International School and Workshop on Probing Hadron Structure at the Electron-Ion Collider

INTERNATIONAL CENTRE for THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

EXCLUSIVE PROCESSES

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UNIVERSITÀ DI PAVIA

INFN Istituto Nazionale di Fisica Nucleare

Map of the Universities in Italy

Map of the Universities in Italy

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Founded in 1361, it is one of the oldest universities in the world

24000 students, 18 departments,
85 different courses
and 18 colleges

Hadron Physics Group
at the Department of Physics:
3 permanent staff
3 PhD students
1 postdoc
a large network of collaborators

Plan of lectures

0. Brief introduction

1. Partonic interpretation of Generalized Parton Distributions (GPDs)

2. Multidimensional picture of the proton in the 1+2D

3. Sum rule for Angular Momentum

4. Form Factors of Energy Momentum Tensor

5. Observables for GPDs

6. Wigner distributions: maps in 3+2D

A few references on GPDs

- M. Diehl, Phys. Rep. 388 (2003) 41
- K. Goeke, M. Polyakov, M. Vanderhaeghen, Prog. Part. Nucl. Phys. 47 (2001) 401
- X. Ji, Ann. Rev. Nucl. Part. Sci. 54 (2004) 413
- A.V. Belitsky, A.V. Radyushkin, Phys. Rept. (2005) 418
- S. Boffi, B. Pasquini, Riv. Nuovo Cim. 30 (2007) 387
- M. Guidal, H. Moutarde, M. Vanderhaeghen, Rept. Prog. Phys. 76 (2013) 066202
- N. D'Hose, S. Niccolai, A. Rostomyan, Eur. Phys. J. A52 (2016) 151
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Recent Review

Eur. Phys. J A52 (2016)

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All Volumes & Issues

The 3-D Structure of the Nucleon

ISSN: 1434-6001 (Print) 1434-601X (Online)

In this topical collection (17 articles)

Editors: M. Anselmino, M. Guidal, P. Rossi

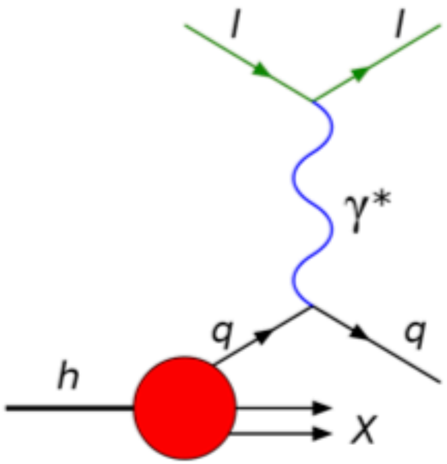
Parton Distribution Functions (PDFs)

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ik^+z^-} \langle p^+, \vec{0}_\perp, \Lambda' | \bar{\psi}(-\frac{z}{2}) \Gamma \psi(\frac{z}{2}) | p^+, \vec{0}_\perp, \Lambda \rangle_{z^+=0, \vec{z}_\perp=0}$$

Depend on

Λ, Λ', Γ: nucleon and quark polarizations

$x = \frac{k^+}{p^+}$: longitudinal momentum fraction

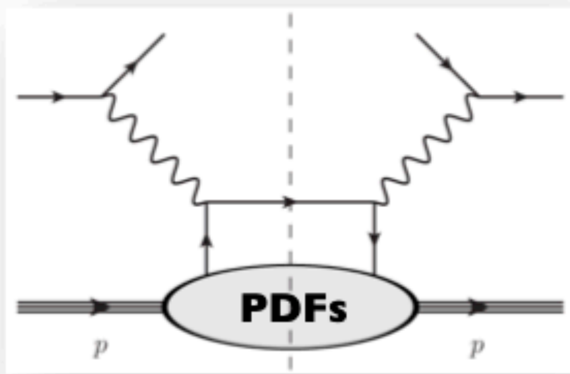


$Q^2 \gg$

$\nu \gg$

$x = \frac{Q^2}{2M\nu}$ finite

Deep Inelastic Scattering



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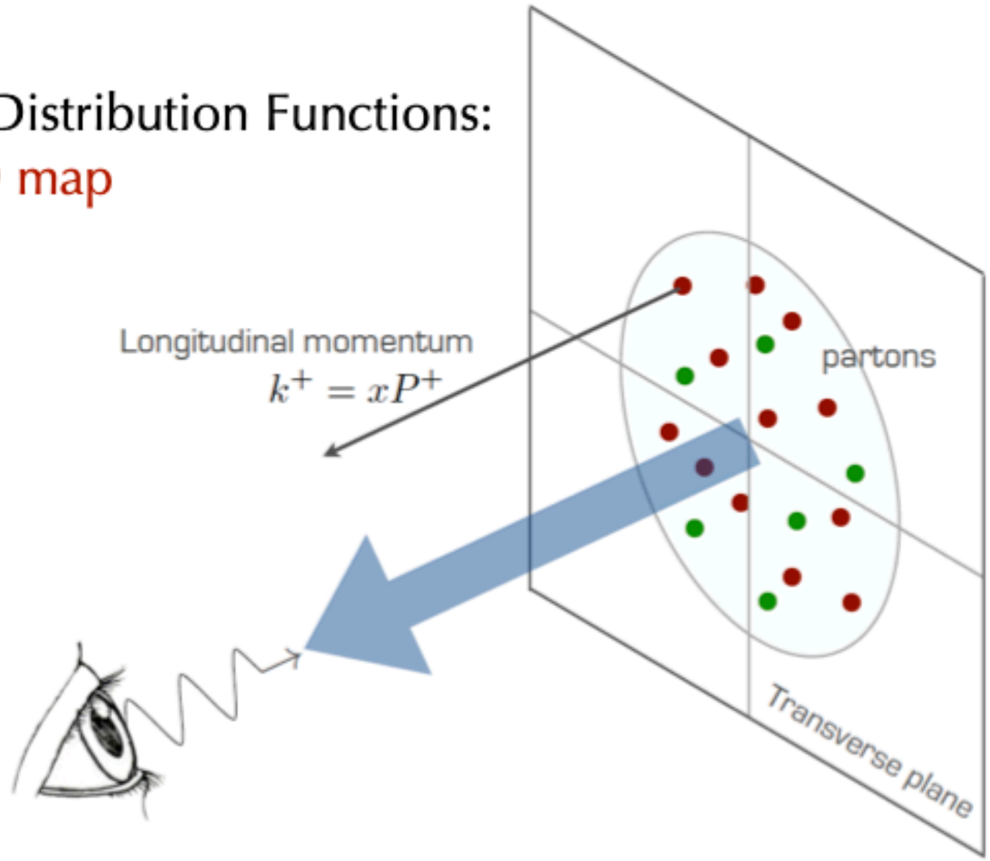
Parton Distribution Functions:
1D+0D map



Generalized Parton Distributions (GPDs)

$$\frac{1}{2}\int\frac{\mathrm{d}z^-}{2\pi}e^{ik^+z^-}\langle p'^+,-\frac{\vec{\Delta}_\perp}{2},\Lambda'|\bar{\psi}(-\frac{z}{2})\,\Gamma\,\psi(\frac{z}{2})|p^+,\frac{\vec{\Delta}_\perp}{2},\Lambda\rangle_{z^+=0,\vec{z}_\perp=0}$$

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non-diagonal matrix elements

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non-diagonal matrix elements

Depend on

Λ, Λ', Γ: nucleon and quark polarizations

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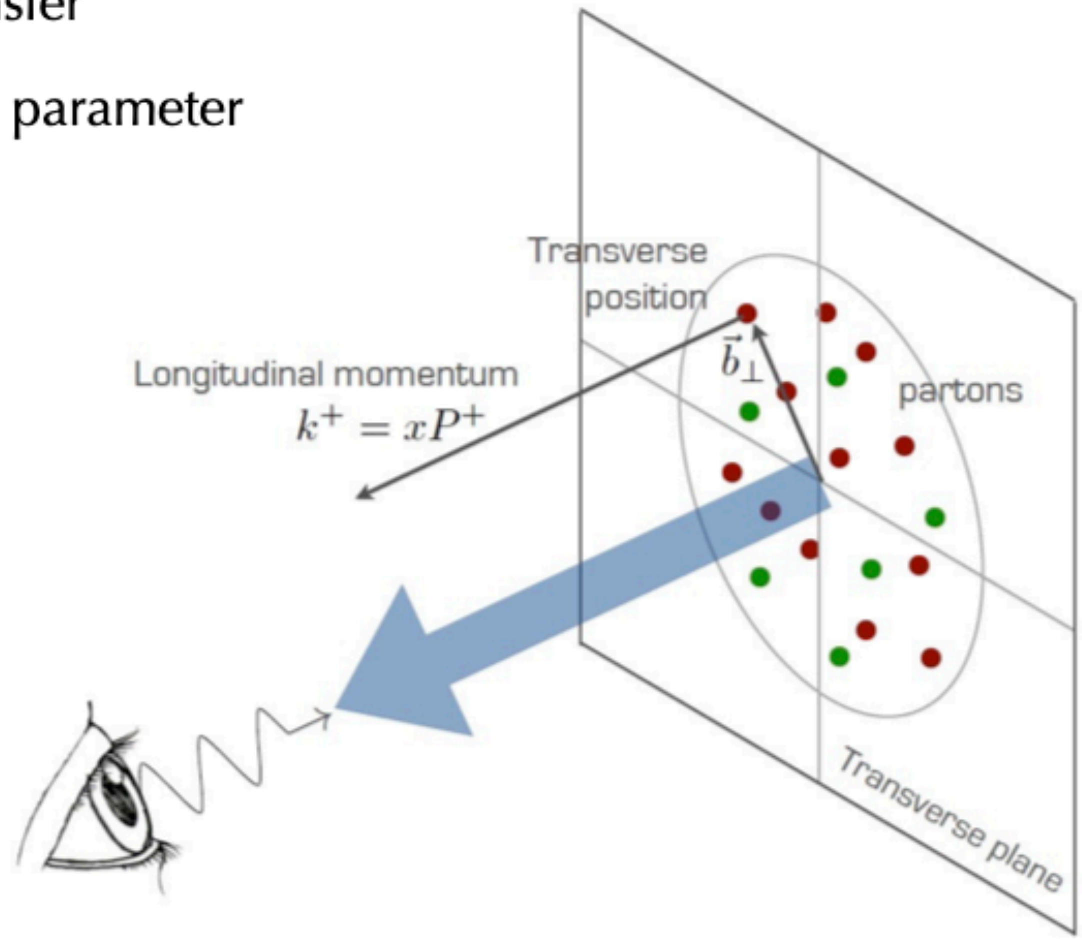
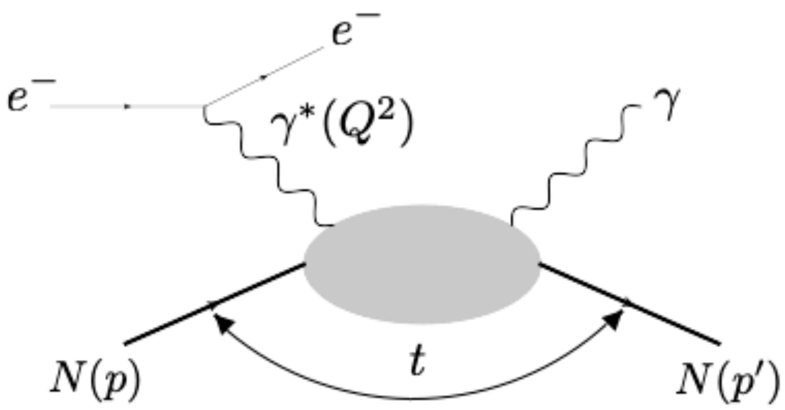
$\vec{b}_\perp \overset{\text{FT}}{\longleftrightarrow} \vec{\Delta}_\perp$: impact parameter



