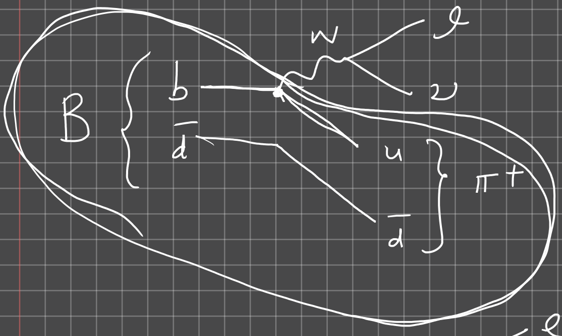
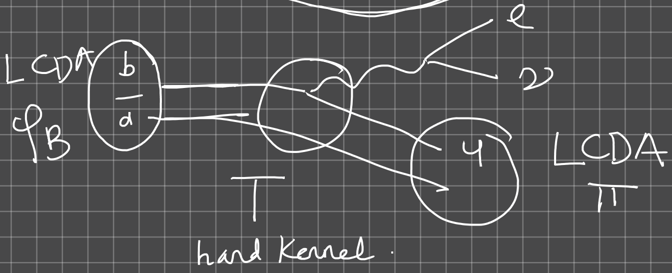
Lecture 3: Flavour and QCD.



$B \to \pi$ form factor
non-perturbative information



$$\varphi_B \otimes T \otimes \varphi_\pi$$

LCDA = light cone distribution amplitude.

$\varphi(u)$ $\qquad B \to p_B$

$$\begin{array}{cc} b & \bar{d} \\ u p_B & (1-u) p_B \end{array}$$

$$\langle P(K) | \underline{\bar{q} \gamma_\mu b} | B(p) \rangle = \left(p_\mu + K_\mu - q_\mu \frac{m_B^2 - m_P^2}{q^2}\right) f_+(q^2) + \frac{m_B^2 - m_P^2}{q^2} q_\mu f_0(q^2)$$

$q^\mu = (p-K)^\mu$. 3FFs f_+, f_0, f_T

7 FFs for $B \to V$ $\quad V, A_{0-3}, T_{1-3}$ $\quad A_3 \Leftrightarrow A_1, A_2$.

Calculation of f_+ in LCSR.

$$\Pi_\mu = i m_b \int d^D x\, e^{-i p_B \cdot x} \langle \pi(p) | T\{\bar{u}(0) \gamma_\mu b(0) \overset{|B\rangle\langle B|}{\uparrow} \bar{b}(x) i \gamma_5 d(x)\} | 0 \rangle$$

$$= (p_B + p)_\mu \Pi_+(p_B^2, q^2) + (p_B - p)_\mu \Pi_-(p_B^2, q^2)$$

$$\Pi_+ = f_B m_B^2 \frac{f_+(q^2)}{m_B^2 - p_B^2} + \int_{s > m_B^2} ds \frac{\rho_{had}}{s - p_B^2} \qquad m_B^2 f_B = m_b \langle 0 | \bar{d}\, i \gamma_5 b | B \rangle$$

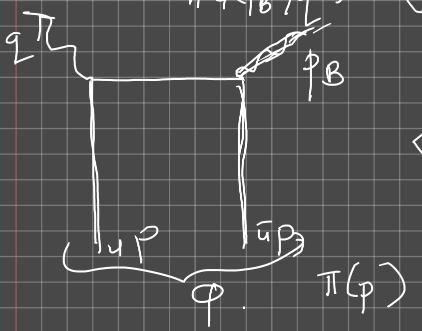
Alternatively in the Euclidean region $p_B^2 - m_B^2$ large + negative light-cone expand around $x^2 = 0$.

$$\Pi_+(p_B^2, q^2) = \sum_n \int du\, T^{(n)}(u, p_B^2, q^2, \mu^2)\, \varphi^{(n)}(u, \mu^2)$$

perturbatively calculable hard kernels

Light cone distribution amplitudes at twist n

At Leading order in α_s and leading twist



$$\Pi_+(p_B^2, q^2) = \underbrace{\frac{f_\pi m_b^2}{2}\int_0^1 du \frac{\varphi(u,\mu^2)}{m_b^2 - u p_B^2 - \bar u q^2}} = \int_0^\infty ds \frac{\rho_{T2}}{s - p_B^2}$$

$$u = \frac{m_b^2 - q^2}{s - q^2}$$

$$\langle \pi(p)|\bar u(0)\gamma_\mu\gamma_5[0,x]d(x)|0\rangle = -i f_\pi p_\mu \int_0^1 du\, e^{i\bar u p\cdot x}\varphi(u,\mu^2) + \dots$$

