Nucleon Structure Properties

em	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N' \vert J^\mu_{\text{em}} \vert N \rangle$	$\longrightarrow$	$Q,\ \mu$
weak	$\partial_\mu J^\mu_{\text{weak}} = 0$	$\langle N' \vert J^\mu_{\text{weak}} \vert N \rangle$	$\longrightarrow$	$g_A,\ g_p$
gravity	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\text{grav}} \vert N \rangle$	$\longrightarrow$	M_N, J, D

$Q_{\text{prot}} = 1.602176487(40) \times 10^{-19}\,\text{C}$

$\mu_{\text{prot}} = 2.792847356(23)\mu_N$

$M_{\text{prot}} = 938.272013(23)\,\text{MeV}$

$g_p = 8 - 12$

$g_A = 1.2694(28)$

$J = \frac{1}{2}$

$$D = \frac{4}{5} d_1 = ??$$

can be accessed from GPDs in hard exclusive reactions

M. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A33 (2018) 1830025

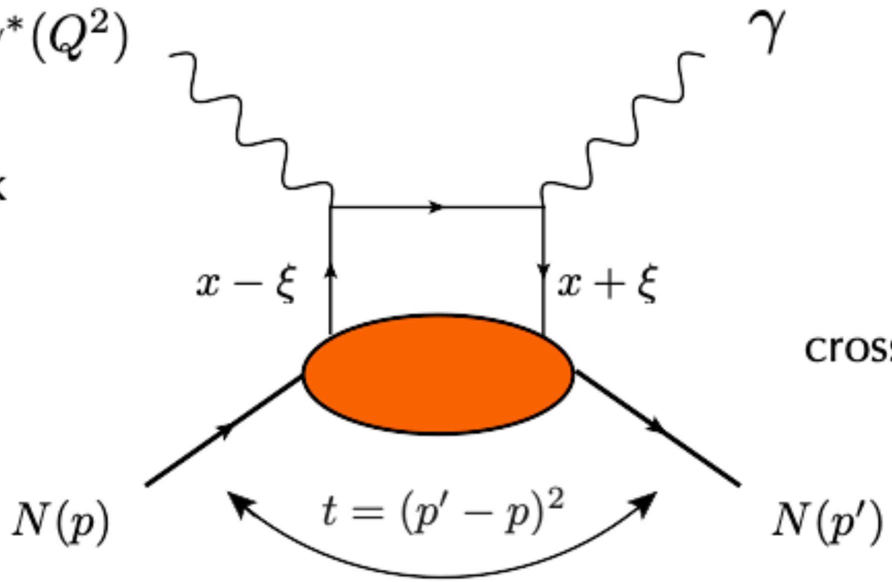
Dispersion relation approach

DVCS at leading twist

x : average fraction of quark longitudinal momentum

ξ : fraction of longitudinal momentum transfer

t : nucleon momentum transfer



$t \ll \nu,\, Q^2$

crossing symmetric variable
$$\nu = \frac{s-u}{4M_N}$$

DVCS tensor at twist 2: $T^{\mu\nu} = \sum_{i=1}^{4} A_i(\nu, t, Q^2)\, O_i^{\mu\nu}$

unpolarized quark

$$A_1 = \mathcal{H} + \mathcal{E}$$

$$A_2 = -\mathcal{E}$$

long. polarized quark

$$A_3 = \tilde{\mathcal{H}}$$

$$A_4 = \tilde{\mathcal{E}}$$

Twist-2 DVCS amplitudes

$$A_i(\xi,t) = \int_0^1 \mathrm{d}x\, F_i^+(x,\xi,t) \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}\right] \qquad (i=1,\ldots,4)$$

- Involve singlet GPDs: $F_i^+ = \{H^+ + E^+, -E^+, \tilde{H}^+, \tilde{E}^+\}$

$$F_i^+(x,\xi,t) = F_i(x,\xi,t) - F_i(-x,\xi,t)$$

- Imaginary part: GPD at $x=\xi$

$$\mathrm{Im}\, A_i(\xi,t) = -\pi F_i^+(\xi,\xi,t)$$

- Real part involves convolution integral:

$$\mathrm{Re}\, A_i(\xi,t) = \mathcal{P} \int_0^1 \mathrm{d}x\, F_i^+(x,\xi,t) \left[\frac{1}{x-\xi} + \frac{1}{x+\xi}\right]$$

*the dependence on Q^2 is implicit

Dispersion relations at fixed t and Q^2

energy variables: $\nu = \frac{s-u}{4M_N} = \frac{Q^2}{4M_N \xi}$ and $\nu' = \frac{Q^2}{4M_N x}$

$A_i(\nu, t)$: analytical functions in the complex ν plane, with cuts on the real axis

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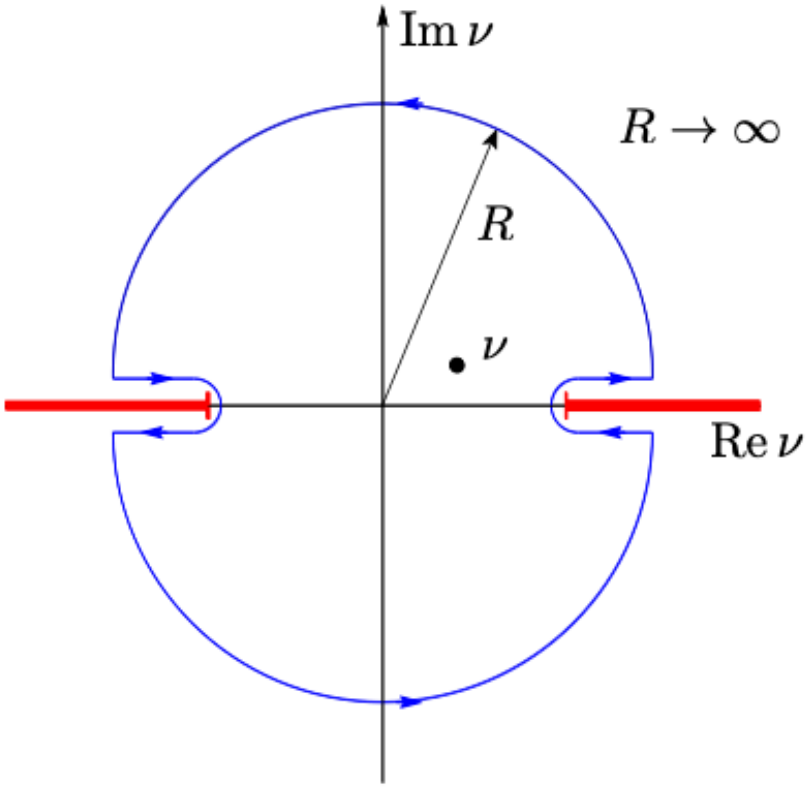

- Cauchy integral formula

$$A_i(\nu, t) = \frac{1}{2\pi i} \oint_C d\nu' \frac{A_i(\nu', t)}{\nu' - \nu}$$

Dispersion relations at fixed t and Q²

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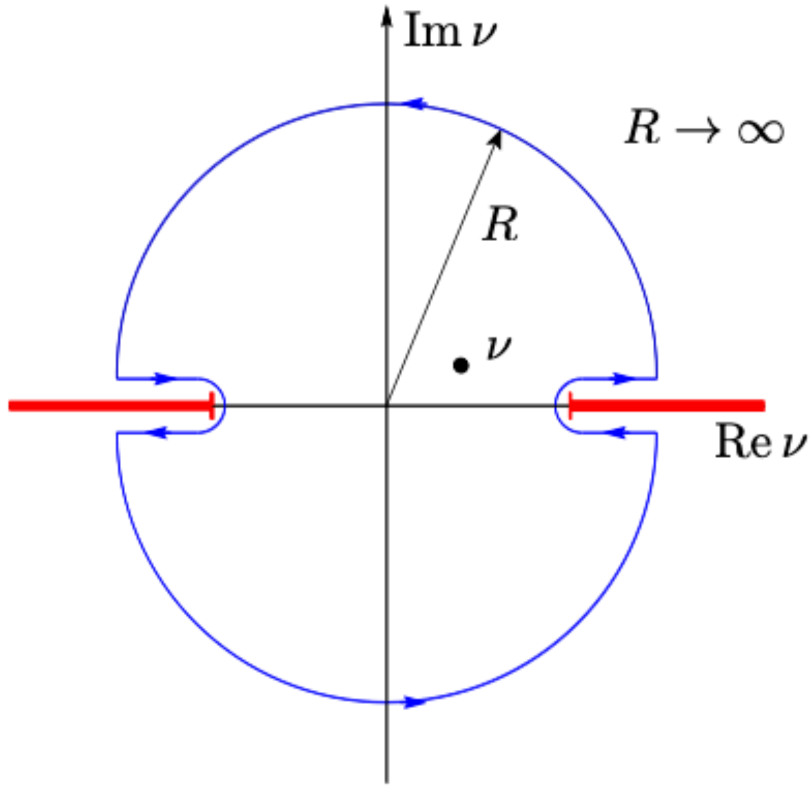

