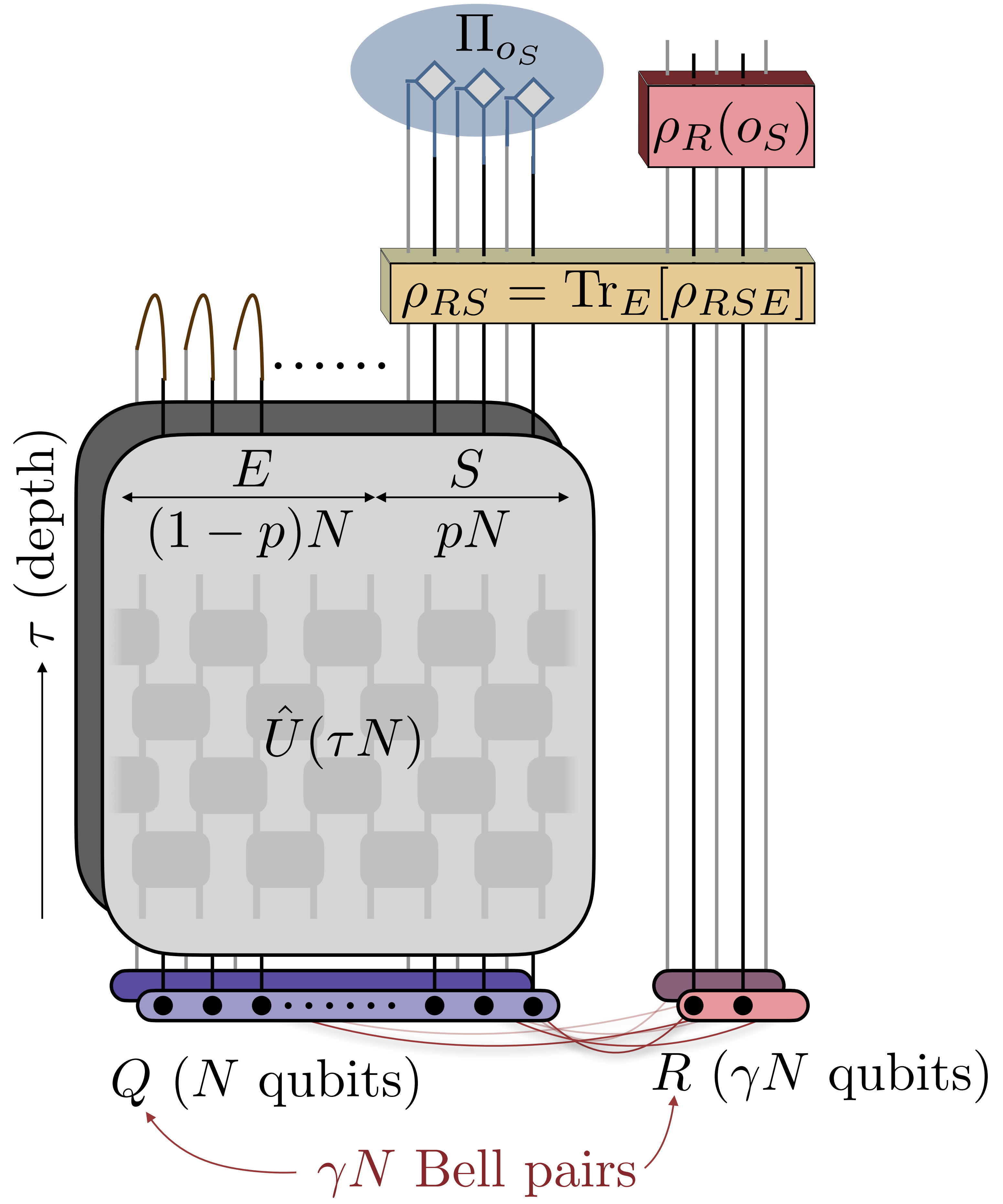
Measurement-invisible quantum correlations in scrambling dynamics

Sthitadhi Roy

International Centre for Theoretical Sciences-TIFR



Alan Sherry and SR, arXiv:2410.24212

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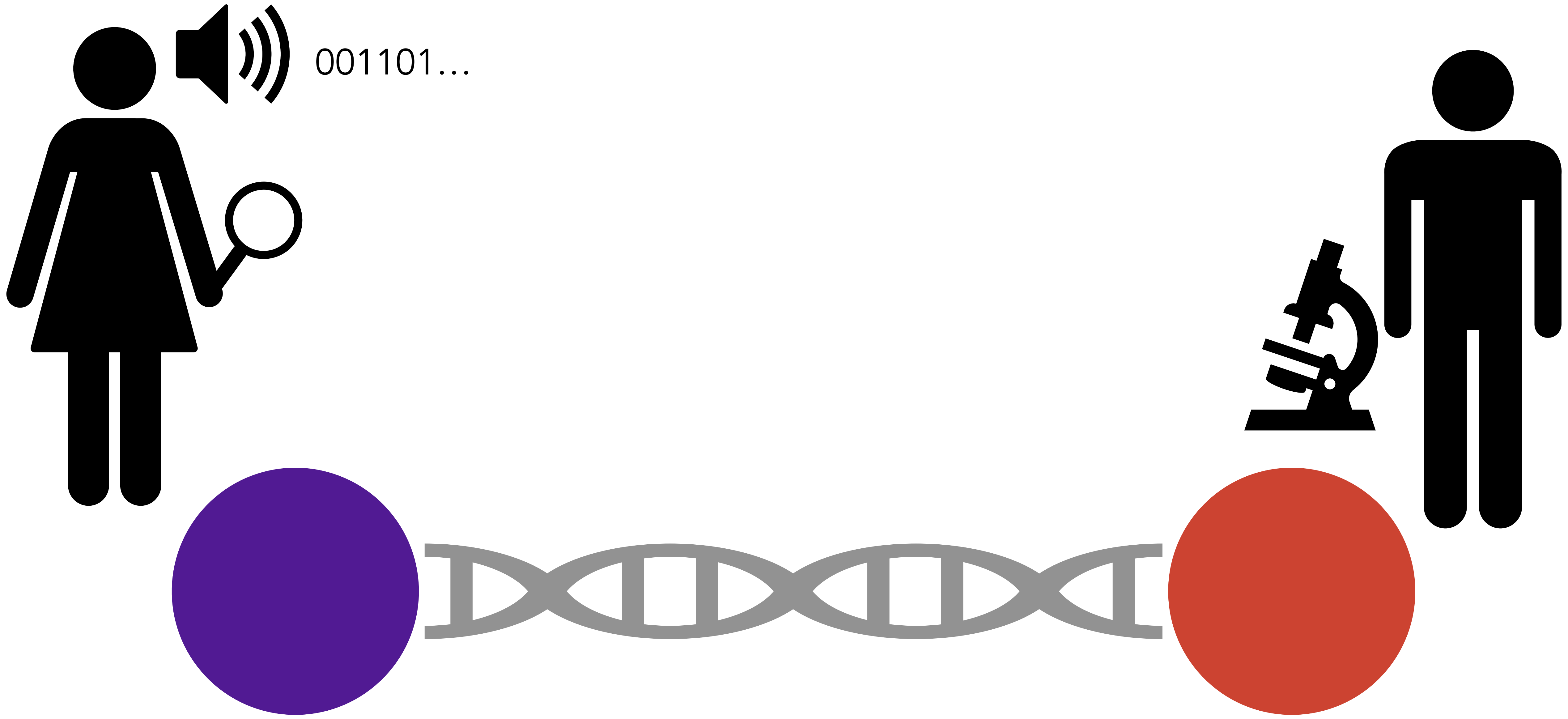
ICTS INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

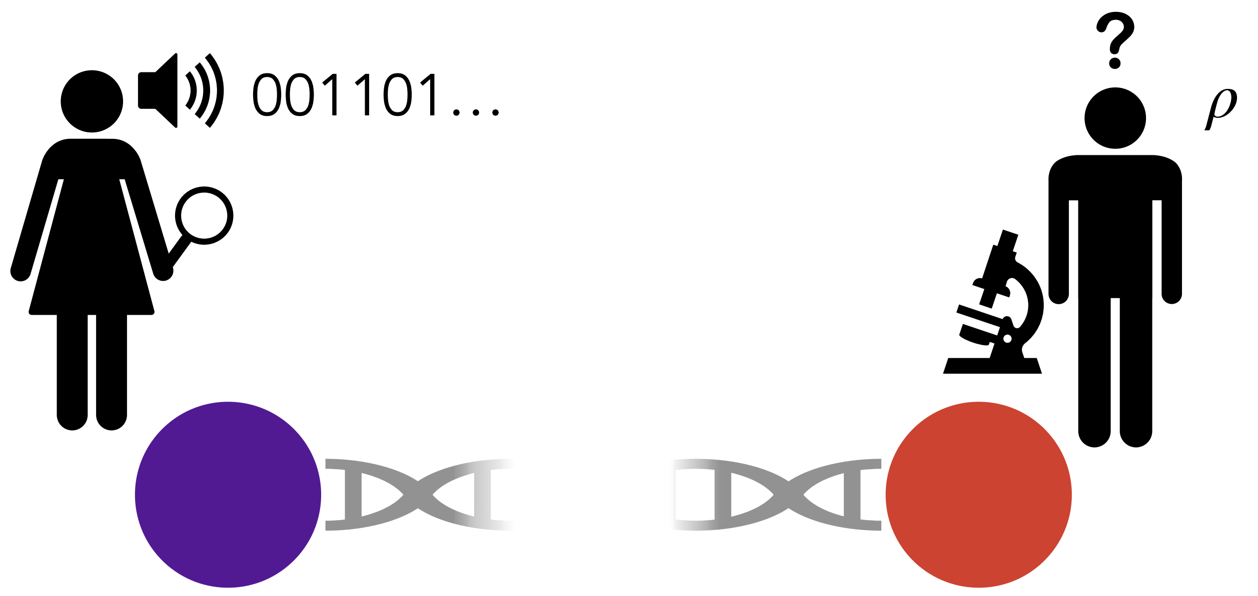
Quantum Trajectories, ICTS-TIFR, January 21, 2025

Summary



- State shared between Alice and Bob
- Alice performs measurements on a part of her state and classically communicates the outcome to Bob
- Bob can do full tomography on his part of the state and see how his state depends on Alice's measurement outcome

Summary



- No entanglement
- Bob's state agnostic to Alice's measurement outcomes

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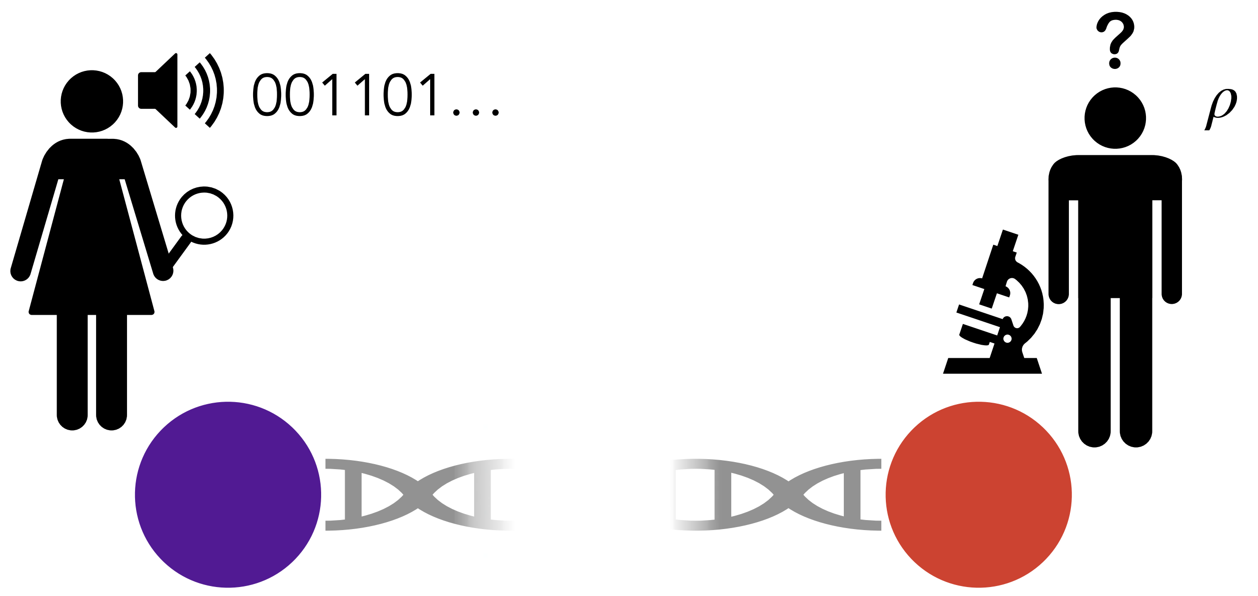

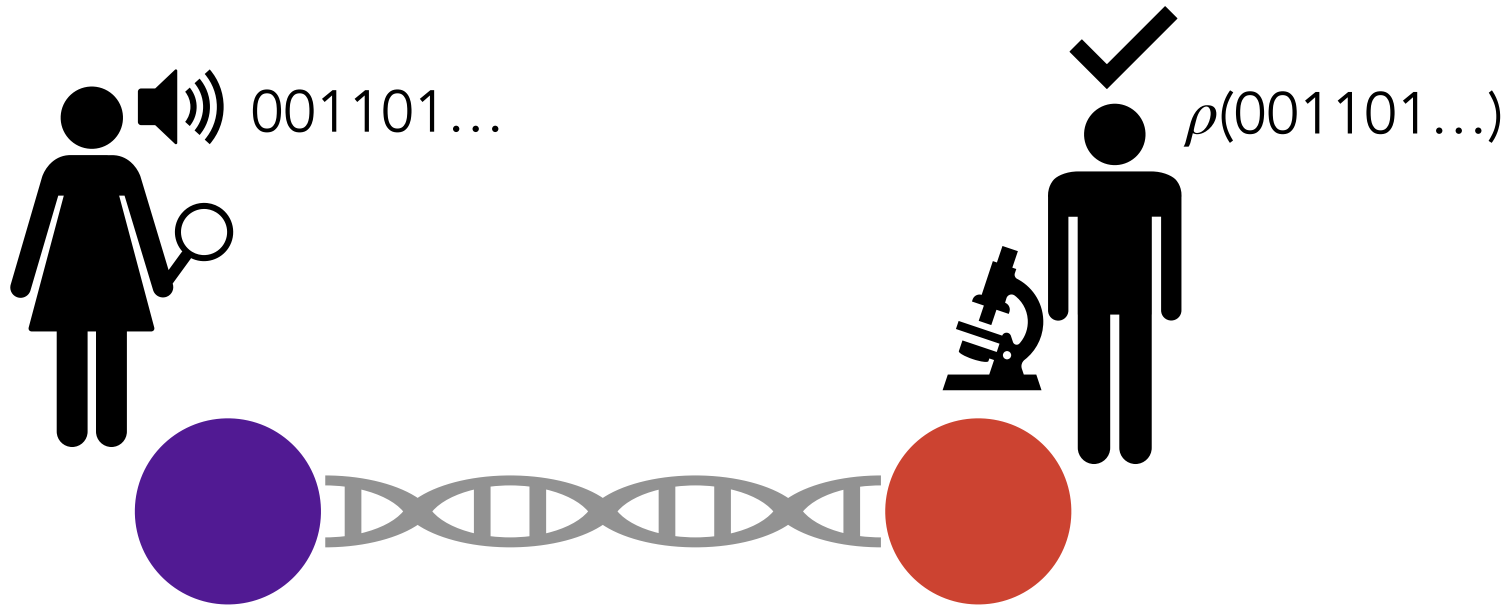

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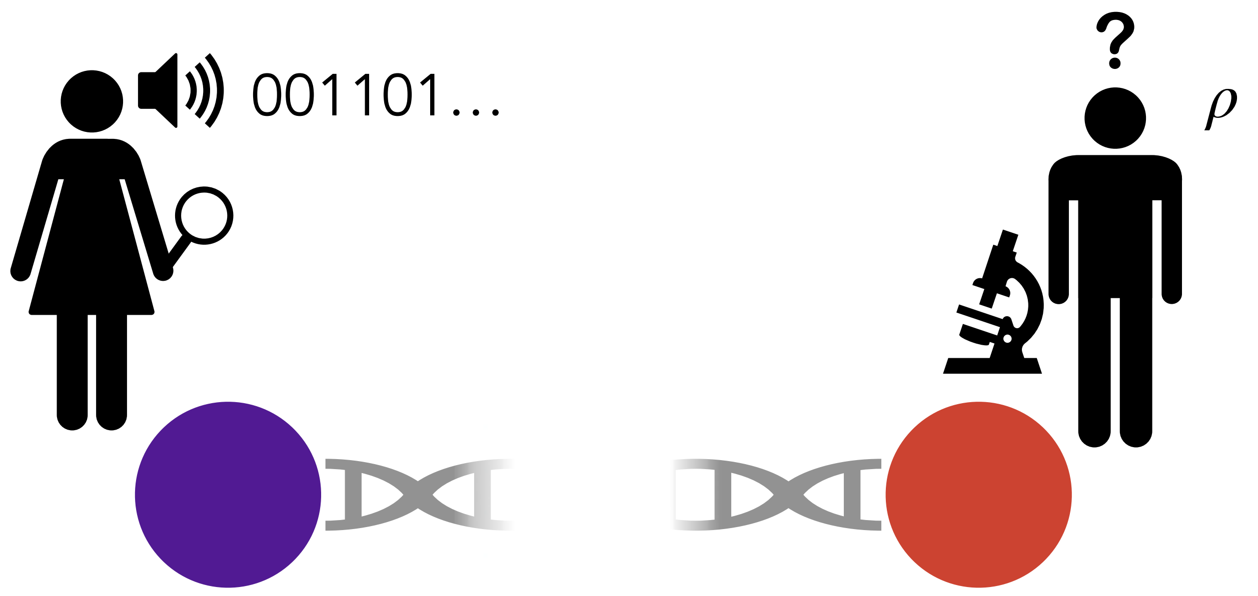

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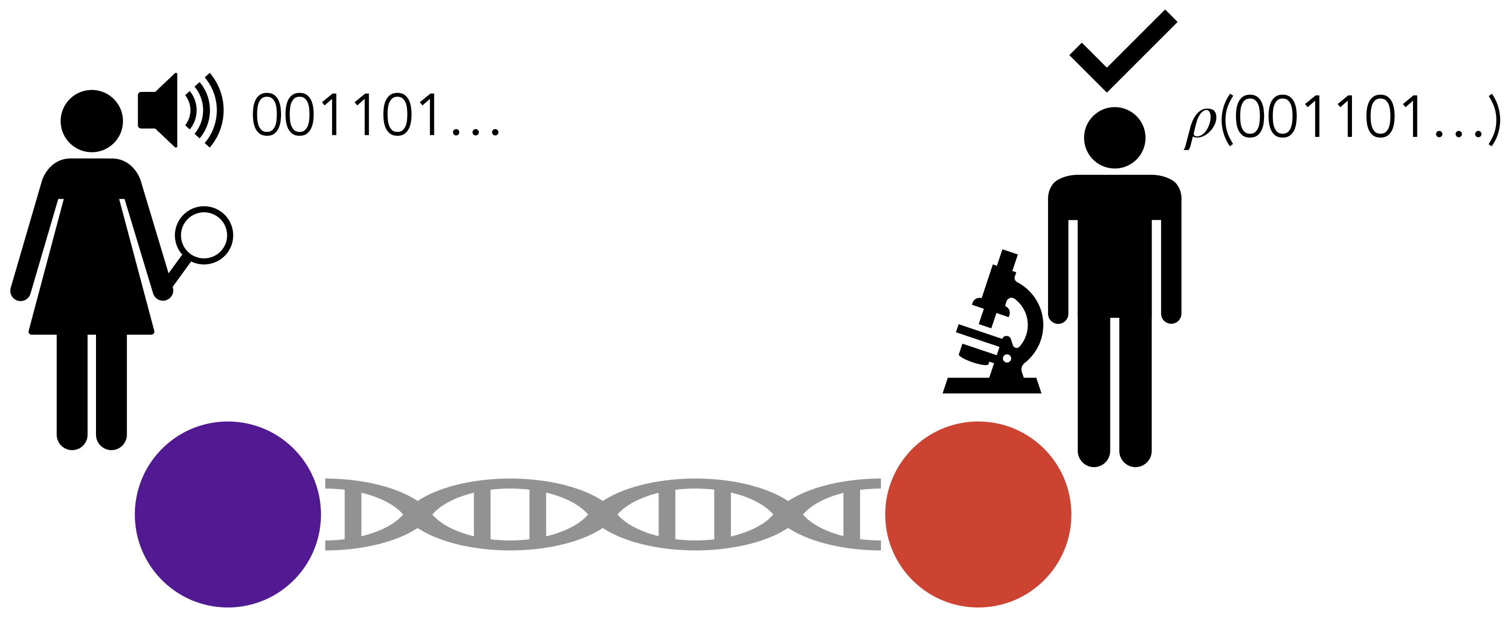

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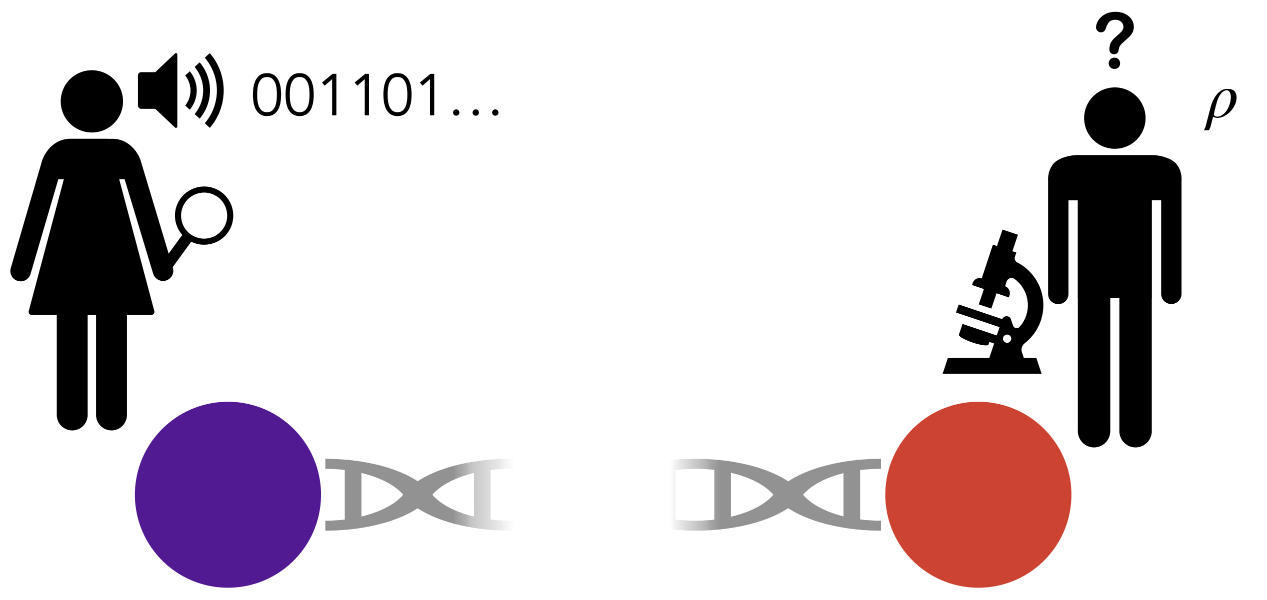

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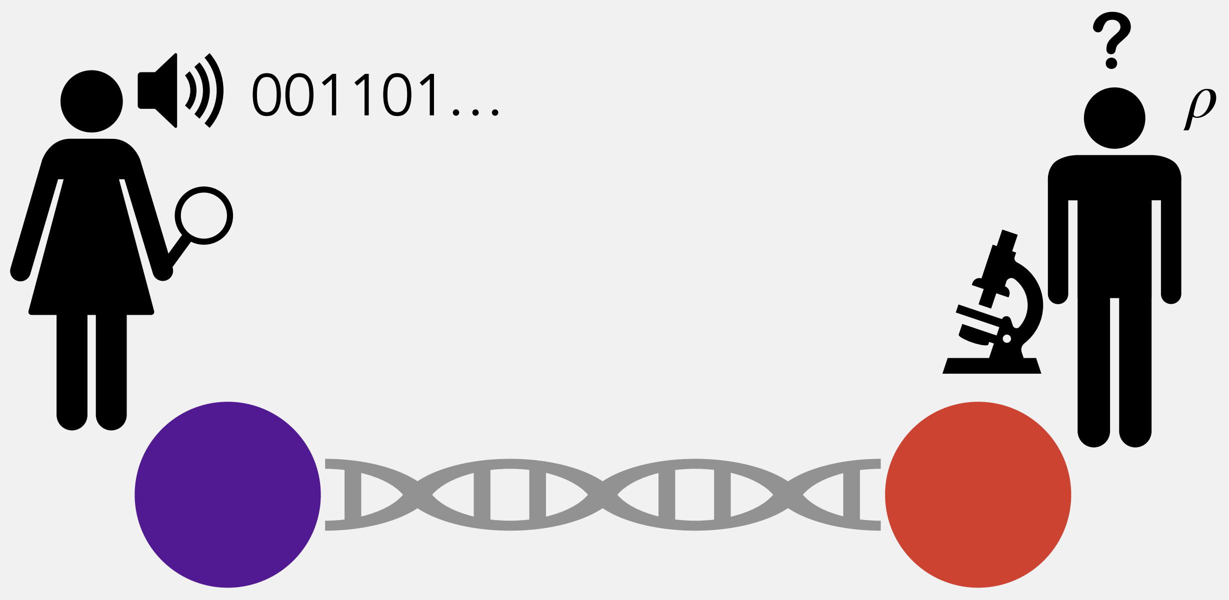