1+2D maps

How to access GPDs



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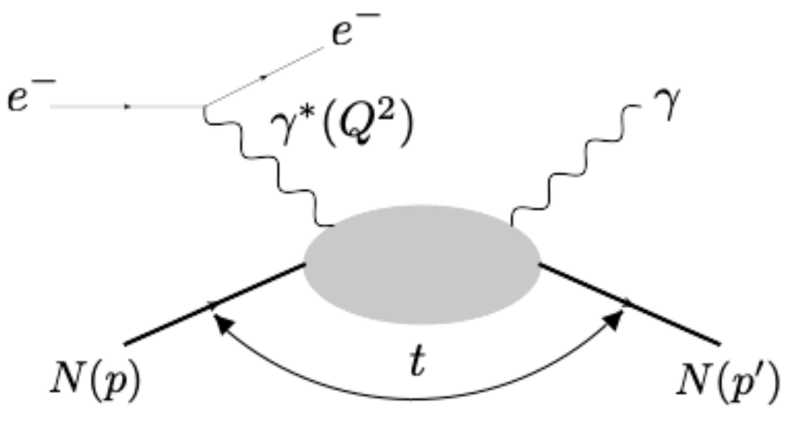
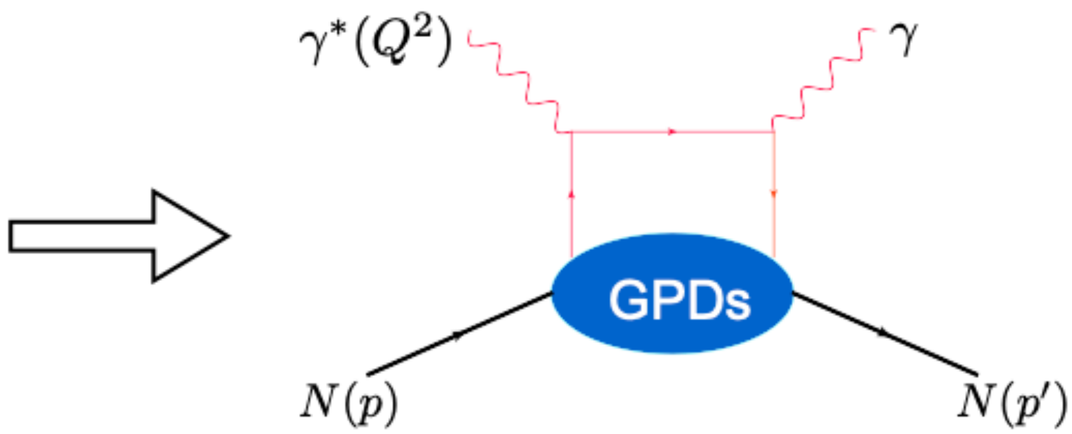
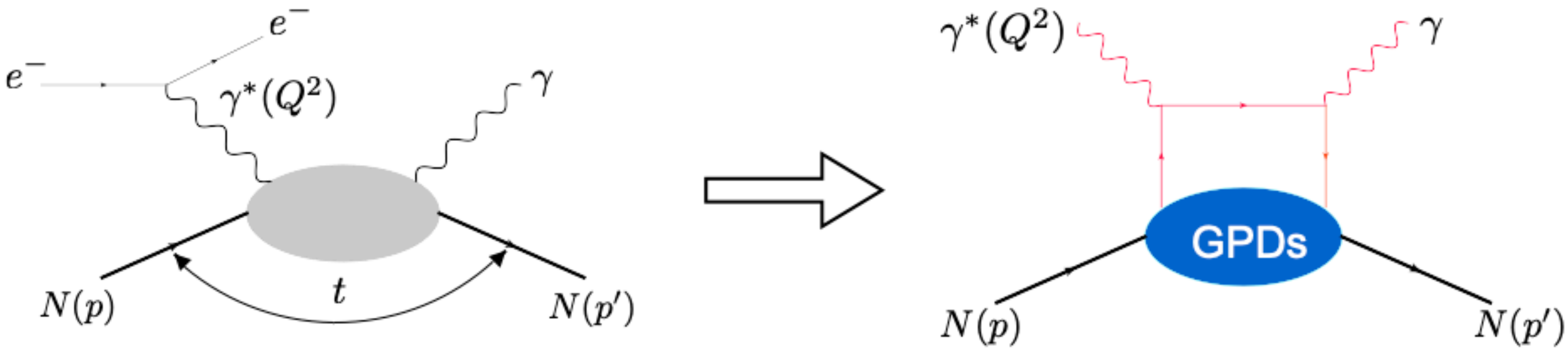




factorization for large Q^2, $|t| << Q^2$, s

$\mathcal{M}$ =[parton Ampl.] $\otimes$ [GPDs]

How to access GPDs



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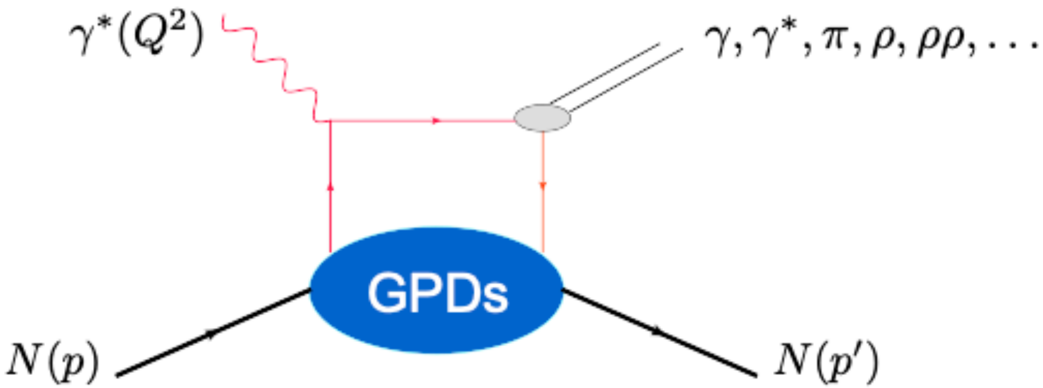
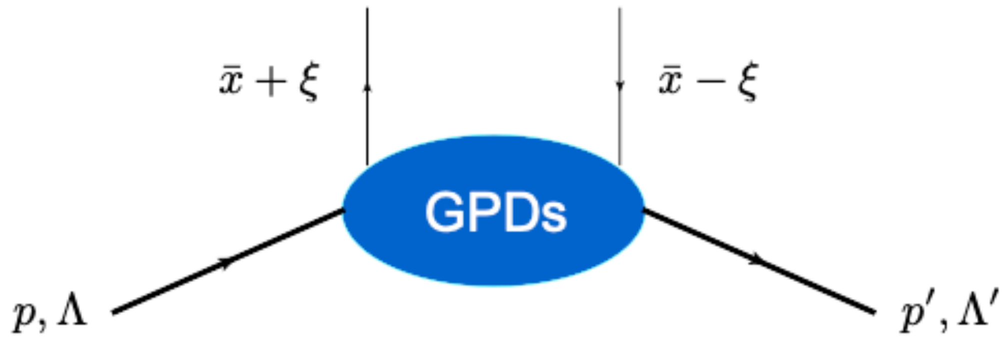


universality: the same GPDs enter a variety of exclusive reactions

Leading-Twist GPDs



$$\Phi^{[\Gamma]}(\bar{x}, \xi, t) = \langle p', \Lambda' | \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-} \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \psi\left(\frac{z}{2}\right) |p, \Lambda\rangle|_{z^+=0, \vec{z}_\perp=0}$$

$$\Gamma = \begin{cases} \gamma^+ & H^q,\ E^q & \text{unpol.} \\ \gamma^+\gamma^5 & \tilde{H}^q,\ \tilde{E}^q & \text{long. pol.} \\ i\sigma^{+i}\gamma^5 & H_T^q,\ E_T^q,\ \tilde{H}_T^q,\ \tilde{E}_T^q & \text{transv. pol.} \end{cases}$$

- $p \neq p' \Rightarrow$ GPDs depend on two momentum fractions $\bar{x}$, ξ, and t

$$\bar{x} = \frac{(k+k')^+}{(p+p')^+} = \frac{\bar{k}^+}{P^+}$$

average fraction of the longitudinal momentum carried by partons

$$\xi = \frac{(p-p')^+}{(p+p')^+} = -\frac{\Delta^+}{2P^+}$$

skewness parameter: fraction of longitudinal momentum transfer

$$t = (p-p')^2 \equiv \Delta^2$$

t-channel momentum transfer squared

Need of a gauge link

$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-}\, \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

not invariant under $\psi(z) \to e^{i\alpha(z)}\psi(z)$

$\Downarrow$

$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-}\, \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\mathcal{U}_{[-\frac{z}{2},\frac{z}{2}]}\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

$$\mathcal{U}(-\tfrac{z}{2},\tfrac{z}{2}) \to e^{i\alpha(-\frac{z}{2})}\,\mathcal{U}(-\tfrac{z}{2},\tfrac{z}{2})\,e^{-i\alpha(\frac{z}{2})}$$

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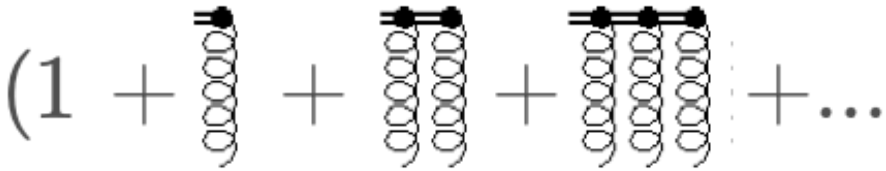
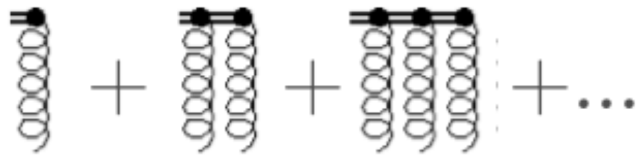
$$\mathcal{U}_{[-\frac{z}{2}\ \frac{z}{2}]} = \mathcal{P}\exp\left[-ig\int_{-\frac{z}{2}}^{\frac{z}{2}} \mathrm{d}\eta^\mu\, A_\mu(\eta)\right] = \sum_{n=0}^{\infty}\frac{1}{n!}(-ig)^n\mathcal{P}\int_{-\frac{z}{2}}^{\frac{z}{2}} \mathrm{d}\eta_n^{\mu_n}\ldots\mathrm{d}\eta_1^{\mu_1}A_{\mu_n}(\eta_n)\ldots A_{\mu_1}(z_1)$$



Wilson line definition for GPDs

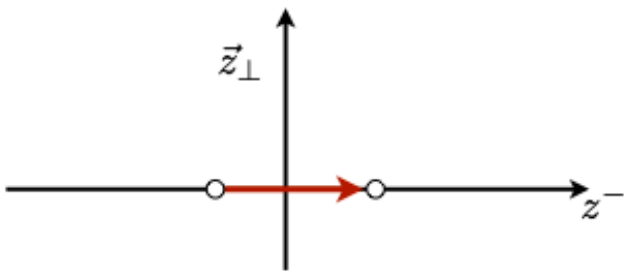
$$\Phi^{[\Gamma]}(\bar{x},\xi,t) = \langle p',\Lambda'| \int \frac{dz^-}{4\pi} e^{i\bar{x}P^+z^-}\bar{\psi}\left(-\frac{z}{2}\right)\Gamma\mathcal{U}_{[-\frac{z}{2},\frac{z}{2}]}\psi\left(\frac{z}{2}\right)|p,\Lambda\rangle\Big|_{z^+=0,\vec{z}_\perp=0}$$

