

Maths Circle Explorations: Session 7

TIFR, Mumbai

21st January 2022

Problem 1

Suppose you are a master of Euclidean geometry. You have started learning trigonometry, and you know the definition of six main trigonometric ratios (sine, cosine, tangent, cotangent, secant and cosecant) as the ratios of appropriate side lengths of right triangle. Without developing the theory of trigonometry any further, prove the following:

1. $4 \sin 20^\circ + \tan 20^\circ = \sqrt{3}$
2. $2 \sin^2 10^\circ + 2 \sin 10^\circ + \sqrt{3} \sin 20^\circ = 1$

Problem 2

1. Show that $\sqrt{2}$ is irrational.
2. Show that $\frac{\sqrt{5}\pm 1}{2}$ is irrational.
3. Is $\sqrt{2} + \sqrt{3} + \sqrt{5}$? irrational ?
4. If $p + q + r$ is rational and $\sqrt{p} + \sqrt{q} + \sqrt{r}$ is also rational. Can p or \sqrt{p} be irrational ?

How do you calculate square root ? Can you write your method and prove its correctness ?

Consider the stated method to find the square root of N .

Make an initial guess $x(0)$ of the square root of N . (i.e. $x(0)^2 \sim N$)

Next assume that you have already found $x(1), x(2), \dots, x(n)$. Now you find $x(n+1)$ by the operations below.

$$\begin{aligned}a(n) &= \frac{N - x(n)^2}{2x(n)} \\b(n) &= x(n) + a(n) \\x(n+1) &= b(n) - \frac{a(n)^2}{2b(n)}\end{aligned}$$

Assertion: The numbers $x(n)$ as n increases come closer and closer to the square root of the number N .

Prove or disprove the Assertion.

Problem 3

Two brothers sold a herd of sheep. Each sheep sold for as many rupees as the number of sheep originally in the herd. The money was then divided in the following manner. First the older brother took 10 rupees, then the younger brother took 10 rupees, then the older brother took another 10 rupees, and so on. At the end of the division the younger brother, whose turn it was, found that there were fewer than 10 rupees left, so he took what remained. To make the division fair, the older brother gave the younger his pen- knife, which was worth an integer number of rupees. How much was the penknife worth ?