# A NEW SEPARATION BETWEEN MONOTONE & NON-MONOTONE CIRCUTIS


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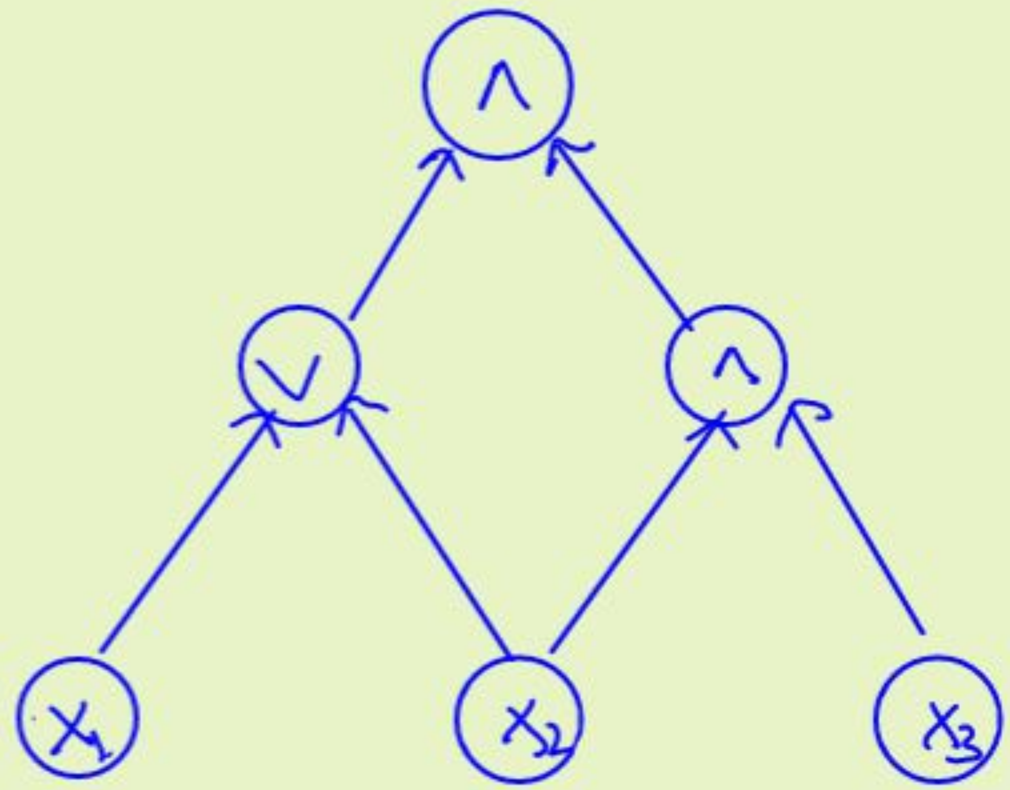
# Monotone Boolean Circuits

