Recent developments in Non-Eq. QFT

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Invitation

- Non-equilibrium field theory is really a very old subject.
- Reminder : Fluid dynamics (arguably the oldest field theory) is a non-equilbrium theory
- Given this fact, we understand surprisingly little : especially Non-eq. QFTs
- One objective way to say this is to point out the relatively few textbooks that cover the subject.

Schwinger Keldysh Path integral

In principle, textbooks give an exact definition of Non-equilibrium QFT via **Schwinger/Keldysh(SK) Path integrals** : Can write down a path integral for evolution of mixed states.

- Ket field ϕ_R and Bra field ϕ_L.
- For a unitary QFT, $S_{SK} = S[\phi_R] - S[\phi_L]$.
- solve with appropriate past/future conditions which set $\phi_R = \phi_L$ at far future .
- gives the following generating function

$$Z_{SK}[J_R, J_L] = \text{Tr}\{\ U[J_R]\ \rho_{initial}\ (U[J_L])^\dagger\ \}$$

Two issues

- computes the (singly) out of time ordered correlators. So this definition is actually incomplete (more on this later).
- Is this a practical object to compute/characterise ?

Schwinger Keldysh Path integral II : Perturbative

- In this business, one quickly learns that SK path integral is rarely used.
- In perturbative QFTs, one often approximates it by Kinetic theory/classical statistical approximations and works with them instead.
- Most of the discussion is usually about linear response/two point correlations/spectral functions
- Much of our understanding is perturbative : no systematic formalism yet to incorporate instantons etc., in real time
- unlike Euclidean case where the situation is much better especially since Lattice QCD : existing approaches try to analytically continue from here
- Not many examples of non-equilibrium dualities (AdS/CFT being a notable exception relating non-equilibrium physics in certain gauge theories to non-equilibrium gravity)

Schwinger Keldysh Path integral III : Holography

- Even in AdS/CFT, we do not actually know how to compute SK path integrals !
- This is the reason why Non-equilibrium gravity in AdS/CFT almost always ignores Hawking radiation effects
- Especially interacting Hawking physics/ BH evaporation is ill-understood quantitatively despite its centrality to information paradox/finite N effects
- What I want to describe today : some modest progress towards addressing these challenges
- Organised into 3 themes :
  - Out of time ordered correlations (OTOCs)
  - Open EFTs
  - Schwinger Keldysh Holography
- New perspectives and puzzles rather than complete answers !

- Surprisingly, SK path integral does not answer every question one can ask of a QFT !
- For example, one can define a generating function

$$Z_{SK}^{(2)}[J_{Ri}, J_{Li}] = \text{Tr}\{ \ (U[J_{L2}])^{\dagger} U[J_{R1}] \ \rho_{initial} \ (U[J_{L1}])^{\dagger} \ U[J_{R2}] \ \}$$

- computes the correlators with a maximum of two time-ordering violations.
- Can string together $2k$ such alternating evolution/inverse evolution operators to define $Z_{SK}^{(k)}$.
- $Z_{SK}^{(k)}$ generates correlators with a maximum of k time-ordering violations.

Hierarchy of OTOCs

- Note that the sequence of generating functions $Z_{SK}^{(k)}$ have a natural and exact hierarchy among them.
- Taking $Z_{SK}^{(k)}$ and setting equal the sources in two adjacent U and $U^\dagger$, we get $Z_{SK}^{(k-1)}$.
- All information in $Z_{SK}^{(k-1)}$ is hence inside $Z_{SK}^{(k)}$
- Think of $Z_{SK}^{(k)}$ as giving a sequence of QFTs which incorporate more and more fine non-equilibrium correlations as we increase k.
- A product of n operators cannot have more than $(n+1)/2$ time-ordering violations
- so all n-pt correlations with any time ordering is captured by $Z_{SK}^{((n+1)/2)}$
- Thus, finer and finer OTO correlations are possible only for higher and higher point functions

arXiv:1701.02820 Felix M. Haehl, RL, Prithvi Narayan, Mukund Rangamani

Hierarchy of OTOCs II

- While this argument is elementary, the result calls for a completely new way of thinking about any non-equilibrium QFT :
  - Correlations in a general state of a QFT have a Russian doll like structure
  - finer correlations do not affect less finer correlations : an exact statement
  - finer correlations show up only in higher point functions with a lot of time ordering violations
- OTO KMS constraints appear in the thermal case

