Far-from-equilibrium universality and spectral functions in the QGP

TECHNISCHE UNIVERSITÄT WIEN

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Extreme Nonequilibrium QCD, ICTS Bangalore

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Talk based on different publications in collaboration with

Berges, Kurkela, Lappi, Mace, Pawlowski, Peuron,

Piñeiro Orioli, Schlichting, Venugopalan

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Literature

Universal classical attractors

Berges, KB, Schlichting, Venugopalan, *PRD 89, 114007 (2014)* ; *PRD 89, 074011 (2014)* ; *JHEP 05, 054 (2014)* ;
PRL 114, 061601 (2015) ; *PRD 92, 096006 (2015)*

KB, Kurkela, Lappi, Peuron, *PRD 100, 094022 (2019)*

Spectral functions of pre-equilibrium QGP

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)* ; KB, Pinerio Orioli, *PRD 101, 091902 (2020)*

Heavy-quark diffusion & gluonic IR excess

KB, Kurkela, Lappi, Peuron, *JHEP 09, 077 (2020)* ; Berges, KB, Mace, Pawlowski, *PRD 102, 034014 (2020)*

Motivation: Heavy-ion collisions (HIC)

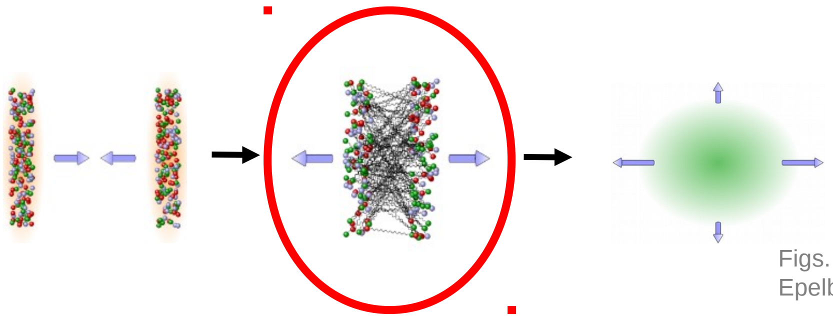
- Main facilities:
  - ➔ Large Hadron Collider (LHC) @ CERN
  - ➔ Relativistic Heavy Ion Collider (RHIC) @ BNL

- Some of the main objectives:
  - ➔ Quantum Chromodynamics (QCD) at high energies
  - ➔ Quark-Gluon plasma (QGP)

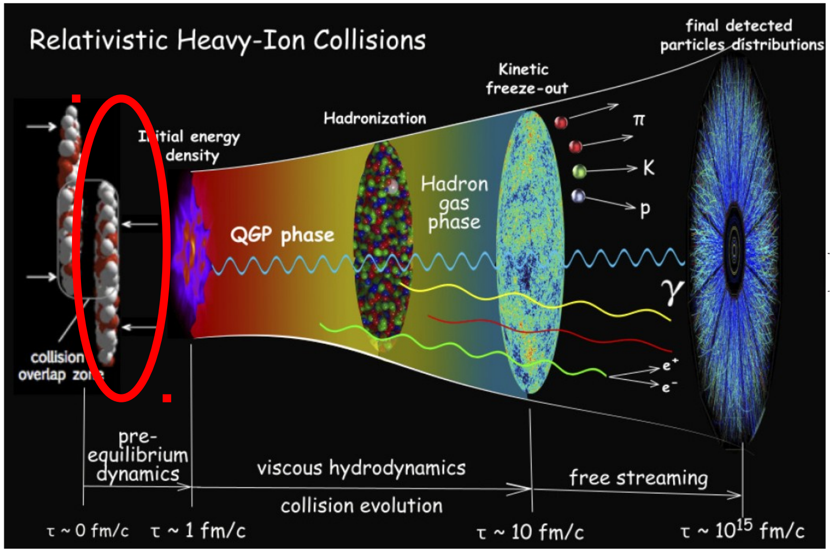
  (QGP thought to have existed in the Early Universe)

LHC @ CERN (Photo: Maximilen Brice)