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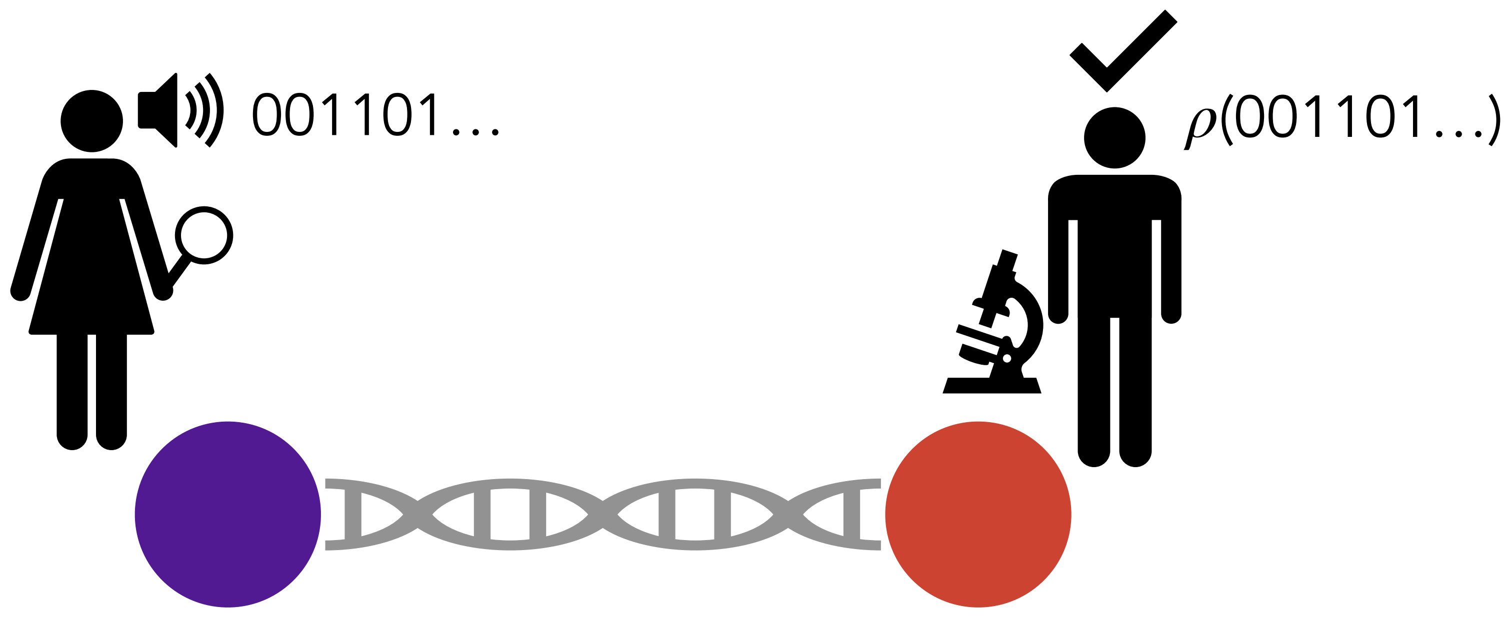
Measurement-invisible quantum correlations



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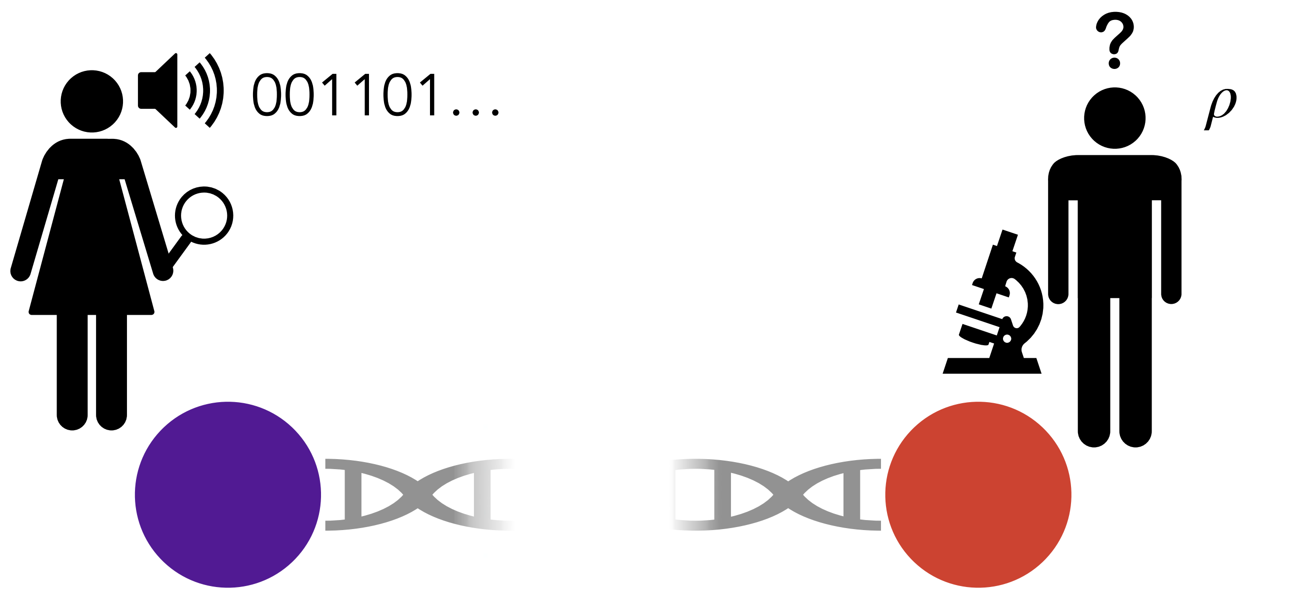

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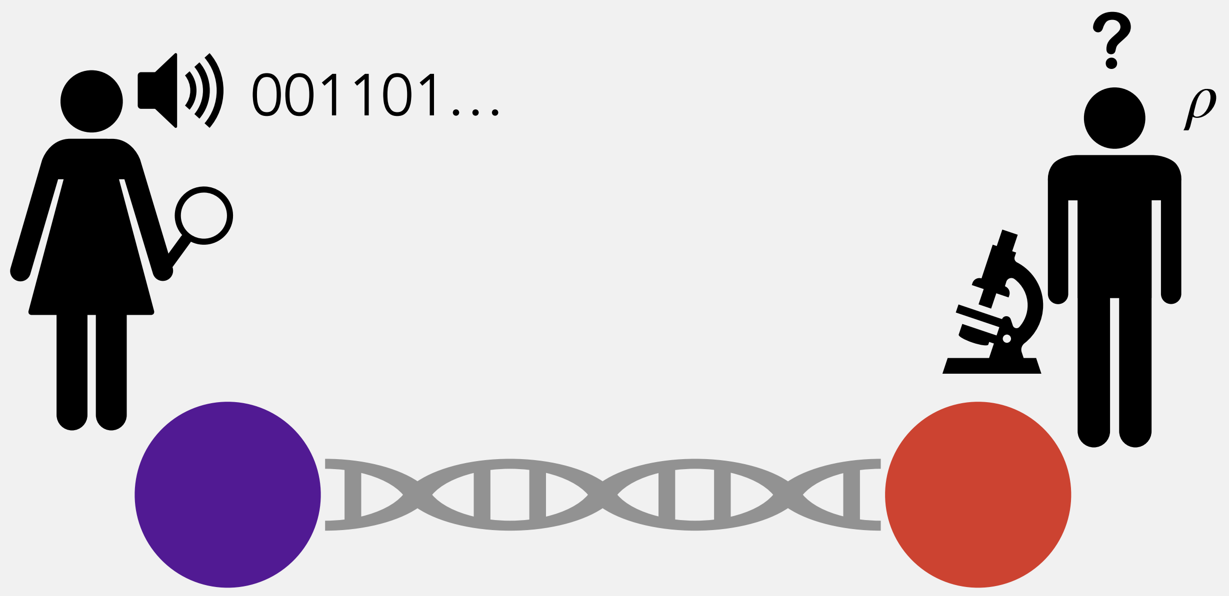

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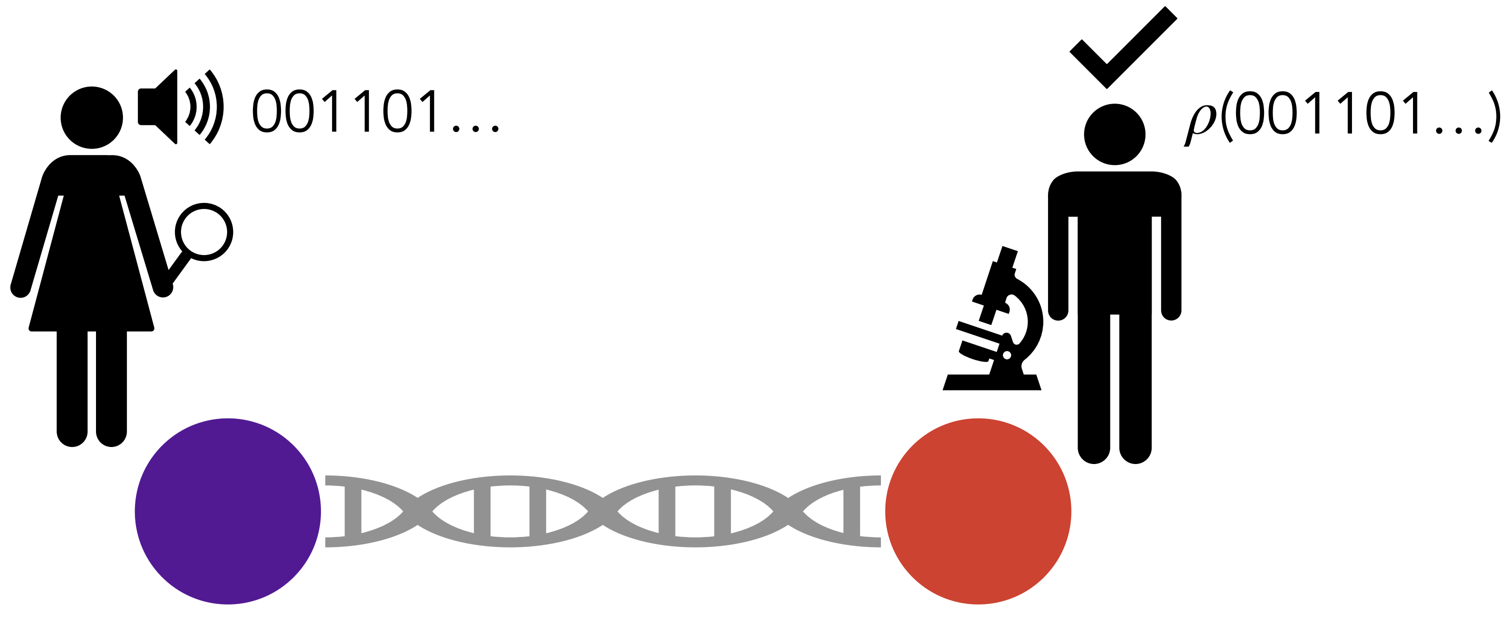

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Key idea: entanglement structure and its response to measurements on subsystem

Entanglement structure

a new paradigm of classifying phases/states of quantum matter

- how much entanglement?

- range of entanglement?

- how multipartite is the entanglement?

Entanglement structure

a new paradigm of classifying phases/states of quantum matter

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  - area-law entangled states have very little entanglement
  - volume-law entangled states have a lot of entanglement

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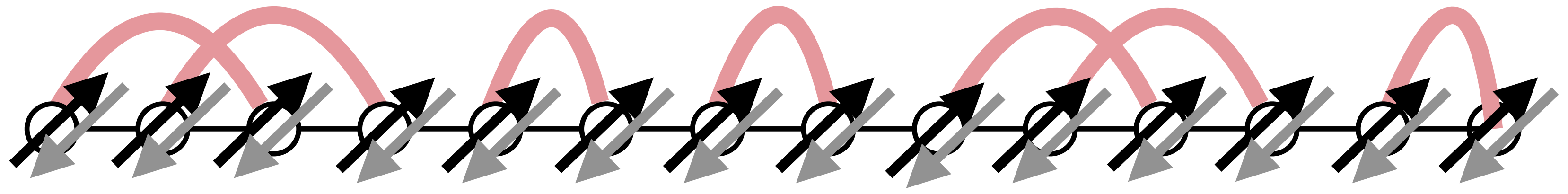
- how multipartite is the entanglement?
    - *few*-partite entanglement — states can be decomposed into direct product of few-body entangled clusters
    - multipartite entanglement — entanglement shared across a large number of degrees of freedom

Area-law entanglement

short-ranged, area-law entangled



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- many-body localised eigenstates in strongly disordered systems

Bauer+Nayak, JSTAT 2013

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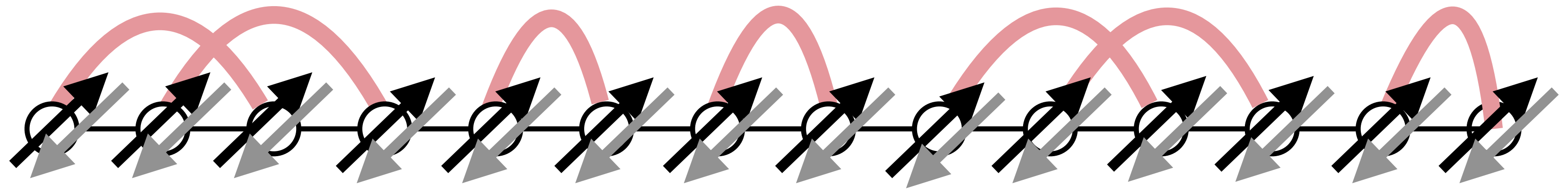

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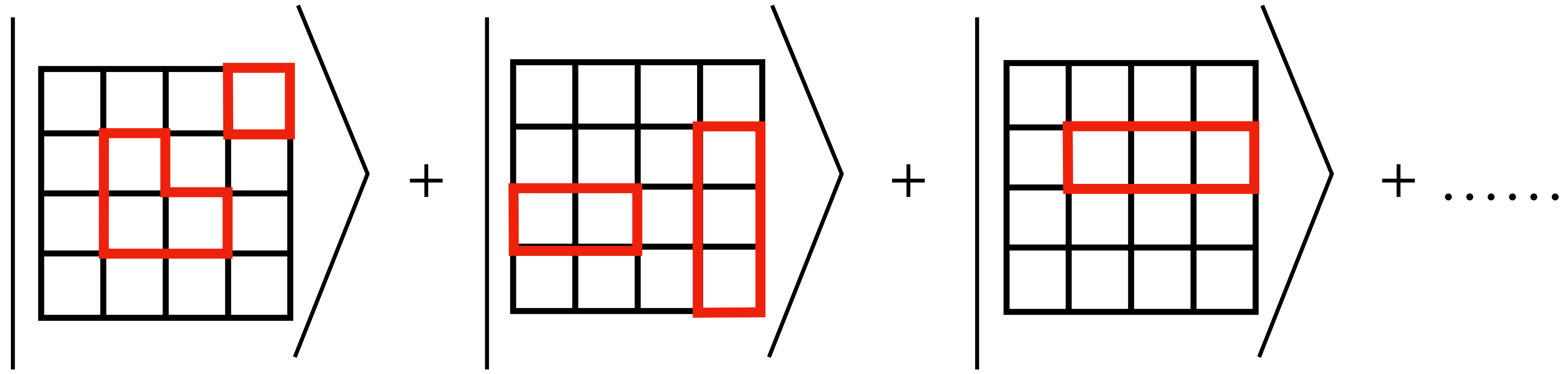

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- entanglement at all lengthscales

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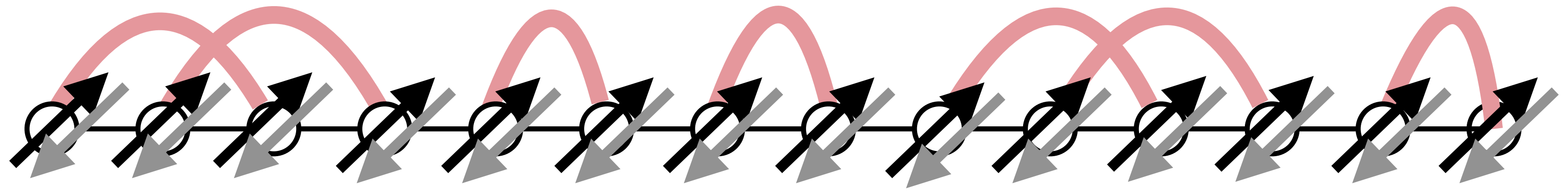

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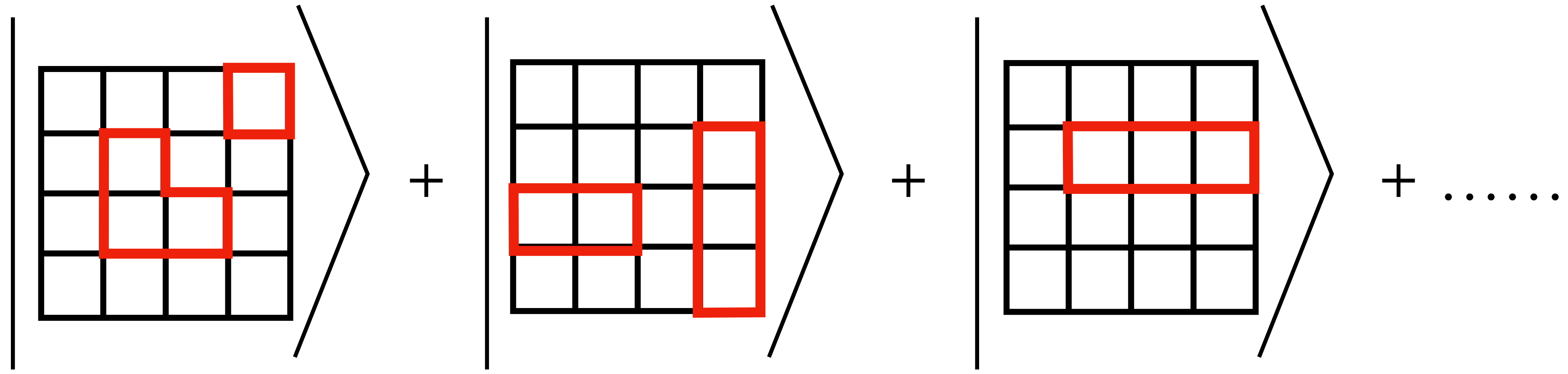

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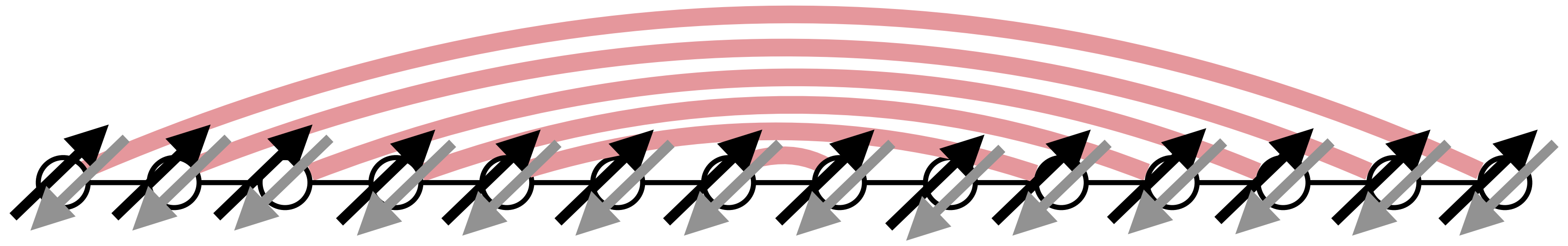

- long-ranged singlets/Bell pairs
- volume-law made up of an extensive number of bipartite entangled objects

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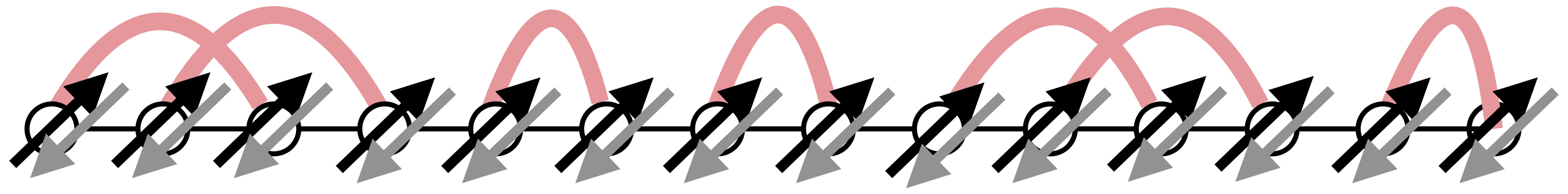

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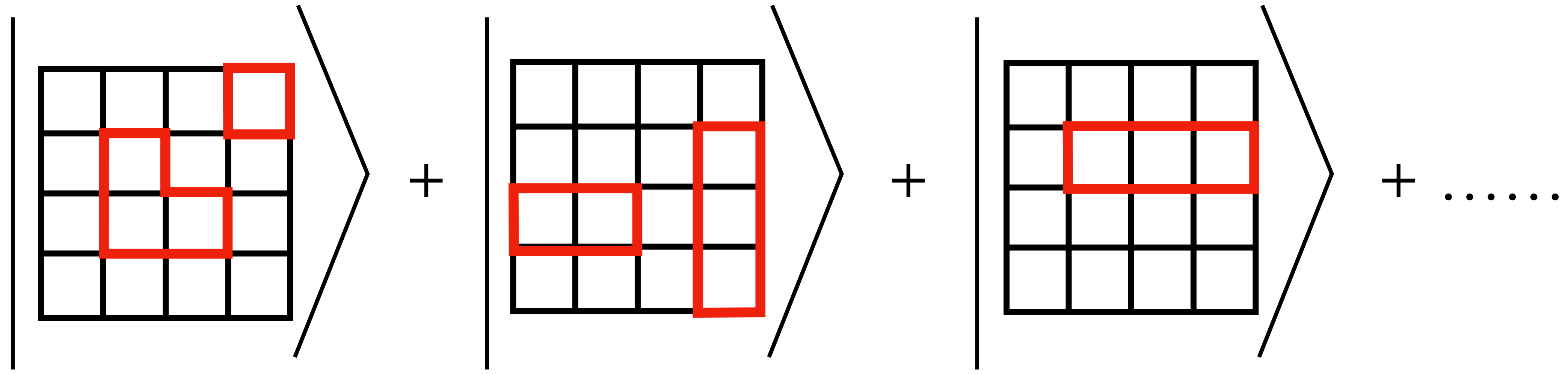

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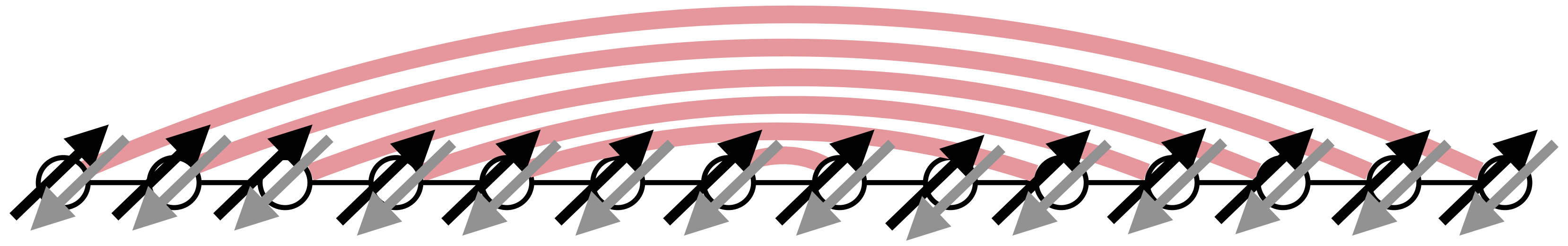

