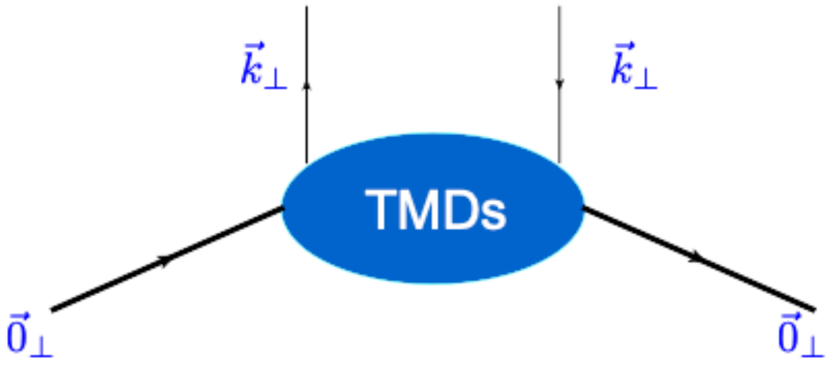
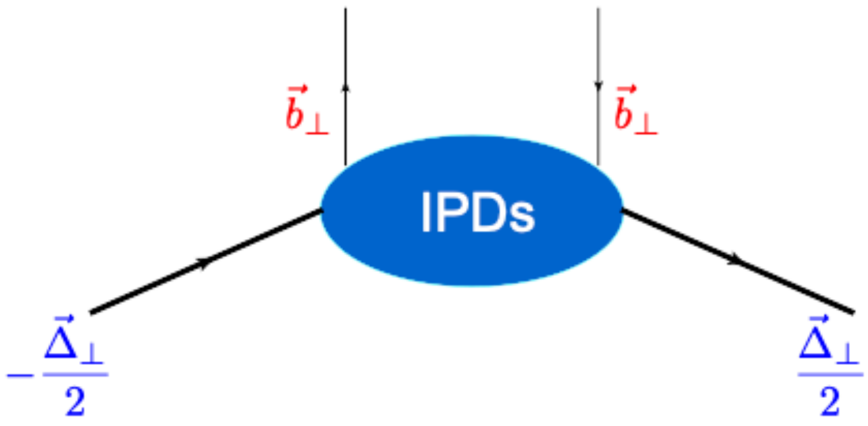
TMDs vs IPDs





$$\int d^2 z_\perp e^{-i\vec z_\perp\cdot\vec k_\perp}\langle 0_\perp|\bar\psi(-\frac{\vec z_\perp}{2})\ldots\psi(\frac{\vec z_\perp}{2})|\vec 0_\perp\rangle$$

$$\int d^2\Delta_\perp e^{-i\vec b_\perp\cdot\vec\Delta_\perp}\langle -\frac{\vec\Delta_\perp}{2}|\bar\psi(\vec b_\perp)\ldots\psi(\vec b_\perp)|\frac{\vec\Delta_\perp}{2}\rangle$$

(longitudinal components are not shown)

difference
of transverse position

average of
transverse momenta

average
position

difference of
transverse momenta

$\vec z_\perp \longleftrightarrow \vec k_\perp$

$\vec b_\perp \longleftrightarrow \vec\Delta_\perp$

TMDs vs IPDs

		quark polarization		
	GPD	U	L	T
nucleon polarization	U	H		$\mathcal{E}_T$
	L		$\tilde{H}$	$\tilde{E}_T$
	T	E	$\tilde{E}$	$H_T, \ \tilde{H}_T$

		quark polarization		
	TMD	U	L	T
nucleon polarization	U	f_1		$h_1^{\perp}$
	L		g_{1L}	$h_{1L}^{\perp}$
	T	$f_{1T}^{\perp}$	g_{1T}	$h_1, \ h_{1T}^{\perp}$

TMDs vs IPDs

ρ_{XY} $X =$ proton pol
$Y =$ quark pol

correlations in $\vec{k}_\perp, \Lambda, \vec{s}_\perp$

$$\rho_{LT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, s^i_\perp\, k^i_\perp \frac{1}{M} h^\perp_{1L}]$$

$\Lambda s^i b^i_\perp$
time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \vec{S}_\perp, \lambda$

$$\rho_{TL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \lambda\, S^i_\perp\, k^i_\perp \frac{1}{M} g^\perp_{1T}]$$

$S^i \lambda b^i_\perp$
time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \Lambda, \lambda$

$$\rho_{LL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, \lambda\, g_{1L}]$$

correlations in $\vec{b}_\perp$, Λ, λ

$$\tilde{\rho}_{TL}(x, \vec{b}_\perp) = \frac{1}{2}[H + \Lambda\, \lambda\, \tilde{H}]$$

correlations in $\vec{k}_\perp, \vec{S}_\perp, \vec{s}_\perp$

$$\rho_{TT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + S^i_\perp\, s^i_\perp\, h_1 + S^i_\perp\, (2k^i k^j - k^2_\perp \delta^{ij})\, s^j_\perp \frac{1}{2M^2} h^\perp_{1T}]$$

correlations in $\vec{b}_\perp$, $\vec{S}_\perp$, $\vec{s}_\perp$

$$\tilde{\rho}_{TT}(x, \vec{b}_\perp) = \frac{1}{2}[H - S^i_\perp\, s^i_\perp (\, H_T - \frac{1}{4M^2} \Delta_b\, \tilde{H}_T) + S^i_\perp\, (2b^i b^j - b^2 \delta^{ij})\, s^j_\perp \frac{1}{M^2} \tilde{H}''_T]$$

Diehl, Haegler, EPJ C44 (2005) 87

TMDs vs IPDs

ρ_{XY} $X =$ proton pol
$Y =$ quark pol

correlations in $\vec{k}_\perp, \Lambda, \vec{s}_\perp$

$$\rho_{LT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, s^i_\perp\, k^i_\perp \frac{1}{M} h^\perp_{1L}]$$

$\Lambda\,\cancel{s^i}\,b^i_\perp$

time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \vec{S}_\perp, \lambda$

$$\rho_{TL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \lambda\, S^i_\perp\, k^i_\perp \frac{1}{M} g^\perp_{1T}]$$

$S^i\,\cancel{\lambda}\,b^i_\perp$

time-reversal odd → GPD=0

correlations in $\vec{k}_\perp, \Lambda, \lambda$

$$\rho_{LL}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + \Lambda\, \lambda\, g_{1L}]$$

correlations in $\vec{b}_\perp,\ \Lambda,\ \lambda$

$$\tilde{\rho}_{TL}(x, \vec{b}_\perp) = \frac{1}{2}[H + \Lambda\, \lambda\, \tilde{H}]$$

correlations in $\vec{k}_\perp, \vec{S}_\perp, \vec{s}_\perp$

$$\rho_{TT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + S^i_\perp\, s^i_\perp\, h_1 + S^i_\perp\, (2k^i k^j - k^2_\perp \delta^{ij})\, s^j_\perp \frac{1}{2M^2} h^\perp_{1T}]$$

correlations in $\vec{b}_\perp,\ \vec{S}_\perp,\ \vec{s}_\perp$

$$\tilde{\rho}_{TT}(x, \vec{b}_\perp) = \frac{1}{2}[H - S^i_\perp\, s^i_\perp (\, H_T - \frac{1}{4M^2} \Delta_b\, \tilde{H}_T) + S^i_\perp\, (2b^i b^j - b^2 \delta^{ij})\, s^j_\perp \frac{1}{M^2} \tilde{H}''_T]$$

Diehl, Haegler, EPJ C44 (2005) 87

TMDs vs IPDs

ρ_{XY} $X =$ proton pol

$Y =$ quark pol

correlations in $\vec{k}_\perp, \vec{S}_\perp$

$$\rho_{TU}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + S_\perp^i \epsilon^{ij} k_\perp^j \frac{1}{M} f_{1T}^\perp]$$

correlations in $\vec{k}_\perp, \vec{s}_\perp$

$$\rho_{UT}(x, \vec{k}_\perp) = \frac{1}{2}[f_1 + s_\perp^i \epsilon^{ij} k_\perp^j \frac{1}{M} h_1^\perp]$$

correlations in $\vec{b}_\perp, \vec{S}_\perp$

$$\rho_{TU}(x, \vec{k}_\perp) = \frac{1}{2}[H - S_\perp^i \epsilon^{ij} b_\perp^j \frac{1}{M} \frac{\partial}{\partial b_\perp^2} E]$$

correlations in $\vec{b}_\perp, \vec{s}_\perp$

$$\rho_{UT}(x, \vec{k}_\perp) = \frac{1}{2}[H - s_\perp^i \epsilon^{ij} b_\perp^j \frac{1}{M} (E_T' + 2\tilde{H}_T')]$$

Model relation TMD ⟷ GPD

unpolarized quark in unpolarized nucleon

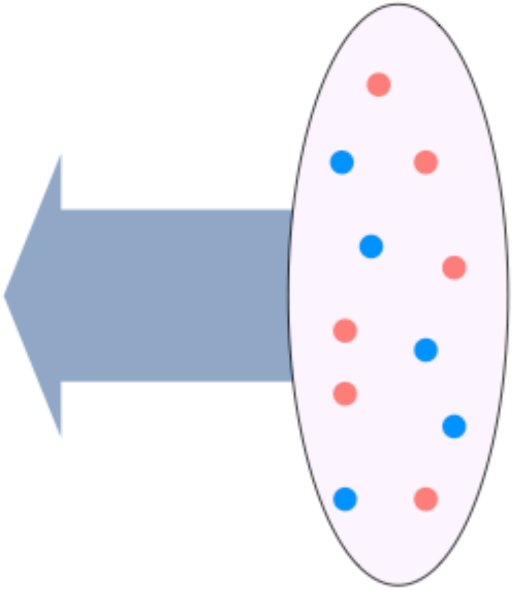


*figure A. Bacchetta

Model relation TMD ⟷ GPD

unpolarized quark in transversely pol. nucleon

Distortion in impact parameter
(related to GPD E)

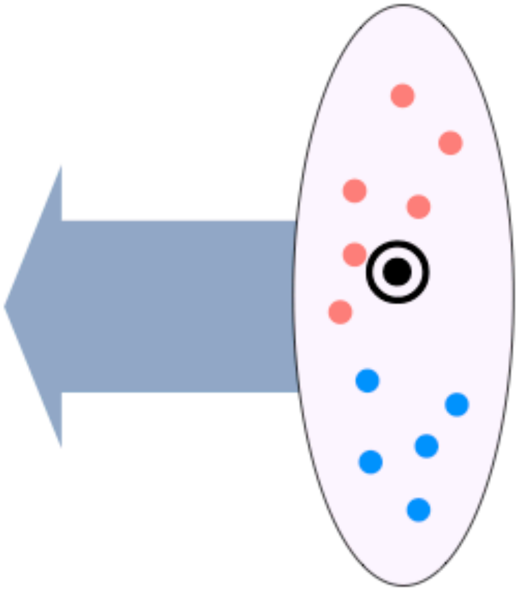
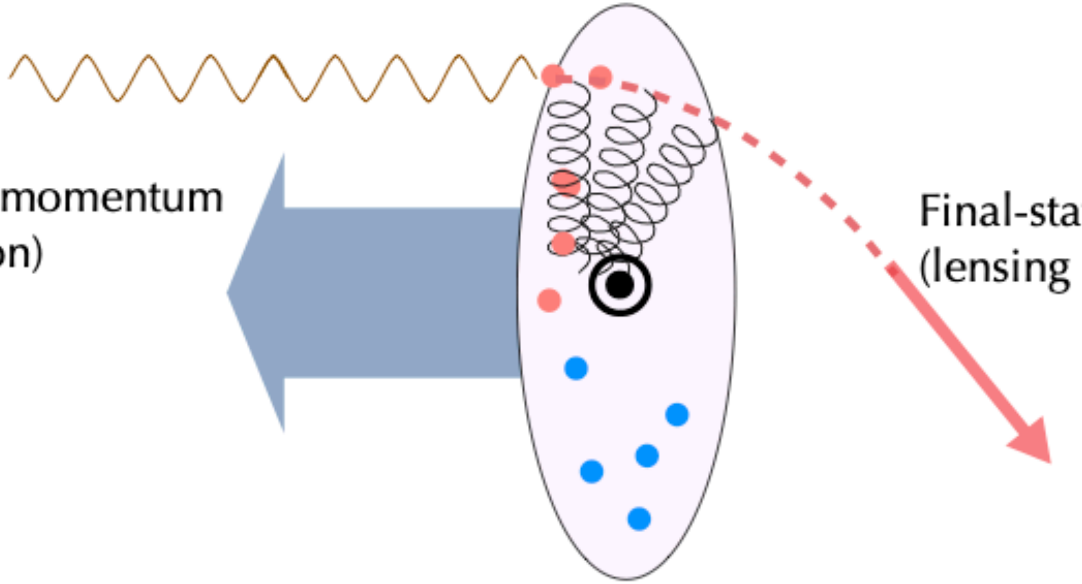


*figure A. Bacchetta

Model relation TMD ⟷ GPD

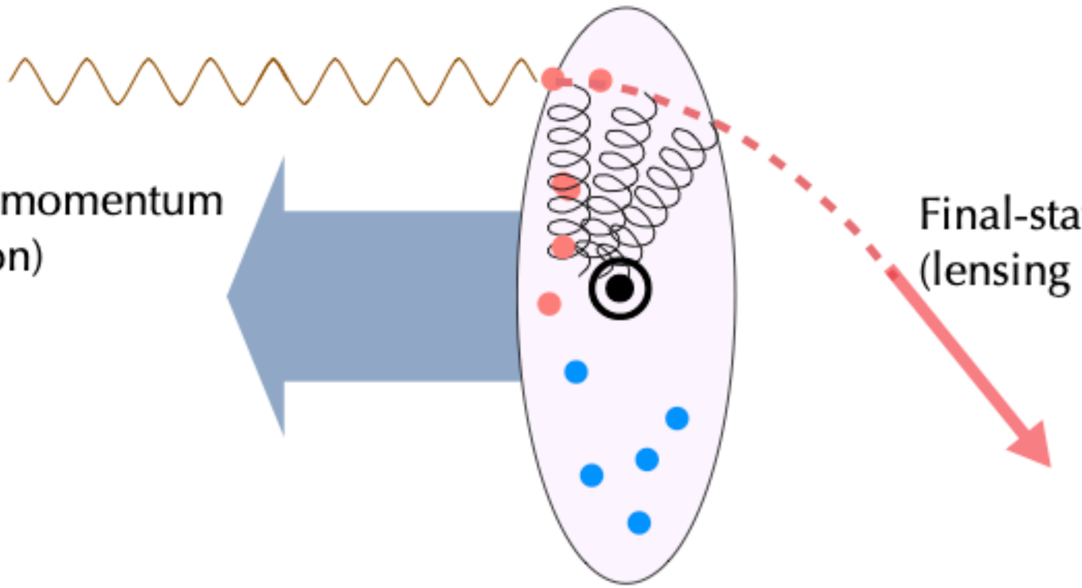


Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

*figure A. Bacchetta

Model relation TMD ⟷ GPD



Distortion in transverse momentum
(related to Sivers function)

Final-state interaction
(lensing function)

*figure A. Bacchetta

$$-\int d^2\vec{k}_T\, k_T^i\, \frac{\epsilon_T^{jk} k_T^j S_T^k}{M}\, \underset{\substack{\uparrow\\ \text{Sivers function}}}{f_{1T}^{\perp q}(x,\vec{k}_T^2)} \;\simeq\; \int d^2\vec{b}_T\, \underset{\substack{\uparrow\\ \text{Lensing function}}}{\mathcal{I}^{q,i}(x,\vec{b}_T)}\, \frac{\epsilon_T^{jk} b_T^j S_T^k}{M} \left(\underset{\substack{\uparrow\\ \text{F.T. of E(x,0,t)}}}{\mathcal{E}^q(x,\vec{b}_T^2)}\right)'$$