$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right] = (1 + \ldots$$



exchange of more than 2 partons
between hard scattering process (H) and soft amplitude (A)
is suppressed except for gluons with polarization A^+

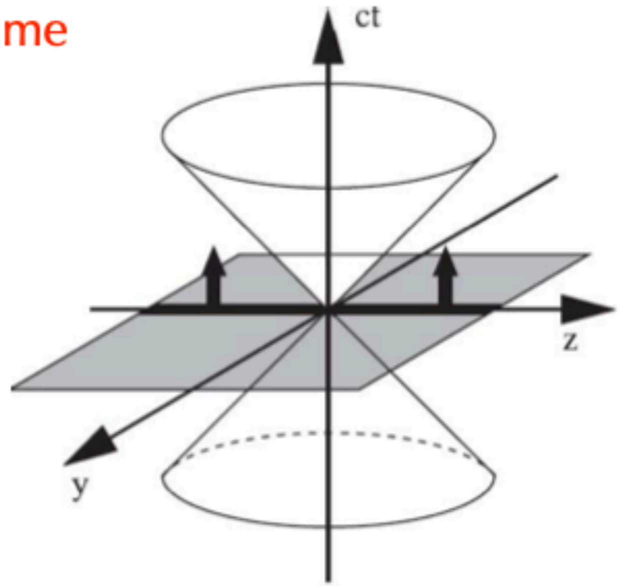
$$\mathcal{U}_{[-\frac{z}{2}\ \frac{z}{2}]} = \mathcal{P}\exp\left[-ig\int_{-\frac{z}{2}}^{\frac{z}{2}} \mathrm{d}\eta^-\ A^+(\eta)\right]_{z^+=0,z_\perp=0}$$



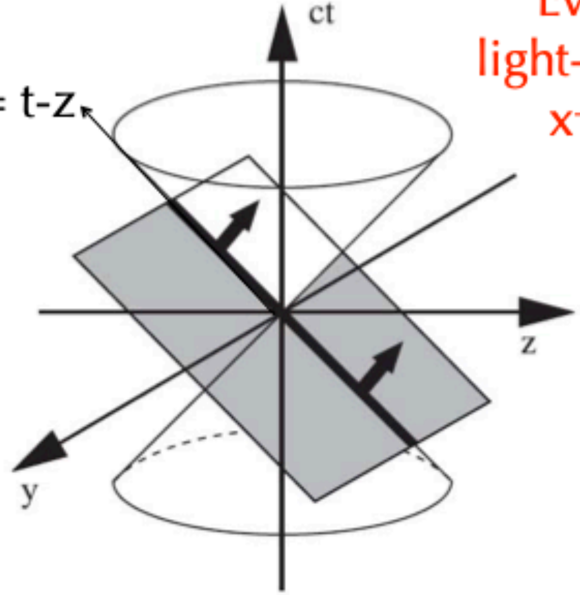
for convenience, choose light-cone gauge: $A^+ = 0$ in which $U = 1$

Partonic interpretation

Evolve in
ordinary time



Instant Form



Evolve in
light-front time
$x^+ = t+z$

Light-Front Form

coordinates

Instant Form		Light-Front Form	
x^0	time	$\frac{x^0 + x^3}{\sqrt{2}}$	time
x^1, x^2, x^3	space	$\frac{x^0 - x^3}{\sqrt{2}},\ \vec{x}_\perp = (x^1, x^2)$	space

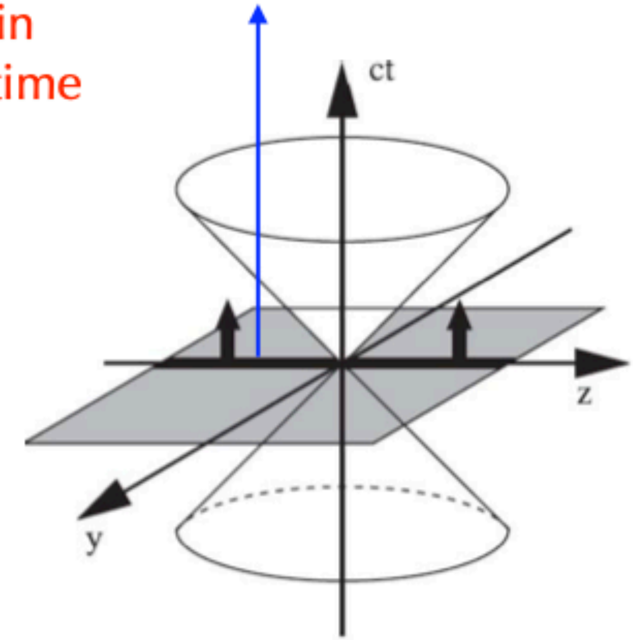
Hamiltonian

$$H = \sqrt{P^2 + M_0^2} + V \qquad P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$$

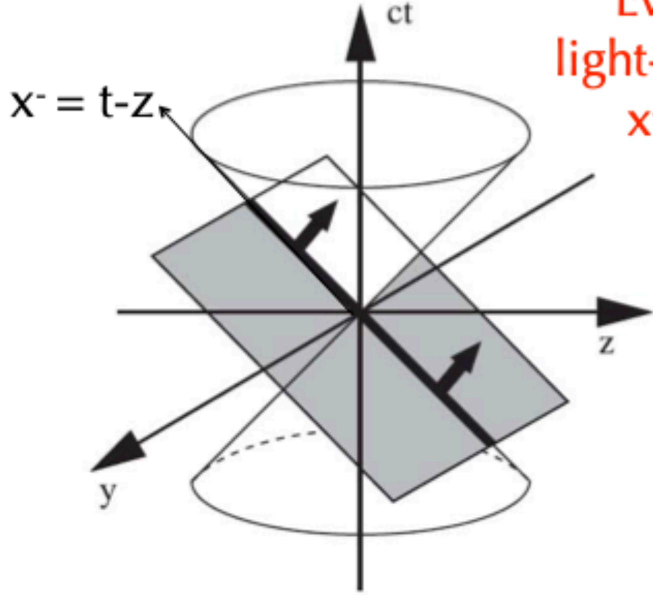
generators of Poincaré group interaction independent

6 7

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)



Instant Form



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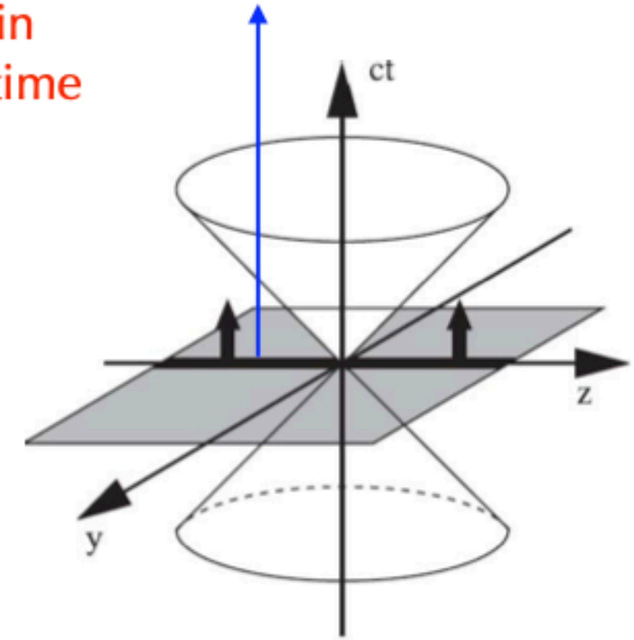
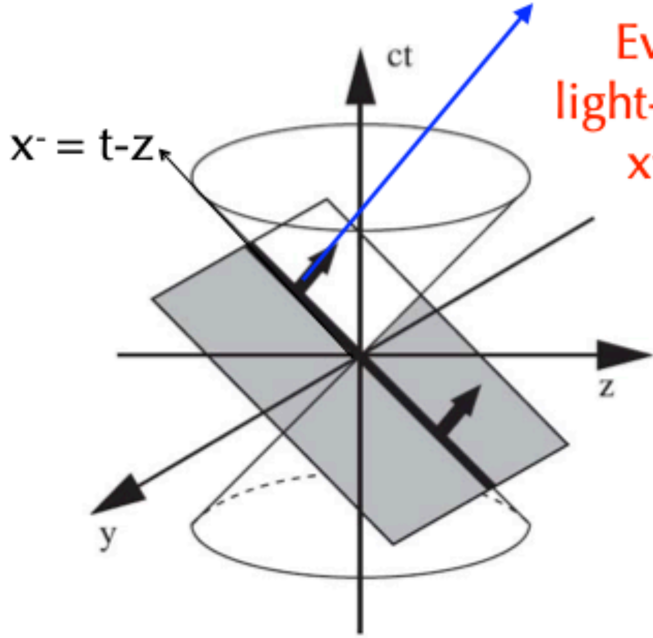


Instant Form

Evolve in light-front time $x^+ = t+z$



Light-Front Form

coordinates

Instant Form		Light-Front Form	
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generators of Poincaré group interaction independent

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P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949)

Good and bad components

- Decompose the four-component fermion field in bad (-) and good (+) components

$$\psi = \psi^+ + \psi^- \qquad \text{with } \psi^+ = P_+\psi \text{ and } \psi^- = P_-\psi$$

- Properties of projector operators: $P_+ = \frac{1}{2}\gamma^-\gamma^+ \quad P_- = \frac{1}{2}\gamma^+\gamma^-$

$$P_+ + P_- = I \qquad (P_+)^2 = P_+ \quad (P_-)^2 = P_- \qquad P_+P_- = P_-P_+ = 0$$

- Projecting the Dirac equation and using the light-cone gauge $A^+ = 0$

$$i\gamma^- \frac{\partial}{\partial x^-}\psi_- = -\vec{\gamma}_\perp \cdot \vec{D}_\perp \psi_+ + m\psi_+ \qquad i\gamma^+ D_+ \psi_+ = -\vec{\gamma}_\perp \cdot \vec{D}_\perp \psi_- + m\psi_-$$

constrained field — independent dynamical degree of freedom

R. Jaffe, hep-ph/9602236

Partonic interpretation of GPDs

- Unpolarized GPDs

$$\Phi^{[\gamma^+]}(\bar{x},\xi,t) = \langle P',\Lambda'| \int \frac{\mathrm{d}z^-}{4\pi} e^{i\bar{x}P^+z^-}\,\bar{\psi}(-\tfrac{z}{2})\,\gamma^+\,\psi(\tfrac{z}{2})|P,\Lambda\rangle|_{z^+=0,\vec{z}_\perp=0}$$

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$$= \sqrt{2}\psi^\dagger(-\frac{z}{2})P_+P_+\psi(\frac{z}{2}) = \sqrt{2}\psi_+^\dagger(-\frac{z}{2})\psi_+(\frac{z}{2})$$

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$$\Longrightarrow\ \psi_+(z^-,\vec{z}_\perp) = \int \frac{dk^+}{2k^+}\frac{d\vec{k}_\perp}{(2\pi)^3}\theta(k^+)\sum_\mu[b_q(w)u_+(w)\exp[-ik^+z^- + i\vec{k}_\perp\cdot\vec{z}_\perp]$$

$$+d_q^\dagger(w)v_+(w)\exp[ik^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp]$$ with $w = (k^+,\vec{k}_\perp,\mu)$