- Cauchy integral formula

$$A_i(\nu, t) = \frac{1}{2\pi i}\oint_C d\nu' \frac{A_i(\nu', t)}{\nu' - \nu}$$

- Crossing symmetry and analyticity

$$A_i(\nu, t) = A_i(-\nu, t) \qquad\qquad A_i(\nu^*, t) = A_i^*(\nu, t)$$

Unsubtracted Dispersion Relations

$$\mathrm{Re}\, A_i(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \mathrm{Im}\, A_i(\nu', t) \frac{\nu' \mathrm{d}\nu'}{\nu'^2 - \nu^2} \qquad (i = 1, \ldots, 4)$$

non-convergent integrals for A_2

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non-convergent integrals for A_2

↓

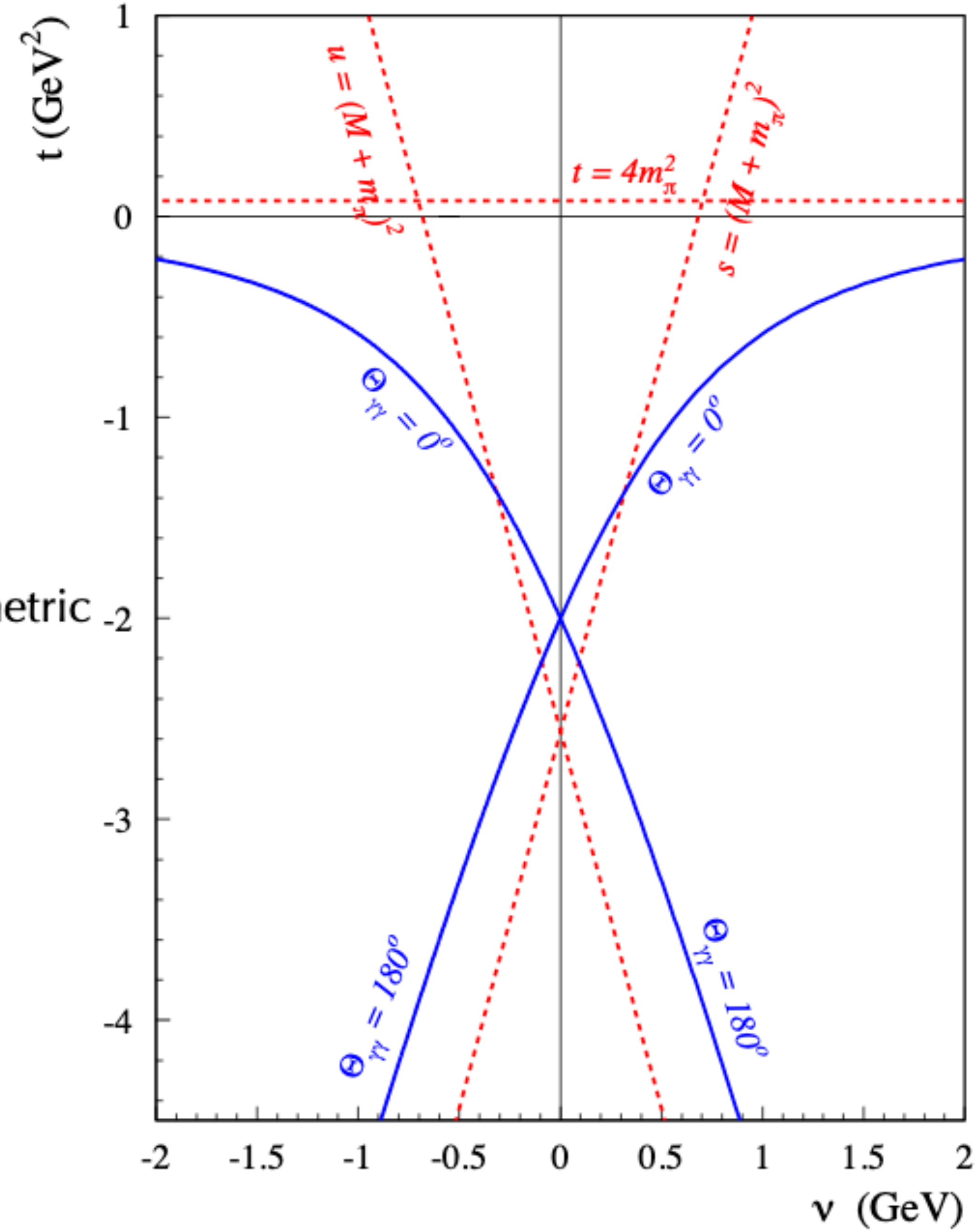
Subtracted Dispersion Relations

$$\mathrm{Re}\, A_2(\nu, t) = A_2(0, t) + \frac{2}{\pi} \nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \mathrm{Im}\, A_2(\nu', t) \frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

↓ subtraction at $\nu = 0$

Fixed

$Q^2 = 2\,\mathrm{GeV}^2$



$\nu = \frac{s-u}{4M_N}$

crossing symmetric variable

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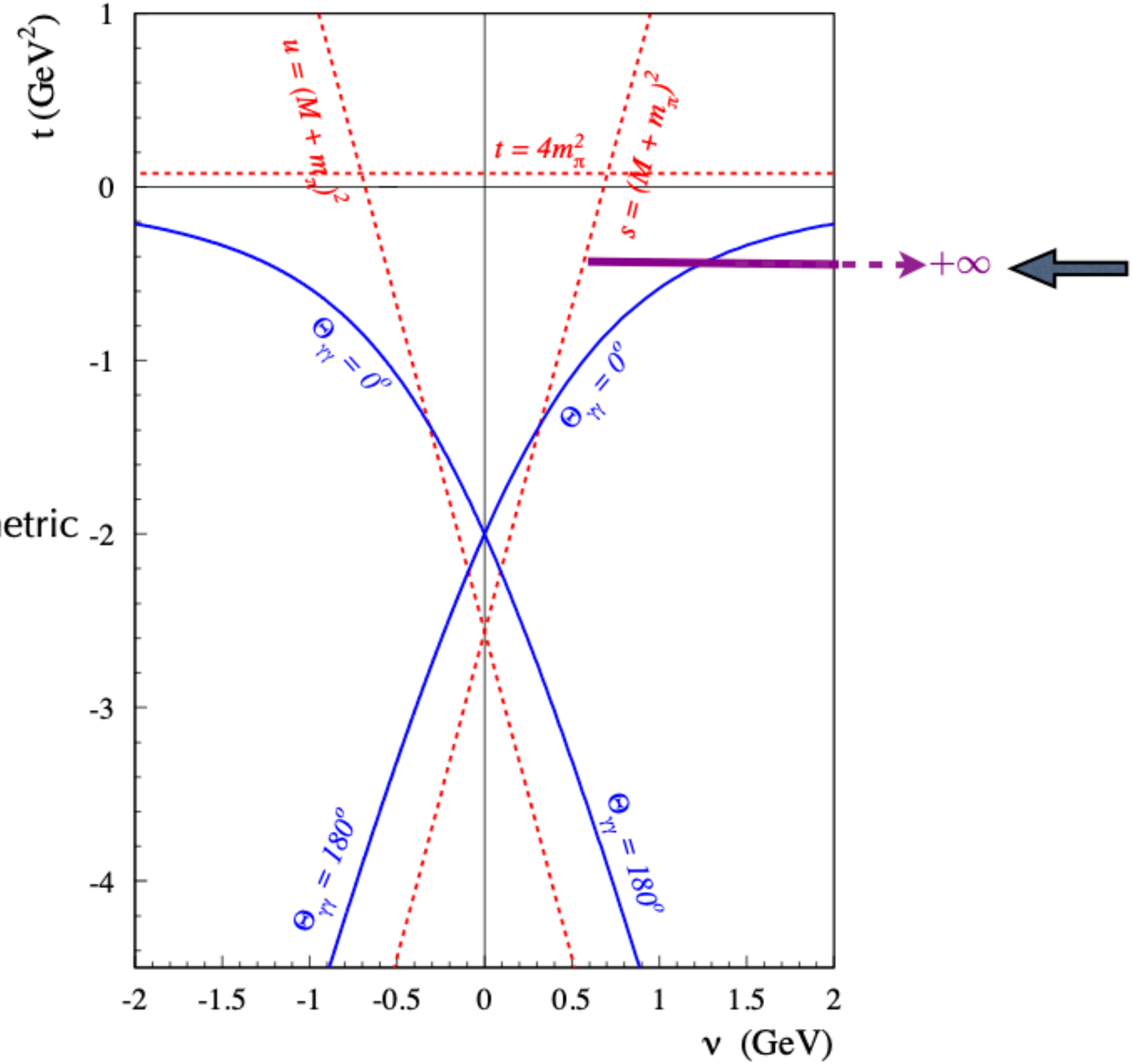
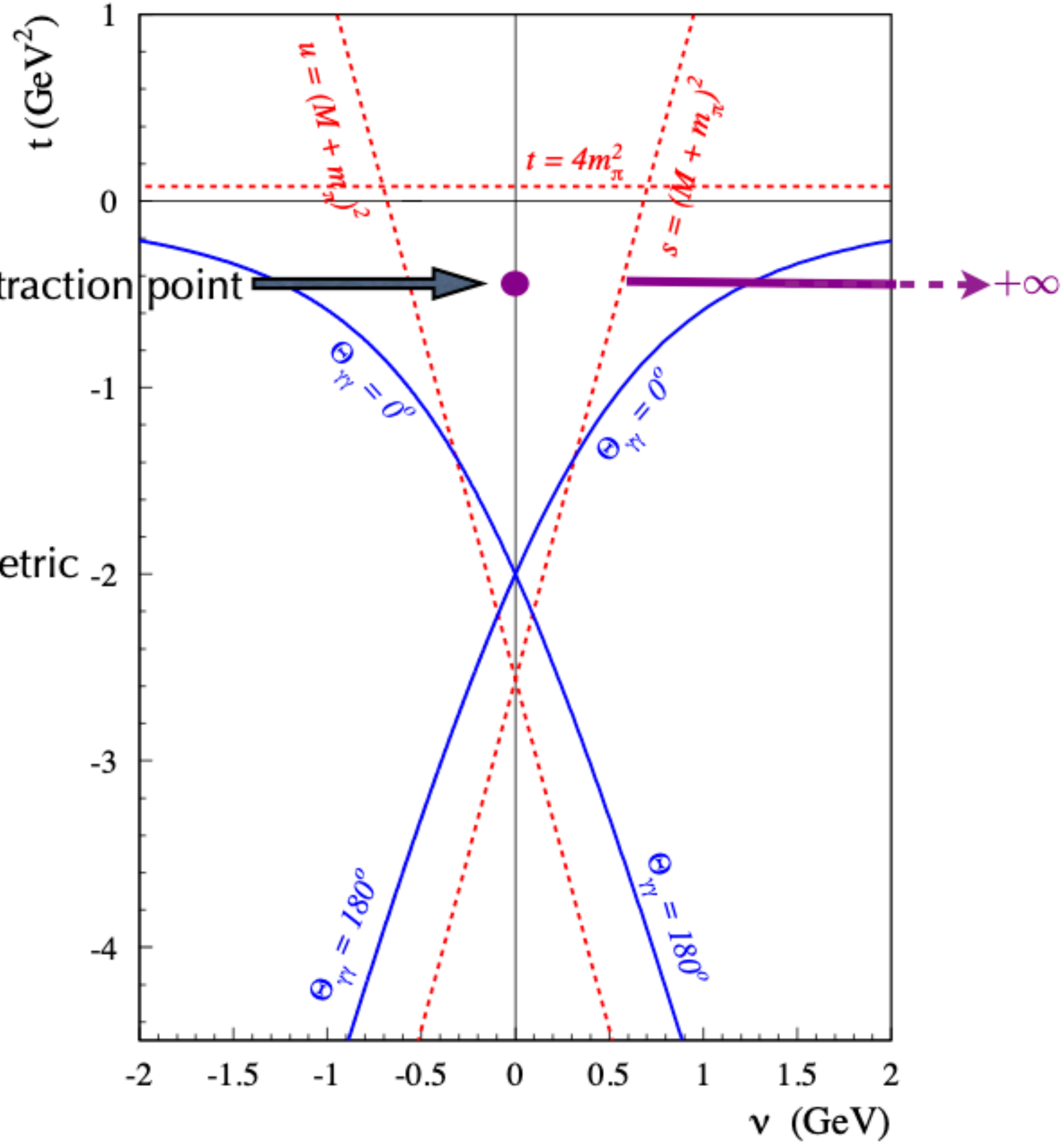