Motivation: Initial stages of HIC



Figs. by T. Epelbaum

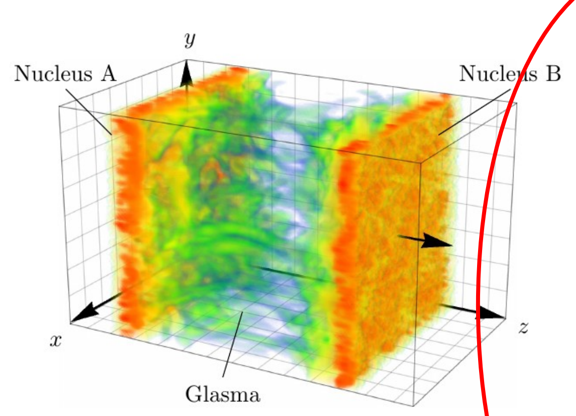


Little Bang by P. Sorensen and C. Shen

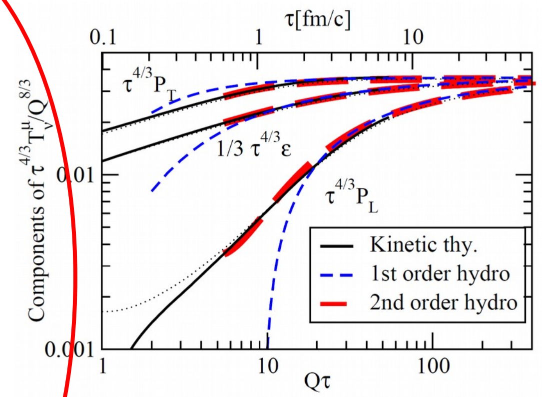
- *How does a pre-equilibrium QGP evolve?*
- Here high energy (weak coupling) limit in HIC considered $g^2 \equiv 4\pi\alpha_s \ll 1$
- Initially (large) gauge fields $A \sim 1/g$
- Thus large occupation numbers $f \sim 1/g^2 \gg 1$

McLerran, Venugopalan (1999); Krasnitz, Venugopalan (1999, 2000, 2001); …

Motivation: Initial stages of HIC



Ipp, Müller, *PLB 771 (2017), 74*



Kurkela, Zhu, *PRL 115, 182301 (2015)*

...

τ

Time axis:

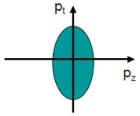
Methods: Classical Yang-Mills simulations — Effective kinetic theory, KØMPØST

Physics: Glasma — Bottom-up scenario

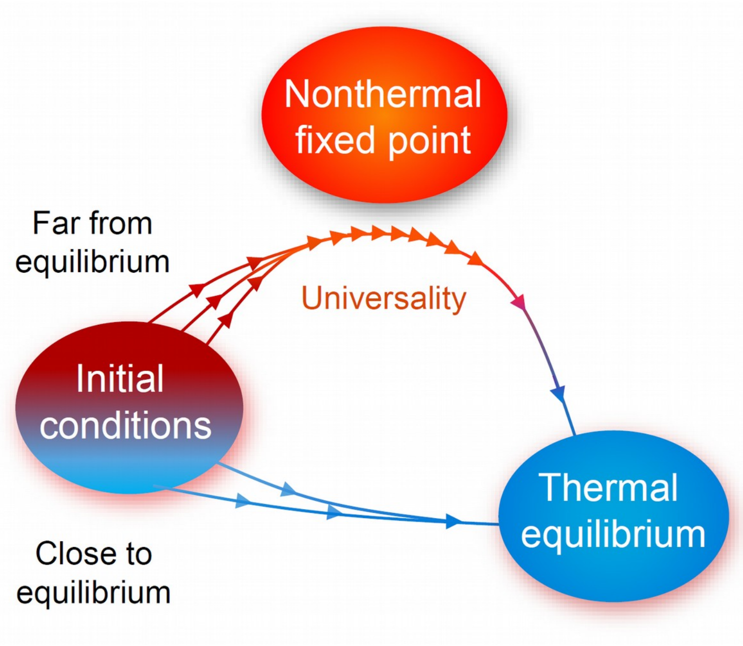
$\Rightarrow$ *Talk by D. Müller* — $\Rightarrow$ *Talk by S. Schlichting*