→ Boolean circuit

De Morgan basis

AND / OR



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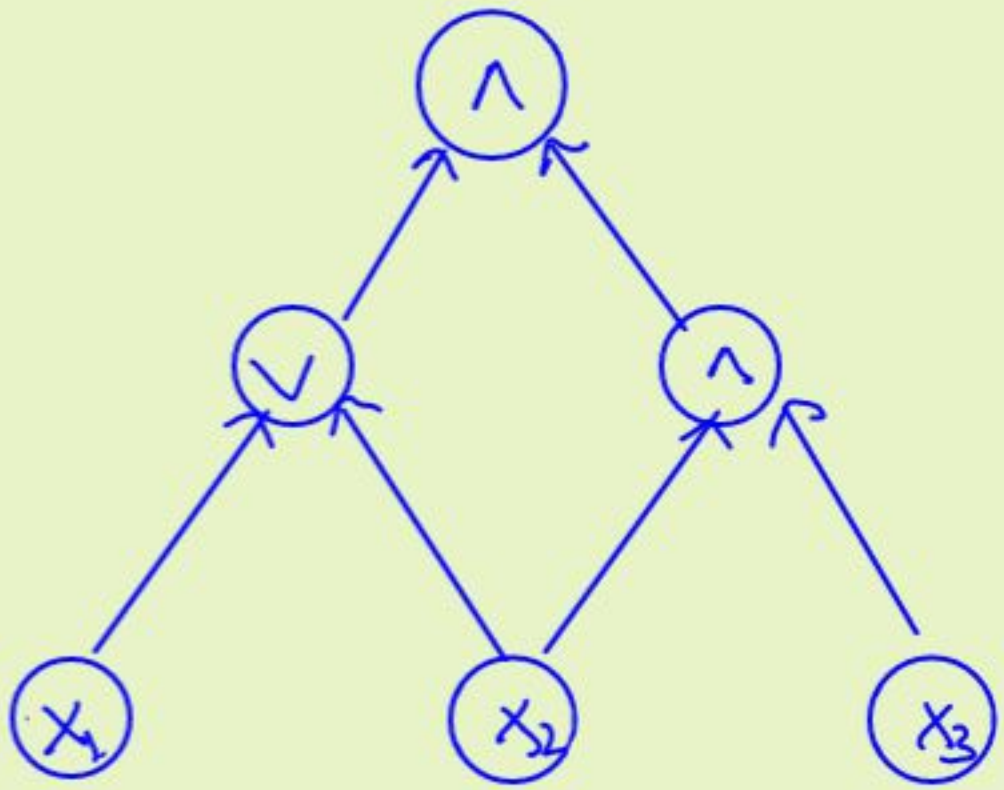
→ Computes monotone

$f:\{0,1\}^n \to \{0,1\}$ → $x \le y \Rightarrow f(x) \le f(y)$



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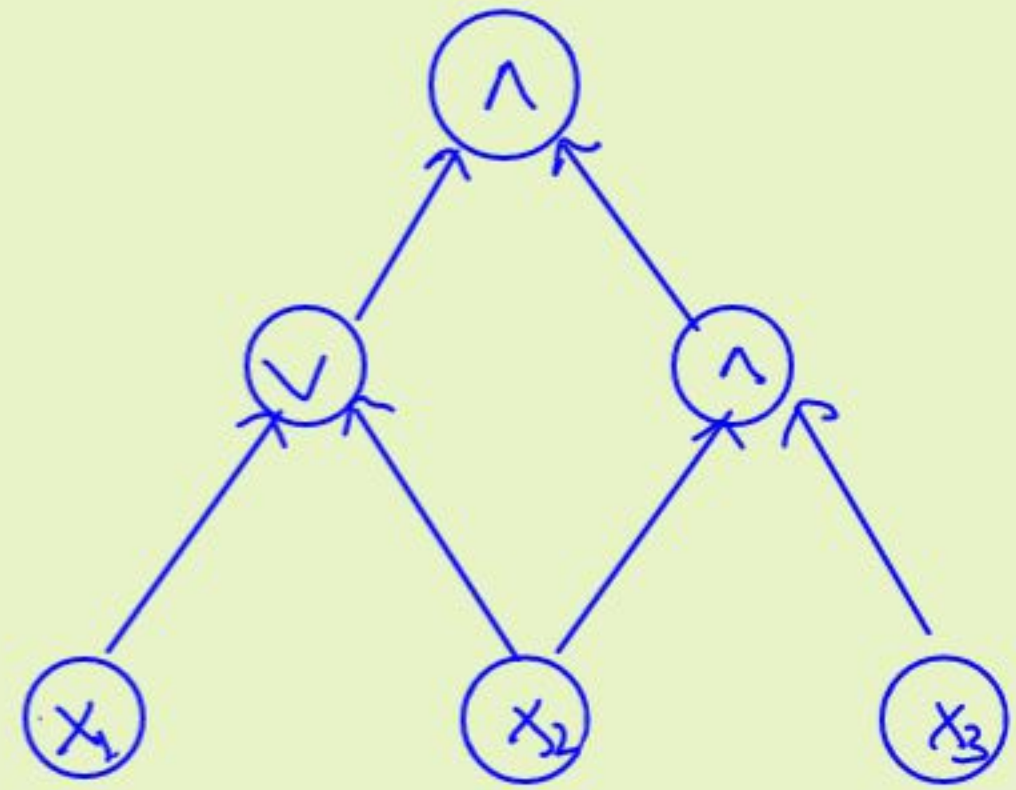
$f : \{0,1\}^n \to \{0,1\}$

