SUM OF SQUARES: FROM FERMAT TO BHARGAVA

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ABSTRACT. Fermat's two-square theorem tells us which natural numbers can be written as the sum of two squares. One can ask a similar question about the sum of three squares and the answer is known and is due to Legendre. Further, Lagrange proved that every natural number can be written as the sum of at most four squares. Another way of stating Lagrange's theorem is that the quadratic form $f(x, y, z, t) = x^2 + y^2 + z^2 + t^2$ in four variables takes on all natural number values. Such quadratic forms are called universal. Ramanujan classified all diagonal universal quadratic forms in four variables. Given a general quadratic form, say $g(x, y, z, t) = ax^2 + by^2 + cz^2 + dt^2$, how does one check if it is universal? In this talk we will provide an answer to this question which is in the form of a theorem originally due to Conway and Schneeberger which was proved in an elegant way by Manjul Bhargava in 2000.