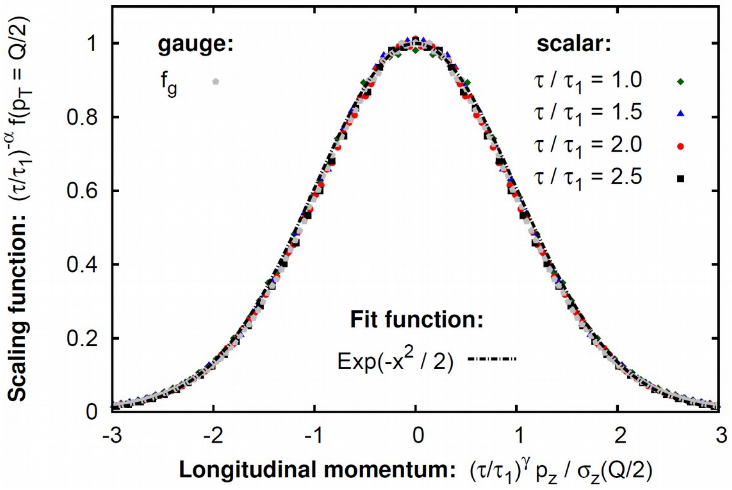
Overlapping region: Over-occupied plasma with finite anisotropy around $\tau \sim 0.1\,\frac{\mathrm{fm}}{c}$

$$f \sim 1/g^2 \gg 1\,,$$



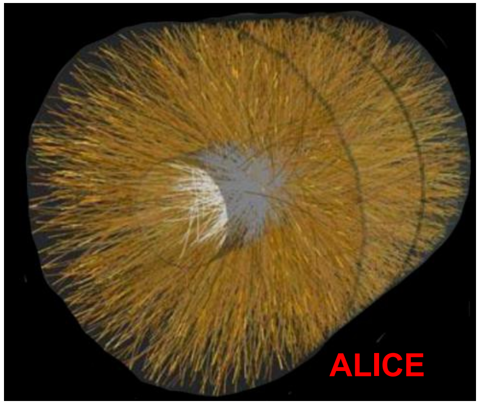
Universal classical attractors in gluonic plasmas





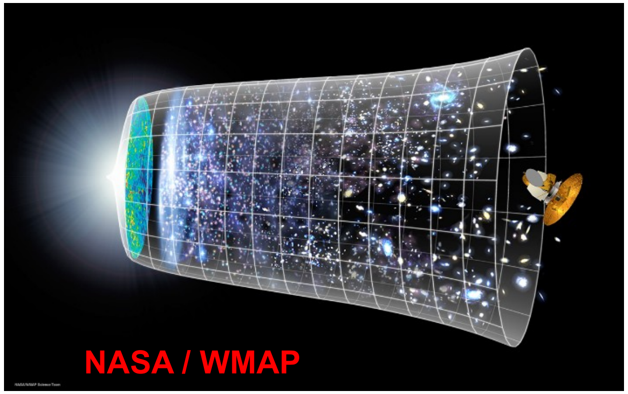
Classical attractors can emerge in

Heavy-ion collisions



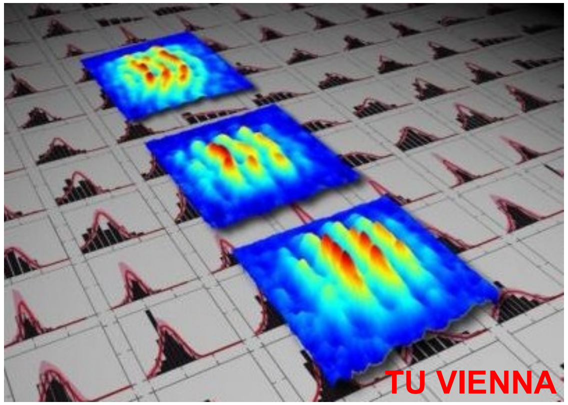
Longitudinally expanding non-Abelian plasmas