ASSUMING Quark hadron duality

$\rho_{had} = \rho_{T2}\,\theta(s - s_0)$ — Continuum threshold.

$$\frac{f_B m_B^2 f_+(q^2)}{m_B^2 - p_B^2} = \int_0^{s_0} ds \frac{\rho_{T2}}{s - p_B^2}$$

$$\left[\hat B \frac{1}{s - p_B^2} = \frac{1}{M^2} e^{-s/M^2}\right]$$

$$f_+(q^2) = \frac{1}{f_B m_B^2}\int_{m_b^2}^{s_0} ds\, \rho_{T2}\, e^{-(s - m_B^2)/M^2}$$

$$\mu = \frac{\sqrt{m_B^2 - m_b^2}}{\sqrt{m_D^2 - m_c^2}}$$

Parameterisation of Form Factors:

Need result over full kinematic range.

LCSR results accurate at low q^2 → expansion in $1/E$

Lattice QCD results accurate at high q^2.

less energy → less "noisy" on lattice.

Need parameterisations for FFs over full range to fit to LCSR + LQCD. ⇒ dominated by low-lying resonances with appropriate quantum numbers

$\langle P|\bar d\gamma_\mu b|B\rangle$

B^* resonance affects shape of form factor

[Resonances with $J^P = 1^-$ for $V, T_1 / f_+ / f_T$

1^+ for $A^{1,2,3}$ T_{23}

0^- for A_0 / f_0]

Review on QCD sum rules Colangelo + Khodjamirian hep-ph/0010175

[hep-ph/0412079 → LCSR for B decays]
Patricia Ball + Roman Zwicky.

$$F_\sigma(q^2) = \sqrt{\frac{q^2}{\lambda}} \sum \varepsilon^{*\mu}_\sigma(q) \langle V(k, \varepsilon(k)) | \bar{q}\gamma_\mu(1+\gamma_5)|B\rangle$$

See <u>1004.3249</u>.

Low lying resonances with masses $\underbrace{(m_B - m_\rho)^2}_{t_-} < m_R^2 < \underbrace{(m_B+m_\rho)^2}_{t_+}$

Pole-type parametrizations:

$$F(q^2) \sim \frac{r_1}{1-q^2/m_R^2} + \frac{r_2}{1-q^2/m_{fit}^2}$$

$$\frac{r_1}{1-q^2/m_{fit_1}^2} + \frac{r_2}{1-q^2/m_{fit_2}^2}$$

$b \to u$ $\qquad$ $b\bar{u}$

Series type parametrizations.

Series $\quad f(t) = \frac{1}{B(t)\phi(t)} \sum_K \alpha_K z^K(t)$

BGL

<u>Boyd Grinstein Lebed.</u>

$$z \equiv \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_{V/P})^2$$

$$t_0^{opt} = t_+\left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right)$$

Optimal t_0 to reduce z in physical range.

$$B(t) = z(m_R^2) \qquad \underline{\sum_K \alpha_K^2 < 1}$$

Simplified Series

<u>BCL</u>

Bourrely Caprini Lellouch.

$$f(t) = \frac{1}{P(t)} \sum \tilde{\alpha}_K z^K(t, t_0)$$

$$P(t) = 1 - t/m_R^2$$

Derive bound on coefficients:

$$\Pi^X_{\mu\nu}(q^2) = i\int d^4x\, e^{iqx}\langle 0|T\{j^X_\mu(x) j^{\dagger X}_\nu(0)\}|0\rangle$$

$$\Pi_I^X = P_I^{\mu\nu}\, \Pi^X_{\mu\nu}(q^2)$$

$$\chi_I^X(n) = \frac{1}{n!}\frac{d^n}{dq^{2n}} \Pi_X(q^2)\Big|_{q^2=0} = \frac{1}{\pi}\int_0^\infty dt \frac{\mathrm{Im}\,\Pi_I^X(t)}{(t-q^2)^{n+1}}\Bigg|_{q^2=0}$$

$$\mathrm{Im}\,\Pi_I^X(q^2) = \frac{1}{2}\sum_\Gamma \int d\rho_\Gamma (2\pi)^4 \delta^4(q-p_\Gamma) P_I^{\mu\nu} \langle 0|j_\mu^X|\Gamma\rangle\langle\Gamma|j_\nu^{\dagger X}|0\rangle$$