→ Any monotone $f$ has a monotone ckt. of size $O(2^n)$



## Monotone Boolean Circuits

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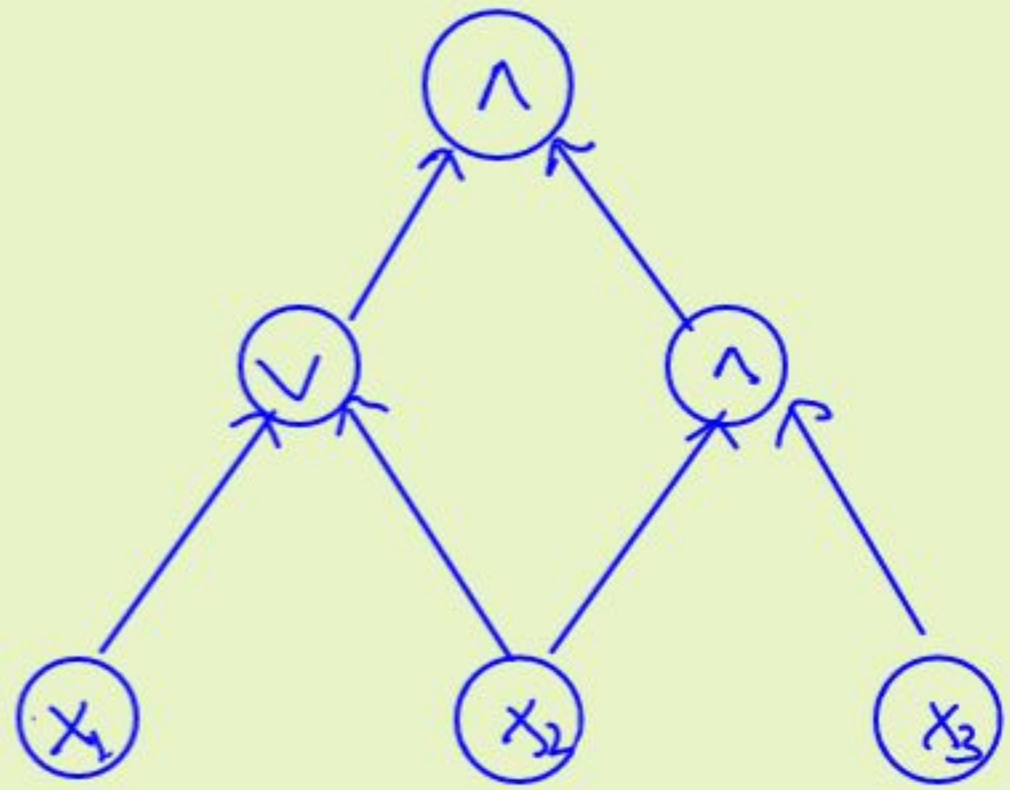
$f : \{0,1\}^n \to \{0,1\}$

→ Any monotone $f$ has a monotone ckt. of size $O(2^n)$

→ Most $f$ need size $2^{\Omega(n)}$

→ Explicit $f$ ?



Razborov '85

**Thm**: $n^{\Omega(\log n)}$ lower bounds for

→ Clique

→ Perfect Matching

Razborov '85

Thm: $n^{\Omega(\log n)}$ lower bounds for

→ Clique ----> Should be exponential!

→ Perfect Matching ---> Monotone vs. non-monotone separation!