$b_q, b_q^\dagger$ annihilation and creation operator of quark

$d_q, d_q^\dagger$ annihilation and creation operator of antiquark

❖Homework: derive the operator structure in the different regions using positivity condition $k^{'+}, k^{+} > 0$
and momentum conservation $k^{+} - k^{'+} = p^{+} - p^{'+} = 2\xi P^{+}$

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$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

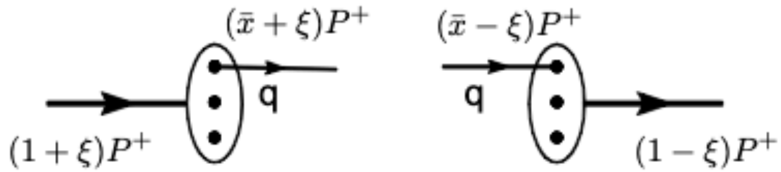
non-diagonal matrix elements of momentum-density matrix

⇓

we loose the probabilistic interpretation of the PDF

we gain information on the quark-momentum correlation

DGLAP region $\xi \le \bar{x} \le 1$



$$\langle N, (1-\xi)P^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)P^+]\, b_\lambda[(\bar{x}+\xi)P^+] | N, (1+\xi)P^+\rangle$$

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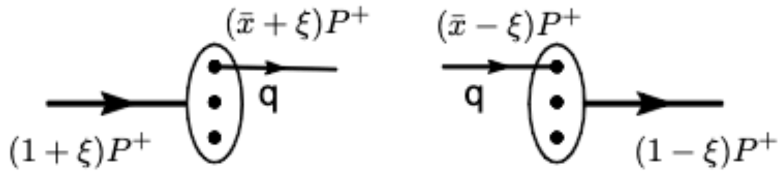
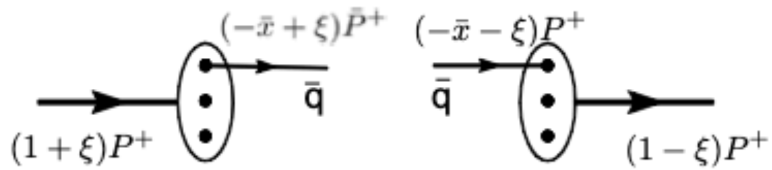


$$\langle N, (1-\xi)P^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)P^+] b_\lambda[(\bar{x}+\xi)P^+] | N, (1+\xi)P^+\rangle$$

DGLAP region $-1 \le \bar{x} \le -\xi$



$$\langle N, (1-\xi)P^+ | d^\dagger_{\lambda'}[(-\bar{x}-\xi)P^+] d_\lambda[(-\bar{x}+\xi)P^+] | N, (1+\xi)P^+\rangle$$

❖Homework: derive the operator structure in the different regions using positivity condition $k'^+, k^+ > 0$ and momentum conservation $k^+ - k'^+ = p^+ - p'^+ = 2\xi P^+$

$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

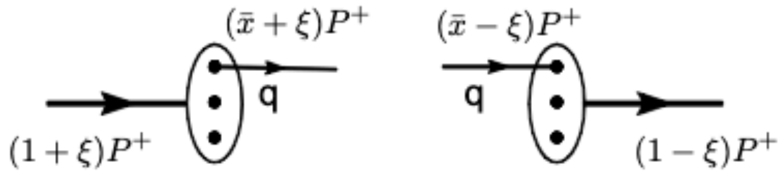
non-diagonal matrix elements of momentum-density matrix

⇓

we loose the probabilistic interpretation of the PDF

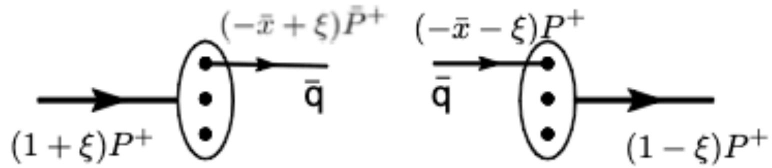
we gain information on the quark-momentum correlation

DGLAP region $\xi \leq \bar{x} \leq 1$



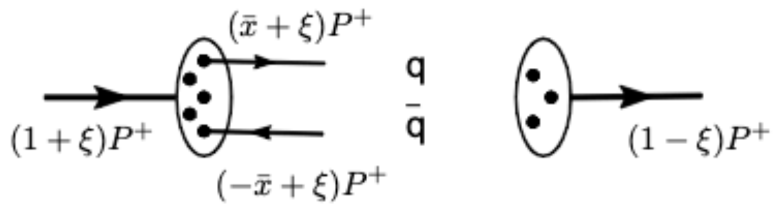
$$\langle N, (1-\xi)P^+ | b^\dagger_{\lambda'}[(\bar{x}-\xi)P^+] b_\lambda[(\bar{x}+\xi)P^+] | N, (1+\xi)P^+ \rangle$$

DGLAP region $-1 \leq \bar{x} \leq -\xi$



$$\langle N, (1-\xi)P^+ | d^\dagger_{\lambda'}[(-\bar{x}-\xi)P^+] d_\lambda[(-\bar{x}+\xi)P^+] | N, (1+\xi)P^+ \rangle$$

ERBL region $-\xi \leq \bar{x} \leq \xi$



$$\langle N, (1-\xi)P^+ | b_{\lambda'}[(\bar{x}+\xi)P^+] d_\lambda[(-\bar{x}+\xi)P^+] | N, (1+\xi)P^+ \rangle$$

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$b,\ b^\dagger$ quarks

$d,\ d^\dagger$ antiquarks

non-diagonal matrix elements of momentum-density matrix

⇩

we loose the probabilistic interpretation of the PDF

we gain information on the quark-momentum correlation

Spin projection

helicity space

$b^\dagger_\uparrow b_\uparrow + b^\dagger_\downarrow b_\downarrow$

$H^q,\ E^q$

$b^\dagger_\uparrow b_\uparrow - b^\dagger_\downarrow b_\downarrow$

$\tilde{H}^q,\ \tilde{E}^q$

transverse-spin space

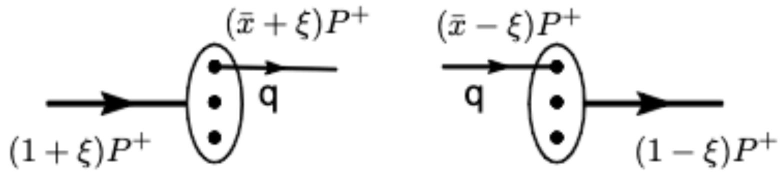
$b^\dagger_\rightarrow b_\rightarrow + b^\dagger_\leftarrow b_\leftarrow$

$H^q_T,\ E^q_T$

$b^\dagger_\rightarrow b_\leftarrow - b^\dagger_\leftarrow b_\rightarrow$

$\tilde{H}^q_T,\ \tilde{E}^q_T$

DGLAP region $\xi \leq \bar{x} \leq 1$



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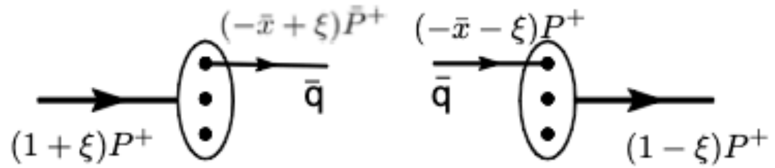

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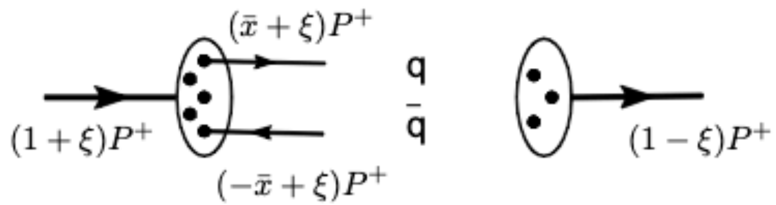


$$\langle N, (1-\xi)P^+ | b_{\lambda'}[(\bar{x}+\xi)P^+] d_\lambda[(-\bar{x}+\xi)P^+] | N, (1+\xi)P^+ \rangle$$

Quark polarization

Nucleon pol.

	U	T_x	T_y	L
U	$\mathcal{H}$	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}_T$	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}_T$	
T_x	$i\,\frac{\Delta_y}{2M}\,\mathcal{E}$	$\mathcal{H}_T + \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}$
T_y	$-i\,\frac{\Delta_x}{2M}\,\mathcal{E}$	$\frac{\Delta_x\Delta_y}{M^2}\,\tilde{\mathcal{H}}_T$	$\mathcal{H}_T - \frac{\Delta_x^2-\Delta_y^2}{2M^2}\,\tilde{\mathcal{H}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}$
L		$\frac{\Delta_x}{2M}\,\tilde{\mathcal{E}}_T$	$\frac{\Delta_y}{2M}\,\tilde{\mathcal{E}}_T$	$\tilde{\mathcal{H}}$

ξ-odd

$$\mathcal{H} = \sqrt{1-\xi^2}\left(H - \frac{\xi^2}{1-\xi^2}\,E\right) \qquad \mathcal{H}_T = \sqrt{1-\xi^2}\left(H_T - \frac{\vec{\Delta}_\perp^2}{2M^2}\,\frac{\tilde{\mathcal{H}}_T}{\sqrt{1-\xi^2}} + \frac{\xi\,\tilde{\mathcal{E}}_T}{\sqrt{1-\xi^2}}\right)$$

$$\mathcal{E} = \frac{E}{\sqrt{1-\xi^2}} \qquad \mathcal{E}_T = \frac{2\tilde{H}_T + E_T - \xi\,\tilde{E}_T}{\sqrt{1-\xi^2}}$$

$$\tilde{\mathcal{H}} = \sqrt{1-\xi^2}\left(\tilde{H} - \frac{\xi^2}{1-\xi^2}\,\tilde{E}\right) \qquad \tilde{\mathcal{H}}_T = -\frac{\tilde{H}_T}{2\sqrt{1-\xi^2}}$$

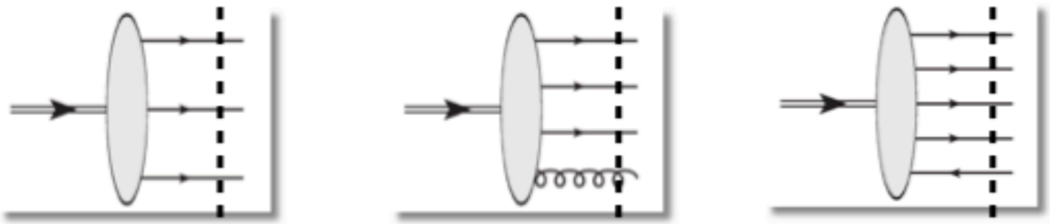
$$\tilde{\mathcal{E}} = \frac{\xi\tilde{E}}{\sqrt{1-\xi^2}} \qquad \tilde{\mathcal{E}}_T = \frac{\tilde{E}_T - \xi\,E_T}{\sqrt{1-\xi^2}}$$

✦ Helicity structure: nucleon (Λ', Λ) quark (λ', λ)

$$U = (++) + (--) \qquad T_x = (-+) + (+-) \qquad T_y = i[(-+) - (+-)] \qquad L = (++) - (--)$$