$\nu = \frac{s-u}{4M_N}$

crossing symmetric variable

DRs in the s-channel at fixed t

Fixed
$Q^2 = 2\,\text{GeV}^2$



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Dispersion relations in terms of GPDs

once subtracted fixed-t DR in the variable x

$$\mathrm{Re}\, A_2(\xi, t) = \Delta(t) + \frac{2}{\pi}\mathcal{P}\int_0^1 \frac{dx}{x}\,\frac{\mathrm{Im}\, A_2(x,t)}{(\xi^2/x^2 - 1)}$$

Anikin, Teryaev (2007); Kumericki-Passek, Mueller, Passek (2008); Diehl, Ivanov (2007); Polyakov, Vanderhaeghen (2008)

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- imaginary part in terms of GPDs: $\mathrm{Im}\, A_2(x,t) = \pi E^+(x, \xi = x, t)$

$$\mathrm{Re}\, A_2(\xi, t) = \Delta(t) - \mathcal{P}\int_0^1 dx E^+(x,x,t)\left[\frac{1}{x-\xi} + \frac{1}{x+\xi}\right]$$

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- difference between convolution and dispersion integrals:

$$\Delta(t) = \mathcal{P}\int_0^1 \mathrm{d}x\left[E^+(x,x,t) - E^+(x,\xi,t)\right]\left[\frac{1}{x-\xi}+\frac{1}{x+\xi}\right]$$

Anikin, Teryaev (2007); Kumericki-Passek, Mueller, Passek (2008); Diehl, Ivanov (2007); Polyakov, Vanderhaeghen (2008)

Subtraction Function

$$\Delta(t) = \mathcal{P}\int_0^1 \mathrm{d}x \left[E^+(x,x,t) - E^+(x,\xi,t)\right] \left[\frac{1}{x-\xi} + \frac{1}{x+\xi}\right]$$

Anikin, Teryaev (2007); Radyushkin (2012)

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⇨ Subtraction function is independent of ξ ⟶ formally put $\xi = 0$

$$\Delta(t) = 2\,\mathcal{P}\int_0^1 \mathrm{d}x\,\frac{1}{x}\,[E^+(x,x,t) - E^+(x,0,t)]$$

Anikin, Teryaev (2007); Radyushkin (2012)

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$$\Delta(t) = 2\,\mathcal{P}\int_0^1 \mathrm{d}x\,\frac{1}{x}\left[E^+(x,x,t) - E^+(x,0,t)\right]$$

⇨ Time-Reversal invariance: GPD even in ξ : $E(x,x,t) = E(x,-x,t)$

$$\Delta(t) = 2\,\mathcal{P}\int_{-1}^1 \mathrm{d}x\,\frac{1}{x}\left[E(x,x,t) - E(x,0,t)\right]$$

Anikin, Teryaev (2007); Radyushkin (2012)

Subtraction Function: relation with D-term

$$\int_{-1}^{1} \frac{\mathrm{d}x}{x}[E(x, x+\xi, t) - E(x, \xi, t)] = \int_{-1}^{1} \frac{\mathrm{d}x}{x} \sum_{n=1}^{\infty} \frac{x^n}{n!} \frac{\partial^n}{\partial \xi^n} E(x, \xi, t)$$

$$= \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \frac{\partial^{n+1}}{\partial \xi^{n+1}} \left[\int_{-1}^{1} \mathrm{d}x x^n E(x, \xi, t)\right]$$

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⇨ Polinomiality of Mellin moments of GPDs

$$\int_{-1}^{1} \mathrm{d}x x^n E(x, \xi, t) = e_0^{(n)}(t) + e_2^{(n)}(t)\xi^2 + \cdots + e_{n+1}^{(n)}(t)\xi^{n+1}$$

$$\int_{-1}^{1} \frac{\mathrm{d}x}{x} [E(x, x+\xi, t) - E(x, \xi, t)] = \sum_{n=0}^{\infty} e_{n+1}^{(n)}(t)$$

Subtraction Function: relation with D-term

$$\int_{-1}^{1} \frac{\mathrm{d}x}{x}[E(x,x+\xi,t) - E(x,\xi,t)] = \int_{-1}^{1} \frac{\mathrm{d}x}{x} \sum_{n=1}^{\infty} \frac{x^n}{n!} \frac{\partial^n}{\partial \xi^n} E(x,\xi,t)$$

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$$\int_{-1}^{1} \frac{\mathrm{d}x}{x}[E(x,x+\xi,t) - E(x,\xi,t)] = \sum_{n=0}^{\infty} e_{n+1}^{(n)}(t)$$

⇨ Highest moment generated by Polyakov-Weiss D-Term $e_{n+1}^{(n)} = -\int_{-1}^{1} \mathrm{d}z z^n D(z,t)$

$$\Delta(t) = 2\mathcal{P}\int_{-1}^{1} \mathrm{d}x \frac{1}{x}\left[E(x,x,t) - E(x,0,t)\right] = -2\int_{-1}^{1} \mathrm{d}z \frac{D(z,t)}{1-z}$$