## Further developments

LB $n^{\Omega((\log n)^c)}$ $\exp(n^{\varepsilon})$

UB

NP Razborov '85

P Razborov '85

# Further developments

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| LB | $n^{\Omega((\log n)^c)}$ | $\exp(n^{\varepsilon})$ |
| UB | | |
| NP | Razborov'85 | AB'87 $\varepsilon = 1/6$ |
| | | Andreev'85, HR'01 $\varepsilon = 1/3$ |
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| | | CGRSS'25 | $\varepsilon = 1/6$ |
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| $AC^0$ | CGRSS'25 | | |

## Main Result

$$M = \begin{array}{c} \begin{array}{cccc} x_1 & \cdots & \cdots & x_n \end{array} \\ \begin{pmatrix} | & & & | \\ v_1 & \cdots & \cdots & v_n \\ | & & & | \end{pmatrix}_{m \times n} \end{array}$$

$$f_M(x_1, \ldots, x_n) = 1 \iff rk\{v_i \mid x_i = 1\} = \text{full} = m$$

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$f_M(x_1, .., x_n) = 1 \iff rk\{v_i \mid x_i = 1\} = \text{full} = m$

Thm: $\exists n = \tilde{O}(m^{3/2})$, $M$ s.t. any monotone Bool. $C$ for $f_M$ has size $\exp(\tilde{\Omega}(\sqrt{m}))$.

# Qualitative differences

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→ Computed by monotone rank program

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[GKRS'19]: separation from monotone span programs

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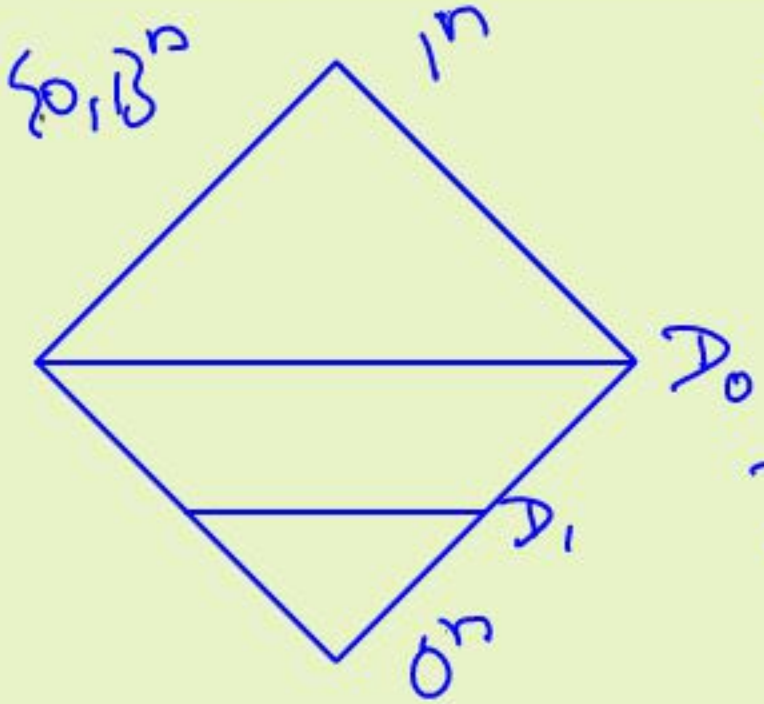
→ Min terms $\iff$ max. ind. sets of a matroid.

Solves an open problem of JS'20.

# Proof Method

$f : \{0,1\}^n \to \{0,1\}$




$D_1$ : hard dist. on 1 inputs

$D_0$ : " " " 0 "

