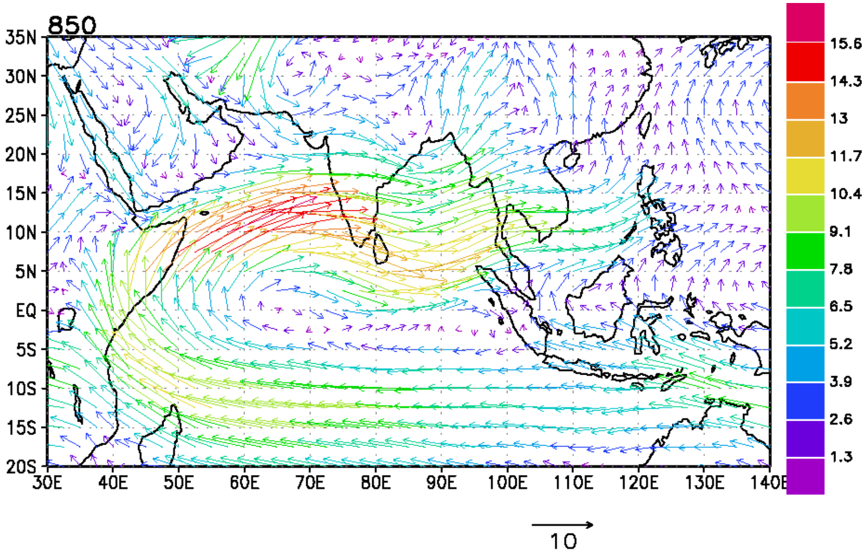
Forced motion in the tropics; Some simple examples

Lecture-4

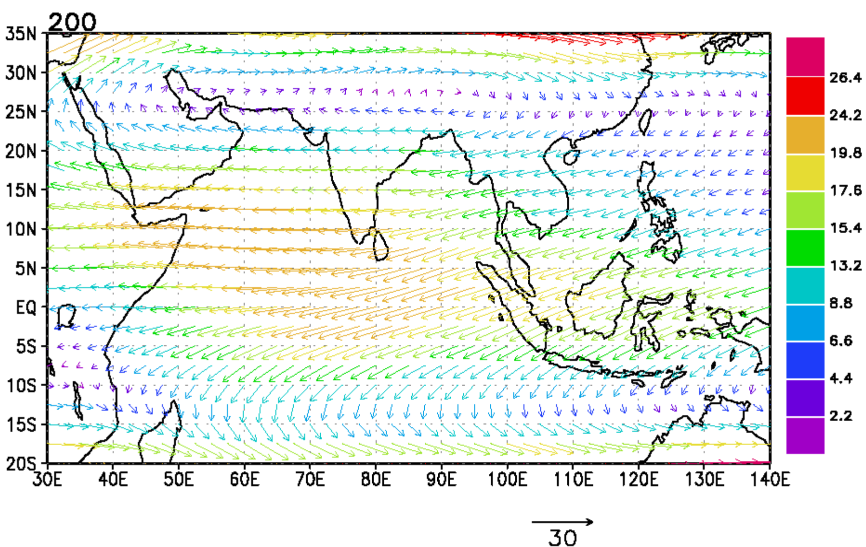
Recap of some Properties of free Equatorial Waves

May, 10, 2022

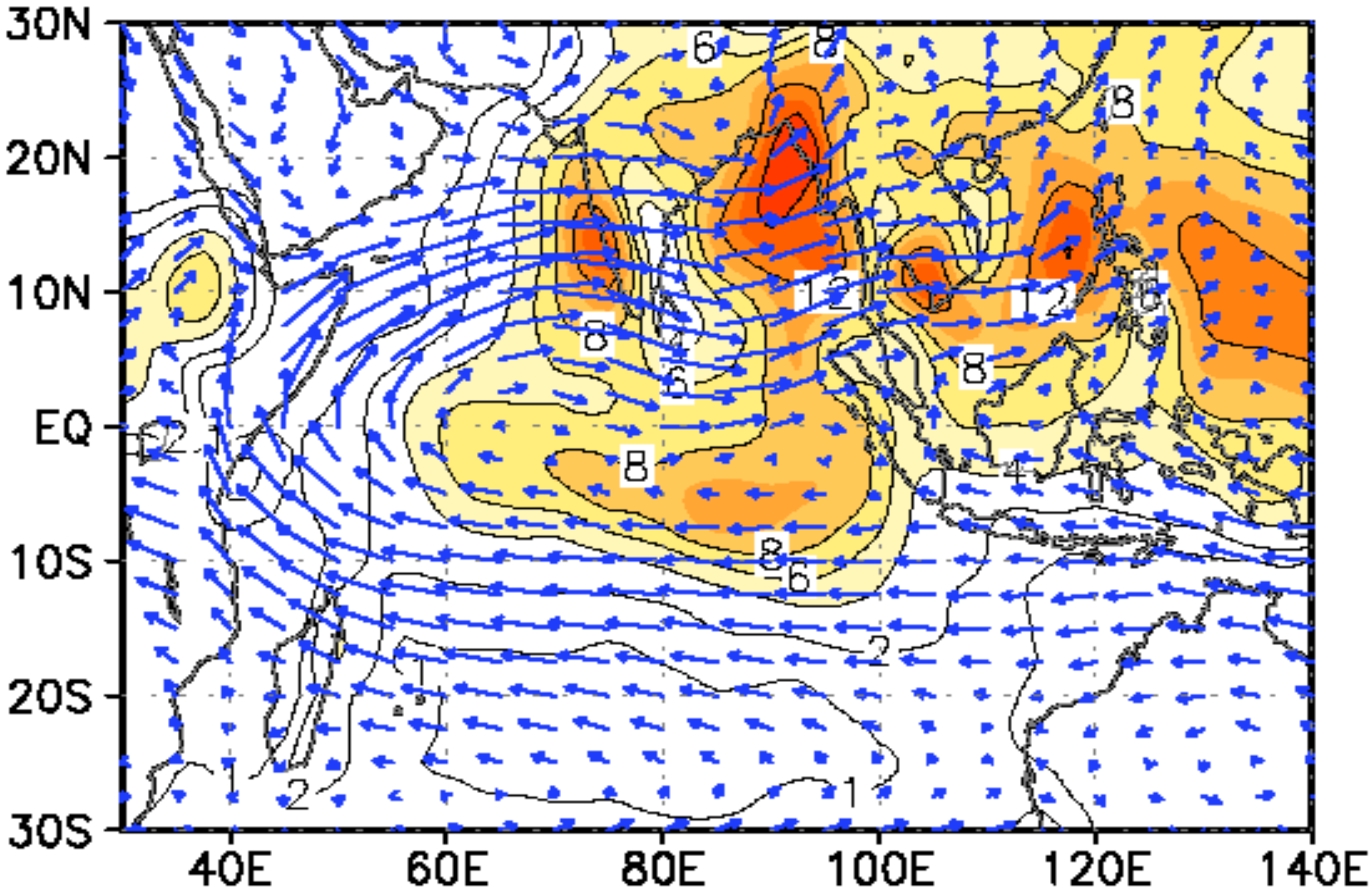
Some examples of forced motion in the atmosphere. Indian monsoon circulation is a classic example



Low level, cross-equatorial flow, south-westerlies and westerly jet over the Arabian sea



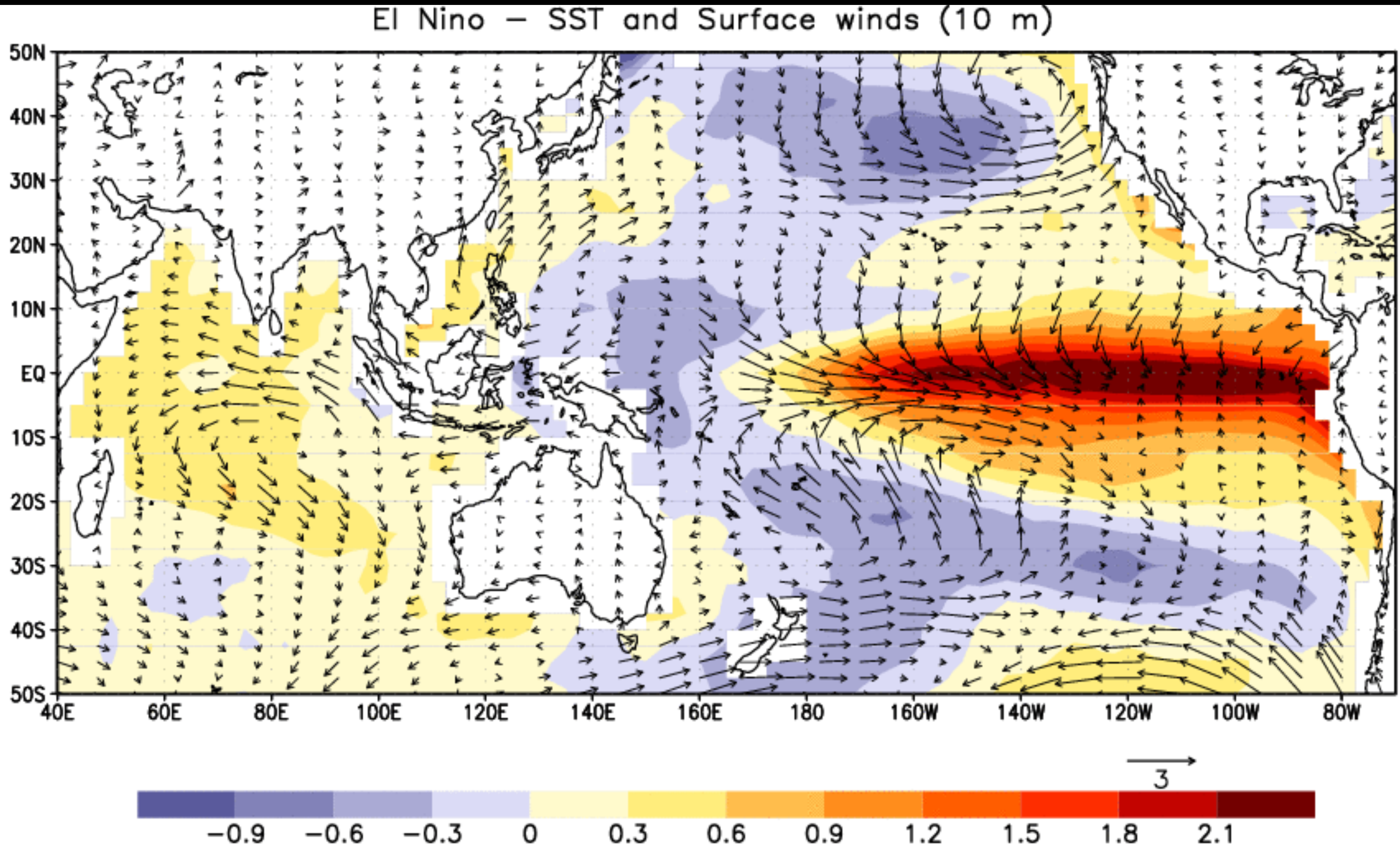
Upper level easterlies, monsoon easterly jet



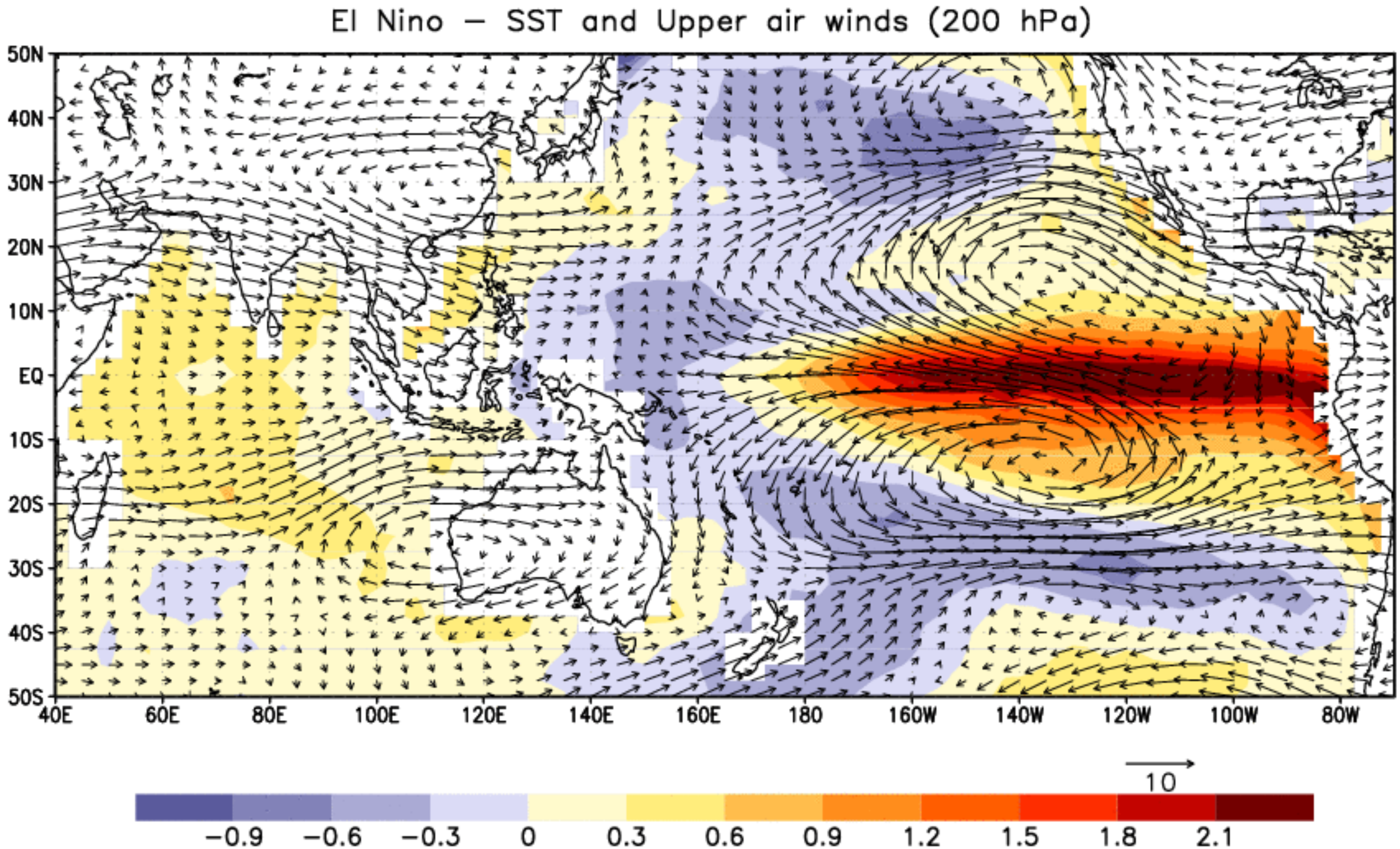
Seasonal mean precipitation (color) and 850 hPa wind vectors during JJAS over the Indian monsoon region.