Subtraction Function: relation with D-term

$$\Delta(t, Q^2) = -2\int_{-1}^{1} \mathrm{d}z \frac{D(z,t)}{1-z}$$

⇨ Gegenbauer expansion of D-term $D(z,t) = (1-z^2) \sum_{\substack{n=1 \\ n\, odd}}^{\infty} d_n(t) C_n^{(3/2)}(z)$

$$\Delta(t) = -4 \sum_{\{n\, odd\}}^{\infty} d_n(t)$$

⇨ Relation to EMT for factors $d_1(t) = 5\, C(t) = \frac{5}{4} D(t)$

Dispersion Relations for DVCS amplitudes

- s-channel subtracted DRs:

$$\mathrm{Re}\, A_2(\nu, t) = \Delta(t) + \frac{2}{\pi}\nu^2 \mathcal{P} \int_{\nu_0}^{\infty} \mathrm{Im} A_2(\nu', t) \frac{\mathrm{d}\nu'}{\nu'(\nu'^2 - \nu^2)}$$

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B. Pasquini, M. Vanderhaeghen, "Dispersion Theory in Electromagnetic Interactions", Ann. Rev. Nucl. Part. Sci., 68 (2018) 75

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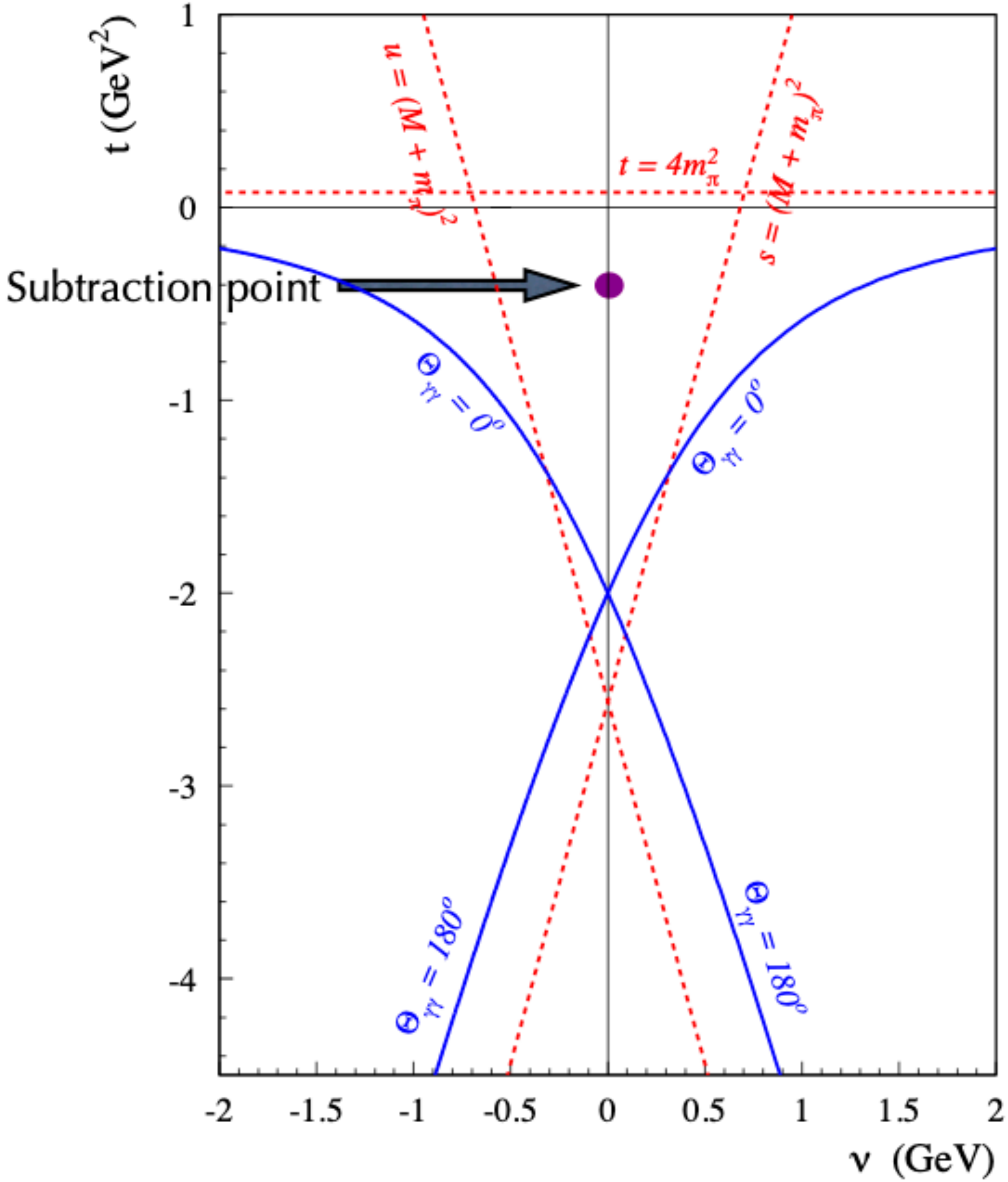
- t-channel DRs for subtraction function

$$-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2$$

$$\Delta(t) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}_t\, A_2(0,t')}{t'-t} + \frac{1}{\pi}\int_{-\infty}^{-a} \mathrm{d}t' \frac{\mathrm{Im}_t\, A_2(0,t')}{t'-t}$$

Fixed $Q^2 = 2\,\mathrm{GeV}^2$



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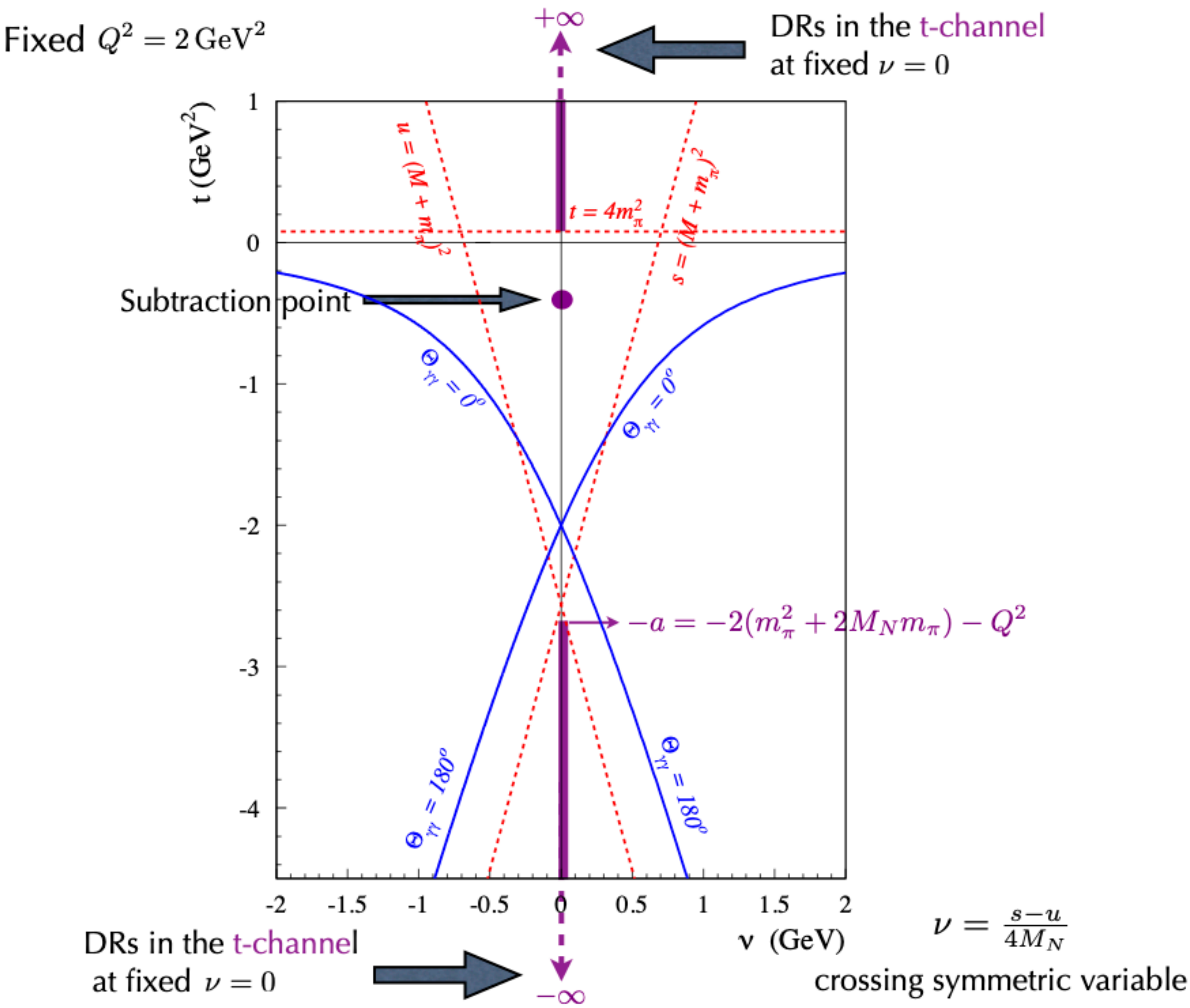
crossing symmetric variable



