Equation informed and data-driven tools for data-assimilation and optimal navigation of turbulent flows

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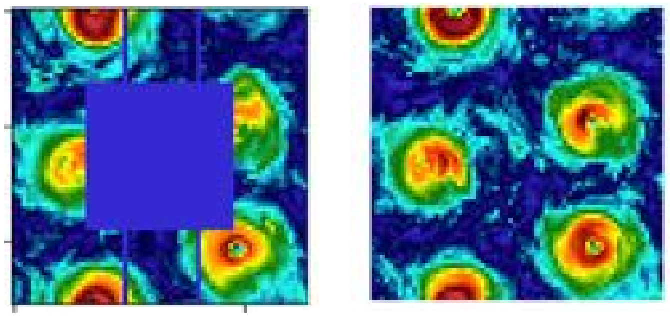
ICTS TPIMP - 2020

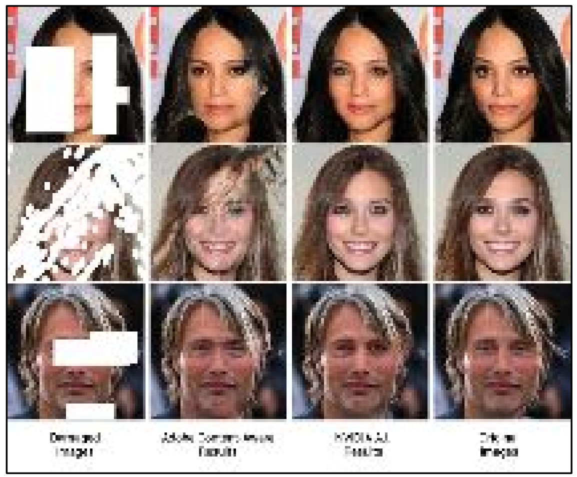
DATA ASSIMILATION <-> INPAINTING

1. EQUATION FREE – MACHINE LEARNING
GENERATIVE-ADVERSARIAL-NETWORK



2. NUDGING (EQUATION INFORMED)

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} + 2\Omega\hat{x}_3\times\boldsymbol{v} = -\nabla p + \nu\nabla^2\boldsymbol{v} + \gamma(1-\hat{M})_{x_3}\odot(\boldsymbol{v}-\boldsymbol{v}_{ref})$$



CREDITS: F. Bonaccorso (IIT, IT), M. Buzzicotti (Univ. Tor Vergata, IT), P. Clark di Leoni (JHU, USA), L. Agasthya (Univ. Tor Vergata, IT)

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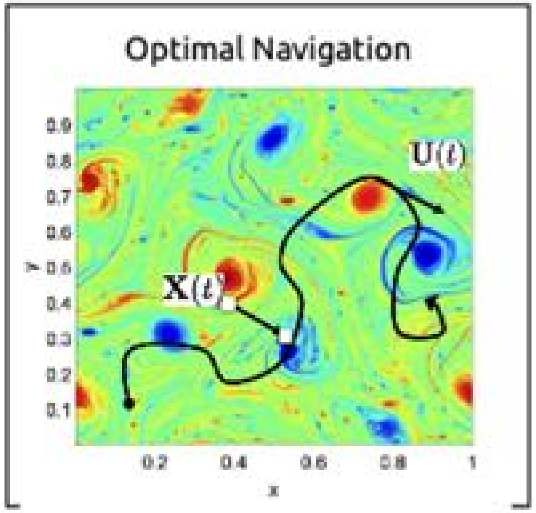
REINFORCEMENT LEARNING
VS OPTIMAL CONTROL

OPTIMAL NAVIGATION

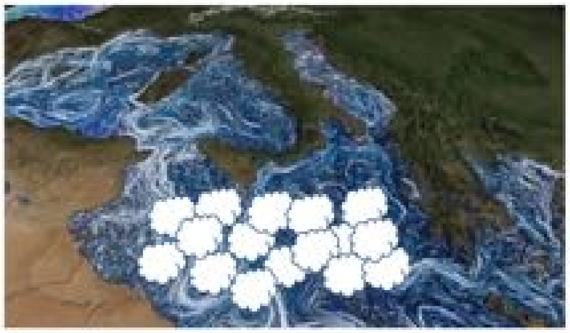


CREDITS: F. Bonaccorso (IIT, IT), M. Buzzicotti (Univ. Tor Vergata, IT), P. Clark di Leoni (JHU, USA), K. Gustafsson (Univ. Gotheborg, SE)

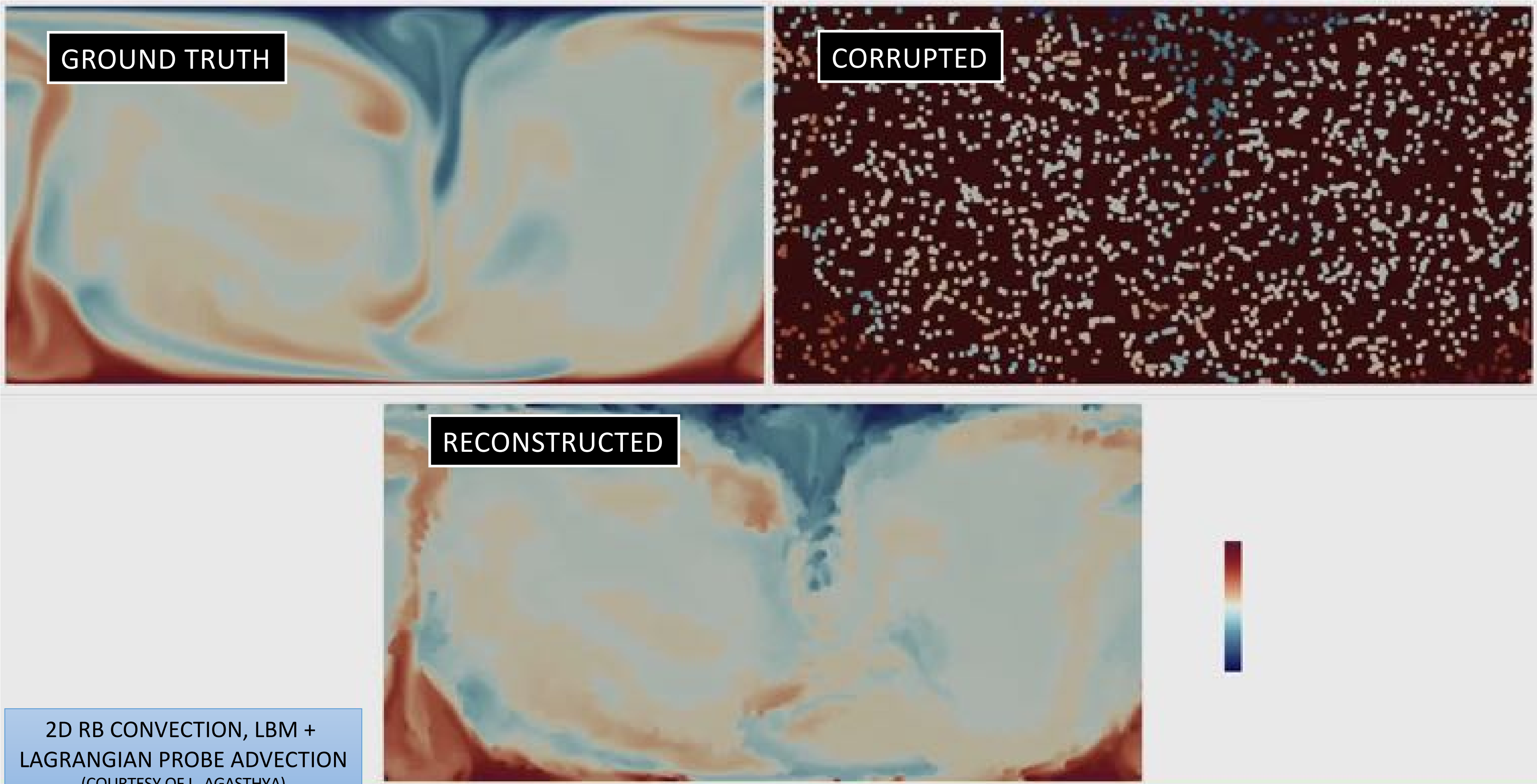
WHY?
FEATURES RANKING: QUALITY AND QUANTITY OF DATA

WHICH FEATURES YOU NEED TO SUPPLY FOR OPTIMAL CLASSIFICATION AND/OR DATA-ASSIMILATION?
-> A WAY TO LEARN ABOUT THE UNDERLYING PHYSICS.

- IS IT BETTER TO INPUT SPATIAL OR TEMPORAL DATA?
- HOW MANY DATA/VARIABLES YOU NEED TO SUPPLY FOR PERFECT RECONSTRUCTION (SYNCHRONIZATION-TO-DATA)?
- CAN YOU GUESS VELOCITY FIELDS BY MEASURING ONLY TEMPERATURE AND/OR VICEVERSA?
- IS IT BETTER TO PROVIDE INFORMATION FROM BOUNDARIES OR BULK?
- FROM LARGE OR SMALL SCALES?
- DO WE NEED TO KNOW THE EQUATIONS?
- HOW TO COMPARE EQUATIONS-BASED AND EQUATIONS-FREE MODELS?

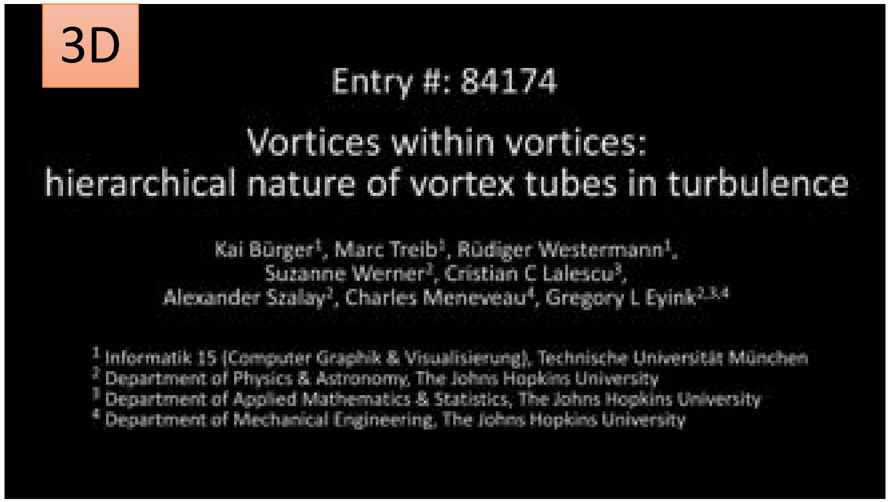

WHY?
FEATURES RANKING: QUALITY AND QUANTITY OF DATA



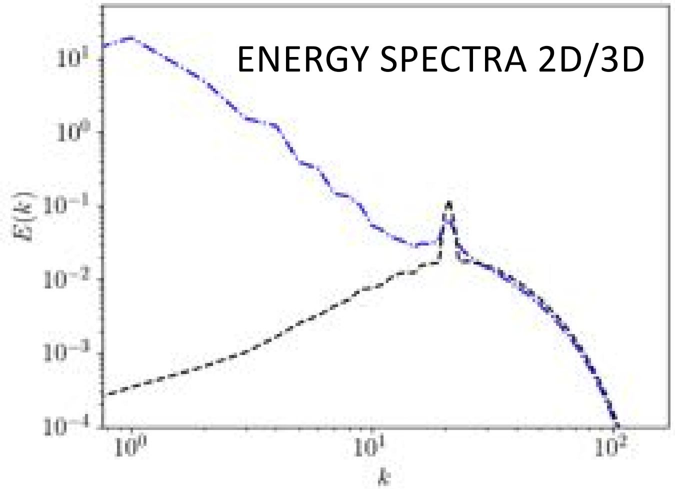
2D RB CONVECTION, LBM + LAGRANGIAN PROBE ADVECTION
(COURTESY OF L. AGASTHYA)

SHORT VISUAL RECAP

(NASA/Goddard Space Flight Center Scientific Visualization Studio)