➔Note that winds are attracted to the rain centers and tries to around them in an anticlockwise manner, consistent with winds being forced by heating associated with the rain

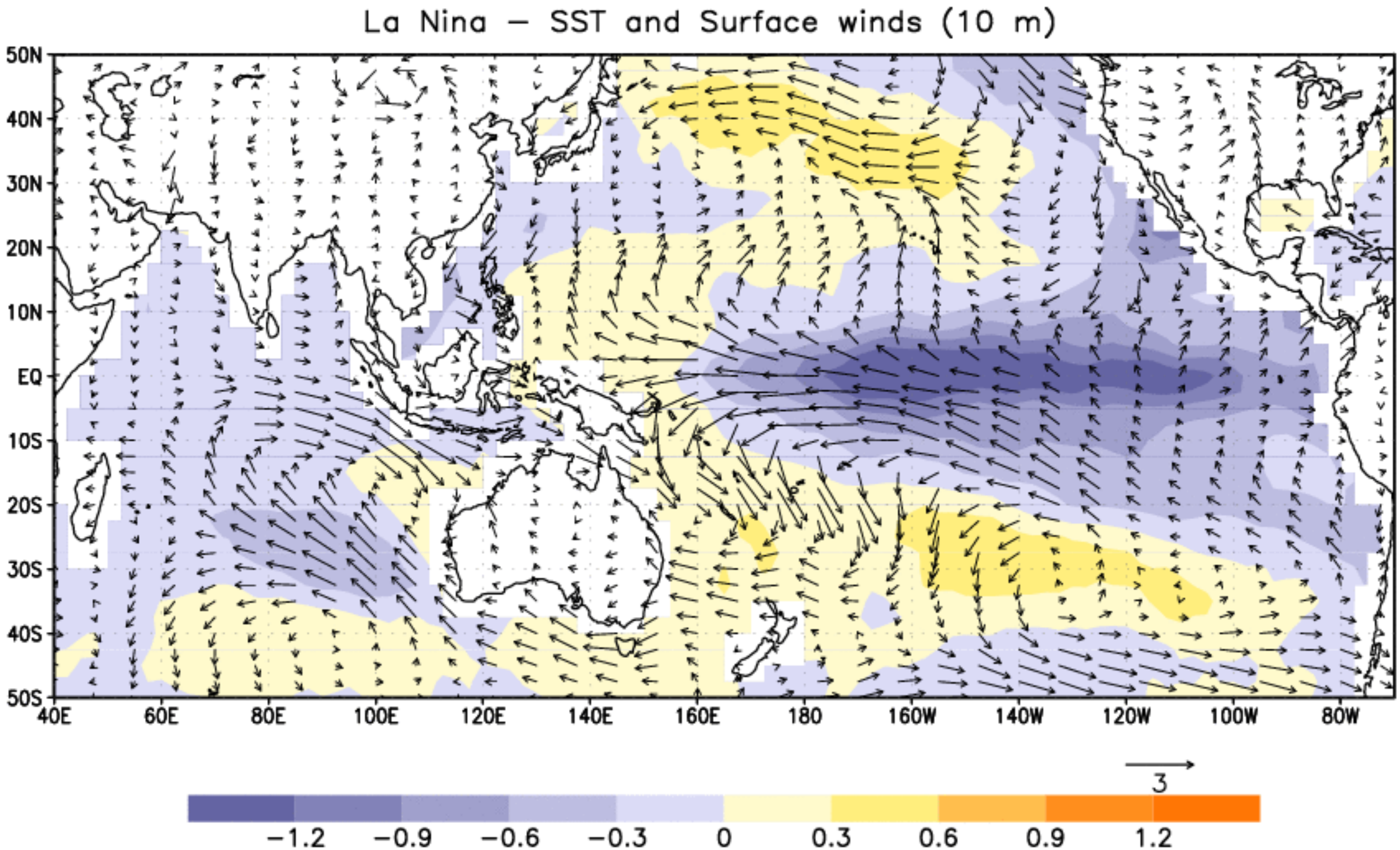
Atmospheric winds associated with ENSO are another example



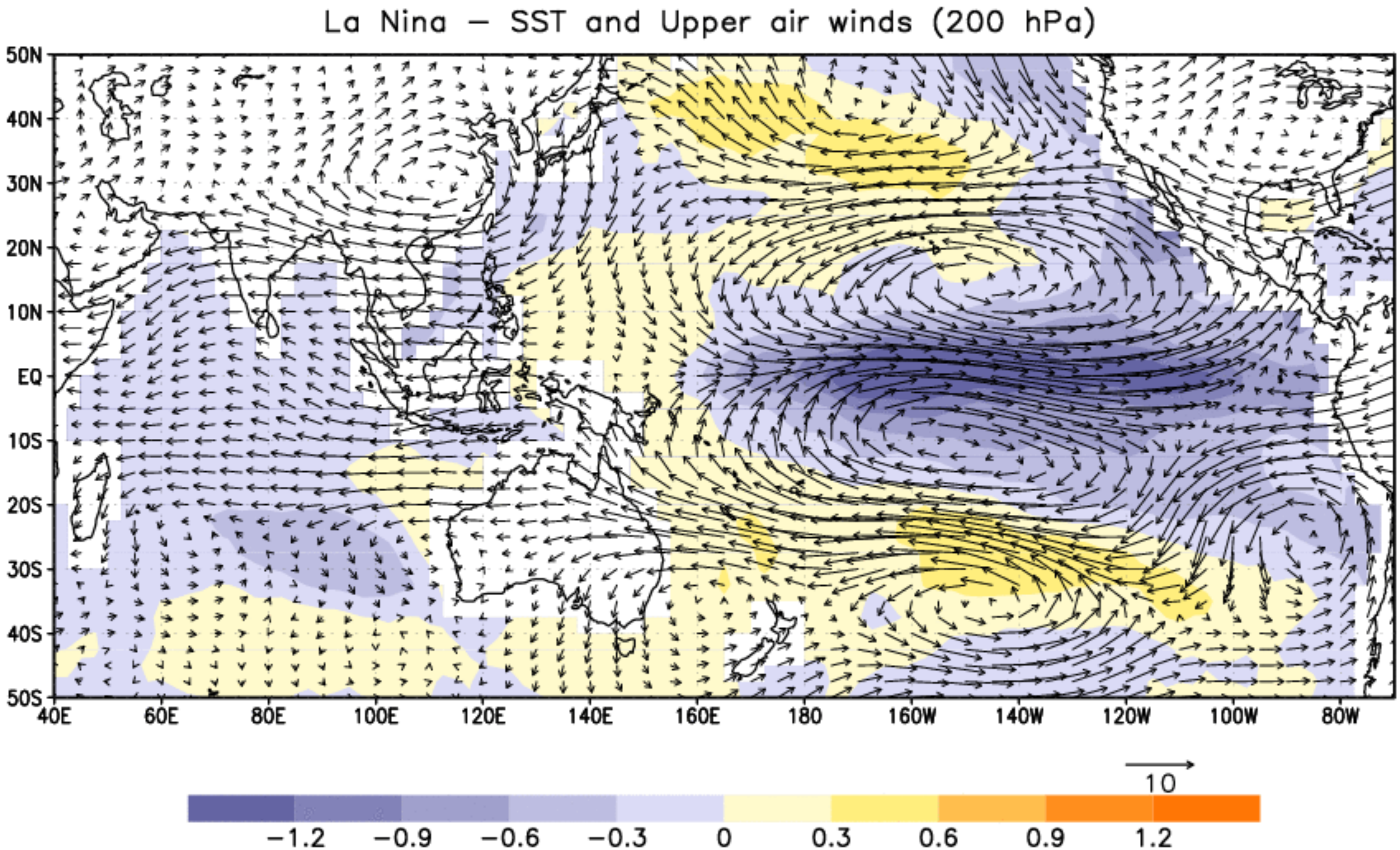
Composite of SST anomalies (shaded) and surface wind vector anomalies associated with a number of El Nino events (during DJF)



Composite of SST anomalies (shaded) and wind vector anomalies associated with a number of El Nino events at 200 hPa (during DJF)



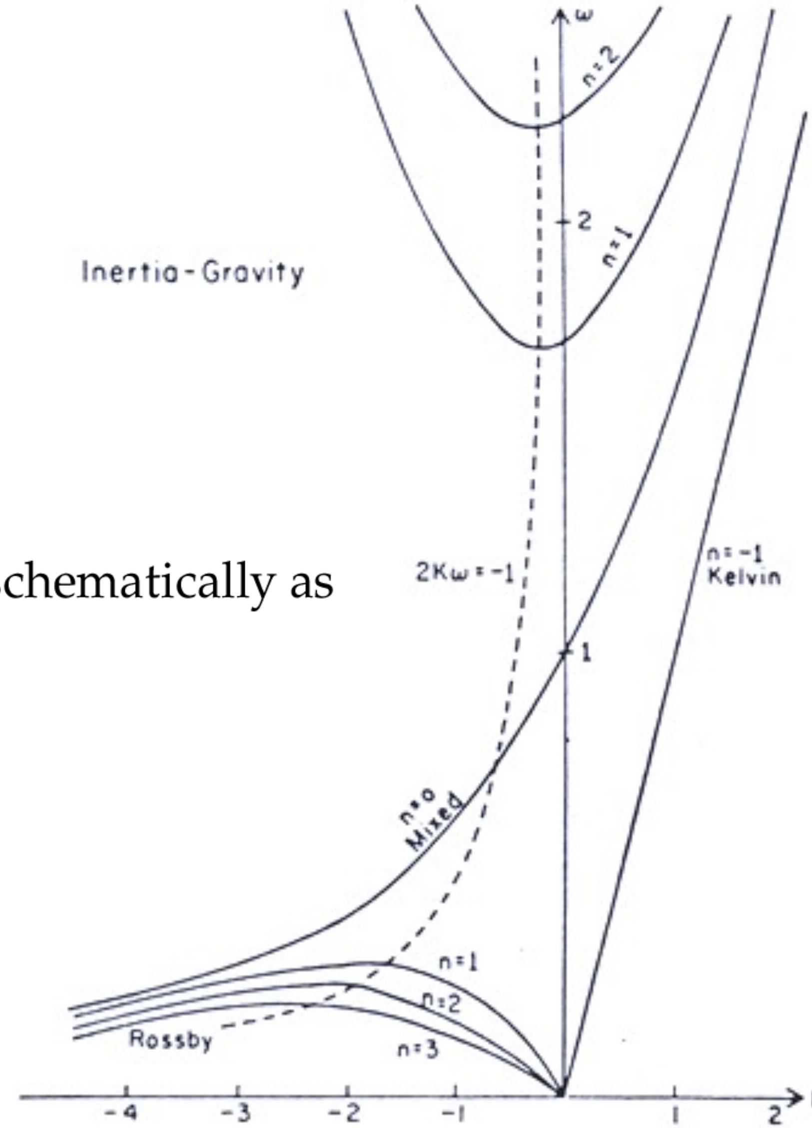
Composite of SST anomalies (shaded) and surface wind vector anomalies associated with a number of La Nina events (during DJF)



Composite of SST anomalies (shaded) and wind vector anomalies associated with a number of La Nina events at 200 hPa (during DJF)

Let us recap some important characters of the Equatorial Waves

The dispersion relation may now be shown schematically as



Spatial structure of the Equatorial Waves

Once v is known , the other eigen functions u and ϕ may be easily obtained.

For $n \geq 1$, Rossby and I G Wave

$$v_n = R_e A_n \psi_n(\eta) e^{i(kx-\sigma t)}$$

$$u_n = R_e i A_n \left[\left(\frac{n}{2}\right)^{1/2} \frac{\psi_{n+1}}{\sigma + k} - \left(\frac{n+1}{2}\right)^{1/2} \frac{\psi_{n-1}}{\sigma + k} \right] e^{i(kx-\sigma t)}$$

$$\phi_n = R_e i A_n \left[\left(-\frac{n}{2}\right)^{1/2} \frac{\psi_{n-1}}{\sigma + k} - \left(\frac{n+1}{2}\right)^{1/2} \frac{\psi_{n+1}}{\sigma + k} \right] e^{i(kx-\sigma t)}$$

- Note that for n=1 Rossby wave **div** ($\delta u/\delta x + \delta v/\delta y$) ➔ $f(\psi_0, \psi_2)$ ➔ Symmetric about the equator

For $n = 0$, Yanai or Mixed wave,

$$v_0 = R_e A_0 \psi_0 e^{i(kx - \sigma t)}$$

$$u_0 = R_e \frac{i\sigma}{\sqrt{2}} A_0 \psi_1 e^{i(kx - \sigma t)}$$

$$\phi_0 = R_e \frac{i\sigma}{(2)^{1/2}} A_0 \psi_1 e^{i(kx - \sigma t)}$$

- For the Yanai wave **div** (δu/δx + δv/δy) ➔ f(ψ_1) ➔ Antisymmetric

For $n = -1$ or Kelvin Wave

$$v = 0$$

$$u_{-1} = R_e A_{-1} \psi_0 e^{i(kx - \sigma t)}$$

$$\phi_1 = R_e A_{-1} \psi_0 e^{i(kx - \sigma t)}$$

- For the Yanai wave **div** (δu/δx + δv/δy) ➔ f(ψ_0) ➔ Symmetric

The following figure is from a very interesting paper by (Wheeler and Kiladis, JAS,1999, vol.56,374pp) on Wavenumber-frequency spectra of twice daily OLR in tropics showing Kelvin, Rossby and MRG waves '