Fixed $Q^2 = 2\,\mathrm{GeV}^2$
$+\infty$
DRs in the t-channel
at fixed $\nu = 0$
t (GeV^2)
$u = (M + m_\pi)^2$
$s = (M + m_\pi)^2$
$t = 4m_\pi^2$
Subtraction point
$\Theta_{\gamma\gamma} = 0^o$
$\Theta_{\gamma\gamma} = 0^o$
$-a = -2(m_\pi^2 + 2M_N m_\pi) - Q^2$
$\Theta_{\gamma\gamma} = 180^o$
$\Theta_{\gamma\gamma} = 180^o$
ν (GeV)
$\nu = \frac{s-u}{4M_N}$
DRs in the t-channel
at fixed $\nu = 0$
$-\infty$
crossing symmetric variable

Dispersion Relations for DVCS amplitudes

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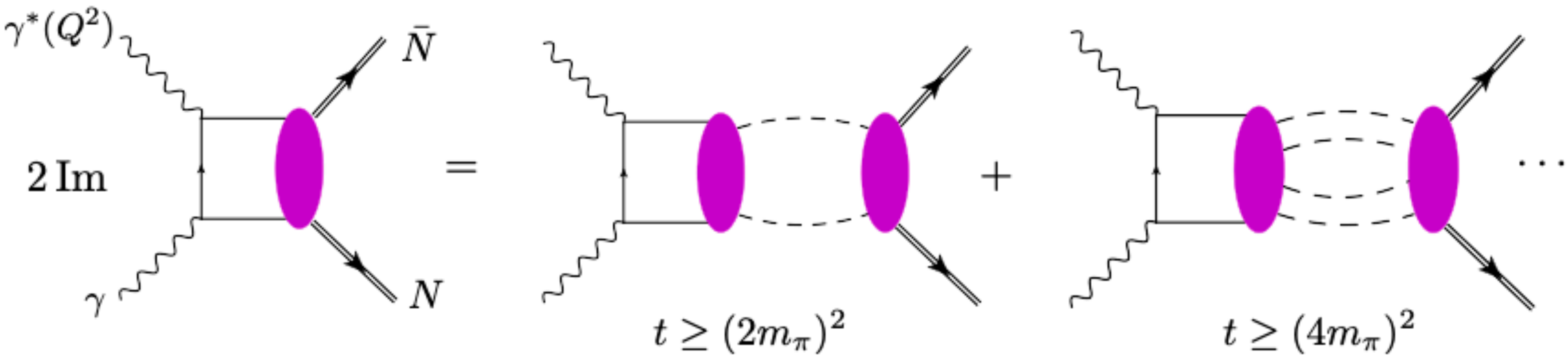
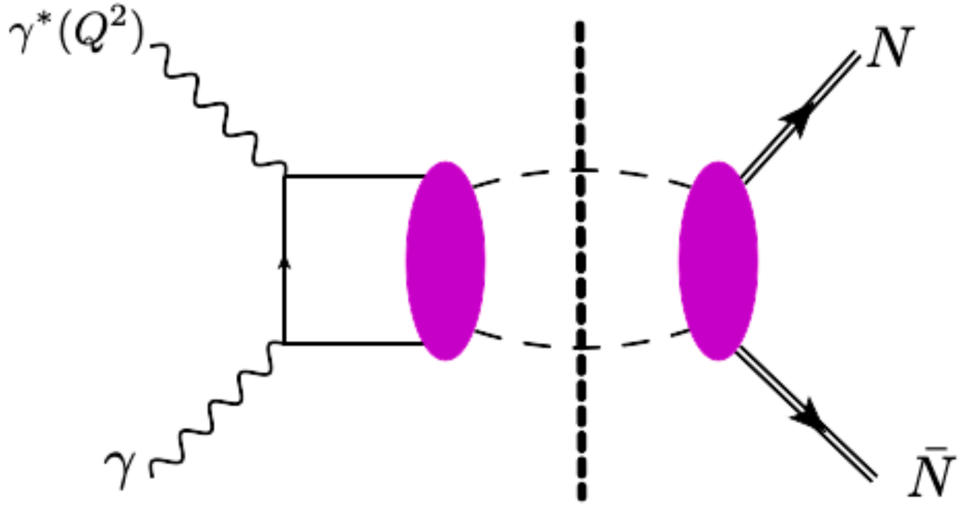
Unitarity relation in t-channel



Unitarity relation in the t-channel: two-pion intermediate state



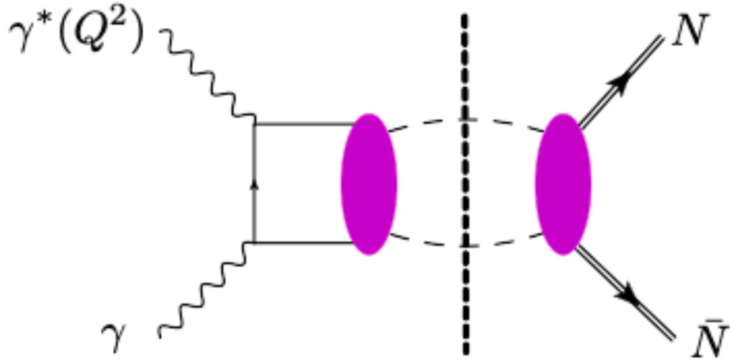
- Charge conjugation and Parity

- Partial wave expansion
  with $\nu = 0 \rightarrow \theta_t = 90^o$

→

two-pion intermediate state with
$I = 0 \qquad J = 0, 2, \cdots$

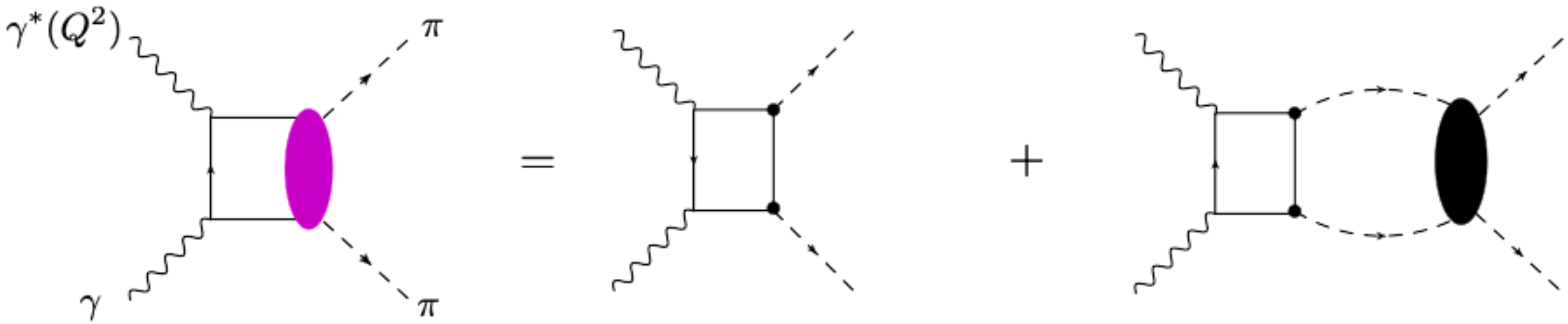
B. Pasquini, M. Polyakov, M. Vanderhaeghen, PLB739 (2014) 133

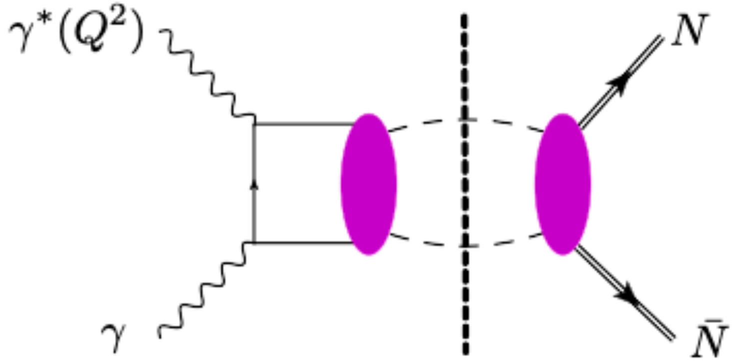


Two-pion intermediate states with $I = 0$ and $J = 0,\ 2$

$$\Delta(t) = -4 \sum_{\substack{n=1 \\ n\, odd}}^{\infty} d_n(t) \longrightarrow \text{DRs for } d_1(t)$$

- $\gamma^*\gamma \to \pi\pi$: two-pion generalized distribution amplitudes $\longrightarrow$ inputs { pion singlet PDF; $\pi\pi$ phase-shifts }