$L = P/V \qquad \Gamma = BL$

$$\geq \frac{1}{2}\int d\rho_{BL} (2\pi)^4 \delta^4(q-p_{BL}) P_I^{\mu\nu}\langle 0|j_\mu^X|BL\rangle\langle BL|j_\nu^{\dagger X}|0\rangle$$

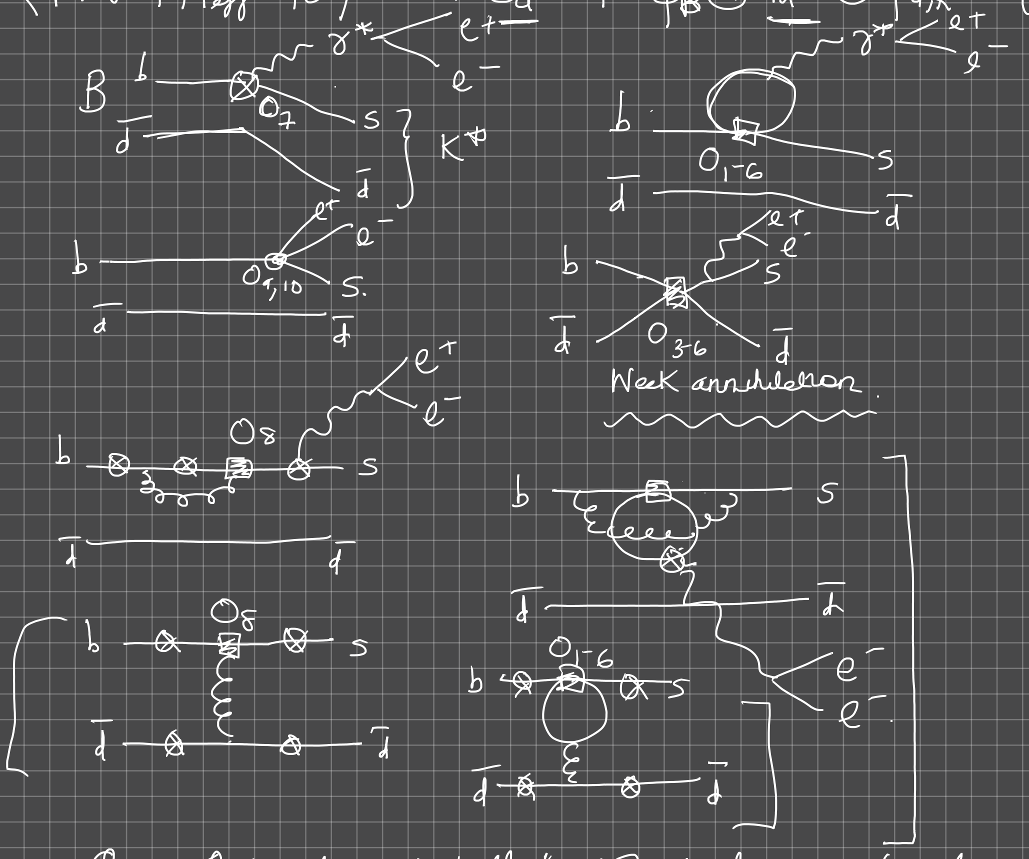
Relate $\langle 0 | j^{x} | BL \rangle \rightarrow \underbrace{\langle B | j^{x} | L \rangle}$

Write in terms of form factors

$$\text{OPE}: \quad \Pi^{x}_{I,\text{OPE}}(q^2) = \sum_{K=1}^{\infty} C^{x}_{I,K}(q^2) \langle O_K \rangle$$

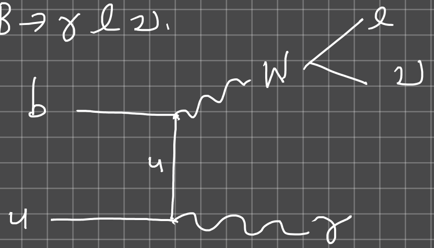
$B \to K^* e^+ e^- \to$ QCD factorization framework

$$\langle M \gamma^* | H^{(i)}_{eff} | B \rangle \sim F C_a^{(i)} + \overbrace{\phi_B \otimes \underline{T_a^{(i)}} \otimes \phi_{a,K^*} + O(1/m}$$



ϕ_B, ϕ_{K^*} not very well known, need more information.

$B \to \gamma l \nu$.



$\nearrow$ 1st moment of ϕ_B

$\lambda_B > 238$ MeV.