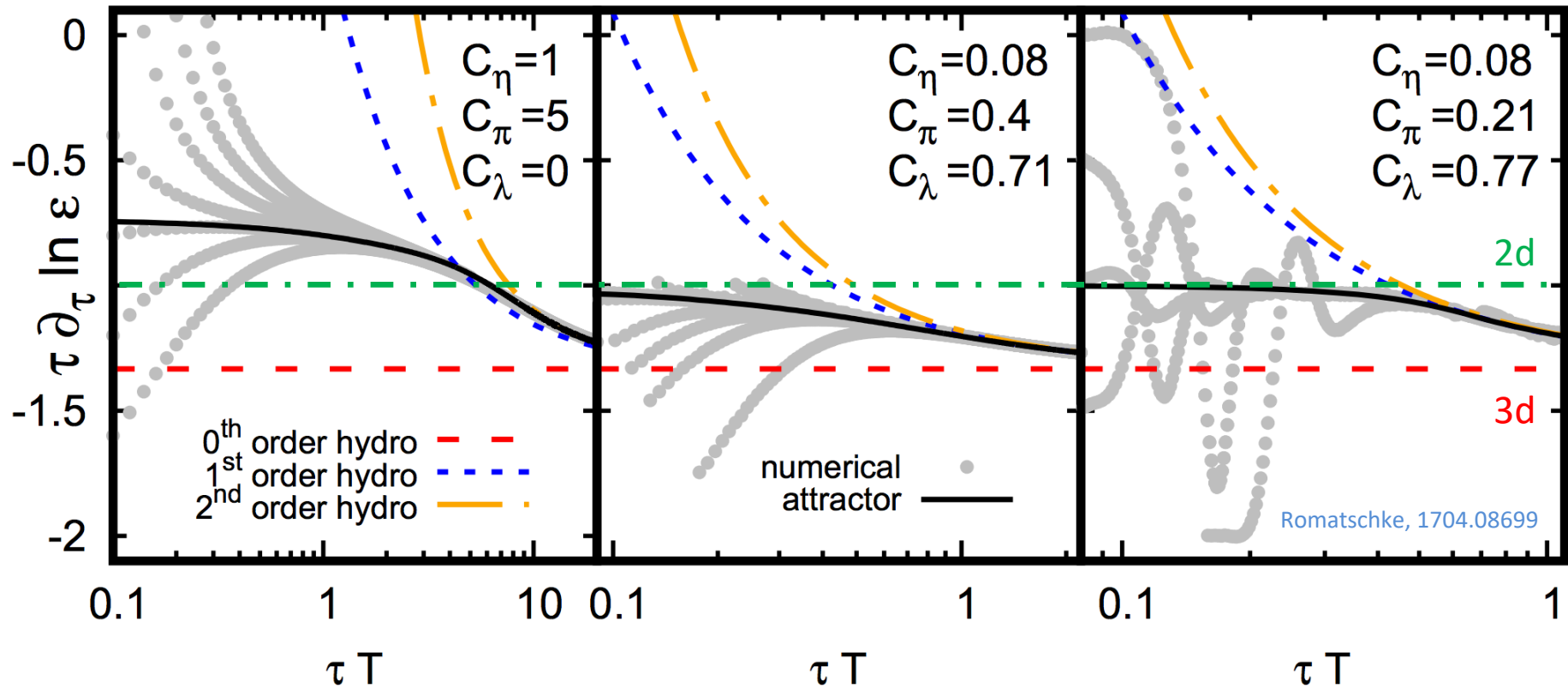
rBRSSS Viscous Hydro

Baier, Romatschke, Son, Starinets, and Stephanov

Exact RTA Boltzmann EQ Sol

Florkowski, Ryblewski, and MS 1304.0665; 1305.7234

AdS/CFT



Quantity plotted is $-2 \frac{\mathcal{P}_L + \mathcal{P}_T}{\mathcal{P}_L + 2\mathcal{P}_T} \rightarrow -4/3$ (ideal fluid)
 $\rightarrow -1$ (2d fluid)

Bjorken Expansion: Exact RTA Solution

- Simple model: Boost-invariant, transversally homogeneous Boltzmann equation in relaxation time approximation (RTA).
- Many results in this model, so we can compare with the literature.