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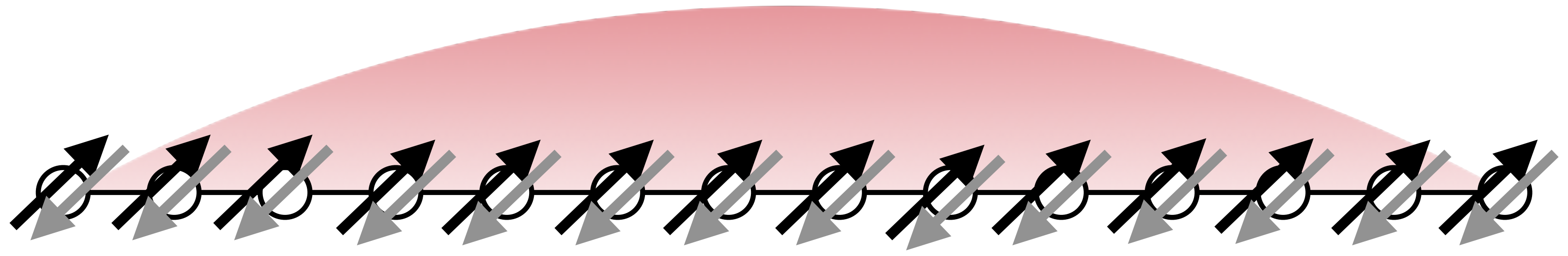
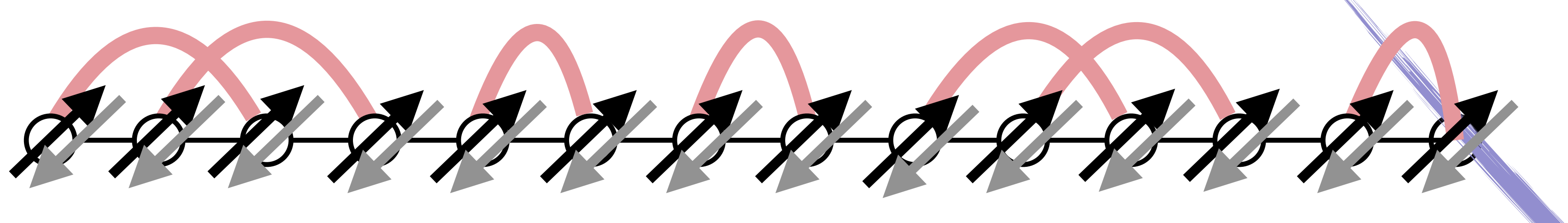


- Haar-random states, eigenstates or time-evolved states of quantum chaotic systems
- every qubit entangled with an extensive number of qubits
- monogamy — bipartite/few-partite measures such as concurrence vanishingly small

Area-law entanglement

short-ranged, area-law entangled



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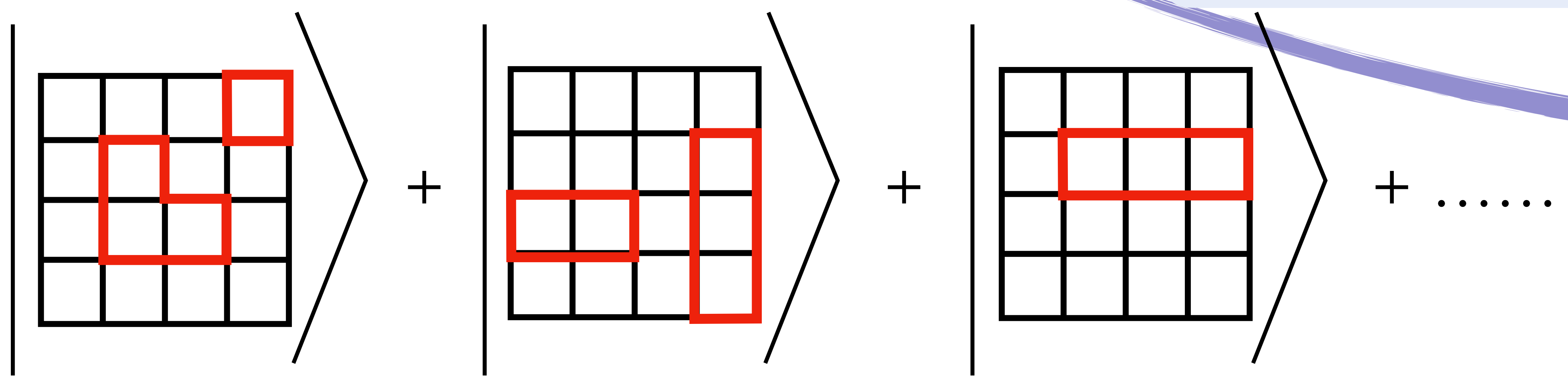

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Scrambling unitary dynamics

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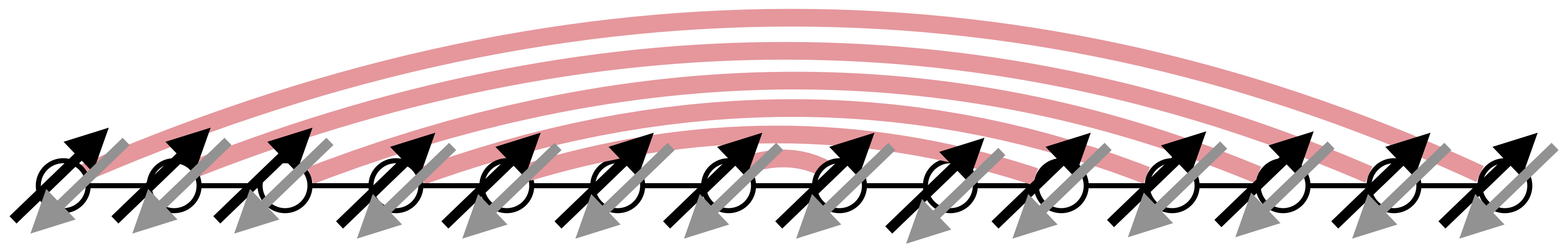

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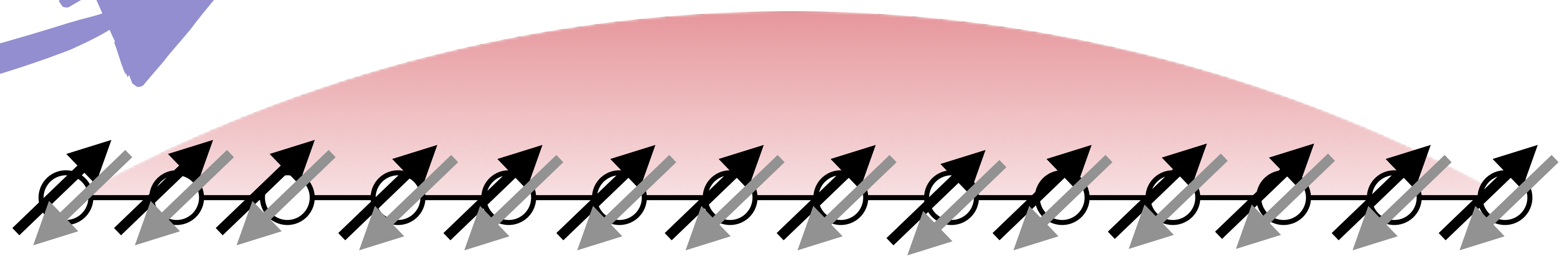

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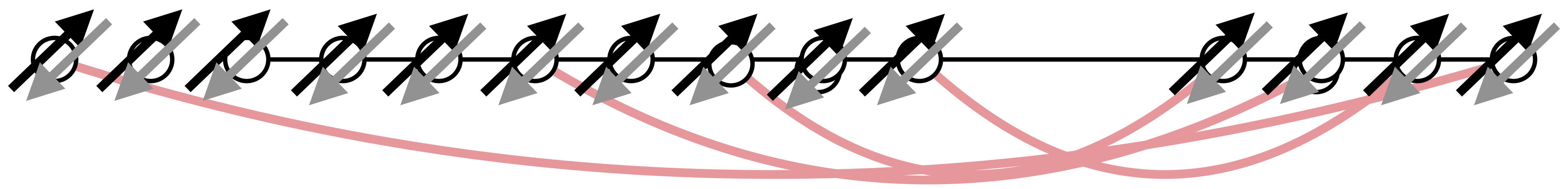
“Transmutation” of quantum entanglement

Scrambling unitary dynamics transmutes local, few-body entanglement into multipartite, volume-law entanglement

- Consider a state which is initially few-partite, volume-law entangled



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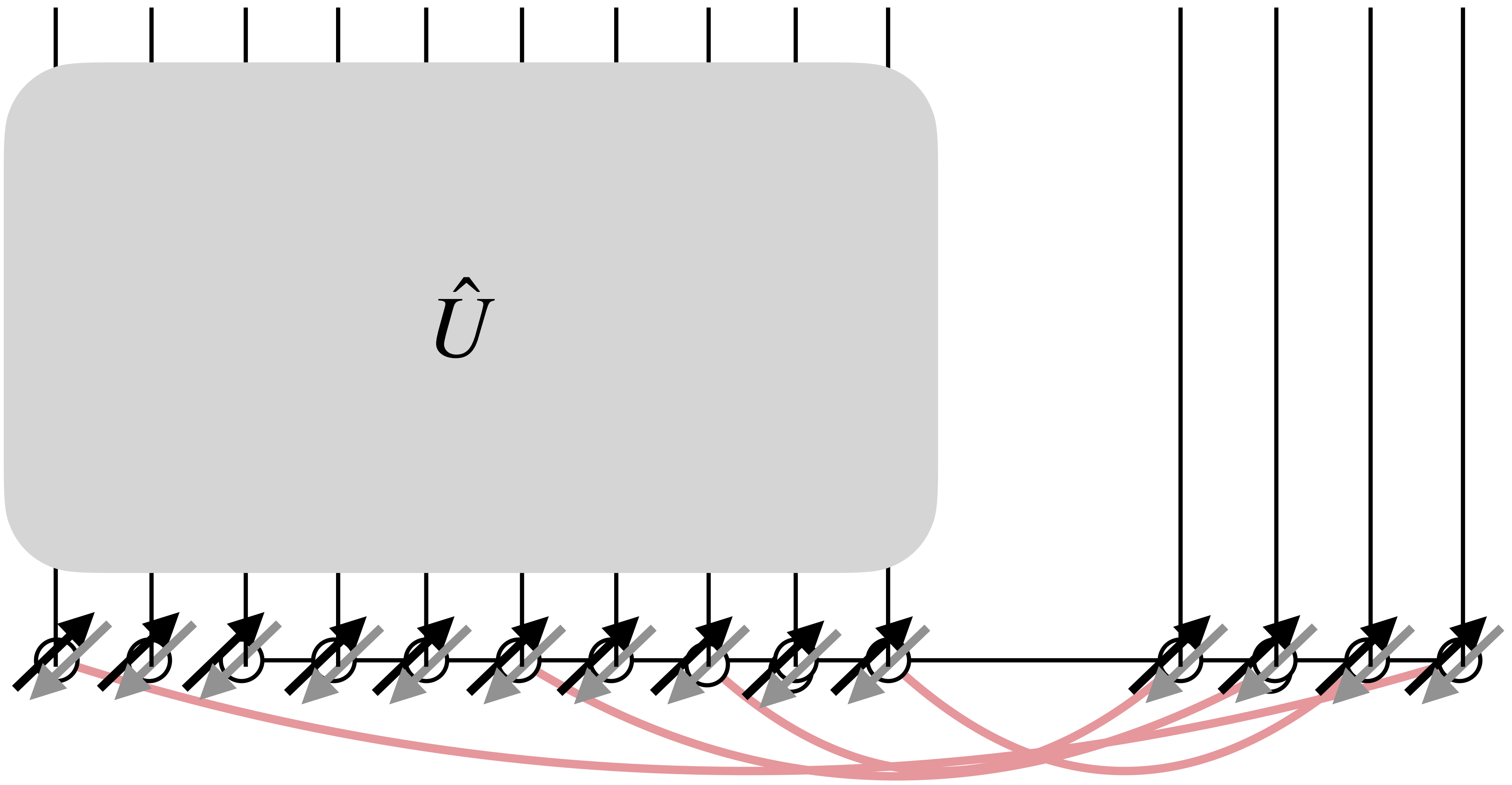

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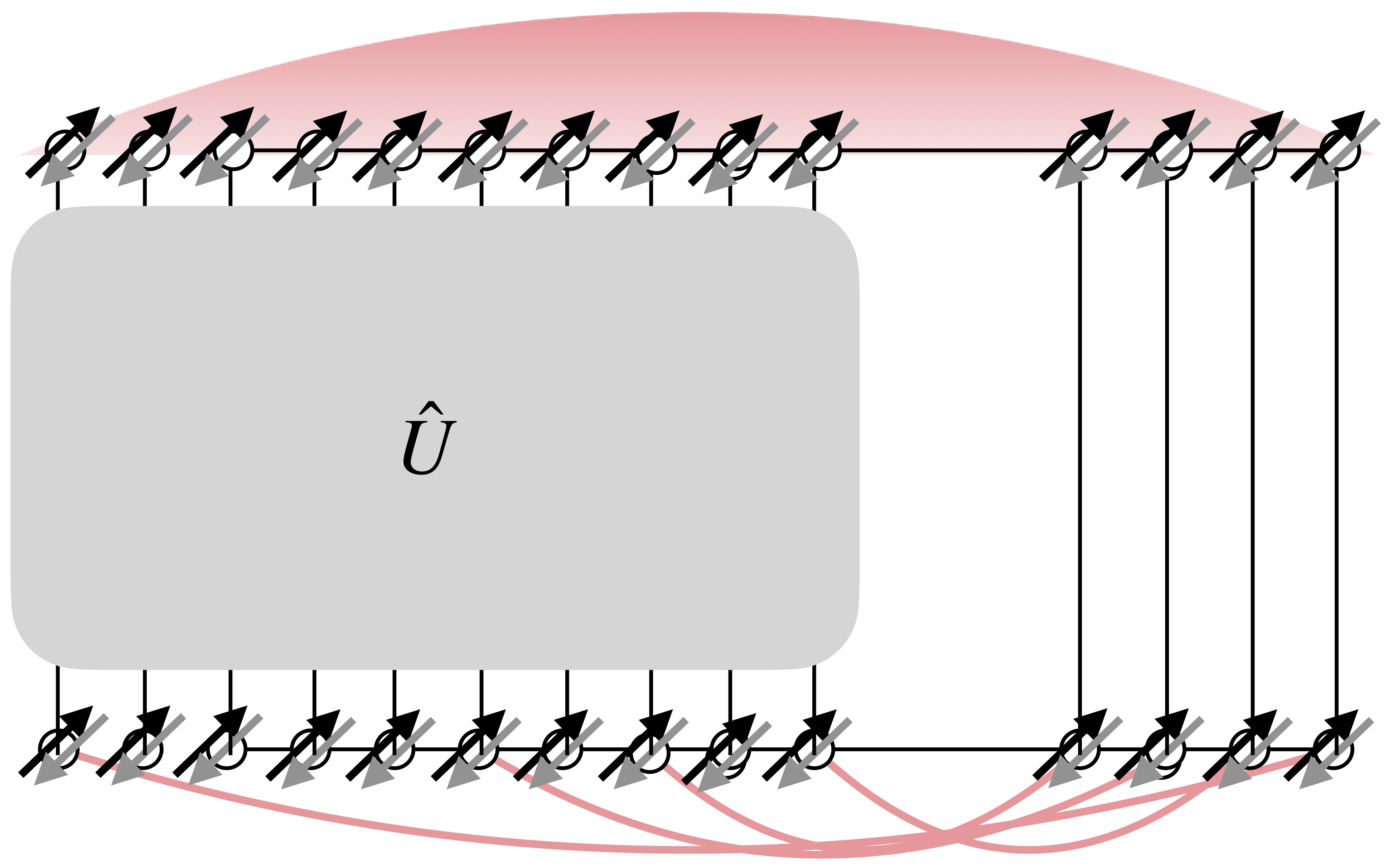

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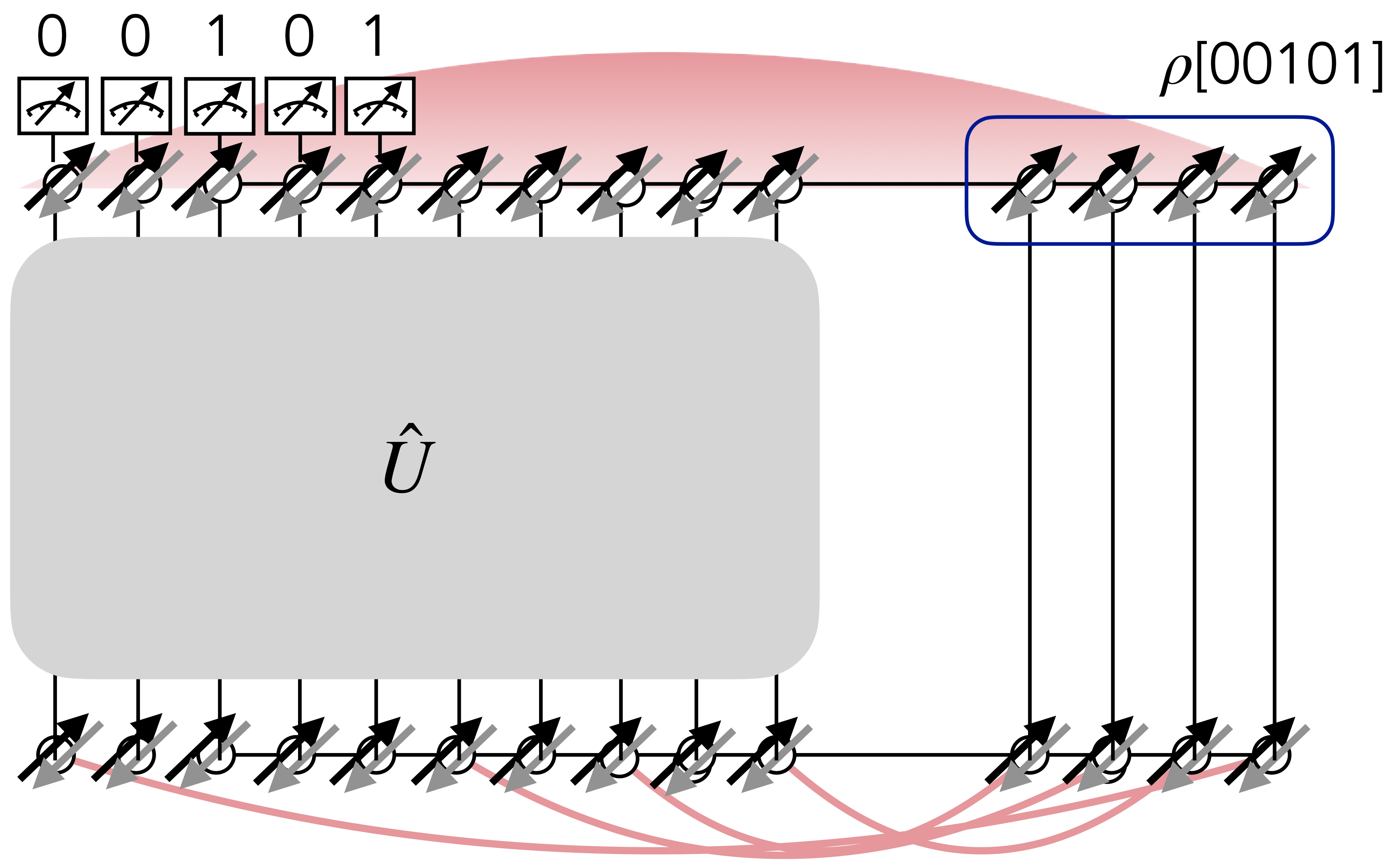

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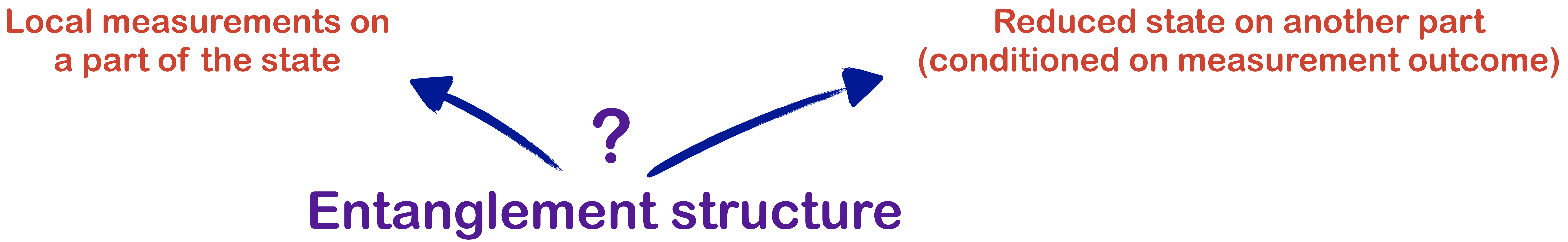
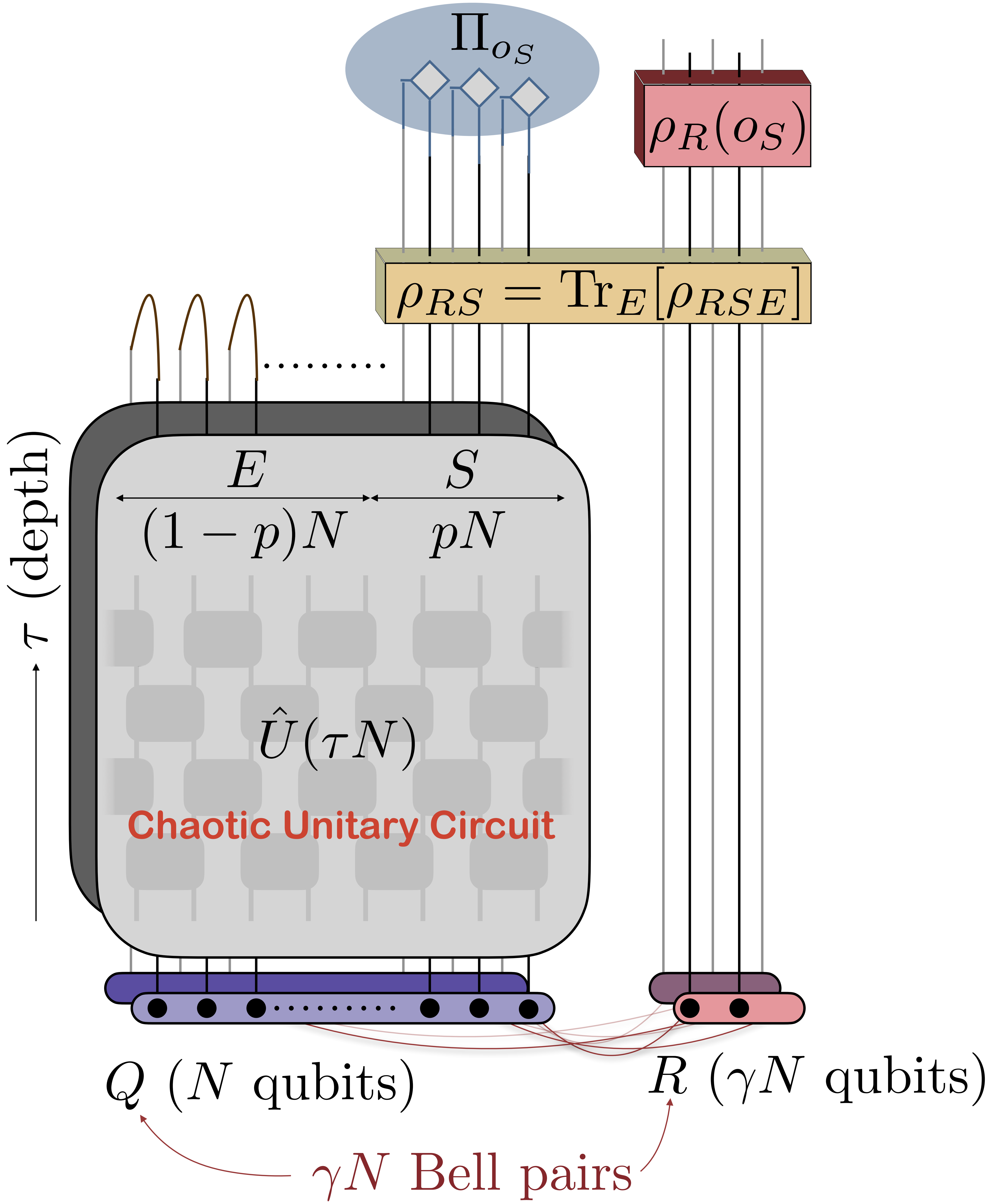