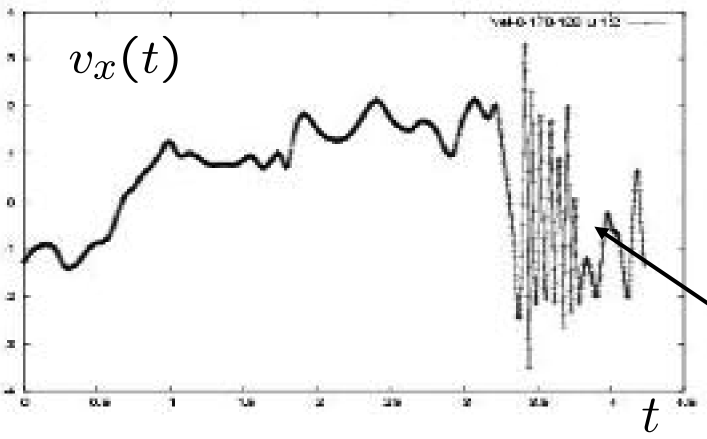


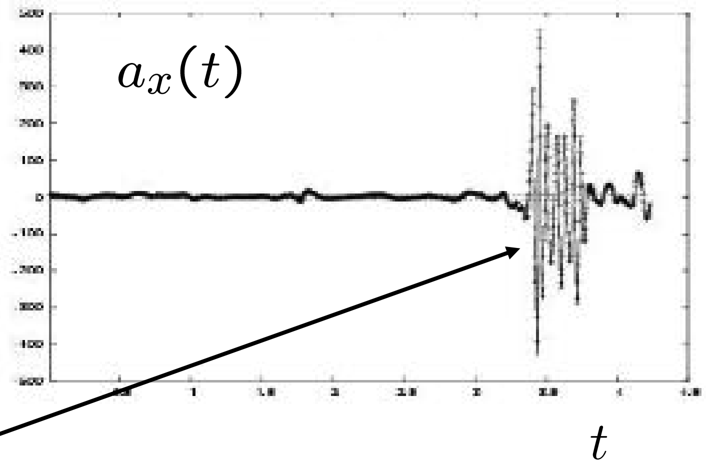
FEATURES RANKING:
QUALITY AND QUANTITY OF DATA

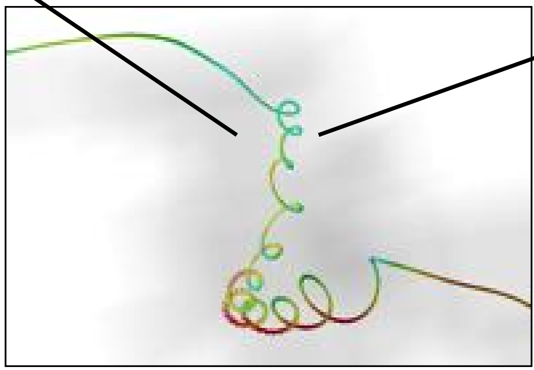


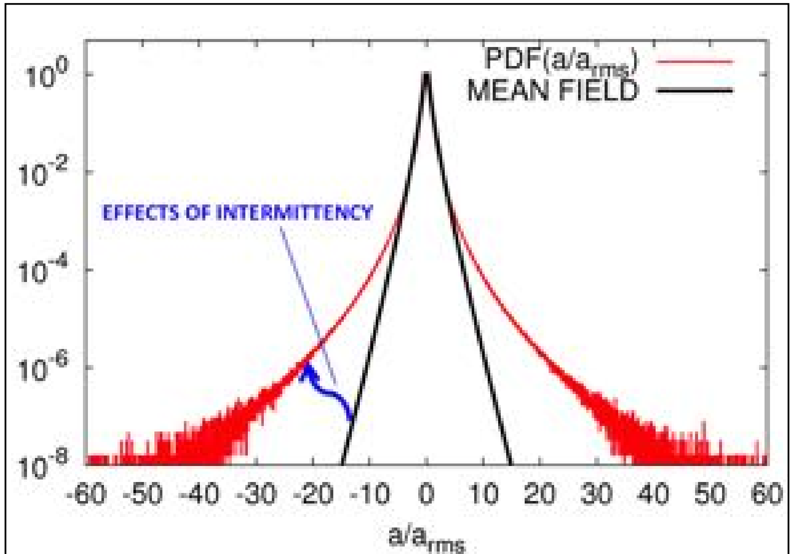
- MULTI-SCALE PHYSICS
- BILLION OF DEGREES OF FREEDOM
- ROUGH NON-DIFFERENTIABLE FIELDS (HOLDER CONTINUOS ONLY)
- NON-GAUSSIAN STATISTICS

EXTREME EVENTS









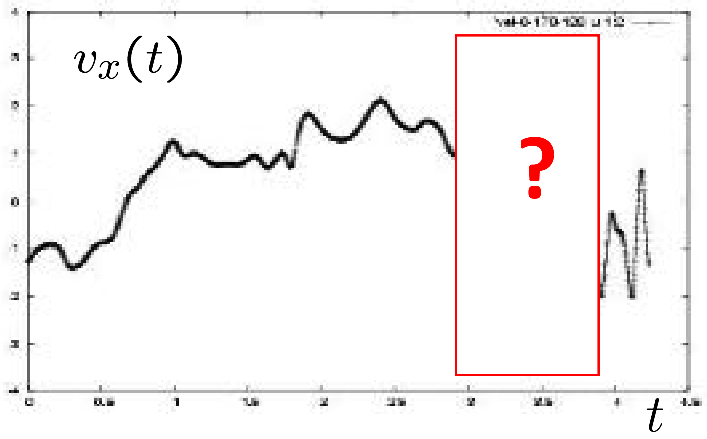
ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04] COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)

L.B., G Boffetta, A Celani, A Lanotte, F Toschi. Particle trapping in three-dimensional fully developed turbulence Physics of Fluids 17 (2), 021701 (2005)

La Porta, G.A. Voth, A.M. Crawford, J. Alexander et al. Fluid particle accelerations in fully developed turbulence. Nature, 409(6823), 1017 (2001)

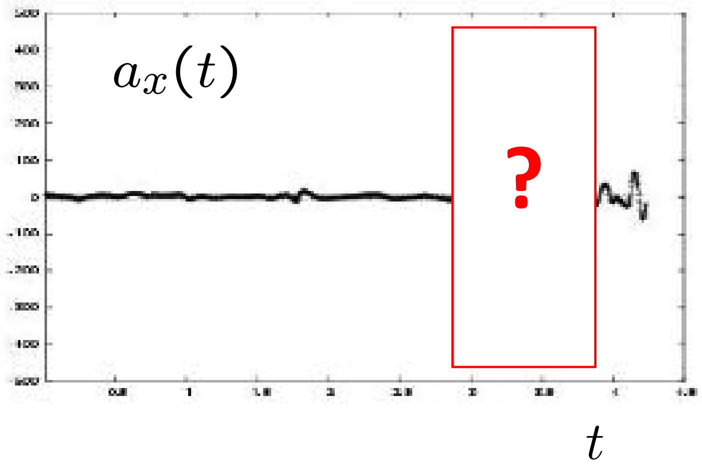
N. Mordant, P. Metz, O. Michel and J.F. Pinton. Measurement of Lagrangian velocity in fully developed turbulence. Phys. Rev. Lett. 87(21), 214501 (2001)

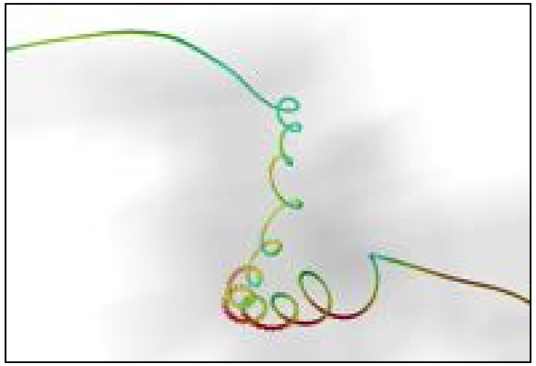
F. Toschi and E. Bodenschatz. Lagrangian Properties of Particles in Turbulence. Annu. Rev. Fluid Mech. 41, 375 (2009)

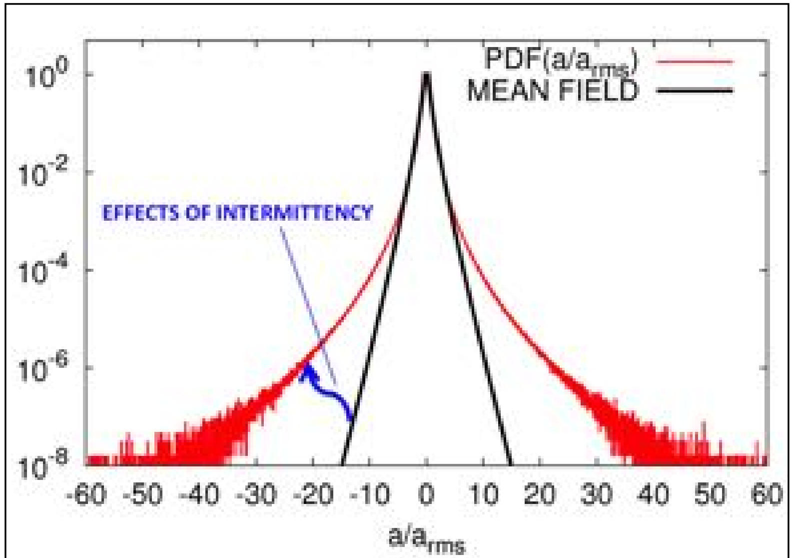


EXTREME EVENTS

Can we reconstruct?







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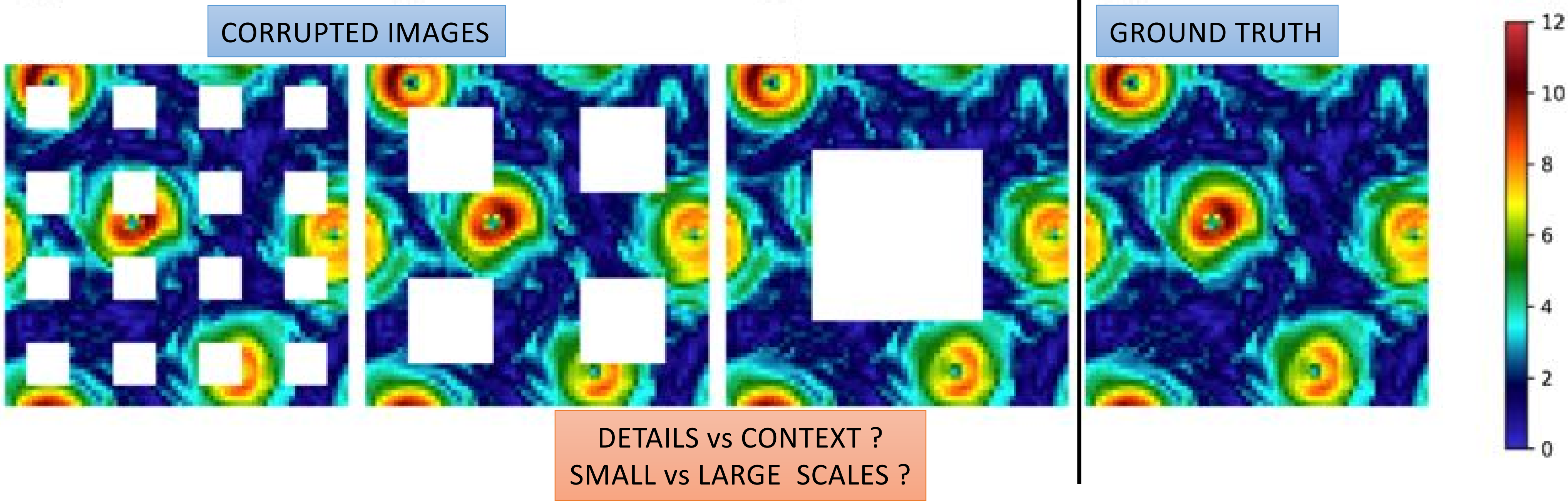
F. Toschi and E. Bodenschatz. Lagrangian Properties of Particles in Turbulence. Annu. Rev. Fluid Mech. 41, 375 (2009)

EULERIAN 2D GAPPY DATA



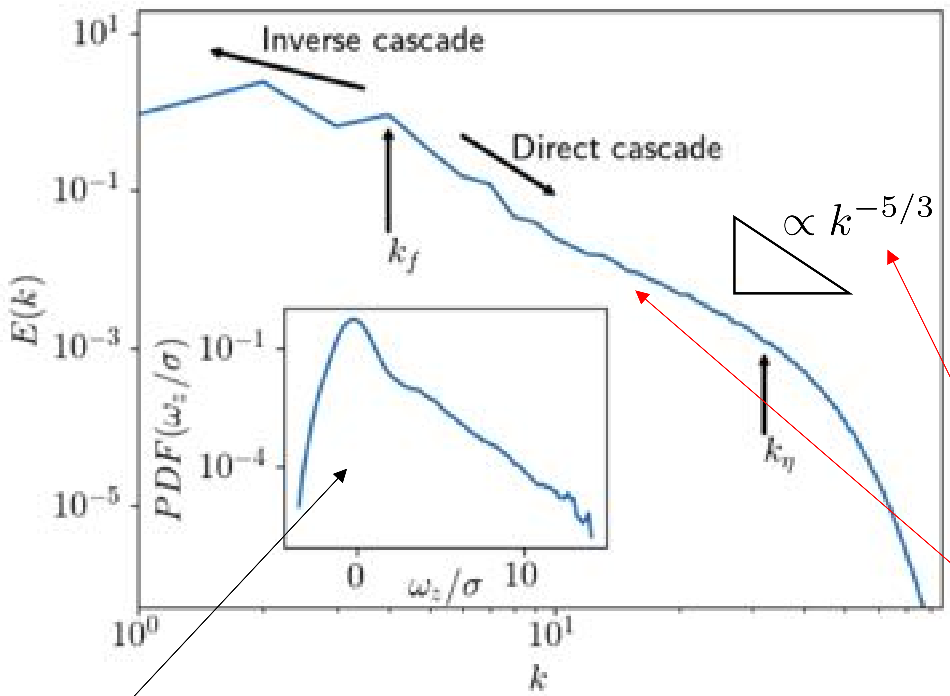
Reconstruction of turbulent data with deep generative models for semantic inpainting from TURB-Rot database