Boltzmann EQ: $p^\mu \partial_\mu f(x, p) = C[f(x, p)]$

RTA: $C[f] = \frac{p_\mu u^\mu}{\tau_{\text{eq}}} \left[f_{\text{eq}}(p_\mu u^\mu, T(x)) - f(x, p) \right]$

Solution:

Massless Particles

W. Florkowski, R. Ryblewski, and MS,
1304.0665 and 1305.7234

Massive Particles

W. Florkowski, E. Maksymiuk,
R. Ryblewski, and MS, 1402.7348

$$f(\tau, w, p_T) = D(\tau, \tau_0) f_0(w, p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\text{eq}}(\tau')} D(\tau, \tau') f_{\text{eq}}(\tau', w, p_T)$$

Time-
dependent
relaxation time

$$\tau_{\text{eq}}(\tau) = \frac{5\bar{\eta}}{T(\tau)}$$

Damping
Function

$$D(\tau_2, \tau_1) = \exp \left[- \int_{\tau_1}^{\tau_2} d\tau \tau_{\text{eq}}^{-1}(\tau) \right]$$

0+1d RTA Exact Solution

$$T^4(\tau) = D(\tau, \tau_0) T_0^4 \frac{\mathcal{H}\left(\frac{\alpha_0 \tau_0}{\tau}\right)}{\mathcal{H}(\alpha_0)} + \int_{\tau_0}^{\tau} \frac{d\tau'}{2\tau_{\text{eq}}(\tau')} D(\tau, \tau') T^4(\tau') \mathcal{H}\left(\frac{\tau'}{\tau}\right)$$

Once this integral equation is solved (by numerical iteration), we can construct the full one-particle distribution function $f(\tau, p)$ and we can compute general moments:

$$f(\tau, w, p_T) = D(\tau, \tau_0) f_0(w, p_T) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\text{eq}}(\tau')} D(\tau, \tau') f_{\text{eq}}(\tau', w, p_T)$$

- GPL'd CUDA code:
personal.kent.edu/~mstrick6/code
- Computes all moments and the full distribution function
- CUDA enables computation on very fine grids (N_tau ~ 4000, N_pt, N_pz ~ 500, 500).

$$\mathcal{M}^{nm}(\tau) = \frac{\Gamma(n+2m+2)}{(2\pi)^2} \left[D(\tau, \tau_0) 2^{(n+2m+2)/4} T_0^{n+2m+2} \frac{\mathcal{H}^{nm}\left(\frac{\alpha_0 \tau_0}{\tau}\right)}{[\mathcal{H}^{20}(\alpha_0)]^{(n+2m+2)/4}} + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\text{eq}}(\tau')} D(\tau, \tau') T^{n+2m+2}(\tau') \mathcal{H}^{nm}\left(\frac{\tau'}{\tau}\right) \right],$$

$$\mathcal{H}^{nm}(y) = \frac{2y^{2m+1}}{2m+1} {}_2F_1\left(\frac{1}{2} + m, \frac{1-n}{2}; \frac{3}{2} + m; 1 - y^2\right).$$

How does one obtain the attractor?

- Let's look at hydrodynamics-like theories for simplicity (e.g. MIS, DNMR, aHydro, etc.)
- Start with the 0+1d energy conservation equation

$$\tau \dot{\epsilon} = -\frac{4}{3}\epsilon + \Pi \quad \Pi = \Pi^\varsigma_\varsigma$$

- Change variables to

$$w = \tau T$$

$$\varphi(w) \equiv \tau \frac{\dot{w}}{w} = 1 + \frac{\tau}{4} \partial_\tau \log \epsilon$$

$$w\varphi \frac{\partial \varphi}{\partial w} = -\frac{8}{3} + \frac{20}{3}\varphi - 4\varphi^2 + \frac{\tau}{4} \frac{\dot{\Pi}}{\epsilon}$$

How does one obtain the attractor?

- Need the evolution equation for the viscous correction.
- To linear order in the shear correction (e.g. MIS, DNMR) one has

$$\dot{\Pi} = \frac{4\eta}{3\tau\tau_\pi} - \beta_{\pi\pi} \frac{\Pi}{\tau} - \frac{\Pi}{\tau_\pi} \quad \text{For DNMR in RTA } \beta_{\pi\pi} = \frac{38}{21}$$

- Plugging this into the energy-momentum conservation equation gives

$$\bar{w}\varphi\varphi' + 4\varphi^2 + \left[\bar{w} + \left(\beta_{\pi\pi} - \frac{20}{3} \right) \right] \varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\bar{w}}{3} = 0$$

$$\bar{w} \equiv \frac{w}{c_\pi} = \frac{\tau T}{5\bar{\eta}} \quad c_{\eta/\pi} \equiv \frac{c_\eta}{c_\pi} = \frac{1}{5}$$

How does one solve for the attractor?