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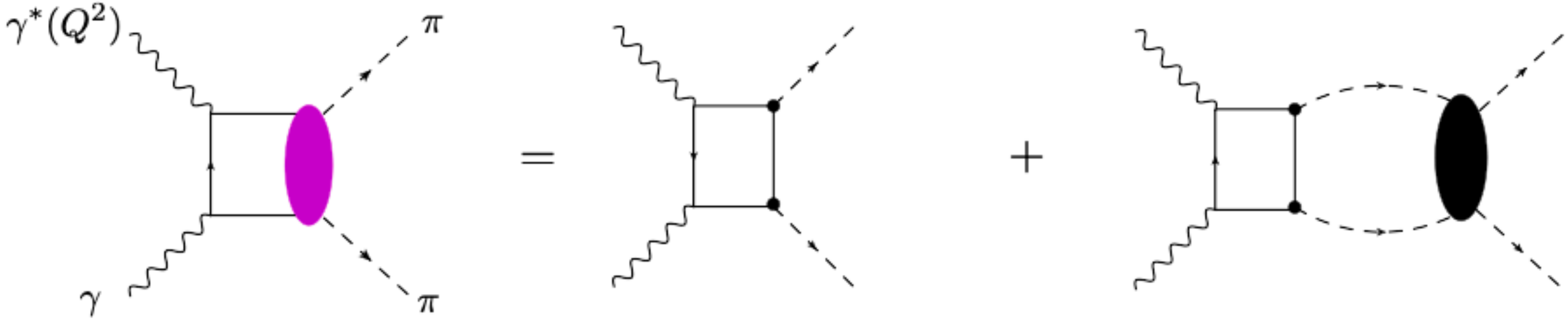
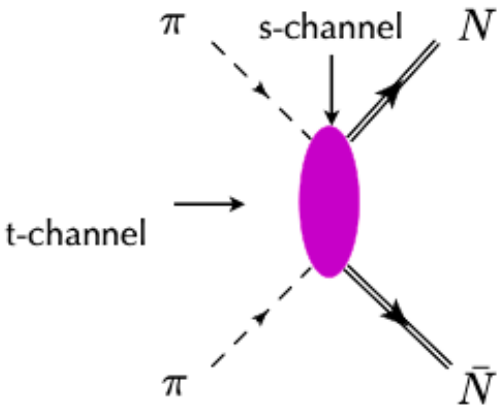


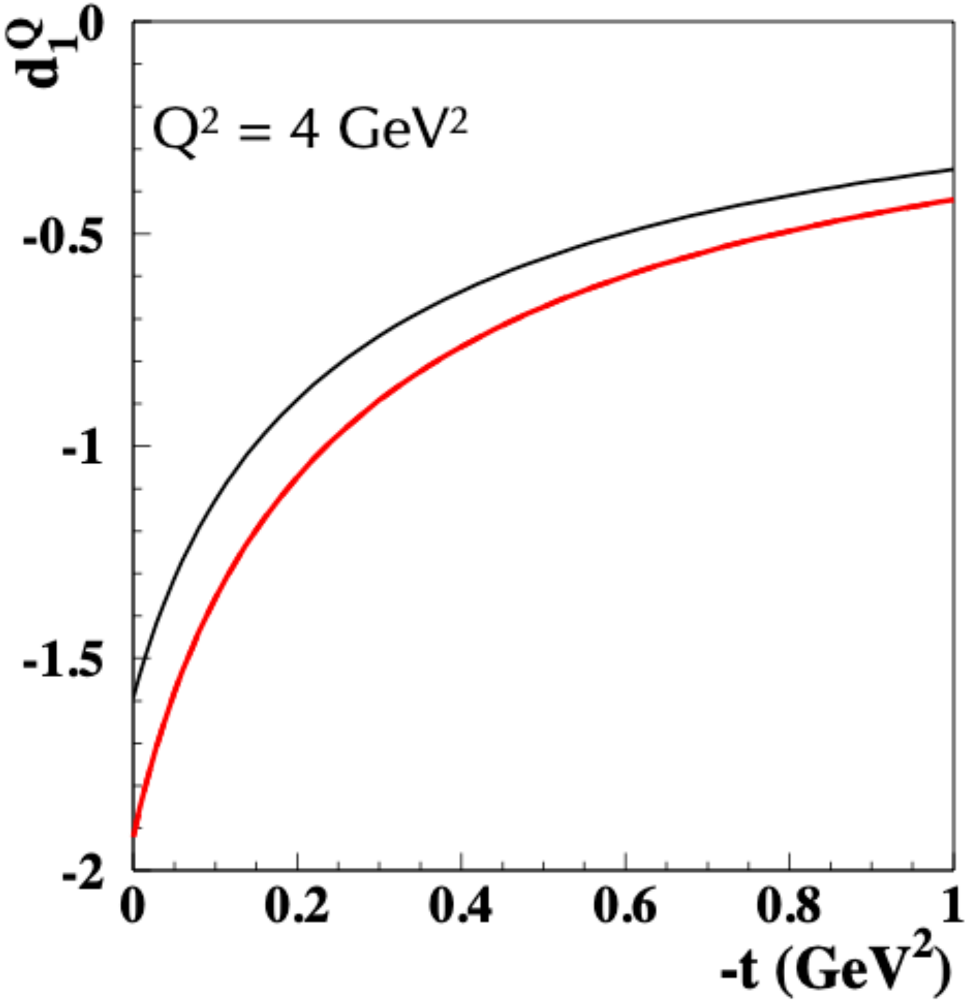
- $\pi\pi \to N\bar{N}$: analytical continuation of s-channel partial-wave helicity amplitudes

$\longrightarrow$ input $\pi\pi$ phase-shifts



D-term form factor: dependence on pion PDFs

Q= u + d



pion PDFs from GRV, ZPC53 (1992)

$d_1(0) = -1.92$

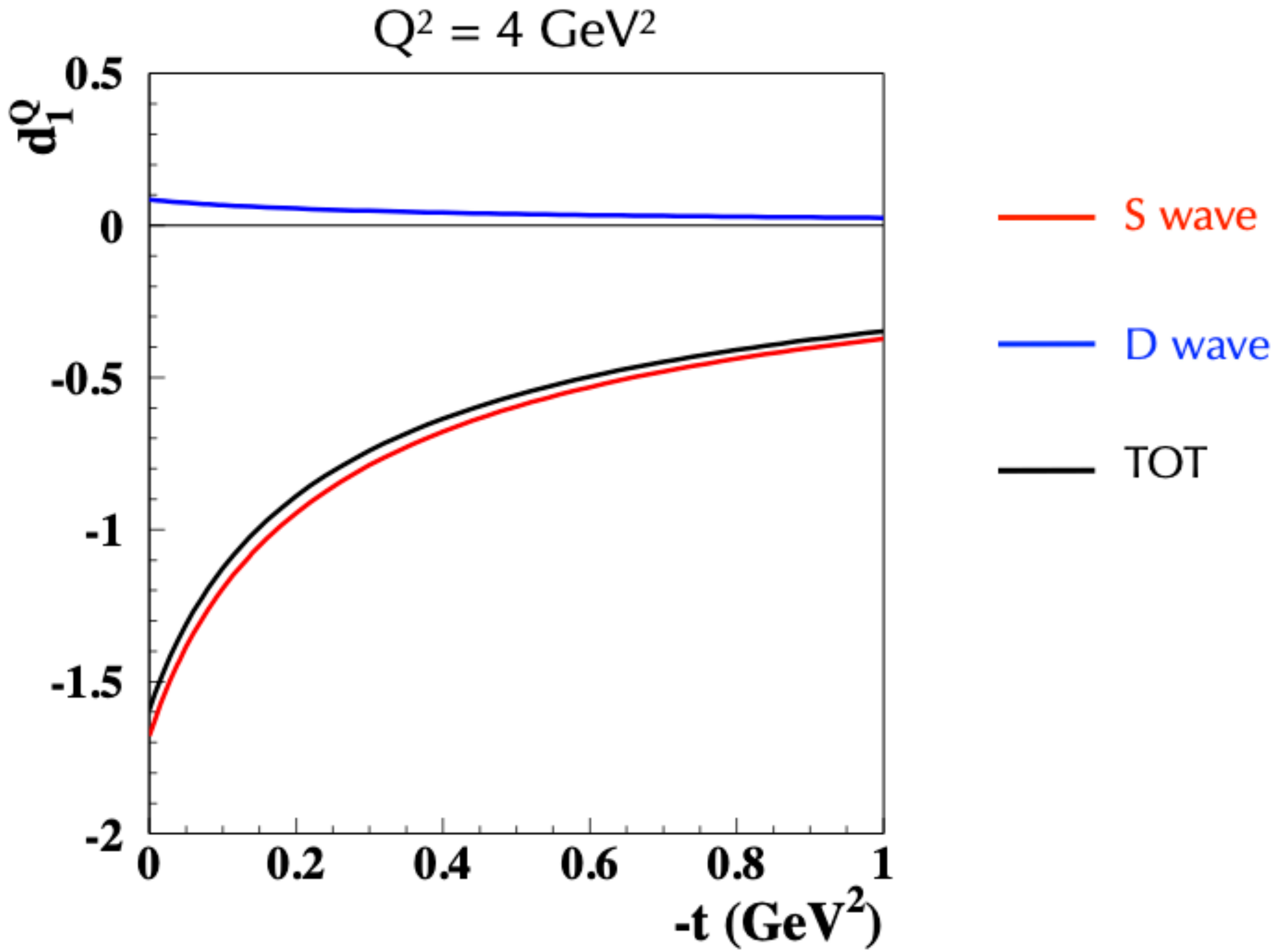
pion PDFs from Owens et al., PRD30 (1984)