*Burkardt, PRD **66** (2002) 114005*

- Relation valid only in restricted class of models, as, for example, the scalar-diquark model

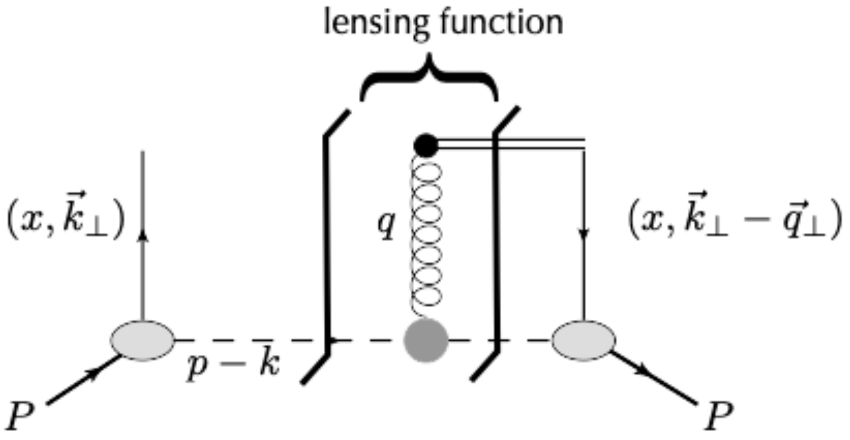
BP, Rodini, Bacchetta, Phys. Rev. D100, 054039 (2019)

Model results

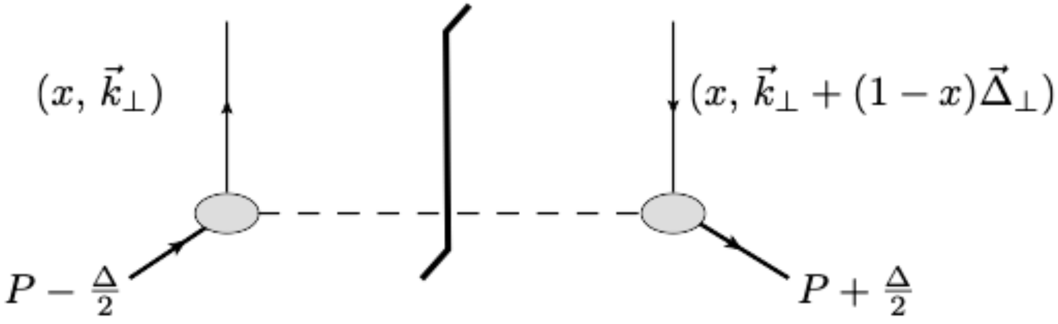
Sivers effect = Lensing function ⊗ IPD

Scalar diquark model:
- two-particle system (one active quark and a scalar spectator)
- perturbative coupling between Wilson line and spectator ⟶ no-helicity flip of the spectator

TMD



IPD



It is violated when considering coupling with the gauge boson that are not helicity conserving (e.g., axial diquark model) or for bound system with more than two constituents

BP, Rodini, Bacchetta, PRD 100, 054039 (2019)

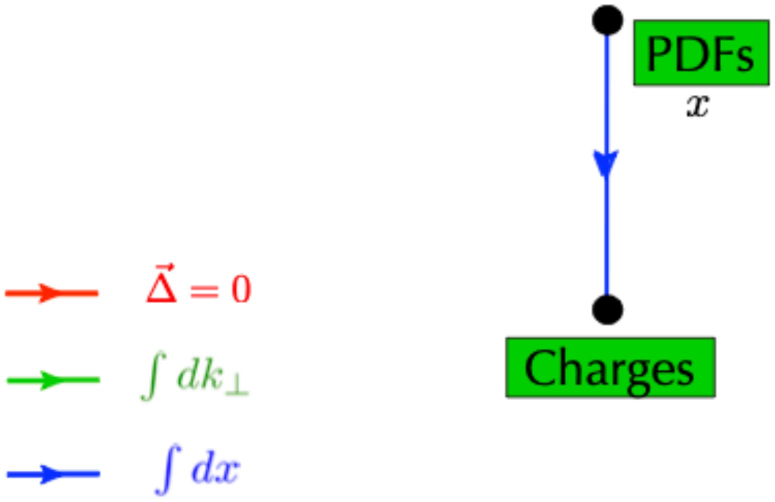
→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

→ $\int dx$

Charges

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



PDFs

x

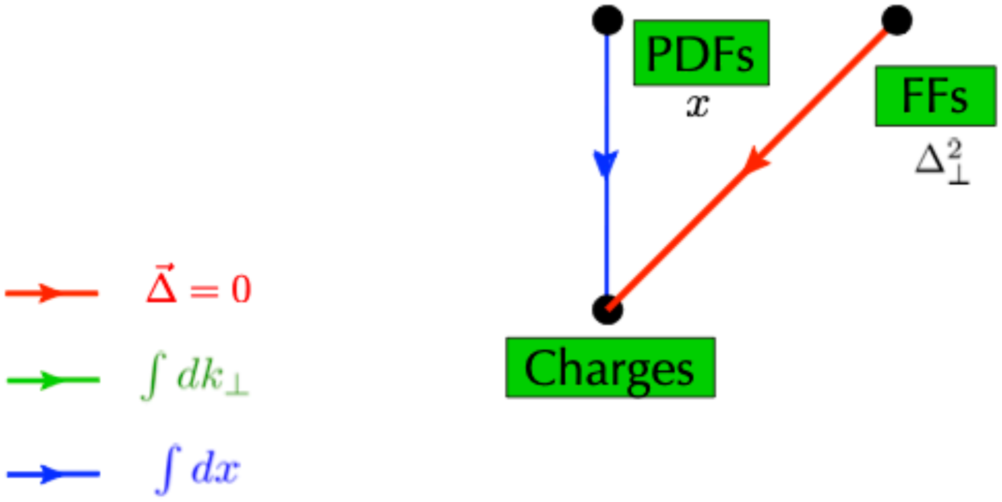
Charges

→ $\vec{\Delta} = 0$

→ $\int dk_\perp$

→ $\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



PDFs

x

FFs

$\Delta_\perp^2$

Charges

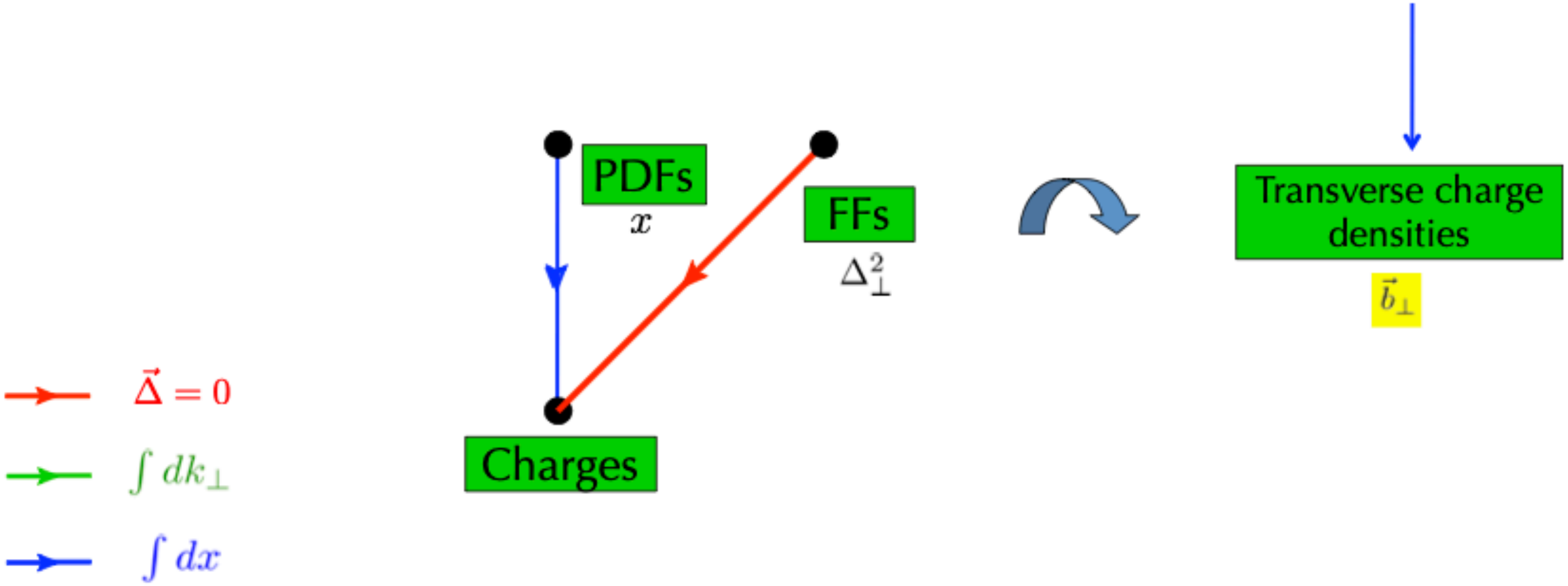
$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier transform at $\xi = 0$ $\Delta_\perp \leftrightarrow b_\perp$



PDFs

x

FFs

$\Delta_\perp^2$

Charges

Transverse charge densities

$\vec{b}_\perp$

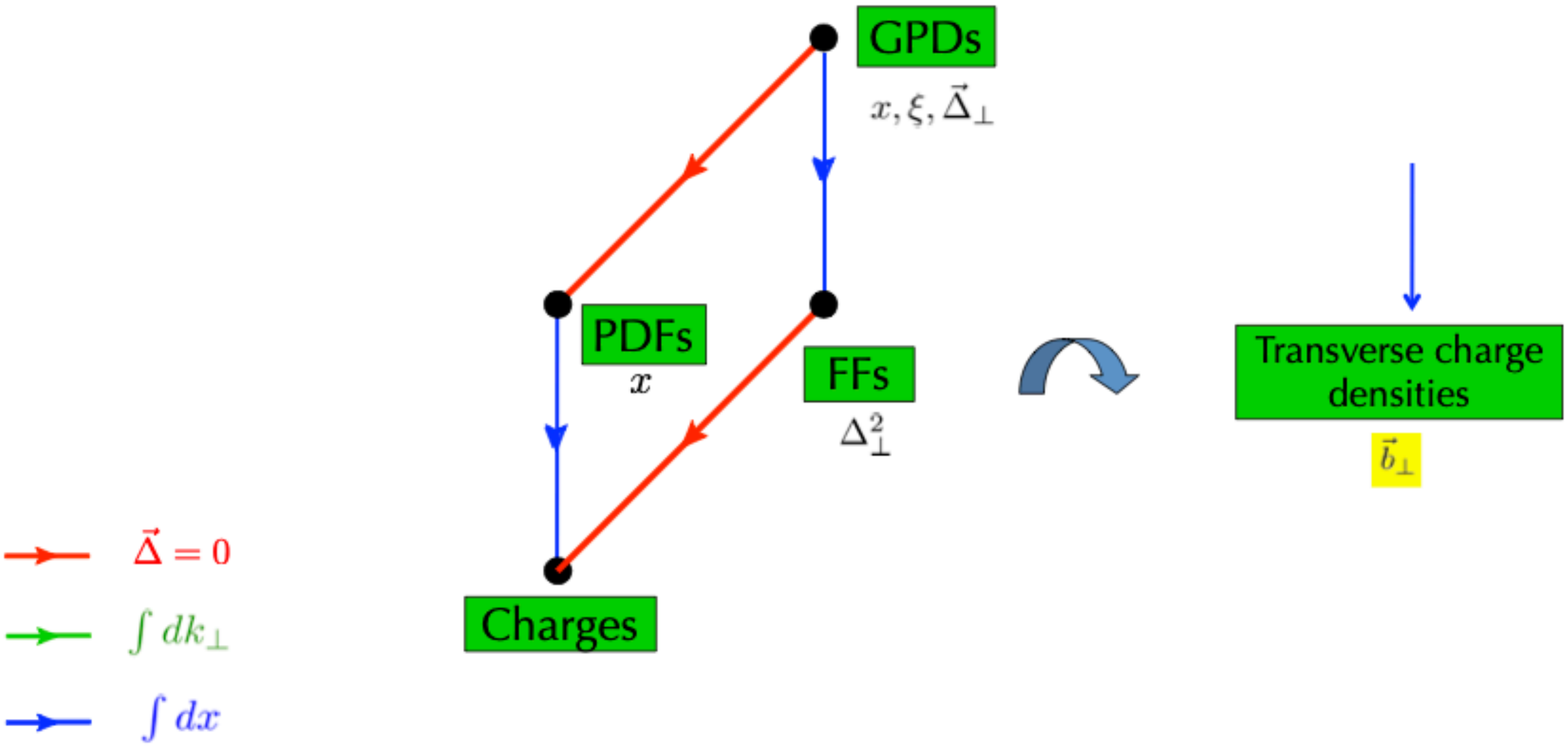
$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier
transform at $\xi = 0$ $\Delta_\perp \leftrightarrow b_\perp$



GPDs
$x, \xi, \vec{\Delta}_\perp$

PDFs
x

FFs
$\Delta_\perp^2$

Charges

Transverse charge densities
$\vec{b}_\perp$

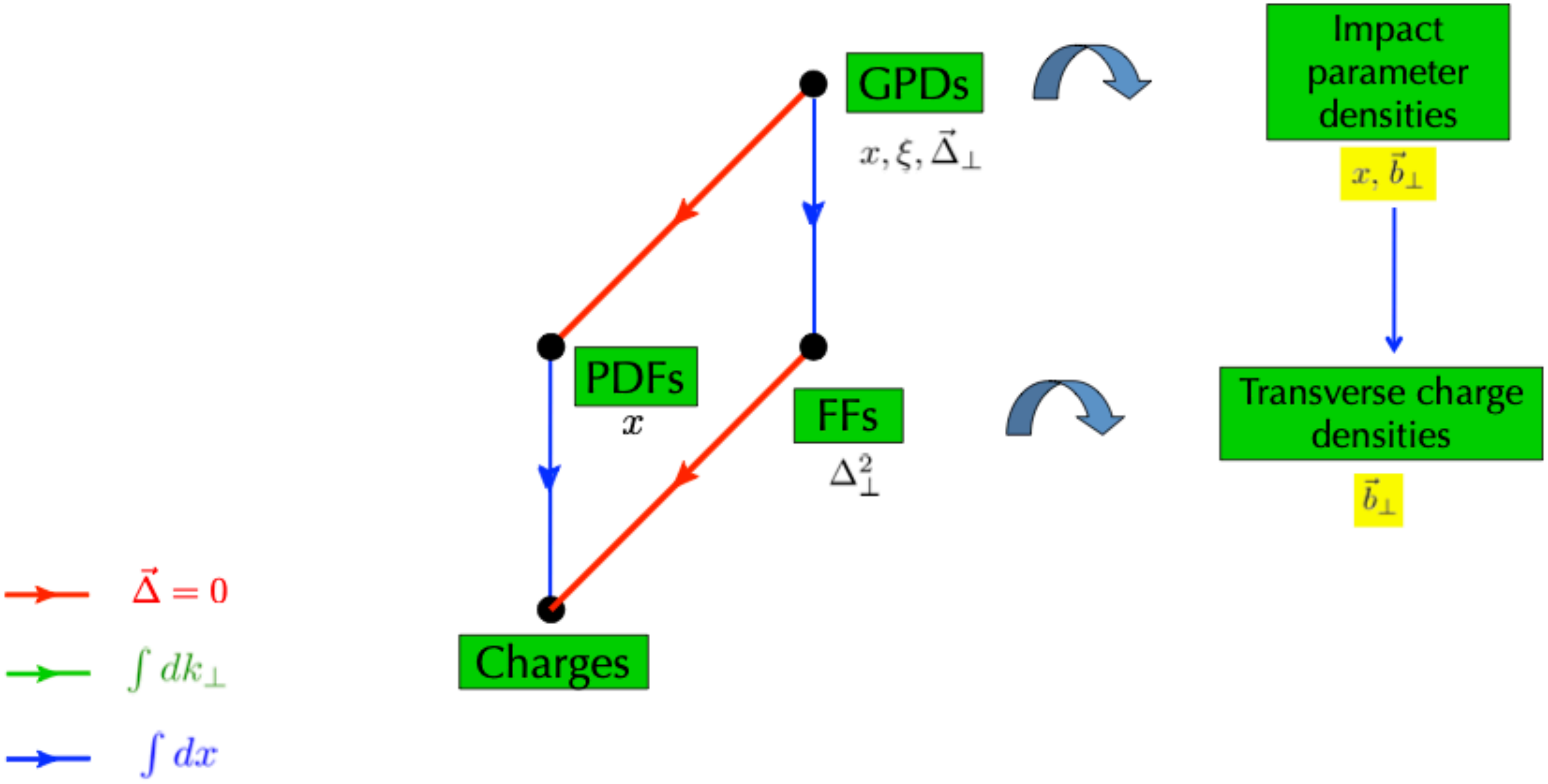
$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier
transform at $\xi = 0$ $\Delta_\perp \leftrightarrow b_\perp$