$$\bar{w}\varphi\varphi' + 4\varphi^2 + \left[\bar{w} + \left(\beta_{\pi\pi} - \frac{20}{3}\right)\right]\varphi - \frac{4c_{\eta/\pi}}{9} - \frac{2}{3}(\beta_{\pi\pi} - 4) - \frac{2\bar{w}}{3} = 0$$

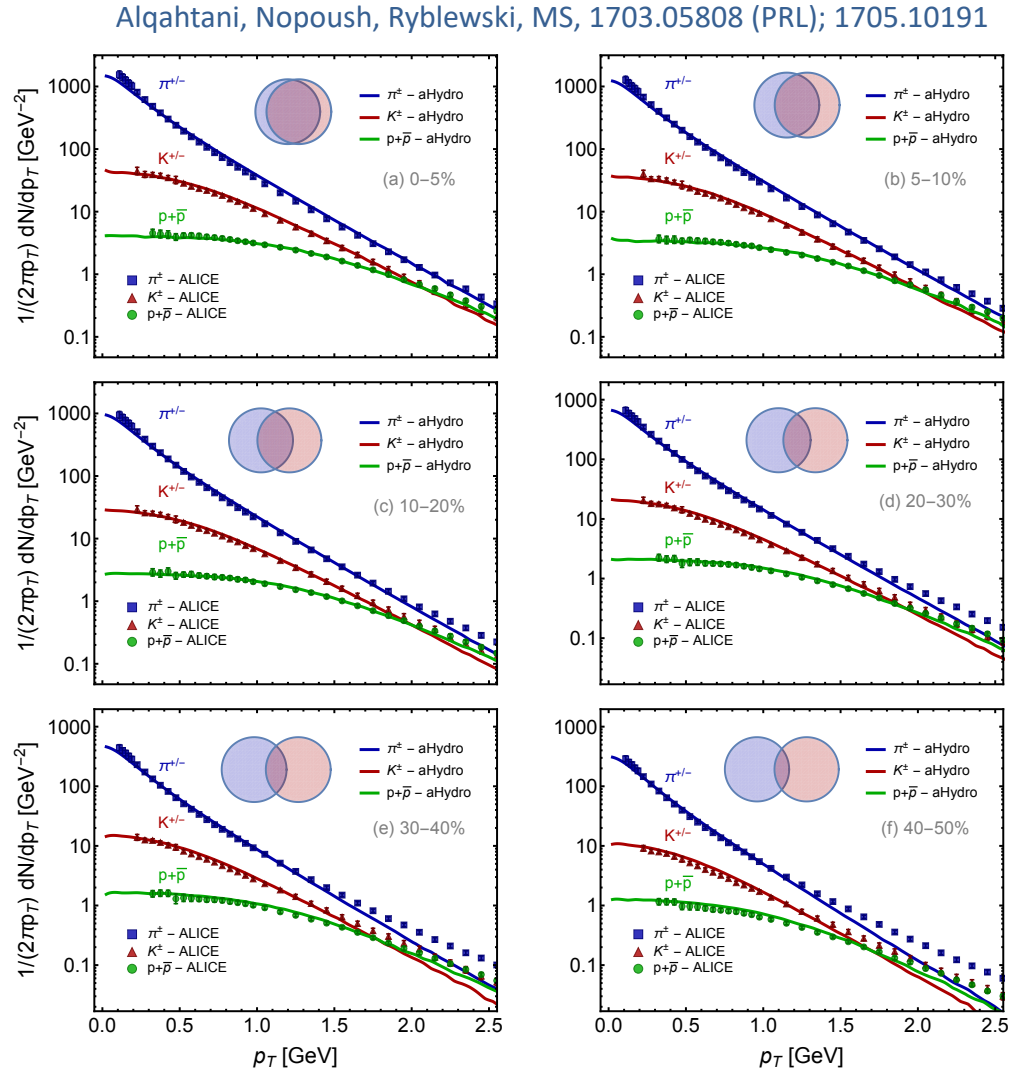
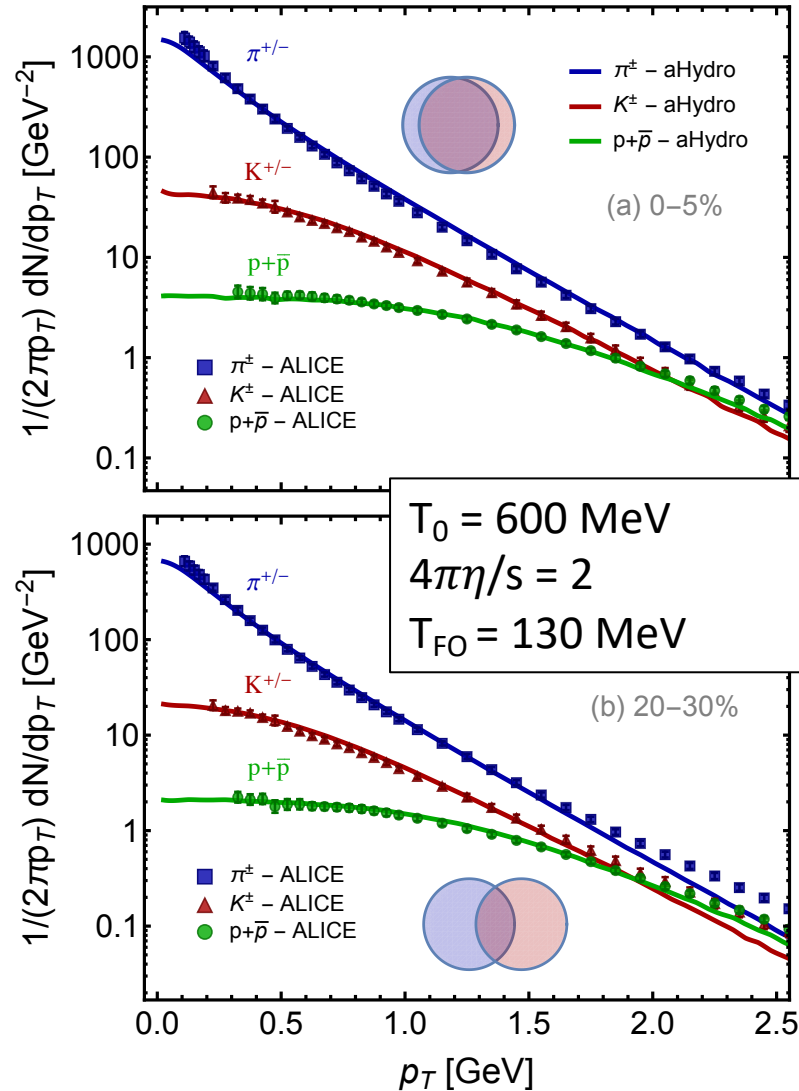
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- First try to approximate using “slow-roll” approx ($\varphi' = 0$)
- From this, we can read off the boundary condition as $w \rightarrow 0$