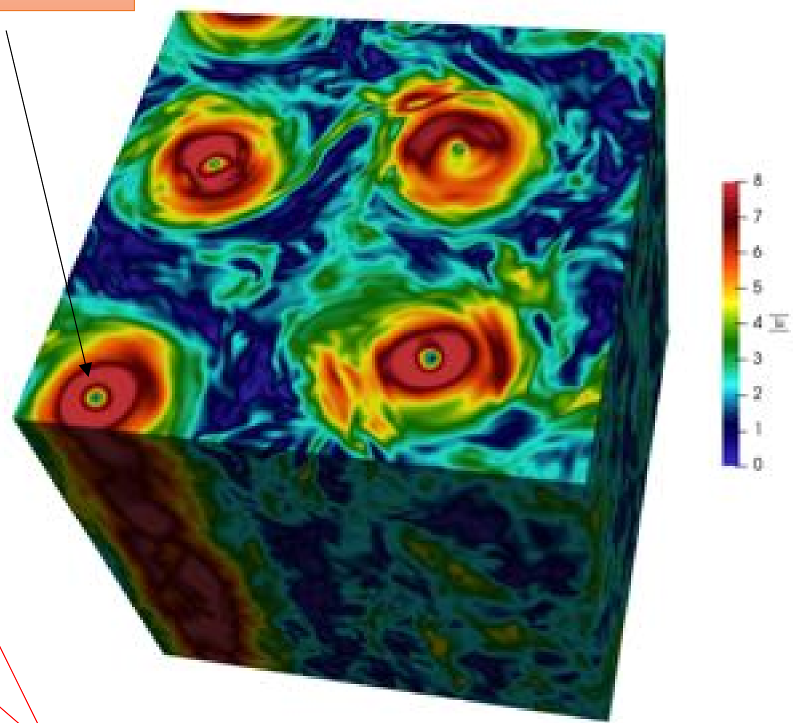
M. Buzzicotti[1], F. Bonaccorso[1,2], P. Clark Di Leoni[3], L. Biferale[1]

Submitted to Physical Review Fluids, arXiv:2006.09179v1

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} + 2\Omega \hat{x}_3 \times \boldsymbol{v} = -\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{v} + \boldsymbol{f}$$

LARGE SCALE



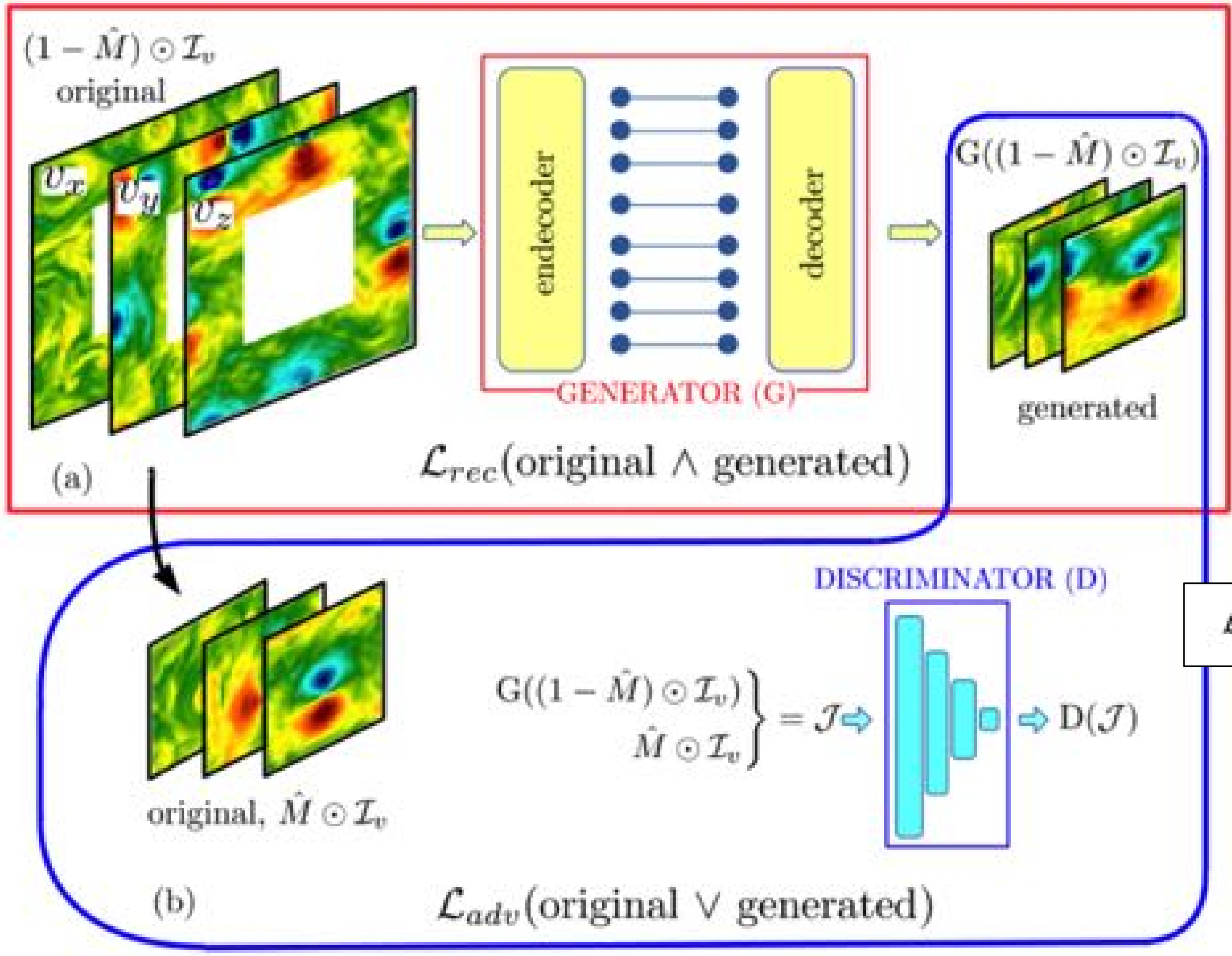


SMALL SCALE

$$E(k) = \sum_{k<\mathbf{k}<k+1} \langle \hat{v}_i(\mathbf{k}) \hat{v}_i(-\mathbf{k}) \rangle_N$$

POWER LAW SPECTRUM:
VELOCITY FIELD IS HOLDER CONTINUOS h=1/3
VAST SCALING RANGE WHERE IT IS NOT DIFFERENTIABLE
+ NON GAUSSIAN FLUCTUATIONS

GENERATIVE ADVERSARIAL NETWORK: CONTEXT ENCODER 1 (CE1)



MINIMIZE:

$$\mathcal{L}_{rec} = \mathbb{E}_{\mathcal{I}_v}\{||\hat{M}\odot\mathcal{I}_v - G[(1-\hat{M})\odot\mathcal{I}_v]||_2\},$$

MAXIMIZE:

$$\mathcal{L}_{adv} = \mathbb{E}_{\mathcal{I}_v}\{\log(D[\hat{M}\odot\mathcal{I}_v]) + \log(1 - D[G[(1-\hat{M})\odot\mathcal{I}_v]])\}$$

D[truth]=1; D[fake]=0

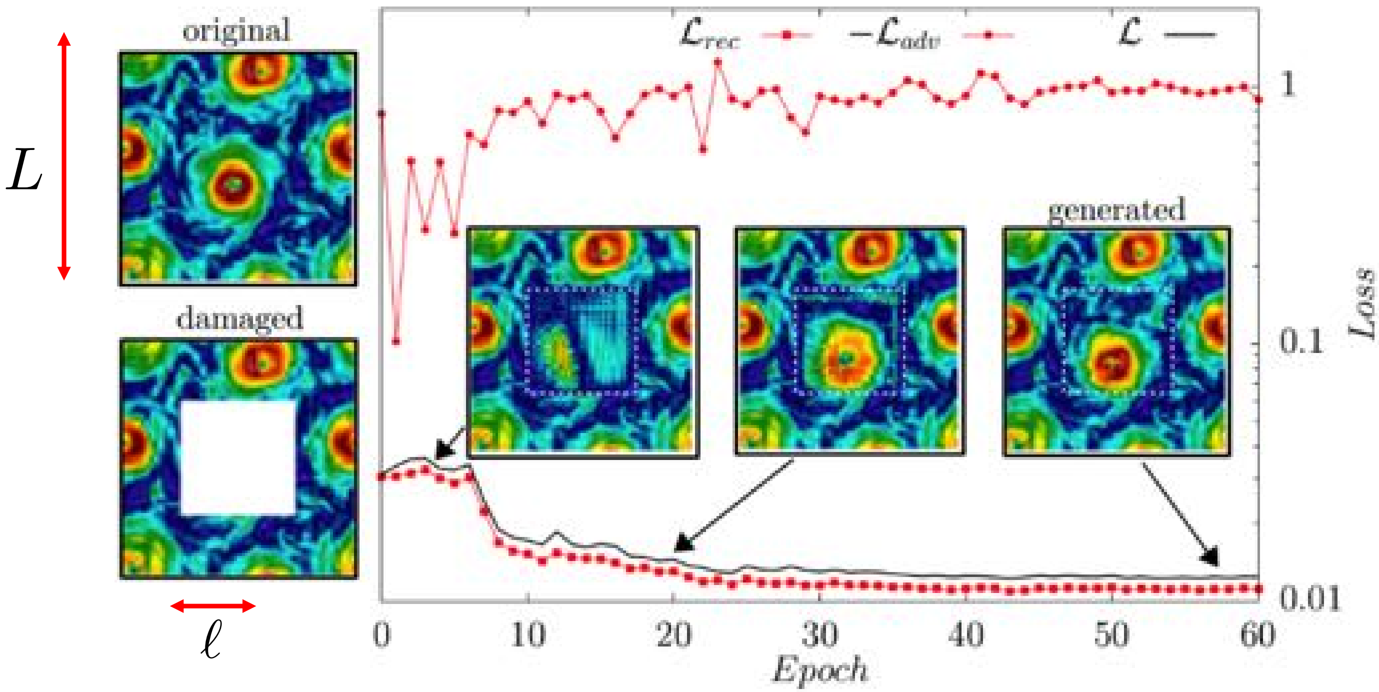
$$\mathcal{L} = \lambda_{rec}\mathcal{L}_{rec} + \lambda_{adv}\mathcal{L}_{adv},$$

[3] Deepak Pathak, Philipp Krahenbuhl, Jeff Donahue, Trevor Darrell, and Alexei A Efros. Context encoders: Feature learning by inpainting. In *Proceedings of the IEEE conference on computer vision and pattern recognition*, pages 2536–2544, 2016.