GPDs

$x, \xi, \vec{\Delta}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Charges

Impact parameter densities

$x, \vec{b}_\perp$

Transverse charge densities

$\vec{b}_\perp$

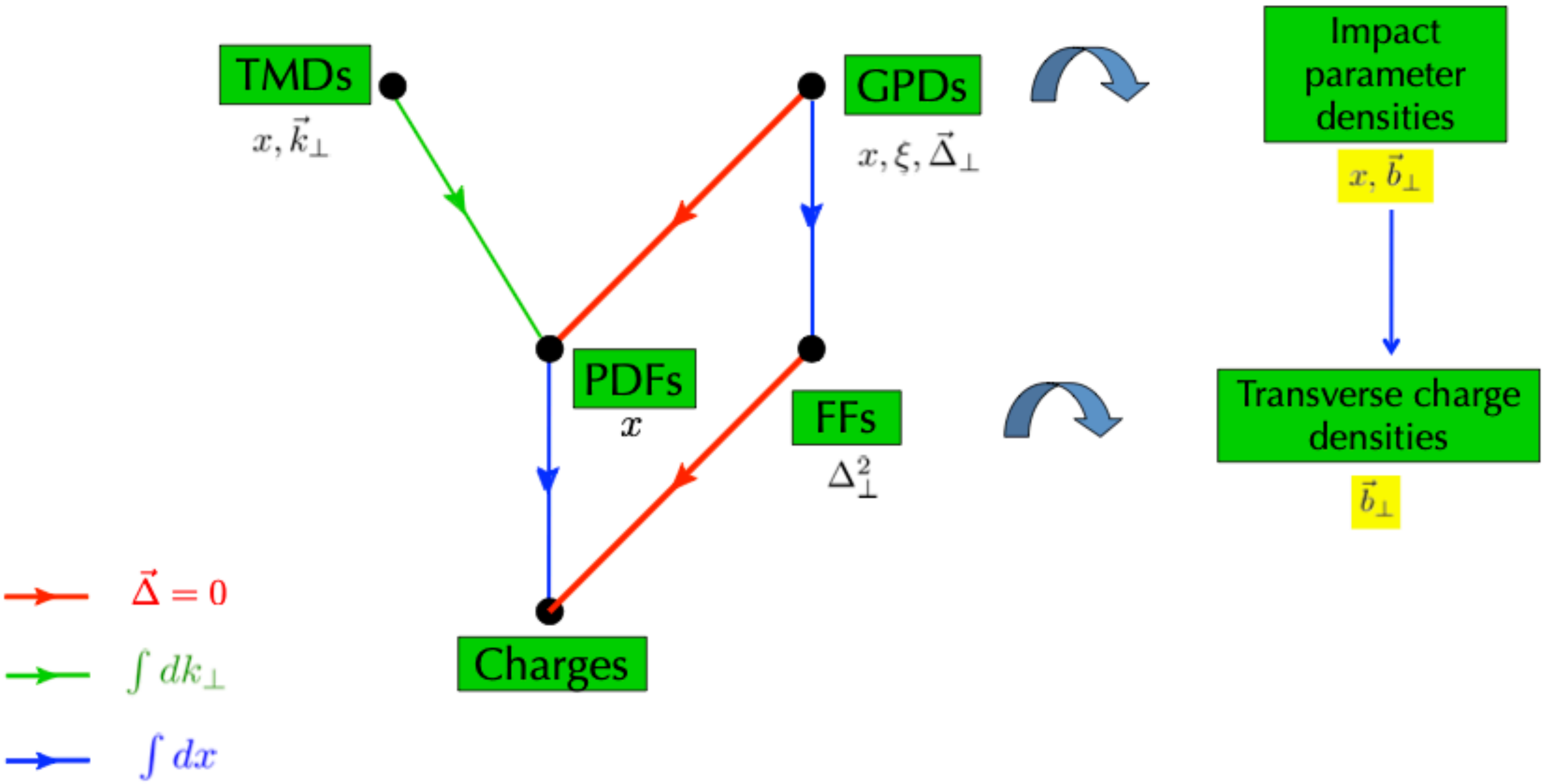
$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier
transform at $\xi = 0$ $\Delta_\perp \leftrightarrow b_\perp$



TMDs $x, \vec{k}_\perp$

GPDs $x, \xi, \vec{\Delta}_\perp$

PDFs x

FFs $\Delta_\perp^2$

Charges

Impact parameter densities $x, \vec{b}_\perp$

Transverse charge densities $\vec{b}_\perp$

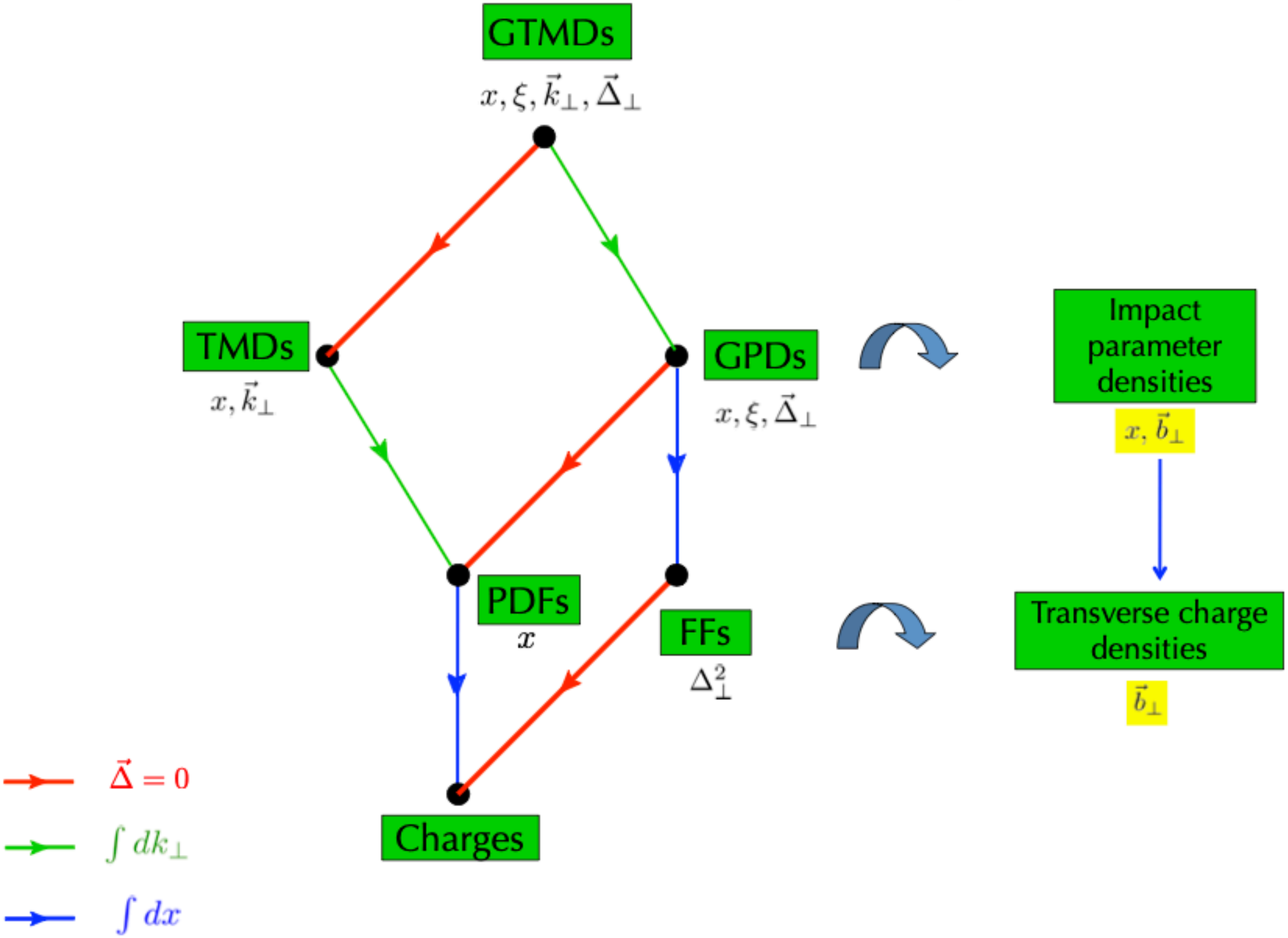
$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

2D Fourier transform at $\xi = 0$ $\Delta_\perp \leftrightarrow b_\perp$



GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

TMDs

$x, \vec{k}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

Impact parameter densities

$x, \vec{b}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Transverse charge densities

$\vec{b}_\perp$

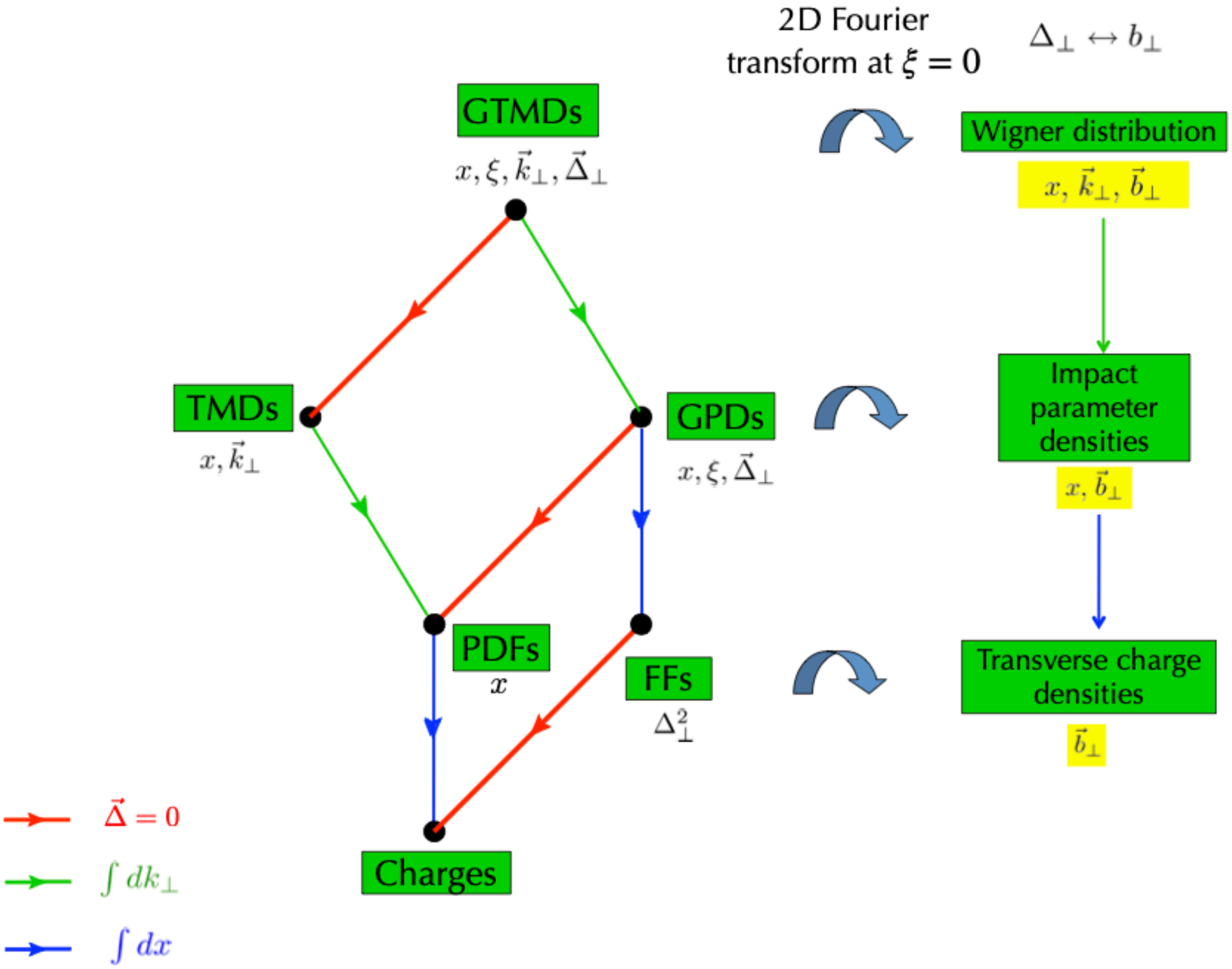
Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



2D Fourier transform at $\xi = 0$ $\quad \Delta_\perp \leftrightarrow b_\perp$

GTMDs
$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

Wigner distribution
$x,\, \vec{k}_\perp,\, \vec{b}_\perp$

TMDs
$x, \vec{k}_\perp$

GPDs
$x, \xi, \vec{\Delta}_\perp$

Impact parameter densities
$x,\, \vec{b}_\perp$

PDFs
x

FFs
$\Delta_\perp^2$

Transverse charge densities
$\vec{b}_\perp$

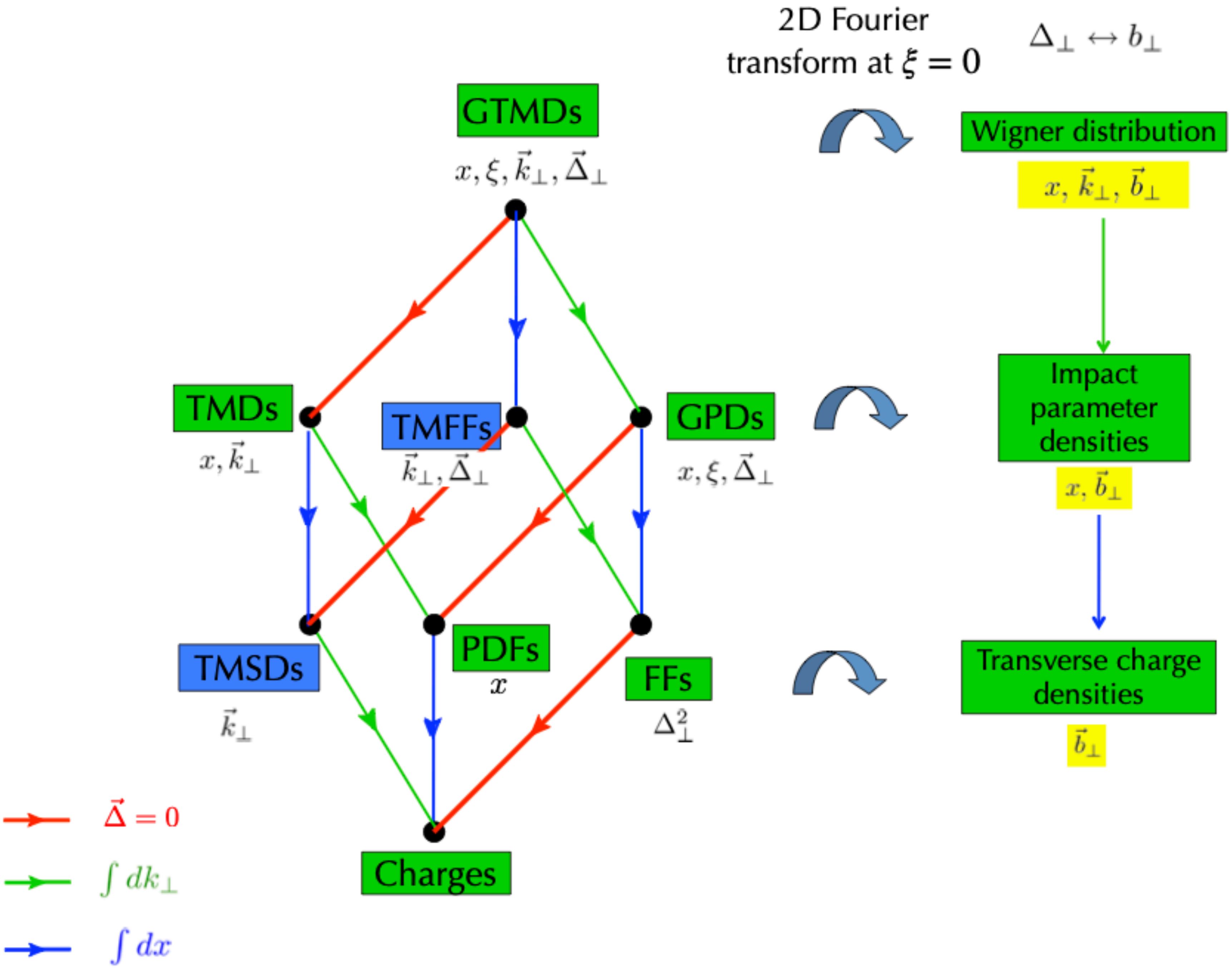
Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