$$\lim_{\bar{w} \rightarrow 0} \varphi(\bar{w}) = \frac{1}{24} \left(-3\beta_{\pi\pi} + \sqrt{64c_{\eta/\pi} + (3\beta_{\pi\pi} - 4)^2 + 20} \right)$$

- Then numerically solve the ODE at the top of the slide

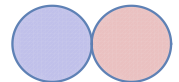
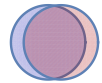
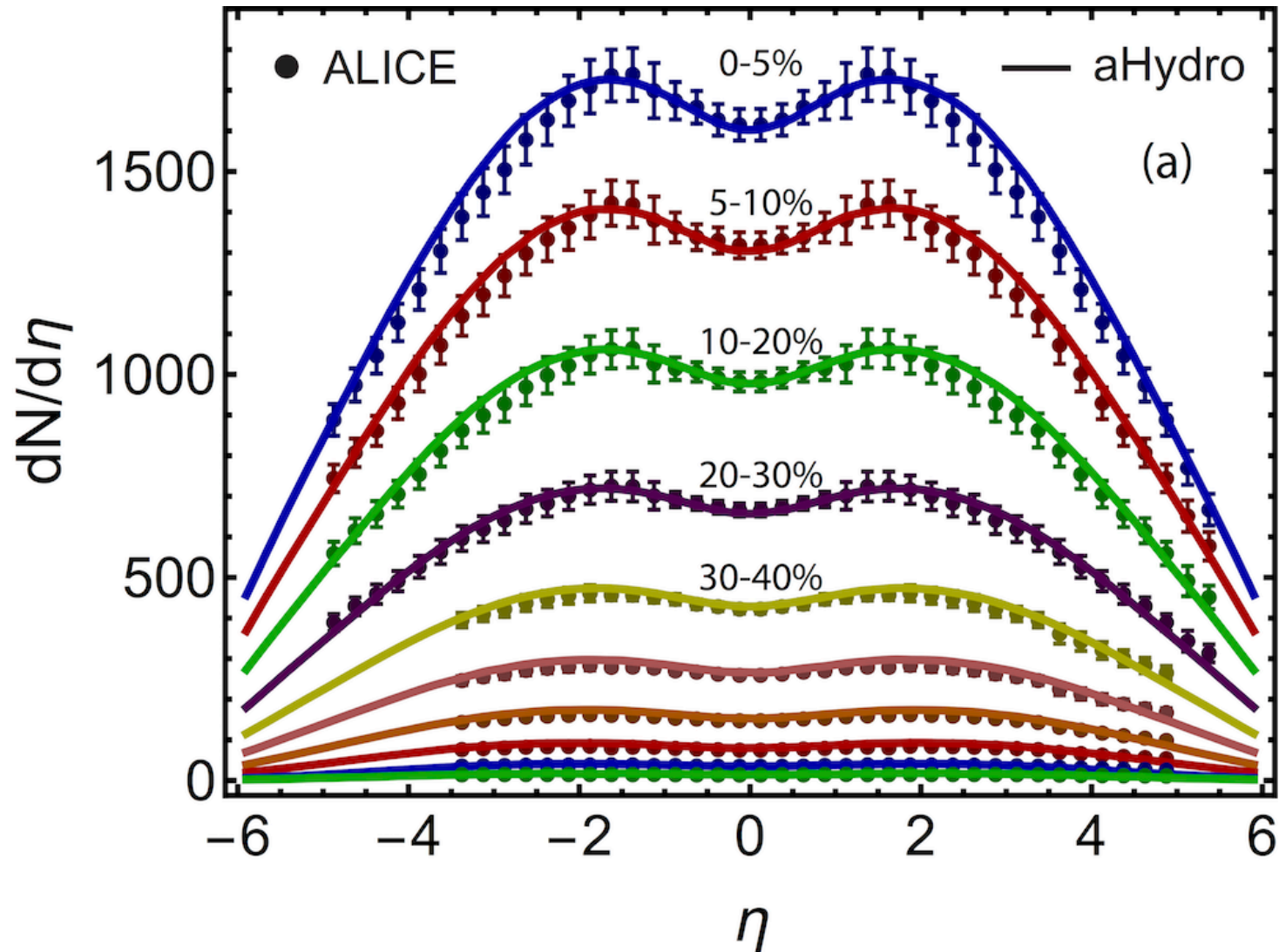
Identified particle spectra