Early Universe



Relativistic scalar systems (O(N), $\lambda(\phi_a\phi_a)^2$)

Ultracold atoms

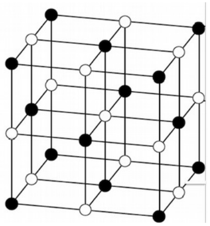


Non-relativistic scalar systems (GPE)

- When couplings $g, \lambda \ll 1$ weak, closed system highly occupied $f \gg 1$
- **Dual description:** class.-stat. simulations for $f \gg 1$, kinetic theory for $f \ll 1/g^2$

"Non-perturbative" vs. "Perturbative"

Classical-statistical simulations



- At initial time t_0, choose quantum (Gaussian) initial conditions (IC):
  Choose $\langle AA \rangle$, $\langle EE \rangle$, with $E_a^j(t,\mathbf{x}) = \partial_t A_j^a(t,\mathbf{x})$
  (formally, this defines a weight functional $W\left[A(t_0), E(t_0)\right]$)
- Solve classical field equations in temporal gauge $A_0 = 0$ on a lattice

$$D_\mu F^{\mu\nu} = 0 \qquad (\text{with } D_\mu = \partial_\mu - igA_\mu\,, \quad F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right])$$

- Use links $U_\mu(t,\mathbf{x}) \approx e^{iga_s A_\mu(t,\mathbf{x})}$ for a gauge covariant formulation
- Evolve fields, obtain observables at time t by averaging over IC

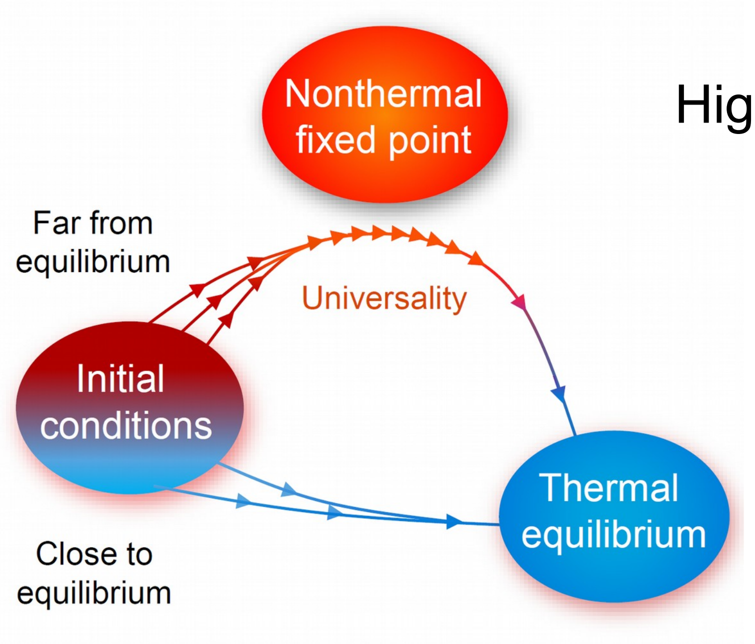
$$O(t) = \langle O_{cl}[A(t), E(t)] \rangle = \int DA(t_0) DE(t_0)\, W\left[A(t_0), E(t_0)\right] O_{cl}\left[A(t), E(t)\right]$$

- Distinguish polarizations in Fourier space

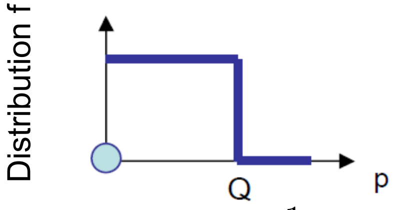
$$E_L(t,\mathbf{p}) = \frac{p_j}{p} E^j(t,\mathbf{p})\,, \qquad E_T^j(t,\mathbf{p}) = \left(E^j(t,\mathbf{p}) - \frac{p^j}{p} E_L(t,\mathbf{p})\right)$$