$d_1(0) = -1.59$

χQSM	Skyrme model	Effective LFWFs
$d_1^Q(0) = -2.35$	$d_1^Q(0) = -4.48$	$d_1^Q(0) = -2.01$
Schweitzer et al., (2007)	*Schweitzer et al., (2007)*	*Mueller and Hwang, (2014)*

D-term form factor: partial-wave decomposition

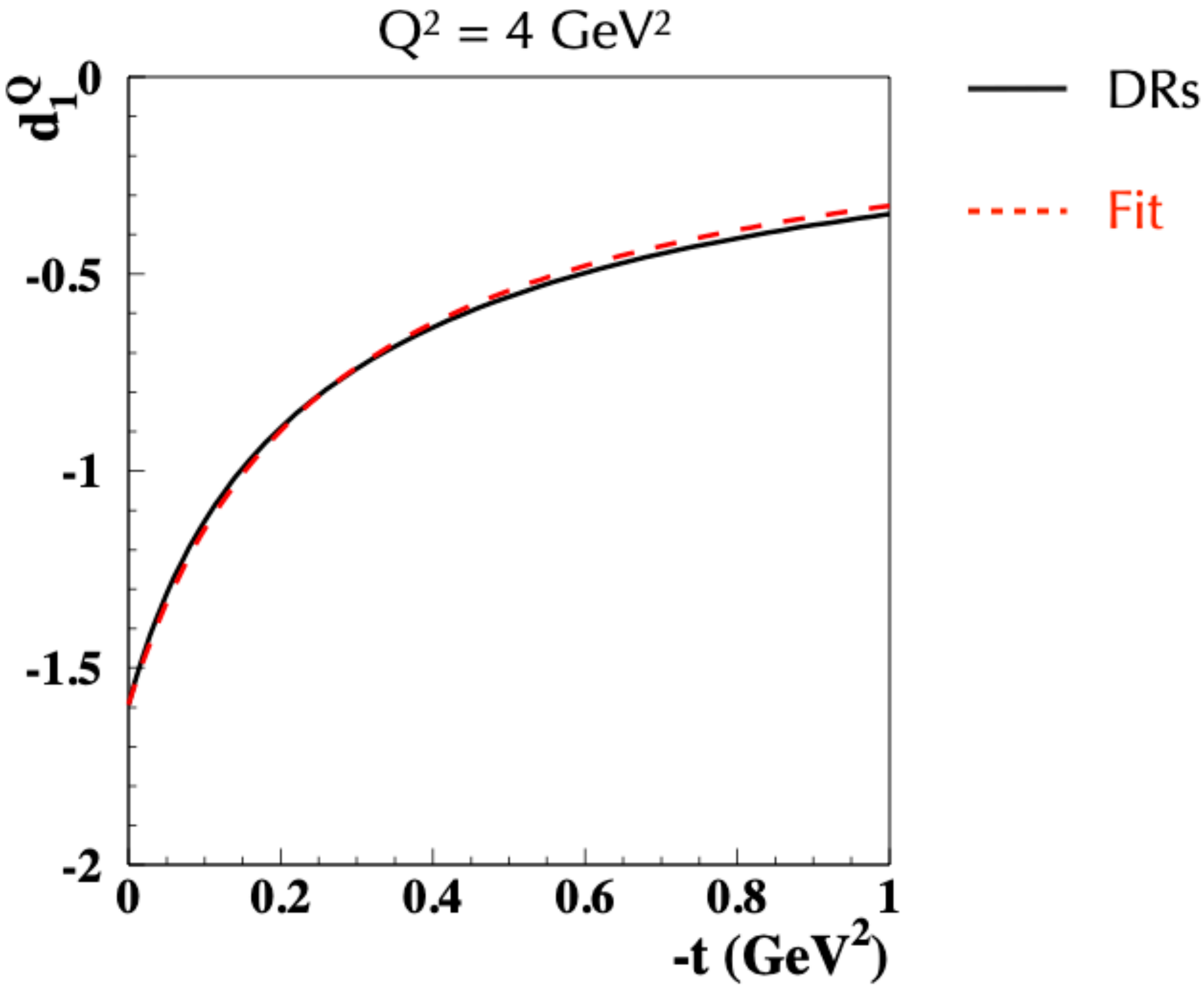
Q= u + d



B. Pasquini, M. Polyakov, M. Vanderhaeghen, PLB739 (2014) 133

D-term form factor: t-dependence

Q= u + d

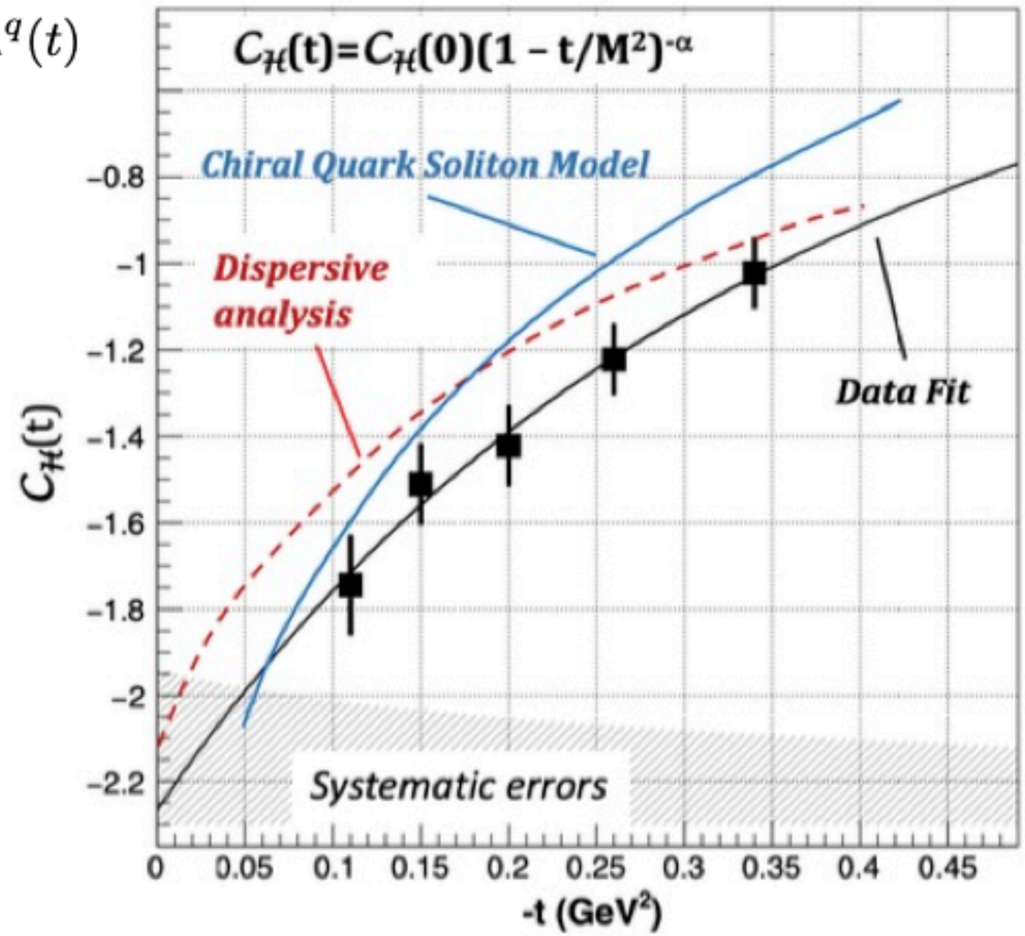


Fit: $d_1(t) = \dfrac{d_1(0)}{[1 - t/(\alpha M_D^2)]^\alpha}$ with $M_D = 0.487\,\text{GeV}$

$\alpha = 0.841$

Extraction of D-term form factor

$$\mathcal{C}_H(t) = -2\sum_q e_q^2 \Delta^q(t)$$



Extraction from data:
- neglecting gluon contribution
- assuming:

$$\mathcal{C}_H(t) = 8\sum_q e_q^2 \sum_{\substack{n=1\\ n\ odd}}^{\infty} d_n(t) \approx \frac{10}{9} d_1^Q(t)$$

Fit to data: $\mathcal{C}_H(t) = \dfrac{\mathcal{C}_H(0)}{(1-t/M^2)^\alpha}$

$\mathcal{C}_H(0) = -2.27 \pm 0.16 \pm 0.36 \qquad \lambda = 2.76 \pm 0.23 \pm 0.48$

$M^2 = 1.02 \pm 0.13 \pm 0.21\,\mathrm{GeV}^2$

Girod, Elouadrhiri, Burkert,Nature 557 (2018) 7705; Burkert, et al., Rev. Mod. Phys. 95 (2023) 014002