Data are from the ALICE collaboration data for **Pb-Pb collisions @ 2.76 TeV/nucleon**

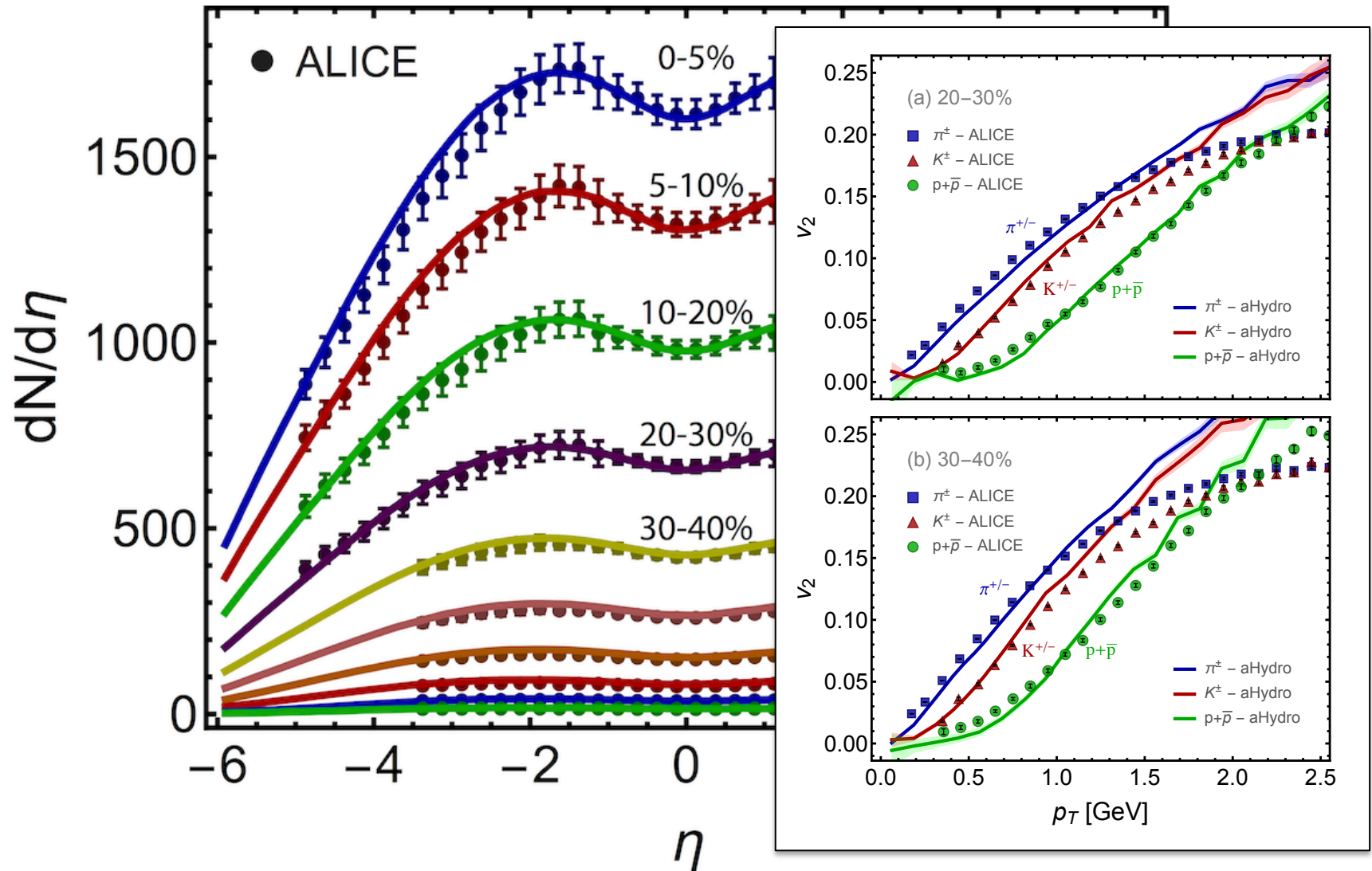
Pb-Pb vs Hydro: Charged particle multiplicities