- Distribution function: $2(N_c^2 - 1) f(t,p) \approx \langle |A_T(t,\mathbf{p})|^2 \rangle\, \omega(t,p) \approx \dfrac{\langle |E_T(t,\mathbf{p})|^2 \rangle}{\omega(t,p)}$

Classical attractors far from equilibrium (for $g^2 \ll 1$)



Highly occupied:



$$f(t_0, p \lesssim Q) \sim \frac{1}{g^2}$$

Nonthermal fixed point (NTFP)

- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

Distribution function (occupation numbers): $f(t,p) = t^\alpha f_s(t^\beta p)$

First experimental observations:

Prüfer et al., *Nature 563, 217 (2018)*

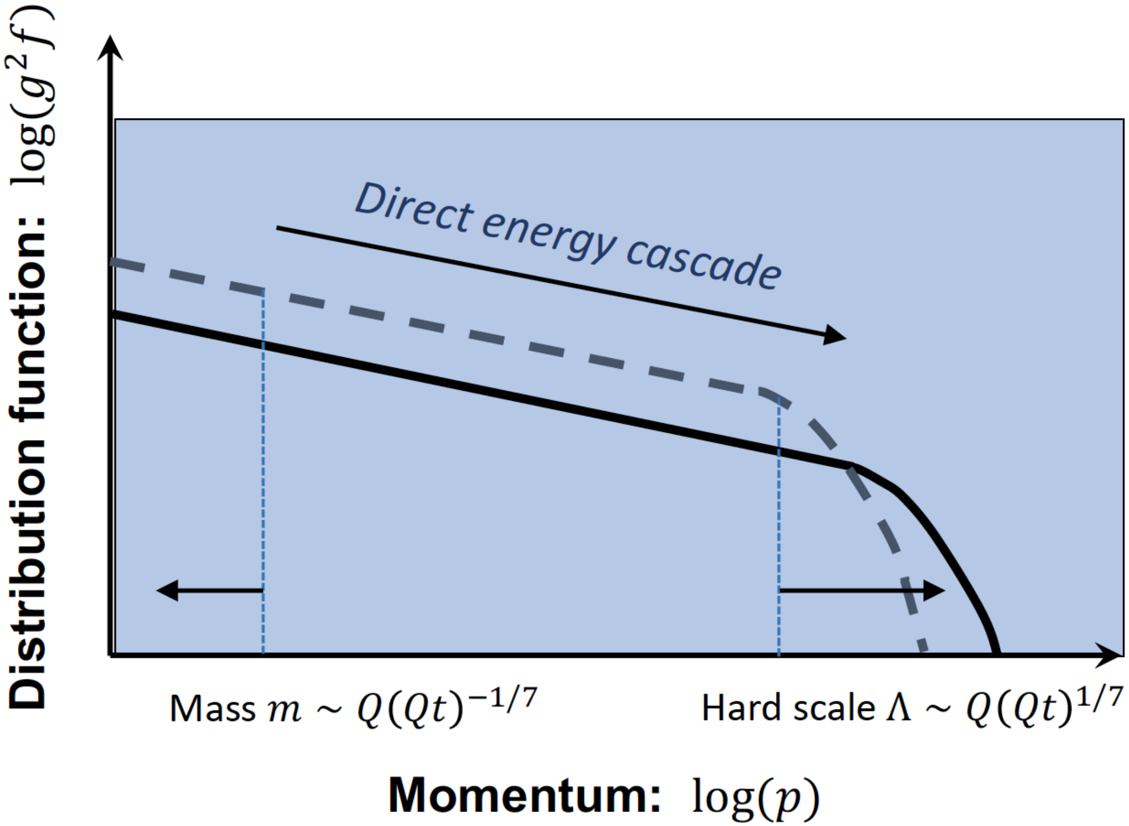
Erne et al., *Nature 563, 225 (2018)*

NTFP: *Micha, Tkachev (2004); Berges, Rothkopf, Schmid (2008)*

Universality far from equilibrium: *Berges, KB, Schlichting, Venugopalan (2015); Piñeiro Orioli, KB, Berges (2015)*

