Bose-Einstein condensation
meets
cavity QED

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ICTS lecture, May 2021
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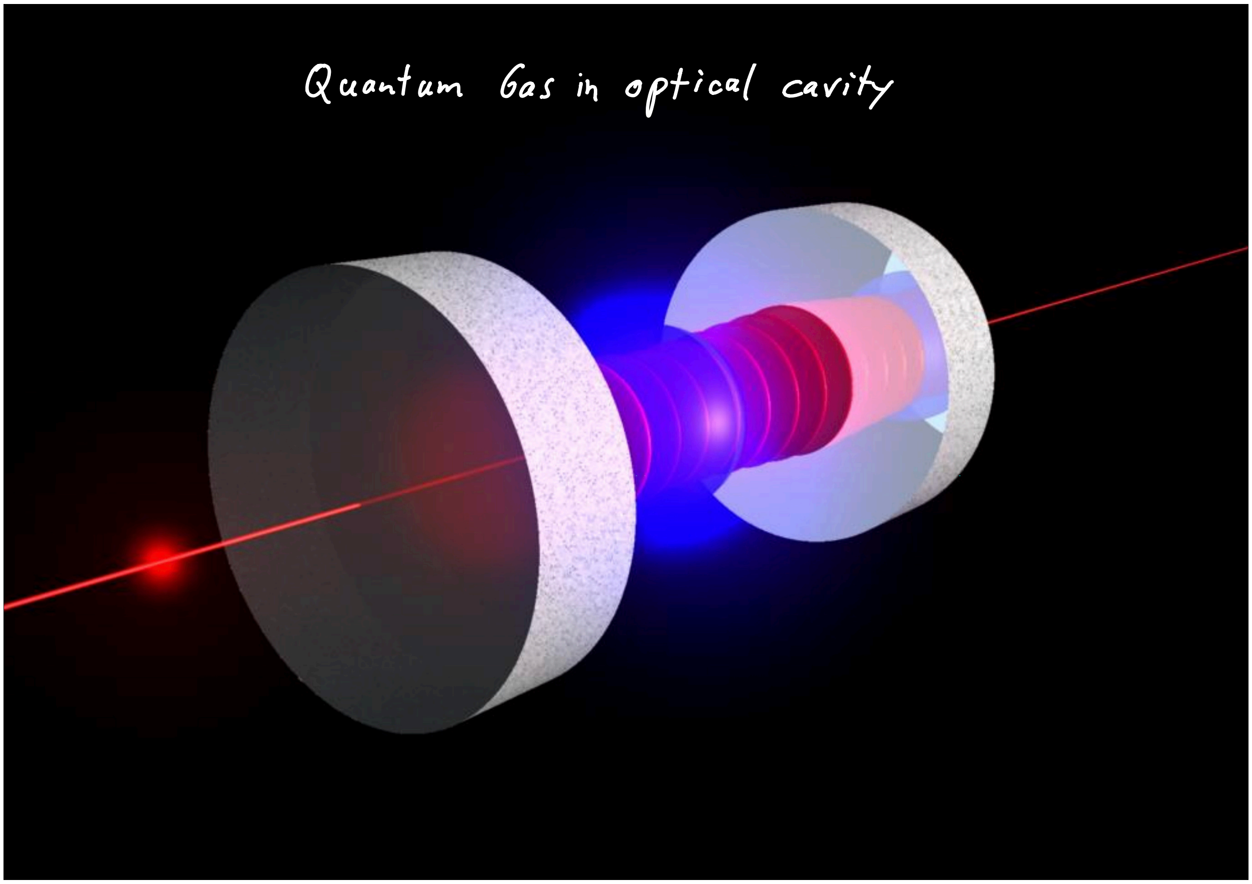


Q: - what do you associate?
- type of physics?
- degrees of freedom?
- types of excitations?
- character of interaction?
- energy scales?
- role of Bose-Einstein condensation?
- different regimes?