  arXiv:1706.08956 Felix M. Haehl, RL, Prithvi Narayan, Amin A. Nizami, Mukund Rangamani

  arXiv:1810.03118 Soumyadeep Chaudhuri, Chandramouli Chowdhury, RL
- Nothing in say kinetic theory suggests such a structure ! Systematic kinetic theory level understanding slowly emerging

  arXiv:1512.07687 Douglas Stanford , arXiv:1609.01251 Igor L. Aleiner, Lara Faoro, Lev B. Ioffe

  arXiv:1703.07353 Aavishkar A. Patel, Debanjan Chowdhury, Subir Sachdev, Brian Swingle

  arXiv:1703.02545 Debanjan Chowdhury, Brian Swingle , arXiv:1804.09182 Grozdanov-Schalm-Scopelliti

What does this mean ? Why should we care ?

- Seems we need an infinite sequence of distribution functions/quasi-particles describing the transport of finer and finer correlations. . .
- What does this mean ? What physics is being captured ? My opinion is we do not yet know. . .
- Compare Schroedinger/von Neumann/EPR's introduction of entanglement in QM or introduction of BHs/CC in GR.
- They are mathematically well-defined objects containing novel info and most probably have experimental consequences.
- What we do know ? OTOCs with 2 time violations can be used to
  - diagnose chaos, study scrambling,
  - to give one measure of how close a theory is to being holographic to classical gravity.

    arXiv:1306.0622 Stephen H. Shenker, Douglas Stanford

    arXiv:1503.01409 Juan Maldacena, Stephen H. Shenker, Douglas Stanford
- **Need for a systematic study and exploration**

Second theme : Open EFTs

- ‘Open’ as in open quantum systems : QFT with ‘environmental’ parts traced out.
- Environment could be a light but fast d.o.f. - think of brownian particle in water.
- Important for many reasons. some are
  1. Understand decoherence and dissipation in QFT.
  2. Cosmology: QFT in dS, fields falling out of dS horizon.
  3. Black Holes : via AdS/CFT, BHs dual to a dissipative subsector of CFT. (BHs$\sim$ open CFT ?)
- Thus there are good reasons to set up a formalism for open QFTs/CFTs study their renormalisation etc.
- For example, necessary for a non-perturbative understanding of emergence of Hydro : can we think of it as a fixed point of RG flow ?

Toy model : Open ϕ^4 Theory

- To be precise, start with a simple model of an open ϕ^4 theory :

$$-\int d^dx\Big[\frac{1}{2}z\,(\partial\phi_{\rm R})^2+\frac{1}{2}m^2\phi_{\rm R}^2+\frac{\lambda_4}{4!}\phi_{\rm R}^4+\frac{\sigma_4}{3!}\phi_{\rm R}^3\phi_L\Big]$$
$$+\int d^dx\Big[\frac{1}{2}z^\star(\partial\phi_{\rm L})^2+\frac{1}{2}m^{\star 2}\phi_L^2+\frac{\lambda_4^\star}{4!}\phi_{\rm L}^4+\frac{\sigma_4^\star}{3!}\phi_{\rm L}^3\phi_{\rm R}\Big]$$
$$+i\int d^dx\Big[z_\Delta\,(\partial\phi_R).(\partial\phi_L)+m_\Delta^2\phi_{\rm R}\phi_{\rm L}+\frac{\lambda_\Delta}{2!2!}\phi_R^2\phi_L^2\Big]$$

arXiv:1704.08335 Avinash, Chandan Jana, R. Loganayagam, Arnab Rudra

- e^{iS} should be invariant under $R\leftrightarrow L$ along with complex conjugation
- Further, this action should vanish when $\phi_R=\phi_L$
- This implies

$$\mathrm{Im}\, z-z_\Delta=0\,,\quad \mathrm{Im}\, m^2-m_\Delta^2=0\,,\quad \mathrm{Im}\,\lambda_4+4\,\mathrm{Im}\,\sigma_4-3\lambda_\Delta=0\,.$$

- Is this preserved under renormalisation ? If yes, why ?