2D Fourier transform at $\xi = 0$

$\Delta_\perp \leftrightarrow b_\perp$

GTMDs

$x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp$

Wigner distribution

$x, \vec{k}_\perp, \vec{b}_\perp$

TMDs

$x, \vec{k}_\perp$

TMFFs

$\vec{k}_\perp, \vec{\Delta}_\perp$

GPDs

$x, \xi, \vec{\Delta}_\perp$

Impact parameter densities

$x, \vec{b}_\perp$

TMSDs

$\vec{k}_\perp$

PDFs

x

FFs

$\Delta_\perp^2$

Transverse charge densities

$\vec{b}_\perp$

Charges

$\vec{\Delta} = 0$

$\int dk_\perp$

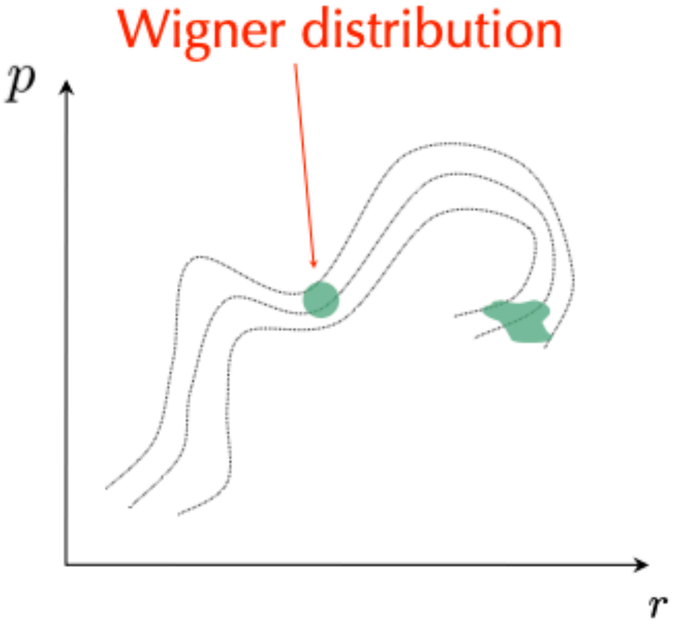
$\int dx$

Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

Phase-Space Distributions in Quantum-Mechanics

Wigner (1932)
Moyal (1949)

Quantum Mechanics



$$\rho_W(r,k) = \int \frac{dz}{2\pi} e^{-ikz} \psi^*(r - \frac{z}{2})\psi(r + \frac{z}{2})$$

$$= \int \frac{d\Delta}{2\pi} e^{-i\Delta r} \phi^*(k + \frac{\Delta}{2} \phi(k - \frac{\Delta}{2})$$

Position-space density

$$|\psi(r)|^2 = \int dk\, \rho_W(r,k)$$

Momentum-space density

$$|\phi(k)|^2 = 2\pi \int dr\, \rho_W(r,k)$$

Quantum average

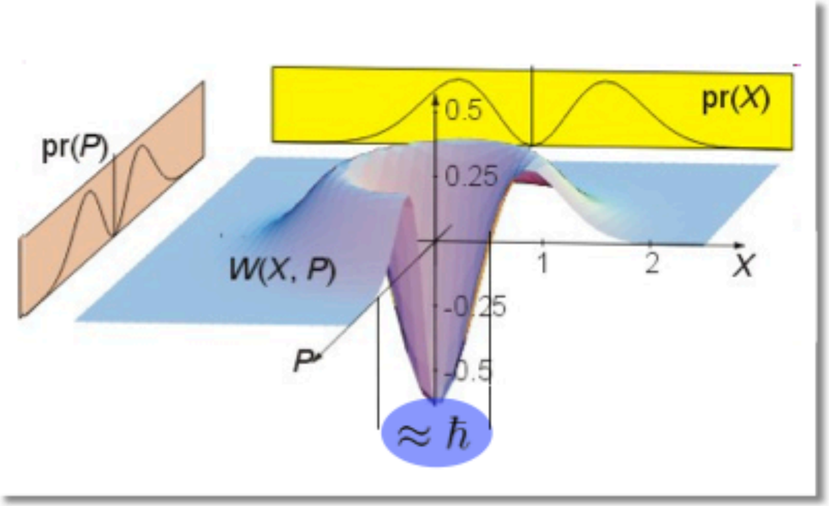
$$\langle \hat{O} \rangle = \int dr\, dk\, O(r,k) \rho_W(r,k)$$

Wigner distributions $(x, \vec{b}_\perp, \vec{k}_\perp)$

- Extend the concept of classical phase-space density

- Phase-space distributions of partons inside the nucleon

- Quasi-probabilistic interpretation



Heisenberg's uncertainty relation

$\Longrightarrow$ Quasi-probabilistic interpretation $\xrightarrow{\hbar \to 0}$ classical density

Wigner Distributions (WDs) in QFT

Quark Wigner operator $$\widehat{W}^{[\Gamma]}(\vec{r},k) = \int \frac{d^4z}{(2\pi)^4} e^{ik\cdot z}\bar{\psi}(\vec{r}-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\vec{r}+\frac{z}{2})$$

Dirac matrix
~ quark polarization

Wilson line

Wigner Distributions (WDs) in QFT

Dirac matrix

~ quark polarization

Quark Wigner operator $$\widehat{W}^{[\Gamma]}(\vec{r}, k) = \int \frac{d^4 z}{(2\pi)^4} e^{ik\cdot z} \bar{\psi}(\vec{r} - \frac{z}{2}) \, \Gamma \, \mathcal{W} \, \psi(\vec{r} + \frac{z}{2})$$

Wilson line

Fixed light-front time $z^+ = 0$ $\Longleftrightarrow$ $\int dk^-$

Wigner Distributions (WDs) in QFT

Dirac matrix

~ quark polarization

Quark Wigner operator $\widehat{W}^{[\Gamma]}(\vec{r},k)=\int\frac{d^4z}{(2\pi)^4}e^{ik\cdot z}\bar{\psi}(\vec{r}-\frac{z}{2})\,\Gamma\,\mathcal{W}\,\psi(\vec{r}+\frac{z}{2})$

Wilson line

Fixed light-front time $z^+=0$ ⟺ $\int dk^-$

WDs
in the Breit frame $\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r},k^+,\vec{k}_\perp)=\frac{1}{2}\int\frac{d^3\vec{\Delta}}{(2\pi)^3}e^{-i\vec{\Delta}\cdot\vec{r}}\langle\frac{\vec{\Delta}}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|-\frac{\vec{\Delta}}{2},\Lambda\rangle$

3+3 D

no semi-classical interpretation

Ji (2003)
Belitsky, Ji, Yuan (2004)

Wigner Distributions (WDs) in QFT

Dirac matrix

~ quark polarization

Quark Wigner operator
$$\widehat{W}^{[\Gamma]}(\vec{r}, k) = \int \frac{d^4 z}{(2\pi)^4} e^{ik\cdot z} \bar{\psi}(\vec{r} - \frac{z}{2})\, \Gamma\, \mathcal{W}\, \psi(\vec{r} + \frac{z}{2})$$

Wilson line

Fixed light-front time $z^+ = 0$ $\Longleftrightarrow$ $\int dk^-$

WDs
in the Breit frame

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r}, k^+, \vec{k}_\perp) = \frac{1}{2} \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \langle \frac{\vec{\Delta}}{2}, \Lambda' | \widehat{W}^{[\Gamma]}(0, k^+, \vec{k}_\perp) | -\frac{\vec{\Delta}}{2}, \Lambda \rangle$$

3+3 D

no semi-classical interpretation

Ji (2003)
Belitsky, Ji, Yuan (2004)

WDs
in the Drell-Yan frame
$(\Delta^+ = 0)$

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp, k^+, \vec{k}_\perp) = \frac{1}{2} \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{b}_\perp} \langle p^+, \frac{\vec{\Delta}_\perp}{2}, \Lambda' | \widehat{W}^{[\Gamma]}(0, k^+, \vec{k}_\perp) | p^+, -\frac{\vec{\Delta}_\perp}{2}, \Lambda \rangle$$

2+3 D

semi-classical interpretation

GTMDs

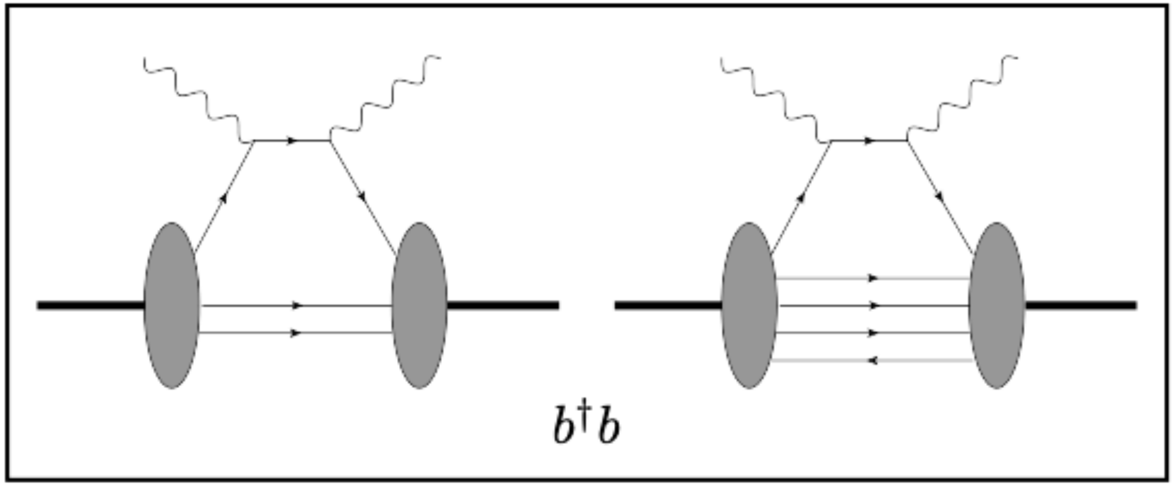
Lorcè, BP (2011)
Lorcè, BP, Xiong, Yuan (2012)

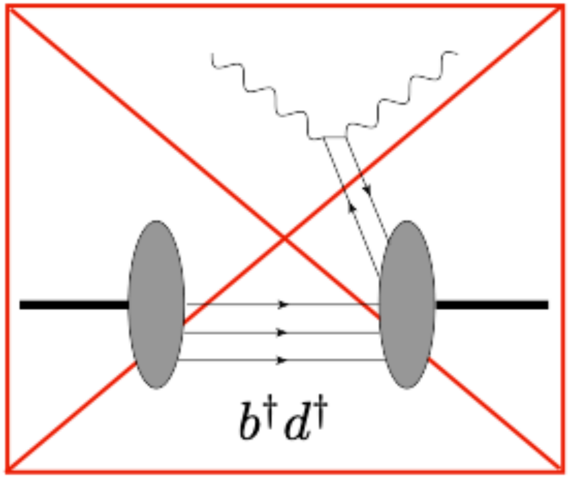
Quasi-probabilistic interpretation

✓ $\int \mathrm{d}r^- \sim \Delta^+ = 0$ → no sensitivity to longitudinal Lorentz contraction

✓ Transverse boosts → no transverse Lorentz contraction

✓ Particle number is conserved in Drell-Yan frame $\Delta^+ = 0$

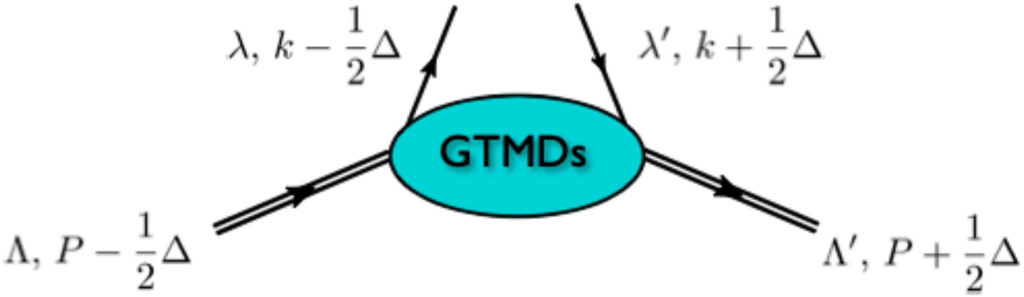




Generalized TMDs

Meißner, Metz, Schlegel, JHEP 0908 (2009)] 56; JHEP 0808 (2008) 38

Lorcé, BP, JHEP 1309 (2013) 138



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\,\mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_{\lambda}(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

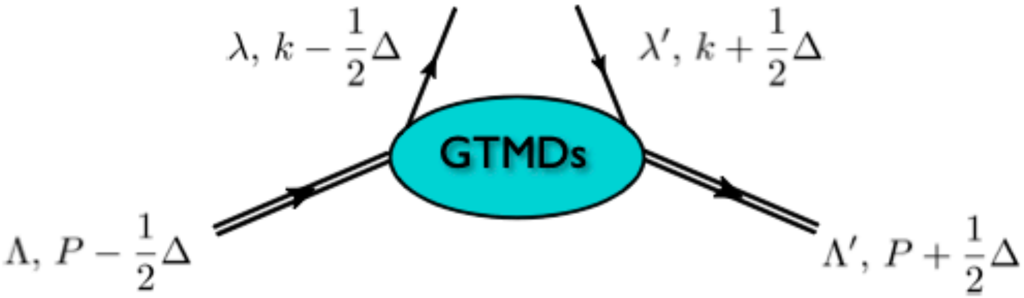
4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

Generalized TMDs

Meißner, Metz, Schlegel, JHEP 0908 (2009)] 56; JHEP 0808 (2008) 38

Lorcé, BP, JHEP 1309 (2013) 138



Quark polarization

$$W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^-\mathrm{d}^2z_\perp}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}^q_{\lambda'}(-\frac{z}{2})\,\Gamma\,\mathcal{W}(-\frac{z}{2},\frac{z}{2}|n)\psi^q_\lambda(\frac{z}{2})|P,\Lambda\rangle e^{i(xP^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)}$$

Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$$W^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp)$$

x: average fraction of quark longitudinal momentum

ξ : fraction of longitudinal momentum transfer

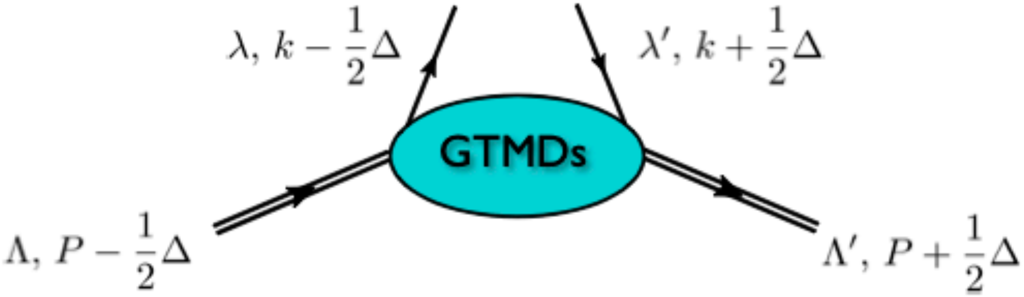
$\vec{k}_\perp$: average quark transverse momentum

$\vec{\Delta}_\perp$: nucleon transverse momentum

Generalized TMDs

Meißner, Metz, Schlegel, JHEP 0908 (2009)] 56; JHEP 0808 (2008) 38

Lorcé, BP, JHEP 1309 (2013) 138



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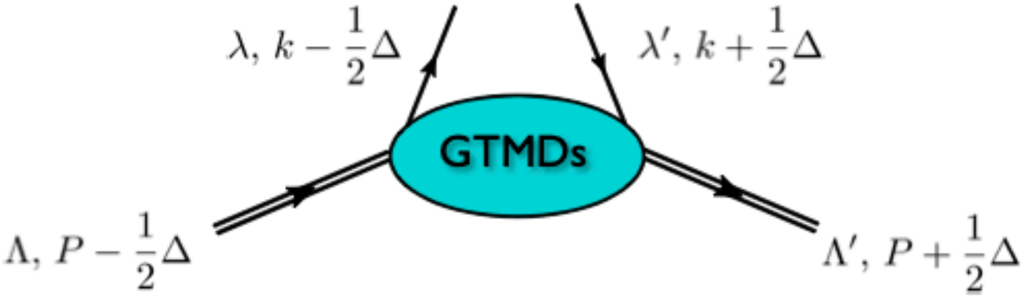

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$$\Big\downarrow \text{ Fourier transform } \vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp \text{ at } \xi = 0$$

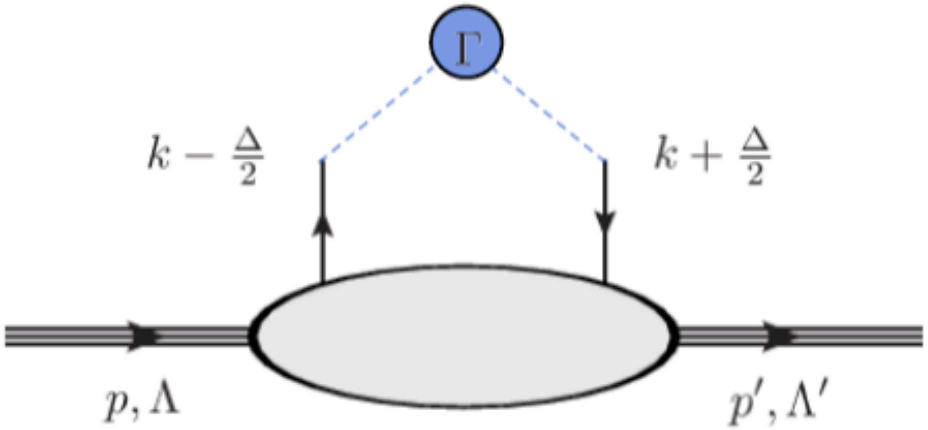
$\tilde{W}^{\Gamma}_{\Lambda',\Lambda}(x,\xi,\vec{k}_\perp,\vec{b}_\perp)$ 32 real Wigner distributions

Transverse phase-space distributions

★ Twist-2: $\boxed{\Gamma_{\text{twist}-2} = \gamma^+, \gamma^+\gamma^5, i\sigma^{j+}\gamma_5}$

quark polarization: **U** **L** **T**

★ Nucleon polarization: **U** **L** **T** → 16 complex GTMDs → 32 real Wigner Distributions



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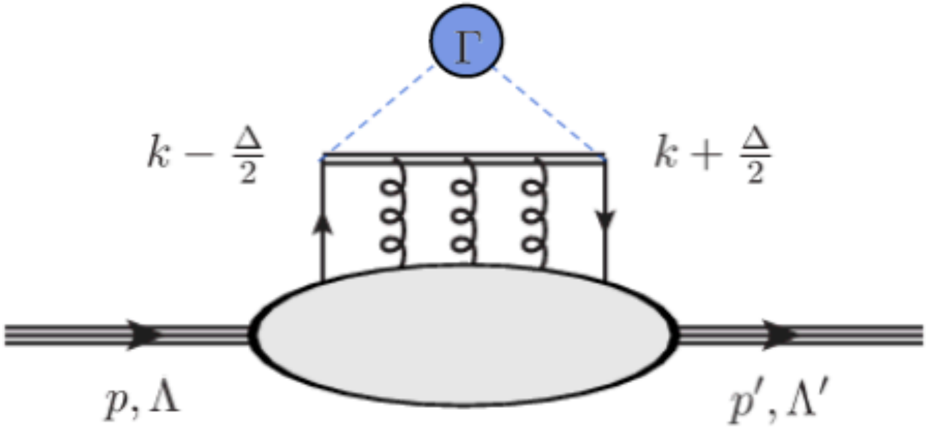
★Gauge link: T-even and T-odd functions



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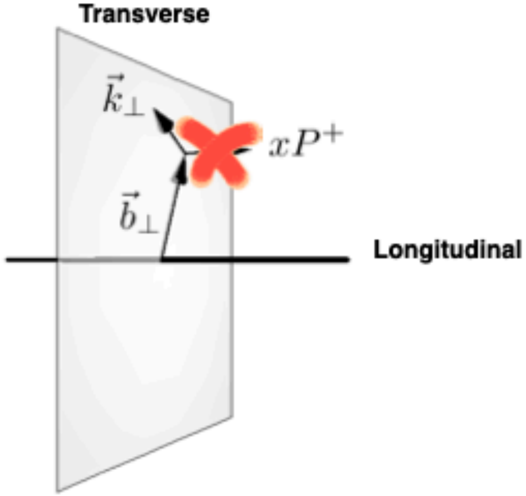

Transverse Phase-Space distributions

$$\rho_X(\vec{k}_\perp, \vec{b}_\perp) = \int \mathrm{d}x\, \rho_X(x, \vec{k}_\perp, \vec{b}_\perp) \;\; X = UU,\, UL, UT,\, LU,\, \dots$$

Angular Correlations

$$\rho_{\vec{S}\vec{S}^q} = \rho_{UU} + S_L\,\rho_{LU} + S_L^q\,\rho_{UL} + S_L S_L^q\,\rho_{LL} + S_T^i\,(\rho_{T^iU} + S_L^q\,\rho_{T^iL}) + S_T^{qi}\,(\rho_{UT^i} + S_L\,\rho_{LT^i}) + S_T^i S_T^{qj}\,\rho_{T^iT^j}$$

quark polarization

nucleon polarization

ρ_X	U	L	T_x	T_y
U	$\langle 1 \rangle$	$\langle S_L^q \ell_L^q \rangle$	$\langle S_x^q \ell_x^q \rangle$	$\langle S_y^q \ell_y^q \rangle$
L	$\langle S_L \ell_L^q \rangle$	$\langle S_L S_L^q \rangle$	$\langle S_L \ell_L^q S_x^q \ell_x^q \rangle$	$\langle S_L \ell_L^q S_y^q \ell_y^q \rangle$
T_x	$\langle S_x \ell_x^q \rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q \rangle$	$\langle S_x S_x^q \rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q \rangle$
T_y	$\langle S_y \ell_y^q \rangle$	$\langle S_y \ell_y^q S_L^q \ell_L^q \rangle$	$\langle S_y \ell_y^q S_x^q \ell_x^q \rangle$	$\langle S_y S_y^q \rangle$

$\boxed{\xi = 0}$

GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	$H_T,\ \tilde{H}_T$

TMD	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

each distribution contains unique information

the distributions in **red** vanish if there is no quark orbital angular momentum

the distributions in **black** survive in the collinear limit

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T_x	$\langle S_x \ell_x^q\rangle$	$\langle S_x \ell_x^q S_L^q \ell_L^q\rangle$	$\langle S_x S_x^q\rangle$	$\langle S_x \ell_x^q S_y^q \ell_y^q\rangle$
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GPD	U	L	T
U	H		$\mathcal{E}_T$
L		$\tilde{H}$	$\cancel{\tilde{E}_T}$
T	E	$\cancel{\tilde{E}}$	$H_T,\ \tilde{H}_T$

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Phase-Space Transverse Modes

$$\rho_X(\vec{k}_\perp|\vec{b}_\perp) = \int dx\, \rho_X(x, \vec{k}_\perp, \vec{b}_\perp; \hat{P} = \vec{e}_z, \eta = +1)\big|_{\vec{b}_\perp \text{ fixed}} \longrightarrow \text{2+2 dimensions}(\vec{b}_\perp, \vec{k}_\perp)$$

Multipole decomposition

$$\rho_X = \sum_{m_k, m_b} \rho_X(m_k, m_b)$$

using PT symmetries

Lorcé, BP, PRD 96, 2016

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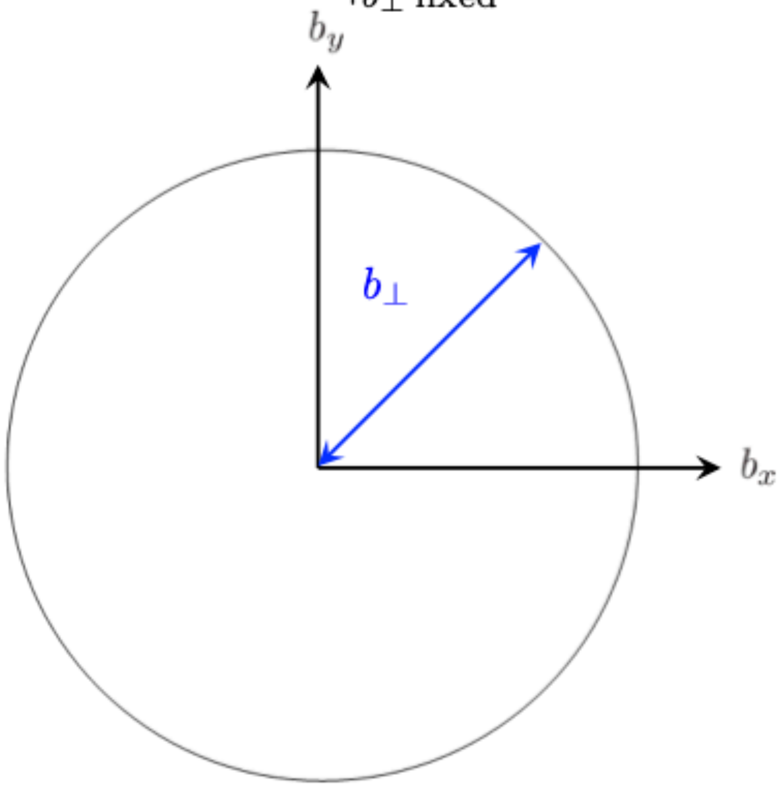

Lorcé, BP, PRD 96, 2016

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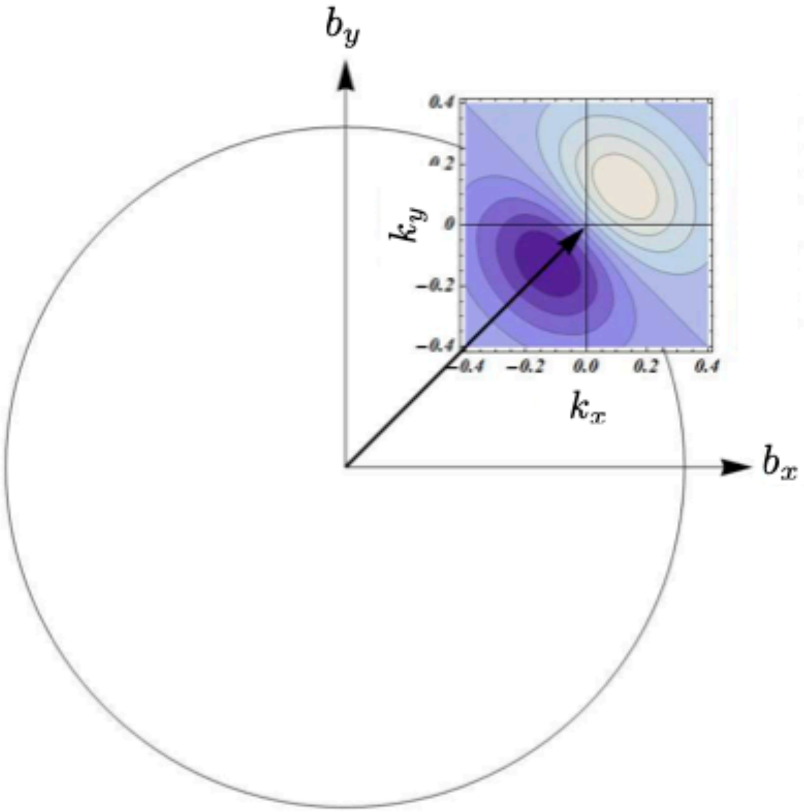

Lorcé, BP, PRD 96, 2016

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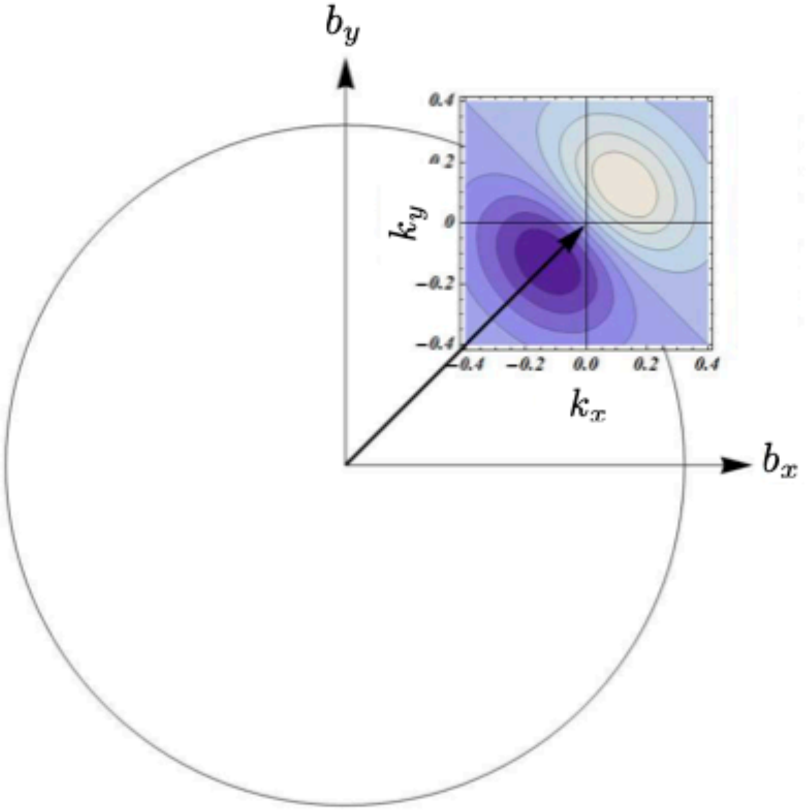