DURING TRAINING

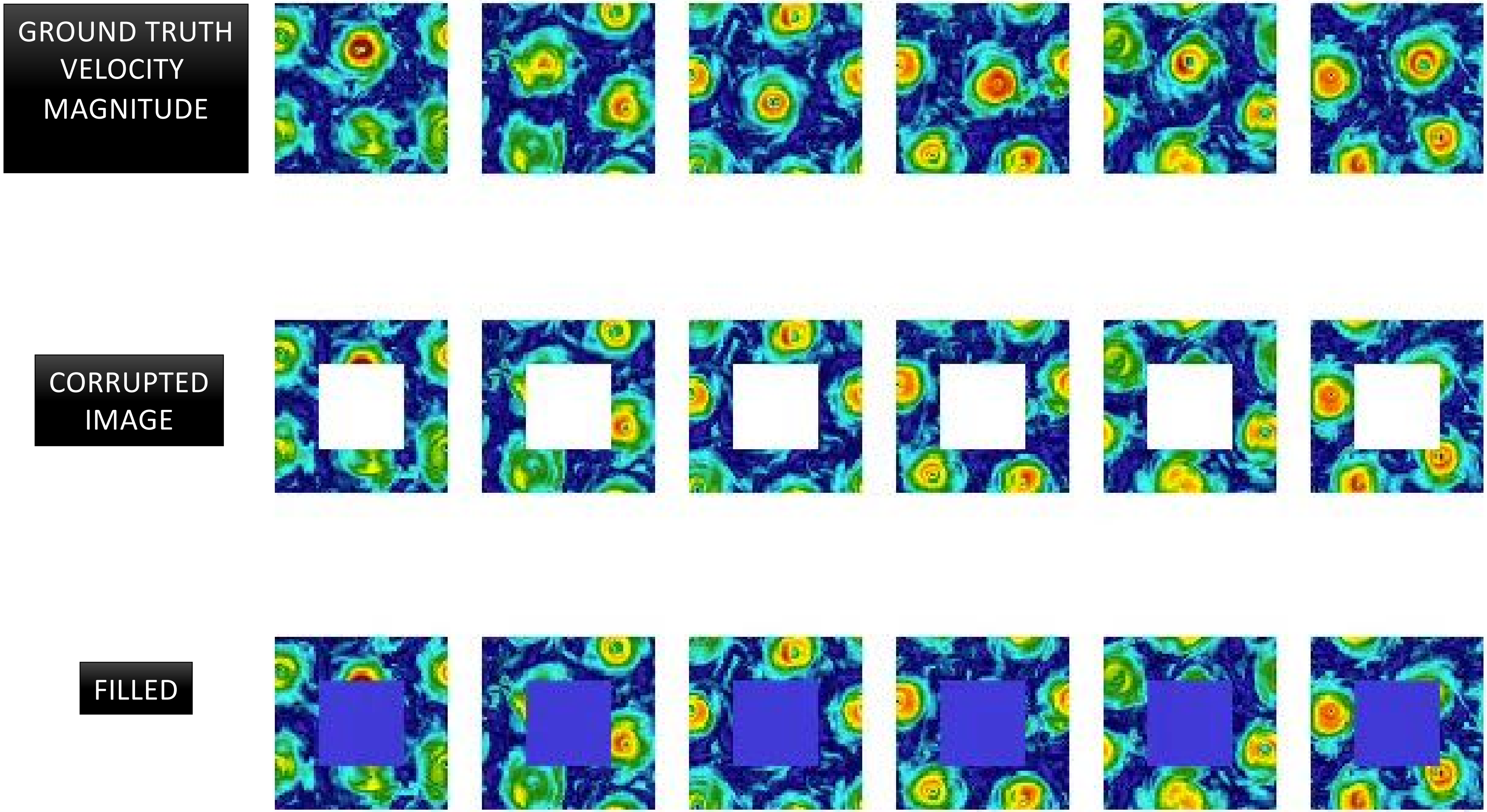
80K 64x64 images of 3 velocity components for training

20K 64x64 images of 3 velocity components for validation



ℓ :much larger than differentiable scale, i.e. velocity fields are rough (no linear interpolation here)

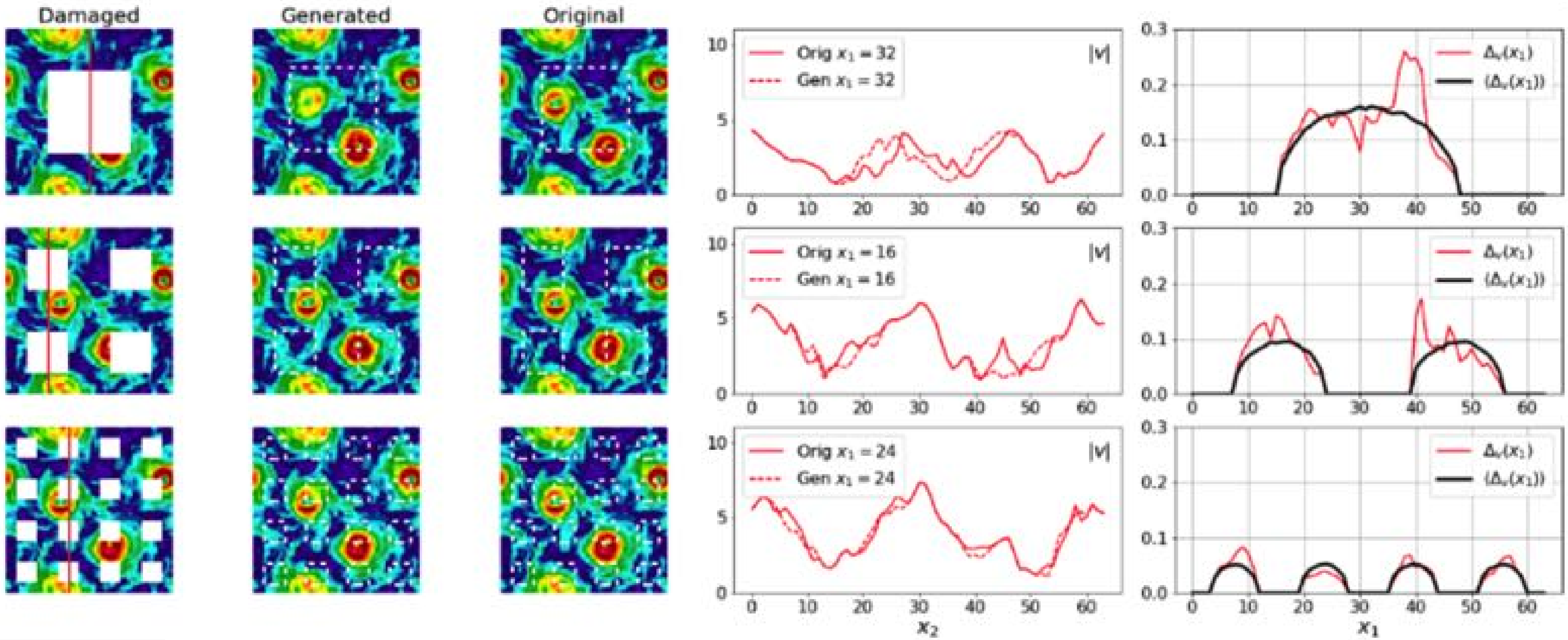
6 DATA ASSIMILATION EXPERIMENTS FOR 2D TURBULENT IMAGES WITH A CONTEXT ENCODER



SCALE DEPENDENCY

$$\frac{\sum_{i=1}^{3}[v_i^{truth}(x_1,x_2) - G_i^v(x_1,x_2)]^2}{E_{tot}}.$$

VELOCITY PROFILE



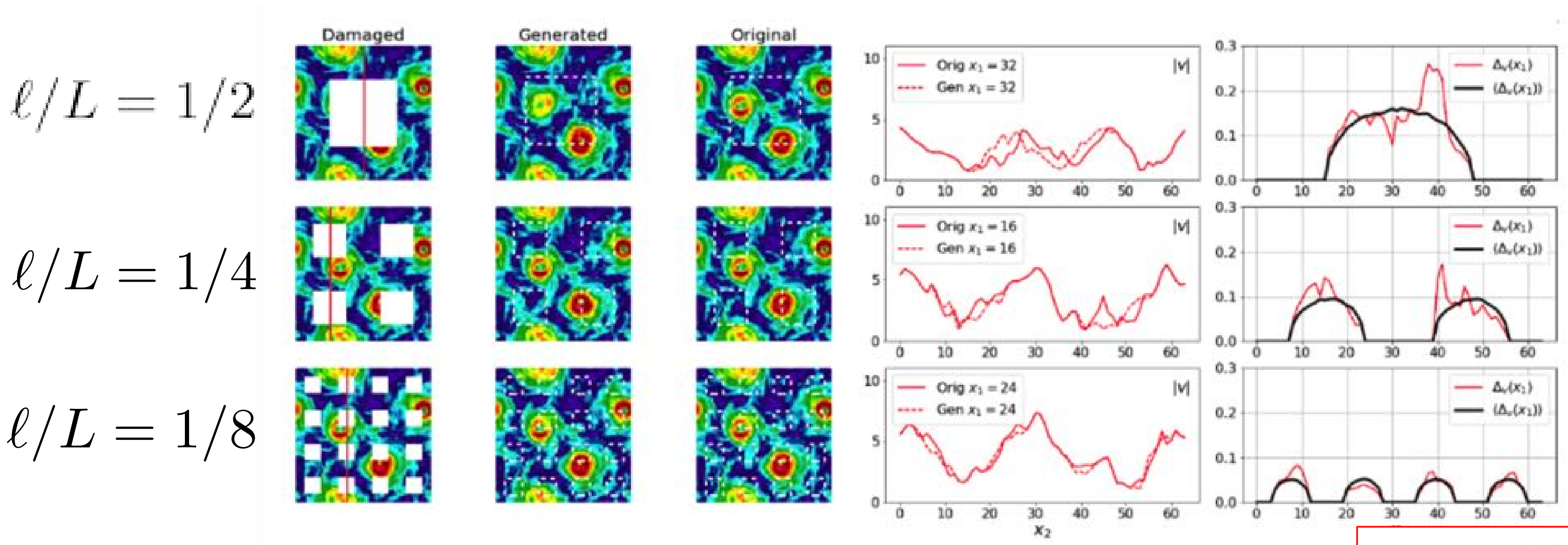
CORRUPTED

INPAINTED

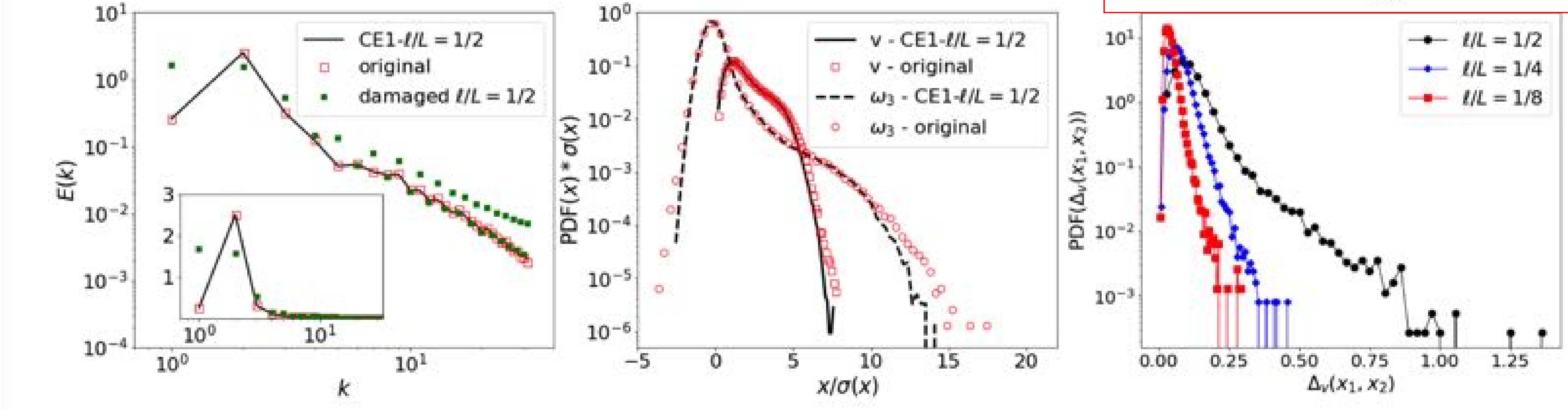
GROUND TRUTH

VELOCITY PROFILE ALONG VERTICAL CUT

L2 ERROR ALONG VERTICAL CUT



$$\Delta_v(x_1, x_2) = \frac{\sum_{i=1}^{3}[v_i^{truth}(x_1, x_2) - G_i^v(x_1, x_2)]^2}{E_{tot}}.$$



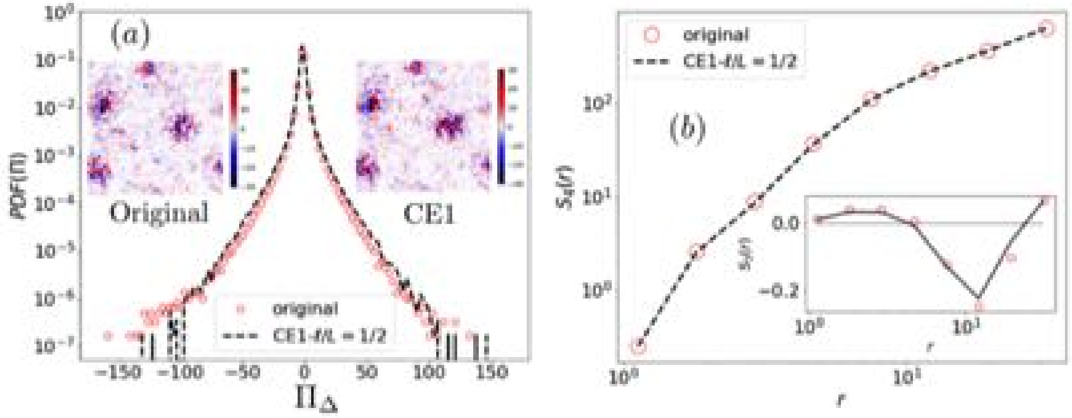


FIG. 9: Panel (a): Comparison of the PDF of the energy transfer Π measured inside the damaged region from both the original (red circles) and reconstructed data using CE1 network (dashed black line). Panel (b): Comparison of the longitudinal structure function, $S_p(r)$, for both original (red circles) and reconstructed data (dashed black lines). In the main panel $S_4(r)$ is presented in log-log scales, while in the inset the 3rd order structure functions, $S_3(r)$ are presented in log-lin scale.