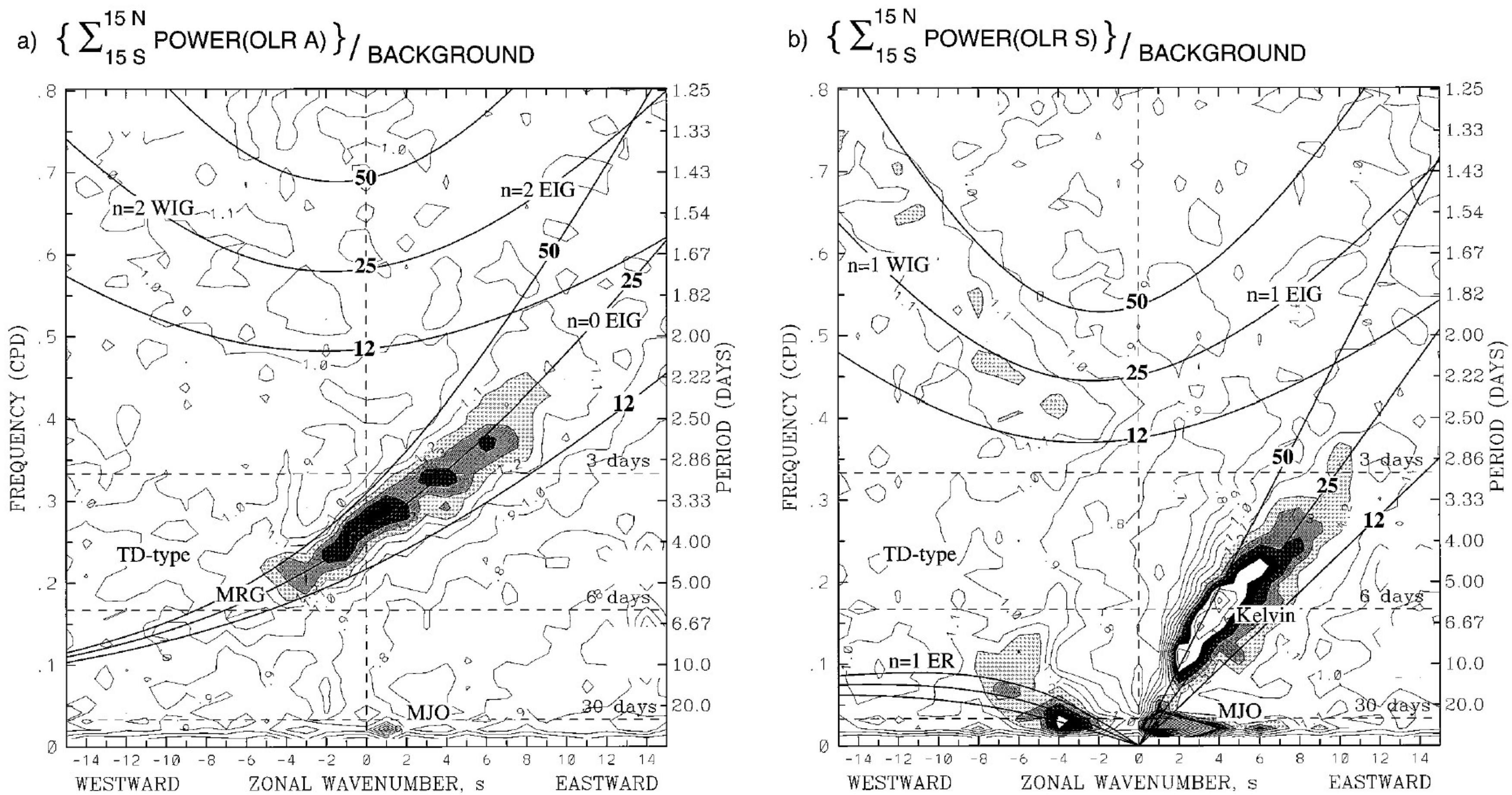


FIG. 3. (a) The antisymmetric OLR power of Fig. 1a divided by the background power of Fig. 2. Contour interval is 0.1, and shading begins at a value of 1.1 for which the spectral signatures are statistically significantly above the background at the 95% level (based on 500 dof). Superimposed are the dispersion curves of the even meridional mode-numbered equatorial waves for the three equivalent depths of h = 12, 25, and 50 m. (b) Same as in panel a except for the symmetric component of OLR of Fig. 1b and the corresponding odd meridional mode-numbered equatorial waves. Frequency spectral bandwidth is 1/96 cpd.

Some comments on waves.

Remember the dispersion relation for equatorial Kelvin Wave

$$\sigma = +k$$

and Rossby Wave

$$\sigma = -\frac{k}{k^2 + (2n+1)}$$

For very long waves $k << 1$

$$C_p(Kelvin) = \frac{\sigma}{k} = +1$$

$$(C_p(Rossby) = \frac{\sigma}{k} = -\frac{1}{3} \qquad for \; n = 1$$

thus the phase speed of the gravest Rossby wave is 1/3 that of the Kelvin wave.

Also for k=0 or very long waves $C_g \approx C_p$

Before moving forward, let me try to answer a question I was asked offline (Dr. R. Shankar) after the last lecture,

- *From extra tropical analysis we found that the Kelvin waves need a boundary for their existence. How then a free Kelvin wave exist in the tropical Atmosphere?*

- This is due to the special nature of the equatorial β-plane where $f(y) = 0 + \beta y$ as against extra tropical β-plane where $f(y) = f_0 + \beta y$
- This character of equatorial β-plane makes the y-structure of all equatorial waves evanescent away from equator on both sides, recall $v(y) \sim \exp(-y^2/2)\psi_n$
- Thus, equatorial β-plane acts like a 'dynamical channel'
- This is why a free equatorially trapped Kelvin wave can exist in the tropics.

Forced Motion in the Atmosphere

So far we have discused free motions. However, most of what we observe in the atmosphere is forced motion. For example, the monsoon circulation is forced by latent heat released associated with monsoon rainfall. In the next fig. the rainfall distribution during June-September and low level wind vectors are shown over the Indian monsoon region. Are the observed wind essentially driven by the latent heat release associated with the rainfall? Can the linear wave theory help us understand this response?.

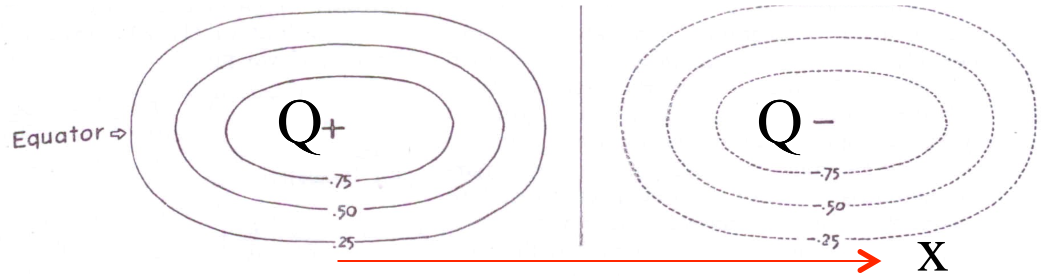
This is essentially a forced stationary motion. Such stationary response arises as a result of balance between forcing and dissipation. In the absence of dissipation, the forcing would continuously generate atmospheric waves and the waves would travel away from the source region and no steady state could result. In the presence of dissipation, the waves gets damped as they propagate away from the source region. When the rate of dissipation is balanced by rate of generation, a steady state or stationary response is set up. The dissipation in the atmosphere comes from turbulent diffussion and radiative damping. For steady response, the equations of motion and continuity equation may be written as

$$
\begin{aligned}
-fv + g\frac{\partial h}{\partial x} &= -au + F_x \\
fu + g\frac{\partial h}{\partial x} &= -av + F_y \\
H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) &= -ah + Q
\end{aligned} \tag{1}
$$

In (1), F_x, F_y represents momentum forcing in the x(east-west) and y(north-south) direction. Q is a source of sink of mass. This essentially represents the heating due to convection. We have added terms $-au$, $-av$ and $-ah$ on the right hand side of (1). These terms represent simple formulation dissipation.

Matsuno, 1966, JMSJ, Quasigeostrophic motions in equatorial area, vol-44, 25-42

Case of idealized Sinusoidal Heat Source in the east-west



If the forcings are also sinusoidal in the x-direciton, we could expand the variables as well as the forcings as e^{ikx}. After nondimensionalization and applying equatorial $\beta - plane$, (1) could be written as

$$
\begin{aligned}
au - yv + ik\phi &= F_x \\
av + yu + \frac{d\phi}{dy} &= F_y \qquad (2) \\
a\phi + iku + \frac{dv}{dy} &= Q
\end{aligned}
$$

B,C: Assuming that forcings are confined to a finite distance form $y = 0$, we can assume

$$u, v, \phi \longrightarrow 0 \qquad at \qquad y \rightarrow \pm\infty$$

We have seen that the free wave solutions form a complete set of eigen functions. We can represent the forced solutions as a superposition of the free solutions. Eq(2) can be symbolically written as

$$(\Omega + \alpha I)\,\chi = \sigma \tag{3}$$

where χ and σ are solution and inhomogeneous terms in (2), I an unit operator and Ω is the operator representing the l.h.s of (2). For the free case, the inhomogeneous terms are not there and $\alpha = i\omega$. Thus, the free solutions are governed by

$$\Omega\xi_m = -i\omega_m\xi_m \tag{4}$$

ξ_m's are the free wave eigen functions. It may be noted that all the eigen functions could be arranged in a single array and labelled by an index m.

Expanding χ and σ in terms of ξ ,

$$\chi = \sum_m a_m\xi_m, \qquad \sigma = \sum_m b_m\xi_m \tag{5}$$

Eq(3) may be written as

$$\sum (\Omega + \alpha I)\, a_m \xi_m = \sum b_m \xi_m$$

Making use of (4), we can write

$$\sum a_m(-i\omega_m + \alpha)\xi_m = \sum b_m \xi_m \tag{6}$$

Since ξ_m's are *orthogonal* with each other, we get the following relation.

$$a_m = \frac{b_m}{\alpha - i\omega_m} \tag{7}$$

Note: α is small, dissipation time scale are typically several days. Thus, only low frequency modes could have resonant response. For example, gravity waves will be damped as their frequency is very high (period of few hours).

Since b_m can be obtained form

$$b_m = \frac{\int \xi_m(y)\sigma(y)dy}{\int |\xi_m(y)|^2 dy}$$

the solution can be easily obtained if we know $\sigma(y)$. Let us take a special case for which

$$\sigma = \begin{pmatrix} 0 \\ 0 \\ Q(y) \end{pmatrix}, \qquad Q(y) = e^{-\frac{y^2}{2}} = \psi_o \tag{8}$$

The solution to this forcing is shown in Fig.

Note:

• Strong zonal flow along the equator. This is due to the strong projection of the forcing on the Kelvin mode. The eastward extention of the surface height contours is due to the eastward propagation of the Kelvin wave.

• Also note that there are two maxima of height (or pressure) symmetric about the equator. We also note that zonal winds are symmetric about the equator but the meridional winds are antisymmetric about the equator. This is due to the n=1 Rossby wave. The forcing could generate a response on this mode.

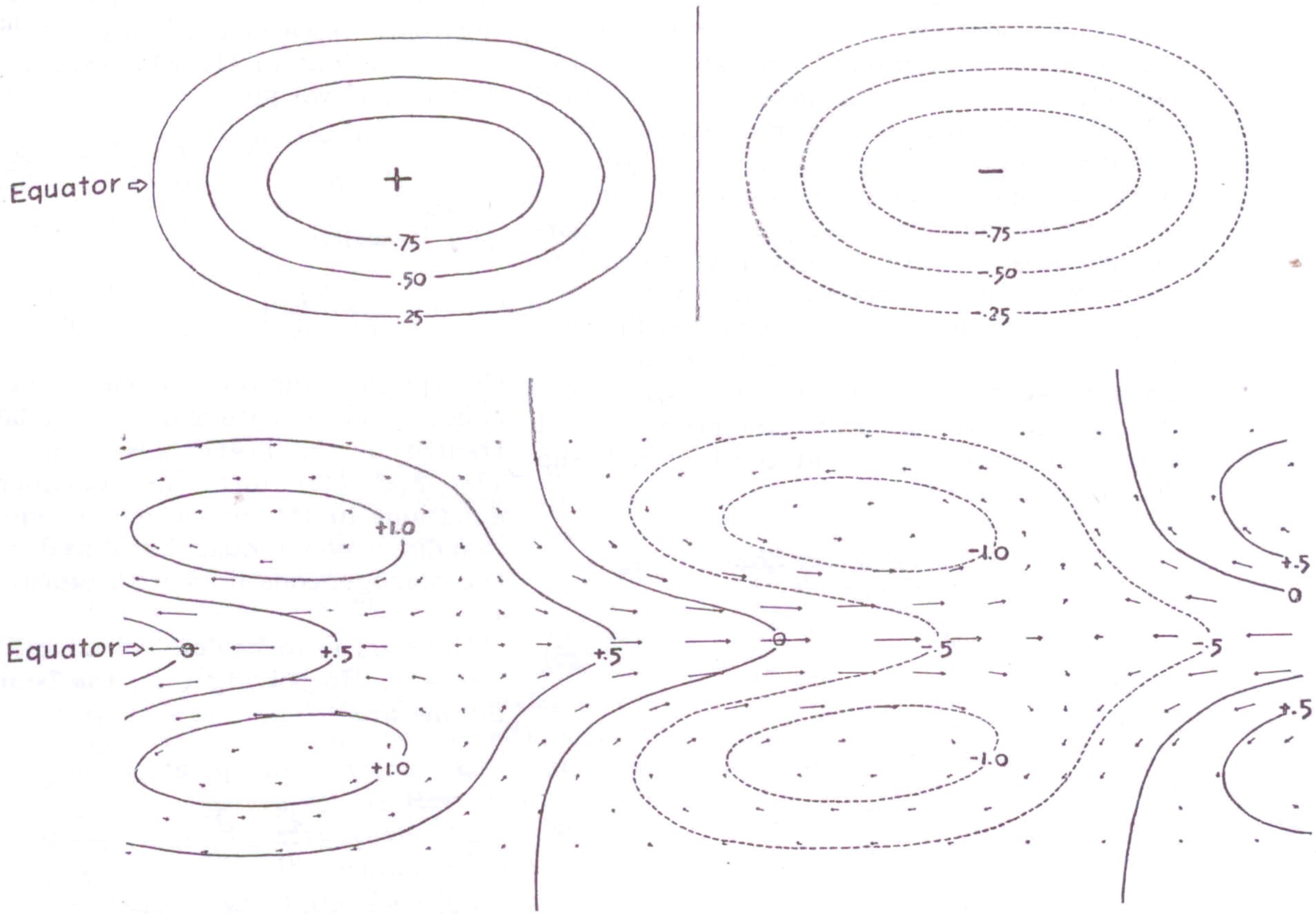


Fig. 9. Stationary circulation pattern (lower) caused by the mass source and sink (upper).

Forced Motion due to a Localized Heat Source

Some simple solutions for heat-induced tropical circulation

Gill A. E., 1980: Some simple solutions for heat-induced tropical circulation, QJRMS,

https://doi.org/10.1002/qj.49710644905

Introduction

- **Implementation of a simple analytical model to elucidate some basic features of the response of the tropical atmosphere to diabatic heating.**

- **Model and Approximations.**

- **Method of solution for different forcings.**

- **Conclusions.**

The Model

- The model is designed to show the response of the tropical atmosphere to diabatic heating .

- How to do it using as simple a model as possible.

- Linear theory for small perturbation (heating rate will be assumed small enough for linear theory to apply)

- Expansion of the forcing function in terms of normal modes expressing the diabatic heating rate as Fourier-type integral over the complete set of modes.

The Model

- Gravest mode has p,u,v vary with height as cos(pi*z/D), w as sin (pi*z/D), and an equivalent H.