Alqahtani, Nopoush, Ryblewski, MS, 1703.05808; 1705.10191



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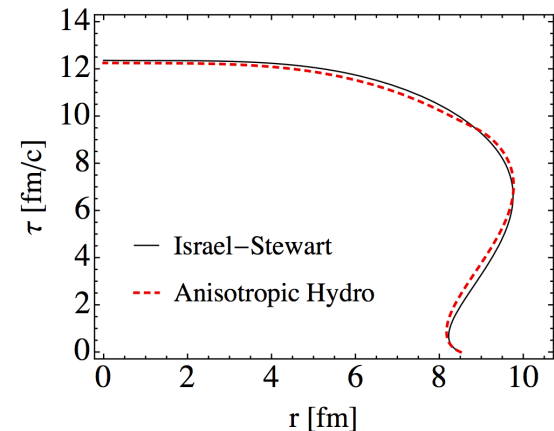
How do hydro models work in practice?

- First, we must specify **initial conditions** for the energy density profile, viscous corrections ($\Pi^{\mu\nu}$), and fluid flow velocity vector in the full 3d volume.
- One then **numerically solves the viscous hydro differential equations** numerically using an advanced PDE solver. 14 DOF are canonically: n , $T^{\mu\nu}$, u^i
- From the full 4d profile, one then extracts a 3-surface called the freeze-out hypersurface Σ

$$\left(p^0 \frac{dN}{d^3p}\right)_i = \frac{\mathcal{N}_i}{(2\pi)^3} \int f_i(x, p) p^\mu d^3\Sigma_\mu$$

aka “**Cooper-Frye freeze-out**”. Canonically, one ^{1+1 example Eqs} takes constant energy density surface.

Importantly, we need a form for f_i on Σ .



- This gives the “**primordial spectra**” which are then fed into a separate hadronic code which takes care of **resonance feed-down, hadronic re-scattering**, etc.

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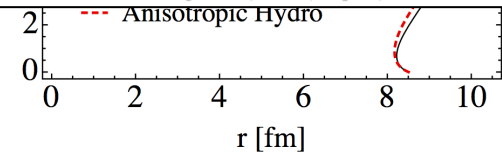
$$\begin{aligned} (\mathcal{E} + \mathcal{P} + \Pi) D_u u^\tau &= -(u^\tau)^2 \left[\partial_\tau (\mathcal{P} + \Pi) - d_\nu \pi_\tau^\nu \right] - u^\tau u^r \left[\partial_r (\mathcal{P} + \Pi) - d_\nu \pi_r^\nu \right], \\ (\mathcal{E} + \mathcal{P} + \Pi) D_u u^r &= -u^\tau u^r \left[\partial_\tau (\mathcal{P} + \Pi) - d_\nu \pi_\tau^\nu \right] - (u^r)^2 \left[\partial_r (\mathcal{P} + \Pi) - d_\nu \pi_r^\nu \right], \\ D_u \mathcal{E} &= -(\mathcal{E} + \mathcal{P} + \Pi) \theta_u - \pi_r^r (1 - v^2)^2 \nabla^\langle \tau u^r \rangle - r^2 \pi_\phi^\phi \nabla^\langle \phi u^\phi \rangle - \tau^2 \pi_\zeta^\zeta \nabla^\langle \zeta u^\zeta \rangle, \end{aligned}$$