Beta fn. of Open ϕ^4

Standard methods applied to 1 loop perturbation theory gives

$$\frac{dm^2}{d\,\ln\mu} = \frac{m^2}{(4\pi)^2}\left[\lambda_4 + 2\sigma_4 - i\lambda_\Delta\right]$$

$$\frac{dm_\Delta^2}{d\,\ln\mu} = \frac{2}{(4\pi)^2}\mathrm{Re}\left[m^2(\lambda_\Delta + i\sigma_4)\right]$$

and

$$\frac{d\lambda_4}{d\,\ln\mu} = \frac{3}{(4\pi)^2}(\lambda_4 + 2\,\sigma_4 - i\lambda_\Delta)(\lambda_4 + i\lambda_\Delta)$$

$$\frac{d\sigma_4}{d\,\ln\mu} = \frac{3}{(4\pi)^2}(\lambda_4 + \sigma_4 + \sigma_4^\star + i\lambda_\Delta)(\sigma_4 - i\lambda_\Delta)$$

$$\frac{d\lambda_\Delta}{d\,\ln\mu} = \frac{1}{(4\pi)^2 i}\left[(\lambda_4 + 2\sigma_4^\star)(\sigma_4^\star + i\lambda_\Delta) + 3i\sigma_4\lambda_\Delta - c.c.\right]$$

Beta fn. preserves Lindblad !

This can be used to give

$$\begin{aligned} &\frac{d}{d\ \ln\mu}(\mathrm{Im}\, m^2 - m_\Delta^2) \\ = \quad &\frac{2}{(4\pi)^2}(\mathrm{Im}\lambda_4 \ + 4\mathrm{Im}\,\sigma_4 - 3\lambda_\Delta)(\mathrm{Re}\, m^2) \end{aligned}$$

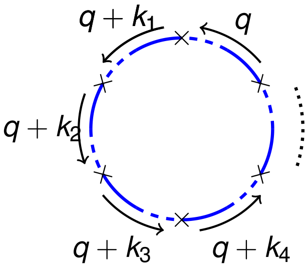
and

$$\begin{aligned} &\frac{d}{d\ \ln\mu}(\mathrm{Im}\,\lambda_4 + 4\mathrm{Im}\,\sigma_4 - 3\lambda_\Delta) \\ = \quad &\frac{6}{(4\pi)^2}(\mathrm{Im}\ \lambda_4 + 4\ \mathrm{Im}\ \sigma_4 - 3\lambda_\Delta)(\mathrm{Re}\ \lambda_4 + 2\mathrm{Re}\ \sigma_4) \end{aligned}$$

So Lindblad conditions once set, do not get corrected (actually to any order in perturbation theory)! So far, so good.

Non-Local divergences

- Not all things are well though
- Consider $D-1$-gon Wightman loop in D space-time dimensional open ϕ^4:



- Thick and dashed lines denote R and L fields

Non-Local divergences in open EFT

$$\int \frac{d^Dq}{(2\pi)^D} 2\pi\ \Theta(q^0)\delta[q^2+m^2] \prod_{j=1}^{D-2} 2\pi\ \Theta(q^0+k_j^0)\delta[(q+k_j)^2+m_j^2].$$

- Naive degree of divergence: $D-2(D-1)=2-D$. Naive Expectation : finite for $D>2$. Turns out to be incorrect !
- Correct counting done after linearising using Dirac delta :

$$\int \frac{d^Dq}{(2\pi)^D} 2\pi\ \Theta(q^0)\delta[q^2+m^2] \prod_{j=1}^{D-2} 2\pi\ \Theta(q^0+k_j^0)\delta[2q\cdot k_j+k_j^2].$$

- The correct counting $D-2-(D-2)=0$

$$\approx \frac{1}{2^{D-2}(2\pi)\Sigma_c}\ln\Lambda.$$

- Σ_c: $\{k_i\}$ Parallelotope volume . $\Sigma_c \sim k^{D-2}$: non-local divergence .

Non-Local divergence II

- Open EFTs have worse divergences than even non-renormalisable theories
- This leads to the breakdown of the standard Wilsonian paradigm in this case.
- Why does this happen ? Underlying the standard paradigm of renormalisation is the operator product expansion (OPE)
- The problem here comes from the fact that OPEs are kind of sick for non-time ordered operator products

  Work in Progress with Subhobrata Chatterjee, Chandan Jana, Arnab Rudra

- We conclude that unless this problem is somehow overcome, it is difficult to make rigorous say the idea of hydro as a RG fixed point.