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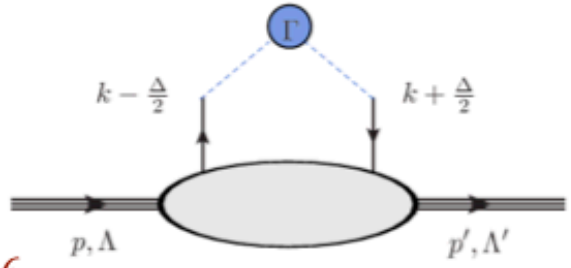
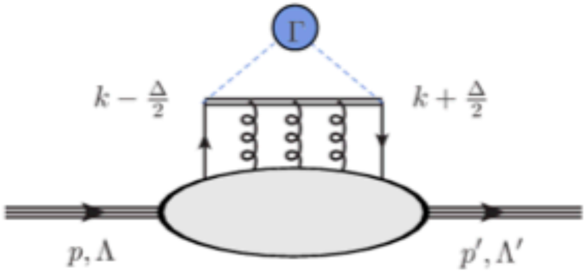
ρ_X^e T-even



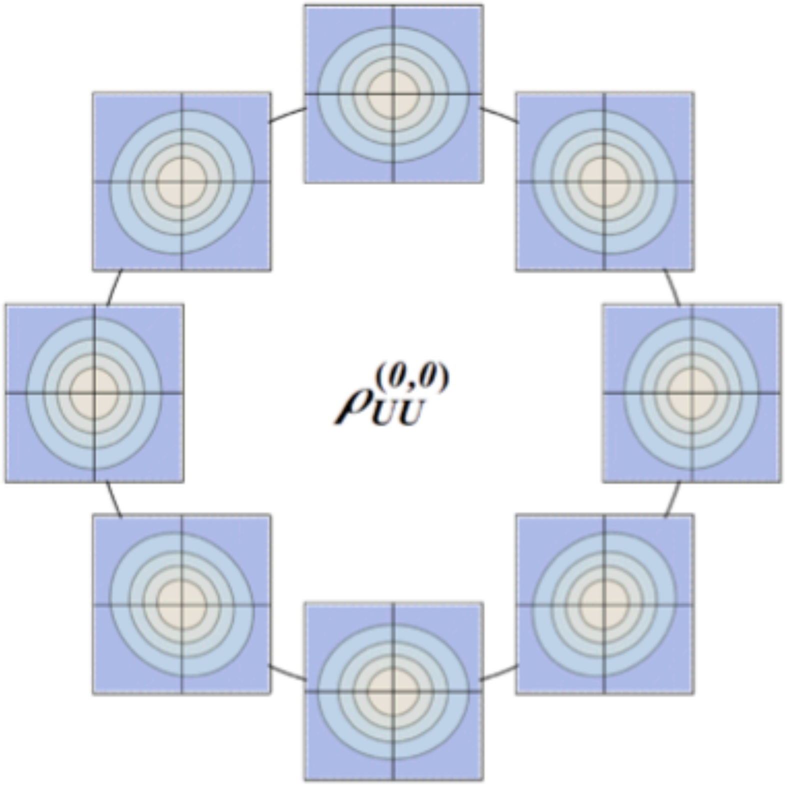
ρ_X^o T-odd



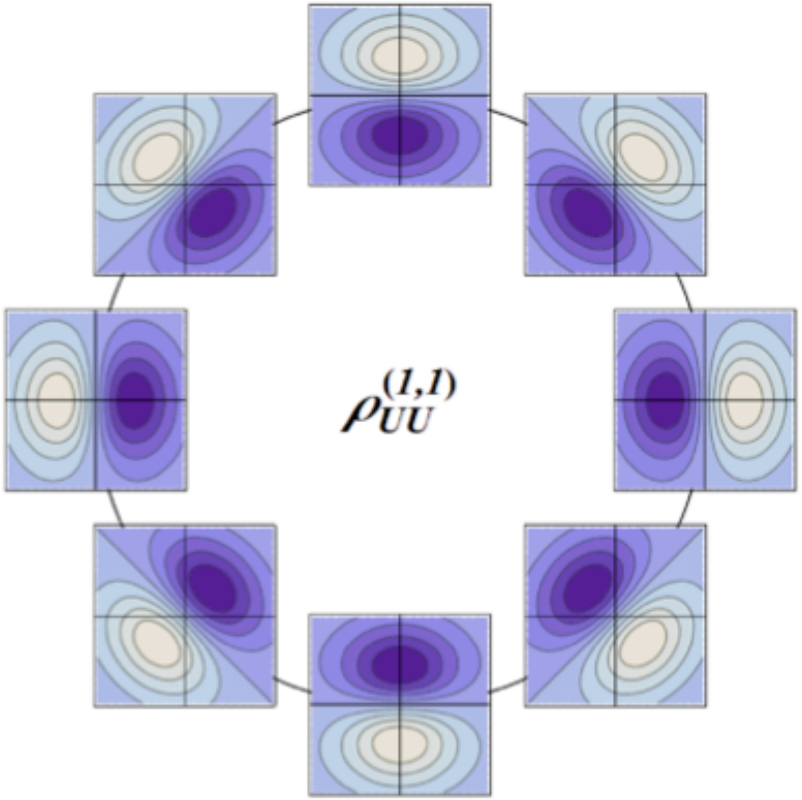
Lorcé, BP, PRD 96, 2016

Unpolarized quarks in unpolarized proton

UU


$\Re e[F_{11}]$

naive time-reversal even


$\Im m[F_{11}]$

naive time-reversal odd

Integral over $k_\perp$ ➔ GPD (monopole)
Integral over $b_\perp$ ➔ TMD (monopole)

no counterpart in the GPD and TMD cases

polar flow $(\vec{k}_\perp \perp \vec{b}_\perp)$ preferred over radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$
bottom-up symmetry ➔ no net OAM

net radial flow $(\vec{k}_\perp \parallel \vec{b}_\perp)$
due to initial/final state interactions

Unpolarized quarks in unpolarized proton

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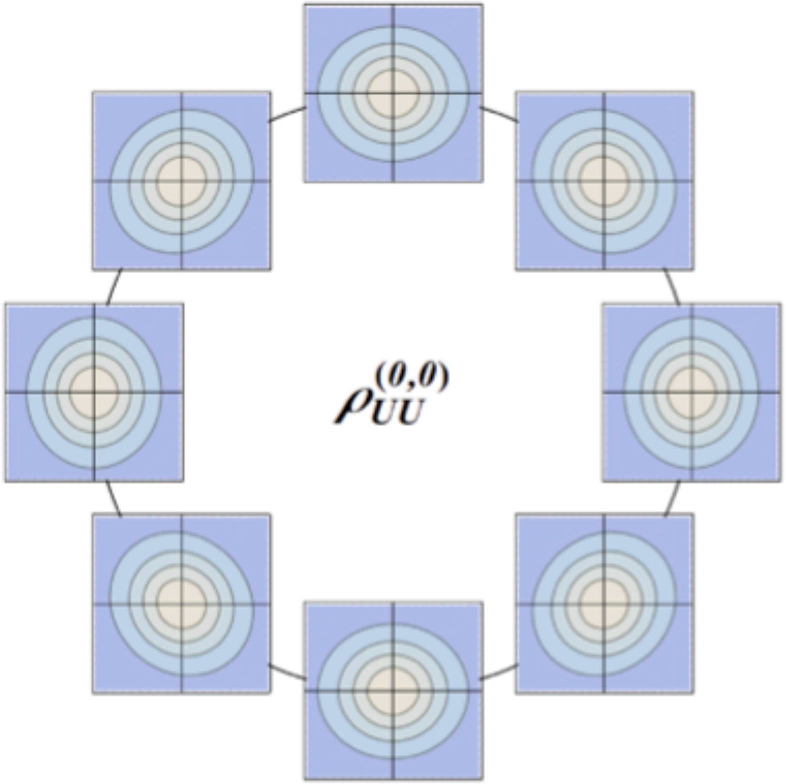

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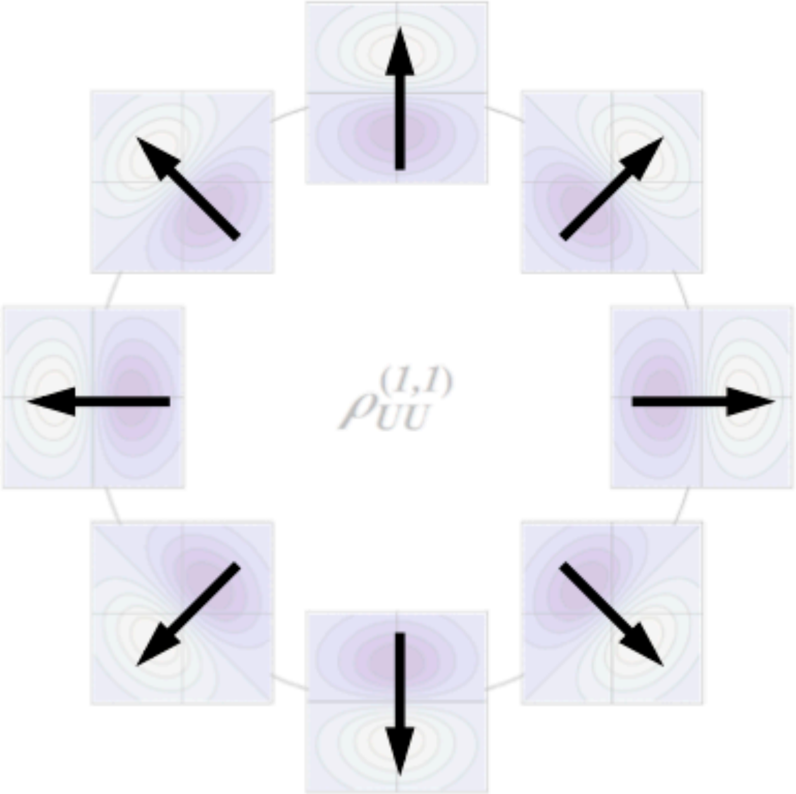

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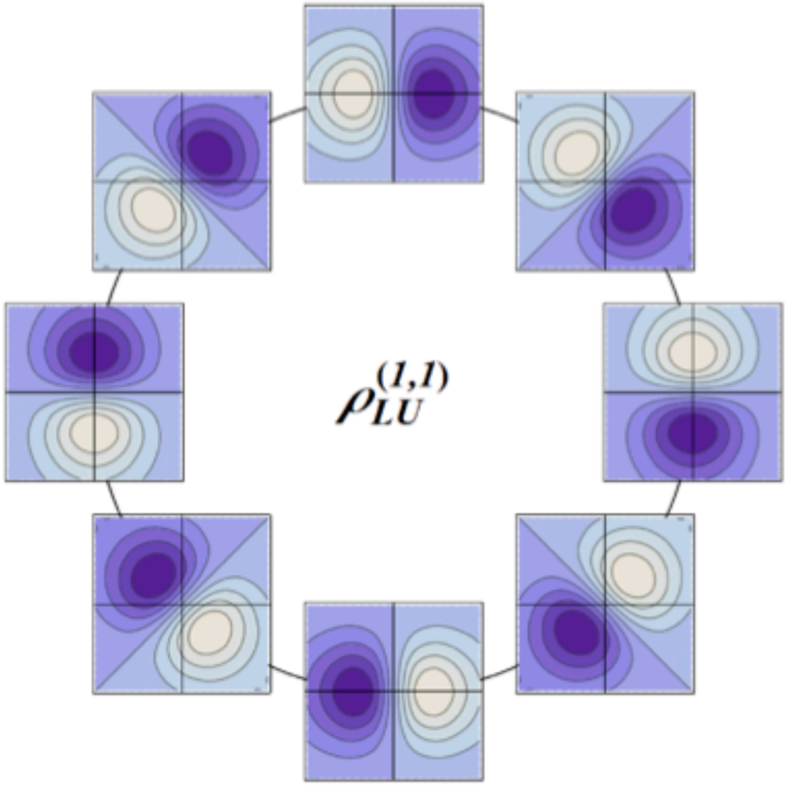
Unpolarized quarks in Longitudinally pol. proton

unique information from GTMDs

$\Re e[F_{14}]$



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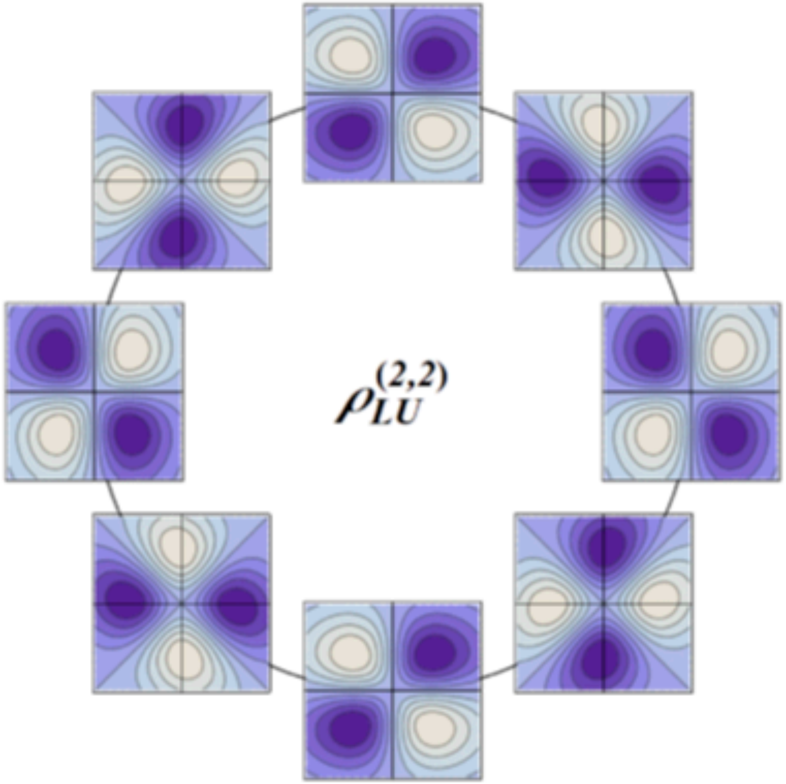


naive time-reversal even

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z$

orbital flow → net OAM correlated S_z with

naive time-reversal odd

$\propto S_z(\vec{b}_\perp \times \vec{k}_\perp)_z\,(\vec{b}_\perp \cdot \vec{k}_\perp)$

spiral flow correlated with S_z

with no-net quark flow

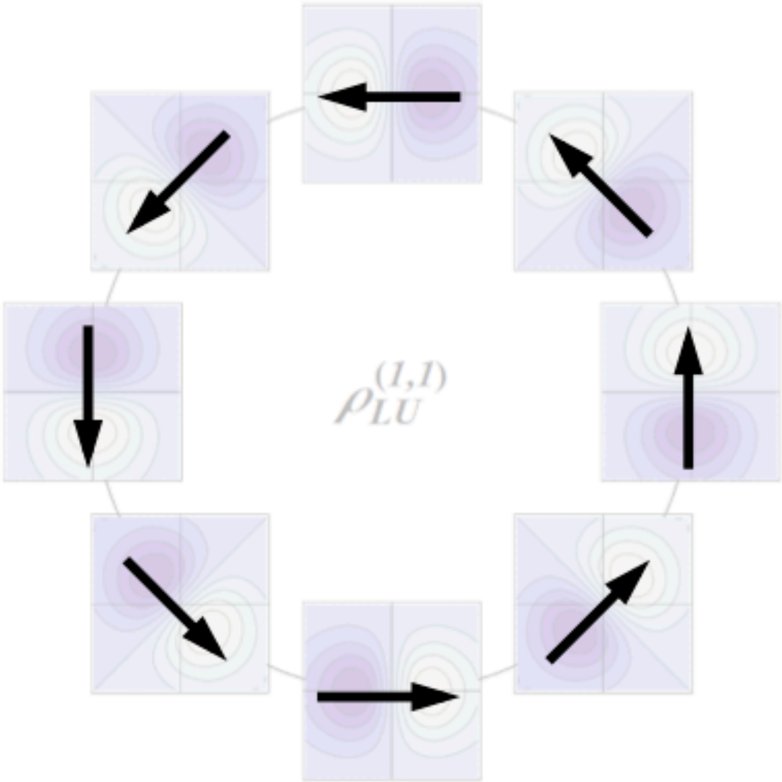
Lorcé, BP, PRD 96, 2016; PRD 84, 2011

LU

Unpolarized quarks in Longitudinally pol. proton

unique information from GTMDs

$\Re e[F_{14}]$



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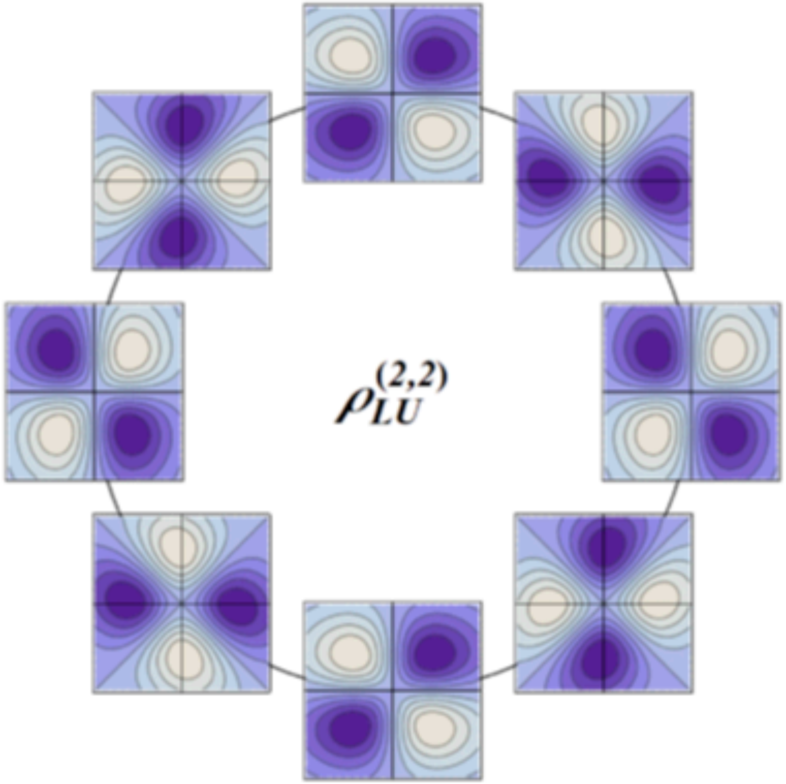