Overview:

1. Single atom in Cavity
2. Coupled BEC - Cavity system
3. Selforganisation and the Dicke transition
4. Recent experiments
   - → Supersolids
   - → Dissipation induced instability

Literature:

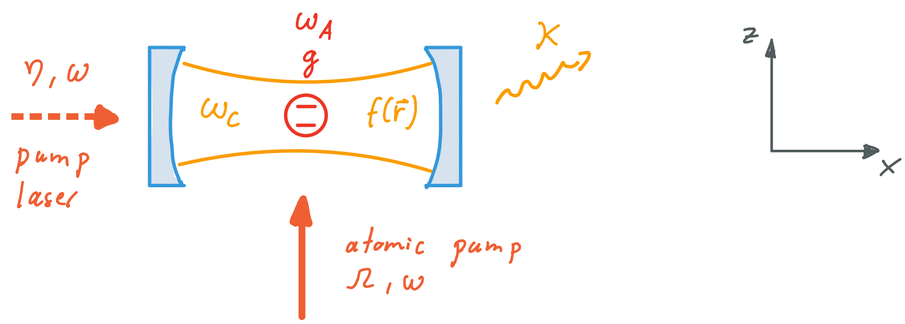
- Exploring the Quantum, Atoms, Cavities and Photons
  S. Haroche and Jean-Michel Raimond, Oxford Press

- H. Ritsch, P. Domokos, F. Brennecke, T. Esslinger
  Cold atoms in cavity-generated dynamical optical potentials
  Reviews of Modern Physics 85, 553 (2013).
  arXiv 1210.0013

1. Single Atom in a Cavity

$$H = H_{JC} + H_{mech} + H_{pump}$$

1.1 A two-level atom in a cavity



Cavity: - a^+ :

- ω_c :

- $f(\vec{r})$:

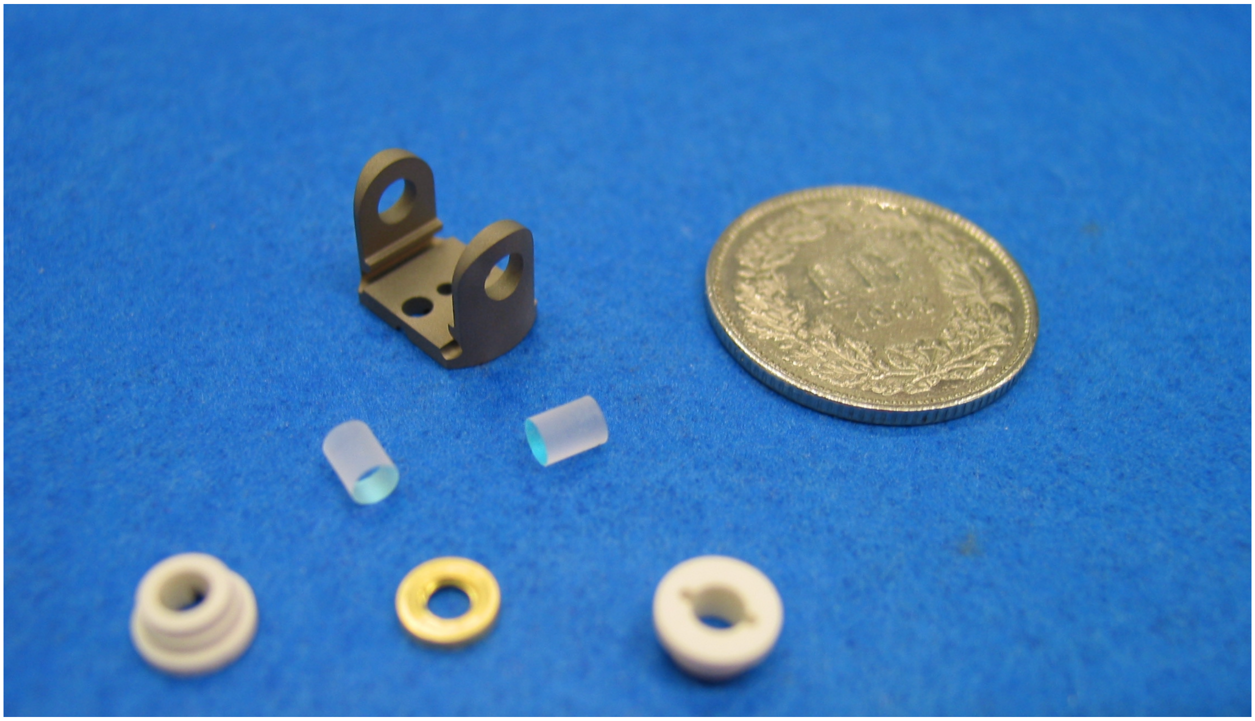
- $g = d\sqrt{\frac{\hbar\,\omega_c}{2\,\varepsilon_0\,V}}$