- Choosing diabatic heating to vary as sin(pi*z/D).

- Shallow-water equations for a single mode describe the complete solution.

- β -plane where f = β y

- Dimensionless equations.

The linear shallow water equations in an equatorial $\beta - plane$ may be written as

$$
\begin{aligned}
\frac{\partial u}{\partial t} - \beta y v &= -\frac{\partial \phi}{\partial x} - \epsilon u \\
\frac{\partial v}{\partial t} - \beta y u &= -\frac{\partial \phi}{\partial y} - \epsilon v \\
\frac{\partial \phi}{\partial t} + gH\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) &= -Q - r\phi
\end{aligned}
\tag{32}
$$

It can be shown that even in a stratified atmosphere these equations also represents the horizontal structure of different vertical normal modes with equivalent depth h_e replacing H in eqn(32)

$$C_0^2 = gH$$

The Model

- **Length, time scales:**
- **$L=(c/2\beta)^{1/2}$**
- **$\tau=(2\beta c)^{-1/2}$**

The Model

$$\frac{\partial u}{\partial t} - \frac{1}{2} y v = -\frac{\partial p}{\partial x} - \varepsilon u \qquad 1.1$$

$$\frac{\partial v}{\partial t} - \frac{1}{2} y u = \frac{\partial p}{\partial y} - \varepsilon v \qquad 1.2$$

$$\frac{\partial p}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q - \varepsilon p \qquad 1.3$$

Q is proportional to the heating rate, the signs being such that if Q is positive, the signs of u, v, p correspond to those at the surface

The Model

Using the so called "Rayleigh friction " and "Newtonian cooling" forms and studying the steady state.

$$\varepsilon u - \frac{1}{2} y v = \frac{\partial p}{\partial x'} \quad 1.4$$

$$\varepsilon v + \frac{1}{2} y u = \frac{\partial p}{\partial y'} \quad 1.5$$

$$\varepsilon p + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q \quad 1.6$$

$$w = \varepsilon p + Q \quad 1.7$$

The math is simplest when the epsilon (damping parameter) for friction is the same as the one for cooling,

The Model

This set of equation can be reduced to a single equation for *v* :

$$\varepsilon^3 v + \tfrac{1}{4}\varepsilon y^2 v - \varepsilon \frac{\partial^2 v}{\partial x^2} - \varepsilon \frac{\partial^2 v}{\partial y^2} - \frac{1}{2}\frac{\partial v}{\partial x} = \varepsilon \frac{\partial Q}{\partial y} - \frac{1}{2} y \frac{\partial Q}{\partial x} \qquad 1.8$$

- **Forcing will be chosen to have a y-scale of order 1.**
- **Forcing has east-west wavenumber k. (2 ε k<<1)**
- **Such forcing has east-west scale large compared with 2 ε**

$$\varepsilon v + \frac{1}{2} y u = \frac{\partial p}{\partial y'}$$

becomes

$$\frac{1}{2} y u = -\frac{\partial p}{\partial y} \qquad 1.9$$

→ **Long wave approximation**

Method of Solution

Recalling the equations after the approximations

$$\varepsilon u - \frac{1}{2} y v = \frac{\partial p}{\partial x'} \quad 1.4$$

$$\frac{1}{2} y u = -\frac{\partial p}{\partial y} \quad 1.9$$

$$\varepsilon p + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q \quad 1.6$$

In order to solve this set of equations, is convenient to introduce two new variables *q* and *r* to replace *p* and *u.*

q= p+u* and *r= p-u

Method of Solution

$$\varepsilon q + \frac{\partial q}{\partial x} + \frac{\partial v}{\partial y} - \frac{1}{2} y v = -Q \quad 1.10$$

$$\varepsilon r - \frac{\partial r}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{2} y v = -Q \quad 1.11$$

$\frac{1}{2} y u = -\frac{\partial p}{\partial y}$ **Can be written in the form** $\frac{\partial q}{\partial y} + \frac{\partial r}{\partial y} + \frac{1}{2} y q - \frac{1}{2} y r = 0$ 1.12

Method of Solution

- **Free solutions of 1.10,1.11,1.12 are parabolic cylinder functions [Dn(y)].**
- **Forced problem solutions can be found :**

$$q = \sum_{n=0}^{\infty} q_n(x) D_n(y) \qquad 1.13$$

Parabolic cylinder functions important properties:

$$D_0, D_1, D_2, D_3 = (1, y, y^2 - 1, y^3 - 3y)\exp(-\tfrac{1}{4} y^2)$$

$$\frac{\partial D_n}{\partial y} + \frac{1}{2} y D_n = n D_{n-1} \qquad \frac{\partial D_n}{\partial y} - \frac{1}{2} y D_n = -D_{n+1}$$

Method of Solution

$$\varepsilon q_0 + \frac{\partial q_0}{\partial x} = -Q_0 \qquad 1.14$$

$n \geq 0$

$$\varepsilon q_{n+1} + \frac{\partial q_{n+1}}{\partial x} - v_n = -Q_{n+1} \qquad n \geq 0 \qquad 1.15$$

$$\varepsilon r_{n-1} - \frac{\partial r_{n-1}}{\partial x} + n v_n = -Q_{n-1} \qquad n \geq 1 \qquad 1.16$$

$$q_1 = 0 \qquad n \geq 1 \qquad 1.17$$

$$r_{n-1} = (n+1) q_{n+1} \qquad n \geq 1 \qquad 1.18$$

Method of Solution

Two cases of our interest:

$$Q(x,y) = F(x)D_0(y) = F(x)\exp(-\tfrac{1}{4}y^2)$$ **Symmetric heating**

$$Q(x,y) = F(x)D_1(y) = F(x)y\exp(-\tfrac{1}{4}y^2)$$ **Antisymmetric heating**

Symmetric Heating

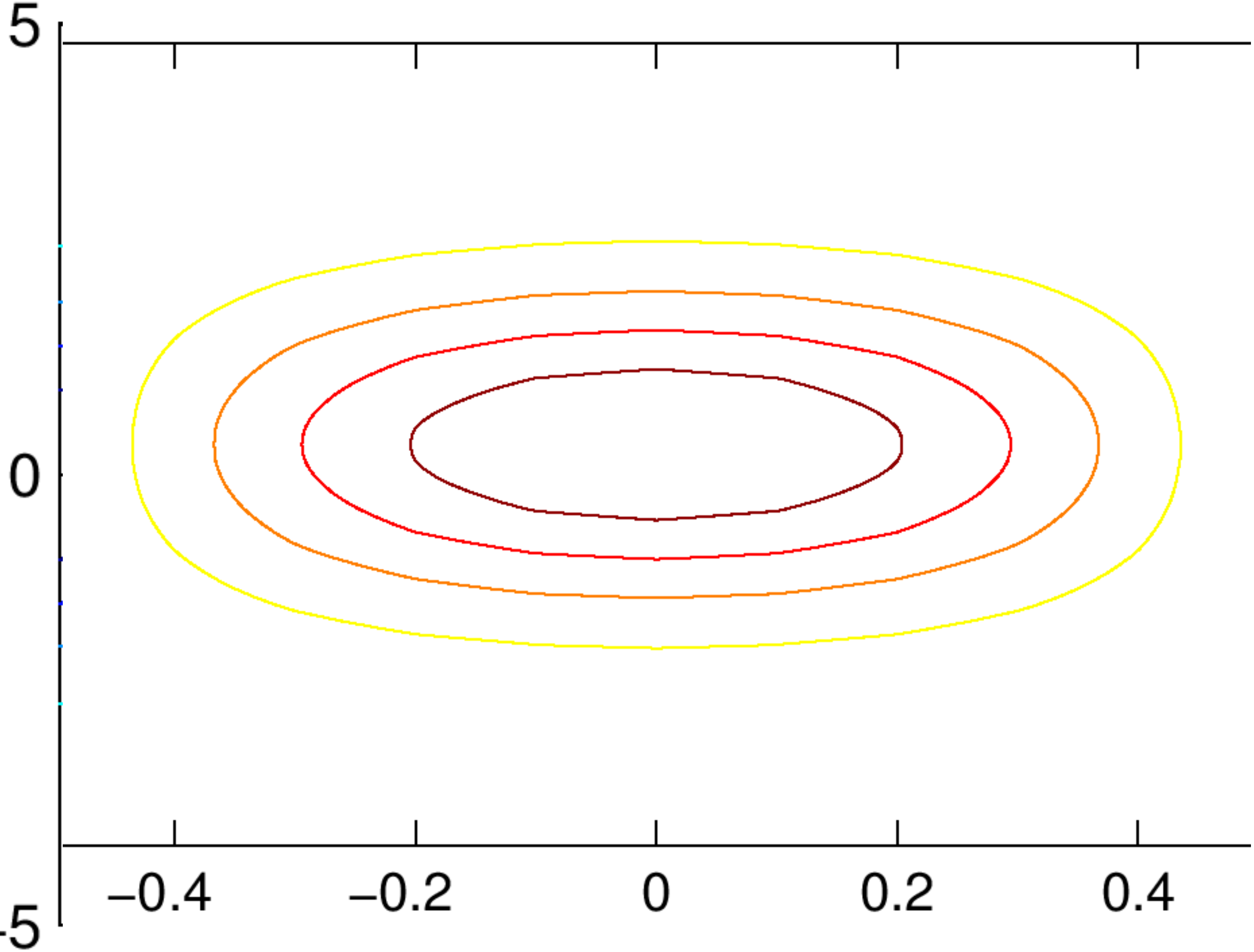
Forcing :

$F(x) = \cos(kx)$ $\quad |x| < L$ and $\quad k = \frac{\pi}{2L}$

$Q_0 = F(x)$ only non-zero coefficient

Two parts of the response:

q_0 n=0 and higher q with n=1


5
0
-5
−0.4
−0.2
0
0.2
0.4

As only Q_0 coefficient is nonzero in the forcing functions, from Eqns (1.14)-(1.18), only q_0 and q_2 are nonzero. All other coefficients of q will not exist. These two coefficients are governed by

$$\frac{dq_0}{dx} + \epsilon q_0 = -F(x) \tag{2.1}$$

and

$$\frac{dq_2}{dx} - 3\epsilon q_2 = F(x) \tag{2.2}$$

with $k = \frac{\pi}{2L}$,

$$F(x) = coskx \qquad |x| < L$$
$$= 0 \qquad |x| > L$$

In the absence of the forcing Eq(2.1) represents an initial perturbation decaying in the +ve x direction while Eq(2.2) represents an initial perturbation decaying in the -ve x direction. Using standard method of solution Eq(2.1) could be solved to give

$$(\epsilon^2 + k^2)q_0(x) = 0 \qquad for \qquad x < -L \qquad (3.1)$$

$$(\epsilon^2 + k^2)q_0(x) = -\epsilon coskx - k[sinkx + e^{-\epsilon(x+L)}] \; for \qquad |x| < L \qquad (3.2)$$

$$(\epsilon^2 + k^2)q_0(x) = -k[1 + e^{-2\epsilon L}]e^{\epsilon(L-x)} \qquad for \qquad \mathrm{mod}\ |x| > L \qquad (3.3)$$

Following a similar procedure, the solution of Eq(2.2) would give

$$(k^2 + 9\epsilon^2)q_2(x) = -k[1 + e^{-6\epsilon L}]e^{3\epsilon(x+L)} \qquad for \qquad |x| < -L \qquad (4.1)$$

$$(k^2 + 9\epsilon^2)q_2(x) = -3\epsilon coskx + k[sinkx - e^{3\epsilon(x-L)}] \qquad for \qquad |x| < L \qquad (4.2)$$

$$(k^2 + 9\epsilon^2)q_2(x) = 0 \qquad for \qquad |x| > L \qquad (4.3)$$

Symmetric Heating

First part of the response q_0 and n=0

$$u = p = \frac{1}{2} q_0(x) \exp(-\frac{1}{4} y^2)$$

$$v = 0$$

$$w = \frac{1}{2}[\varepsilon q_0(x) + F(x)] \exp(-\frac{1}{4} y^2)$$

WALKER CIRCULATION

Symmetric Heating

Second part of the solution n=1

$$p = \tfrac{1}{2} q_2(x)(1 + y^2)\exp(-\tfrac{1}{4} y^2)$$

$$u = \tfrac{1}{2} q_2(x)(y^2 - 3)\exp(-\tfrac{1}{4} y^2)$$

$$v = [F(x) + 4\varepsilon q_2(x)]y\exp(-\tfrac{1}{4} y^2)$$

$$w = \tfrac{1}{2}[F(x) + \varepsilon q_2(x)](1 + y^2)\exp(-\tfrac{1}{4} y^2)$$

Long planetary waves propagating westwards

Symmetric Heating

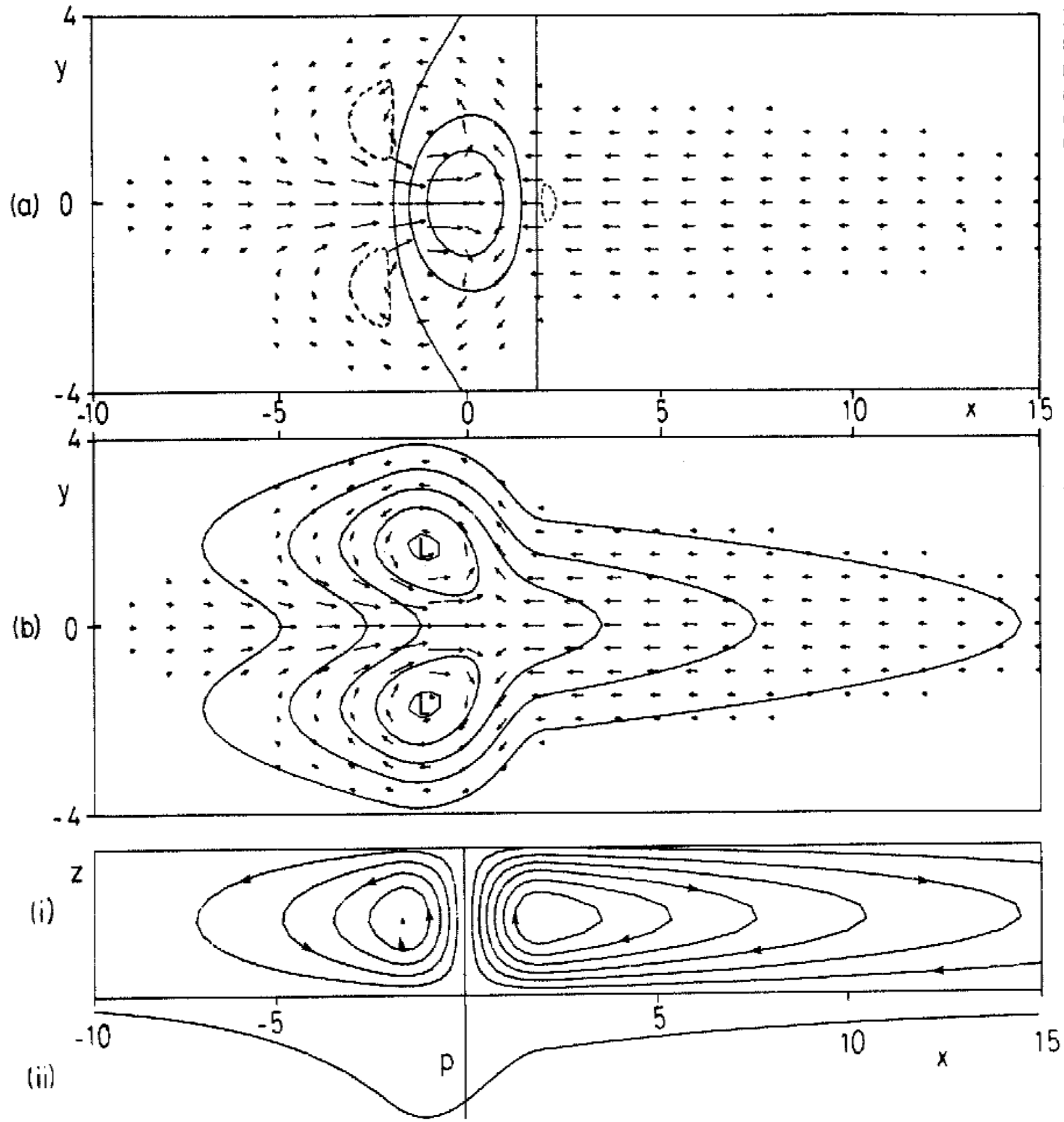


Figure 1. Solution for heating symmetric about the equator in the region |x| < 2 for decay factor ε = 0·1.