Classical attractors: direct energy cascade

in isotropic 3+1D gluonic plasmas



Self-similar evolution

$$f(t,p) = t^\alpha f_s(t^\beta p)$$

During evolution

$$\Lambda = \langle p \rangle \ll T \,, \;\; f \gg 1$$

Turbulent-like power law spectrum

$$f_s(p) \sim p^{-k}$$

Universal exponents

$$\alpha = \quad 4\beta$$

$$\beta = -1/7$$

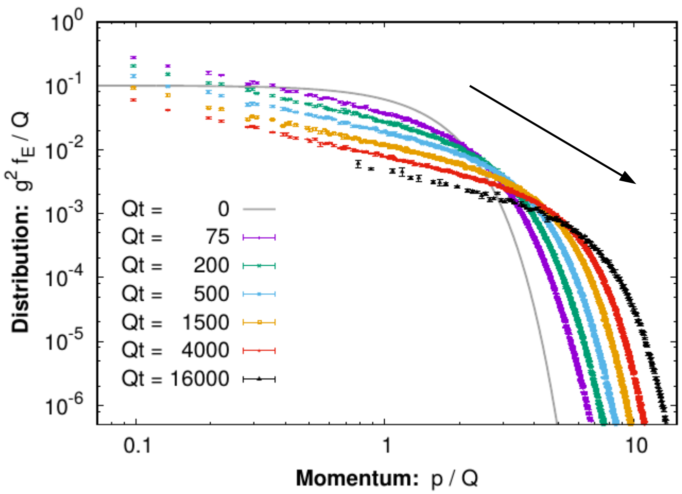
Berges, Scheffler, Sexty (2009); Kurkela, Moore (2011, 2012); Berges, Schlichting, Sexty (2012); Schlichting (2012); Berges, KB, Schlichting, Venugopalan (2014); York, Kurkela, Lu, Moore (2014); …

Classical attractor in gluonic 2+1D and Glasma-like theories

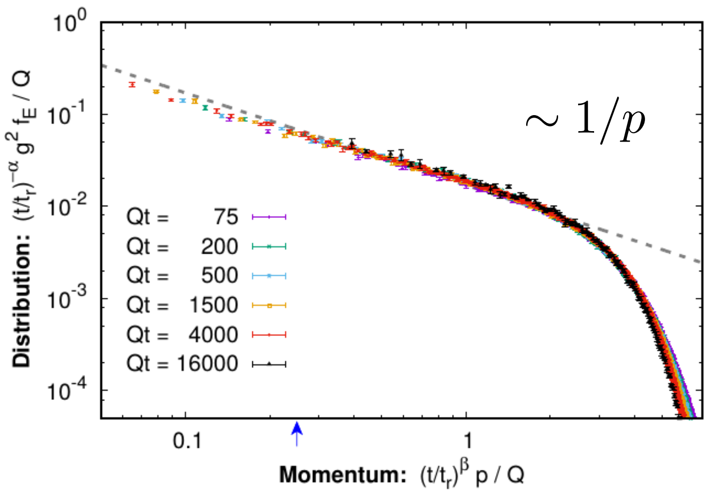
But no expansion, over-occupied initial conditions

KB, Kurkela, Lappi, Peuron, *PRD 100, 094022 (2019)*

Original



Rescaled



Evolution

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

Universal exponents:

$\alpha = \quad 3\beta$ Energy conservation $\int d^d p\, p\, f(t,p) \approx const$

$\beta = -1/5$ Different from 3+1D (there -1/7)

(→ Although for 2+1D, HTL framework breaks down at soft scale m,
kinetic arguments lead to observed exponents, quasiparticles exist!)