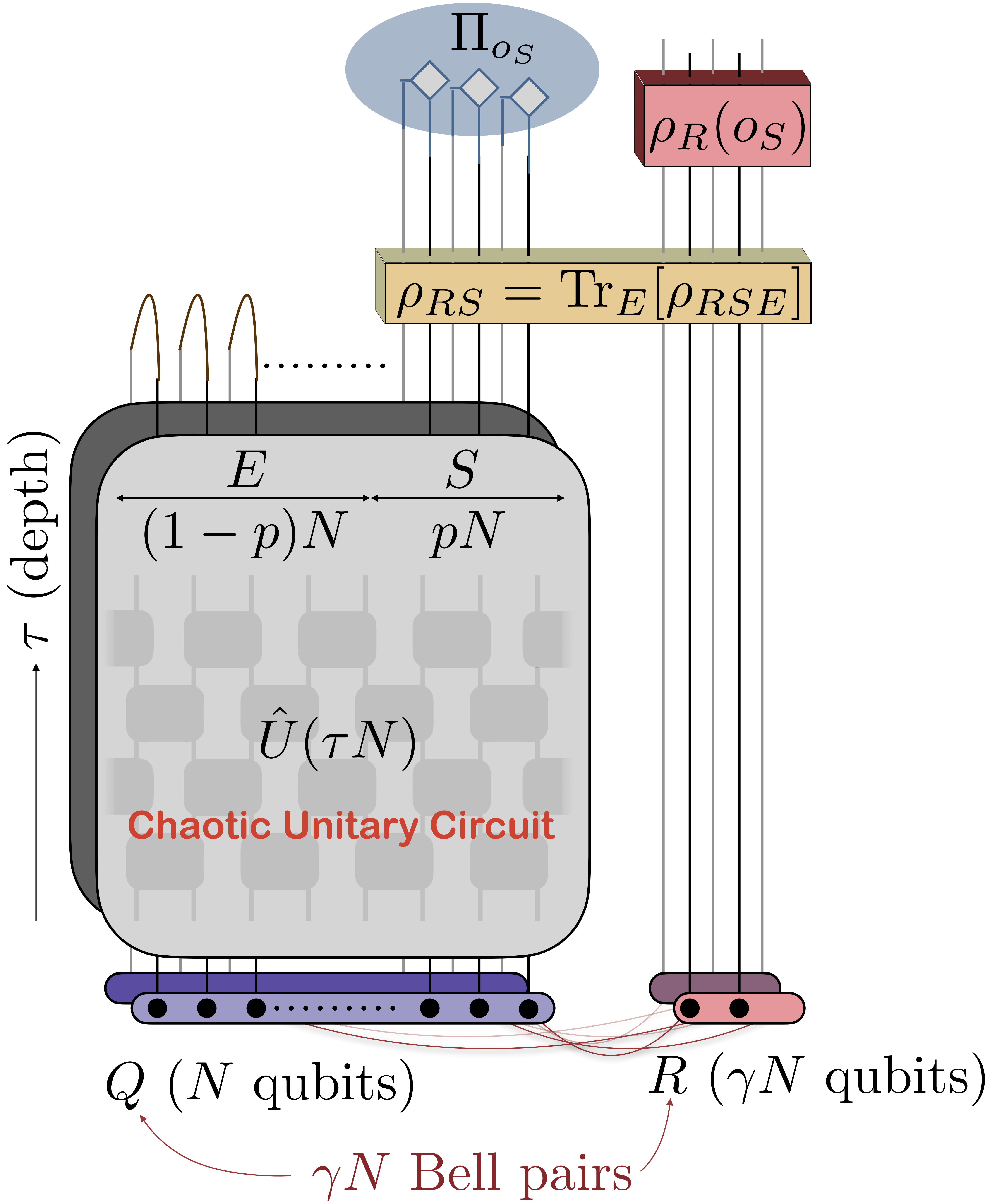
Setup and definitions



Setup and definitions

- State of the system after the unitary $\equiv \rho_{RSE} = |\psi_{RSE}\rangle\langle\psi_{RSE}|$
- Subsystem E is traced out leaving a mixed states between R and S denoted by ρ_{RS}
- Measurements on the subsystem S with outcome o_S with probability $p(o_S)$
  - conditional state on R denoted by $\rho_R(o_S)$
- Reduced density matrix of R denoted by ρ_R

$$\rho_R = \sum_{o_S} p(o_S)\rho_R(o_S)$$



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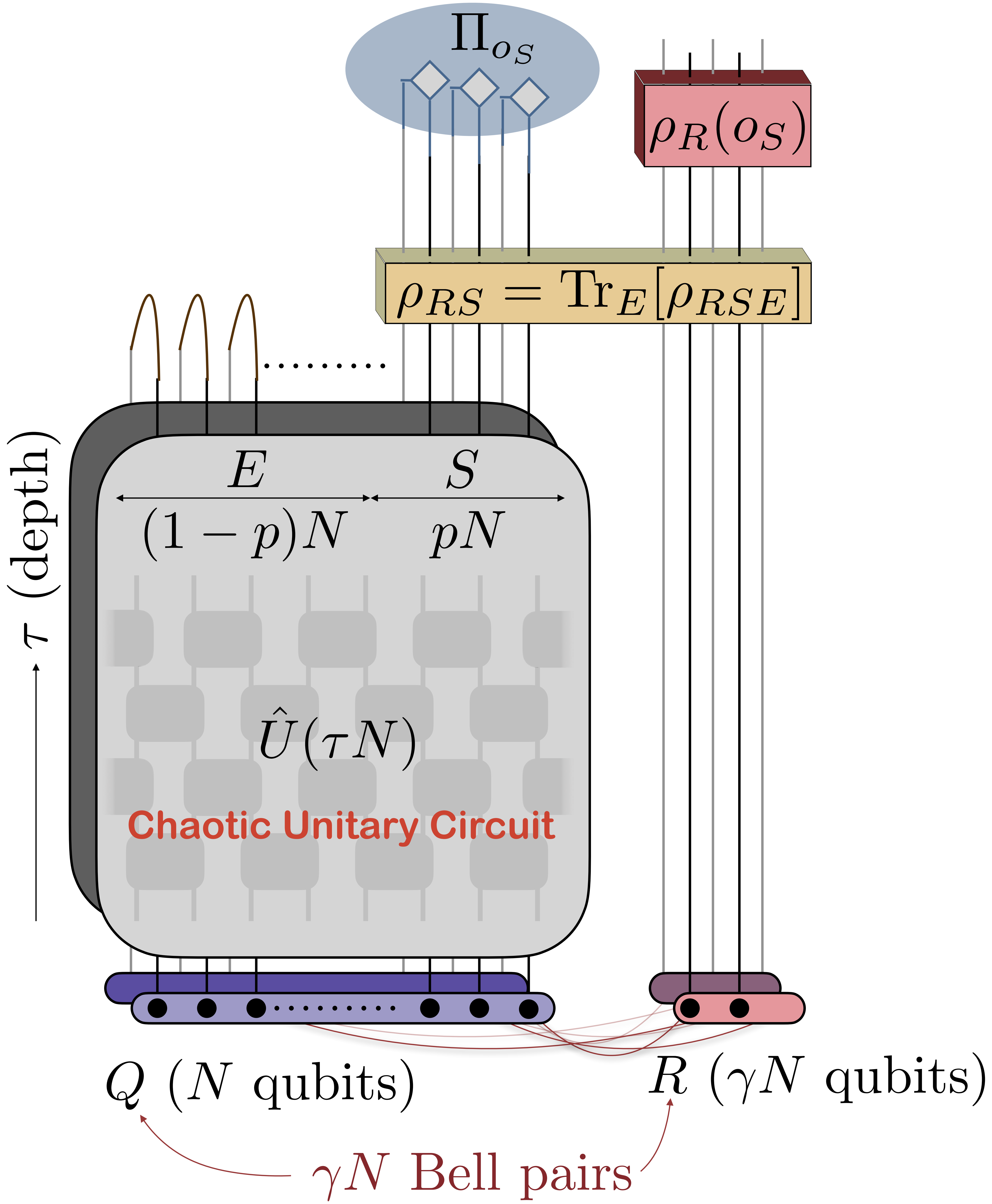
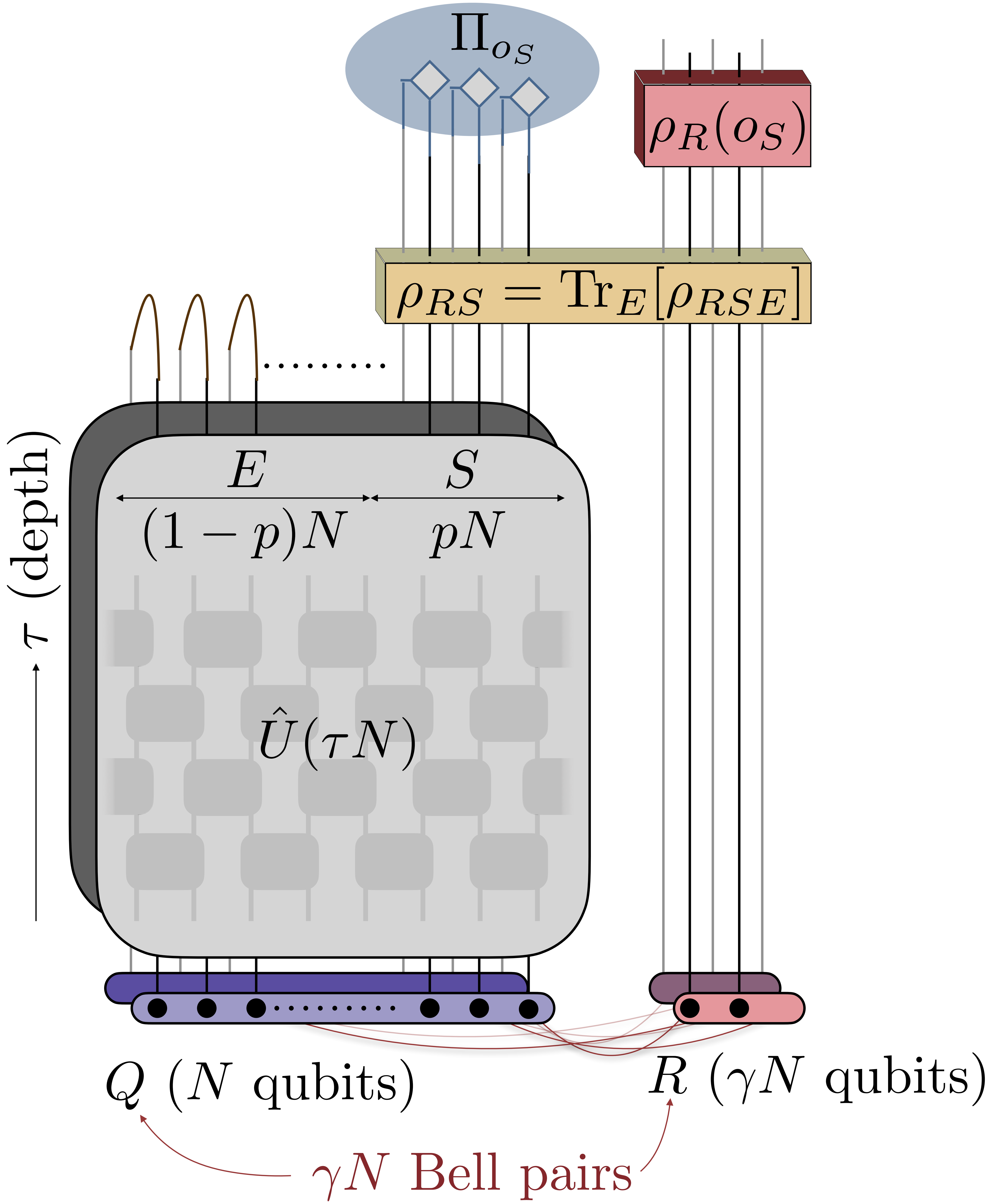
- Entanglement between R and $S \cup E$ remains $\gamma N \ln 2$ at all times
- $\rho_R \propto \mathbb{1}$ at all times



‣ **Are R and S entangled with each other?**

‣ **Is the ensemble $\{p(o_S),\ \rho_R(o_S)\}$ of states in R non-trivial?**



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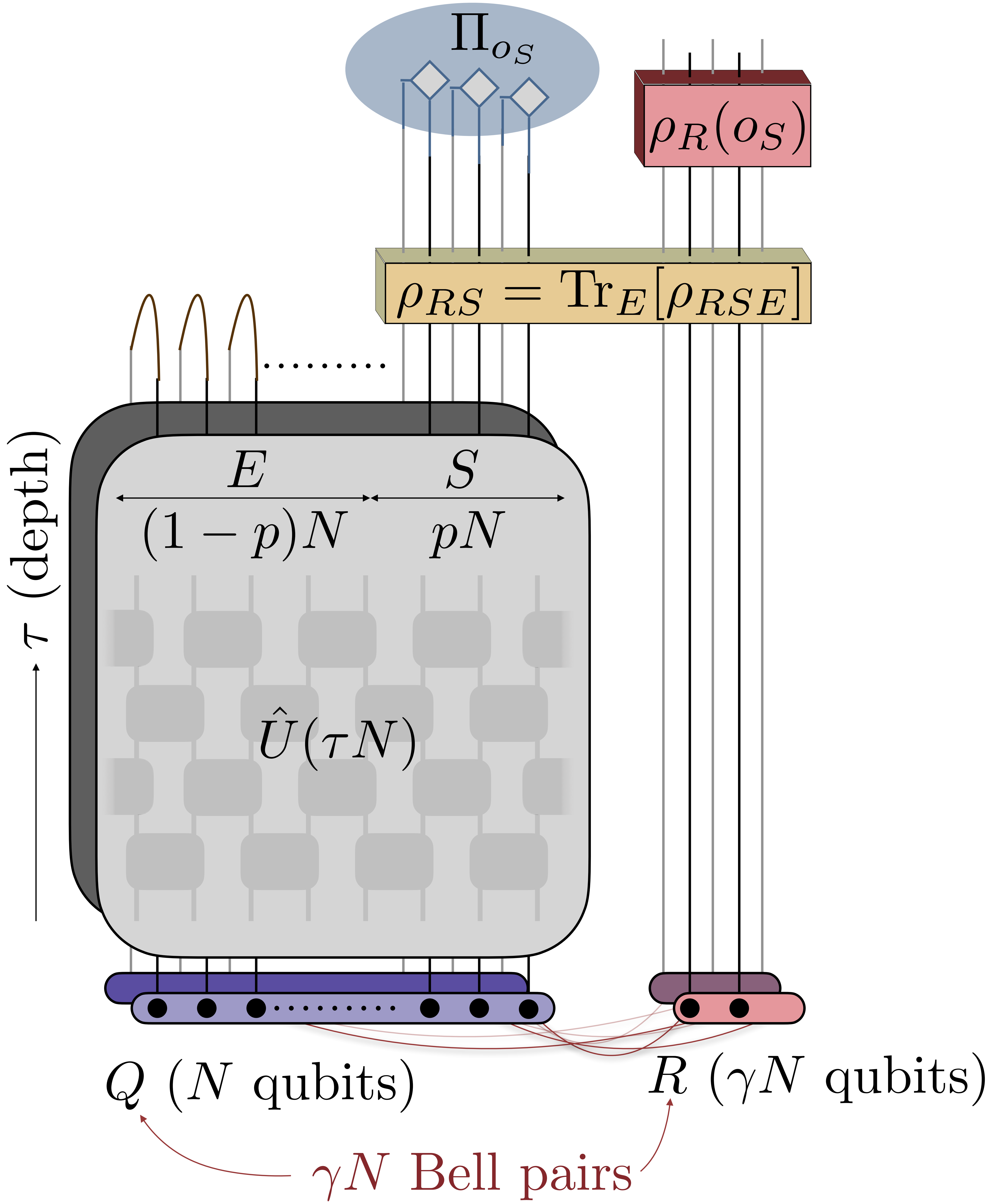
- How do the answers to the above questions depend on the parameters τ, p, and γ in the thermodynamic limit ($N \to \infty$)?
  - phase diagram in terms entanglement structure and response to measurements

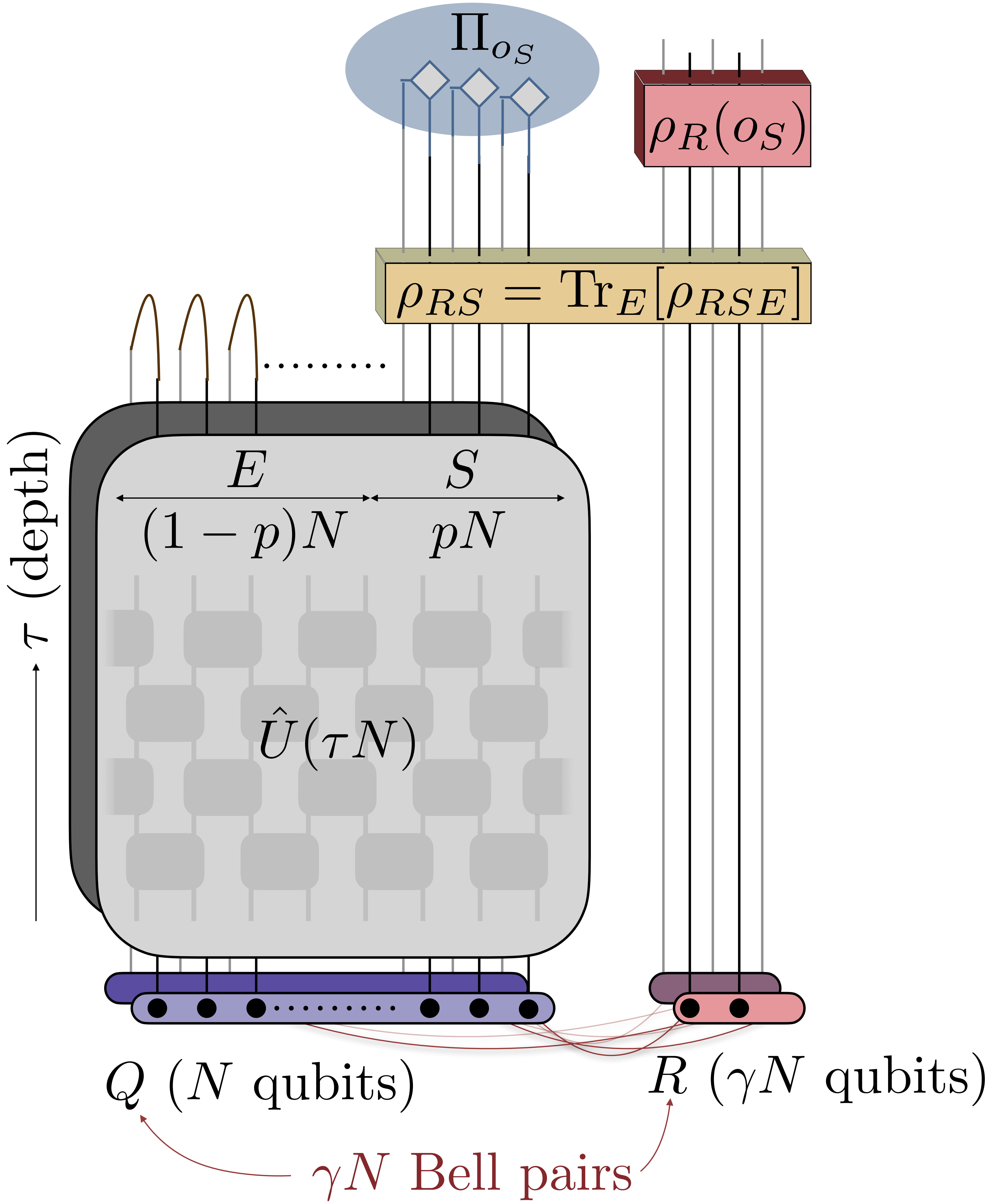


Entanglement between R and S

- Quantify entanglement between R and S via Logarithmic Negativity Vidal+Werner, PRA 2002; Plenio PRL 2005

$$\mathcal{N}_{RS} = \ln \left|\left|\rho_{RS}^{\mathsf{T}_S}\right|\right|_1$$

- Peres-Horodecki criterion:
  - ρ_{RS} is separable $\Rightarrow$ $\mathcal{N}_{RS} = 0$
  - Contrapositive: $\mathcal{N}_{RS} \neq 0 \Rightarrow R$ and S are entangled



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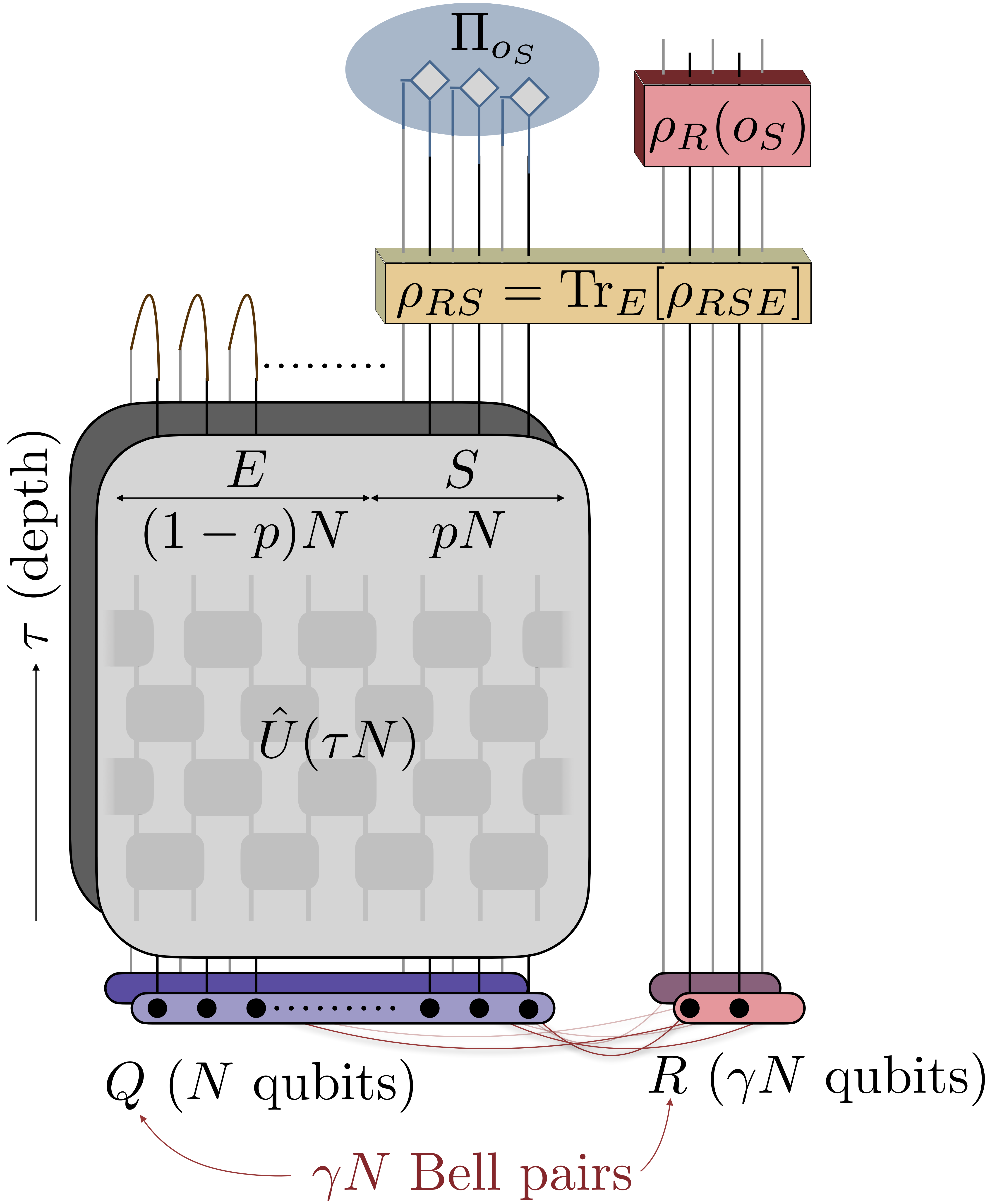
  - Contrapositive: $\mathcal{N}_{RS} \neq 0 \Rightarrow R$ and S are entangled
    - Separability for mixed states: $\rho_{RS} = \sum_i p_i \rho_R^{(i)} \otimes \rho_S^{(i)}$
    - $\mathcal{N}_{RS} = 0$ does not necessarily imply separability; counter example: bound entanglement
    - for our purposes $\mathcal{N}_{RS} = 0$ is a working criterion for being disentangled



Measurement visibility

- **Projected ensemble:** $\{p(o_S),\ \rho_R(o_S)\}$ ensemble of states in R conditioned on measurement outcomes in S

Goldstein et al. J. Stat. Phys. 2006, Cotler et al. PRXQ 2023, Mark et al. PRX 2024

- If $\rho_R(o_S)$ is agnostic to o_S then we will have $\rho_R(o_S) = \rho_R \ \forall \ o_S$