$V = \int d^3\vec{r}\,|f(\vec{r})|^2$

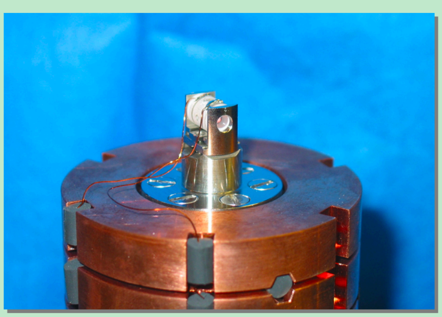
d: dipole along cavity mode polarisation

effective cavity mode volume

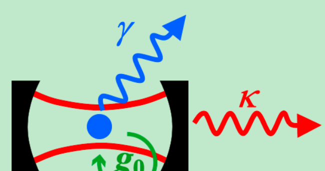
- κ :



High-Finesse Optical Cavity



Length: L=178 µm
Mode waist: w=26 µm
Finesse: F=300.000



g_0 = 2π 10.4 MHz
γ = 2π 3.0 MHz
κ = 2π 1.4 MHz

Atom:

$|e\rangle$

$|g\rangle$

γ:

$\sigma^+ = |e\rangle\langle g|$

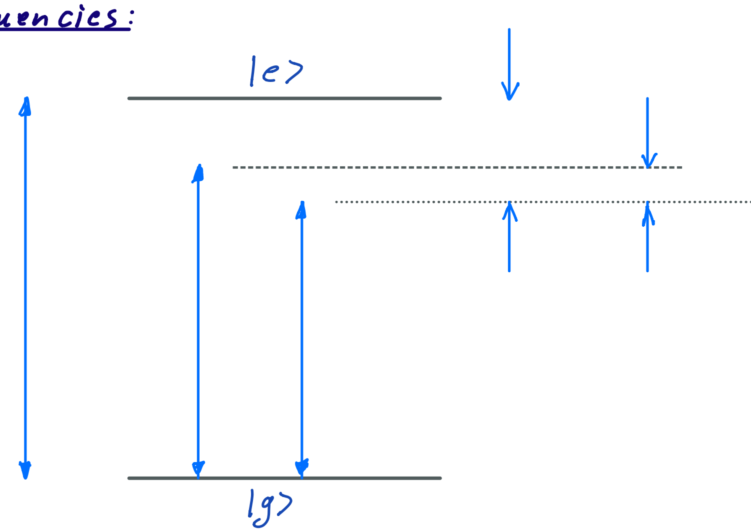
$$\omega_R \equiv \frac{\hbar k^2}{2m}$$

$\sigma_z = \frac{1}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$

$\sigma^+ = \sigma_x + i\sigma_y$

$k = \frac{\omega}{c}$

Frequencies:



ω_A : atomic transition frequency

ω_c : cavity resonance frequency (closest to pump)

ω : pump laser frequency

detunings: $\Delta_A = \omega - \omega_A$

$\Delta_c = \omega - \omega_c$

Jaynes - Cummings Hamiltonian

$$\frac{H_{JC}}{\hbar} = -\Delta_C a^+ a - \Delta_A(\vec{r})\sigma^+\sigma^- + i g\left(\sigma^+ a f(\vec{r}) - f^*(\vec{r}) a^+ \sigma\right)$$

-
- $\Delta_A(\vec{r}) = \Delta_A - \Delta_S(\vec{r})$

dipolar approximation

rotating wave approximation

differential AC-Starkshift by auxiliary field, e.g. optical trap

External degrees of freedom

$$H_{mech} = \frac{\vec{p}^2}{2m} + V_{cl}(\vec{r})$$

Pump:

$$\frac{H_{pump}}{\hbar} = \underbrace{i\eta(a^+ - a)} + \underbrace{i\Omega h(\vec{r})(\sigma^+ - \sigma)}$$

Dissipation:

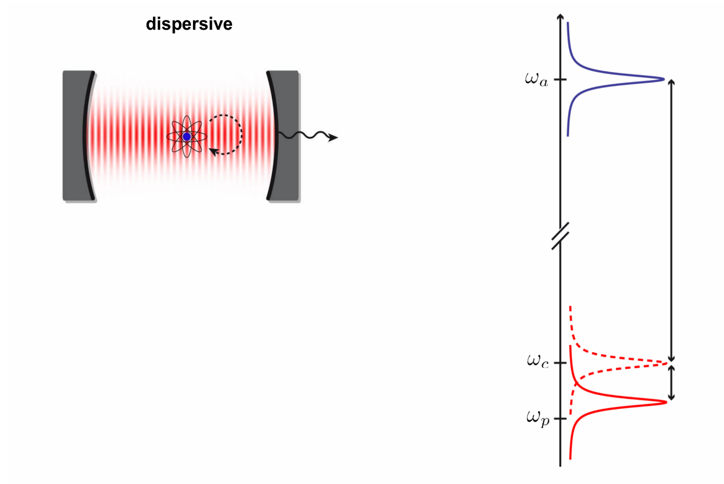
$$\dot{\rho} = -\frac{i}{\hbar}[\mathcal{H}, \rho] + \mathcal{L}\rho$$

Heisenberg-Liouville eqn.

ρ: density operator

$\mathcal{L}$: Liouville operator in Born-Markov approx.

1.2 Dispersive Regime



Equations of motion:

$$\dot{\sigma}^- = i\,\Delta_A \sigma^- - g\, f(\vec{r})\, 2\sigma_z a - \Omega h(\vec{r})\, 2\sigma_z - \gamma \sigma^-$$ atomic polarization

$$\dot{a} = i\,\Delta_C a - g\, f(\vec{r})\,\sigma^- - \kappa a + \eta$$ cavity field

- low saturation and large detuning

$\sigma_z = \frac{1}{2}\left(|e\rangle\langle e| - |g\rangle\langle g|\right)$

- atoms driven from transvers direction ($\Omega \neq 0$)
  determine steady state:

$$\dot{\sigma}^- = i\,\Delta_A \sigma^- - g\, f(\vec{r})\, 2\sigma_z a - \Omega h(\vec{r})\, 2\sigma_z - \gamma \sigma^-$$

$$\rightarrow \sigma^- = \frac{g f(\vec{r}) a + \Omega h(\vec{r})}{-i\Delta_A + \gamma}$$

later we will neglect γ

"atomic polarization is slaved to the cavity mode"

- cavity field

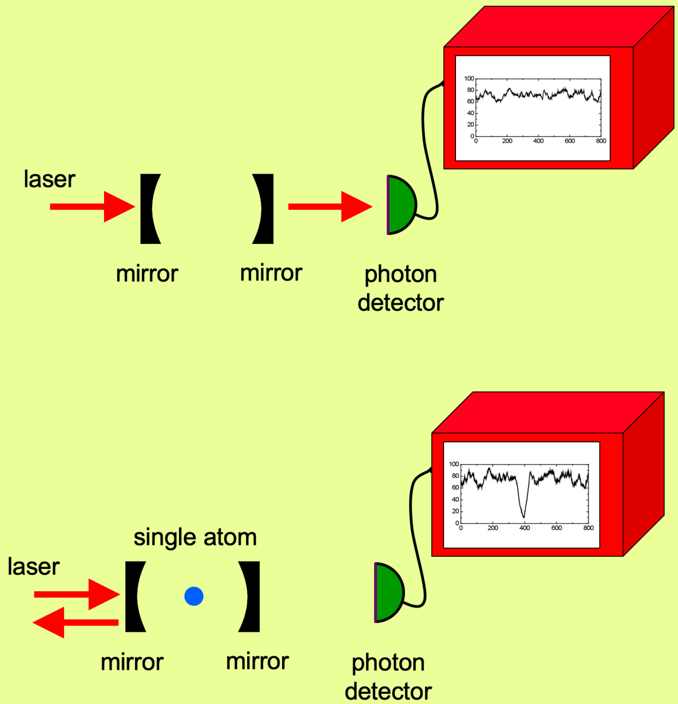
$$\dot{a} = i\Delta_C a - \frac{g^2 f^2(\vec{r}) a + \overbrace{g\Omega f(\vec{r}) h(\vec{r})}}{-i\Delta_A + \gamma} - \kappa a + \eta$$

<u>Effective Hamiltonian for dispersive coupling ($\kappa = 0$)</u>

- shares same equations of motion

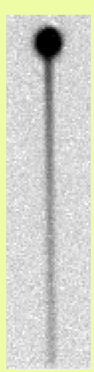
$$\frac{H_{eff}}{\hbar} = -(\Delta_C - U_0 |f(\vec{r})|^2) a^\dagger a + \eta_{eff} h(\vec{r}) (f^*(\vec{r}) a^\dagger + f(\vec{r}) a)$$