Third theme : Holographic Open QFTs

- What can holography say about all these ?
- A simple example realisable in AdS/CFT is this : an external probe field Ψ coupled to strongly interacting, large N gauge theory environment (with fields A) via a single trace scalar operator O.

$$S = \int d^d x \; L[\Psi] + LCFT[A] + \Psi \; O.$$

- dual to open QFT of a bulk scalar field Φ heated by blackholes
- Recent development : a formalism to engineer Open QFTs in holography.
- This is good news ! holographic black hole baths are very good baths (compared to weakly coupled descriptions) leading to local open EFTs.
- A clear setting to address long-standing conceptual problems within open QFTs.

The gravitational setup

- Take a planar AdS$_{d+1}$ black brane
- The scalar operator, O in CFT , maps to a scalar field Φ on this background with a contact self-interaction

$$-\int d^{d+1}x \sqrt{-g} \left\{ \frac{1}{2} g^{AB} \partial_A \Phi \partial_B \Phi + \frac{m^2}{2} \Phi^2 + \frac{\lambda_n}{n!} \Phi^n \right\}$$

arXiv:2004.02888 Chandan Jana, RL, Mukund Rangamani

- Self interaction λ_n in bulk models the 1/N-suppressed non-linearities in the gauge theory.
- To do an SK path integral, bulk should be a doubled gravitational SK (grSK) geometry.

The gravitational SK(grSK) geometry

- To do an SK path integral, double this geometry and identify at future time-like boundary .
- Geometry along the time direction: AdS boundaries asymptote to CFT SK contour. Skenderis, van Rees (arXiv:0805.0150, 0812.2909, 0902.4010)
- What to we do at the horizon ?
- This is also a future boundary for the exterior of black brane : suggests that we should identify.
- A conjecture on how to do this was given by

  Crossley-Glorioso-Liu (1812.08785)
- This conjecture reproduces KMS relations for any n point functions :

  arXiv:1906.07762 Bidisha Chakrabarty, Joydeep Chakravarty, Soumyadeep Chaudhuri, Chandan Jana, RL, Akhil Sivakumar

Holographic open ϕ^n EFT

- The interacting bulk scalar theory can be solved in the afore-mentioned geometry and its effective action gives a open ϕ^n theory.
- The gravitational physics is here is of k Hawking fluctuations scattering against $(n-k)$ in-falling modes.
- Study of this kind of this kind of physics is new in non-equilibrium field theory as well as in gravity.
- On perturbative side, this is tantamount to an analysis of Schwinger-Dyson equations for n point functions keeping all fluctuations
- This is beyond current capabilities even using kinetic theory approximations !
- But, in gravity this can be done and leads to novel non-linear FDTs :

arXiv:2004.02888 Chandan Jana, RL, Mukund Rangamani

Fluctuating hydro from Holography

- Many generalisations beyond this toy model is possible : we can include Fermions for example

  Work in progress with Krishnendu Ray,Akhil Sivakumar

- Can one go beyond this toy model and get a hydrodynamic theory with fluctuations ?
- Answer seems to be Yes ! with many more new ideas
  Work in progress with

  Jewel Kumar Ghosh,Siddharth G Prabhu, Mukund Rangamani,Akhil Sivakumar, Vishal Vijayan

- This and many other questions are now within our reach !

‘Open’ questions

- Talked about three new developments in non-eq QFT
  - Out of time ordered correlations (OTOCs)
  - Open EFTs
  - Schwinger Keldysh Holography
- It is also interesting to consider their intersection : OTOCs seem to admit their own open EFTs

  arXiv:1807.09731 Soumyadeep Chaudhuri, RL

  arXiv:1811.01513 Bidisha Chakrabarty, Soumyadeep Chaudhuri, RL

  arXiv:1905.08307 Bidisha Chakrabarty, Soumyadeep Chaudhuri
- Barely scratched the surface of these new developments. Many unanswered questions/ challenges
- How to **derive** open EFTs : problem of initial state, IR divergences

Thank you !