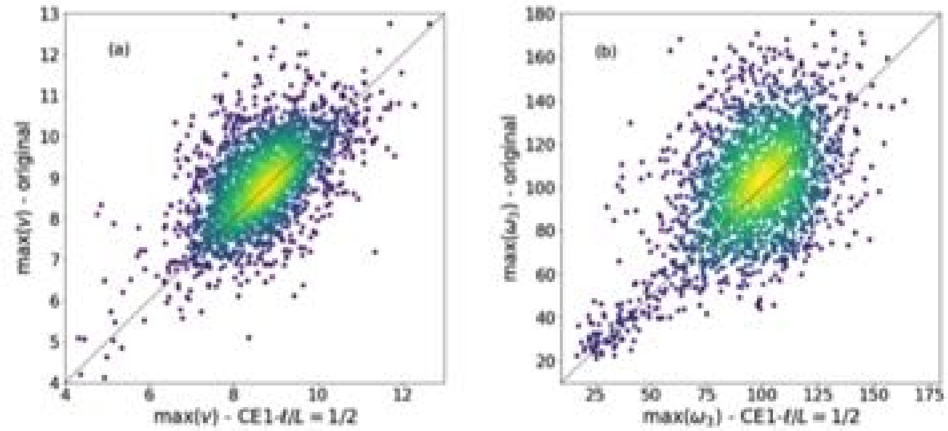
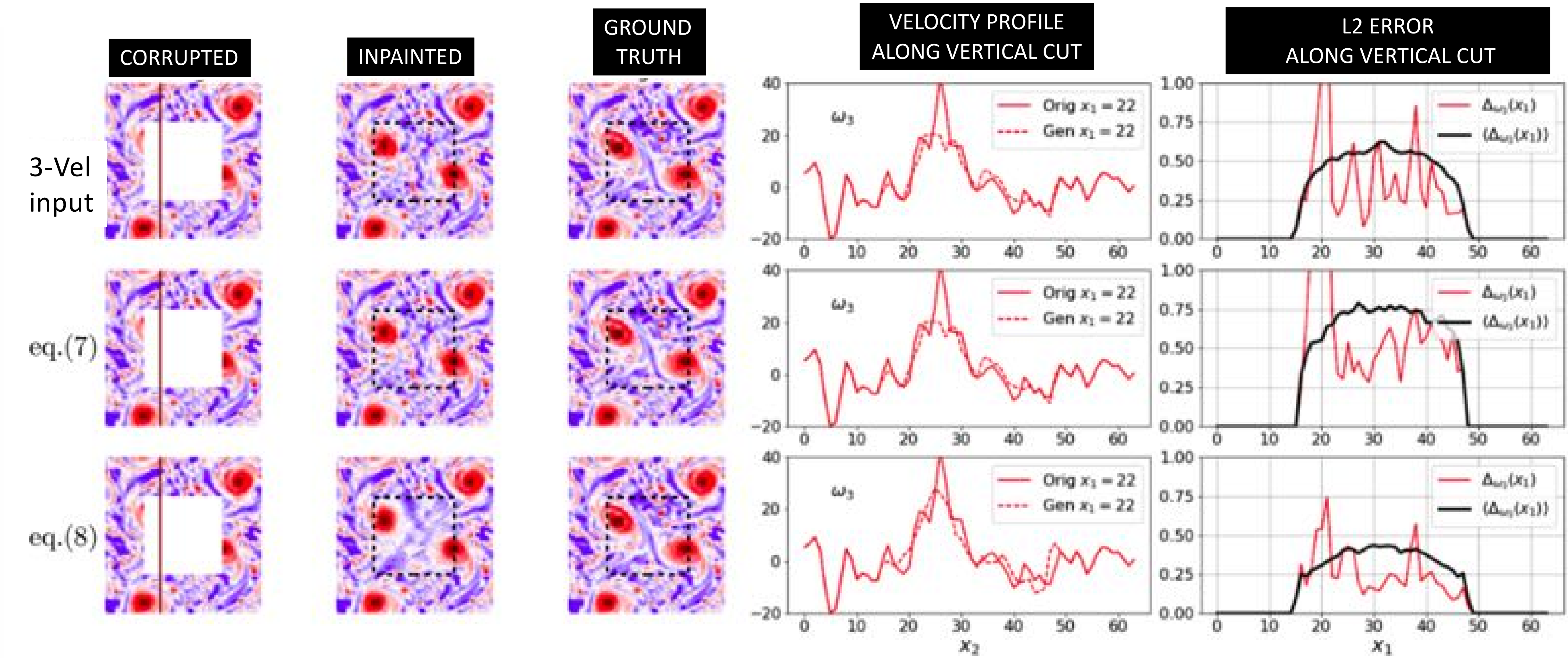


FIG. 10: Scatter plot of the maximum values inside the reconstructed region of the (a) velocity and (b) vorticity calculated from the original data and the one produced by CE1. Colours are proportional to the density of points in the scatter plot.

FEATURES RANKING: VORTICY DATA ASSIMILATION AT CHANGING INPUT CHANNELS AND COSTS

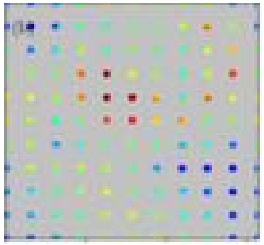
$$\mathcal{L}_{rec} = \mathbb{E}_{\mathcal{I}_\omega} ||\hat{M} \odot (\mathcal{I}_\omega - G[(1-\hat{M}) \odot \mathcal{I}_\omega])||_2 \tag{7}$$

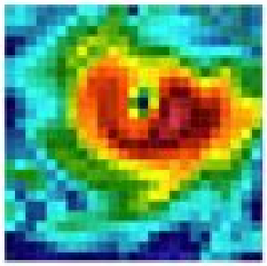
$$\mathcal{L}_{rec} = \alpha \mathbb{E}_{\mathcal{I}_v} \{||\hat{M} \odot (\mathcal{I}_v - G[(1-\hat{M}) \odot \mathcal{I}_v])||_2\} + (1-\alpha)\mathbb{E}_{\mathcal{I}_v} \{||\hat{M} \odot (\omega_3 \odot \mathcal{I}_v - \omega_3 \odot G[(1-\hat{M}) \odot \mathcal{I}_v])||_2\} \tag{8}$$

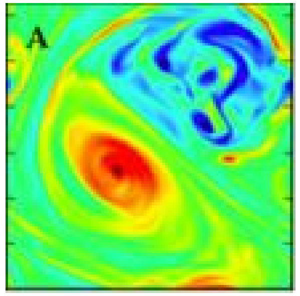


NUDGING: AN **EQUATION-INFORMED UNBIASED** TOOL TO ASSIMILATE AND RECONSTRUCT TURBULENCE DATA/PHYISICS BY ADDING A DRAG TERM AGAINST PARTIAL FIELD MEASUREMENTS

$$\mathbf{v}_N = G[\mathbf{v}]$$



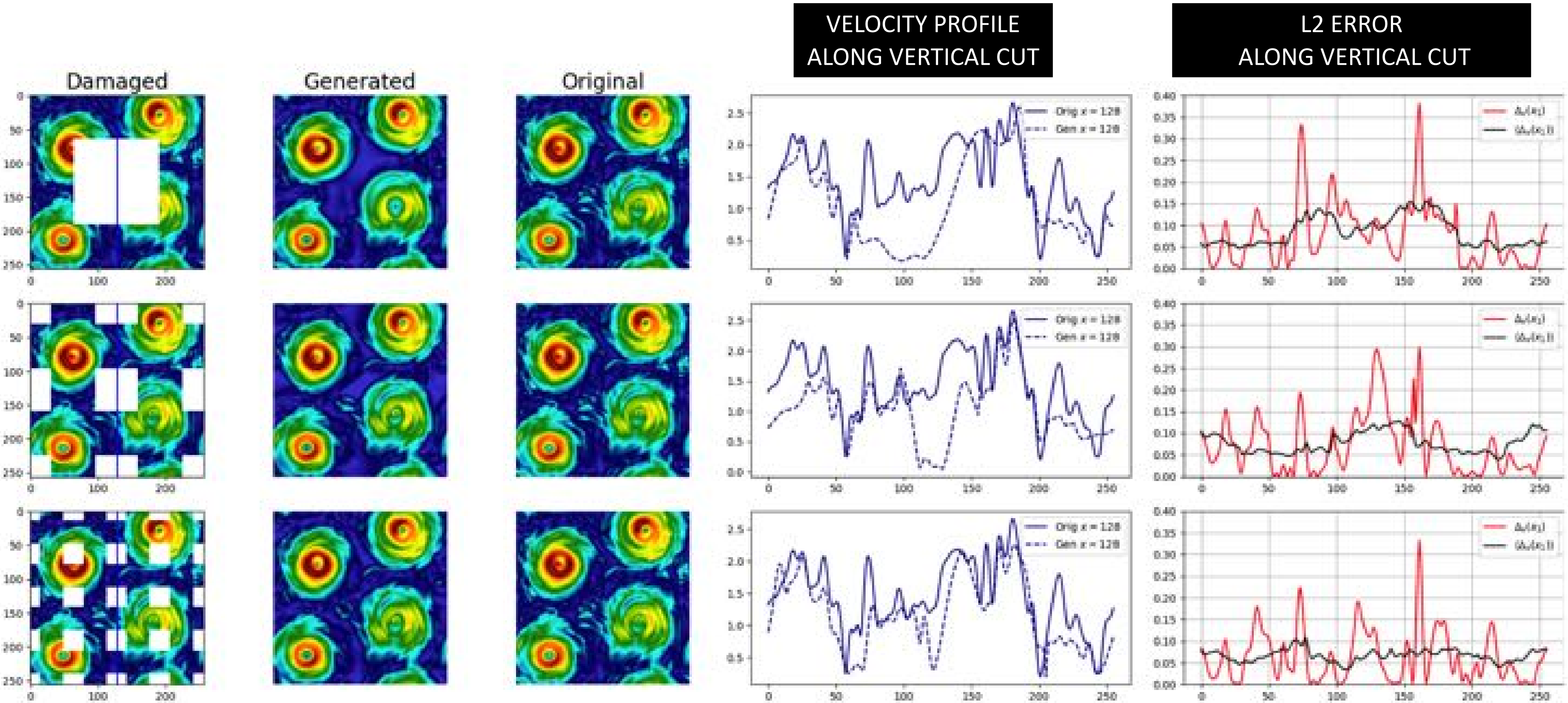




$$\begin{cases} \partial_t \mathbf{v} + \mathbf{v}\cdot\partial_{\mathbf{x}}\mathbf{v} + \partial_{\mathbf{x}} P - \nu\Delta\mathbf{v} = 2\mathbf{v}\times\boldsymbol{\Omega} + \mathcal{S}\mathbf{v} + \alpha g\hat{\mathbf{z}}T + \mathcal{F} - N(\mathbf{v}_N - \mathbf{v}) \\ \partial_x \mathbf{v} = 0 \end{cases}$$

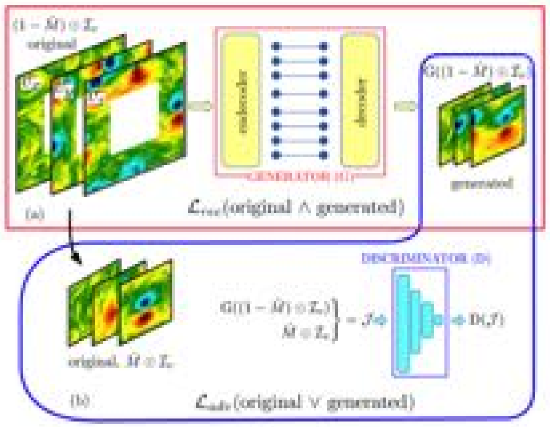
C.C. Lalescu, C. Meneveau and G.L. Eyink. Synchronization of Chaos in Fully Developed Turbulence. Phys. Rev. Lett. 110, 084102 (2013)
A. Farhat, E. Lunasin, and E.S. Titi. Abridged Continuous Data Assimilation for the 2d Navier-Stokes Equations Utilizing Measurements of Only One Component of the Velocity Field. J. Math. Fluid Mech. 18(1), 1 (2016)

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} + 2\Omega \hat{x}_3 \times \boldsymbol{v} = -\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{v} + \gamma (1 - \hat{M})_{x_3} \odot (\boldsymbol{v} - \boldsymbol{v}_{\text{ref}})$$



Patricio Clark Di Leoni, Andrea Mazzino, and Luca Biferale. Synchronization to Big Data: Nudging the Navier-Stokes Equations for Data Assimilation of Turbulent Flows. *Physical Review X*, 10(1):011023, February 2020. Publisher: American Physical Society.

