# Maths Circle India 

TIFR-STCS Maths Circle Team

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## 1 Aggressive Chess

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After their chess practice, Arun and Barun play the following game on a standard chess board. They place a queen on the top left corner and take turns moving the queen. The constraint is that they cannot move the queen up or to the left, and they have to otherwise follow the chess rules for moving the queen. So, at each step, they can move the queen either right, or down, or diagonally to the "south east" (i.e., equal number of steps to the right and down). On their turn, each player has to move the queen: you are not allowed to pass on your move. The player who is forced to move the queen to the bottom right corner loses.

If you wanted to win the game, would you play first or second? What if the board is just $2 \times 2$ ? $3 \times 3 ? 4 \times 4$ ?


A $2 \times 2$ chessboard

## 2 Digital communication

Kiron has a new game for Arun and Barun. Initially, Kiron shows them a square table, like the one below.

Arun and Barun can look at the table, and discuss their strategy for the game before hand. After their discussions, Kiron gives them each a copy of the table, and takes them to separate rooms. She

| 0 | 1 |
| :--- | :--- |
| 0 | 0 |

An example game table
then secretly chooses an entry in the table, and tells Arun only the row of the table in which the entry lies, and separately tells Barun only the column of the table in which the entry lies.

After their initial discussion, Arun and Barun can now communicate only through an app specially designed by Kiron, which only allows them to communicate in rounds. In each round, only one of Arun and Barun can send a message to the other, and the message can be either "YES", or "NO" or "STOP". Once the "STOP" message is sent, the game stops and one of Arun and Barun is supposed to tell Kiron the value ( 0 or 1 ) that was in the entry chosen by her. To win, they only need to tell her the value of the entry, not necessarily the position of the entry in the table.

Arun and Barun (of course) want to win the game after communicating for the smallest possible number of rounds. Kiran, on the other hand, wants to make them take as long as possible. Note that Kiron knows the strategy they will follow. So she will try to choose the entry that causes Arun and Barun to take as many rounds as possible.

What do you think is the smallest number of rounds for the table above for Arun and Barun to win the game, no mater what Kiron does? What about some of the tables below?

| 1 | 0 | 0 | 0 |
| :--- | :--- | :--- | :--- |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

Figure 1: Game table

| 0 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Figure 2: Game table

| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

Figure 3: Game table

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Figure 4: Game table

| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

Figure 5: Game table

| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |

Figure 6: Game table