Classical attractors: heavy-ion collisions (shortly after collision)

Berges, KB, Schlichting, Venugopalan, *PRD 89, 114007 (2014) ; PRD 89, 074011 (2014)*

Self-similar evolution

$$f(t, p_T, p_z) = t^{\alpha} f_s(t^{\beta} p_T, t^{\gamma} p_z)$$

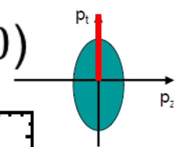
Universal exponents

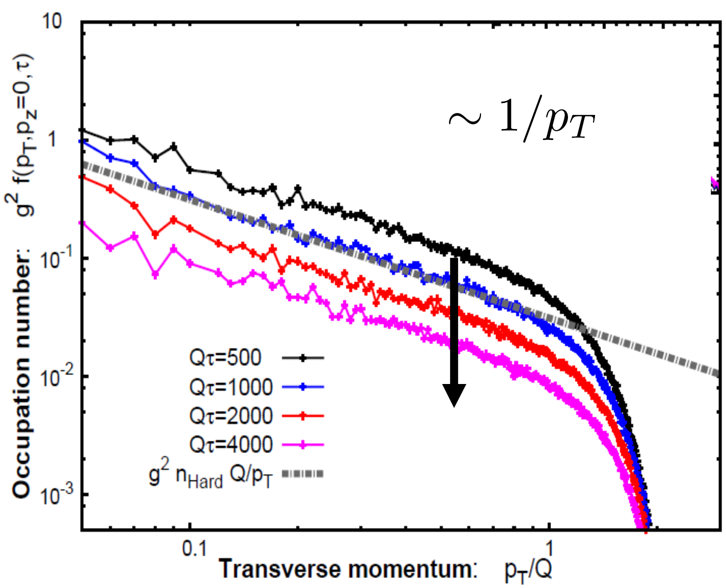
$$\alpha \simeq -2/3$$
$$\beta \simeq 0$$
$$\gamma \simeq 1/3$$

Expanding metric:

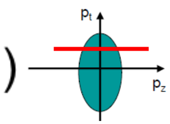
$$\tau = \sqrt{t^2 - z^2}, \quad \eta = \text{atanh}\left(\frac{z}{t}\right)$$
$$g_{\mu\nu} = \text{diag}\left(1, -1, -1, -\tau^2\right)$$