CONTEXT ENCODER 1



CNN-GAN

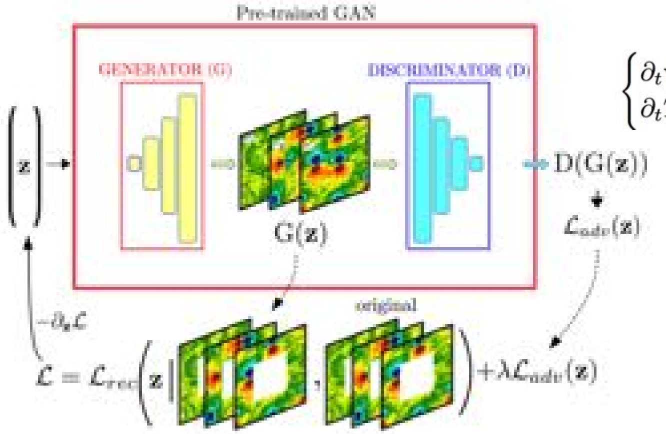
-EQUATION-FREE

GENERATION OF MISSING DATA ONLY

+ONCE TRAINED -> INSTANTANEOUS

+MIXED INPUT FEATURES

CONTEXT ENCODER 2



PRETRAINED CNN-GAN

-EQUATION-FREE

GENERATION OF FRAME & MISSING DATA

-NEW MINIMIZATION FOR EACH DA

+MIXED INPUT FEATURES

NUDGING

$$\begin{cases}\partial_t \mathbf{v} + \mathbf{v}\cdot\partial_{\mathbf{x}}\mathbf{v} + \partial_{\mathbf{x}} P - \nu\Delta\mathbf{v} = 2\mathbf{v}\times\mathbf{\Omega} + \mathcal{S}\mathbf{v} + \alpha g\hat{\mathbf{z}}T + \mathcal{F} - N(\mathbf{v}_N - \mathbf{v}) \\ \partial_t T + \mathbf{v}\cdot\partial_{\mathbf{x}} T - \chi\Delta T = \mathcal{G} v_z + \mathcal{L} - N_T(T_N - T)\end{cases}$$

+EQUATION-INFORMED

GENERATION OF FRAME & MISSING DATA

-NEW 3D DNS FOR EACH DA

-RESTRICTED INPUT FEATURES

Optimal control of point-to-point navigation in turbulent flows using Reinforcement Learning

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University of Rome 'Tor Vergata'
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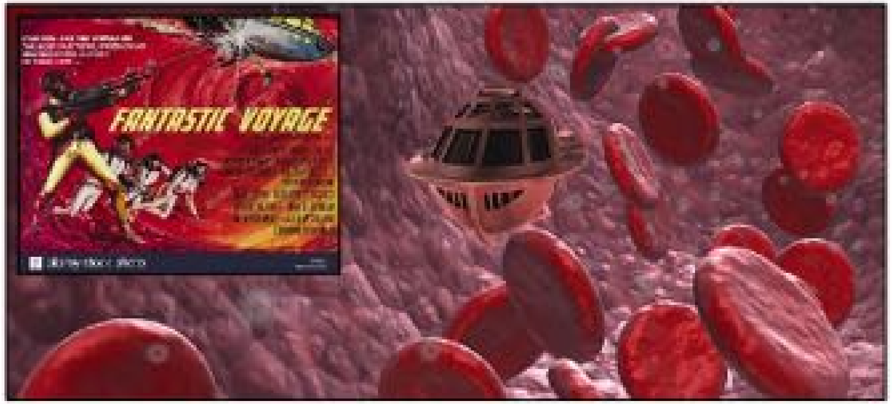
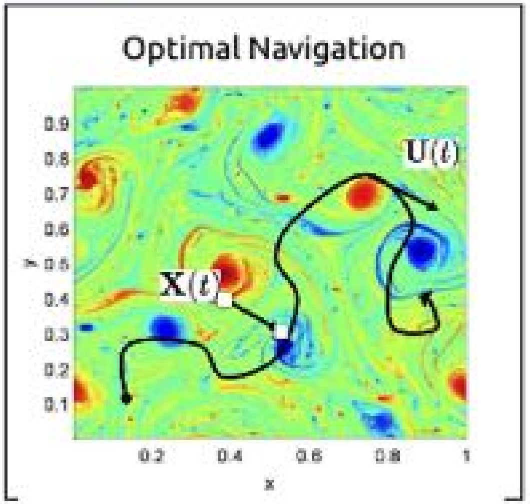
michele.buzzicotti@roma2.infn.it





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https://gdp.ucsd.edu/ldl/

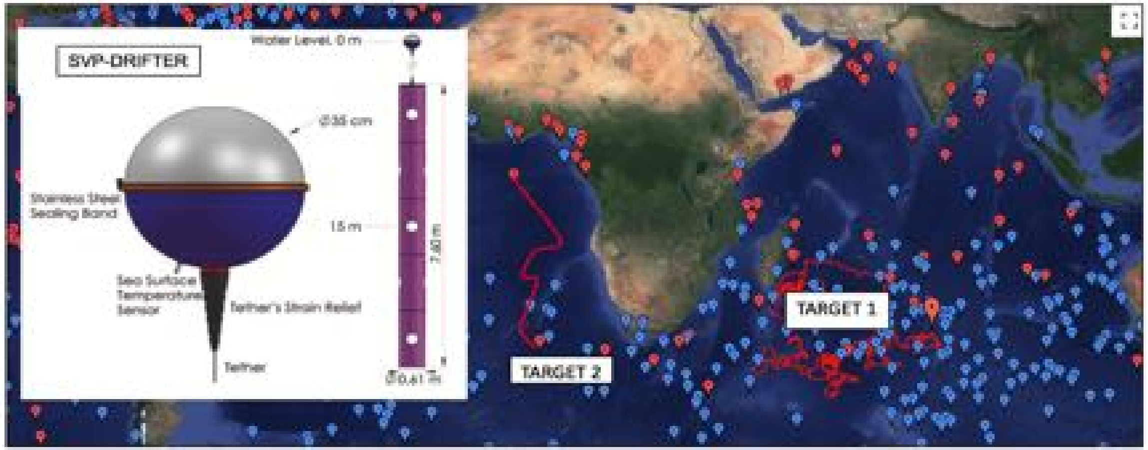
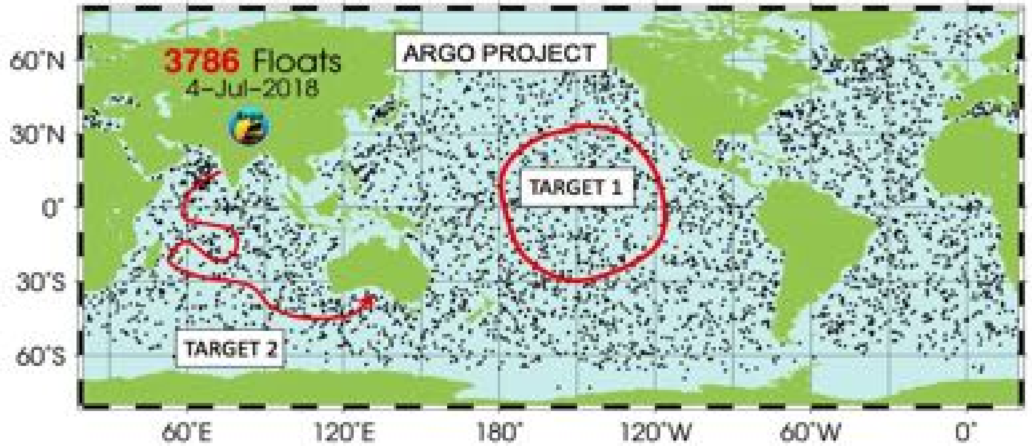


FIG. 1: INSTANTANEOS WORLDWIDE DISTRIBUTION OF DRIFTERS FROM THE GLOBAL DRIFTER MAP PROGRAM [www0]. WE HIGHLIGTHED IN RED TWO POTENTIAL TARGETS: (1) KEEP THE PROBES INSIDE A GIVEN REGION OR (2) MINIMISING THE NAVIGATION TIME AMONG TWO END-POINTS (ZERMELO PROBLEM). INSET: A SKETCH OF THE DRIFTER WITH THE LONG DROUGE AT 15M DEPTH.

FIG. 2: INSTANTANEOS WORLDWIDE DISTRIBUTION OF DRIFTERS. WE HIGHLIGTHED IN RED TWO POTENTIAL TARGETS: (1) KEEP THE PROBES INSIDE A GIVEN REGION OR (2) OPTIMISING THE NAVIGATION AMONG TWO END-POINTS



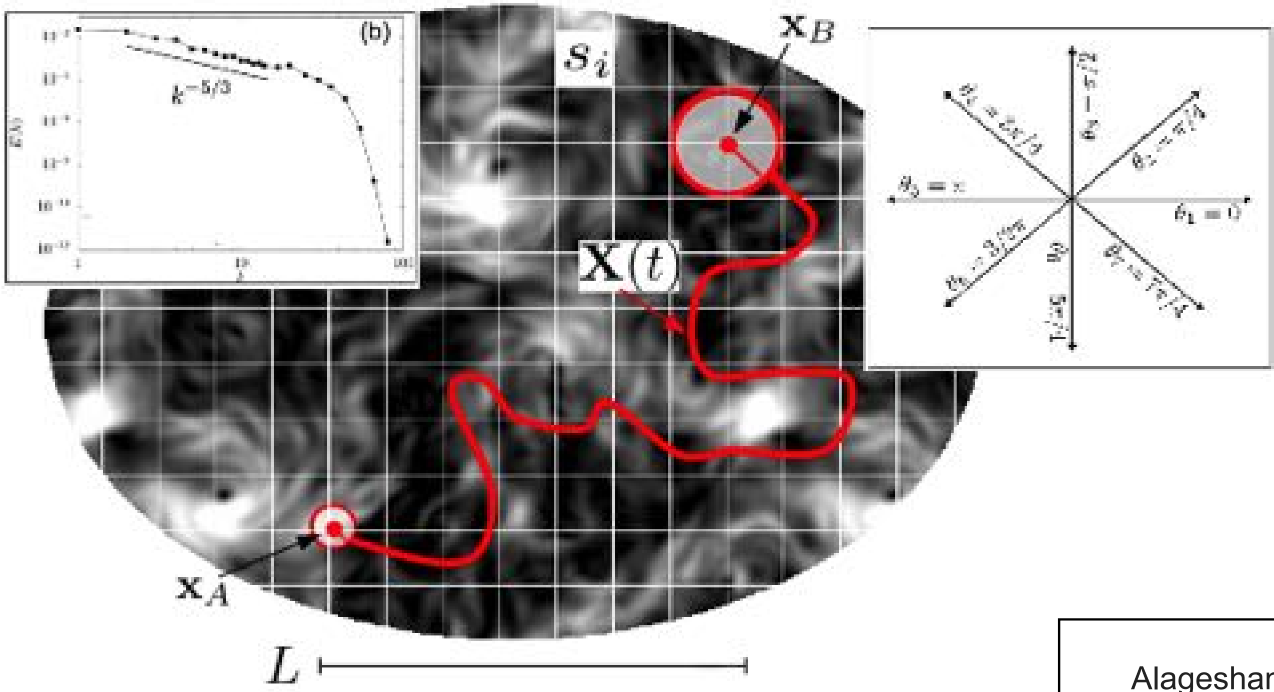
GLOBAL DRIFTER PROGRAM
ARGO PROGRAM

D. Roemmich, G.C. Johnson, S. Riser, R. Davis et al.
The Argo Program: Observing the global ocean with profiling floats.
Oceanography 22, 34 (2009)