Light-cone Fock expansion

$$|P\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \ldots$$



Fixed light-cone time $x^+=0$

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Light-cone Fock expansion

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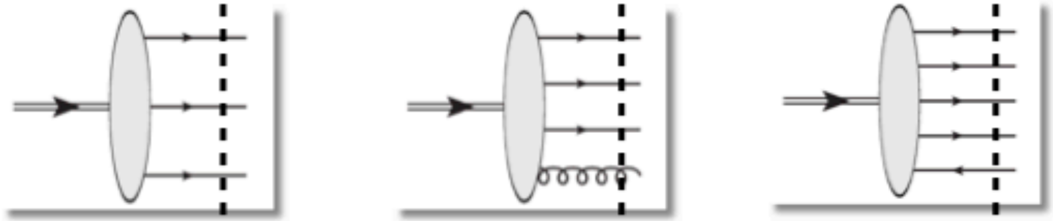


Fixed light-cone time $x^+=0$

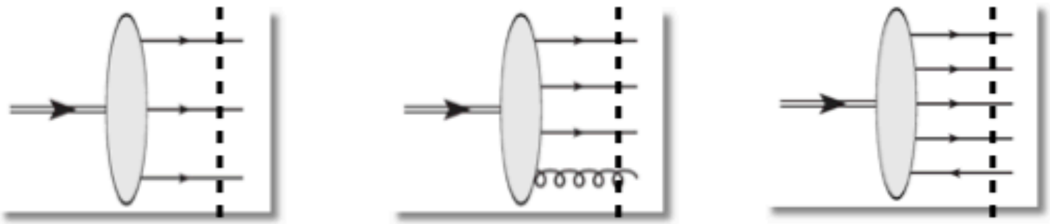
Light-front wave functions: Probability Amplitude for the N,β Fock state

$$|(P^+,\vec{P}_\perp),\Lambda\rangle = \sum_{N,\beta}[dx]_N[d\vec{k}_\perp]_N\Psi^{\Lambda}_{N,\beta}(x_i,\vec{k}_{\perp\, i})|N,\beta;(x_iP^+,x_i\vec{P}_\perp+\vec{k}_{\perp\, i}),\lambda_i\rangle$$

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Light-cone Fock expansion

$$|P\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \dots$$



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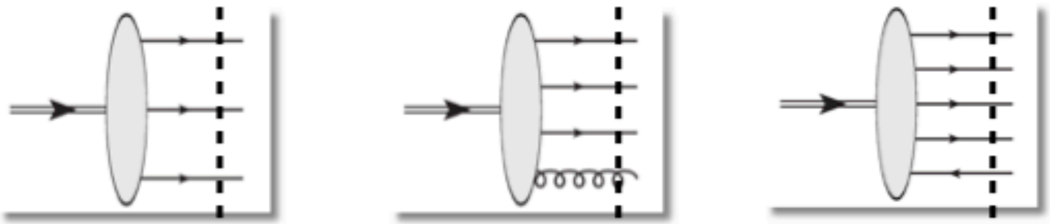
Internal variables: $x_i = \frac{p_i^+}{P^+}$ $\sum_{i=1}^{N} x_i = 1$ $\sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp$

Frame Independent

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Light-cone Fock expansion

$$|P\rangle = \Psi_{qqq}\,|qqq\rangle + \Psi_{qqqg}\,|qqqg\rangle + \Psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \ldots$$



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Frame Independent

Eigenstates of parton light-front helicity

$$\hat{S}_{iz}\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N} = \lambda_i\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N}$$

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Light-cone Fock expansion

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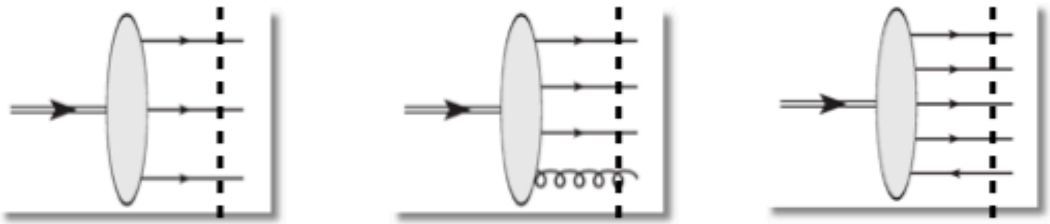

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Light-front wave functions: Probability Amplitude for the N,β Fock state

$$|(P^+, \vec{P}_\perp), \Lambda\rangle = \sum_{N,\beta}[dx]_N[d\vec{k}_\perp]_N \Psi^{\Lambda}_{N,\beta}(x_i, \vec{k}_{\perp\, i})|N, \beta; (x_iP^+, x_i\vec{P}_\perp + \vec{k}_{\perp\, i}), \lambda_i\rangle$$

Internal variables: $x_i = \dfrac{p_i^+}{P^+}$ $\qquad \sum_{i=1}^{N} x_i = 1 \qquad \sum_{i=1}^{N} \vec{k}_{i\perp} = \vec{0}_\perp$

Frame Independent

Eigenstates of parton light-front helicity

$$\hat{S}_{iz}\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N} = \lambda_i\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N}$$

Eigenstates of total OAM

$$\hat{L}_z\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N} = \ell_z\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N}$$

$A^+ = 0$ **gauge**

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Light-cone Fock expansion

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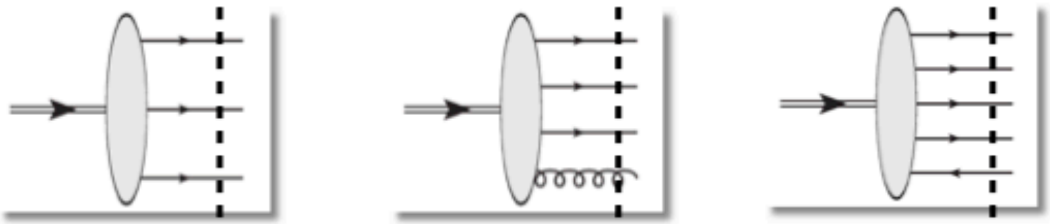

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Internal variables: $x_i = \dfrac{p_i^+}{P^+}$ $\quad \displaystyle\sum_{i=1}^{N} x_i = 1$ $\quad \displaystyle\sum_{i=1}^{N}\vec{k}_{i\perp} = \vec{0}_\perp$

Frame Independent

Eigenstates of parton light-front helicity

$$\hat{S}_{iz}\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N} = \lambda_i\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N}$$

$$\Lambda = \sum_{i=1}^{N}\lambda_i + \ell_z$$

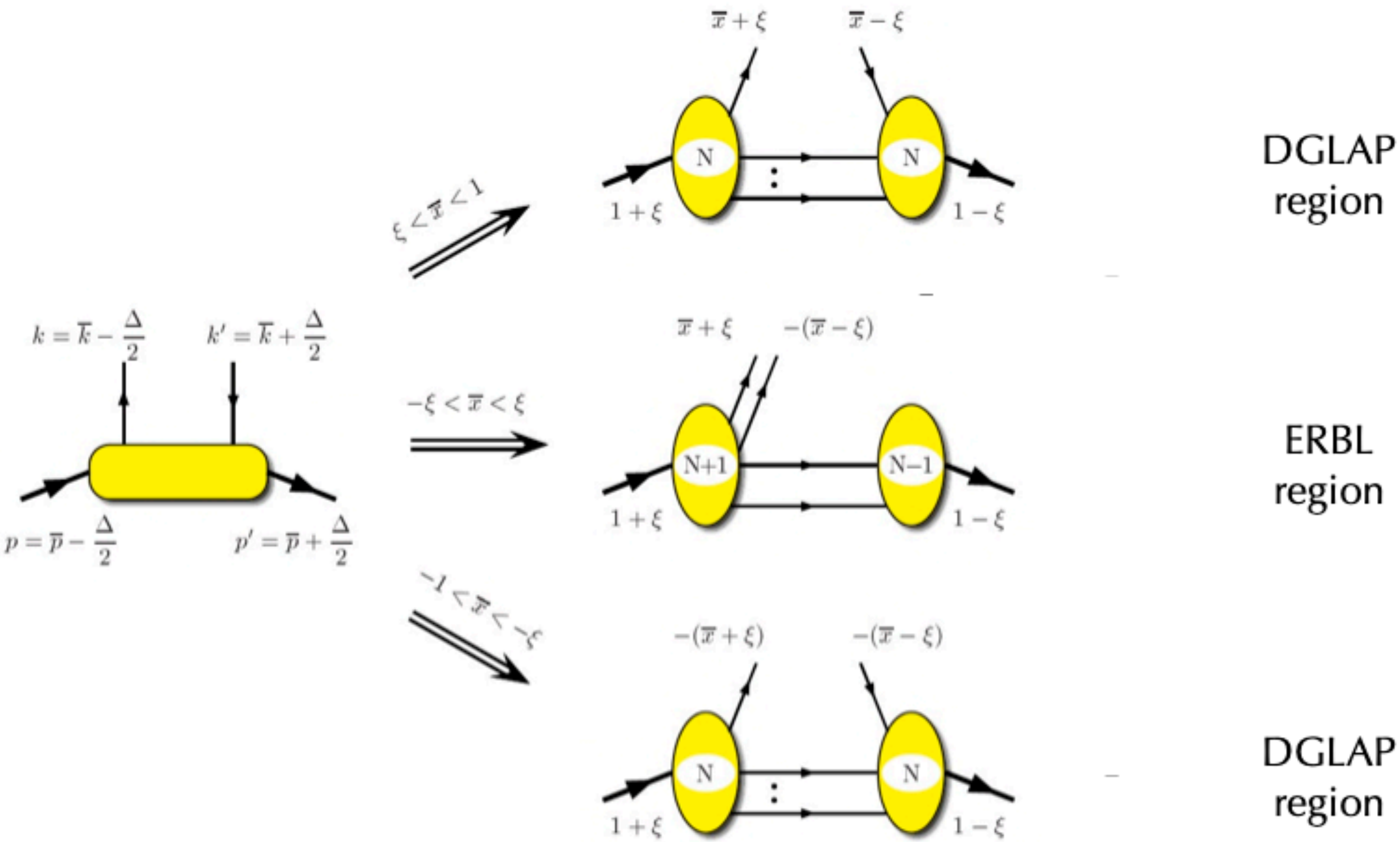
Eigenstates of total OAM

$$\hat{L}_z\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N} = \ell_z\Psi^{\Lambda}_{\lambda_1\ldots\lambda_N}$$

⚠ $A^+ = 0$ **gauge**

Brodsky, Pauli, Phys. Rept. 301 (1998) 299

Light-Front Wave Function Overlap Representation



GPDs $\sim \sum_N \int [d^3k]_N \, \Psi^*_N(k'_N)\Psi_N(k_N)\delta(\ldots)$ interference of probability amplitudes

PDFs $\sim \sum_N \int [d^3k]_N \, |\Psi_N(k_N)|^2\delta(\ldots)$ probability density

Diehl, Feldmann, Jakob, Kroll, NPB596, 2001
Diehl, Hwang, Brodsky, NPB596, 2001
Boffi, Pasquini, NPB649, 2003

Properties of GPDs

- Forward limit: ordinary parton distributions

$H^q(x, \xi = 0, t = 0) = q(x)$ unpolarized quark distributions

$\widetilde{H}^q(x, \xi = 0, t = 0) = \Delta q(x)$ long. polarized quark distributions

