Maths Circle India: Module 8, Session 1
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Friday the 13th

As per Western superstition, *Friday the 13th* is considered an unlucky day. It occurs when the thirteenth day of a month in the *Gregorian calendar* (the calendar that we use) falls on a Friday. The superstition around Friday the 13th is so deep-rooted that there is a series of horror/mystery movies on this theme. If you would like to know more about the history of Friday the 13th, please refer to the following Wikipedia link:

https://en.wikipedia.org/wiki/Friday_the_13th.

Here are a few observations on Friday the 13th:

- 2023 has two such days: 13th January and 13th October.
- 2022 had just one Friday the 13th (on May 13).
- 2015 had three such days: 13th February, 13th March, and 13th November.

Keeping the above observations in mind, the following questions arise naturally:

(a) Will every calendar year have at least one Friday the 13th?

(b) Can there be a calendar year in which there are more than three such days?

Problem 1: *Answer the above questions.*

A Shortcut for Computing Squares

Suraj is very scared of computations. His friend Selim, on the other hand, loves computations and knows many cool computational tricks. One day, Selim taught Suraj a shortcut for computing the square of any positive integer whose rightmost digit is 5.

We illustrate this computational shortcut with a few examples. Suppose we would like to compute the square of 65. Look at the integer that is obtained when we remove the rightmost digit - that will be 6 in this case. Multiply this integer with its successor. That will give us $6 \times (6 + 1) = 6 \times 7 = 42$. Therefore, we can conclude that

$$65 \times 65 = 4225.$$

Let us take another example. Suppose we want to calculate 95×95. Note that the number obtained by removing the rightmost digit is 9. When we multiply 9 with its successor, we get $9 \times 10 = 90$. Therefore

$$95 \times 95 = 9025.$$

Similarly, we can compute $85 \times 85 = 7225$, $35 \times 35 = 1225$, $55 \times 55 = 3025$, $105 \times 105 = 11025$, etc.

The general method can be described as follows. If N is a positive integer whose rightmost digit is 5, then we first remove this digit and obtain the integer M. Then we calculate $P = M(M+1)$. Finally, we obtain N^2 by writing the number P followed by the digit 2 and then the digit 5.

Problem 2: *Show mathematically that the method suggested by Selim always works.*

Rectangles Inside a Polygon

Suppose $\mathcal{P}$ is a regular polygon with 418 sides. Consider all the quadrilaterals whose vertices are also vertices of $\mathcal{P}$

Problem 3: *How many of the above quadrilaterals are rectangles?*