V. Matching of the 2-Jet Current

As a very important application of our formalism, we now reconsider the process $e^+e^- \to \gamma^* \to 2$ jets. What is the proper representation of the vector current $\bar{\psi}\gamma^\mu\psi$ in SCET? The naive guess

$$\bar{\psi}\gamma^\mu\psi \;\longrightarrow\; \underbrace{\bar{\xi}_{\bar{n}}}_{\text{anti-collinear quark}}\,\gamma_\perp^\mu\,\underbrace{\xi_n}_{\text{collinear quark}}$$

is not gauge invariant in SCET, because ξ_n and $\xi_{\bar{n}}$ transform under <u>different</u> sets of residual gauge transformations (see reading assignment in part 2).

A gauge-invariant current operator is:

$$(\bar{\xi}_{\bar{n}}\,W_{\bar{c}})(0)\;\gamma_\perp^\mu\;(W_c^\dagger\,\xi_n)(0)$$

To see this, note that:

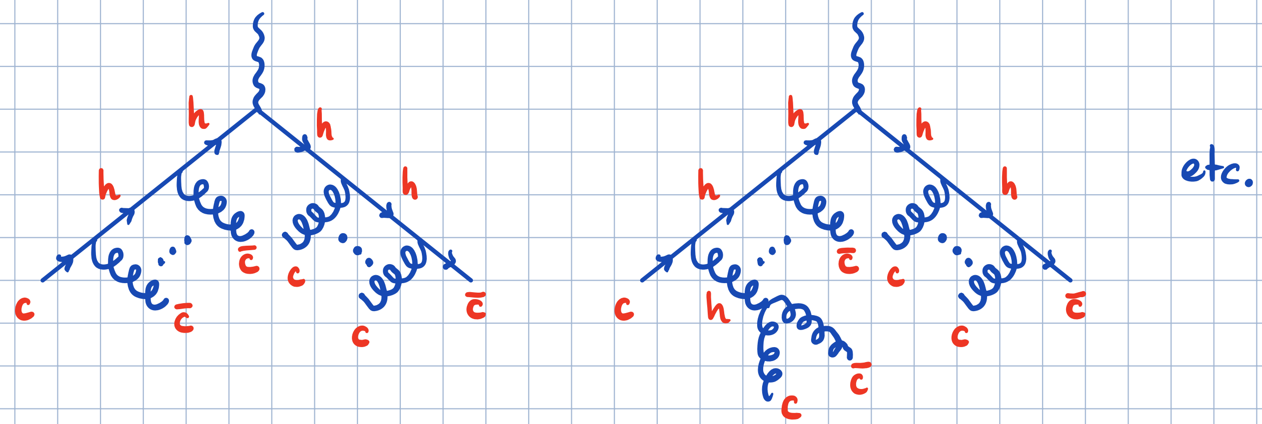
$$W_c^\dagger(x)\,\xi_n(x) \xrightarrow{U_c} W_c^\dagger(x)\,U_c^\dagger(x)\,U_c(x)\,\xi_n(x)$$
$$= W_c^\dagger(x)\,\xi_n(x)$$

$$\xrightarrow{U_{us}} U_{us}(x_-)\,W_c^\dagger(x)\,U_{us}^\dagger(x_-)\,U_{us}(x_-)\,\xi_n(x)$$
$$= U_{us}(x_-)\,W_c^\dagger(x)\,\xi_n(x)$$

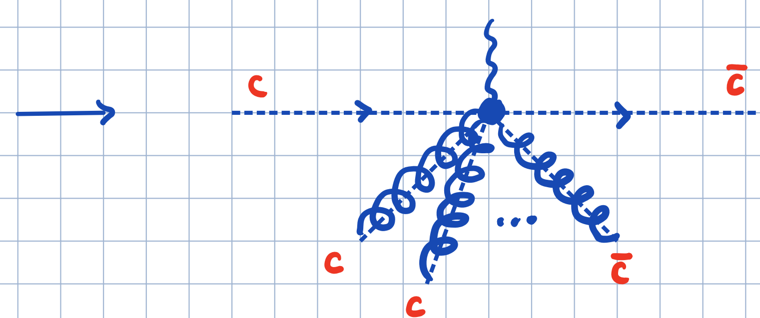
The anti-collinear field $\bar{\xi}_{\bar{n}}W_{\bar{c}}(x)$ transforms in a similar way.