$H_T^q(x, \xi = 0, t = 0) = h_1(x)$ transv. polarized quark distributions

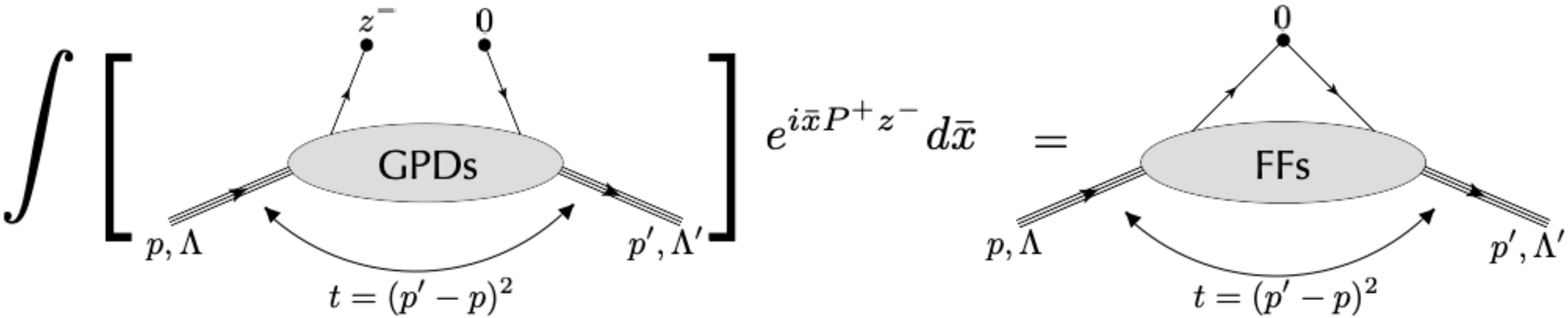
$x \in [-1, 1]:$ $x > 0$: quarks $x < 0$: antiquarks

analogous relations for gluons, except for transversity distribution

- all the other GPDs do NOT appear in inclusive DIS $\Longrightarrow$ new information

- They all depend on the renormalisation scale ($\mu^2 = Q^2$)
with different evolution equations in the DGLAP and ERBL regions

Properties of GPDs



$$\int_{-1}^{1} d\bar{x}\, H^q(\bar{x}, \xi, t) = F_1^q(t)$$ Dirac Form Factor

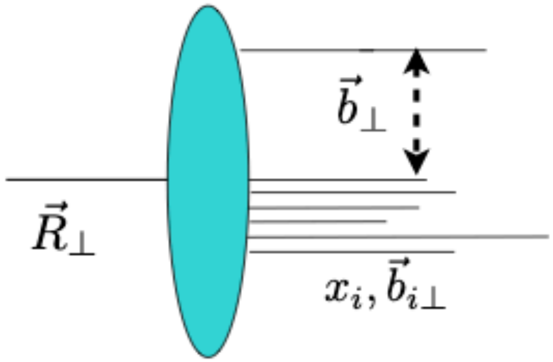
$$\int_{-1}^{1} d\bar{x}\, E^q(\bar{x}, \xi, t) = F_2^q(t)$$ Pauli Form Factor

$$\int_{-1}^{1} d\bar{x}\, \widetilde{H}^q(\bar{x}, \xi, t) = G_A^q(t)$$ Axial Form Factor

$$\int_{-1}^{1} d\bar{x}\, \widetilde{E}^q(\bar{x}, \xi, t) = G_P^q(t)$$ Pseudoscalar Form Factor

- matrix elements of local operators
  $\longrightarrow$ can be calculated on the lattice
- renormalisation scale independent
- Lorentz invariance
  $\longrightarrow$ ξ independent

Impact Parameter Space



- average transverse position of the partons

$$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+} \quad (i = q, \bar{q}, g)$$

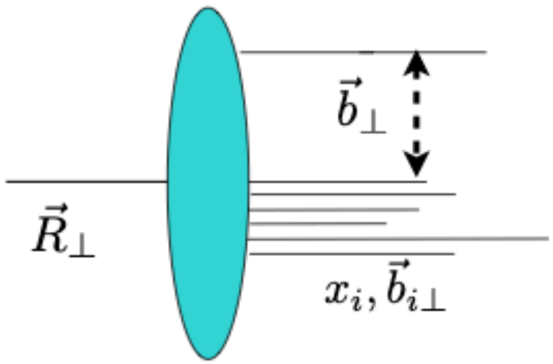
- $b_\perp$: transverse distance between the struck parton and the centre of momentum of the hadron

[Burkardt, 2003]

Isomorphism between Galilei and subgroup of Light-Front operators

Galilei transformation:	Transverse boost:
$m_i \to m_i$ $\qquad \vec{p}_i \to \vec{p}_i - m_i \vec{v}$	$p_i^+ \to p_i^+$ $\qquad \vec{p}_{\perp i} \to \vec{p}_{\perp i} - p_i^+ \vec{v}$
Center of mass:	Center of plus momentum:
$\vec{r}_* = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$	$\vec{R}_\perp = \frac{\sum_i p_i^+ \vec{b}_{\perp i}}{\sum_i p_i^+}$

Impact Parameter Space



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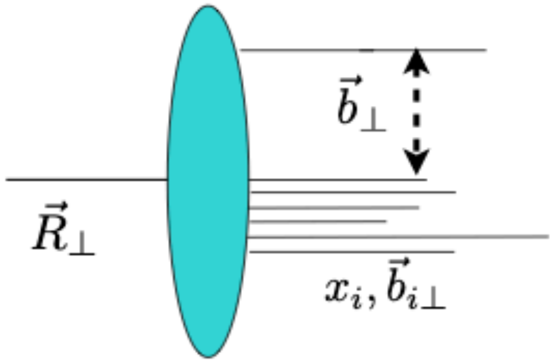

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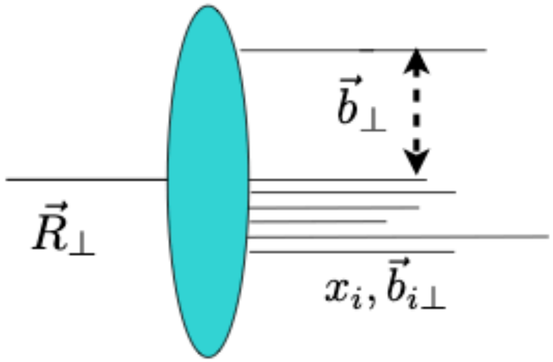
[Burkardt, 2003]

- Localized wave packet in the transverse plane

$$|p^+, \vec{R}_\perp = \vec{0}_\perp, \lambda\rangle = \mathcal{N} \int \frac{d^2\vec{p}_\perp}{(2\pi)^2} |p^+, \vec{p}_\perp, \lambda\rangle \quad \text{with} \quad |\mathcal{N}|^2 \int \frac{d^2\vec{p}_\perp}{(2\pi)^2} = 1$$

Impact Parameter Space



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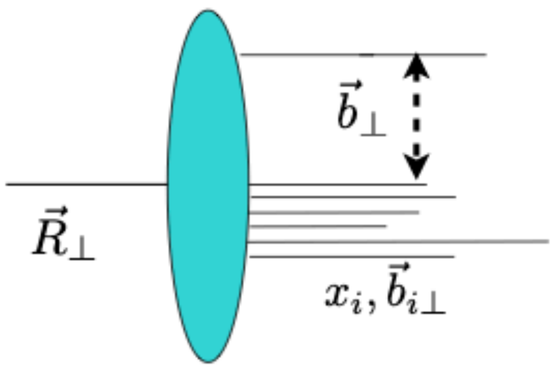
- Impact parameter dependent GPD (IPD)

$$H(x, 0, \vec{b}_\perp) = \langle p^+, \vec{R}_\perp = 0, \Lambda| \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \bar{\psi}(-\frac{z^-}{2}, \vec{b}_\perp)\gamma^+\psi(\frac{z^-}{2}, \vec{b}_\perp)|p^+, \vec{R}_\perp = 0, \Lambda\rangle$$

Impact Parameter Space



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- Impact parameter dependent GPD (IPD) $\Longrightarrow$ number density of (anti)quark with longitudinal momentum x and transverse position $\vec{b}_\perp$

$$H(x, 0, \vec{b}_\perp) = \langle p^+, \vec{R}_\perp = 0, \Lambda| \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \bar{\psi}(-\frac{z^-}{2}, \vec{b}_\perp)\gamma^+\psi(\frac{z^-}{2}, \vec{b}_\perp)|p^+, \vec{R}_\perp = 0, \Lambda\rangle$$

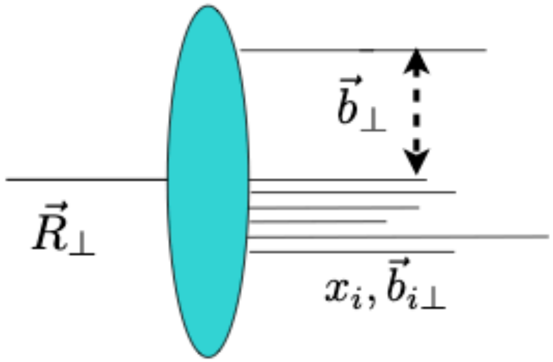
x>0

$$H(x, 0, \vec{b}_\perp) \sim \sum_\lambda \left| b_\lambda(xp^+, \vec{b}_\perp)|p^+, \vec{R}_\perp = 0, \Lambda\rangle \right|^2 \geq 0$$

x<0

$$H(x, 0, \vec{b}_\perp) \sim -\sum_\lambda \left| d_\lambda^\dagger(xp^+, \vec{b}_\perp)|p^+, \vec{R}_\perp = 0, \Lambda\rangle \right|^2 \leq 0$$

Impact Parameter Space



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x<0

$$H(x, 0, \vec{b}_\perp) \sim -\sum_\lambda \left| d^\dagger_\lambda(xp^+, \vec{b}_\perp)|p^+, \vec{R}_\perp = 0, \Lambda\rangle \right|^2 \leq 0$$

- IPDs=Fourier transform of GPDs (homework)

$$H^q(x, 0, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x, 0, t) e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} \qquad (t = -\Delta_\perp^2)$$

Homework

$$H(x,0,\vec{b}_\perp) = \langle p^+, \vec{R}_\perp = 0, \Lambda | \hat{O}_q(x,\vec{b}_\perp) | p^+, \vec{R}_\perp = 0, \Lambda \rangle$$

where $\hat{O}_q\ (x,\vec{b}_\perp) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \bar{\psi}(-\frac{z^-}{2},\vec{b}_\perp)\gamma^+\psi(\frac{z^-}{2},\vec{b}_\perp)$

Homework

$$H(x,0,\vec{b}_\perp) = \langle p^+, \vec{R}_\perp = 0, \Lambda | \hat{O}_q(x,\vec{b}_\perp) | p^+, \vec{R}_\perp = 0, \Lambda \rangle$$

$$\text{where } \hat{O}_q\ (x,\vec{b}_\perp) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \bar{\psi}(-\frac{z^-}{2}, \vec{b}_\perp)\gamma^+ \psi(\frac{z^-}{2}, \vec{b}_\perp)$$

- Insert the expression for the localized wave packet in the transverse plane

$$|p^+, \vec{R}_\perp = \vec{0}_\perp, \lambda\rangle = \mathcal{N} \int \frac{d^2\vec{p}_\perp}{(2\pi)^2} |p^+, \vec{p}_\perp, \lambda\rangle \quad \text{with} \quad |\mathcal{N}|^2 \int \frac{d^2\vec{p}_\perp}{(2\pi)^2} = 1$$

$$H(x,0,\vec{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2 p_\perp}{(2\pi)^2} \int \frac{d^2 p'_\perp}{(2\pi)^2} \langle p^+, \vec{p}\,'_\perp, \Lambda | \hat{O}_q(x,\vec{b}_\perp) | p^+, \vec{p}_\perp, \Lambda \rangle$$