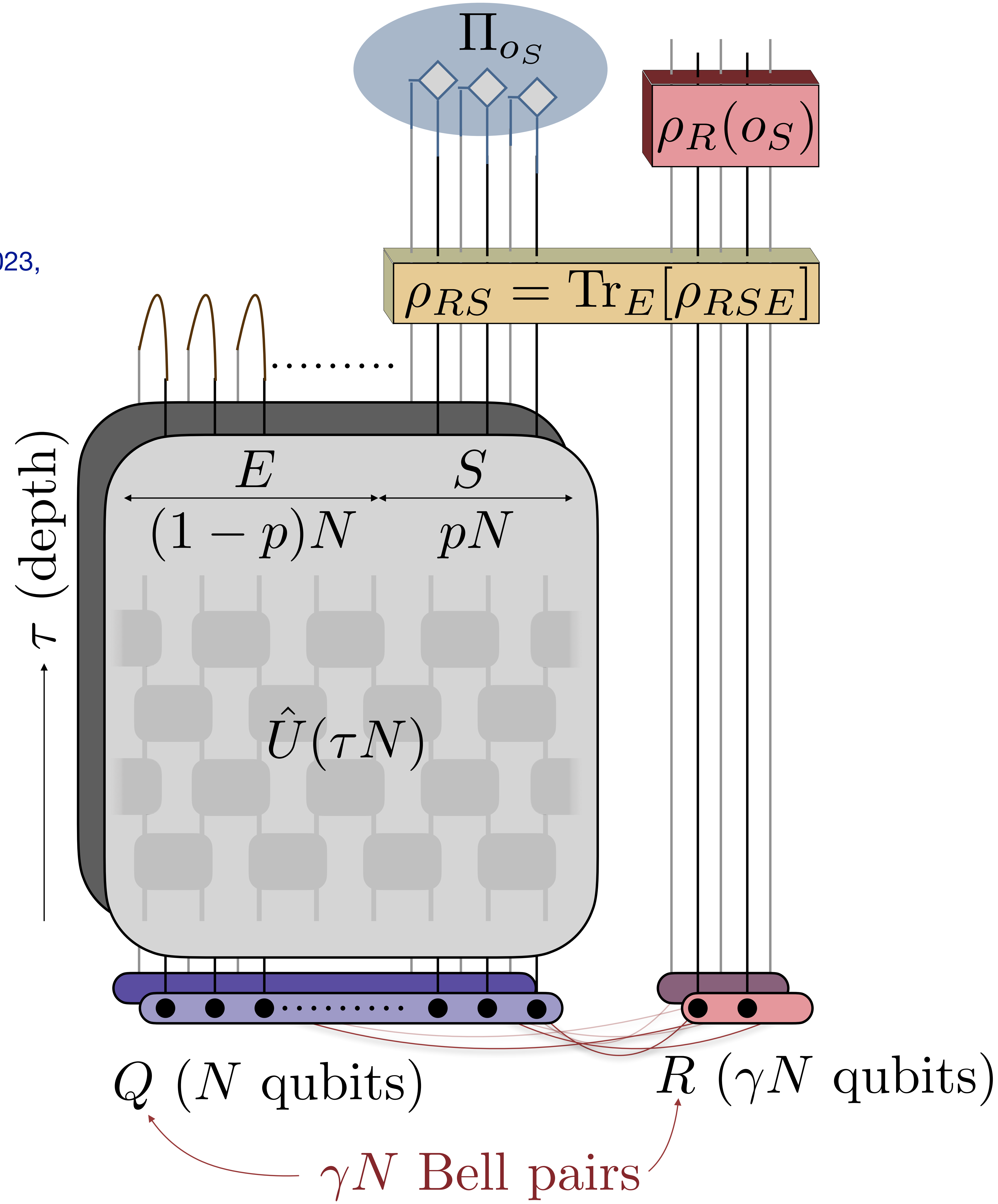
- Natural measure for measurement-visibility is therefore the spread of the ensemble over o_S

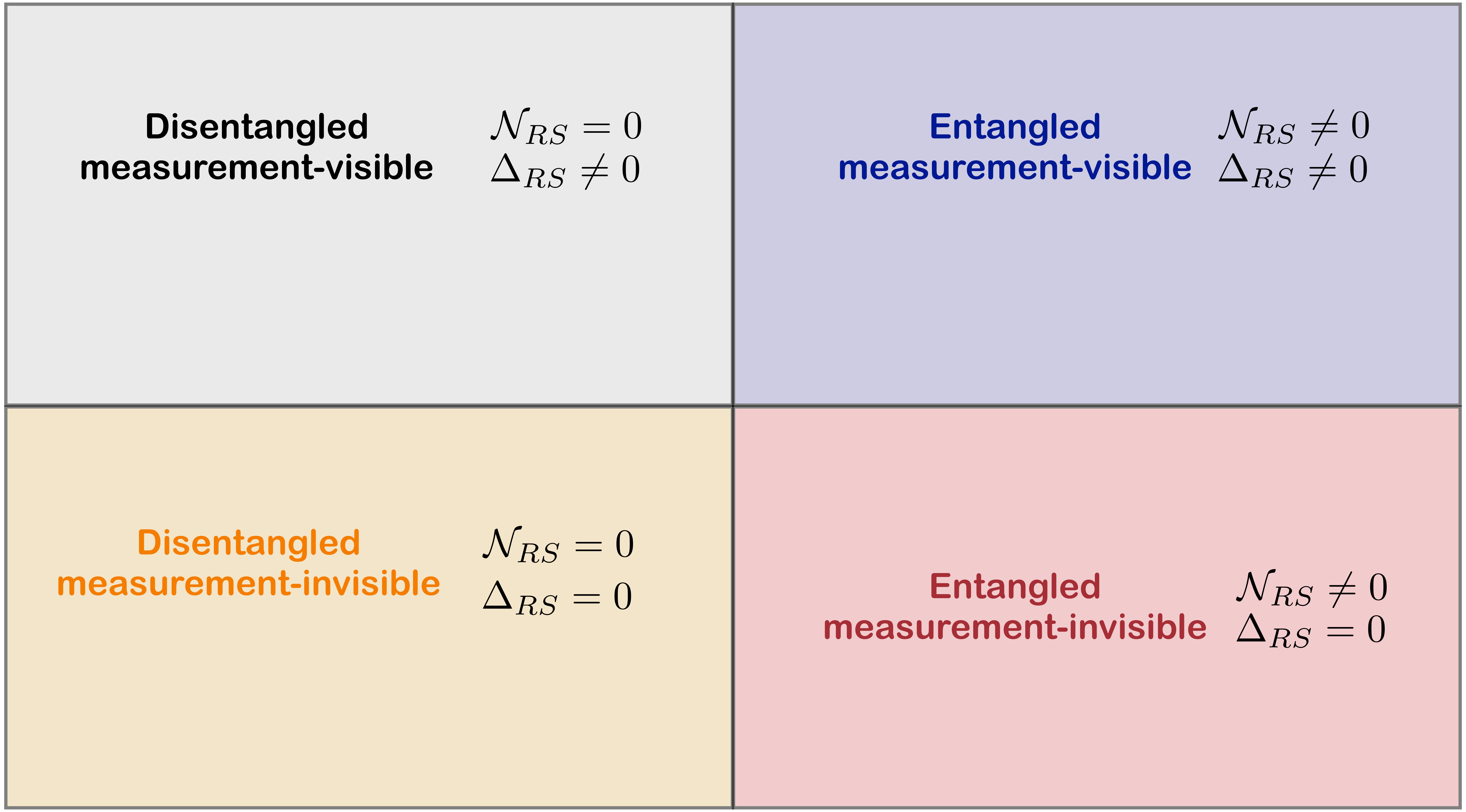
- Quantified by

$$\Delta_{RS} = \sum_{o_S} p(o_S)\, ||\rho_R(o_S) - \rho_R||_1$$

- Measurement-invisibility

$$\Delta_{RS} = 0 \ \Leftrightarrow \rho_R(o_S) = \rho_R \ \forall \ o_S$$

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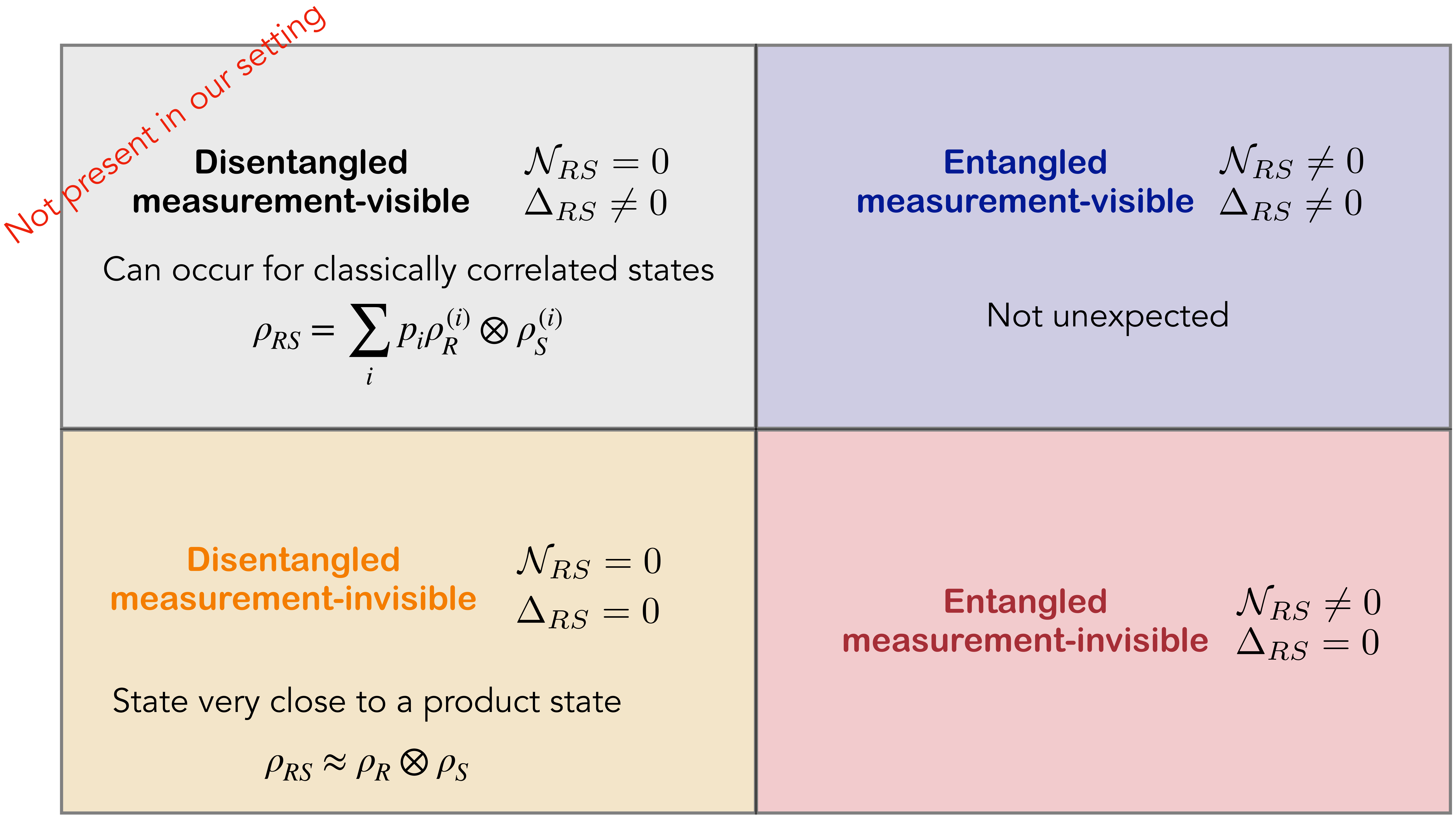
Not present in our setting
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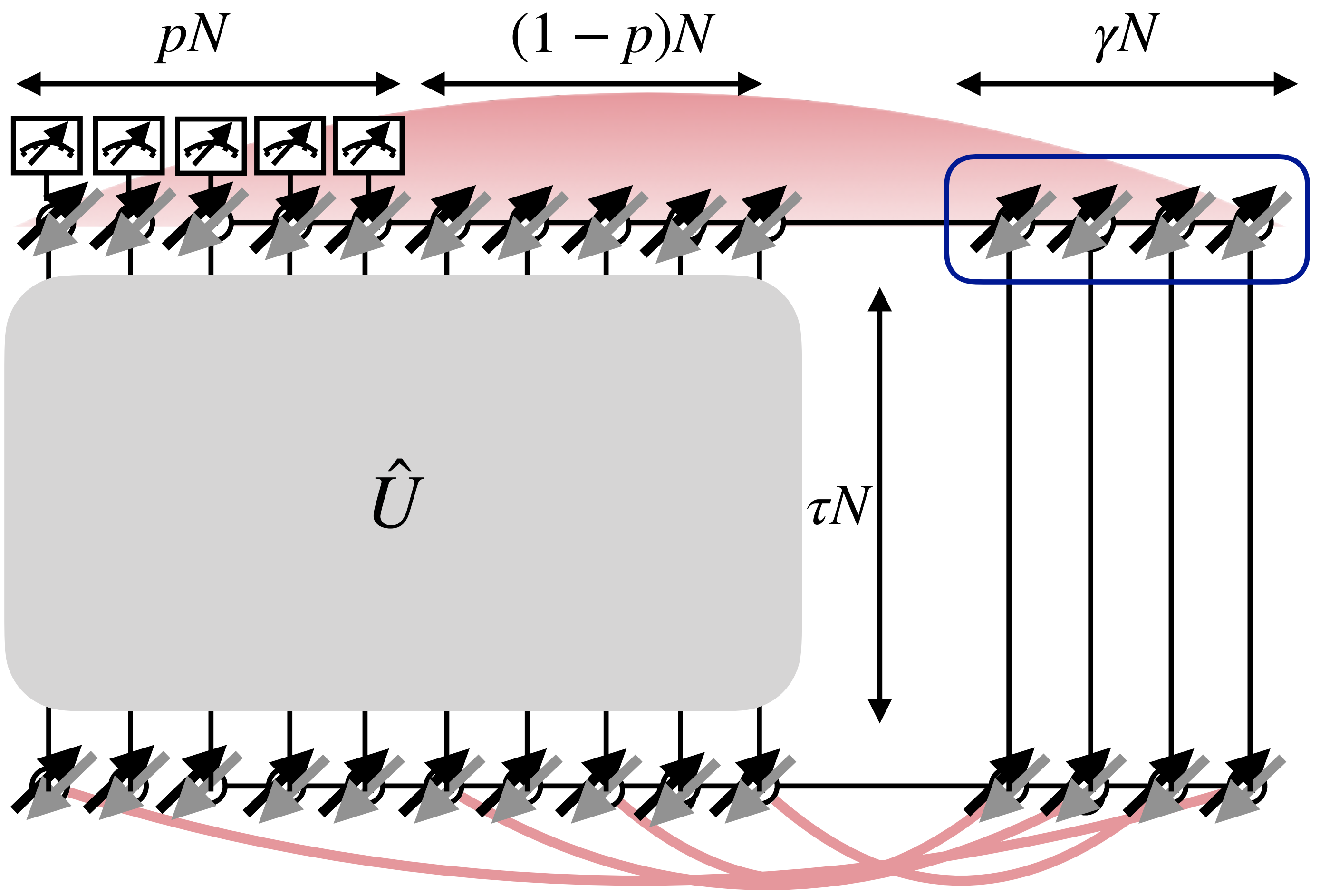
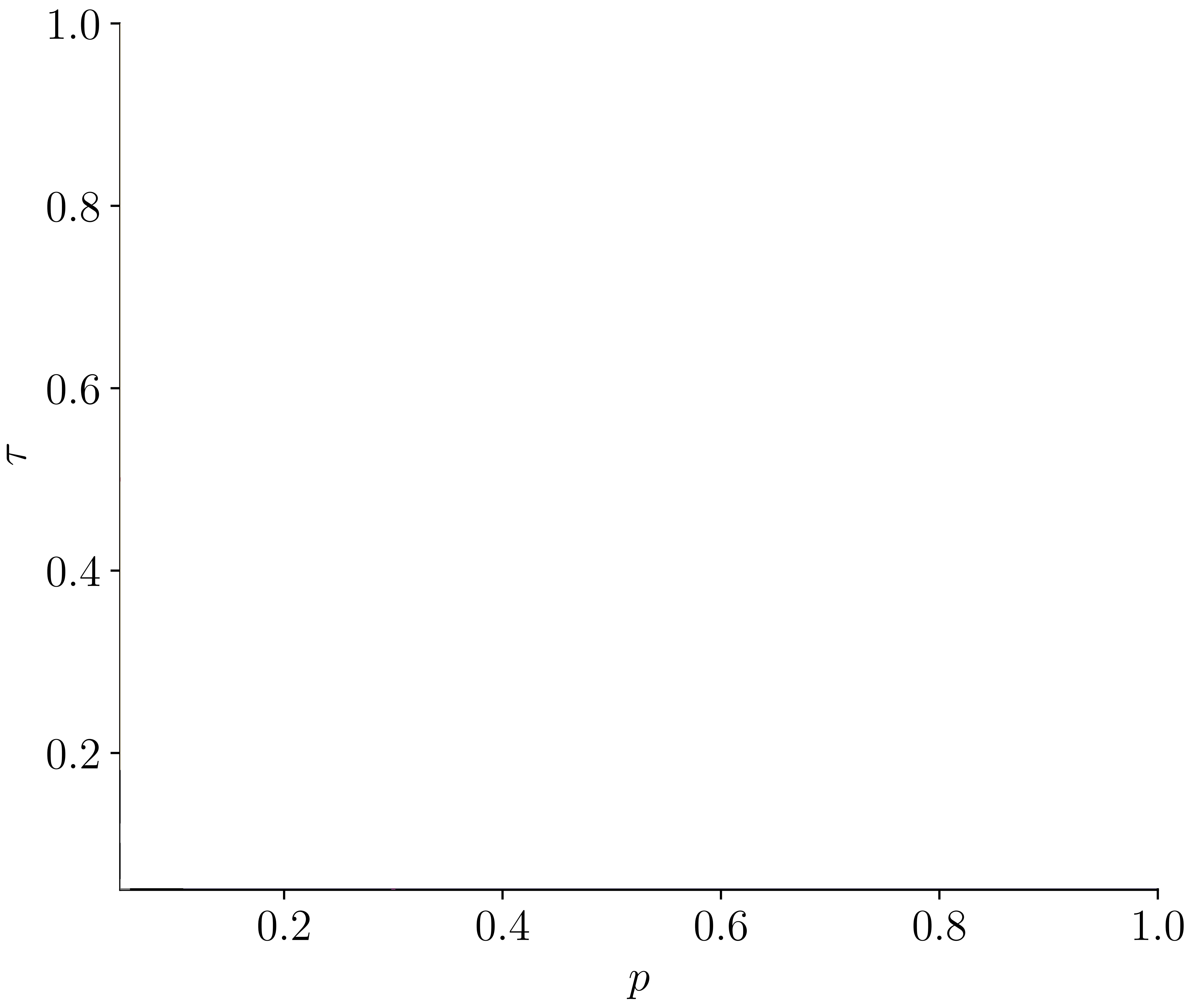
$\mathcal{N}_{RS} \neq 0$
$\Delta_{RS} = 0$

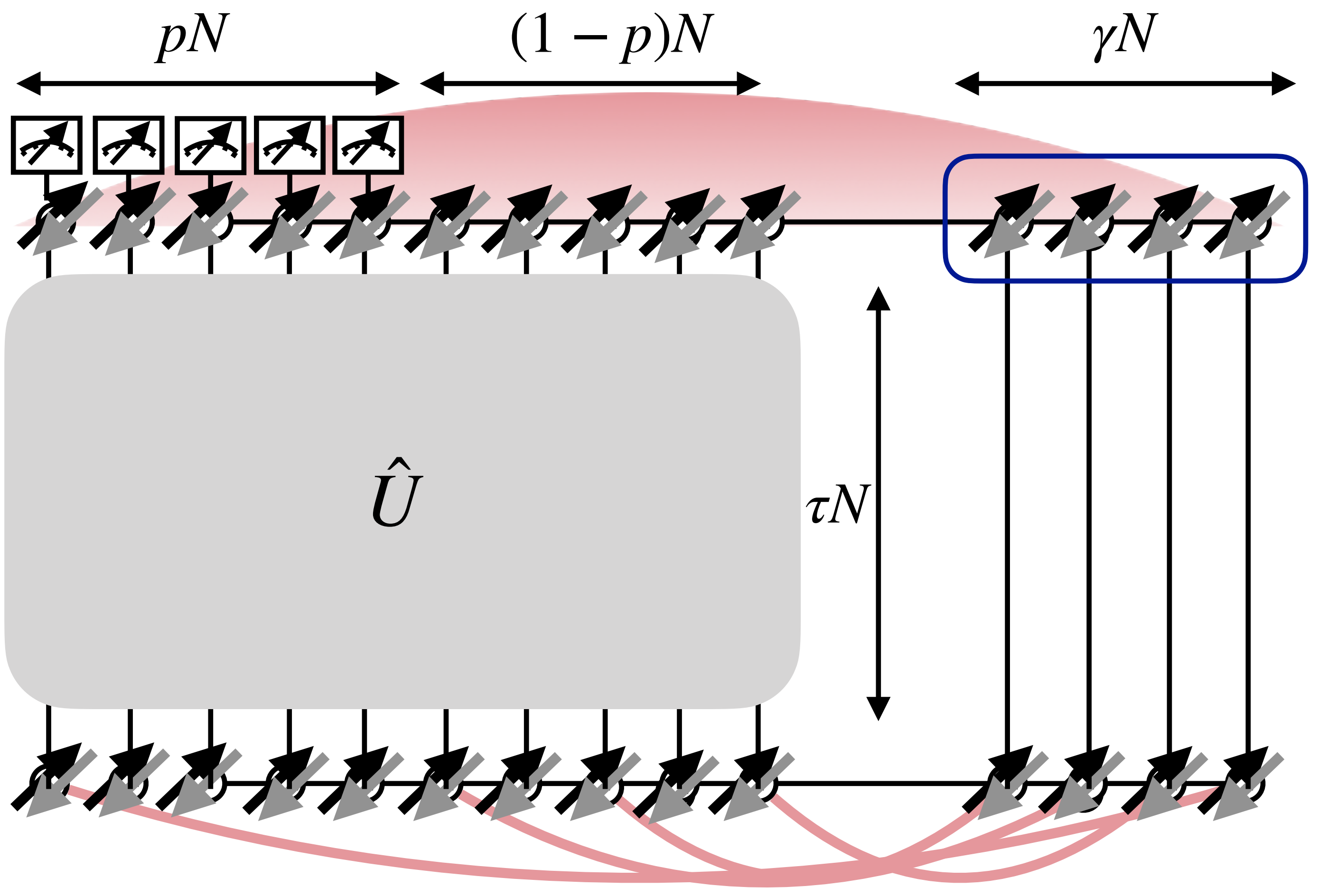
Not present in our setting **Disentangled measurement-visible** $\mathcal{N}_{RS} = 0$, $\Delta_{RS} \neq 0$ Can occur for classically correlated states $\rho_{RS} = \sum_i p_i \rho_R^{(i)} \otimes \rho_S^{(i)}$	**Entangled measurement-visible** $\mathcal{N}_{RS} \neq 0$, $\Delta_{RS} \neq 0$ Not unexpected
Disentangled measurement-invisible $\mathcal{N}_{RS} = 0$, $\Delta_{RS} = 0$ State very close to a product state $\rho_{RS} \approx \rho_R \otimes \rho_S$	**Entangled measurement-invisible** $\mathcal{N}_{RS} \neq 0$, $\Delta_{RS} = 0$