(a) Contours of vertical velocity *w* (solid contours are 0, 0·3, 0·6, broken contour is −0·1) superimposed on the velocity field for the lower layer. The field is dominated by the upward motion in the heating region where it has approximately the same shape as the heating function. Elsewhere there is subsidence with the same pattern as the pressure field.

(b) Contours of perturbation pressure *p* (contour interval 0·3) which is everywhere negative. There is a trough at the equator in the easterly régime to the east of the forcing region. On the other hand, the pressure in the westerlies to the west of the forcing region, though depressed, is high relative to its value off the equator.
Two cyclones are found on the north-west and south-west flanks of the forcing region.

(c) The meridionally integrated flow showing (i) stream function contours, and (ii) perturbation pressure. Note the rising motion in the heating region (where there is a trough) and subsidence elsewhere. The circulation in the right-hand (Walker) cell is five times that in each of the Hadley cells shown in (c).

Asymmetric Heating

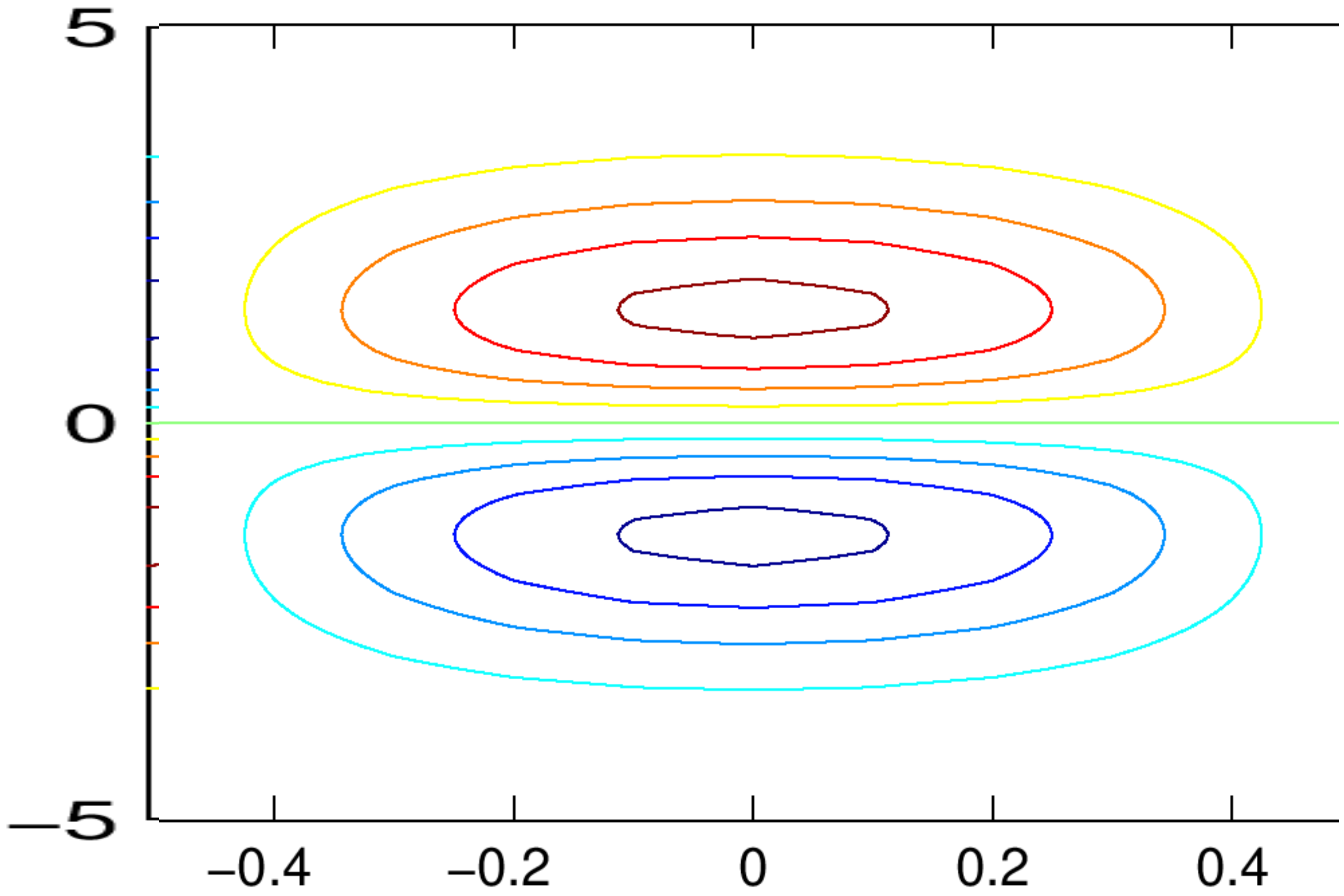
Asymmetric Forcing $Q(x,y) = F(x)D_1(y) = F(x)y\exp(-\frac{1}{4}y^2)$

The only non-zero coefficient Qn corresponds to n=1

$$Q_1 = F(x)$$

Again, there are two parts to the response:

n=0-----mixed planetary-gravity wave---- $q_1 = 0$, $v_0 = Q_1$



Spatial structure of a localized Antisymmetric Heating

Assymetric Heating

Again there will be two parts of the solution. The first part represents long n=0 mixed Rossby-gravity wave for which we shall get,

$q_1 = 0 \qquad\qquad v_0 = Q_1$

This part shows no effect outside the forcing region as the long MRG wave do not propagate. (Recall the dispersion relation figure)

The second part is the long n=2 planetary wave for which we shall get

$$v_2 = \frac{dq_3}{dx} + \epsilon q_3 \tag{42.1}$$

$$r_1 = 3q_3 \tag{42.2}$$

$$\frac{dq_3}{dx} - 5\epsilon q_3 = Q_1 \tag{42.3}$$

Asymmetric Heating

$$p = \tfrac{1}{2} q_3(x) y^3 \exp(-\tfrac{1}{4} y^2)$$

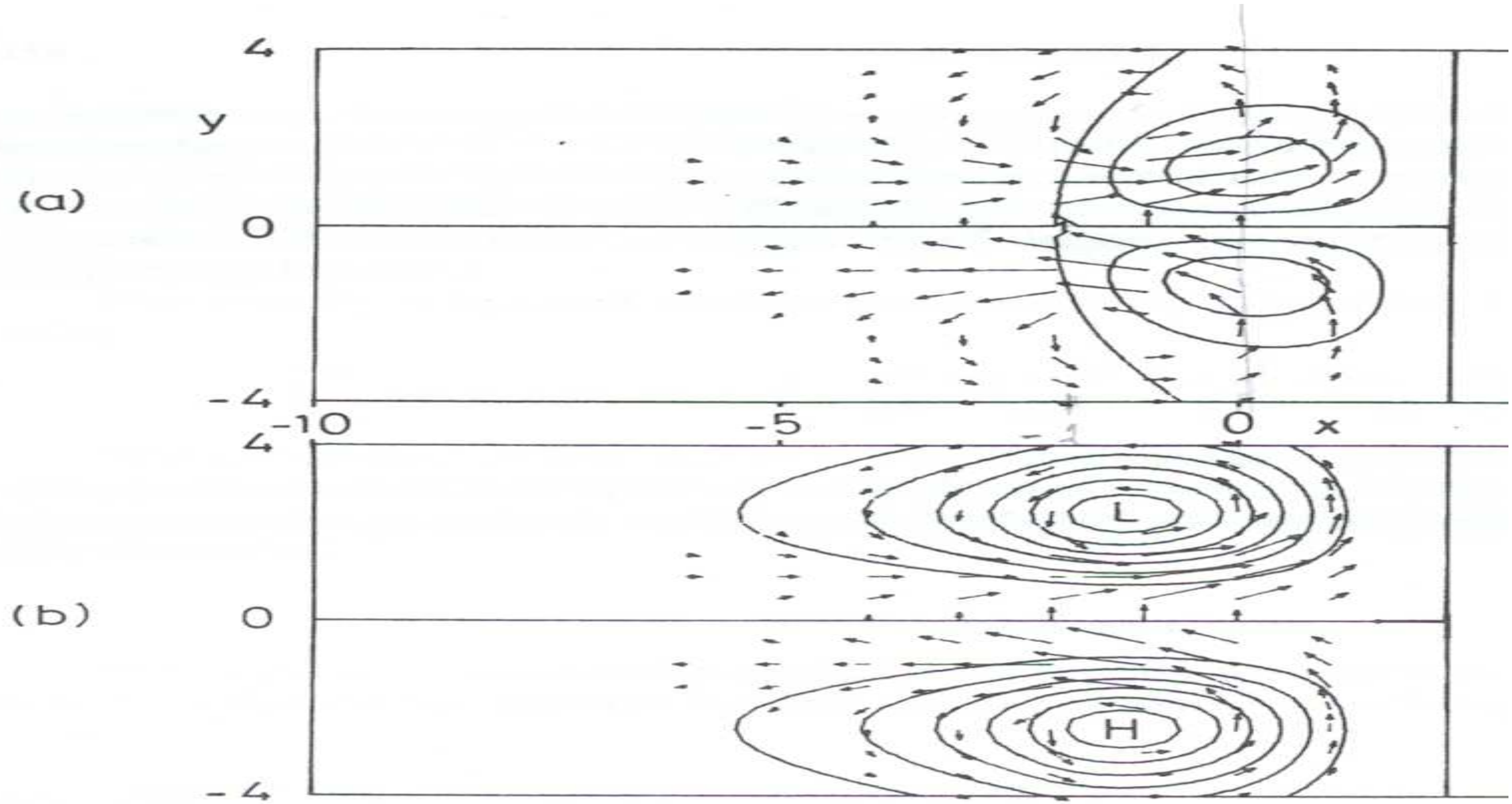
$$u = \tfrac{1}{2} q_3(x)(y^3 - 6y) \exp(-\tfrac{1}{4} y^2)$$

$$v = [F(x) y^2 + 6\varepsilon q_3(x)(y^2 - 1)] \exp(-\tfrac{1}{4} y^2)$$

$$w = [F(x) y + \tfrac{1}{2} \varepsilon q_3(x) y^3] \exp(-\tfrac{1}{4} y^2)$$

n=2--- planetary wave

Asymmetric Heating



Further solutions

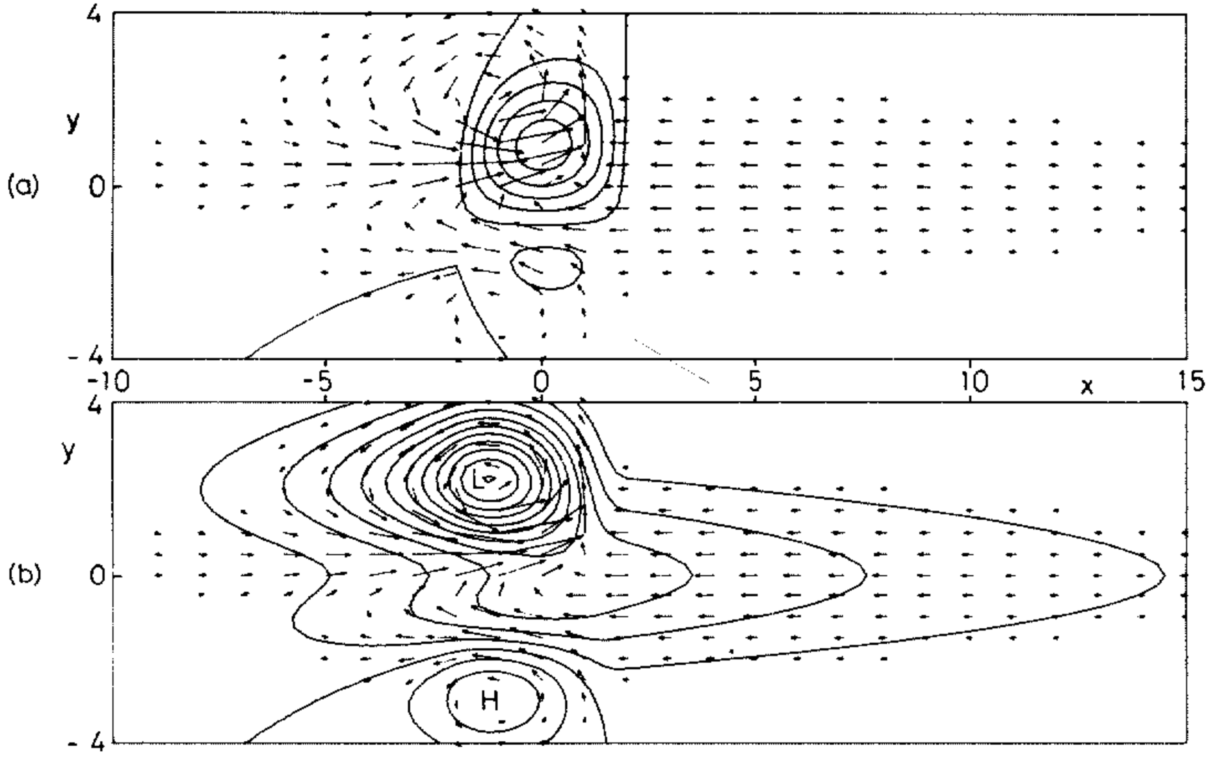


Figure 3. Solution obtained by adding the solutions shown in the two previous figures, corresponding to heating which is confined to the region $|x| < 2$ and is mainly concentrated to the north of the equator.
(a) Contours of vertical velocity w (contour interval 0·3) showing the dominance of the heating north of the equator. The flow to the east of the forcing region is the same as in Fig. 1, this being provided entirely by the symmetric part of the heating. West of the forcing, the westerly inflow is concentrated between the equator and $y = 2$. An easterly flow is found south of the equator.
(b) Contours of perturbation pressure p (contour interval 0·3). The pattern is dominated by a low on the western flank of the heating region and by the equatorial trough. A high is found in the southern hemisphere.