and

$$\begin{aligned} \tau_\Pi D_u \Pi + \Pi &= -\zeta \theta_u - \delta_{\Pi\Pi} \Pi \theta_u - \lambda_{\Pi\pi} \left[2r^2 \pi_\phi^\phi \nabla^\langle \phi u^\phi \rangle + 2\tau^2 \pi_\zeta^\zeta \nabla^\langle \zeta u^\zeta \rangle \right. \\ &\quad \left. + r^2 \pi_\zeta^\zeta \nabla^\langle \phi u^\phi \rangle + \tau^2 \pi_\phi^\phi \nabla^\langle \zeta u^\zeta \rangle \right], \\ \tau_\pi D_u \pi_\phi^\phi + \pi_\phi^\phi &= -2r^2 \eta \nabla^\langle \phi u^\phi \rangle - \delta_{\pi\pi} \pi_\phi^\phi \theta_u - \frac{\tau_{\pi\pi}}{3} \left[-r^2 \pi_\phi^\phi \nabla^\langle \phi u^\phi \rangle + 2\tau^2 \pi_\zeta^\zeta \nabla^\langle \zeta u^\zeta \rangle \right. \\ &\quad \left. + r^2 \pi_\zeta^\zeta \nabla^\langle \phi u^\phi \rangle + \tau^2 \pi_\phi^\phi \nabla^\langle \zeta u^\zeta \rangle \right] - r^2 \lambda_{\pi\Pi} \Pi \nabla^\langle \phi u^\phi \rangle, \\ \tau_\pi D_u \pi_\zeta^\zeta + \pi_\zeta^\zeta &= -2\tau^2 \eta \nabla^\langle \zeta u^\zeta \rangle - \delta_{\pi\pi} \pi_\zeta^\zeta \theta_u - \frac{\tau_{\pi\pi}}{3} \left[2r^2 \pi_\phi^\phi \nabla^\langle \phi u^\phi \rangle - \tau^2 \pi_\zeta^\zeta \nabla^\langle \zeta u^\zeta \rangle \right. \\ &\quad \left. + r^2 \pi_\zeta^\zeta \nabla^\langle \phi u^\phi \rangle + \tau^2 \pi_\phi^\phi \nabla^\langle \zeta u^\zeta \rangle \right] - \tau^2 \lambda_{\pi\Pi} \Pi \nabla^\langle \zeta u^\zeta \rangle, \end{aligned}$$

where

$$\begin{aligned} -d_\nu \pi_\tau^\nu &= v^2 \partial_\tau \pi_r^r + v \partial_r \pi_r^r + \pi_r^r \left[\partial_\tau v^2 + \partial_r v + \frac{v^2}{\tau} + \frac{v}{r} \right] + \frac{1}{\tau} \pi_\zeta^\zeta, \\ d_\nu \pi_r^\nu &= v \partial_\tau \pi_r^r + \partial_r \pi_r^r + \pi_r^r \left[\partial_\tau v + \frac{v}{\tau} + \frac{2-v^2}{r} \right] + \frac{1}{r} \pi_\zeta^\zeta. \end{aligned}$$