D(t) form factor from data

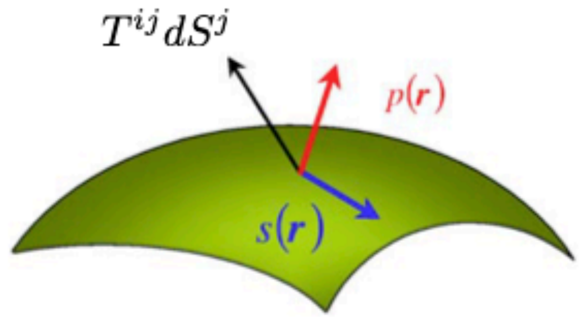
➡ Fourier transform in coordinate space

$$T^Q_{ij}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\,\delta_{ij}$$

$s(\vec{r})$ → shear forces

$p(\vec{r})$ → pressure

"mechanical properties" of nucleon



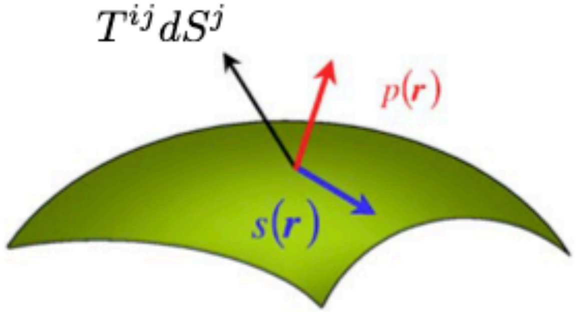
D(t) form factor from data

➡ Fourier transform in coordinate space

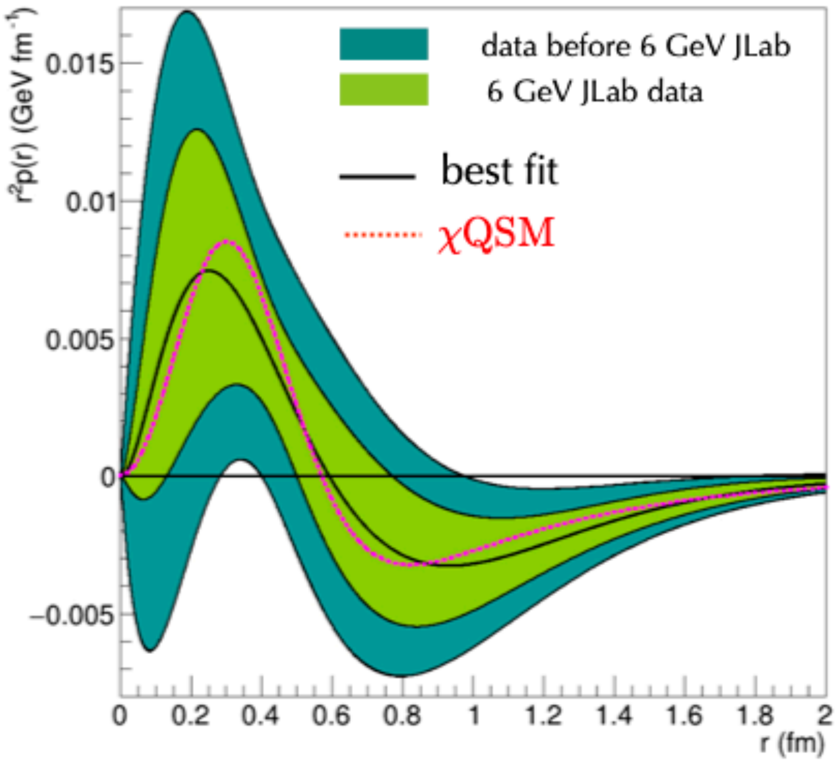
$$T_{ij}^{Q}(\vec{r}) = s(\vec{r})\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(\vec{r})\,\delta_{ij}$$

shear forces ($s(\vec{r})$) — pressure ($p(\vec{r})$)

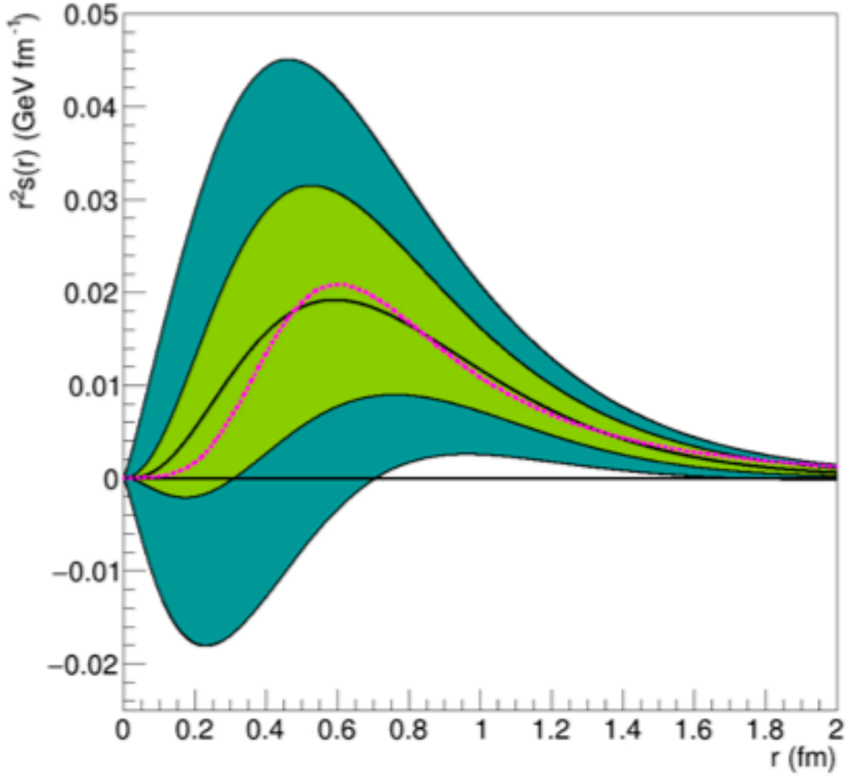
"mechanical properties" of nucleon



$$p(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}D(r)$$



$$s(r) = -\frac{1}{4M_N} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr}D(r)$$

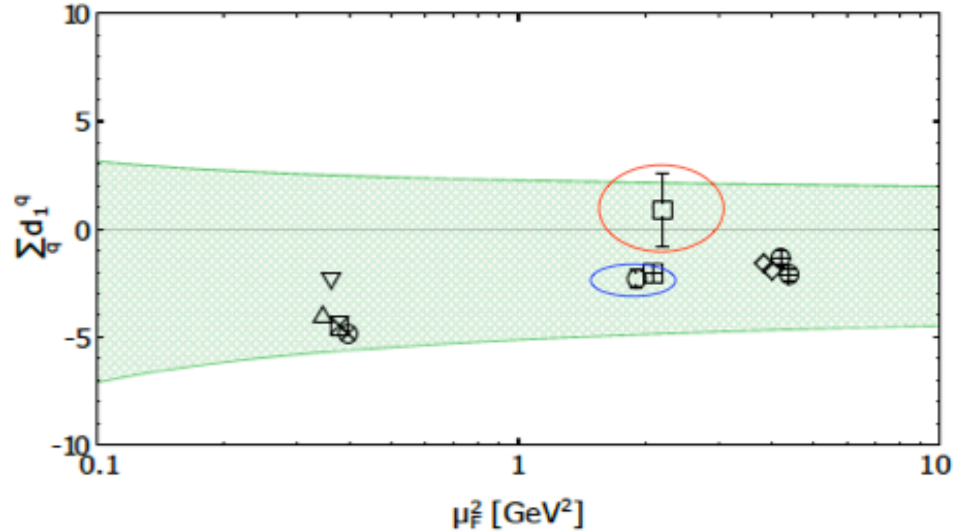


Girod, Elouadrhiri, Burkert, Nature 557 (2018) 7705; Polyakov, Schweitzer, IJMA 33 (2018) 1830025

Necessary to verify model assumptions in the exp extraction
with more data coming from JLab, COMPASS and the future EIC, EIcC

Kumericki, Nature 570 (2019) 7759; Dutrieux et al, Eur. Phys. J. C81 (2021) 4

global fit to DVCS data
with artificial neural networks



CLAS data, with fixed param.,
Girod et al.

CLAS data, with neural networks
Kumericki

$$\sum_q d_1^q < 0$$

in all model calculations
for a stable proton

Marker in Fig. 3	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
□	0.88 ± 1.69	2.2	2	from experimental data
◇	-1.59	4	2	t-channel saturated model
	-1.92	4	2	t-channel saturated model
△	-4	0.36	3	χQSM
▽	-2.35	0.36	2	χQSM
⊠	-4.48	0.36	2	Skyrme model
⊞	-2.02	2	3	LFWF model
⊗	-4.85	0.36	2	χQSM
⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
	-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)