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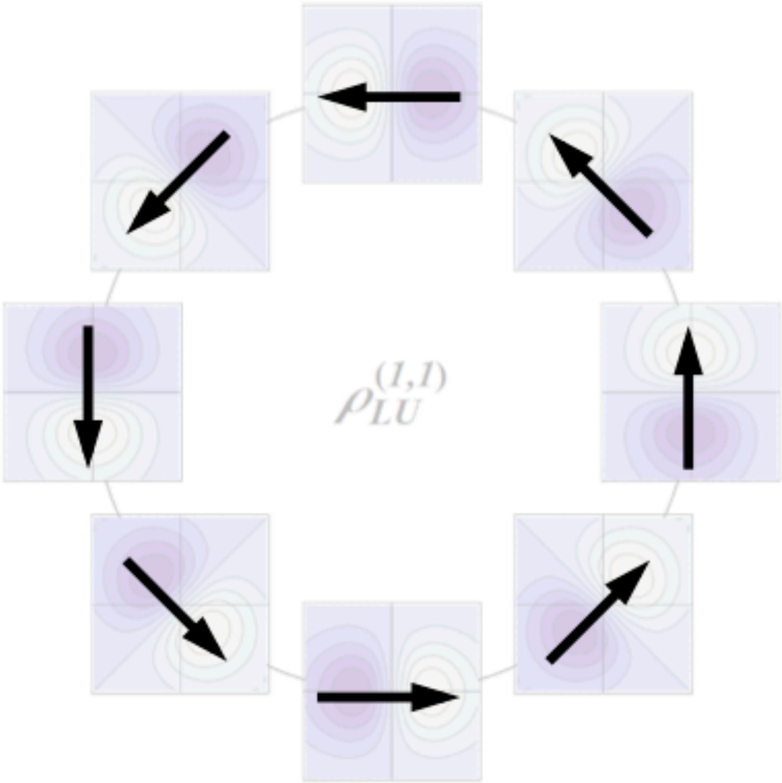
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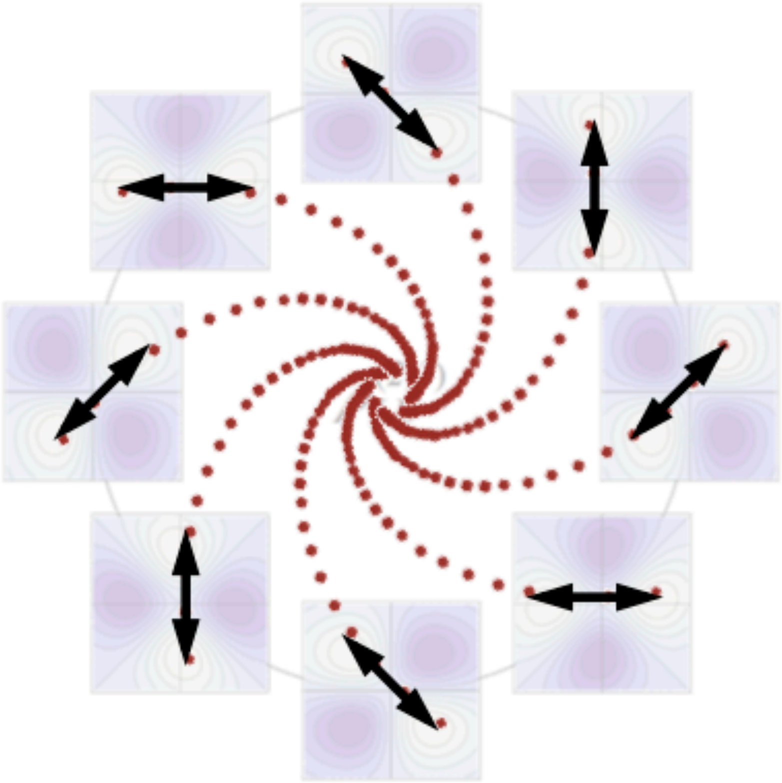
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Lorcé, BP, PRD 96, 2016; PRD 84, 2011

Orbital angular momentum of the proton from Wigner functions

$$l_z^q = \int \mathrm{d}x\mathrm{d}^2\vec{k}_\perp \mathrm{d}^2\vec{b}_\perp (\vec{b}_\perp \times \vec{k}_\perp) \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp)\rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp \, \vec{k}_\perp \, \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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- intuitive definition of OAM

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- intuitive definition of OAM
- mutually orthogonal components of quark position and momentum

→ no conflict with uncertainty principle

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- the integrand $l_z^q(x)$ represents the OAM density

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  ⟶ no conflict with uncertainty principle
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- same equation for both canonical (staple-like link) and Ji (straight link) OAM

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- the integrand $l_z^q(x)$represents the OAM density
- same equation for both canonical (staple-like link) and Ji (straight link) OAM
- equation holds also for gluon OAM

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Orbital angular momentum of the proton from Wigner functions

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$$l_z^q = \int \mathrm{d}^2\vec{b}_\perp \vec{b}_\perp \times \langle \vec{k}_\perp^q \rangle \longrightarrow \langle \vec{k}_\perp(\vec{b}_\perp)\rangle = \int \mathrm{d}x\mathrm{d}\vec{k}_\perp\, \vec{k}_\perp\, \rho_{LU}^{q,\mathcal{W}}(\vec{b}_\perp, \vec{k}_\perp, x)$$

- intuitive definition of OAM
- mutually orthogonal components of quark position and momentum

  ⟶ no conflict with uncertainty principle
- the integrand $l_z^q(x)$represents the OAM density
- same equation for both canonical (staple-like link) and Ji (straight link) OAM
- equation holds also for gluon OAM
- it can be calculated in LQCD *Engelhardt, PRD95 (2017) 094505*

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

Lorcé, BP, Xiong, Yuan, PRD 85 (2012) 114006

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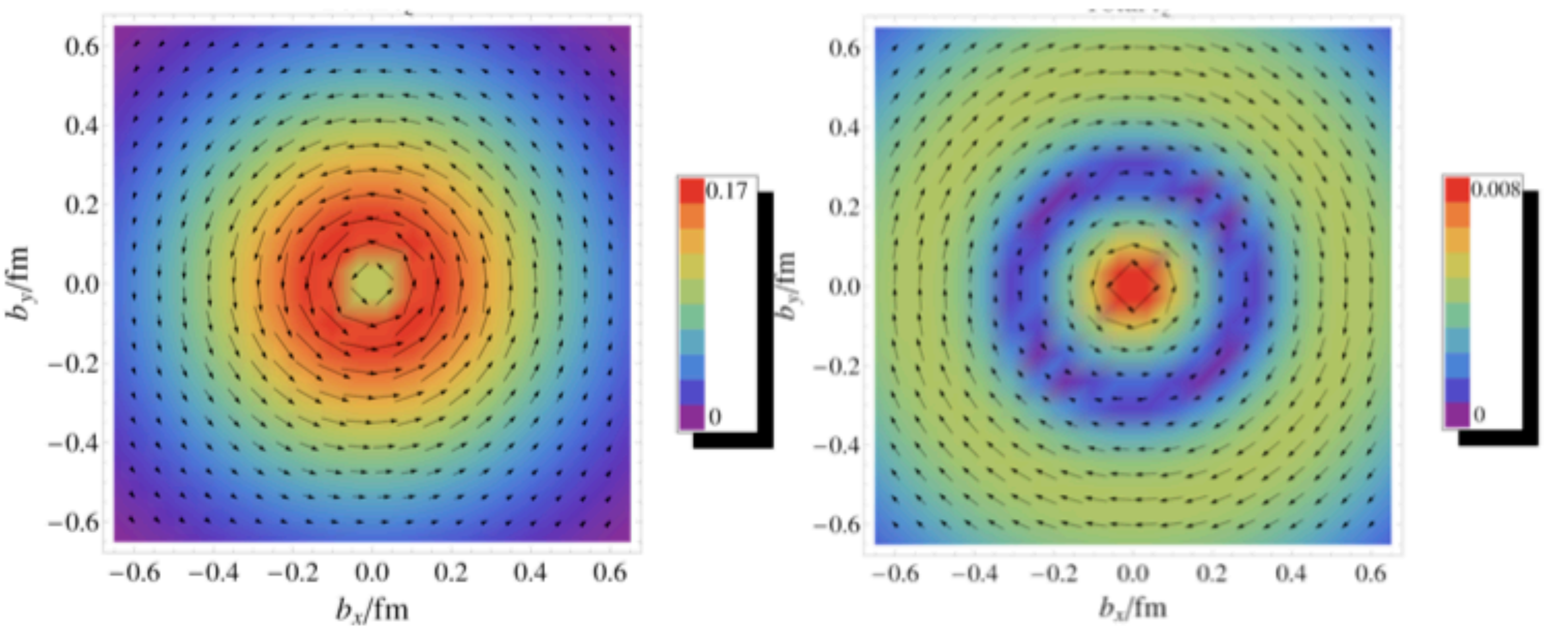

→ Proton spin

→ u-quark OAM

← d-quark OAM

Lorcé, BP, PRD 84 (2011) 014015

Hatta, PLB 708 (2012) 186

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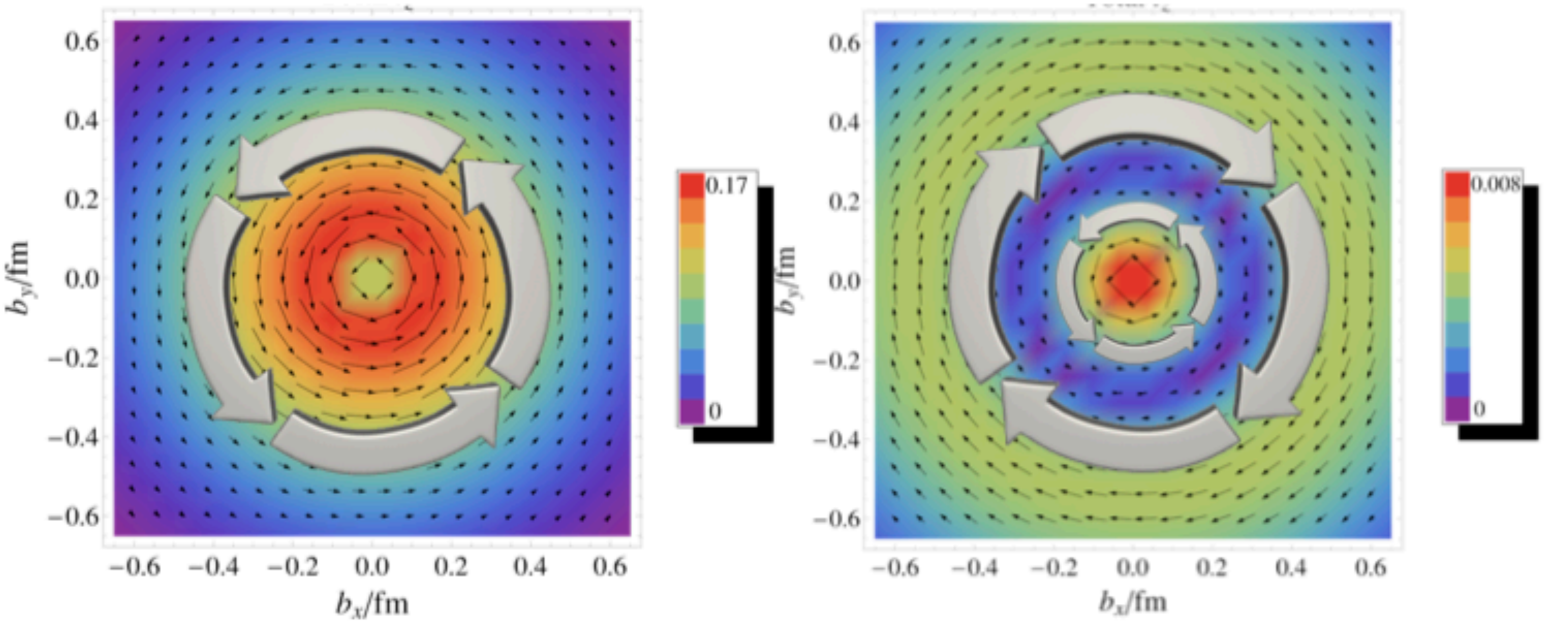

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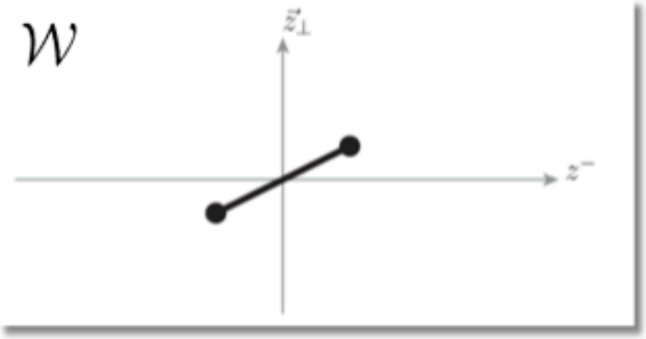
[Lorcé, BP (2011)]
[Lorcé, BP, Xiong, Yuan(2011)]

Kinetic

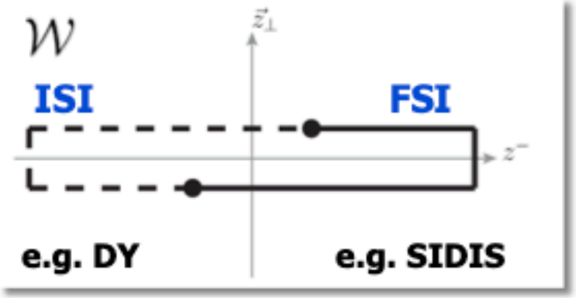
[Ji (1997)]