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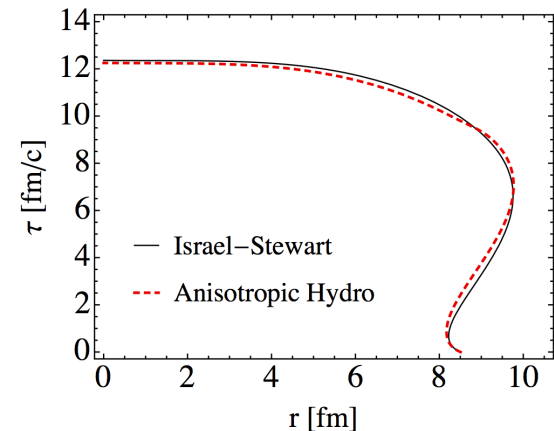
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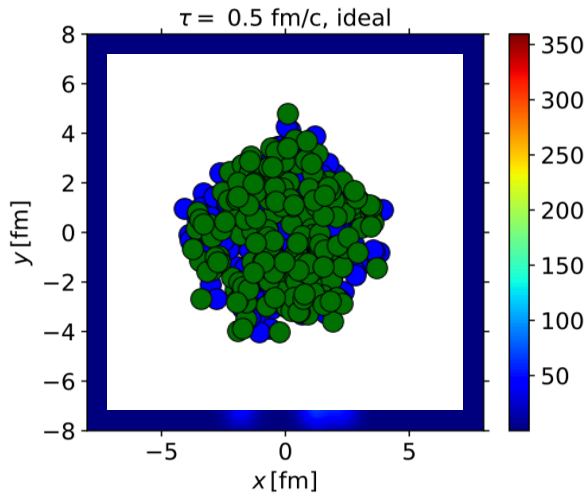
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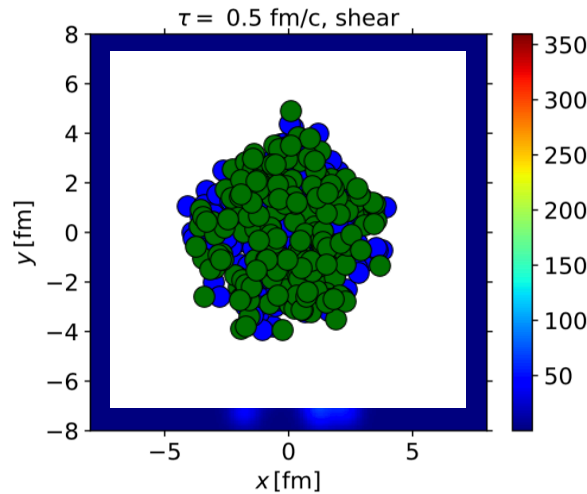
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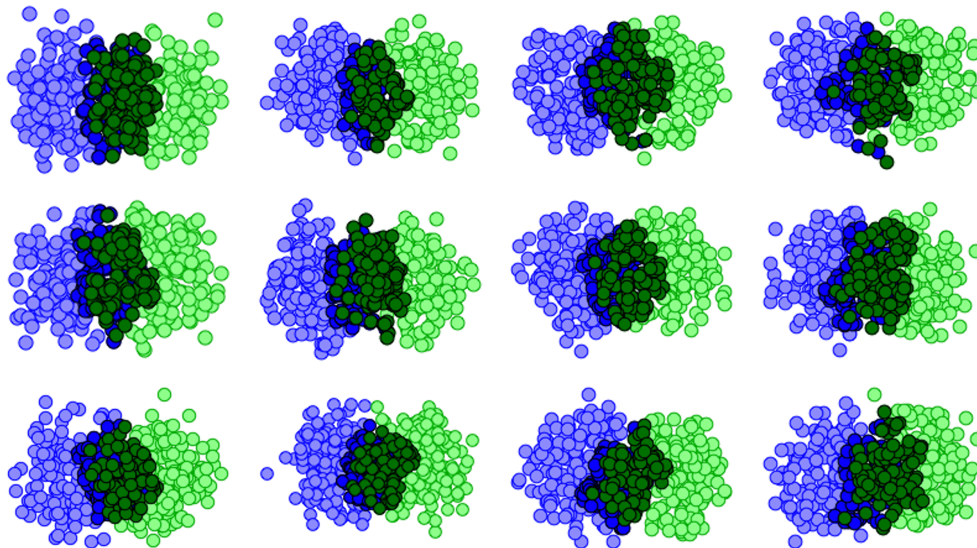
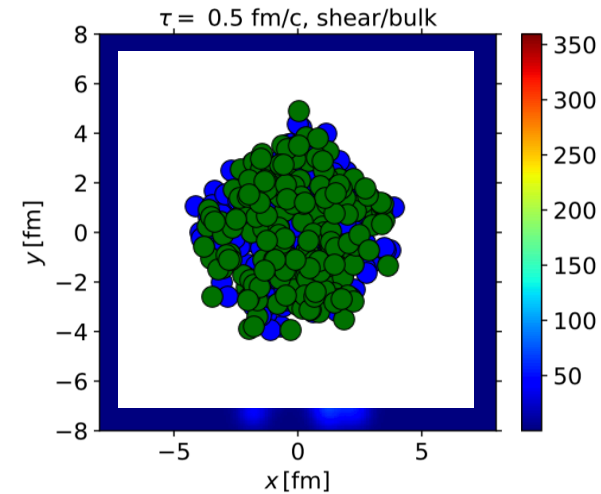
Ideal Hydro



Shear only ($\eta/s = 0.2$)



Shear + Bulk



- Variety of initial conditions (ICs) on the market. For simplicity, here I only discuss Monte-Carlo Glauber IC.
- Top shows animation of sampling # of participants in a central collision ($b = 0 \text{ fm}$).
- Left shows a set of fluctuating initial conditions for $b = 8 \text{ fm}$.

QGP momentum anisotropy cartoon

