Adiabatic elimination and laser classes:

Maxwell-Bloch eqns

$$\frac{dA}{dt} = -\left(\frac{1}{2\tau_{cav}} - i\delta_{cav}\right)A + i\frac{\omega}{2\epsilon}P$$

$$\frac{dP}{dt} = -(\Gamma - i\delta)P - i\frac{d^2}{\hbar}A\Delta n$$

$$\frac{d\Delta n}{dt} = -\frac{1}{\tau}(\Delta n - \Delta n_0) - \frac{i}{\hbar}(A^*P - AP^*)$$

Single mode Laser eqns.

Perfectly describes the interaction between the electromagn. field and two level system. (2) shows that the field induces a polarization (atomic coherence) when the atom has population inversion. ($\Delta n \neq 0$).

Phase of induced pol. depends the phase of the field. with a sign. that changes with the sign of inversion.

amplification or attenuation | lead to | this pol. will emit a field with

depending on sign of Δn. | moreover, this field will act on atoms which have a nonzero pol to alter their population.

Dynamical variables, A, P, Δn. nonlinearly coupled

can exhibit quasiperiodicity and chaos.

Formal equivalence with Lorentz's eqns butter fly chaos.

Laser dynamic classes.

The three equation have some similarities.
contain a relaxation terms wi lifetimes τ_{car}, τ, Γ^{-1}
These times — characteristic of time needed to reach the steady state
Depending on how they compare, one classifies three distinct classes of lasers.

Class C laser. all lifetimes: same order of magnitude.
Example: NH_3 Maser. (far infrared)
It has been shown to exhibit deterministic chaos.

Class B laser τ_{car}, $\tau \gg \Gamma^{-1}$ and τ_{car} is not much larger than τ. (comparable)
Ex: CO_2 laser
In this cas P responds very quickly to the changes of Δn and A. P reaches the steady state value instantly and we can write $\frac{dP}{dt} = 0$.
This allows to express P as a function of Δn and A and eliminate P from the equations $\Longrightarrow$ Adiabatic elimination

Class A laser $\tau_{car} \gg \tau, \Gamma^{-1}$ Most gas lasers and dye lasers belong to this class
We can adiabatically eliminate Δn and P whose response times are much shorter than the response time of the field.

Next we focus on class B and class A lasers.

Adiabatic elimination of $\mathcal{P}$.

$$-(\Gamma - i\delta)\mathcal{P} - i\frac{d^2}{\hbar}\Delta n\, A = 0 \qquad \frac{d\mathcal{P}}{dt} = 0$$

$$\mathcal{P} = -i\frac{d^2}{\hbar}\frac{1}{\Gamma - i\delta} A \Delta n.$$ substitute in eqn for Δn

$$\frac{d\Delta n}{dt} = -\frac{1}{\tau}(\Delta n - \Delta n_0) - \frac{i}{\hbar}(A^* \mathcal{P} - A\mathcal{P}^*)$$

Ⓐ $$\frac{d\Delta n}{dt} = \frac{1}{\tau}\left(\Delta n_0 - \Delta n - \frac{I}{I_{sat}}\Delta n\right)$$

$$\frac{dI}{dt} = 2\epsilon_0 n_0 c_0 \left(A^* \frac{dA}{dt} + c.c\right)$$

we used

$$I_{sat} = \frac{\epsilon_0 c_0 n_0 \hbar^2}{d^2 \Gamma} \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - A}(\Gamma^2 + \delta^2)$$

$$I = 2\epsilon_0 n_0 c_0 |A|^2$$

$\Rightarrow$ Ⓑ $$\frac{dI}{dt} = \frac{I}{\tau_{cav}}\left(\frac{\Delta n}{\Delta n_{th}} - 1\right)$$

where $$\Delta n_{th} = \frac{n_0}{\sigma \tau_{cav} c_0} = \frac{\Pi}{\sigma L_{cav.}}$$

Π - intensity losses per cav. roundtrip

Eqns Ⓐ & Ⓑ constitute the so called Statz and de Mars equations

Adiabatic elimination of polarization reducing the equations only for the pop. in Δn and intensity $\Rightarrow$ Rate equation approximation.

Evolution equations in terms of <u>no of photons</u> and <u>atoms</u>:

It is useful to rewrite eqns in terms of no of photons inside the cavity and the no of atoms in the excited level.

<u>No of photons</u>. (intracav)
$$F = \frac{I}{\hbar \omega} \frac{n_0 L_{cav}}{c_0} \cdot S$$

S laser beam uniform section.

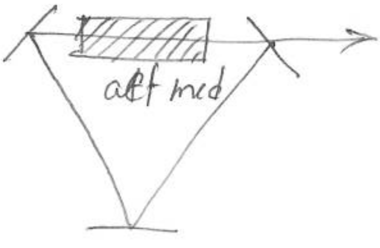
<u>Population</u> <u>inversion</u>
$$\Delta N = V_{cav} \Delta n = \underline{L_{cav} S,} \Delta n$$

With these new notations

$$\boxed{\begin{aligned} \frac{dF}{dt} &= -\frac{F}{\tau_{cav}} + \varkappa F \Delta n \\ \frac{d\Delta N}{dt} &= -\frac{1}{\tau}(\Delta N - \Delta N_0) - \varkappa F \Delta N \end{aligned}}$$

$$\varkappa = \frac{c_0}{n_0} \frac{\sigma}{V_{cav}}$$

Case where the active medium does not fill the cavity



Eqns remain valid with the following change.

