

Maths Circle India: Module 8, Session 2
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1 Division

- (a) Let a and b be two positive integers. Show that there are integers q and r such that $b = qa + r$, where $r = 0$ or $0 < r < a$.
- (b) Now let a be a positive integer and b be any integer. Show that there are integers q and r such that $b = qa + r$, where $r = 0$ or $0 < r < a$.
- (c) Now let a and b be two integers where a is negative. Show that there are integers q and r such that $b = qa + r$, where $r = 0$ or $0 < r < -a$.

2 Chessboard and Dominoes

- (a) A domino is a rectangle consisting of 2 unit squares. Take a chessboard (that has 64 unit squares) and cut the squares of two opposite corners from it. The rest has 62 unit squares. Is it possible to cut this portion into 31 dominoes?

[**Hint:** Try to use that it is a chessboard.]

- (b) Find all pairs of squares such that after removing them, the remaining portion of the chessboard can be cut into 31 dominoes.

3 A Palace with Thousand Doors

A palace has 1000 doors all of which were closed at the beginning. There are 1000 security guards in this palace. The first security guard opened all the doors. The second security guard closed the 2^{nd} , 4^{th} , 6^{th} , \dots , 1000^{th} doors. The third security guard changed the state of the 3^{rd} , 6^{th} , 9^{th} \dots , 999^{th} doors (i.e., he closed the open doors among these and opened the closed ones). The fourth guard changed the state of 4^{th} , 8^{th} , 12^{th} , \dots , 1000^{th} doors, and so on. Finally, the thousandth security guard changed the state of the 1000^{th} door.

How many doors were open in the end?