Phase diagram



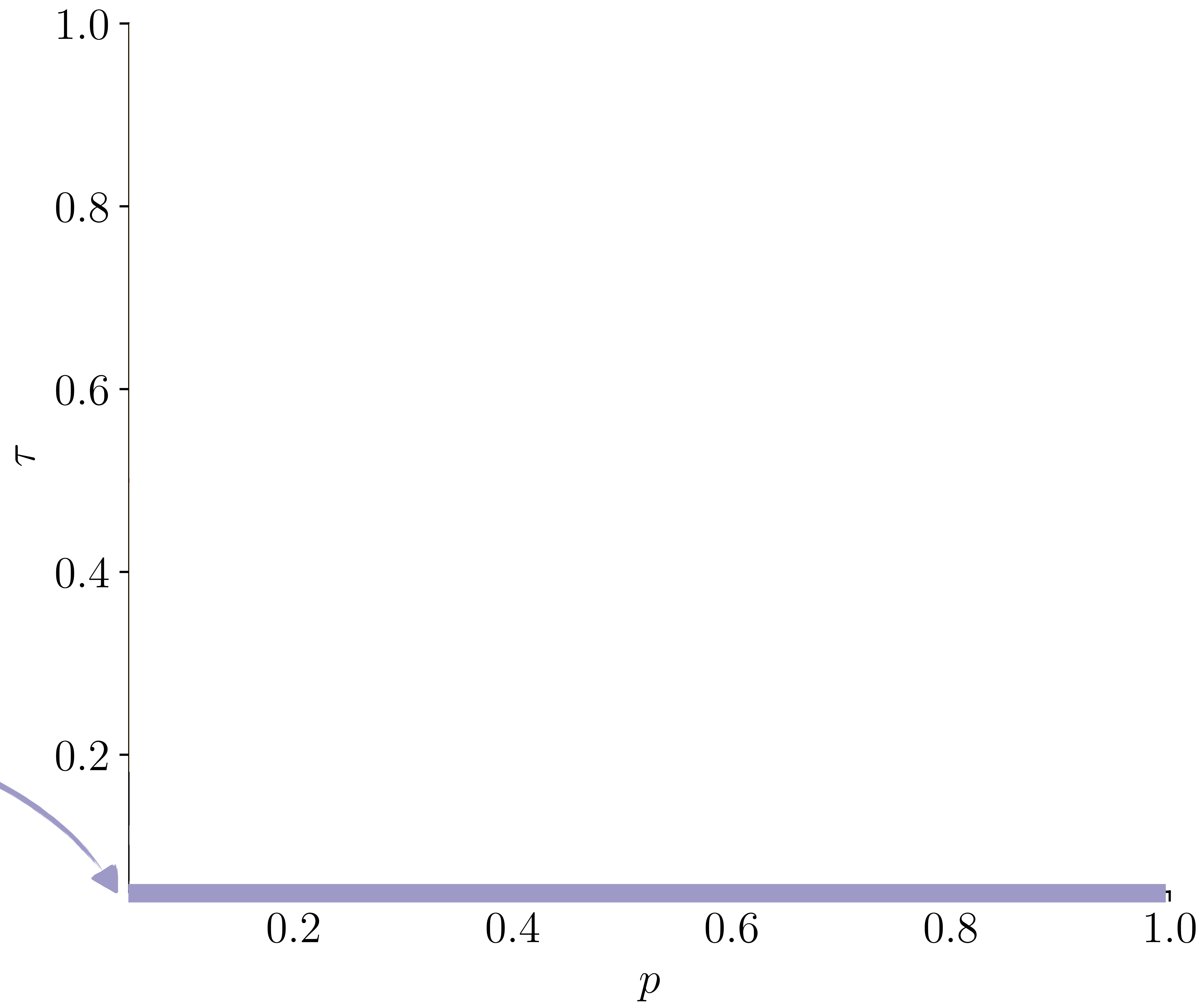


Phase diagram

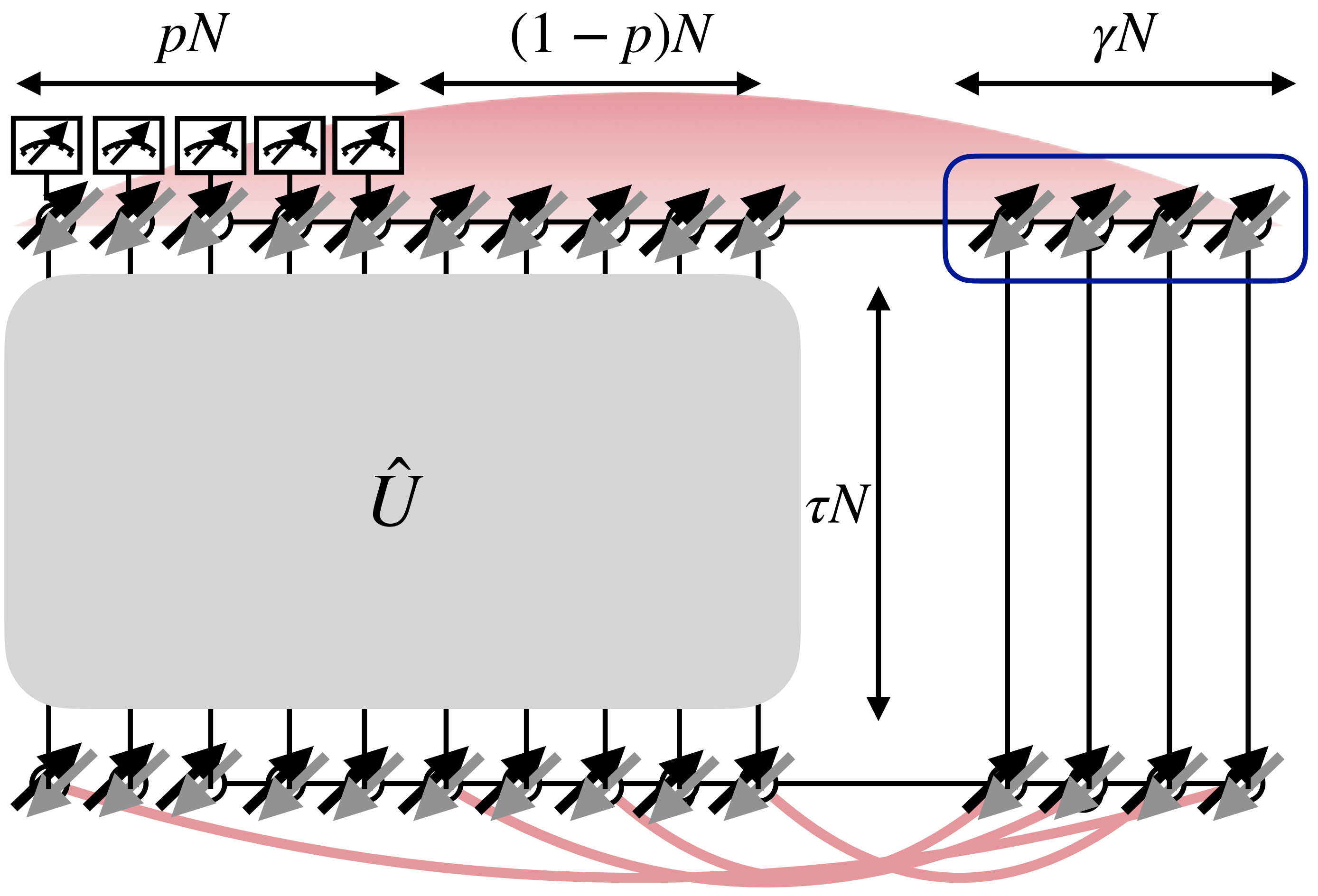


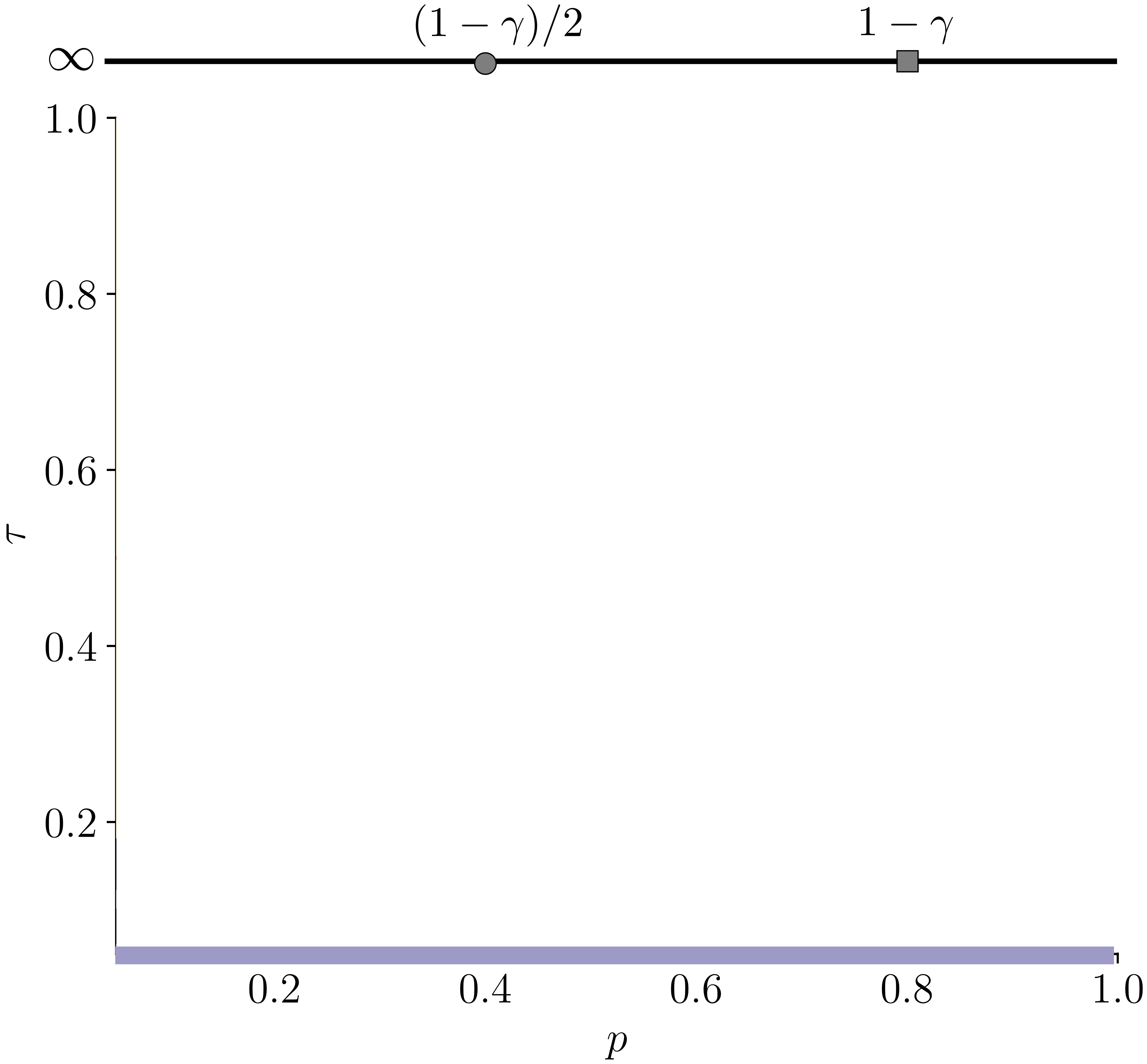
$\tau = 0$: **Entangled, measurement-visible**

- Finite fraction of measured sites necessarily form Bell pairs with qubits in R
- State of R necessarily sensitive to the measurement outcome
- R and S entangled by construction

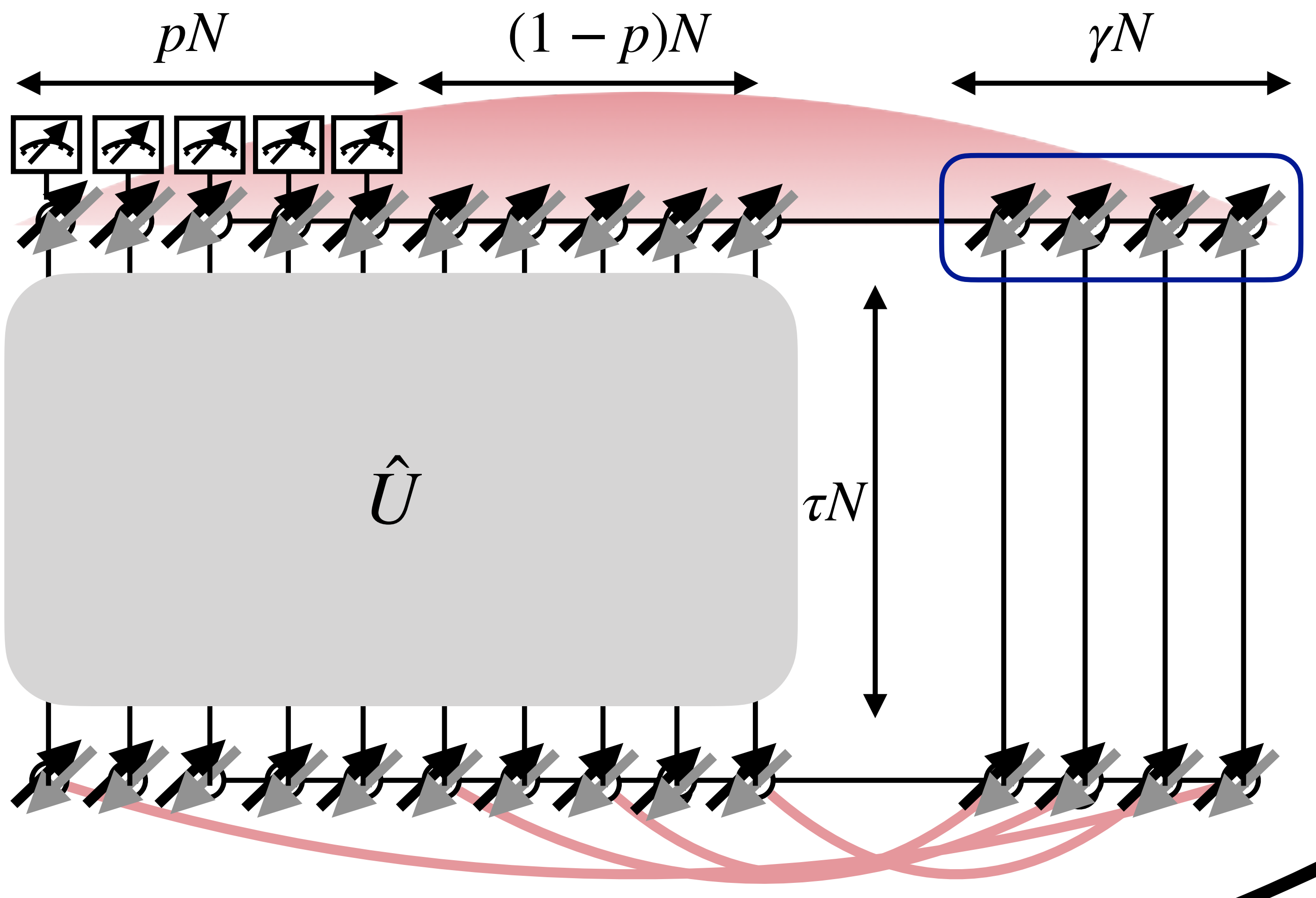


Phase diagram



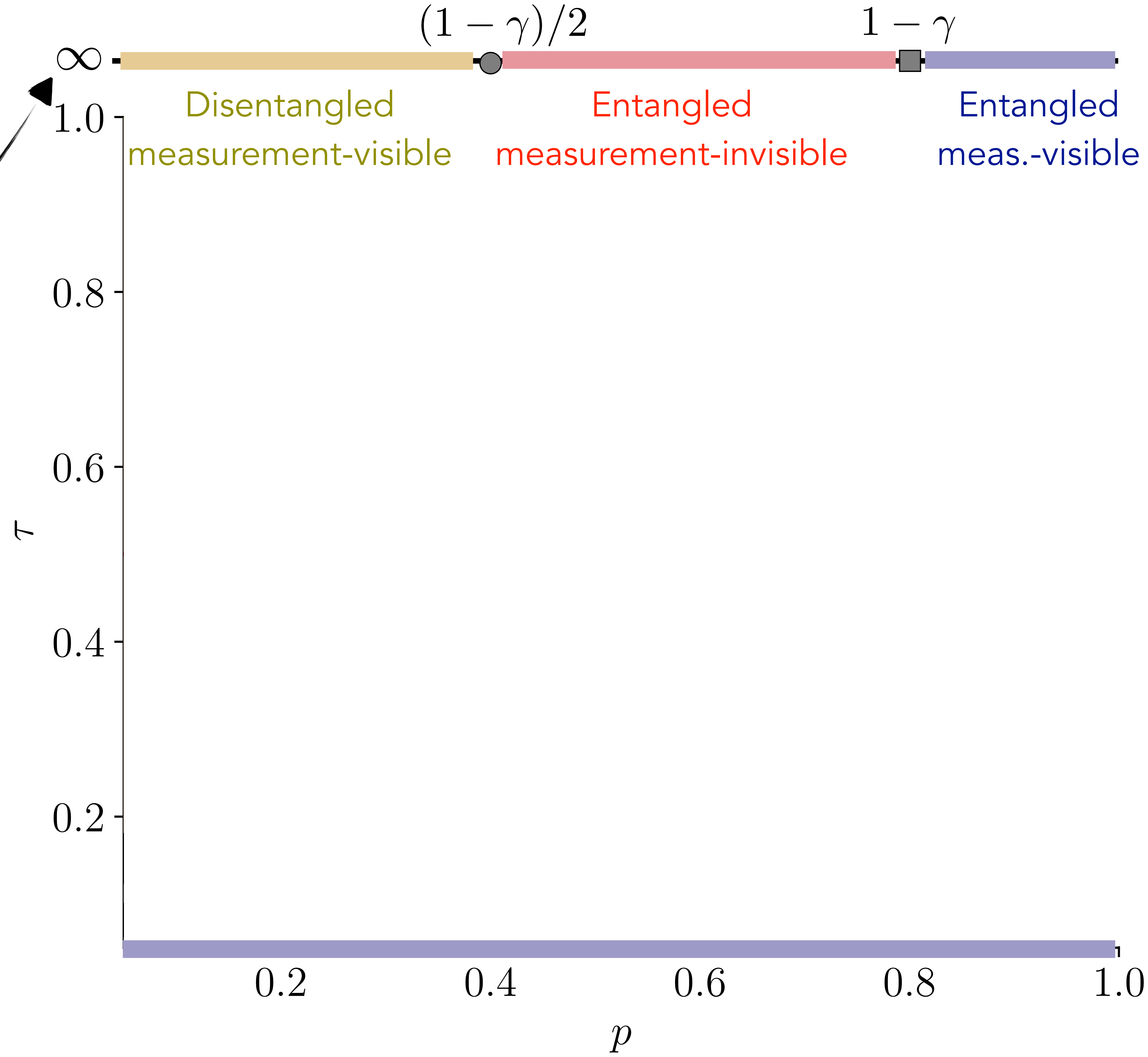


Phase diagram

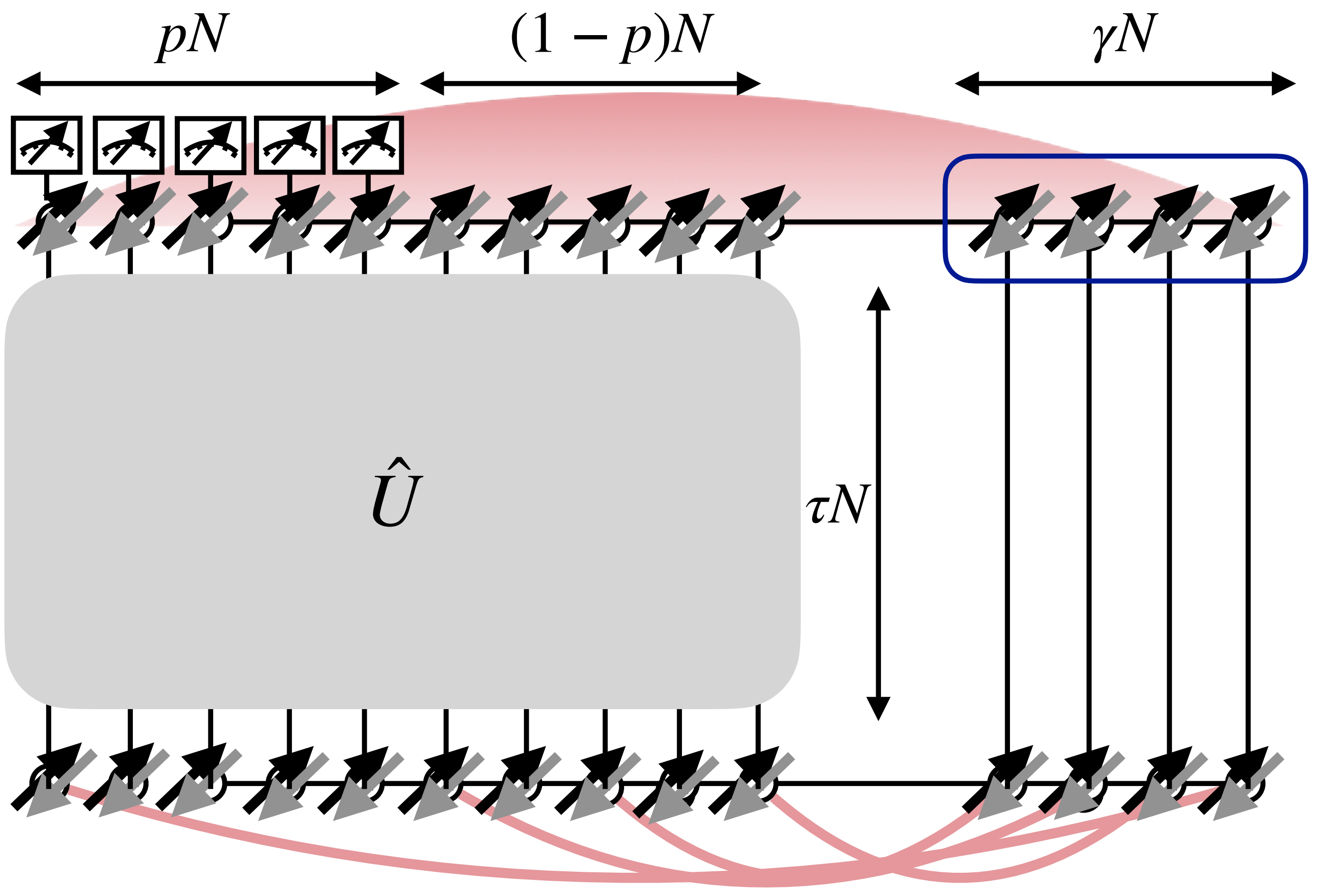


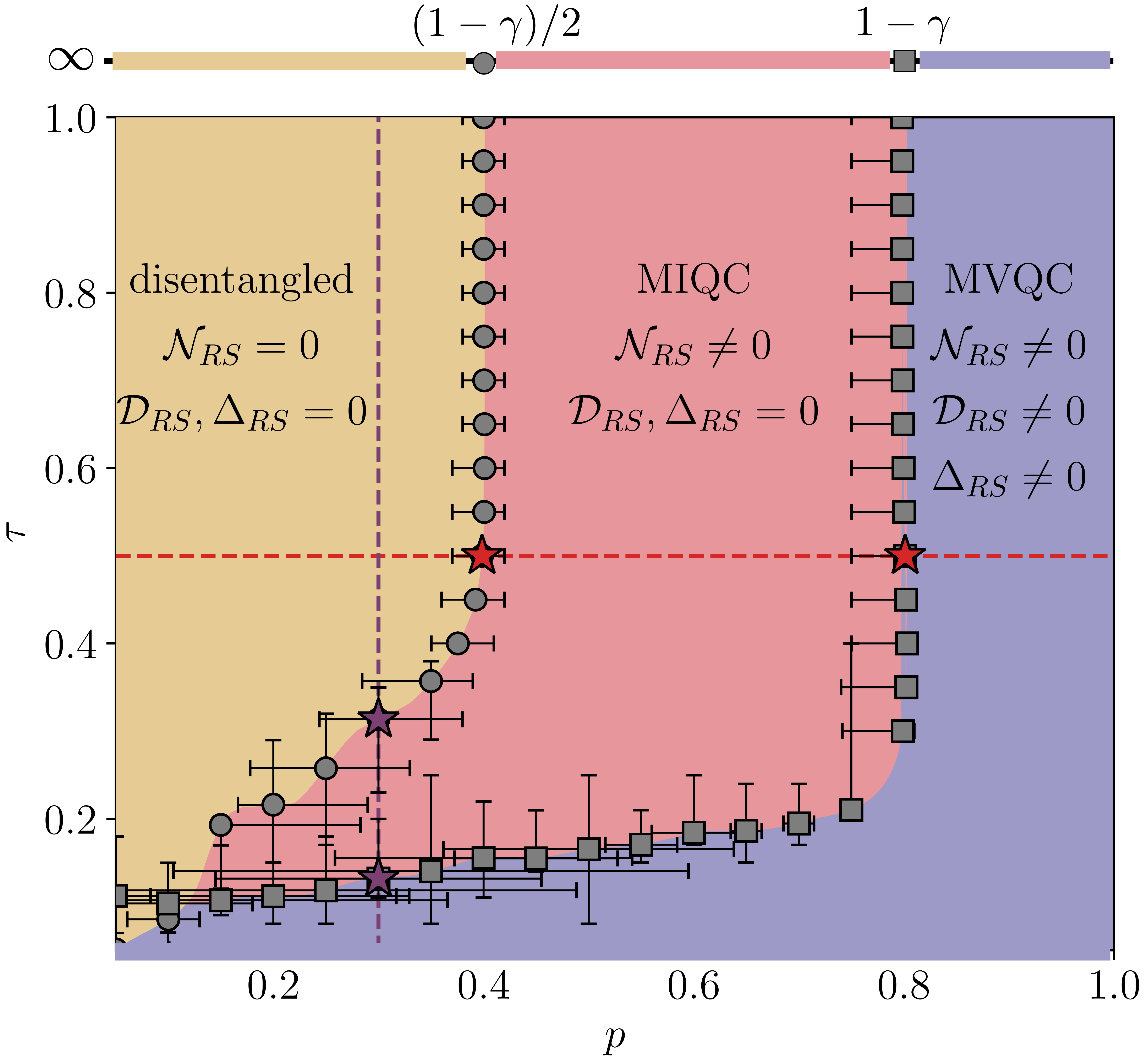
$\tau = \infty$: two transitions

- For $\tau = \infty$, $\hat{U}$ is well approximated by a Haar-random unitary, analytically tractable
- The entanglement/quantum correlations are genuinely multipartite — shared across an extensively large number of qubits