Homework

$$H(x,0,\vec{b}_\perp) = \langle p^+, \vec{R}_\perp = 0, \Lambda | \hat{O}_q(x,\vec{b}_\perp) | p^+, \vec{R}_\perp = 0, \Lambda \rangle$$

$$\text{where } \hat{O}_q\ (x,\vec{b}_\perp) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \bar{\psi}(-\frac{z^-}{2},\vec{b}_\perp)\gamma^+\psi(\frac{z^-}{2},\vec{b}_\perp)$$

- Insert the expression for the localized wave packet in the transverse plane

$$|p^+,\vec{R}_\perp = \vec{0}_\perp, \lambda\rangle = \mathcal{N}\int \frac{d^2\vec{p}_\perp}{(2\pi)^2}|p^+,\vec{p}_\perp,\lambda\rangle \quad \text{with} \quad |\mathcal{N}|^2\int \frac{d^2\vec{p}_\perp}{(2\pi)^2} = 1$$

$$H(x,0,\vec{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2p_\perp}{(2\pi)^2}\int \frac{d^2p'_\perp}{(2\pi)^2}\langle p^+,\vec{p}\,'_\perp,\Lambda|\hat{O}_q(x,\vec{b}_\perp)|p^+,\vec{p}_\perp,\Lambda\rangle$$

- Use translation invariance

$$\langle p^+,\vec{p}\,'_\perp,\Lambda|\hat{O}_q(x,\vec{b}_\perp)|p^+,\vec{p}_\perp,\Lambda\rangle = e^{i\vec{b}_\perp\cdot(\vec{p}_\perp - \vec{p}\,'_\perp)}\langle p^+,\vec{p}\,'_\perp,\Lambda|\hat{O}_q(x,\vec{0}_\perp)|p^+,\vec{p}_\perp,\Lambda\rangle$$

Homework

$$H(x,0,\vec{b}_\perp) = \langle p^+, \vec{R}_\perp = 0, \Lambda | \hat{O}_q(x,\vec{b}_\perp) | p^+, \vec{R}_\perp = 0, \Lambda \rangle$$

$$\text{where } \hat{O}_q\,(x,\vec{b}_\perp) = \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \bar{\psi}(-\frac{z^-}{2}, \vec{b}_\perp)\gamma^+\psi(\frac{z^-}{2}, \vec{b}_\perp)$$

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- Change variables of integration:

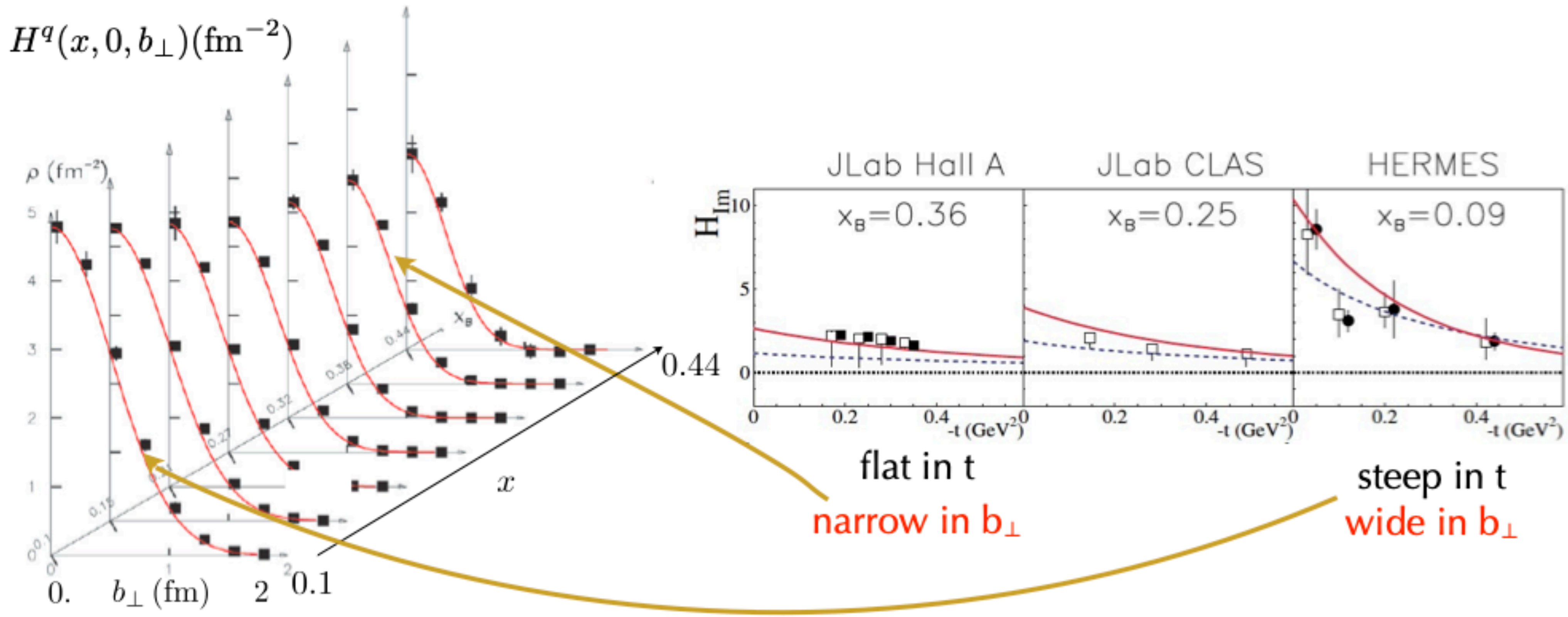
$$\int d^2p_\perp \int d^2p'_\perp = \int d^2\Delta_\perp \int d^2P_\perp \quad \text{where} \quad \vec{\Delta}_\perp = \vec{p}'_\perp - \vec{p}_\perp \;\; \vec{P}_\perp = \vec{p}'_\perp + \vec{p}_\perp$$

$$H(x,0,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \langle p^+, \vec{p}'_\perp, \Lambda | \hat{O}_q(x,\vec{0}_\perp) | p^+, \vec{p}_\perp, \Lambda \rangle$$

The unpolarized GPD H

$$H^q(x,0,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x,0,t)\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \qquad (t=-\vec{\Delta}_\perp^2)$$

extrapolation from data



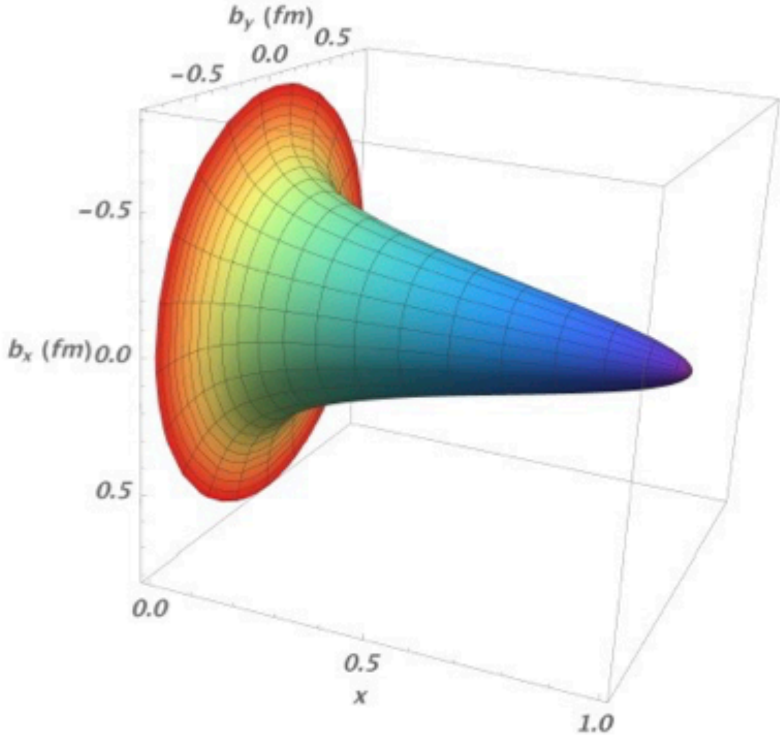
Guidal et al., Rep. Prog. Phys. ***76*** *(2013) 066202*

The unpolarized GPD H

$$H^q(x,0,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H^q(x,0,t) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} \qquad (t = -\vec{\Delta}_\perp^2)$$

↓ extrapolation from data

$$\langle \vec{b}_\perp^2(x) \rangle = \frac{\int d^2\vec{b}_\perp \, \vec{b}_\perp^2 \, H(x,0,b_\perp)}{\int d^2\vec{b}_\perp H(x,0,b_\perp)}$$

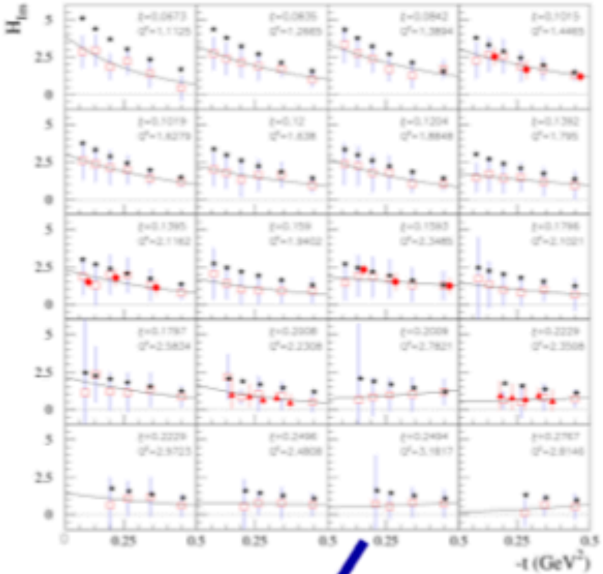


As x ⟶ 1, the active parton carries all the momentum and represents the centre of momentum

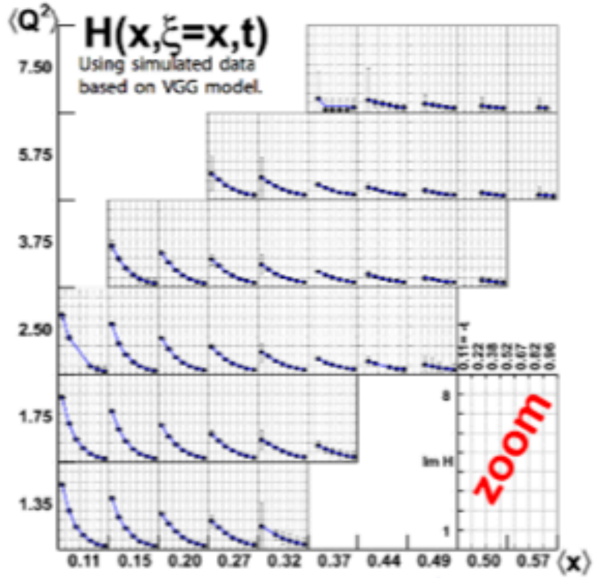
Dupré et al., PRD95(2017)011501

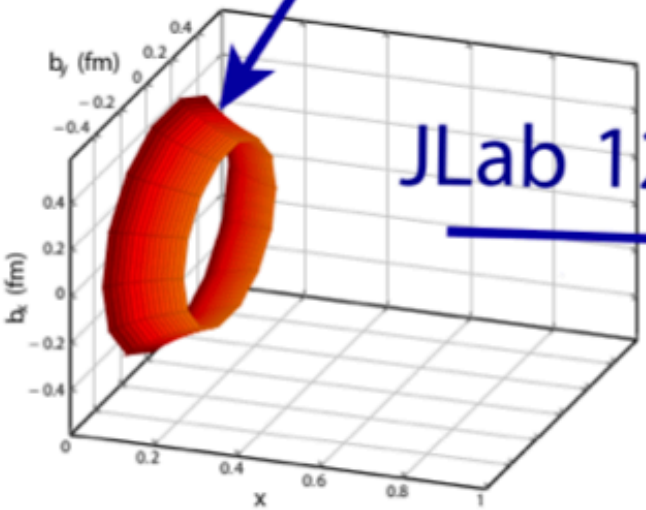
The unpolarized GPD H

Düpré-Guidal-Vanderhaeghen-PRD **95** 011501 (R) (2017)