Response to sum of a Symmetric and an Antisymmetric Heating

Conclusions

- **Interpretation of the equatorial part of the flow in the lower troposphere with a simple analytical model.**

- **Walker circulation driven only by the response to symmetric heating.**

- **Hadley circulation driven by the heating region (asymmetric heating).**

- **Solutions were found for only one mode. Solutions for semi-infinite atmosphere can be found superposing solutions for different modes.**

Some comments on the Gill Model

• Recall the Gill's solutions for the 'symmetric heating' and 'assymmetric heating' for $\epsilon = 0.1$. Let us examine the dissipation time scale associated with this nondimensional ϵ.

$$\epsilon^* \tau = \epsilon \ (nondimnesional)$$

$$\epsilon^* \ (dimensional) = \frac{\epsilon}{\tau} = \frac{0.1}{0.25days} = \frac{1}{2.5days}$$

Thus, the dissipation time scale used is about 2.5 days. This value was used to get realistic horizontal scale of the wind anomalies.

• What would happen to the spatial structure of those solutions if ϵ was 0.05 or 0.2 ?

• Gill's model was initially interpreted as the first baroclinic mode representation of the free atmosphere, so that we could interpret the above solution as applicable to the lower atmosphere. Therefore, we should interpret the wind response as wind at lower atmosphere.eg. 850 hPa. However, the dissipation in the free atmosphere is small and dissipation time scale is much larger than 2.5 days. Therefore, there is a slight inconsistency.

• On the other hand, such dissipation rate would be realistic in the planetary boundary layer(PBL). Therefore, if we could interpret the Gill's model as a 'boundary layer model', then the dissipation reate will be justified. Then we could interpret the results as surface wind and pressure. However, the first baroclinic mode assumption and the corresponding gravity wave speed ($C_o^2 = 60ms^{-1}$) may not be applicable in the PBL.

• The Gill type model essentially represents the response of latent heating. The low-level wind obtained from such a model has been used to force ocean models in many coupled model. Most of the convective heating takes place above the PBL. So, Gill model actually gives wind and pressure at the top of the PBL. These act as forcing for the PBL and produce PBL circulation and surface wind. In applying Gill's model to force an ocean, the assumption is that the wind at the top of the atmosphere can be extrapolated to the surface to give the surface wind (albeit with some what reduced strength).

In addition to the forcing at the top of the PBL coming from large scale circulation, there is an additional forcing within the boundary layer to produce surface wind. This is the pressure gradient associated with the underlying SST. Essentially the air above hot SST is lighter (due to sensible heat exchange) and hence there is a low pressure there. On the other hand, the air over the colder(negative) SST is heavier asd a higher pressure is located there. This pressure gradient itself could drive some surface wind.

Lindzen and Nigam (1987, J. Atmos. Sci, vol. 45, 2440-2458) developed a boundary layer model to simulate surface wind where only forcing associated with the SST gradients was used.

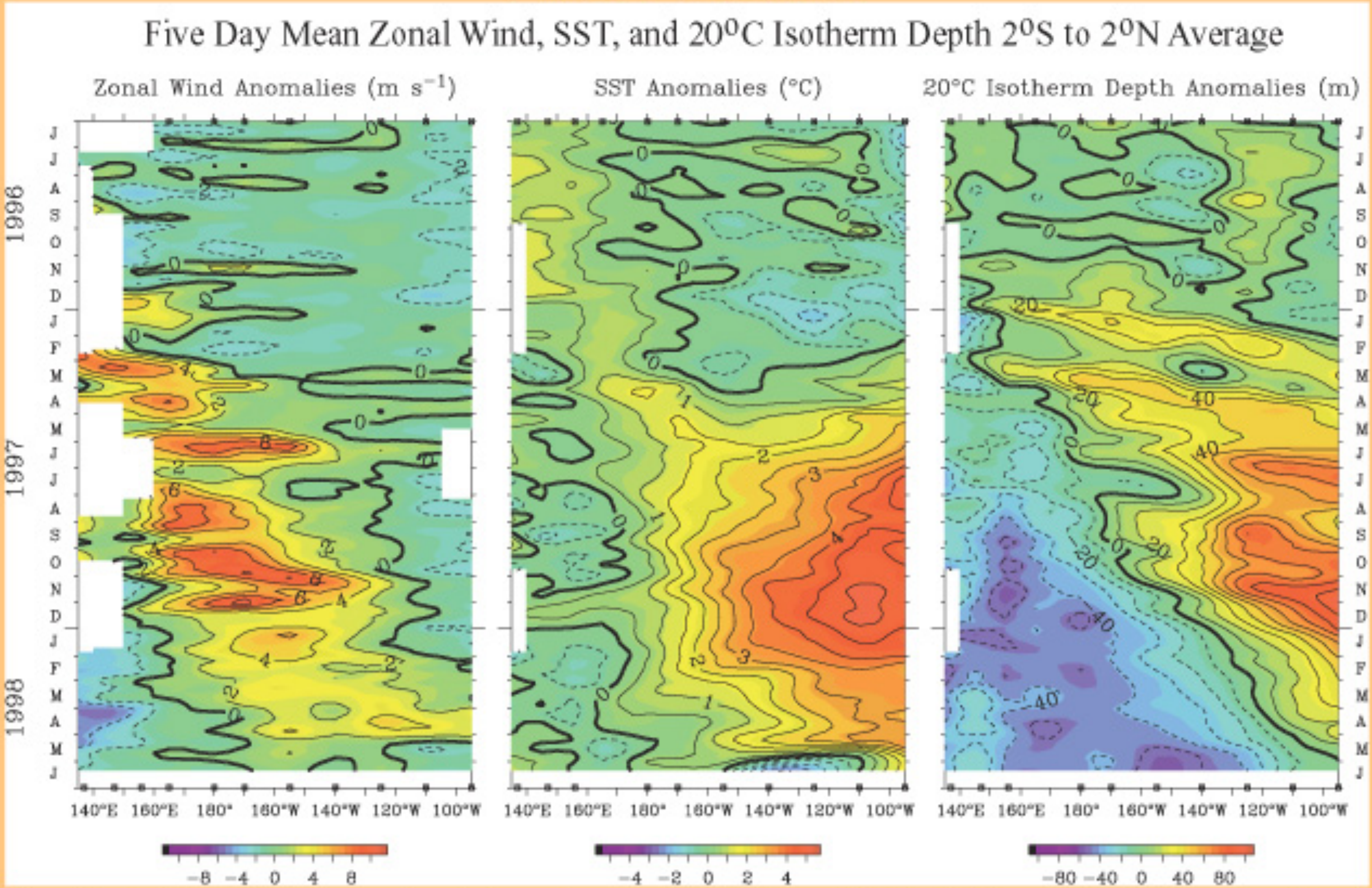
Neelin (1989, J. Atmos. Sci, 46, 2466-2468) showed that with a proper scaling, the Gill's equations could be transformed to the PBL equation derived by Lindzen and Nigam. Therefore, with proper modification of parameter the Gill's model can be interpreted as a boundary layer model and thus, it justifies use of the Gill model to obtain surface winds.

- In reality both the convective forcing and the SST-gradient forcing would contribute in forcing the observed surface wind. In fact, ina paper with Saji (Saji and Goswami, 1996, Q.J.R.M.S, 122,23-53) we have actually demonstrated this.

Application of Gill's Model

- Show composite(or typical) surface wind during a typical El Nino

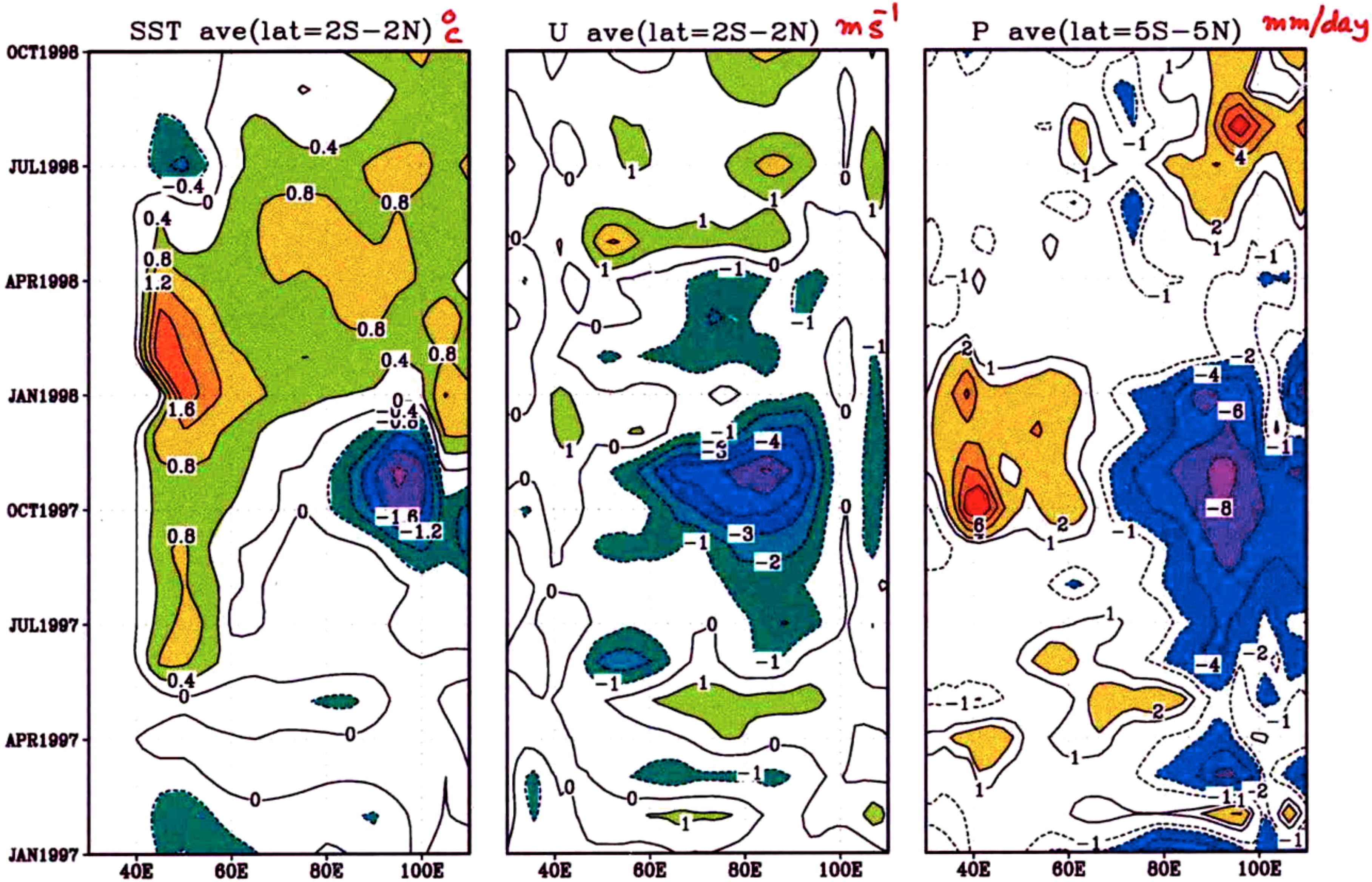
ENSO 1997/98



Time / longitude sections of anomalies in the surface zonal winds (in m s^{-1}), SST (in °C) and 20°C isotherm depth (in m) for the past 24 months. Analysis is based on 5-day averages between 2°N-2°S of moored time series from the TAO array. Anomalies are relative to monthly climatologies cubic spline fitted to 5-day intervals (COADS winds, Reynolds SST, CTD/XBT 20°C depths). Positive winds are westerly. Squares on the abscissas indicate longitudes where data where available at the start of the time series (top) and at the end of the time series (bottom). The TAO array is presently supported by the US (NOAA Office of Global Programs), Japan (JAMSTEC), Taiwan (NSC), Korea (STA) and France (ORSTOM). Further information is available from Dr. M.J. McPhaden (NOAA/PMEL) (courtesy of NOAA/PMEL).

AV/G1-99-7

THE 1997-98 SPECIAL EVENT IN THE IO



event and associated precipitation anomalies. Indicate association of easterlies and westerlies consistent wiht expected response associated with precipitation anomalies.

- Show a Hovmoller diagram of equatorial precipitation anomalies and zonal wind anomalies associated with ENSO. Again indicate general correspondence between precipitation forcing and surface zonal wind.

- Also show the 1996-98 time series of SST, precip and zonal wind in the Indian Ocean.

