Maths Circle India

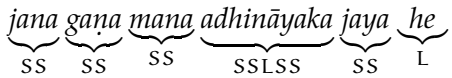
TIFR-STCS Maths Circle Team

Session 2: April 29, 2022
DRAFT

1 *And the two are mixed*

More than two thousand years ago, Piṅgala, while describing patterns of prosody in poems, asked how many prosody patterns there are of a given length.

Such a pattern consists of some "long" units, each of which counts as two units of time, and some "short" units, each of which counts as one unit of time. As an example, the pattern for the first line of the national anthem is given below, and has a total length of 16:



How many patterns are there of total length 1? 2? 3? …10? …16? …20?

2 How many friends?

It's lunchtime at school, and Ravi has been telling Hasan all about the party he went to the night before. Hasan, however, has begun to suspect that some of the stories that Ravi has been telling him about the party are probably made up on-the-spot, and could not possibly have happened. After all, could lemonade-flavoured-milk – the brand-new drink that Ravi tasted at this party, and of which he cannot stop singing praises – really exist? But, then, Hasan can't just contradict him, since he wasn't at the party. So he is looking hard for clues.

"Do you know, Hasan, I asked everyone at the party how many other people at the party they were friends with (including me of course!), and wrote down those numbers. Then, I also counted how many of the people at the party were my own friends, and wrote down that number too. And would you believe it! All those numbers were different from each other!", Ravi continued.

Hasan finally had the clue he wanted: "Caught you! Now I know you have been making up stories about the party!"

What did Hasan catch?

3 So much to read!

Arun, Barun and Kiron from last time have now taken to reading electronic books. Unfortunately, the story books they like are large, and their electronic devices aren't so new, so they are afraid they would soon run out of storage space.

Arun notices that the books they read are composed of letters, numbers, punctuation marks and spaces, of course arranged in a sequence that makes sense. He then notices that sometimes, he can express some longer strings using much smaller expressions. For example, he claims that the screaming sound of "AAAAAAAAAAAAAAAAAAA!" that the hero in their last story made on seeing a lizard could as well be described in just four symbols as "20A!". He tells this to Barun, and they both get excited. After several days of labour, they come back to Kiron and claim that they have written two computer programs: the first, which they call "Shrinker", is supposed to take any of their story books, and output a "shorter" book (also composed of letters, numbers, spaces and punctuation marks, but not necessarily readable by humans), and "Deshrinker" (which will take as input one of the "shrunk" books given as output by "Shrinker" and output the original book from which "Shrinker" produced that output). They offer the programs to Kiron if she would share with them her new story book, *Kironmala*.

Kiron thinks for a few moments, tells them that such programs cannot exist, and that they must have made some error, and then goes back to reading her storybook.

Is she right?

4 Moving ahead

Credits: Arjan Cornelissen, Nikhil Mande and Subhasree Patro

The kingdom of Phoolpatti is now no more at war, and the citizens are now organising their annual games festival. The queen, now freed from worrying about wars and bridges, is also back at her hobby of designing mathematical games. Her latest game takes the following form. First, she creates a map which must satisfy certain conditions. Here are the conditions.

1. A map consisting of just one point is a valid map. In this case, this single point is called the *root* of the map.

2. If M_1 and M_2 are two valid maps with roots at points p_1 and p_2 respectively, then the map M comprising, in addition to the old maps M_1 and M_2, a new point r along with arrows $r \to p_1$ and $r \to p_2$ is also a valid map. The new point r is then declared to be the root of M.

Can you create some such maps? How many maps are there with just one point? Just two points? Just three points?...Just eight points?

Once she has such a map M, the game is played between two players: the *painter* and the *planner*. First, note that for any point p in a valid map M, there are either zero or two arrows going out of p. The painter's job is to colour one arrow blue and the other red, at each point p in the map which has two arrows going out. Note that the painter can choose at each point which of the two arrows he would colour red and which he would colour blue.

Once the painter is done painting, the *planner*'s job is to look at the painting and ask the painter to "travel" from the *root* of the map M to one of the points in the map that has no arrows going out of it. The planner can choose any such point, and the painter can only travel in the direction of the arrows. The planner's score is the number of red arrows that the painter has to follow.

The painter's goal in the game is to minimize the *planner*'s score, while the planner's goal (of course) is to maximize her score.

What is the best the painter can do in the example maps you constructed? Is there a simple method for finding out this "best possible score" given the description of the map?