$D_1$ : supp. on wt. $w \ll n$

$D_0$ : sunflower lemma

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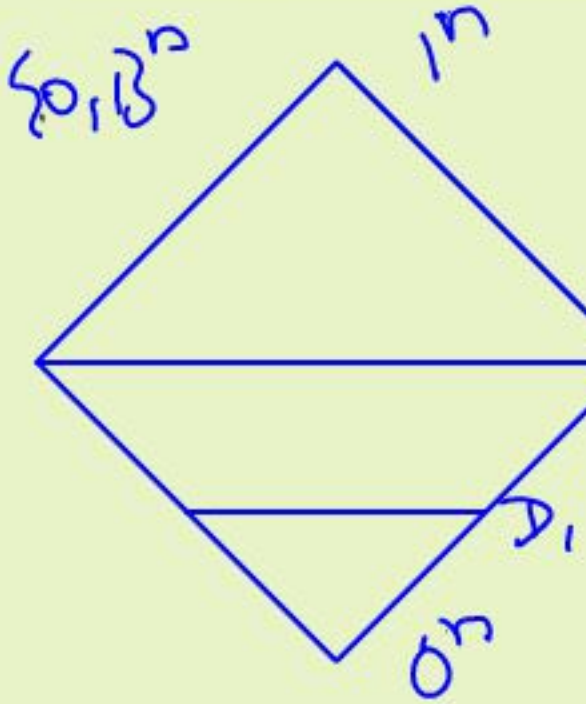


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$D_1$ is $k$-spread: $\Pr_{x \sim D_1}\left[\bigwedge_{i \in I} x_i = 1\right] \leq O\left(\frac{w}{n}\right)^{|I|} \ \forall\, |I| \leq k$

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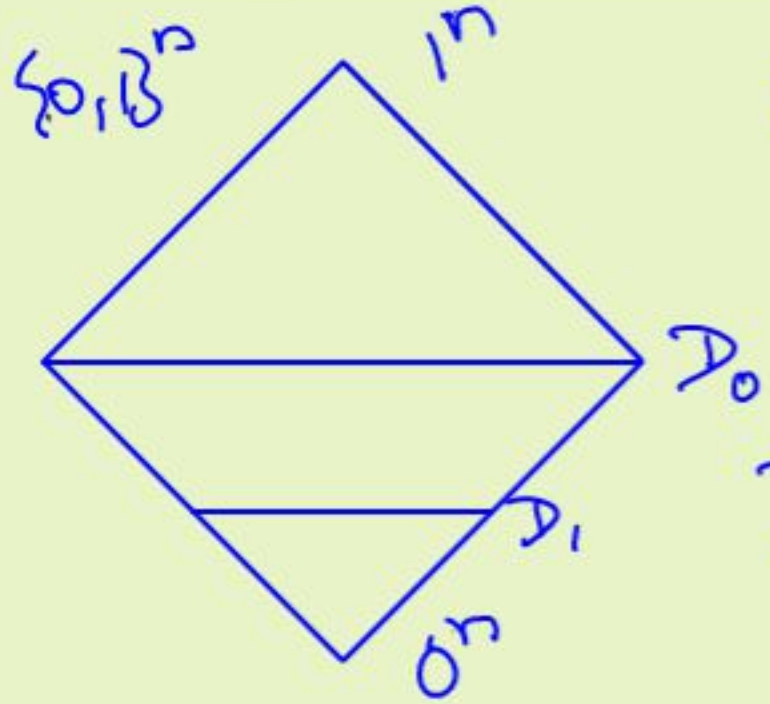


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$D_0$ has an $(r, 2k)$- : $\quad F = \bigvee_{i=1}^{N} T_i \quad |T_i| = \ell \leq 2k$, $\ N \geq r^{\ell}$

sunflower lemma

$\Rightarrow \exists F' = \bigvee_{i=1}^{p} T_i$ : $\Pr_{x \sim D_0}\left[F'(x) \neq \bigcap_{i=1}^{p} T_i(x)\right] \leq \frac{1}{n^{10k}}$.