- Early models of Pacific Surface wind. eg. that of Zebiak (1982, A simple atmospheric model of relavance to El Nino, J. Atmos. Sci. 39, 2017-2027). He used a model similar to Gill with a steady mean backward flow.

$$\epsilon u + \bar{U} u_x - \frac{1}{2} y v = -p_x$$
$$\epsilon v + \bar{U} v_x + \frac{1}{2} y u = -p_y \qquad (1)$$
$$\epsilon p + \bar{U} p_x + u_x + v_y = -Q$$

He assumed that heating anomalies in the atmosphere are essentially due to SST anomalies. The basis of the arguement is that increase in SST causes increase in evaporation and assuming that the tropical atmosphere is nearly saturated, the excess moisture is available for condensation and release of latent heat. Linearising the Clausius-Clapeyron equation about a mena SST of $\bar{T}$ it can be shown that

$$\Delta e \propto \Delta T (\frac{b}{\bar{T}^2}) e^{-b/\bar{T}} \qquad (2)$$

where,

ΔT $\longrightarrow$SST anomaly

Δe $\longrightarrow$change in evaporation

b is constant $\sim$ 5400

$\bar{T}$ is mean SST.

therefore,

$$Q \propto \Delta T(\frac{b}{\bar{T}^2})e^{-b/\bar{T}} = \alpha \Delta T \tag{3}$$

Using such a formulation of atmosphere heating, which is directly proportional to the SST anomaly, he tried to simulate surface wind anomalies in the Pacific associated with El Nino. He found that to get correct scale of the wind anomalies he had to use $\epsilon \sim 0.3$ i.e dissipation time scale of about one day and still the simulation had large errors in the east. (discuss)

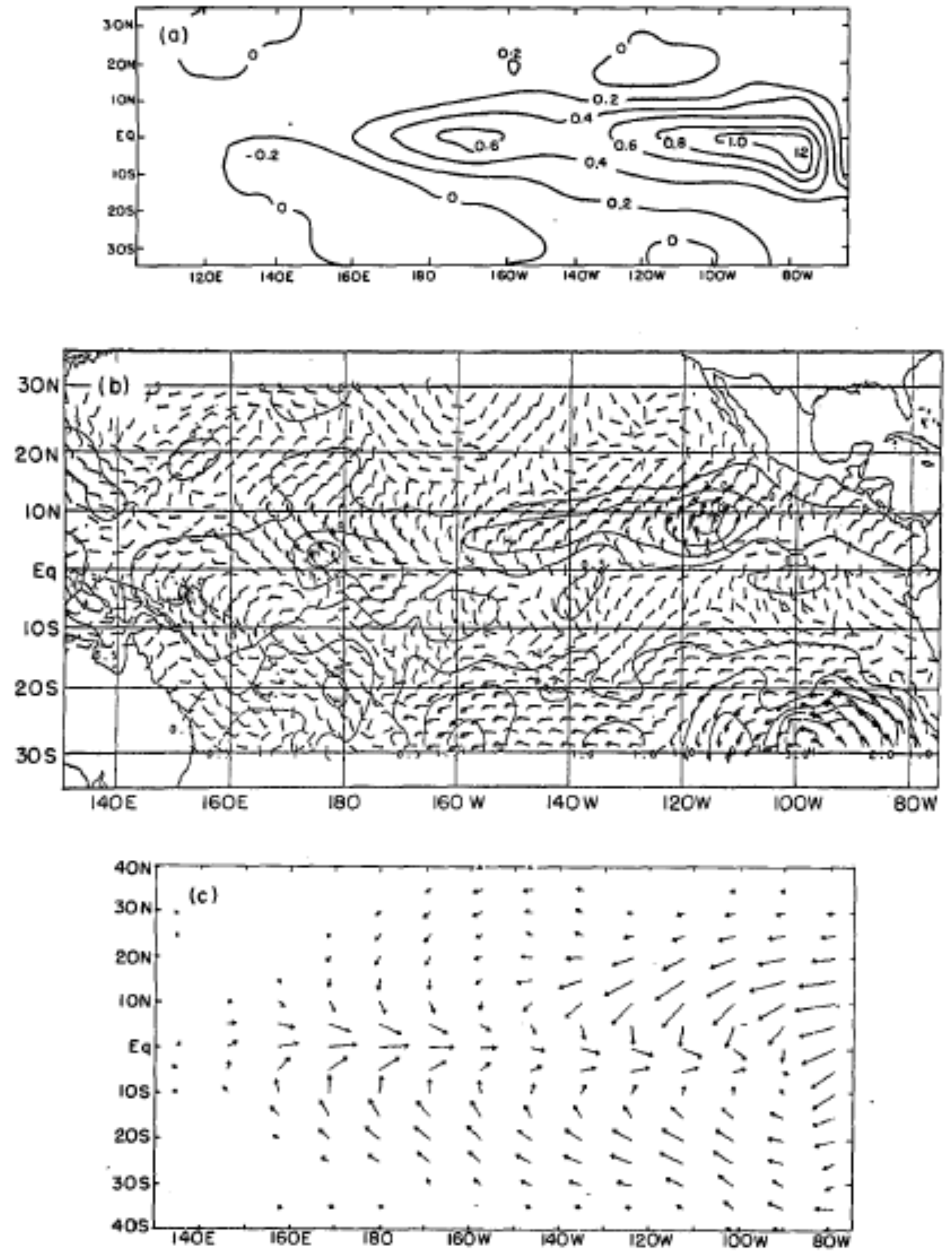


FIG. 6a. Model heating, based on the composite SST anomalies for the period centered around June, during the coastal warming.

FIG. 6b. Composite wind anomalies for the same time period as in 6a.

FIG. 6c. Model results for the heating in 6a.

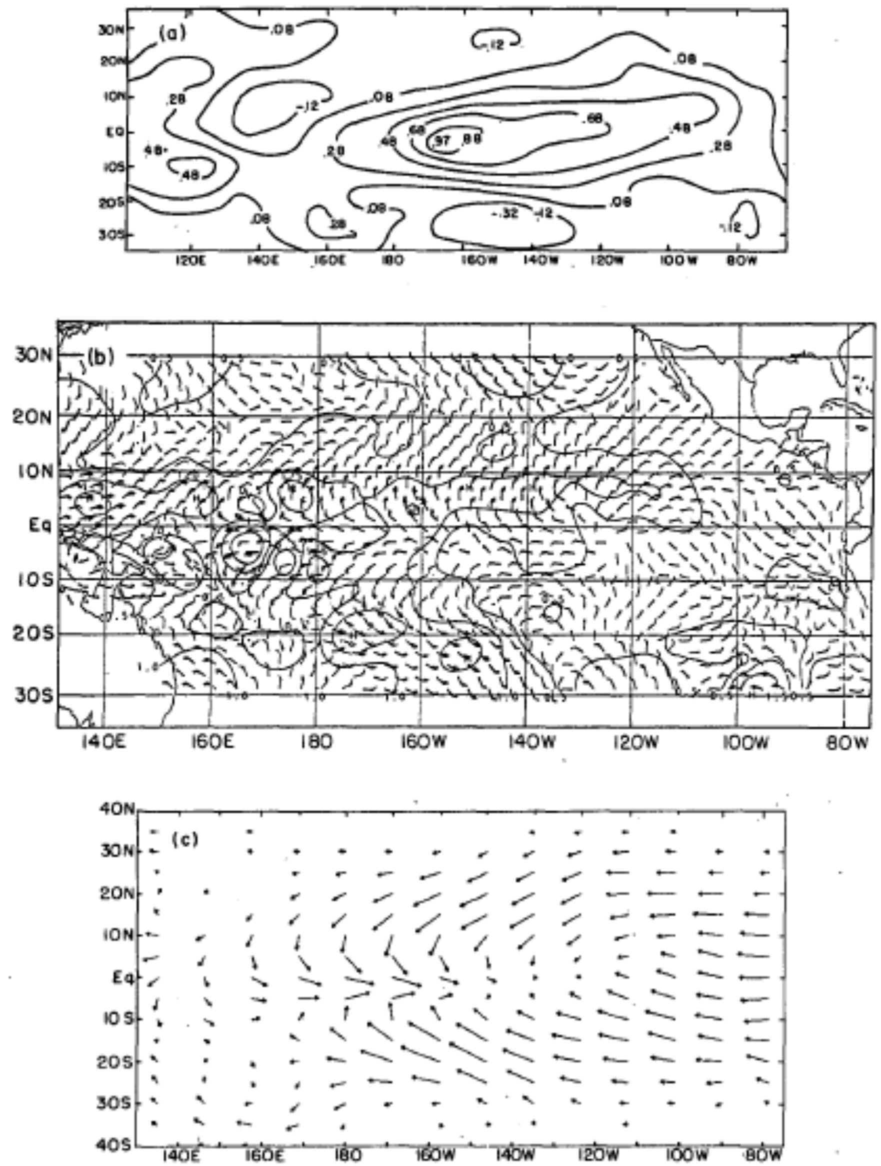


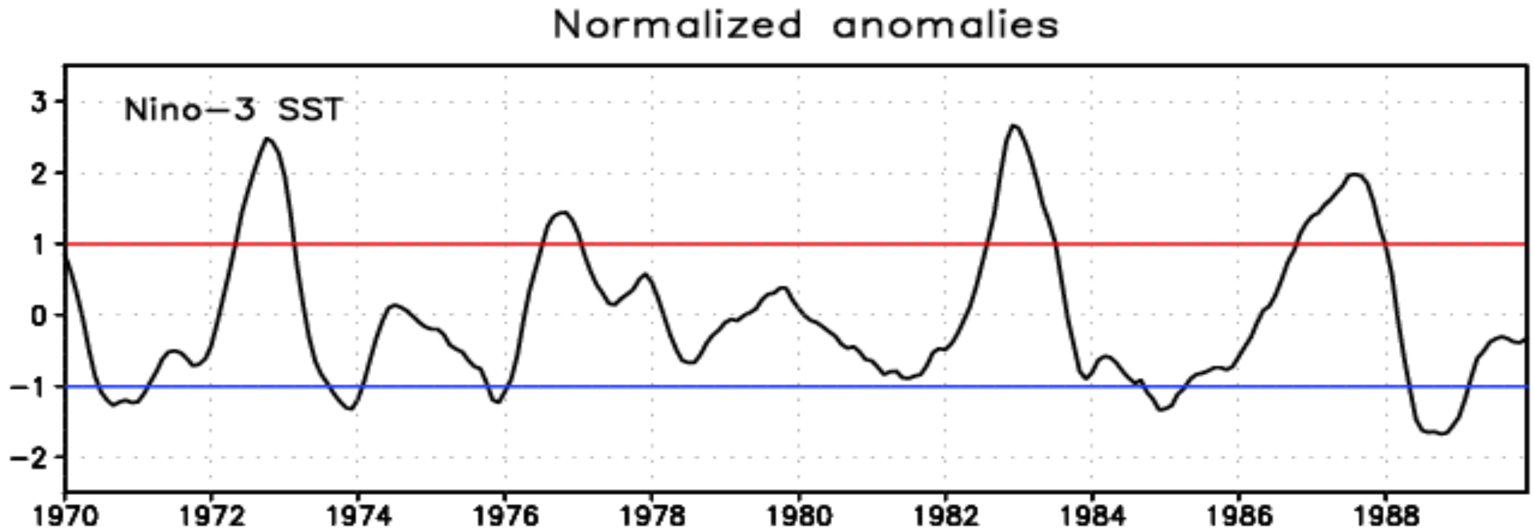
FIG. 7a. Model heating, based on the composite SST anomalies for the period centered around February, following the coastal warming.

FIG. 7b. Composite wind anomalies for the same time period as in 7a.

FIG. 7c. Model results for the heating in 7a.

Why does the response of atmosphere associated with El Nino SST anomalies not match observed low level winds during an El Nino?

Composites of atmospheric winds at surface (10 m) and 200 hPa, surface pressure, precipitation as well as SST anomalies during peaks of El Nino and La Nina.