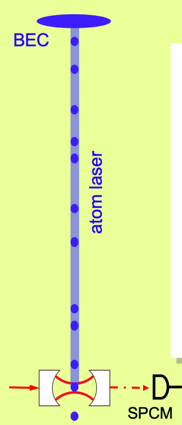
Atom Counter

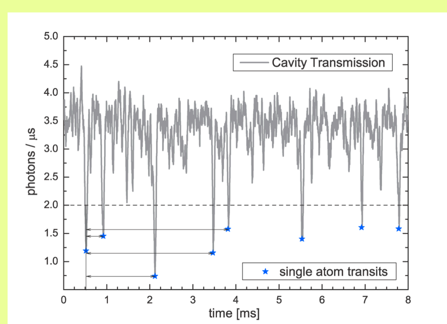


H. Mabuchi et al., Opt. Lett. **21**, 1393 (1996)

Single Atom Transits



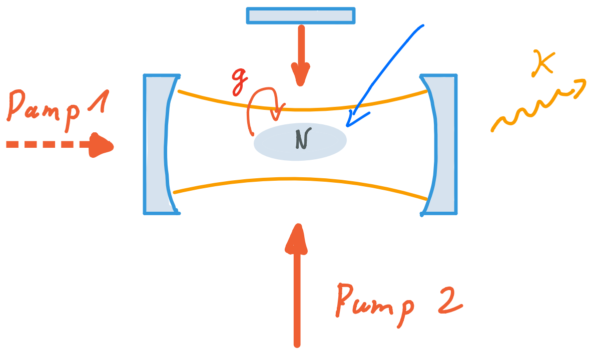


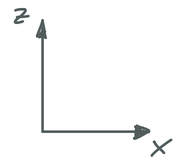


$$g^{(2)}(\tau) = P_c(t+\tau\,|\,t) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I\rangle^2}$$

A. Öttl, S. Ritter, M. Köhl, and T. Esslinger, Phys. Rev. Lett. 95, 090404 (2005).

2.1 Coupled BEC-cavity system





$$H = H_A + H_C + H_{AC}$$

Atoms:

- large detuning $\rightsquigarrow$

- atoms described by matter-wave field operator

$$H_A = \int d^3\vec{r}\, \Psi^\dagger(\vec{r}) \left[\mathcal{H}^{(1)} + \underbrace{\frac{U}{2} \Psi^\dagger(\vec{r})\, \Psi(\vec{r})}_{} \right] \Psi(\vec{r})$$

$$\Psi^\dagger(\vec{r}) = \sum_k \psi_k^*(\vec{r})\, c_k^\dagger$$

complete orthonormal basis of single-particle wave function

a_s: s-wave scattering length

Cavity:

- single mode
- (driven by coherent field)

$$H_c = -\hbar \Delta_c a^\dagger a \left(+ i \hbar \eta (a^\dagger - a)\right)$$

rotating frame: ω

$\Delta_c = \omega - \omega_c$

Interaction:

$$H_{AC} = \int d^3r \, \underline{\Psi}(\vec{r}) \Big[\underbrace{\qquad\qquad\qquad}_{\mathrm{I}} + \underbrace{\qquad\qquad\qquad}_{\mathrm{II}} \Big] \Psi(\vec{r})$$

I: light shift

maximum atomic light shift for a single intracavity photon

II: scattering:

-
-

maximum scattering for single atom

Ω: Rabi frequency of transverse pump laser

Dissipation:

Heisenberg - Langevin equation

$$\frac{d}{dt} a = - i [a, H] - \kappa a + \xi$$

a: operator, not expectation value

ξ: Gaussian noise operator maintains commutation relation for photon operators in presence of cavity decay

$T = 0$: $\langle \xi(t) \xi^\dagger(t') \rangle = 2\,\delta(t-t')$

(other dissipation channels via atomic cloud)

Heisenberg:
$|\psi^H\rangle = e^{iHt} |\psi(t)\rangle$

Mean field description

- cavity field mediates global coupling
  → mean field approach suitable

$\hat{\Psi}(\vec{r},t)$ is in Heisenberg representation

α : coherent cavity field $\alpha = \langle a \rangle$

N_c : number of condensate atoms

$\varphi(\vec{r})$: condensate wave function (normalized to 1)