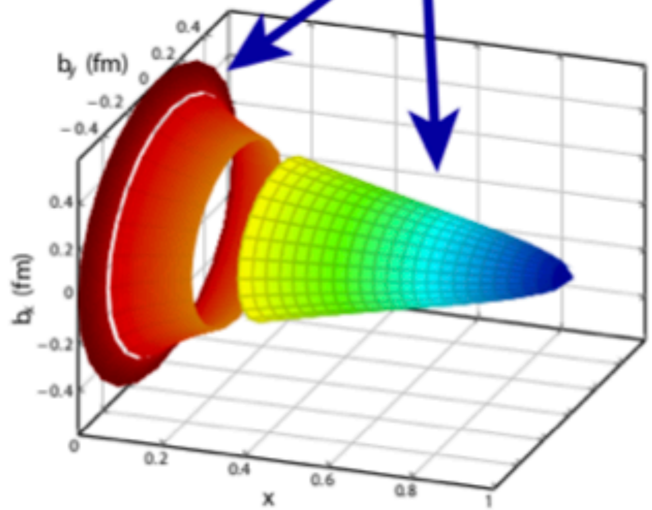


CLAS12 projections E12-06-119 with DVCS A_{UL} and A_{LU}





JLab 12 GeV



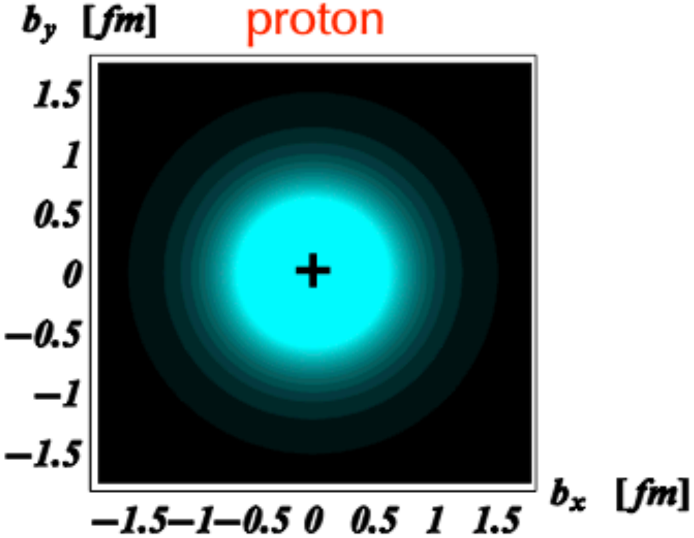
Courtesy of R. Dupré, M. Vanderhaeghen and M. Guidal

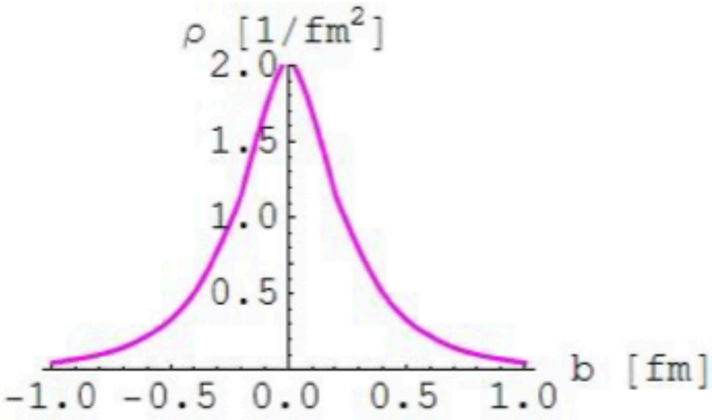
Charge density of partons in the transverse plane

$$\rho^q(b_\perp) = e_q \int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} \int dx H^q(x,0,\Delta_\perp^2) = \int d^2\Delta_\perp e^{i\Delta_\perp \cdot b_\perp} F_1^q(\Delta_\perp^2)$$

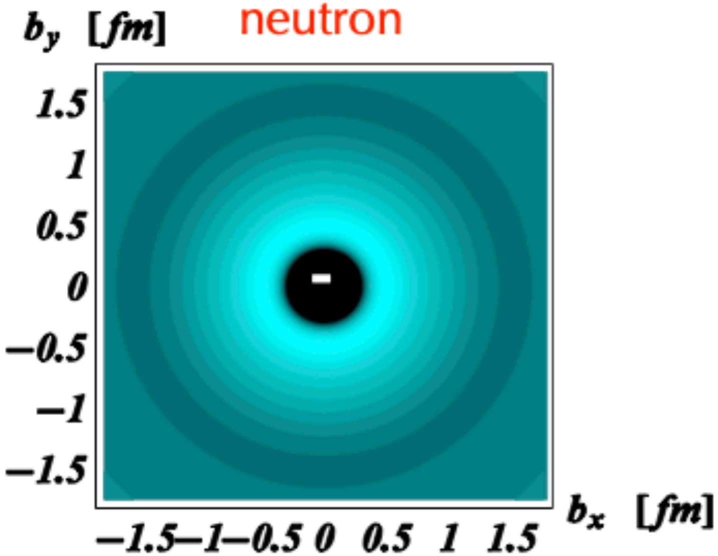
Infinite-Momentum-Frame Parton charge density in the transverse plane

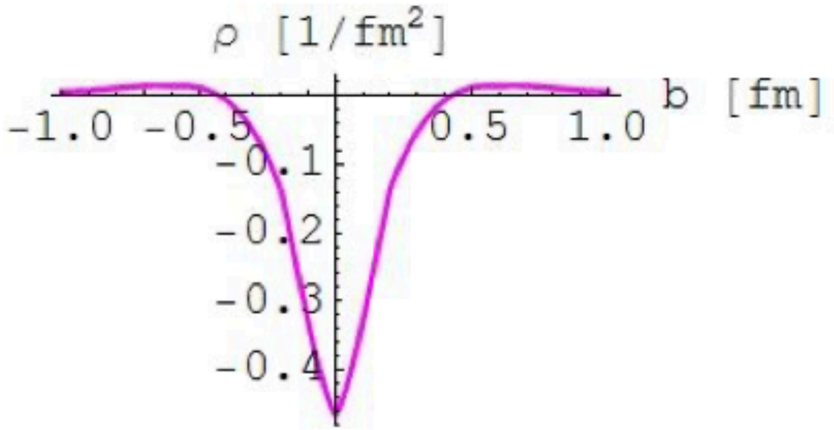
proton





neutron





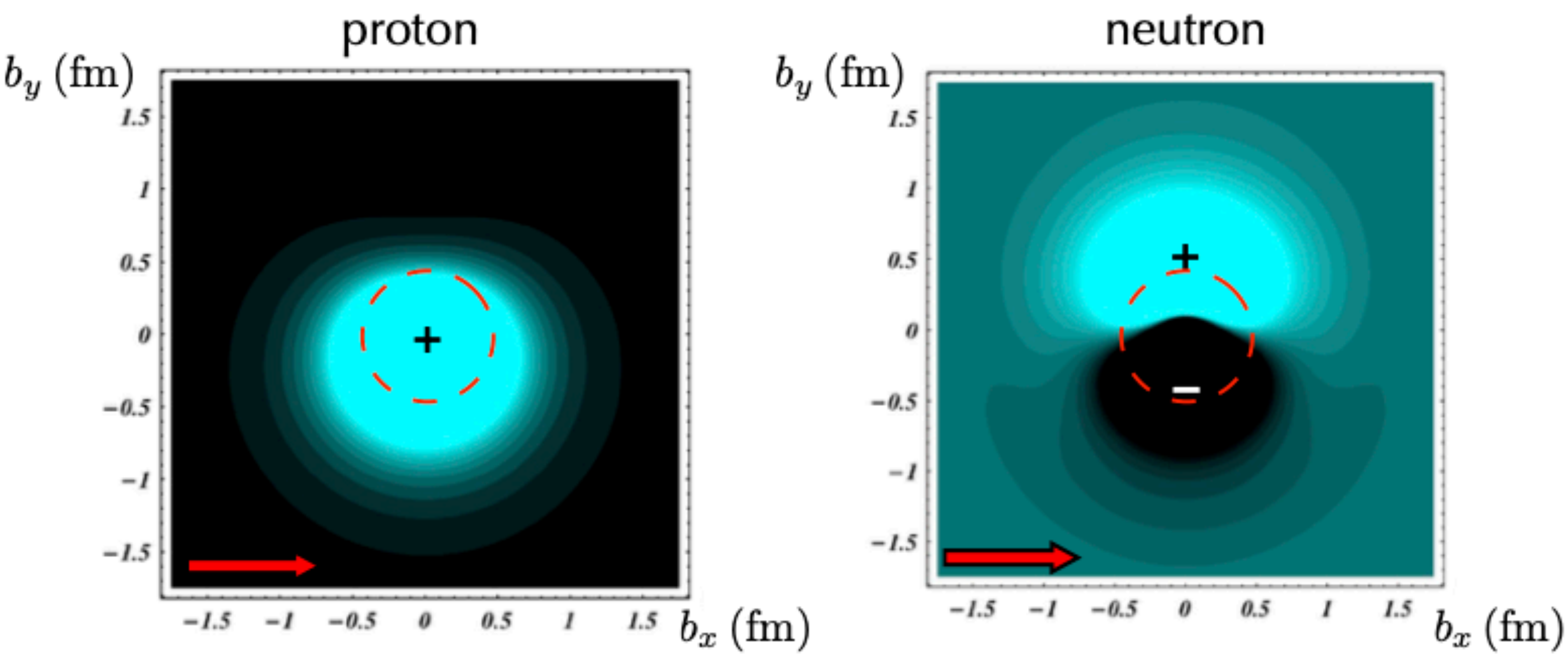
Miller (2007); Burkardt (2007)

Electromagnetic Form Factors

Transversely polarized proton

$$\rho_T(\vec{b}_\perp) = \rho(\vec{b}_\perp) + \sin(\phi_b - \phi_s) \int \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(Qb_\perp) F_2(Q^2)$$

monopole — dipole



nucleon polarized in the x direction

C. Carlson, and M. Vanderhaeghen, Phys. Rev. Lett. 100 (2008) 032004

Take-home message

Parton distribution functions ⟷ **Generalized Parton distributions**

Parton distribution functions

enter at cross section level
in fully inclusive reactions

squared wave functions

→ probability density

Generalized Parton distributions

enter at amplitude level
in fully exclusive reactions

correlate wave function with different parton configurations

→ quantum-mechanical interference terms

provide a decomposition in x of form factors

at $\xi = 0$ and in impact parameter space ($\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$) we recover the probabilistic interpretation for the position of partons in the transverse plane with longitudinal momentum x