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Thm [this work based on CKR'20, CGRSS'25]:

Any monotone Boolean ckt. for $f$ has size

$$\left(\frac{c \cdot n}{w r}\right)^{k}$$

# Sunflower lemmas

$D_0$ has an $(r,2k)$-sunflower lemma: $\quad F = \bigvee_{i=1}^{N} T_i \quad |T_i| = \ell \le 2k$, $N \ge r^\ell$

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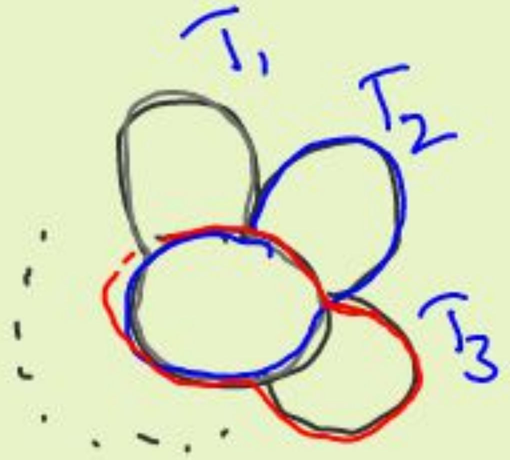
$D_0 = \mathcal{U}_n$ (uniform on $\{0,1\}^n$)

**Obs**: Sufficient to find a combinatorial sunflower with $c2^{\ell} k \log n$ petals.



## Sunflower lemmas

$\mathcal{D}_0$ has an $(r,2k)$-sunflower lemma: $F = \bigvee_{i=1}^{N} T_i$, $|T_i| = \ell \le 2k$, $N \geq r^\ell$

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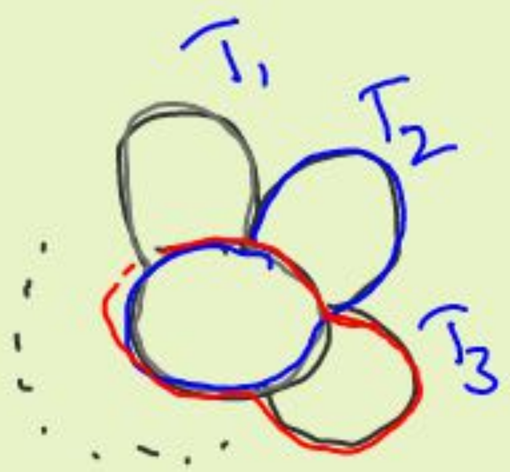


ER'61: Need $r \geq 2^\ell k \log n$

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Rossman'11: Sufficient to find "nearly disjoint" subfamilies.

ALWZ'19: "Spread" subfamilies $\Rightarrow$ sunflower lemma

ALWZ'19, BCW'21, Rao'25: $r \geq c \cdot k \log n$ sufficient.

# Proving lower bounds for $f_M$

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$f_M(x_1, \ldots, x_n) = 1 \iff \{v_i \mid x_i = 1\}$ full rank.

$D_1$ is $k$-spread: $\Pr_{x \sim D_1}\left[\bigwedge_{i \in I} x_i = 1\right] \le O\left(\frac{w}{n}\right)^{|I|} \ \forall |I| \le k$

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$D_1$: $S \in_R \binom{[n]}{m}$ $x = \mathbb{1}_S$

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$D_0$ has uniform marginals & $t$-wise independence

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$\Longrightarrow$ Sunflower lemma for $D_0$

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Bazzi's thm + Sunflowers for $U_n$

# Explicit Matrix

$$\begin{pmatrix} 1 & \cdots & 1 \\ \alpha_1 & & \alpha_n \\ \vdots & \cdots & \vdots \\ \alpha_1^{t-1} & & \alpha_n^{t-1} \end{pmatrix} \qquad \alpha_i \in \mathbb{F}_{2^{\log n}}$$

# Explicit Matrix

$$\begin{pmatrix} 1 & & 1 \\ \alpha_1 & & \alpha_n \\ \vdots & \cdots & \vdots \\ \alpha_1^{t-1} & & \alpha_n^{t-1} \end{pmatrix} \qquad \alpha_i \in \mathbb{F}_{2^{\log n}}$$

$\mathbb{F}_2$-matrix: replace $\alpha_i$ by $M_i \in \mathbb{F}_2^{\log n \times \log n}$

representing $x \in \mathbb{F}_{2^{\log n}} \mapsto x \cdot \alpha_i \in \mathbb{F}_{2^{\log n}}$.

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Thank you!