(A) $\tau_{cav} = \frac{L_{cav,opt}/c_0}{\Pi} = \frac{\text{Duration cav roundtrip}}{\text{Losses per roundtrip}}$

$L_{cav,opt}$ — optical length of the cavity.

(B) $\Delta N = V_a \Delta n$ $\quad V_a$ — volume occupied by the laser mode in the active med.

Presence of the same term $\varkappa F \Delta N$ in both the eqns for F and ΔN implies
when a photon is emitted by stim. emission an atom gets deexited.

Till now we restricted ourselves to 4-level situation which was simple and enabled us to recover the heuristic equations we started off earlier.

The general case will be treated with the so called rate equations.

Rate equations : 3- and 4-level systems. General case.

↓

these equations describe the exchange of energy quanta between matter and light | without taking into account the phase coherence, nor the atomic coherence.

—— 2

—— 1

N_1, N_2 dimless population.

$$\frac{dN_2}{dt} = -\sum_{i\neq 2}\left(\gamma_{2i}^{r} + \gamma_{2i}^{nr}\right)N_2 - \varkappa F N_2 + \sum_{j\neq 2}\left(\gamma_{j2}^{r} + \gamma_{j2}^{nr}\right)N_j + \varkappa F N_1$$

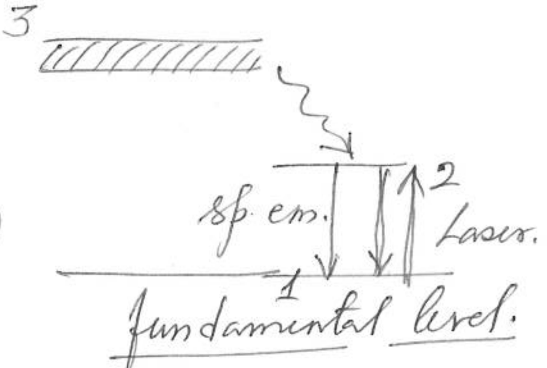
$$\frac{dN_1}{dt} = -\sum_{i\neq 1}\left(\gamma_{1i}^{r} + \gamma_{1i}^{nr}\right)N_1 - \varkappa F N_1 + \sum_{j\neq 1}\left(\gamma_{j1}^{r} + \gamma_{j1}^{nr}\right)N_j + \varkappa F N_2$$

$r \longrightarrow$ radiative
nr — non radiative. $\gamma_{ij} \longrightarrow$ decay rates from level i to level j

Solving such systems — almost impossible

$\Longrightarrow$ 3-level, 4-level systems.

Three-level system.



level 1 - fundamental level of considered atom.
level 2 - often (but not always) the first excited level.
level 3 - intermediate level to pump level 2.

3-level system closed.

$N_1 + N_2 = N$.

We suppose that decay of level 3 fast enough to make $N_3 = 0$.

In case when levels 1 and 2 are nondegenerate

$$\frac{dN_2}{dt} = W_p N_1 - A N_2 - \varkappa F \Delta N$$

$$\frac{dN_1}{dt} = -W_p N_1 + A N_2 + \varkappa F \Delta N$$

$N_1 + N_2 = N$

$N_1 - N_2 = -\Delta N$

$N_1 = \frac{N - \Delta N}{2}$

$N_2 = \frac{N + \Delta N}{2}$.

W_p — pumping probability / unit time

A — spont em. probability

$$\frac{d}{dt}(N_2 - N_1) = 2W_p N_1 - 2A N_2 - 2\varkappa F \Delta N$$

$$= 2W_p\left(\frac{N-\Delta N}{2}\right) - 2A\left(\frac{N+\Delta N}{2}\right) - 2\varkappa F \Delta N$$

$$\boxed{\frac{d\Delta N}{dt} = (W_p - A)N - (W_p + A)\Delta N - 2\varkappa F \Delta N},$$

Can be written in the usual form.

$$\frac{d\Delta N}{dt} = \frac{1}{\tau}\left(\Delta N_0 - \Delta N - \frac{I}{I_{sat}}\Delta N\right)$$

Provided. $\tau = (W_p + A)^{-1}$

$$\Delta N_0 = N\frac{W_p - A}{W_p + A}, \quad I_{sat} = \frac{\hbar\omega}{2\sigma\tau}.$$

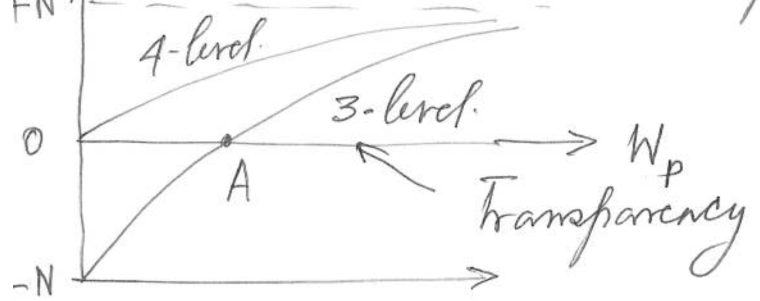
In the absence of the field ($I=0$)
steady state solution $\Delta N = \Delta N_0$

$$\Delta N_0 = N \frac{W_p - A}{W_p + A} \Longrightarrow \text{ for } W_p = 0 \Rightarrow \Delta N_0 = -N$$

all atoms in lower level.

