Geometry of tropical varieties

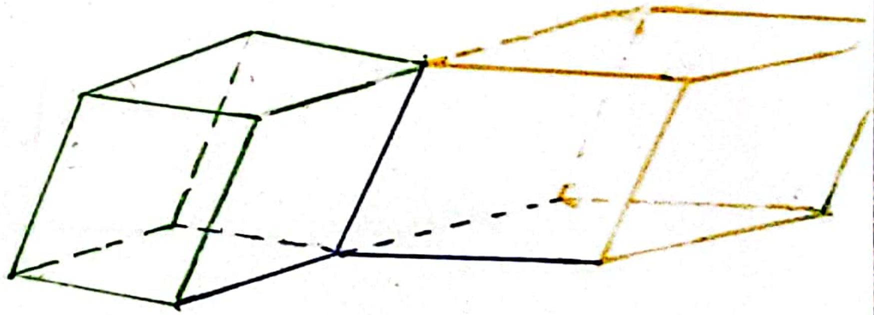
Lecture 4

01/07/2022

objective

- Place the preceding results in a broader context
- Provide a global generalization

§ 7. Canonical Compactifications



Σ fan in $\mathbb{R}^n$

Y corresponding fanfold

Σ defines a compactification $\overline{Y}$ of Y.

$\overline{\Sigma}$ corresponding Polyhedral structure

Theorem (A.–Piquerez)

$$A^k(\Sigma) \simeq H^{k,k}(\overline{\Sigma}).$$

§8. Tropical cohomology

Tropical variety — topological space X endowed with an atlas of charts

$$\begin{array}{ccc} U & \longrightarrow & V \\ \cap & & \cap \\ X & & \overline{\Sigma} \end{array} \qquad \Sigma \text{ tropical fan}$$

such that transition maps are integral linear.

Typically $Y \subseteq \mathbb{R}^n$ rational polyhedral space + balancing condition $\qquad X = \overline{Y}$ canonical compactificati[on]

Ω^p_X : sheaf of tropical holomorphic p-forms.

$$H^{p,q}(X) := H^q(X, \Omega^p_X)$$

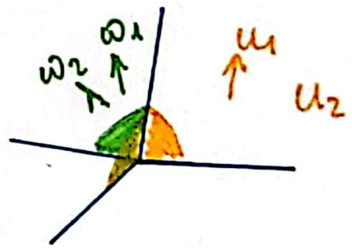
$$H^k(X) = \bigoplus_{p+q=k} H^{p,q}(X)$$

Definition

Let X be a tropical variety endowed with a polyhedral structure.

F_p sheaf of tropical holomorphic p-multivectors

Ω^p_X dual of F_p



$u_1 \wedge u_2$

$\omega_1 \wedge \omega_2$

$v_1 \wedge v_2$

Scanné avec CamScanner

§ 9. Tropical Hodge theory

Theorem (A.–Piquerez)

X tropical variety, compact, local charts shellable

Then,

(Poincaré duality) $\quad H^{p,q}(X) \times H^{d-p,d-q}(X) \longrightarrow H^{d,d}(X) \simeq \mathbb{Q}$

induces a duality

Assume X Kähler, ω Kähler form.

(Hard Lefschetz) $\quad \omega^{d-k} : \quad H^k(X) \xrightarrow{\simeq} H^{2d-k}(X)$

$$H^{p,q}(X) \xrightarrow{\simeq} H^{d-q,d-p}(X)$$

(Hodge symmetry) $\quad H^{p,q}(X) \xrightarrow{\simeq} H^{q,p}(X)$

(Hodge-Riemann) $\quad H^{p,q}(X) \times H^{p,q}(X) \to \mathbb{Q}$

$$\alpha, \beta \longmapsto \alpha \cup \bar{\beta} \cup \omega^{d-p-q}$$

is definite on $P^{p,q} := \mathrm{Ker}\left(\omega^{d-k+1} : H^{p,q}(X) \to H^{d-q+1,d-p+1}(X)\right)$.