For the special choice $x=0$ the operator is invariant under all three types of gauge transformations ($c, \bar{c}$, us). The case $x \neq 0$ is a bit more tricky, since it requires a multipole expansion of the collinear and anti-collinear fields themselves (↪ see problem set 2).

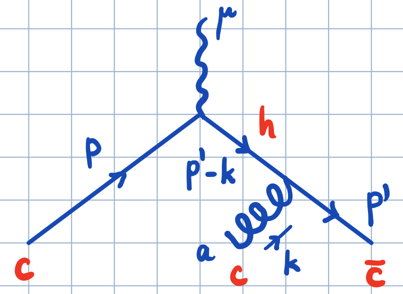
The (anti-) collinear Wilson lines are required by gauge invariance, but what do they represent physically? In fact, they account for an infinite set of QCD graphs of the type:



Despite the many hard propagators, these diagrams contain leading-power pieces in λ, which in the EFT are accounted for by the Wilson lines.



To see how this works, consider the attachment of a single collinear gluon to an anti-collinear quark:



$$p' \sim (1, \lambda^2, \lambda)$$

$$k \sim (\lambda^2, 1, \lambda)$$

$$\epsilon \sim (\lambda^2, 1, \lambda)$$

$$= \bar u(p')\, i g_s\, \not\epsilon(k)\, t_a\, \frac{i(\not p' - \not k)}{(p'-k)^2}\, \gamma^\mu u(p)$$

$$\underset{\lambda^2}{p'^2} + \underset{\lambda^2}{k^2} - \big(\underset{\lambda^0}{n\cdot p'}\; \underset{\lambda^0}{\bar n\cdot k} + \underset{\lambda^2}{\bar n\cdot p'}\; \underset{\lambda^2}{n\cdot k} + 2\underset{\lambda}{p'_\perp}\cdot \underset{\lambda}{k_\perp}\big)$$

$$= \bar u(p')\, i g_s \Big(\underset{\lambda^0}{\bar n\cdot\epsilon}\, \frac{\not n}{2} + \underset{\lambda^2}{n\cdot\epsilon}\, \frac{\not{\bar n}}{2} + \underset{\lambda}{\not\epsilon_\perp} \Big)\, t_a\, \frac{i}{(p'-k)^2}$$

$$\times \Big(\bar n\cdot(\underset{\lambda^2}{p'} - \underset{\lambda^0}{k})\, \frac{\not n}{2} + n\cdot(\underset{\lambda^0}{p'} - \underset{\lambda^2}{k})\, \frac{\not{\bar n}}{2} + \underset{\lambda}{(\not p' - \not k)_\perp} \Big)\, \gamma^\mu\, u(p)$$

$$= \bar u(p')\, \underbrace{\frac{\not n \not{\bar n}}{4}}_{P_n = \overline{P_{\bar n}}}\, (-g_s t_a)\, \frac{\bar n\cdot\epsilon\; \cancel{n\cdot p'}}{-\cancel{n\cdot p'}\; \bar n\cdot k}\, \gamma^\mu\, u(p) + \text{power-suppressed terms}$$

$$\gamma^\mu = \not n\, \frac{\bar n^\mu}{2} + \cancel{\not{\bar n}\, \frac{n^\mu}{2}} + \gamma_\perp^\mu$$

drop

drop, since $\not n\, u(p) \approx 0$ is suppressed

$$= \underbrace{\bar u(p')\, \overline{P_{\bar n}}}_{\text{from } \bar\xi_{\bar n} = \bar\psi_{\bar c}\, \overline{P_{\bar n}}} \boxed{\left(\frac{g_s t_a\, \bar n\cdot\epsilon(k)}{\bar n\cdot k} \right)}\, \gamma_\perp^\mu\, \underbrace{P_n\, u_n(p)}_{\text{from } \xi_n = P_n \psi_c} + \ldots$$

The highlighted factor is nothing but the one-gluon matrix element of the collinear Wilson line $W_c^\dagger$:

$$\langle 0|\, W_c^\dagger(0)\,|\,\epsilon, k\rangle = \langle 0|\, \bar{\mathrm{P}} \exp\left(-ig_s \int_{-\infty}^{0} dt\; \bar{n}\cdot A_c(t\bar{n})\right) |\,\epsilon, k\rangle$$

opposite ordering than for W_c

$$= -ig_s\, t_a\, \bar{n}\cdot\epsilon(k) \int_{-\infty}^{0} dt\; e^{-it\,\bar{n}\cdot k} = \frac{g_s\, t_a\, \bar{n}\cdot\epsilon(k)}{\bar{n}\cdot k} \quad ✓$$