- zeroth order in fluctuations

$$i\hbar \frac{\partial}{\partial t} \varphi(r,t) = \Big[-\frac{\hbar^2 \nabla^2}{2m} + V_0(\vec{r}) + N_c u |\varphi(\vec{r},t)|^2$$

$$+$$

$$+ \qquad \Big] \varphi(\vec{r},t)$$

-
-

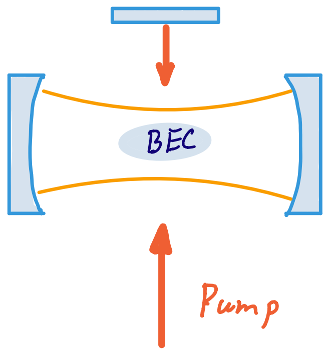
$$i\frac{\partial}{\partial t}\alpha(t) = \Big[-\Delta_c + \overbrace{N_c U_0 \langle\cos^2(kz)\rangle} - i\kappa\Big]\alpha(t)$$

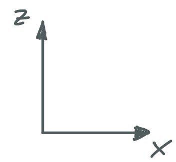
$$(+\, i\eta) \quad + N_c\, \eta_{eff} \langle \cos(kx)\cos(kz)\rangle$$

using $\langle f(\vec{r})\rangle = \int d^3r\, f(\vec{r})\, |\psi(\vec{r},t)|^2$

- inclusion of fluctuations to leading order:

2.2 Self-organisation of Bose-Einstein condensate and the Dicke phase transition



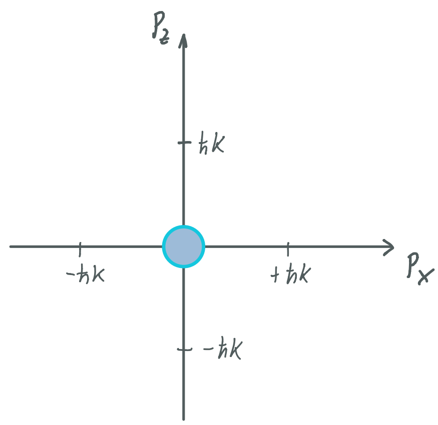


neglect lattice potential due to pump beam

$$H = H_C + H_A + H_{AC} \qquad \text{with } \eta = 0$$

Two-mode expansion:

In momentum space:



Insert $\Psi = \psi_0 \hat{c}_0 + \psi_1 \hat{c}_1$ into many-body Hamiltonian, we obtain

- neglect atomic collisions

up to a constant term

$$\frac{\hat{H}}{\hbar} = \underbrace{\omega_0 \hat{J}_z + \omega a^\dagger a + \frac{\lambda}{\sqrt{N}} (a^\dagger + a)(\hat{J}_+ + \hat{J}_-)}_{\mathrm{I}:} + \underbrace{U_0 M c_1^\dagger c_1 a^\dagger a}$$

- describes coupling between N two-level systems with transition frequency $\omega_0 =$ and a bosonic field mode with frequency $\omega = -\Delta_c + NU_0/2$

- bosonic mode operators:

- total atom number :

- ladder operator :
(creation of collective excitation)

collective spin :

- collective coupling strength:

- Hamiltonian is invariant under parity transformation:

(Hepp-Lieb)

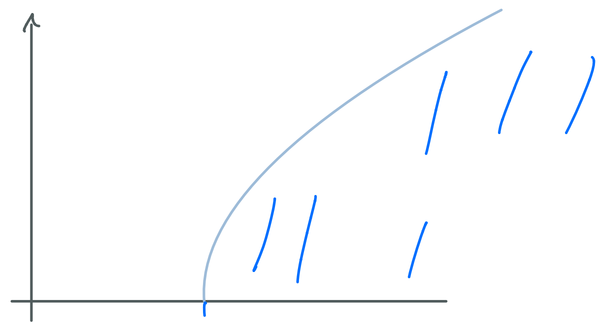
Thermodynamic properties of Dicke model: Dicke phase transition

1973: Hepp + Lieb, Wang + Hioe } RWA

2007: Carmichael n RWA + open system

Dicke model exhibits a 2nd order phase transition in the thermodynamic limit from a normal to a phase.

Phase diagram:



RWA: $\sqrt{\omega_0 \omega}$

normal phase:

superradiant phase:

checkerboard density