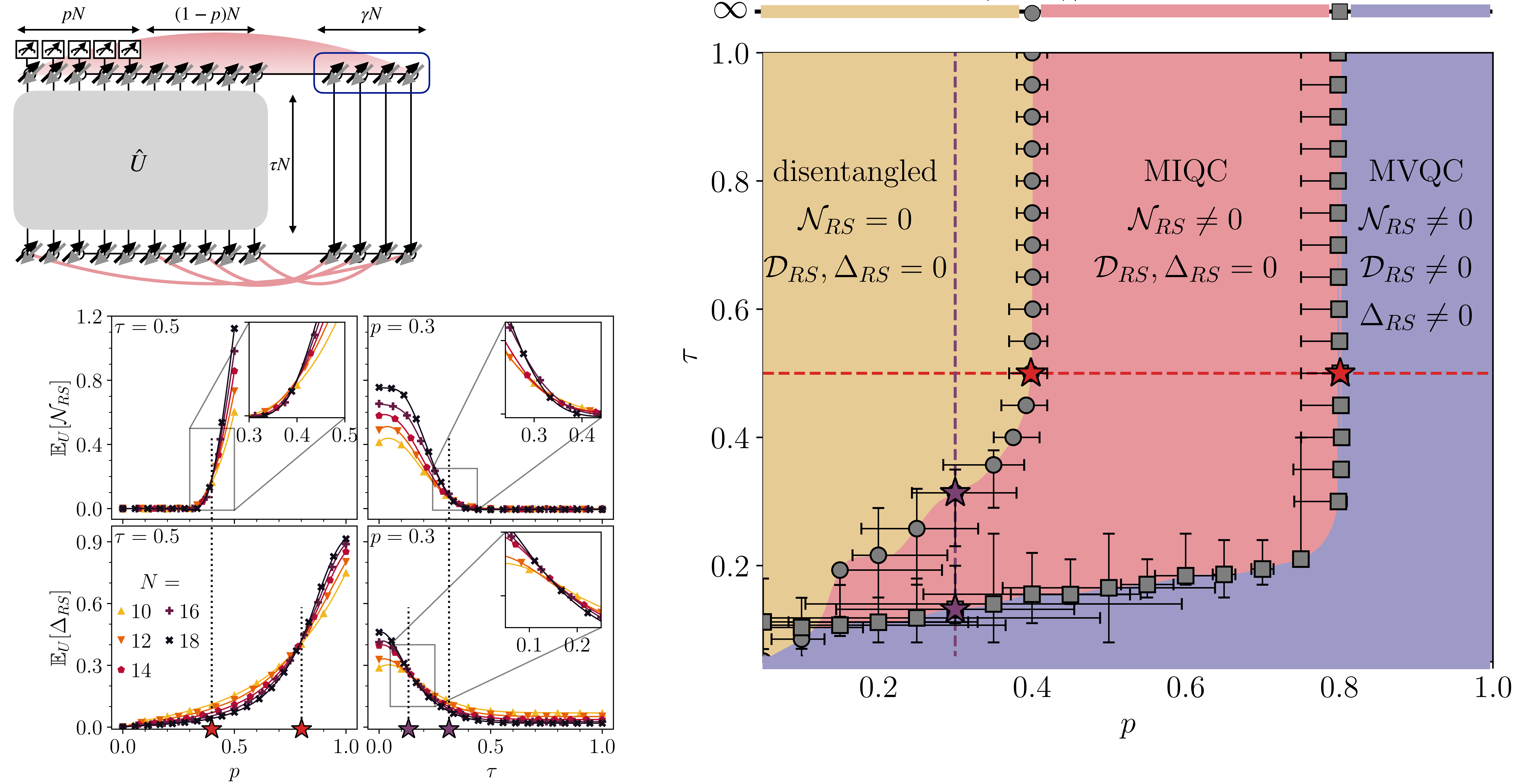


Phase diagram

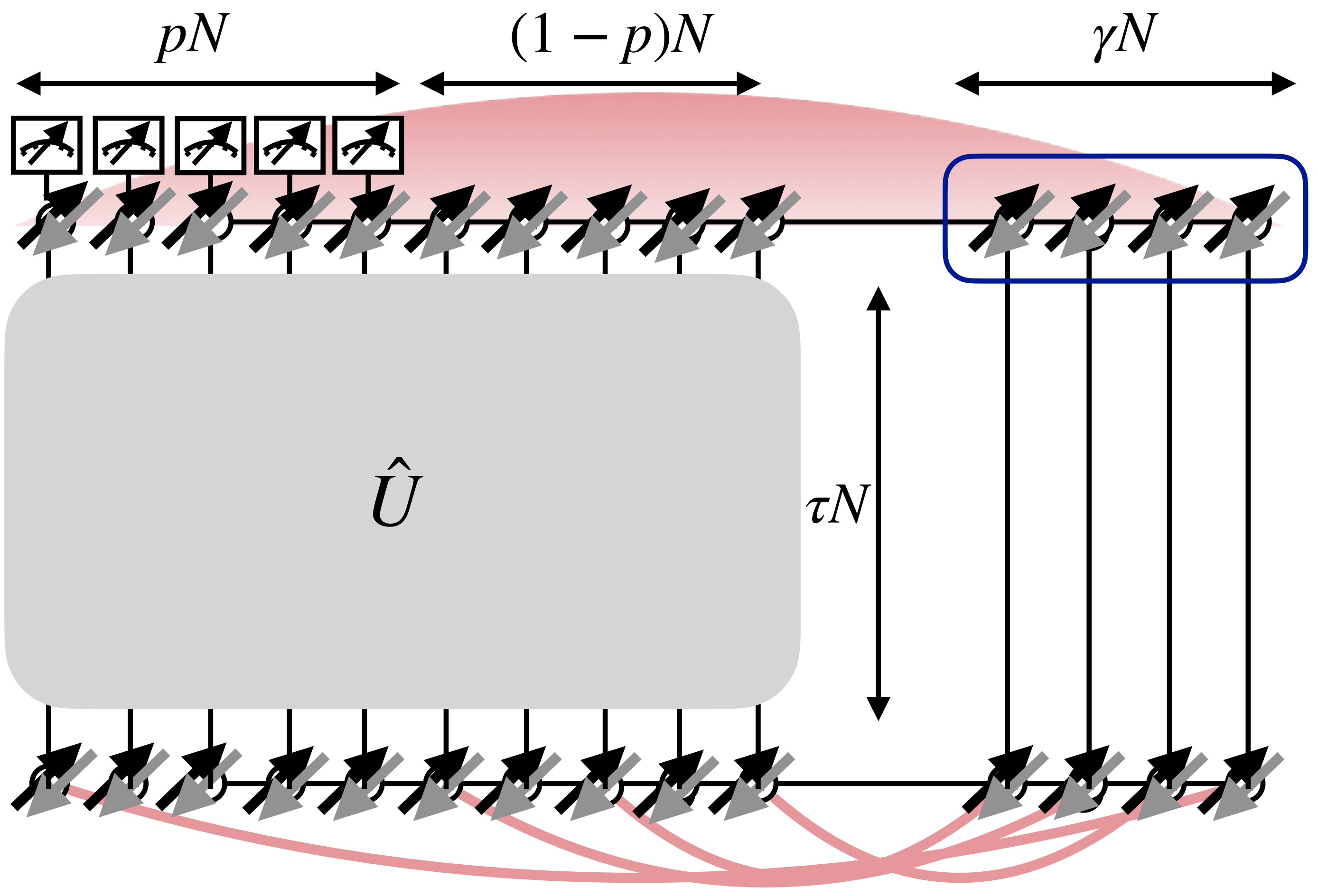




Phase diagram

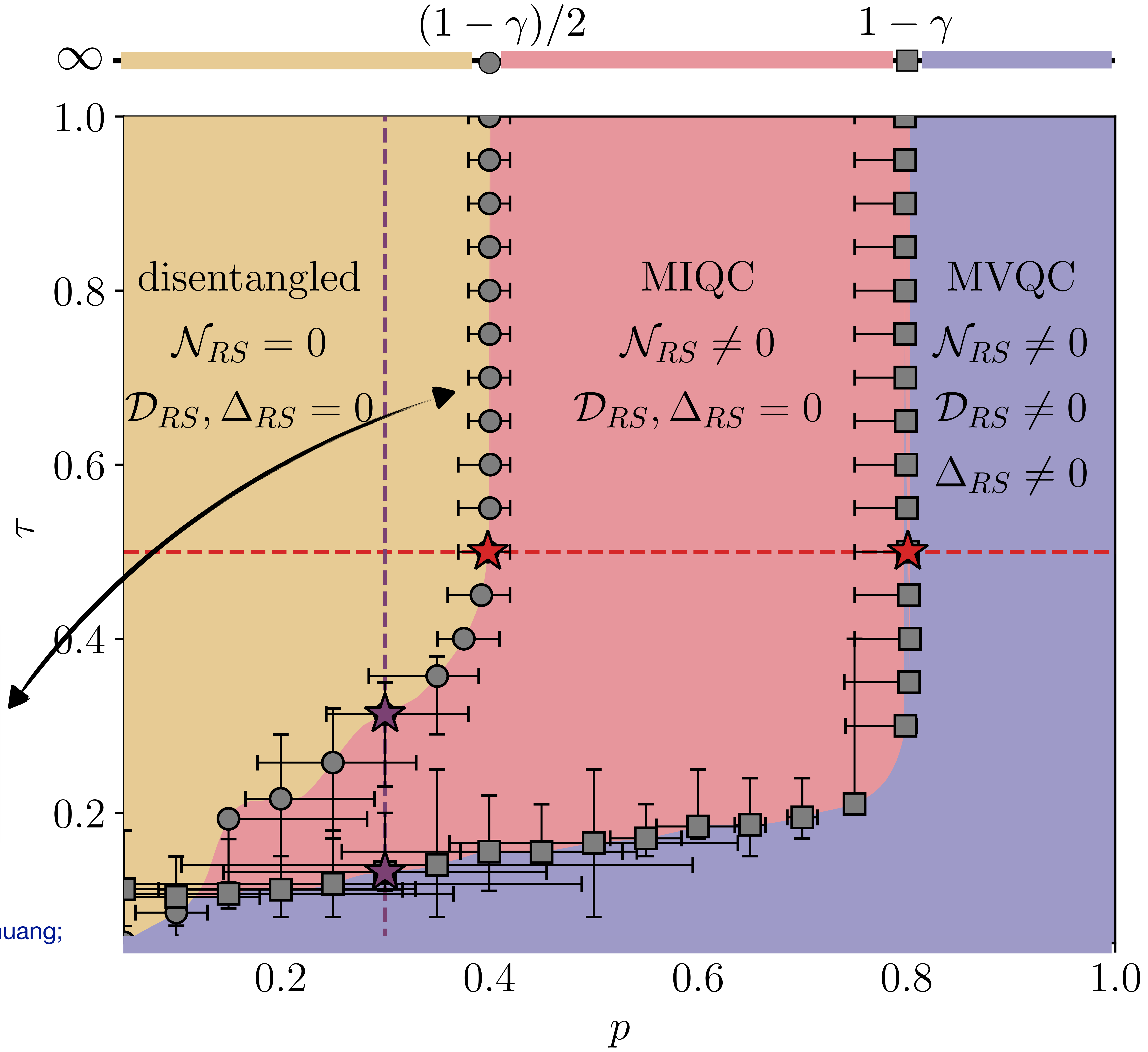


Phase diagram

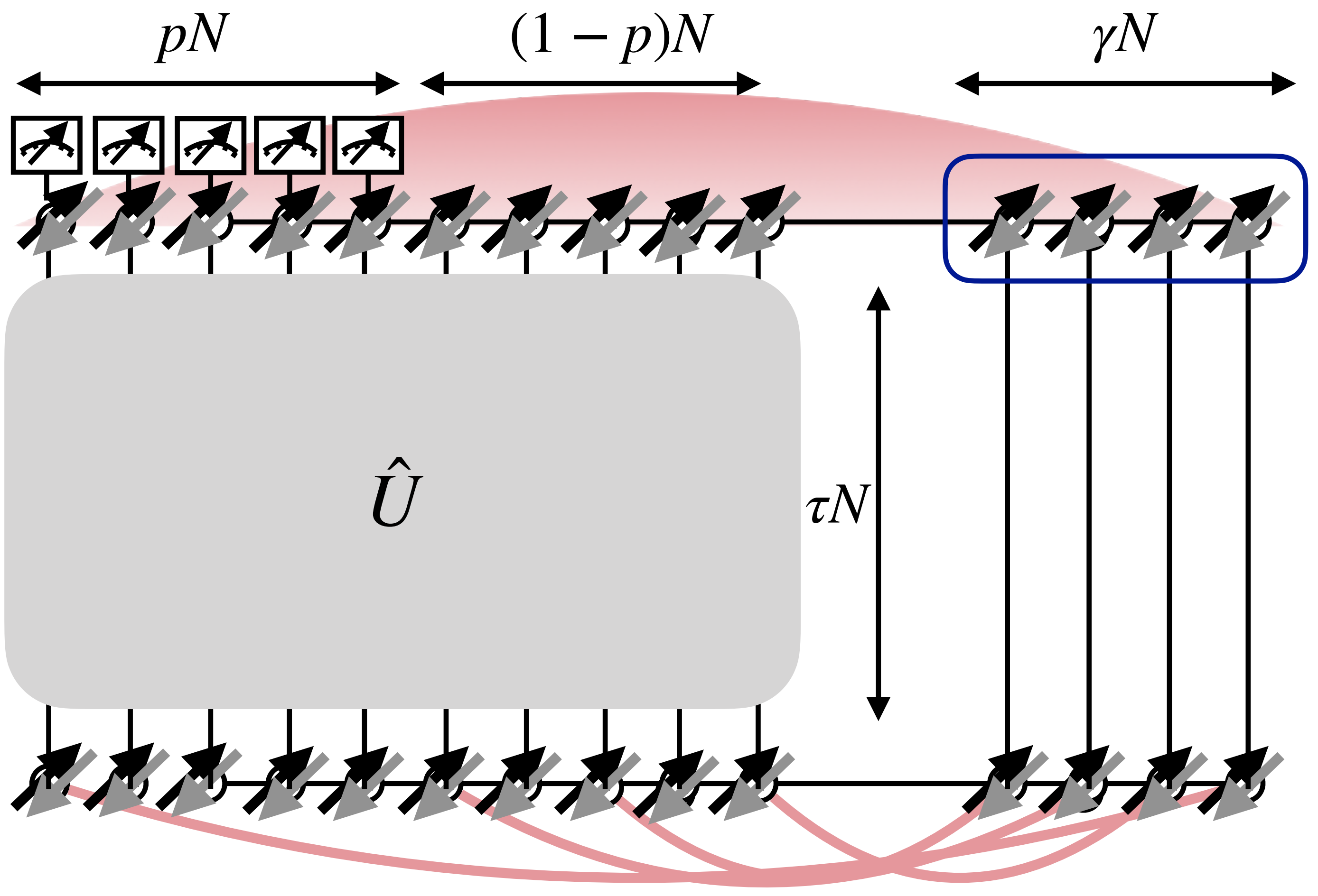


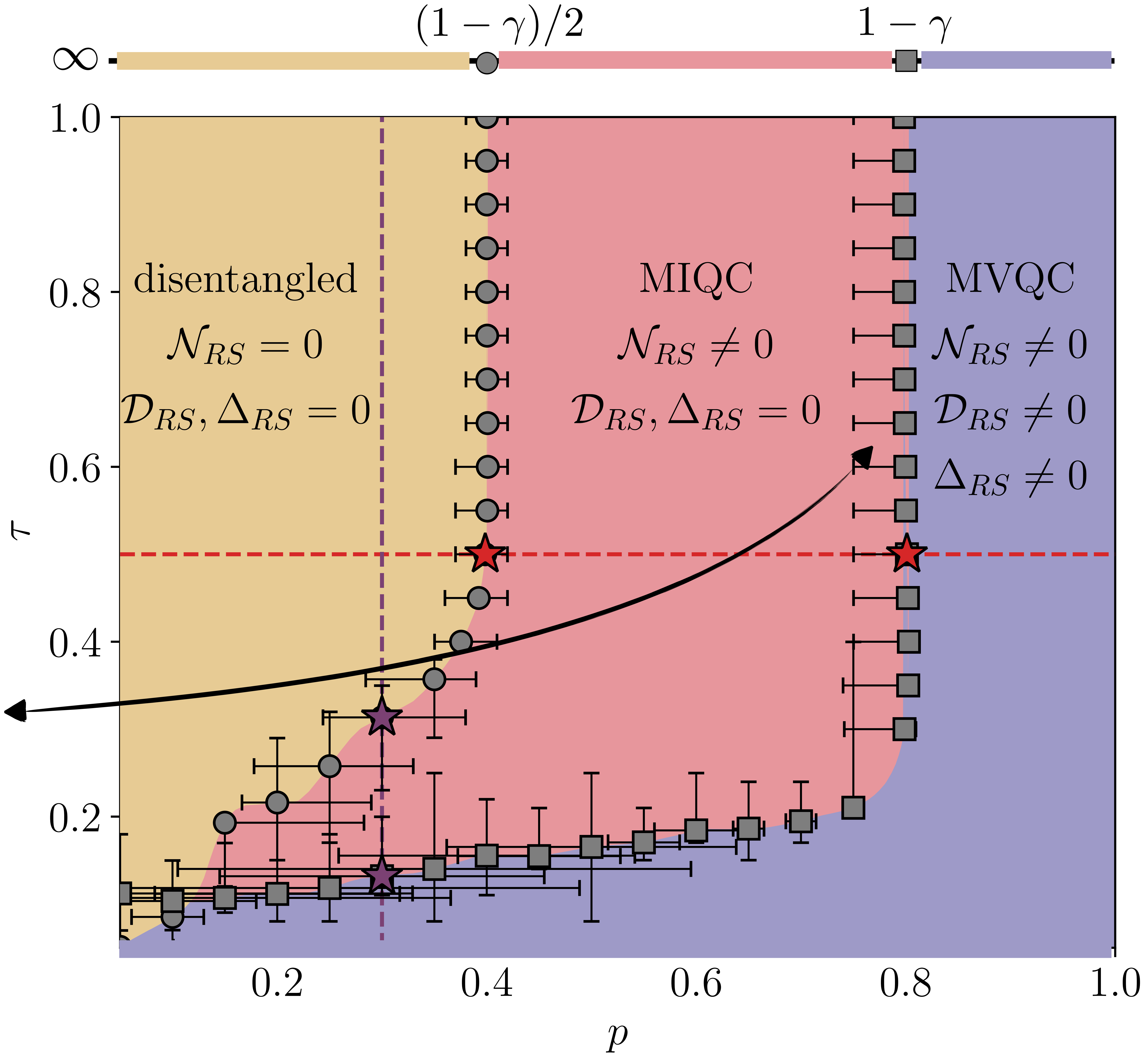
- **The entanglement transition = decoupling transition**
- Tracing out a sufficiently large number of qubits annihilates all quantum correlations between R and S
- Not enough remaining qubits to mediate the long-ranged, multipartite entanglement between R and S

See also, textbook by Nielsen+Chuang; Choi et al PRL 2020



Phase diagram





- **Measurement-invisibility transition**
- Number of qubits traced out NOT large enough to kill entanglement between R and S
- Yet the multipartite nature of the entanglement means it is not visible to 1-local measurements on S

Implication # 1

A stronger version of the steering statement for mixed states

Schrödinger, Mat. Proc. Cam. Phil. Soc.,1935, 1936;
Wiseman et al., PRL 2007

Quantum Steering

- Alice does measurements on her part of the system and classically communicates the result to Bob
- Bob can do full tomography on his part of the state conditioned on measurement outcomes of Alice
- Alice needs to convince Bob that the state is quantum entangled (and Alice wasn't using prior knowledge of Bob's partial states)
- All bipartite entangled pure states are necessarily steerable

Gisin, Phys. Lett. A 1991,1992;
Popescu+Rohrlich, Phys. Lett. A 1992

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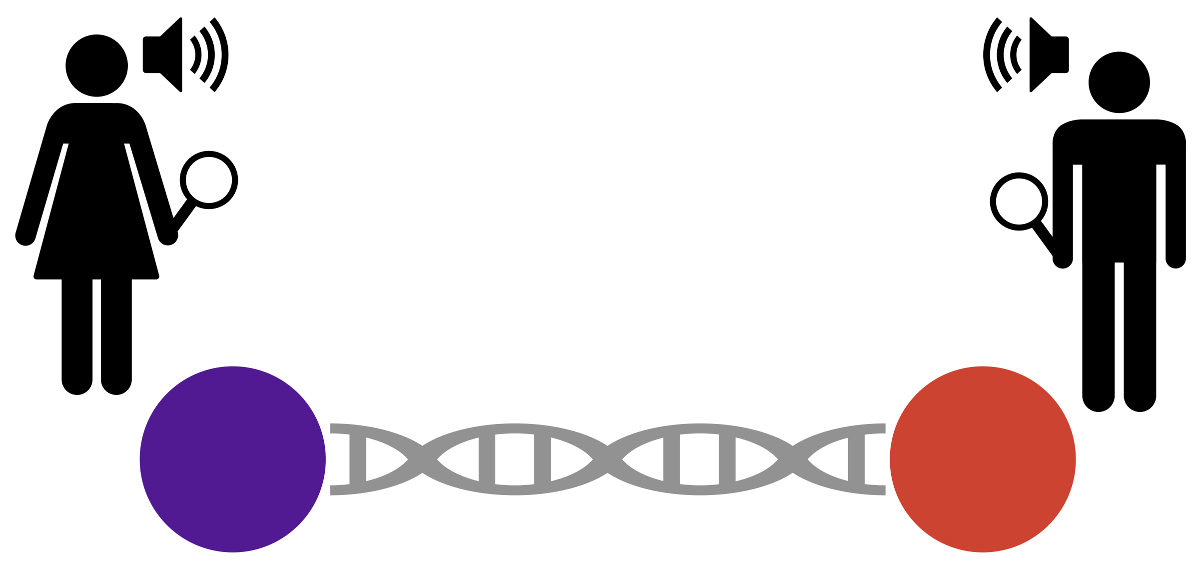
Meas. Invisible Quantum Correlated phase

- State of Bob independent of Alice's measurement outcome
- State appears uncorrelated to Bob and therefore Alice fails in her task
- Alice fails to convince Bob not only about entanglement but also quantum correlations
- Entangled and yet 'unsteerable' — possible as the state is mixed

Implication # 2

Factorisation of probabilities of bitstring probabilities



- Alice measures observable $\hat{O}_A$ with outcome o_A
- Bob measures observable $\hat{O}_B$ with outcome o_B
- Joint distribution of the measurement outcomes
  $P(o_A, o_B) = \mathrm{Tr}[(\Pi_{o_A} \otimes \Pi_{o_B})\rho]$
- Conditional probability $P(o_A \,|\, o_B) = \mathrm{Tr}[\rho_A(o_B)\Pi_{o_A}]$

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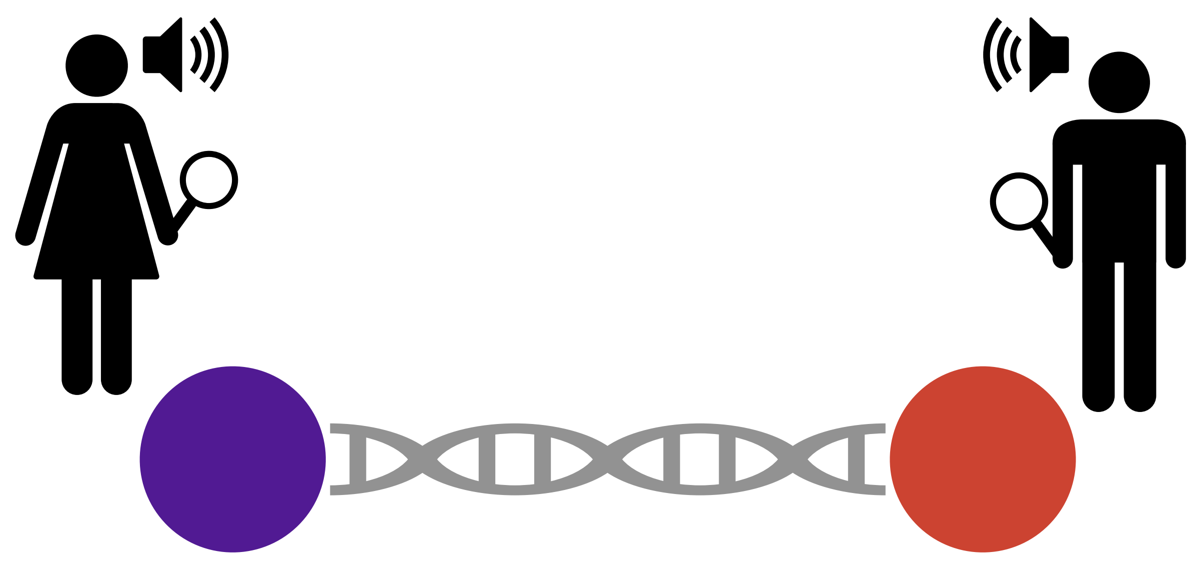