[Ji, Xiong, Yuan (2012)]
[Burkardt (2012)]

Canonical

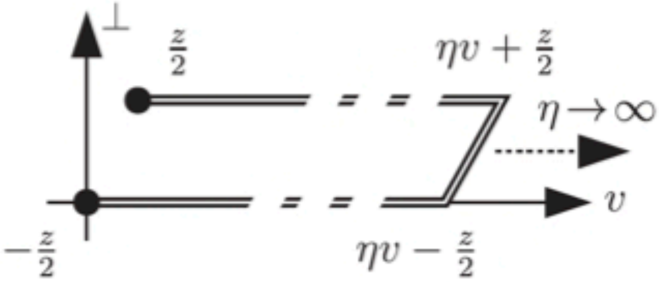
[Jaffe, Manohar (1990)]



[Hatta (2012)]

difference between the two definitions can be interpreted as
the change in the quark OAM as the quark leaves the target in a DIS experiment
[M. Burkardt (2013)]

Lattice calculation



Continuous interpolation between the Ji limit $\eta = 0$ and the Jaffe-Manohar (canonical) limit $\eta \to \infty$

Staple direction off the light-cone
light-cone limit for $\hat{\zeta} = \frac{v \cdot P}{\sqrt{|v^2|}\sqrt{|P^2|}} \to \infty$

M. Engelhardt, Phys. Rev. D95, 094505 (2017)
M. Engelhardt et al., PRD102, 074505 (2020)

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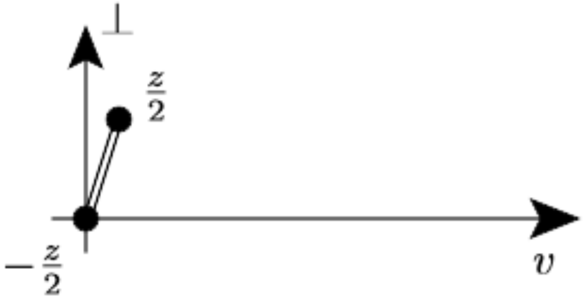

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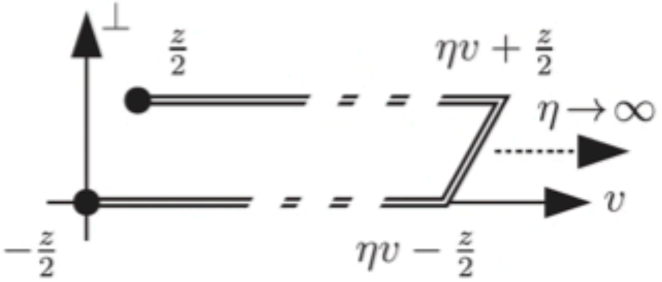

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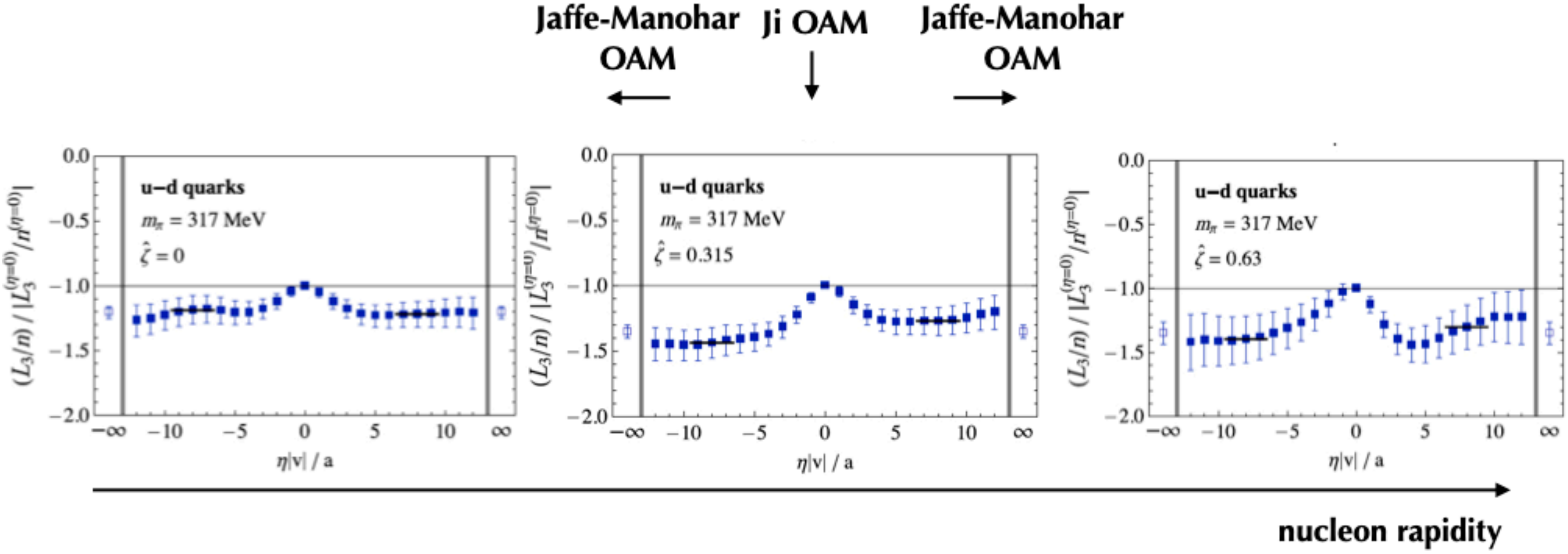
M. Engelhardt, Phys. Rev. D95, 094505 (2017)
M. Engelhardt et al., PRD102, 074505 (2020)


Jaffe-Manohar OAM ←

Ji OAM ↓

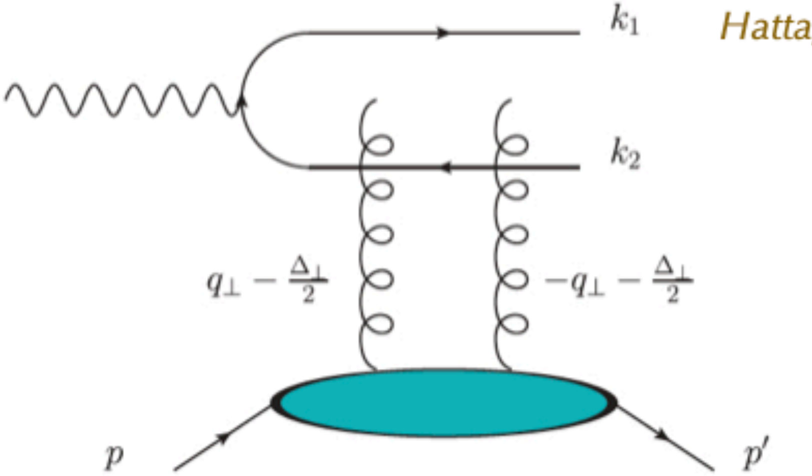
Jaffe-Manohar OAM →

nucleon rapidity →

Observables for GTMDs and Wigner functions

Diffractive Exclusive back-to-back dijet production in ℓN / ℓA collisions

Hatta, Xiao, Yuan, PRL 116 (2016) 202301



$$\vec{\Delta}_\perp \approx -(\vec{k}_{\perp,1} + \vec{k}_{\perp,2}) \qquad \vec{k}_\perp \sim \vec{P}_\perp = \frac{(\vec{k}_{\perp,1} - \vec{k}_{\perp,2})}{2} \qquad |\vec{P}_\perp| \gg |\vec{k}_{\perp,1} + \vec{k}_{\perp,2}|$$

- Reconstruction of full dijet kinematics and measure the azimuthal modulations in the angle between $\vec{\Delta}_\perp$ and $\vec{P}_\perp$

- At small x: sensitivity to gluon GTMDs

- Estimates in the CGC effective field theory suggest that modulations are maximum some tens of percent level

Mäntysaari, Mueller, Schenke, PRD99 (2019) 074004; Boer, Setyadi, PRD104 (20121) 074006

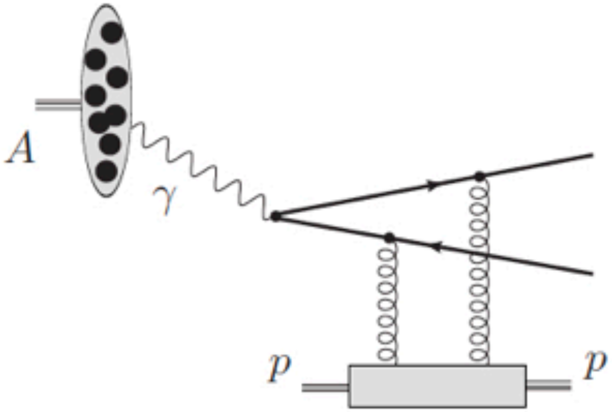
- With proton polarization one may access $F_{1,4}^g$

Hatta, Nakagawa, Xiao, Yuan, Zhao, PRD 95 (2017) 114032; Ji, Yuan, Zhao, PRL 118 (2017) 192004

Observables for GTMDs and Wigner functions

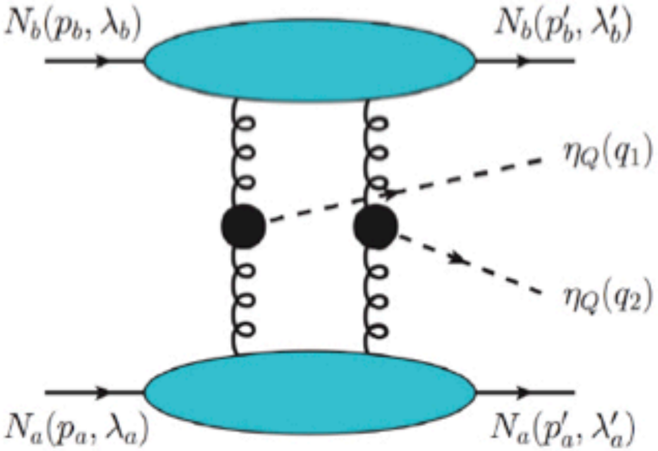
Exclusive dijet production in pA UPC (gluon GTMDs)

Hagiwara, Hatta, Pasechnik, Tasevsky, Teryaev, PRD 96 (2016) 034009



Exclusive double quarkonia production in hadronic collisions (gluon GTMDs)

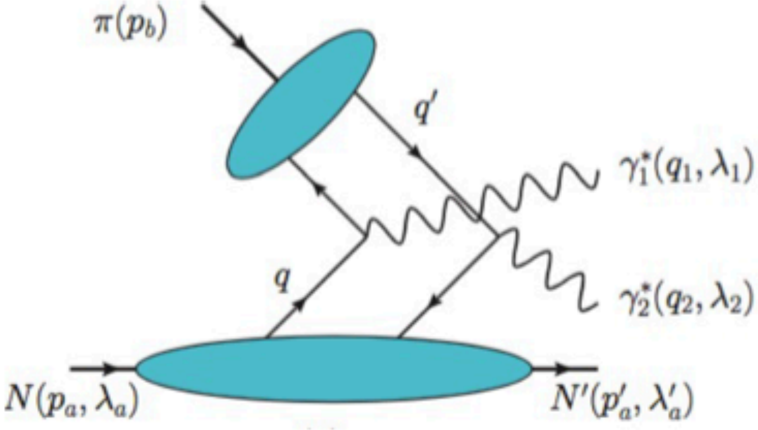
Bhattacharya, Metz, Ojha, Tsai, Zhou, arXiv:1802.10550 *Boussarie, Hatta, Xiao, Yuan, PRD 98 (2015) 074015*



Observables for GTMDs and Wigner functions

Exclusive pion-nucleon double Drell-Yan (quark GTMDs)

Bhattacharya, Metz, Zhou, PLB 771 (2017) 396



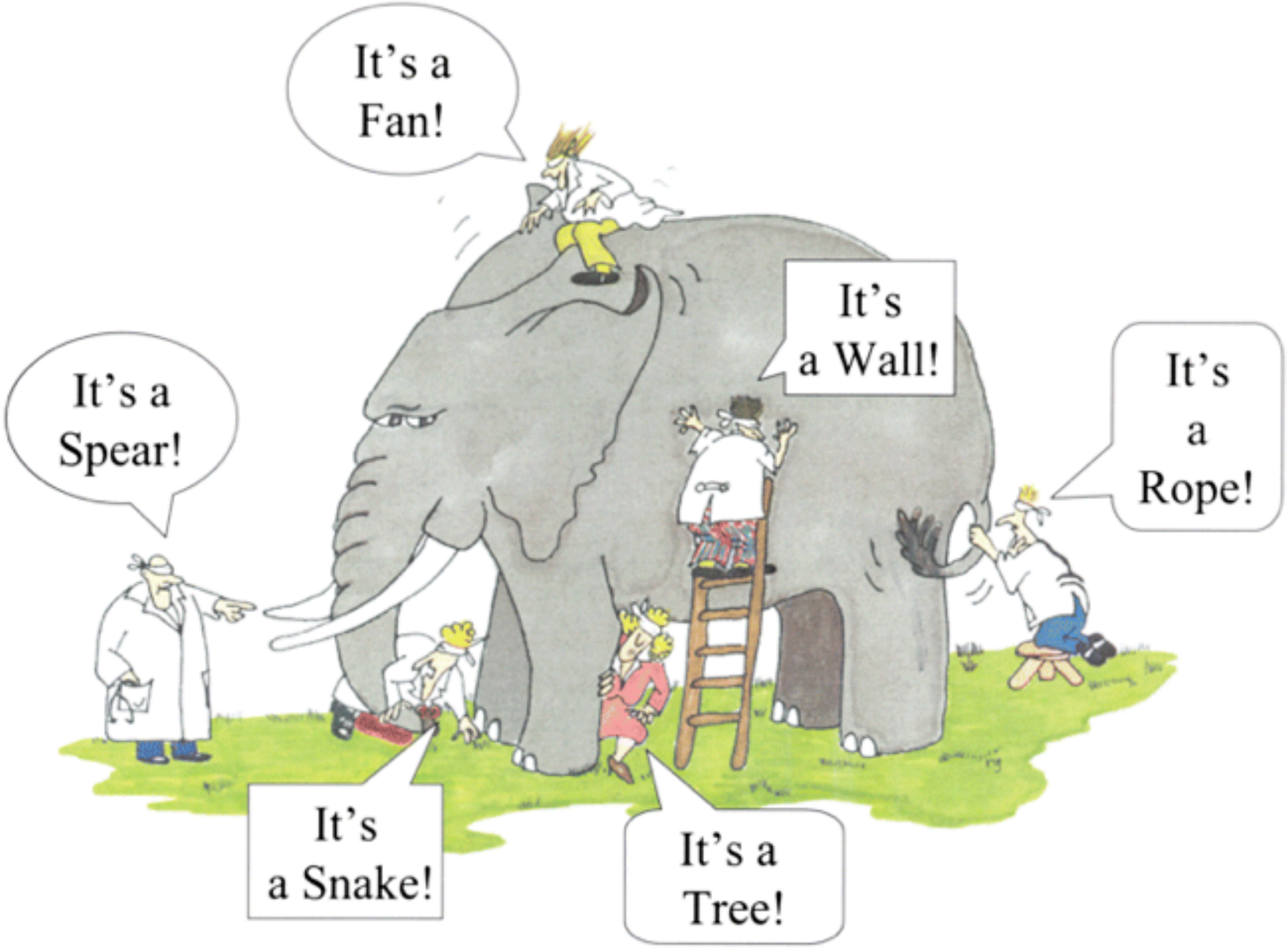
- In leading order is sensitive to ERBL region only

- Low count rate (amplitude $T \sim \alpha_{\rm em}^2$)

-New proposal to access quark GTMDs: exclusive π^0 production off the proton with longitudinally polarized proton

Bhattacharya, Zheng, Zhou, arXiv: 2312.01309

The blind men and the elephant



Different observables in different kinematical regimes
need to talk to each other
to reconstruct the full picture of the nucleon