Zermelo's problem: Optimal point-to-point navigation in 2D turbulent flows using Reinforcement Learning

L. Biferale,[1] F. Bonaccorso,[1,2] M. Buzzicotti,[1] P. Clark Di Leoni,[1,3] and K. Gustavsson[4]

Chaos: An Interdisciplinary Journal of Nonlinear Science
29.10 (2019): 103138.
arXiv preprint:1907.08591



$$\begin{cases} \dot{\boldsymbol{X}}_t = \boldsymbol{u}(\boldsymbol{X}_t) + \boldsymbol{U}^{ctrl}(\boldsymbol{X}_t) \\ \boldsymbol{U}^{ctrl}(\boldsymbol{X}_t) = V_s \boldsymbol{n}(\boldsymbol{X}_t) \end{cases}$$

$$\boldsymbol{n}(\boldsymbol{X}_t) = (\cos[\theta_t], \sin[\theta_t]) \, ,$$

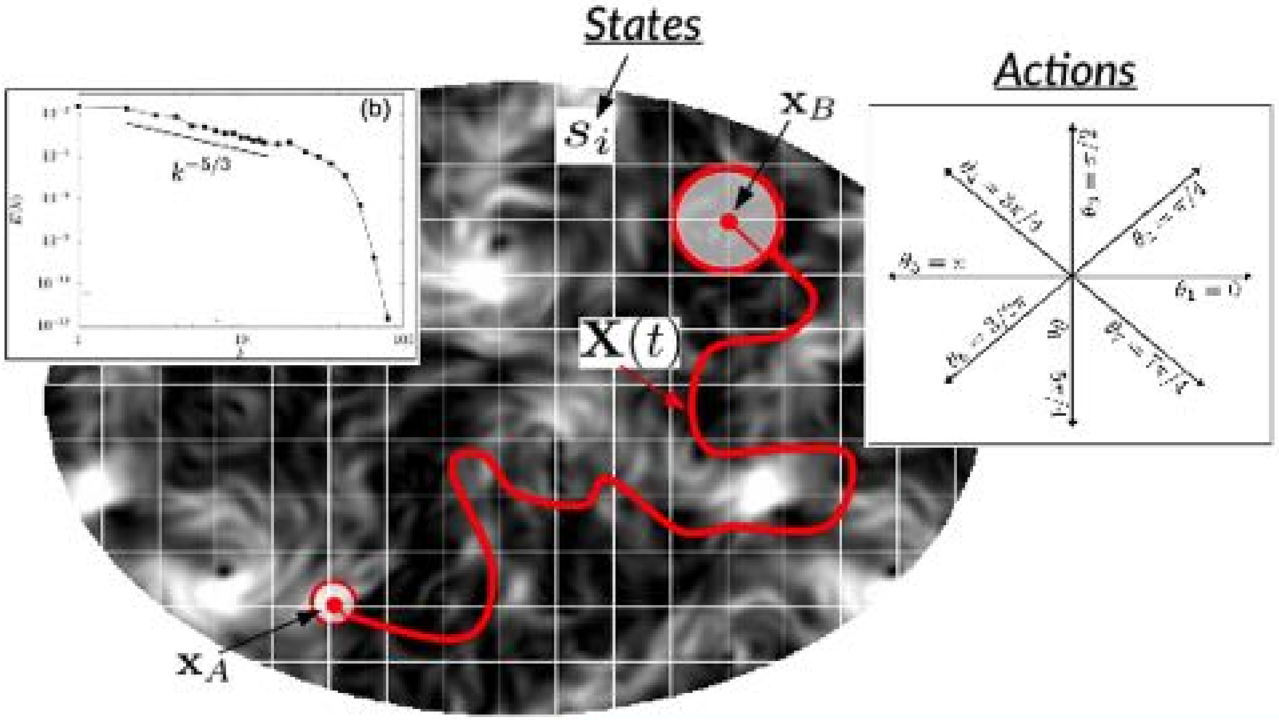
V_s → <u>Navigation speed is small compared to the velocity of the underling flow!</u>

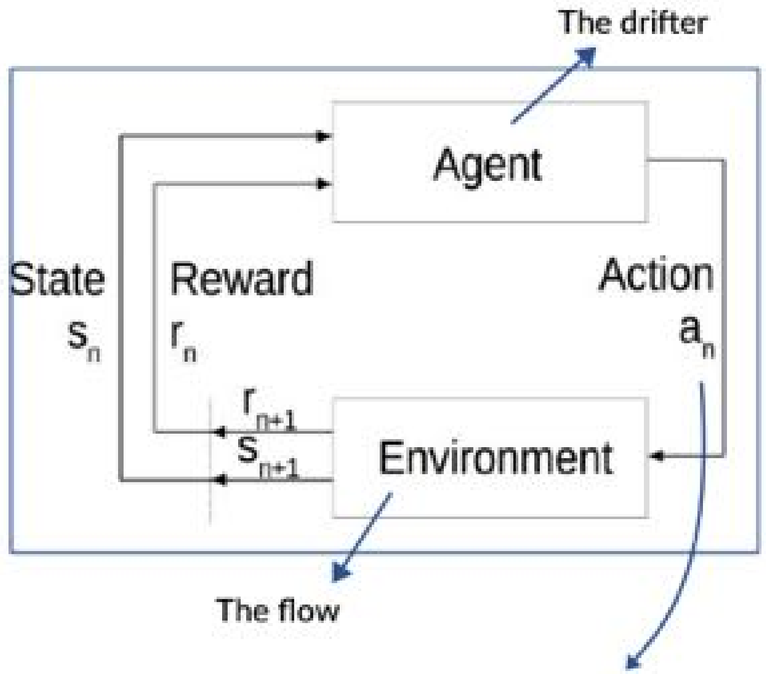
Alageshan, J. K., Verma, A. K., Bec, J., & Pandit, R. (2020). Machine learning strategies for path-planning microswimmers in turbulent flows. *Physical Review E*, *101*(4), 043110.

E. Zermelo, "Über das navigationsproblem bei ruhender oder veränderlicher windverteilung," ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik **11**, 114–124 (1931).

A. E. Bryson and Y. Ho, *Applied optimal control: optimization, estimation and control* (New York: Routledge, 1975).

Reinforcement Learning; *Policy Gradient Methods*





Parameterized policy:

$$\pi(a_j|s_i, \boldsymbol{q}) = \frac{\exp h(s_i, a_j, \boldsymbol{q})}{\sum_{k=1}^{N_a} \exp h(s_i, a_k, \boldsymbol{q})}$$

Parameterized state value function:

$$\hat{v}(s_i, \boldsymbol{w}) = \sum_{j=1}^{N_s} w_j \delta_{j,i}$$

Reward

$$r_t = -\Delta t$$

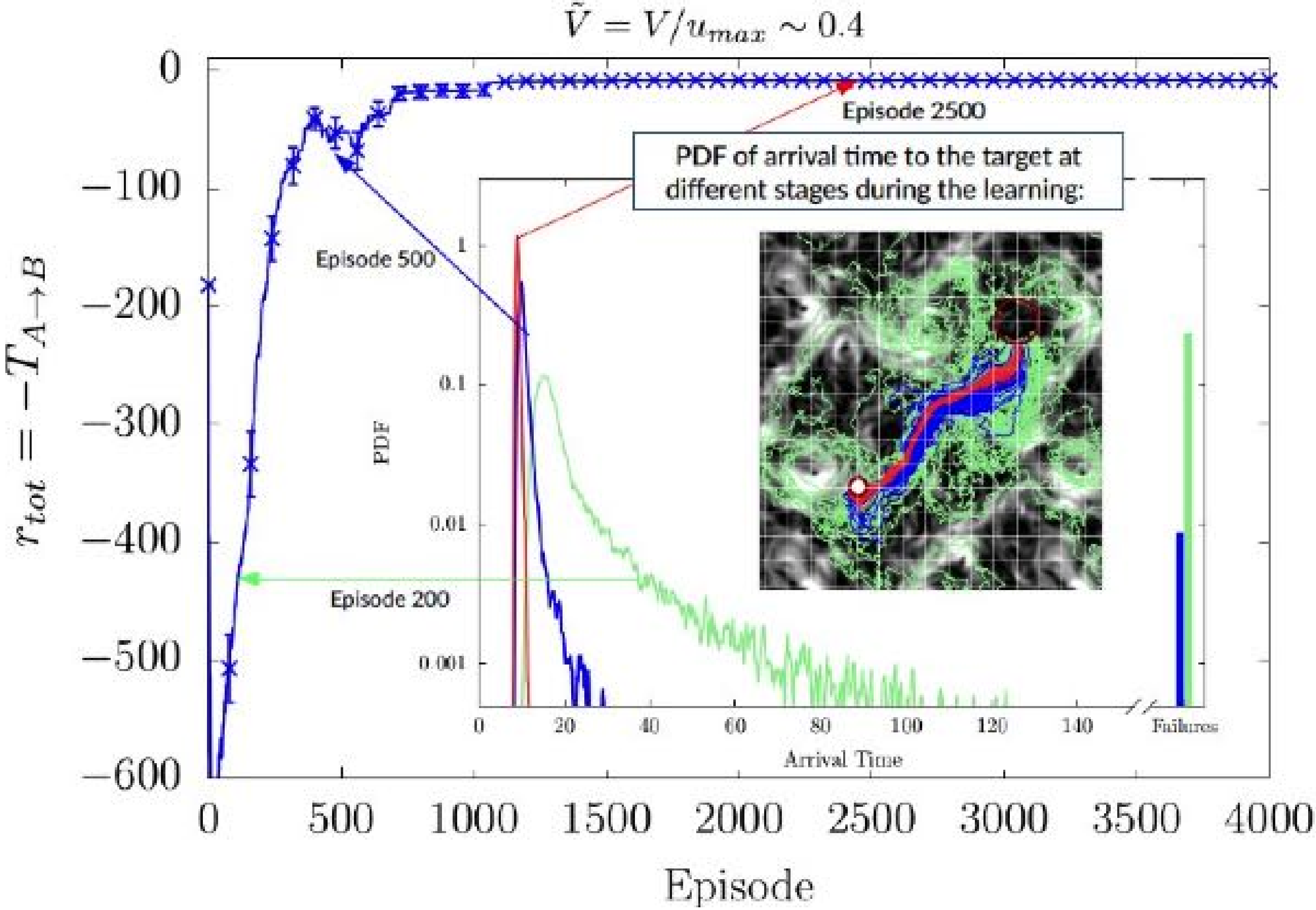
$$r_{tot} = -T_{A \to B}$$

Actor-Critic algorithm

$$\begin{cases} \boldsymbol{q}_{t+\Delta t} = \boldsymbol{q}_t + \alpha_t \beta_t \nabla_{\boldsymbol{q}} \ln(\pi(a_t|s_t, \boldsymbol{q}_t)) \\ \boldsymbol{w}_{t+\Delta t} = \boldsymbol{w}_t + \alpha'_t \beta_t \nabla_{\boldsymbol{w}} \hat{v}(s_t, \boldsymbol{w}_t) \end{cases}$$

$$\beta_t = [\hat{r}_{t+\Delta_t} - \hat{v}(s_t, \boldsymbol{w}_t)] \to \text{ baseline}$$