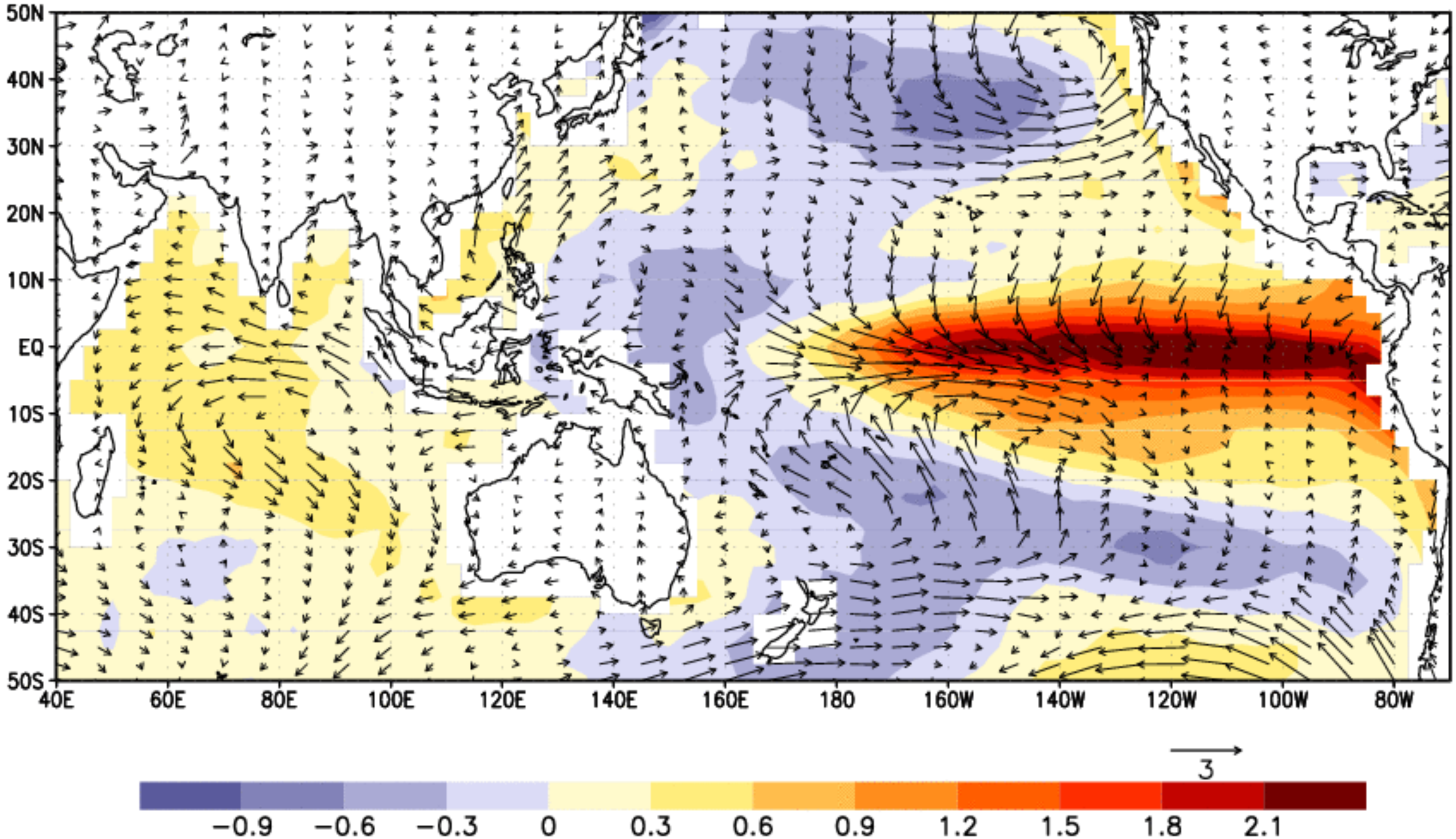


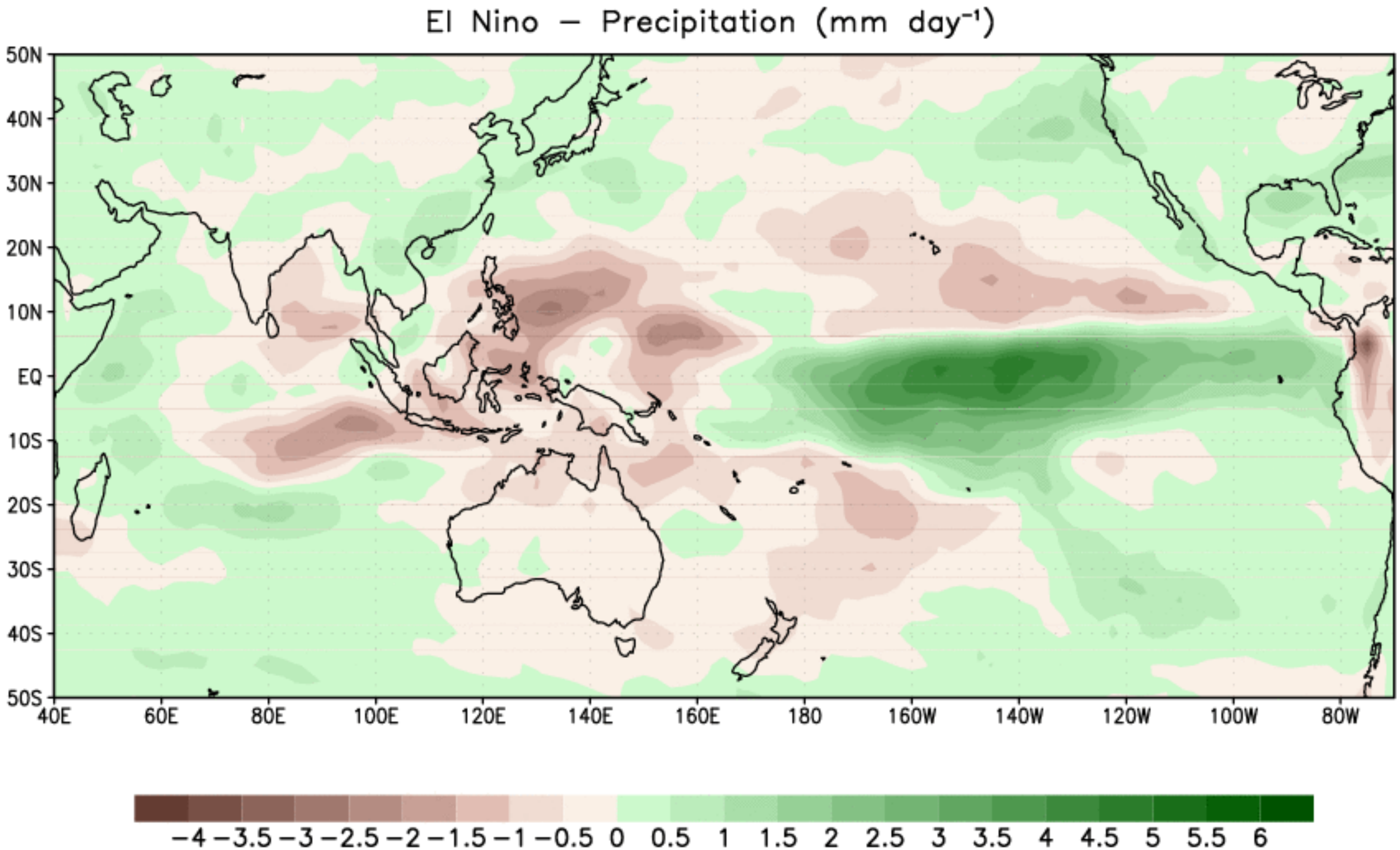
Composites :

Identify the peak months of El Nino/La Nina.

Average other fields for those months at all points.

El Nino – SST and Surface winds (10 m)





El Nino – Precipitation (mm day⁻¹)
50N
40N
30N
20N
10N
EQ
10S
20S
30S
40S
50S
40E
60E
80E
100E
120E
140E
160E
180
160W
140W
120W
100W
80W
−4 −3.5 −3 −2.5 −2 −1.5 −1 −0.5 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6

Do we see these waves predicted by theory in the Atmosphere ?

Wavenumber-frequency spectra of twice daily OLR in tropics showing Kelvin, Rossby and MRG waves (Wheeler and Kiladis, JAS,1999, vol.56,374pp)

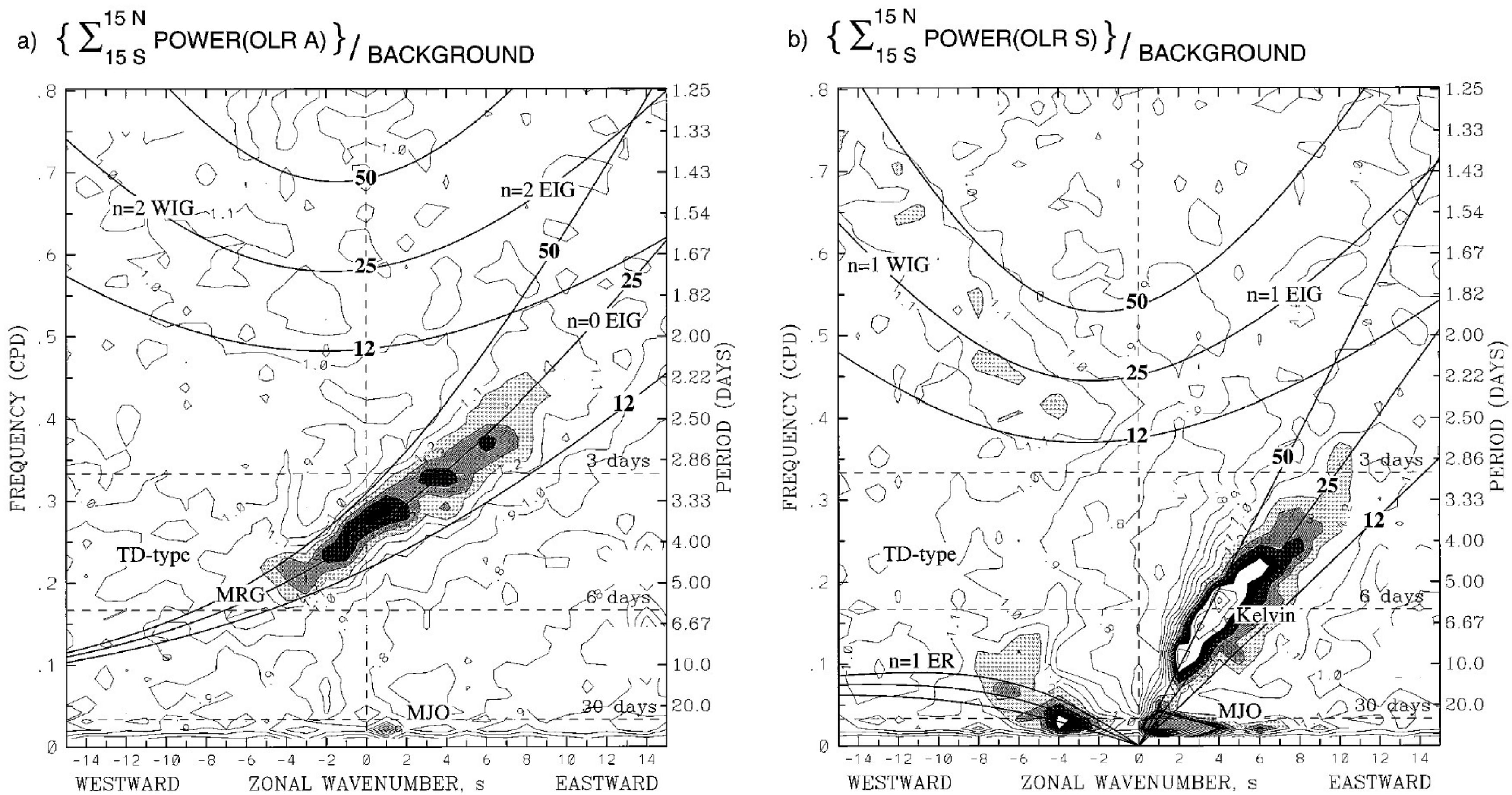


FIG. 3. (a) The antisymmetric OLR power of Fig. 1a divided by the background power of Fig. 2. Contour interval is 0.1, and shading begins at a value of 1.1 for which the spectral signatures are statistically significantly above the background at the 95% level (based on 500 dof). Superimposed are the dispersion curves of the even meridional mode-numbered equatorial waves for the three equivalent depths of $h = 12$, 25, and 50 m. (b) Same as in panel a except for the symmetric component of OLR of Fig. 1b and the corresponding odd meridional mode-numbered equatorial waves. Frequency spectral bandwidth is 1/96 cpd.

Moist Kelvin wave in the Atmosphere

So far we have discussed the free normal modes of the tropical atmosphere and examined some steady solutions due to externally imposed steady forcing. However, in most intersting phenomena, the heating distribution may be internally determined by the flow field itself. Let us discuss one such phenomenon namely the eastward propagating Madden and Jullian Oscillation. Discuss

1. Disucss Madden Jullian Oscillation from observation.

The main characteristics of the mode are

1. Propagates eastward and goes around the globe in about 40 days. The phase propagation in the easter Indian Ocean and Western Pacific are smaller but it is faster in the eastern Pacific. therefore, average phase speed

$$\sim \frac{360^{\circ} lat}{40 days} = \frac{360 \times 110 \times 10^3 m}{40 \times 24 \times 3600} \sim 11 m s^{-1}$$

2. Fluctuations of the zonal component of wind is large while that of the meridional wind is negligible.
3. It is equatorial trapped
4. First baroclinic vertical structure
5. Horizontal (zonal, wave length is about wave number one.

So, the phenomenon has many characteristics of a Kelvin wave. If it is dry Kelvin wave it should follow the dry dissipation relation (nondimensional).

$$w = k$$
$$\frac{1}{\bar{T}} = \frac{1}{\lambda}$$
$$\frac{T^*}{\widetilde{T}} = \frac{\lambda^*}{\widetilde{L}}$$

$$T^* = \frac{\lambda^* \widetilde{T}}{\widetilde{L}} = \frac{2\pi R \times 0.25}{1100Km} \approx 9days$$

or phase speed should have been about $35ms^{-1}$. Thus the observed phase speed is far too slow for a dry free Kelvin wave. Thus, it cannot be a dry Kelvin wave. However, it could be a moist Kelvin wave in which wave is associated with convergence and divergence of moisture.

For this purpose, Let us start with a little more general model. It is a two layer formulation with a first baroclinic mode vertical structure. The temperature equation is written in the middle layer and linearized form of moisture equation is added.

Moist Tropical waves

To understand this let us try to understood how the waves could be modified by internally generated heating. Tropical atmosphere is very moist. Convergence associated with the waves can produce low level moisture convergence (upward motion) and left the moisture to lifting condensation level and initiate free convection. This is known as Wave-CISK.

$$Q = -\alpha(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})$$

Let us put this in the basic equatorial β-plane equations(in nondimensional unit)

$$\frac{\partial u}{\partial t} - y = -\frac{\partial \phi}{\partial x} \tag{1}$$

$$\frac{\partial v}{\partial t} + y = -\frac{\partial \phi}{\partial y} \tag{2}$$

$$\frac{\partial \phi}{\partial t} + (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = -Q = +\delta(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) \tag{3}$$

or

$$\frac{\partial \phi}{\partial t} + (1-\delta)(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = 0 \tag{4}$$

Recall that the parameter in nondimensional term the divergence term in eq(3) is associated with a static stability parameter and is proportional to the gravity wave speed(C_0). The Wave-CISK essentially lead to a reduction of the effective static stability and hence reduction in the characteristic speed of the waves.

The Moist Kelvin wave

Let us write the basic equations in terms of potential temperature perturbation (θ) instead of geopotential perturbation(ϕ)

$$\frac{\partial u}{\partial t} - yv = \frac{\partial \theta}{\partial x} - \epsilon u$$

$$\frac{\partial v}{\partial t} + yu = \frac{\partial \theta}{\partial y} - \epsilon v$$

$$\frac{\partial \theta}{\partial t} - (1-\alpha)(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) = -\Lambda u - \epsilon \theta$$

Let $(1-\alpha) = \Gamma$= effective static stability

Λu = represents heating due to evaporation caused by wind (evaporation-wind feedback)

For Kelvin wave, v=0 therefore,

$$\frac{\partial u}{\partial t} + \epsilon u = \frac{\partial \theta}{\partial x}$$

$$yu = \frac{\partial \theta}{\partial y}$$

$$\frac{\partial \theta}{\partial t} - \Gamma \frac{\partial u}{\partial x} + \Lambda u + \epsilon \theta = 0$$

$v = 0$, $R = \epsilon$ (change in notation)

Let us consider $(u, \theta) \sim e^{i(kx - wt)}$

From equations(4), we get

$$-iwu + \epsilon u = k\theta \tag{6.1}$$

$$\frac{1}{2} yu = \frac{\partial \theta}{\partial y} \tag{6.2}$$

$$-iw\theta - \Gamma iku + \Lambda u + \epsilon \theta = 0 \tag{6.3}$$

from eqn(6.3)

$$(-iw+\epsilon)\theta = (ik\Gamma - \Lambda)u$$

or

$$u = \frac{-iw+\epsilon}{ik\Gamma - \Lambda}\theta \tag{7}$$

substituting(7) in (6.1)

$$(-iw+\epsilon)\frac{-iw+\epsilon}{ik\Gamma - \Lambda}\theta = ik\theta$$

or

$$(-iw+\epsilon)^2 = (ik\Gamma - \Lambda)ik \tag{8}$$

In the absence of dissipation $\epsilon = 0$

$$-w^2 = (-k^2\Gamma - ik\Lambda) = -k(k\Gamma + i\Lambda)$$

or

$$w^2 = k(k\Gamma + i\Lambda) \tag{9}$$

Thus there are two possibilities

1) $\Gamma > 0$

CISK is not able to overcome cooling by ascent. In this case in absence of $\epsilon - w$ feedback $\Lambda = 0$, w is real and the Kelvin mode is stable. For $\Lambda \neq 0$, Let us assume $w = w_r + iw_i$.

therefore $(w_r + iw_i)^2 = k^2\Gamma + ik\Lambda$

or,

$$w_r^2 - w_i^2 = k^2\Gamma \tag{10.1}$$

and

$$2w_r w_i = k\Lambda \tag{10.2}$$

therefore

$$w_r^2 - \frac{k^2\Lambda^2}{4w_r^2} = k^2\Gamma$$

or

$$4w_r^4 - 4w_r^2k^2\Gamma - k^2\Lambda^2 = 0$$

or

$$w_r^4 - w_r^2k^2\Gamma - \frac{k^2\Lambda^2}{4} = 0$$

or

$$w_r^2 = \frac{k^2\Gamma}{2} \pm \frac{1}{2}(k^4\Gamma^2 + k^2\Lambda^2)^{1/2}$$

therefore,

$$w_r = \pm\left[\frac{k^2\Gamma \pm (k^4\Gamma^2 + k^2\Lambda^2)^{1/2}}{2}\right]^{1/2} \qquad (12)$$