In order to bleach the level, i.e, to make it transparent (one has to apply | pump rate $W_p = A$

$\Longrightarrow$ System turns into an amplifier $\Delta N_0 > 0$ iff $W_p > A$



Moreover, one notices a factor 2 in the eqn.

$$I_{sat} = \frac{\hbar\omega}{2\sigma\tau}.$$

this corresponds to a decrease in the saturation intensity by a factor 2. with respect to the 4-level system.

$\Longleftarrow$ Due to the fact that each emitted photon decreases N_2 by 1, increases N_1 by 1. and thus decrease ΔN by 2.

In terms of photon numbers and no of inverted atoms eqns for a 3-level laser

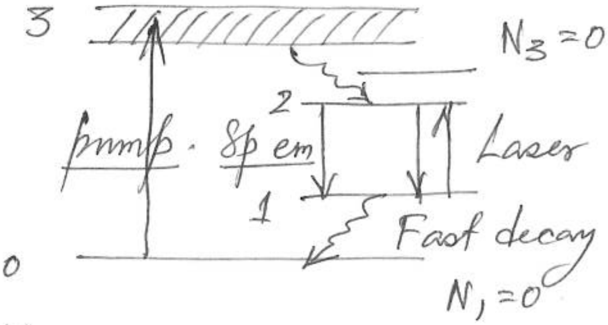
$$\frac{dF}{dt} = -\frac{F}{\tau_{cav}} + \varkappa F \Delta N$$

$$\frac{d\Delta N}{dt} = -\frac{1}{\tau}\left(\Delta N - \Delta N_0\right) + 2\varkappa F \Delta N$$

Four-level system.

Assume: levels ③ and ① decay fast enough.

$N_1 \sim N_3 = 0$



System is closed. $N_0 + N_2 = N$

$$\frac{dN_2}{dt} = W_p N_0 - A N_2 - \varkappa F N_2 \qquad N_0 + N_2 = N$$

$$\frac{dN_0}{dt} = - W_p N_0 + A N_2 + \varkappa F N_2$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = W_p N - (W_p + A)\Delta N - \varkappa F \Delta N$$

$$\Rightarrow \quad \frac{d\Delta N}{dt} = \frac{1}{\tau}\left(\Delta N_0 - \Delta N - \frac{I}{I_{sat}}\Delta N\right)$$

where

$$\boxed{\begin{aligned} \tau &= (W_p + A)^{-1} \\ \Delta N_0 &= N \frac{W_p}{W_p + A} \\ I_{sat} &= \frac{\hbar\omega}{\sigma\tau} \end{aligned}}$$

In absence of field $I = 0$ steady state $\Delta N = \Delta N_0$

$\Delta N_0 = 0$ if $W_p = 0$.

as soon as $W_p > 0$ $\Rightarrow$ medium exhibits gain, $\Delta N_0 > 0$

shown in fig.

NO factor 2. since st. emission of photon decreases ΔN by 1 since level 1 remains always empty.

$2^* = 1$ 4-lev.
$= 2$ 3-lev.

Standard Eqn.

$$\frac{dF}{dt} = - \frac{F}{\tau_{cav}} + \varkappa F \Delta N$$

$$\frac{d\Delta N}{dt} = - \frac{1}{\tau}(\Delta N - \Delta N_0) - 2^* \varkappa F \Delta N$$

Inclusion of spont. emission.

$$\frac{dF}{dt} = -\frac{F}{\tau_{car}} + \kappa\left[(F+\underline{\underline{1}})N_2 - F N_1\right]$$

↳ comes from Q. Mech.

Pumping scheme -

(a) Radiative pumping. (Optical pumping)

flash lamp, ion lasers Ar^+, Kr^+ N_2 laser excimer lasers.

Example: Ruby, Nd-YAG. (Nd^{3+}; $Y_3Al_2O_3$) Yittrium alum. garnet.

(b) Electronic pumping. using DC or r.f. electric discharge.

Ex: He-Ne; He-Cd., Ar^+, Kr^+ N_2-laser., excimer lasers, CO_2 las.

(c) Thermal pumping. Hydrodynamic expansion CO_2 laser, CO laser.

(d) Chemical pumping. by exothermic chemical combustion, or by fast combustion. (I, HF, HCl, HBr; CO.)

(e) Pumping by injection of carriers. $\Longrightarrow$. current thro' p-n junction semiconductor lasers.

(f) pumping by Heavy particles; Ion beam, fission products etc.