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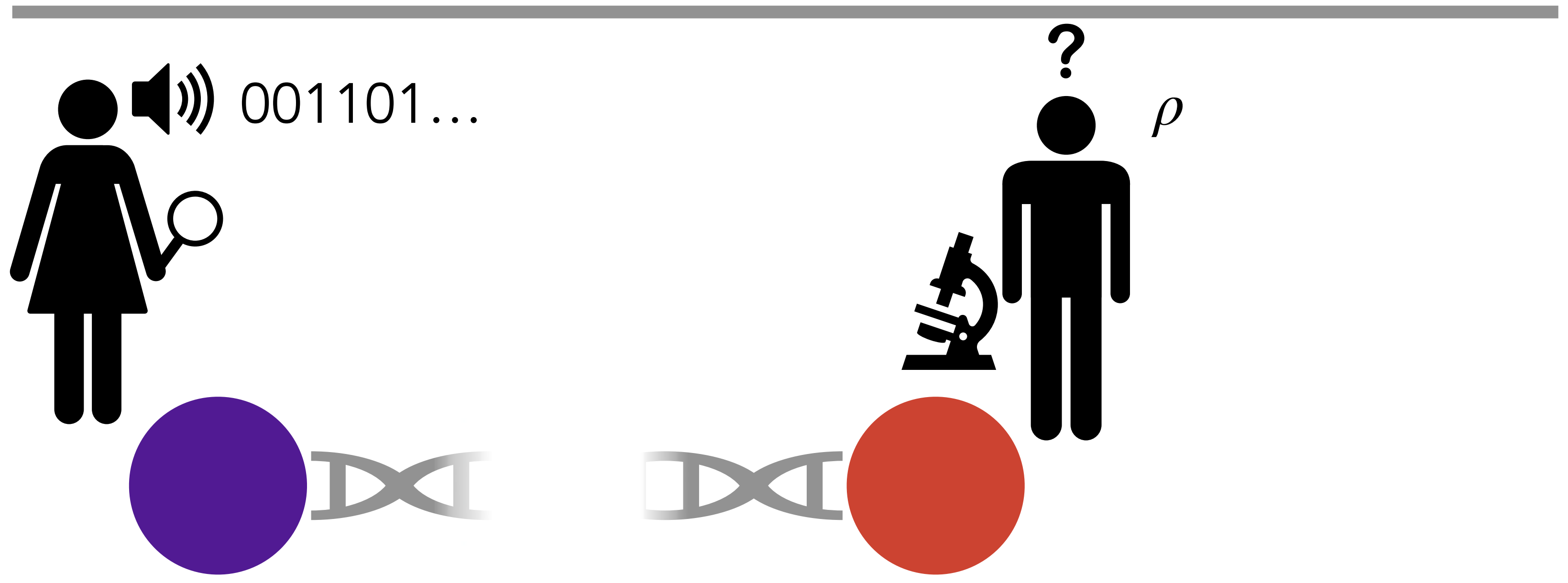
- Conditional probability $P(o_A | o_B) = \mathrm{Tr}[\rho_A(o_B)\Pi_{o_A}]$ $\xrightarrow[\text{In the MIQC phase}]{\rho_A(o_B) = \rho_A}$ $\mathrm{Tr}[\rho_A \Pi_{o_A}] = P(o_A)$

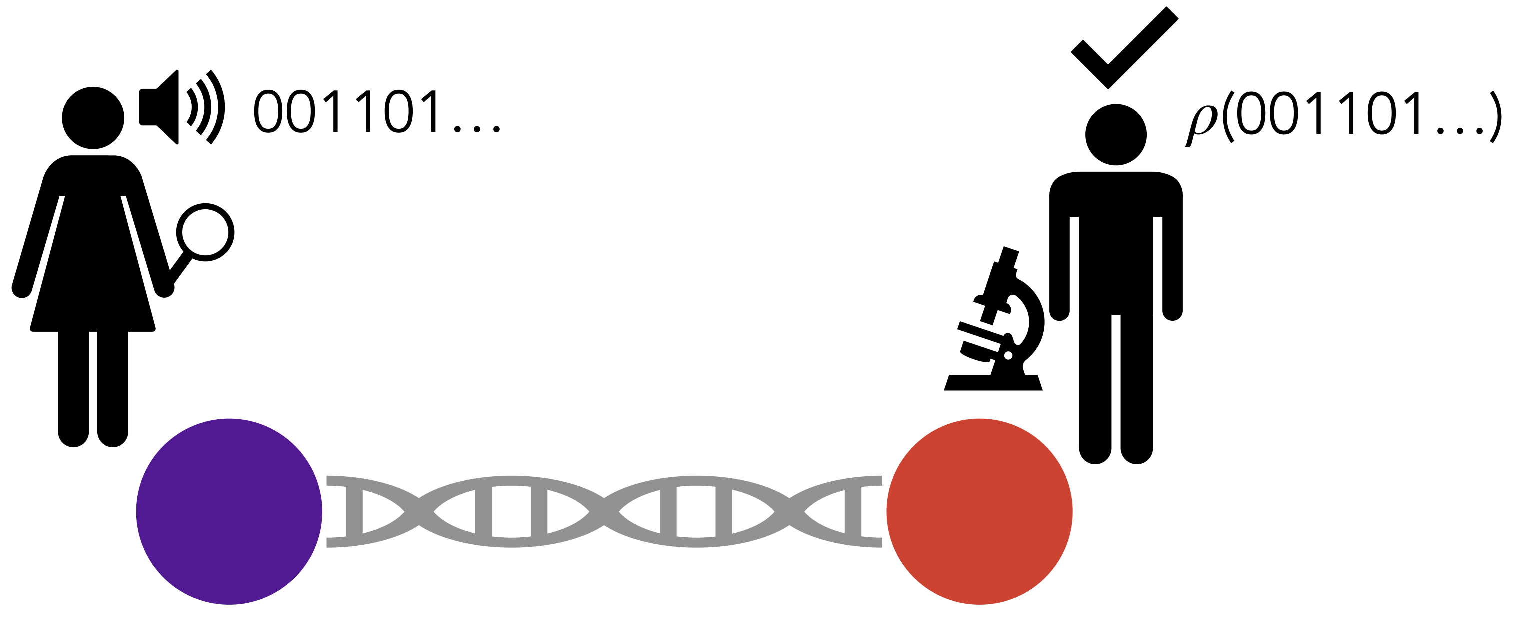
$$P(o_A, o_B) = P(o_A)P(o_B)$$

Summary

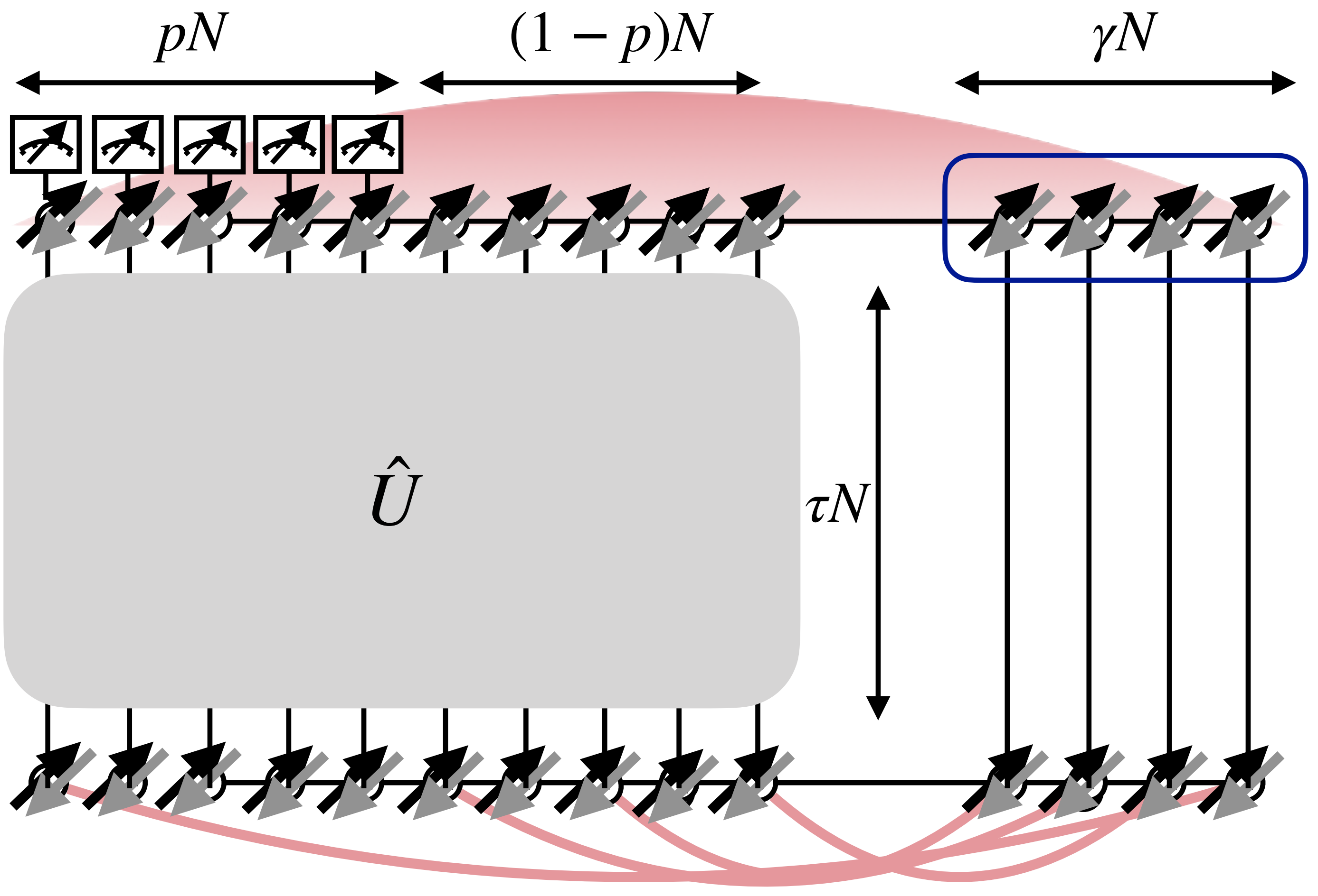


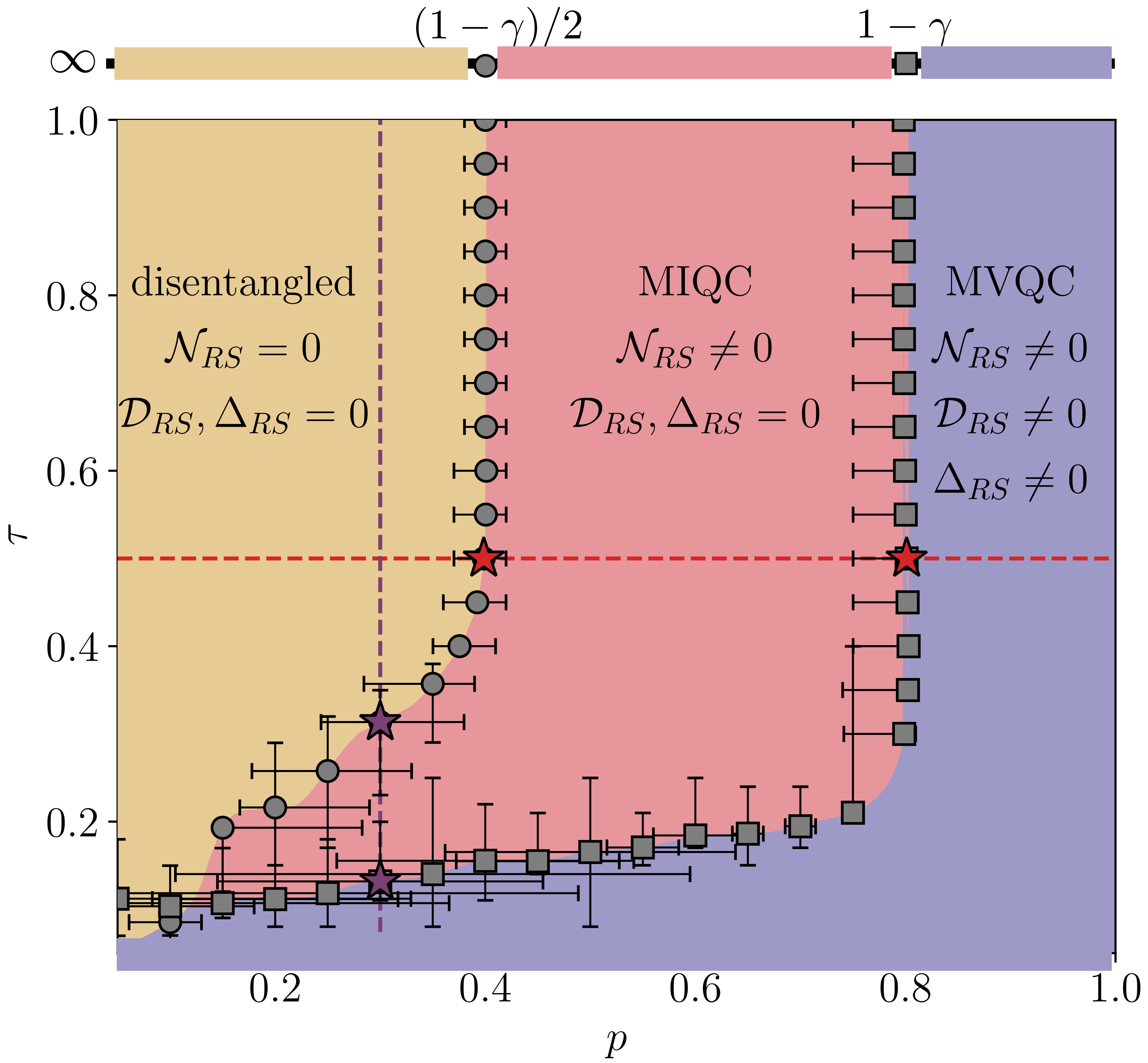
Disentangled
measurement-visible

Entangled
measurement-invisible



Entangled
meas.-visible





Alan Sherry and SR, arXiv:2410.24212

Outlook

works in progress with A. Sherry, S. Mandal, and P.W. Claeys

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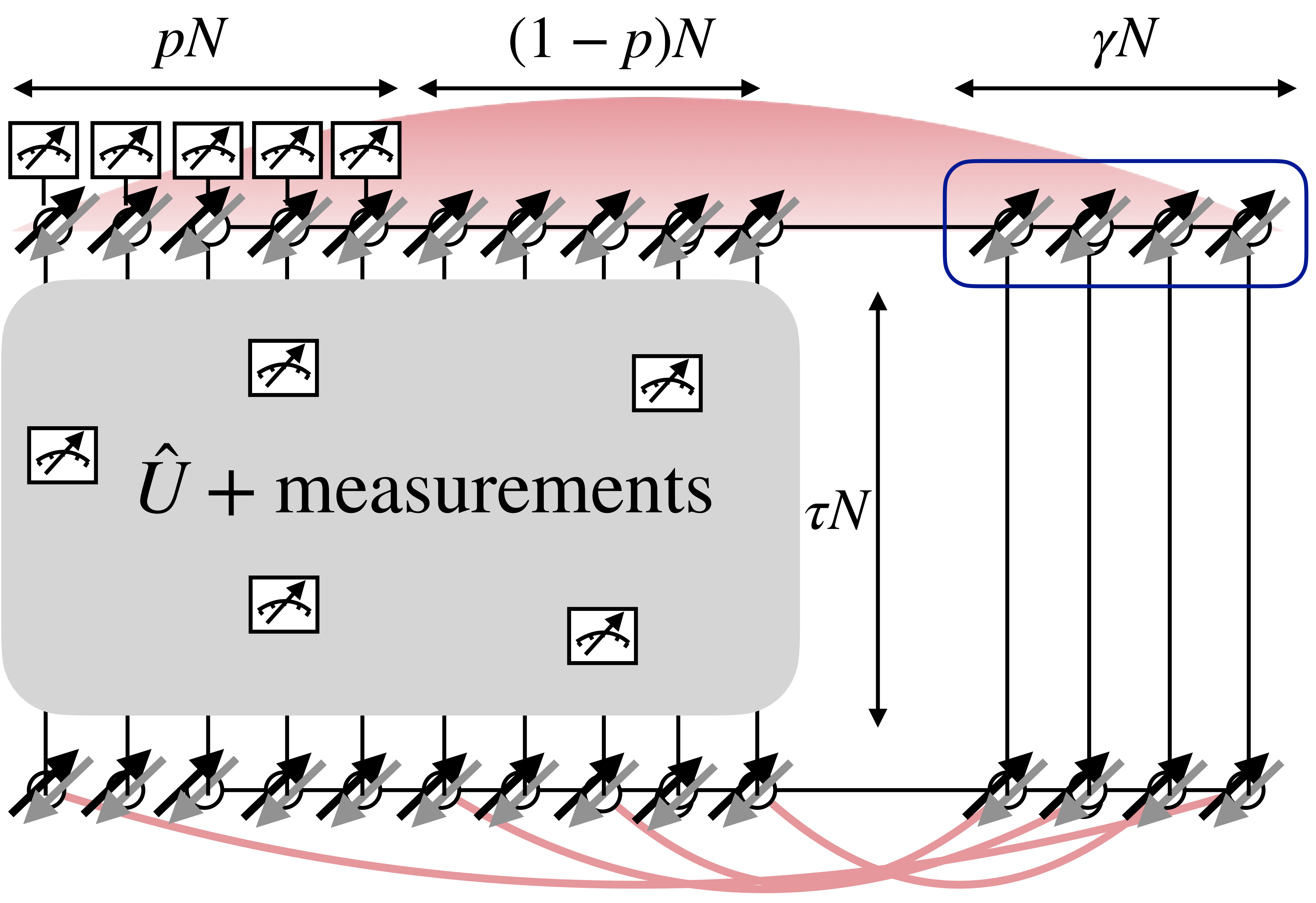

- Fate of the phase diagram in the presence of bulk measurements
- What happens across the measurement-induced entanglement transition?
- Is decoupling or measurement-invisibility the root of the stability of the volume-law phase?

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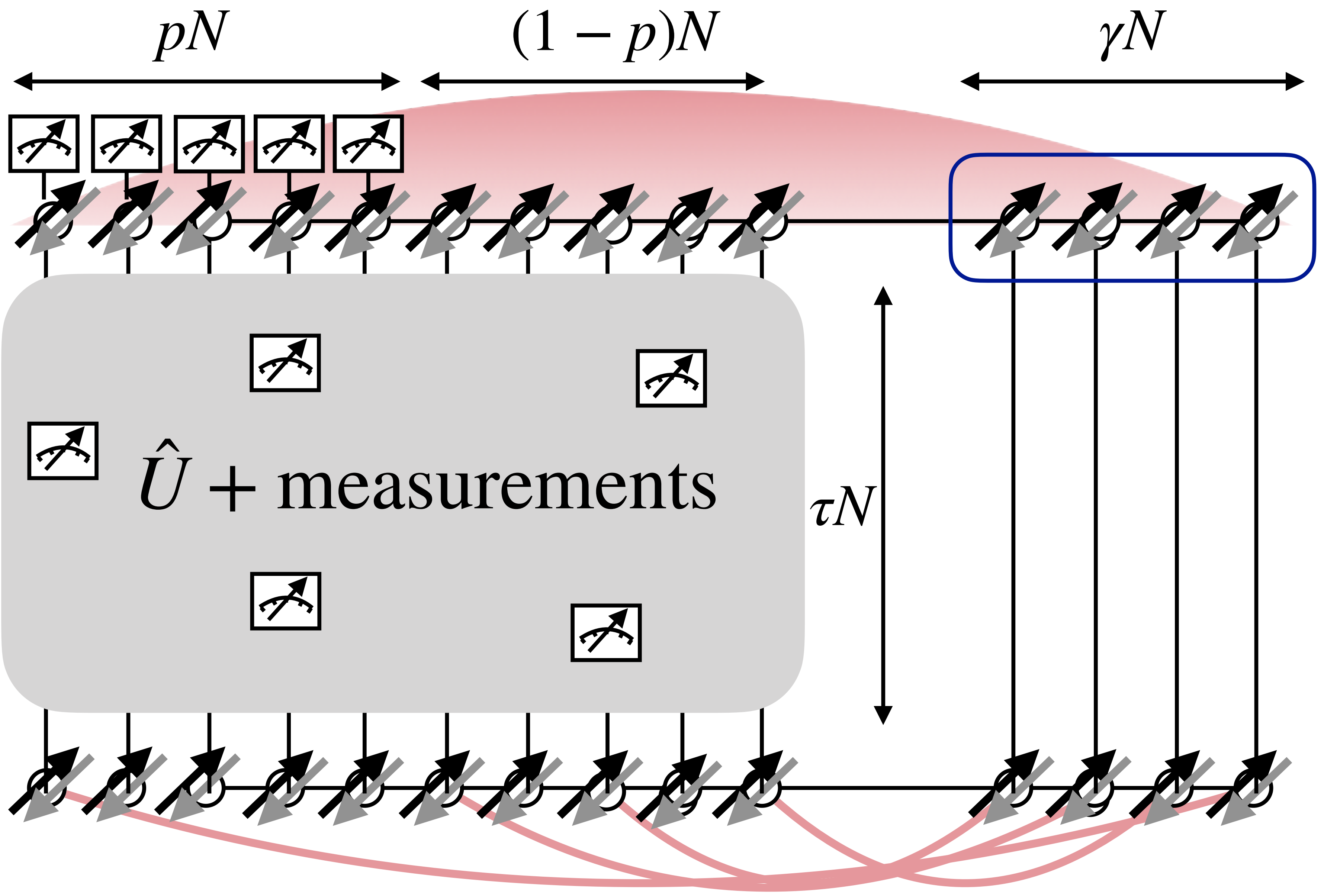

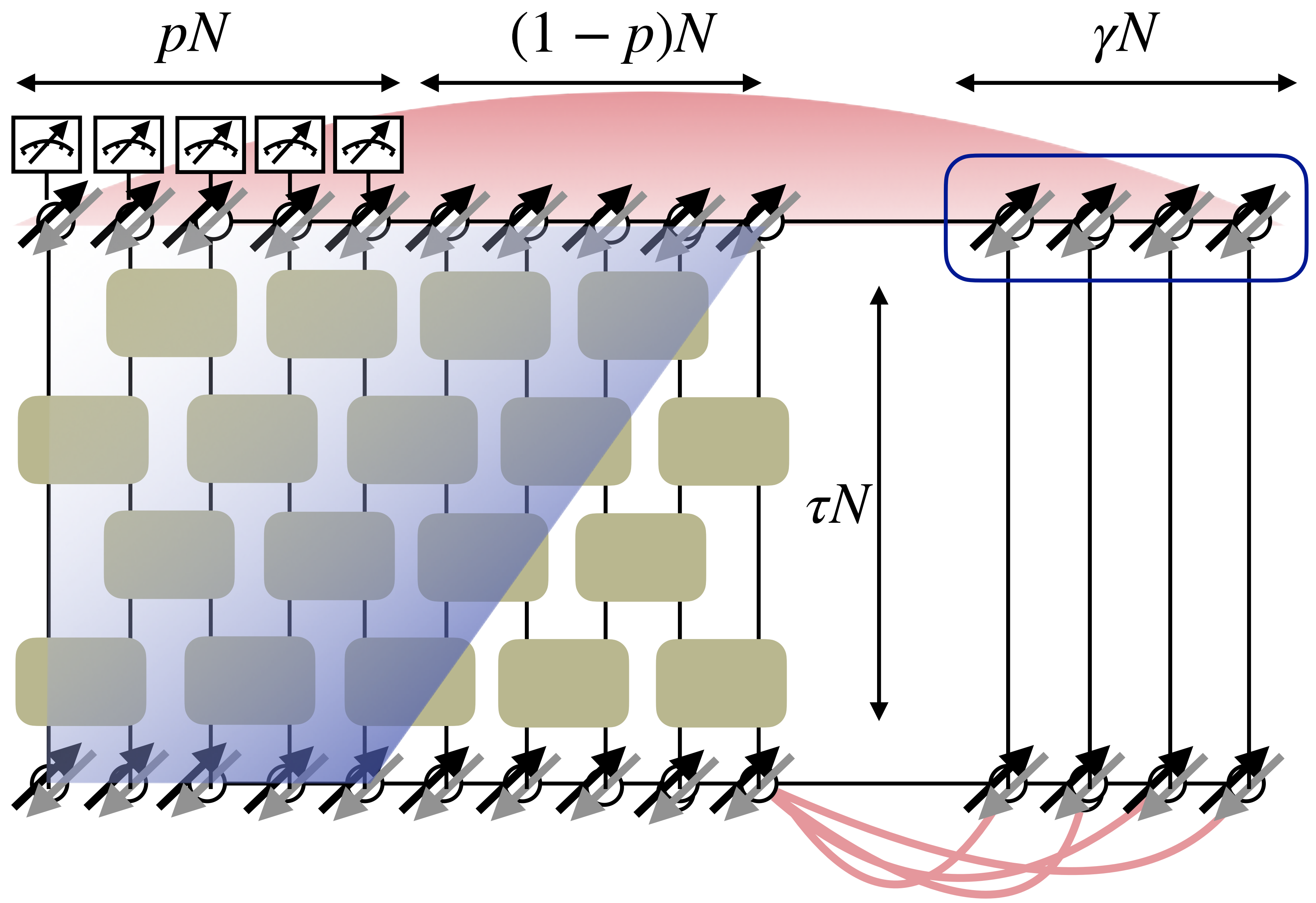

- Fate of the phase diagram in the presence of bulk measurements
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- Circuits with local structure which manifestly have a light-cone
- Measurement-visibility as a signature of space-time profile of information spreading?
- Analogous information to OTOCs but operator independent?

works in progress with A. Sherry, S. Mandal, and P.W. Claeys