V_s

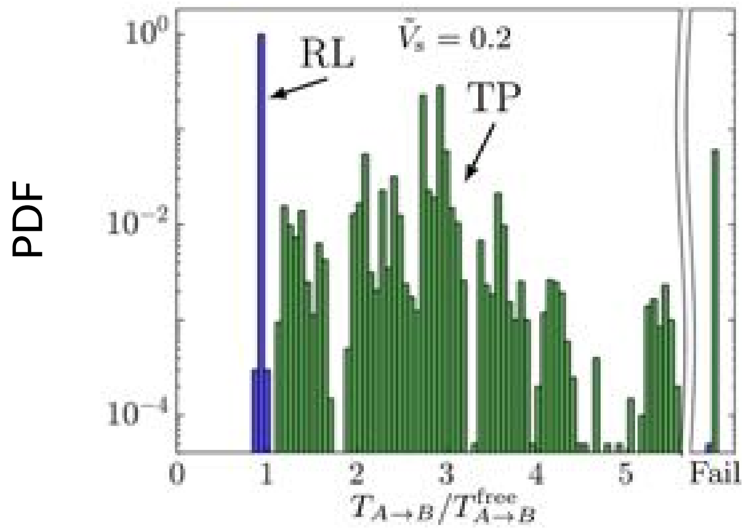


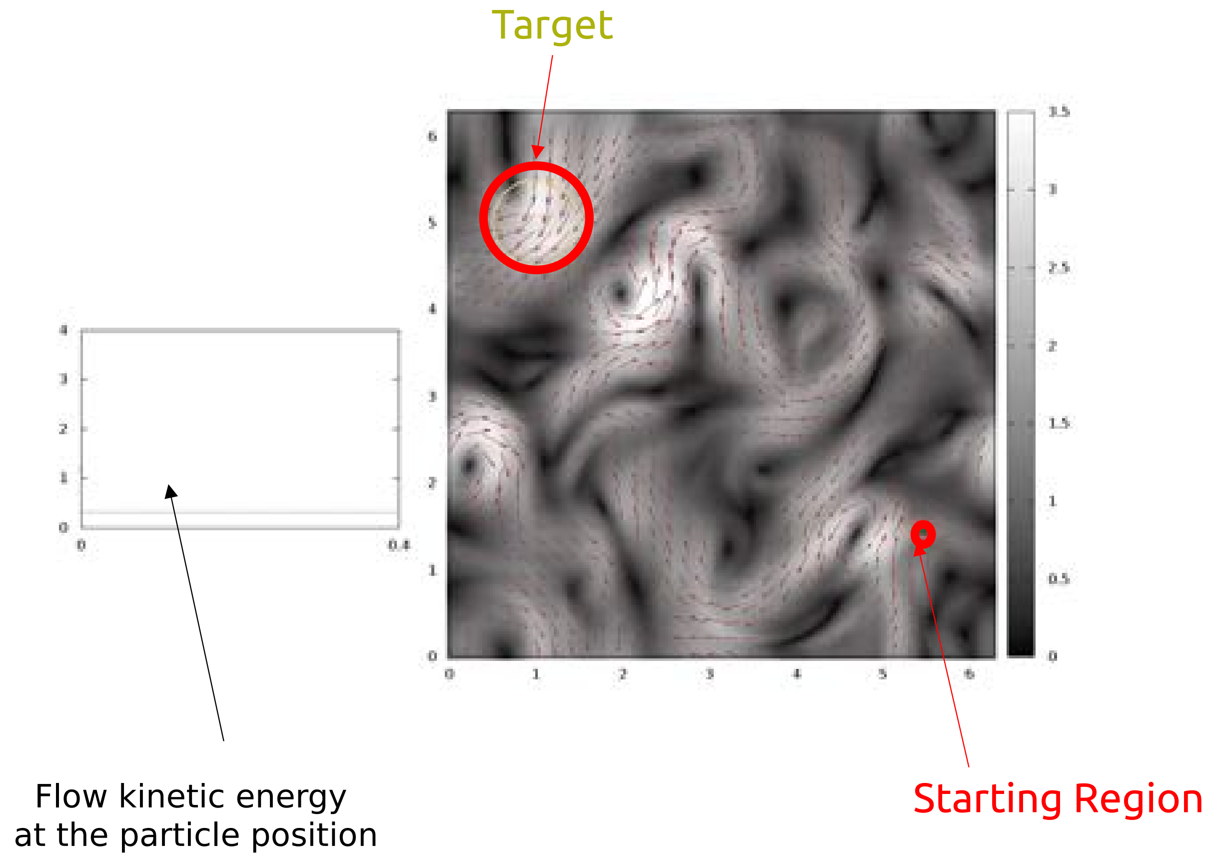
$\tilde{V} = V/u_{max} \sim 0.4$
$r_{tot} = -T_{A \to B}$
0
−100
−200
−300
−400
−500
−600
0
500
1000
1500
2000
2500
3000
3500
4000
Episode
Episode 2500
Episode 500
Episode 200
PDF of arrival time to the target at different stages during the learning:
PDF
1
0.1
0.01
0.001
0
20
40
60
80
100
120
140
Failures
Arrival Time

TIME-DEPENDENT 2D TURBULENT FLOWS

REINFORCEMENT LEARNING (BLUE) VS TRIVIAL POLICY (GREEN) $\tilde{V}_s = 0.2$

Comparison with a Trivial Policy (TP)





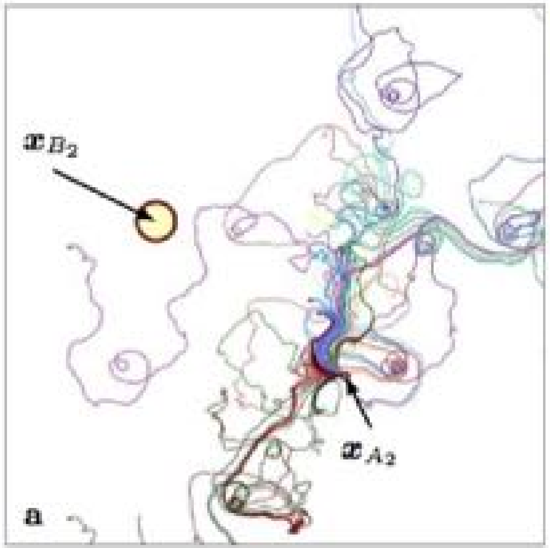
COMPARISON RL VS OPTIMAL NAVIGATION

A. E. Bryson and Y. Ho, Applied optimal control: optimization, estimation and control (New York: Routledge, 1975).

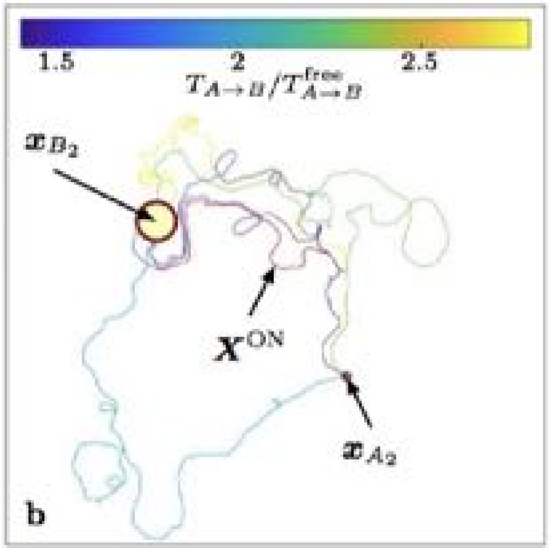
Time independent flow

$$\begin{cases}\dot{\boldsymbol{X}}_t = \boldsymbol{u}(\boldsymbol{X}_t) + \boldsymbol{U}^{ctrl}(\boldsymbol{X}_t) \\ \boldsymbol{U}^{ctrl}(\boldsymbol{X}_t) = V_s \boldsymbol{n}(\boldsymbol{X}_t)\end{cases} \qquad \boldsymbol{n}(\boldsymbol{X}_t) = (\cos[\theta_t], \sin[\theta_t]) , \qquad A_{ij} = \partial_i u_j$$

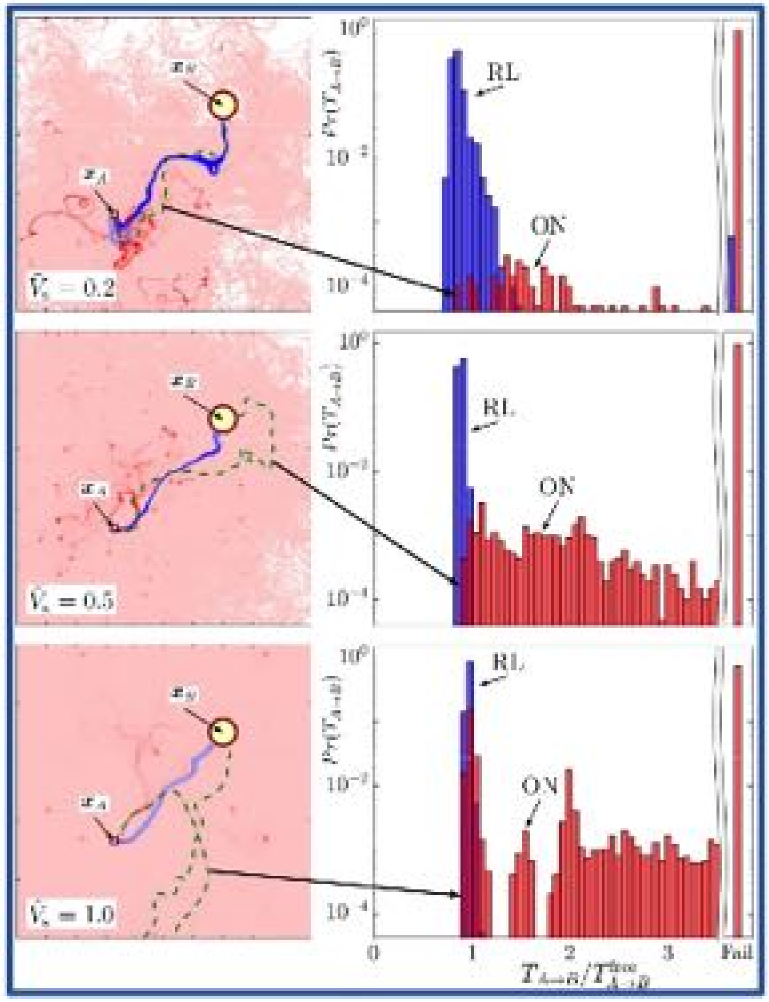
$$\dot{\theta}_t = A_{21} \sin^2 \theta_t - A_{12} \cos^2 \theta_t + (A_{11} - A_{22}) \cos \theta_t \sin \theta_t ,$$



1000 trials (all failures)



100k trials (10 successes)



- EQUATION-FREE VS EQUATION-INFORMED MODELS TO CLASSIFY, ASSIMILATE OR NAVIGATE TURBULENT DATA
- BOTH UNBIASED (NO NEED TO KNOW THE PRIOR DISTRIBUTION)
- PROBING QUALITY AND QUANTITY OF INFORMATION
- SUPPLY AN 'AUGMENTED REALITY' FLOW REPRESENTATION OUT OF A SET OF ROUGH MEASUREMENTS (GEO OR BIO DATA)

(1) Reconstruction of turbulent data with deep generative models for semantic inpainting from TURB-Rot database. M. Buzzicotti, F. Bonaccorso, P. Clark di Leoni, L.B. submitted to Physical Review Fluids (2020) arXiv:2006.09179v1

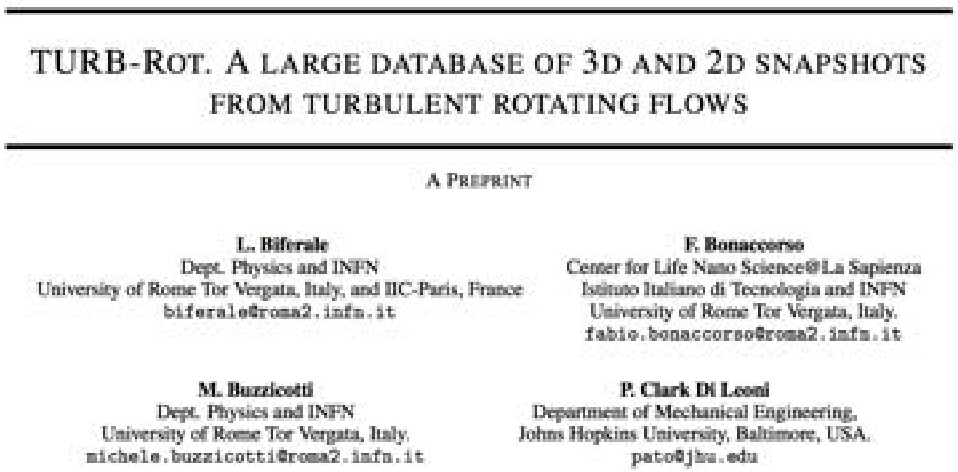
(2) Inferring flow parameters and turbulent configuration with physics-informed data-assimilation and spectral nudging. P. Clark Di Leoni, A. Mazzino, L.B. Phys. Rev. Fluids 3, 104604, 2018

(2) Synchronization to big-data: nudging the Navier-Stokes equations for data assimilation of turbulent flows. P. Clark Di Leoni, A. Mazzino, L.B. PRX in press 2020, arXiv:1905.05860

(3) Zermelo's problem: Optimal point-to-point navigation in 2D turbulent flows using Reinforcement Learning. L.B., F. Bonaccorso, M. Buzzicotti, PC Di Leoni, K Gustavsson Chaos 29, 103138, 2019

(4) Smart Inertial Particles. S. Colabrese, K. Gustavsson, A. Celani, L.B. Physical Review Fluids 3 (8), 084301, 2018

(5) Flow navigation by smart microswimmers via reinforcement learning. S. Colabrese, K. Gustavsson, A. Celani, L.B. Physical Review Letters 118 (15), 158004, 2017



TURB-ROT. A LARGE DATABASE OF 3D AND 2D SNAPSHOTS FROM TURBULENT ROTATING FLOWS

A PREPRINT

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arXiv:2006.07469v1

https://smart-turb.roma2.infn.it/