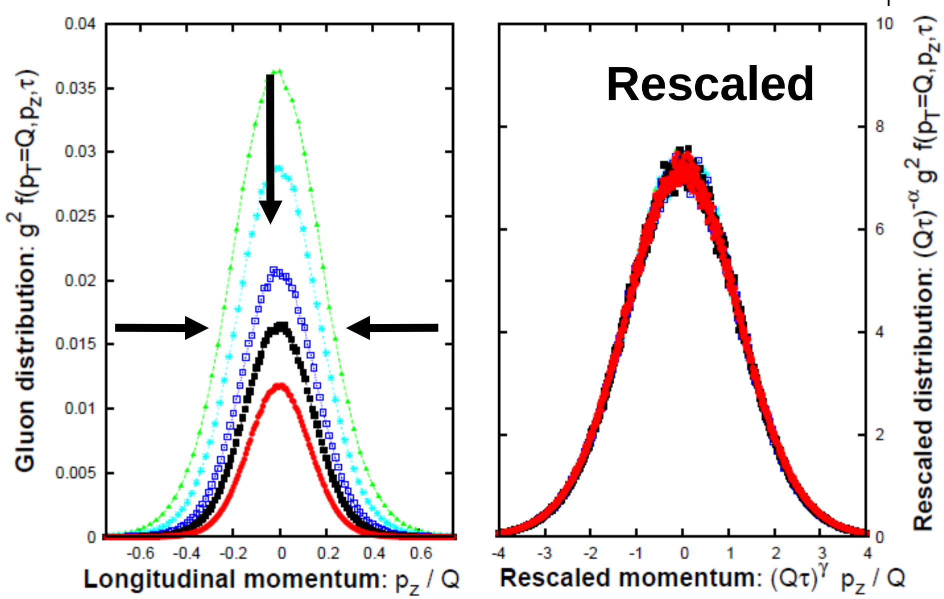
Transverse distribution $(p_z = 0)$





Longitudinal distribution $(p_T = Q)$



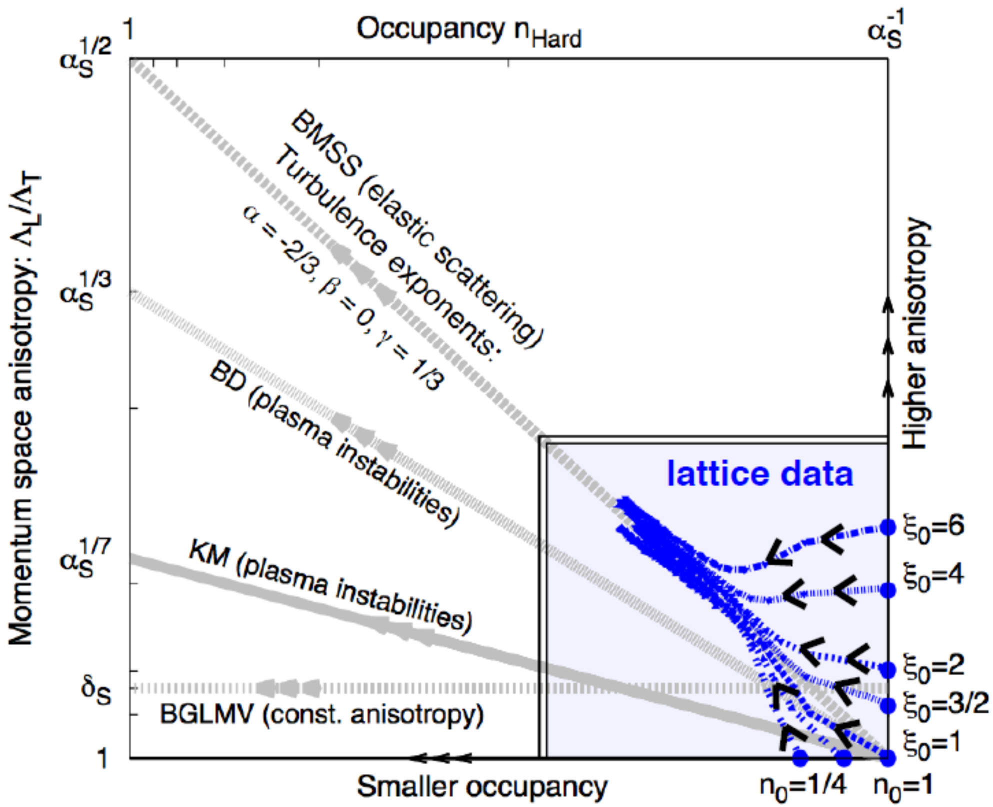


Classical attractors: Distinguishing kinetic descriptions

Real-time lattice simulations

Berges, KB, Schlichting, Venugopalan,

PRD 89, 114007 (2014), PRD 89, 074011 (2014)



Kinetic descriptions

- Baier, Mueller, Schiff, Son (BMSS), (2001)
- Bodeker (BD), (2005)
- Kurkela, Moore (KM), (2011)
- Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

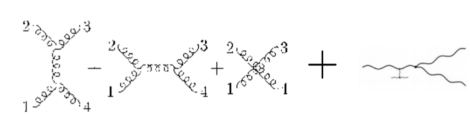
- Numerics consistent with 1st stage of BMSS scenario

  Baier, Mueller, Schiff, Son, *PLB 502, 51 (2001)*

- Effective kinetic theory (EKT)

  Arnold, Moore, Yaffe, *JHEP 0301, 030 (2003)*

$$\frac{\mathrm{d}f}{\mathrm{d}\tau} - \frac{p_z}{\tau}\frac{\partial f}{\partial p_z} =$$



- Used as standard description now

Classical attractors: Universality class far from equilibrium

Universality with scalar $\lambda\phi^4$ field theory in suitable p region

Berges, KB, Schlichting, Venugopalan,
PRL 114, 061601 (2015), Editors' Suggestion
PRD 92, 096006 (2015)

Self-similar evolution

$$f(t, p_T, p_z) = t^\alpha f_s(t^\beta p_T, t^\gamma p_z)$$

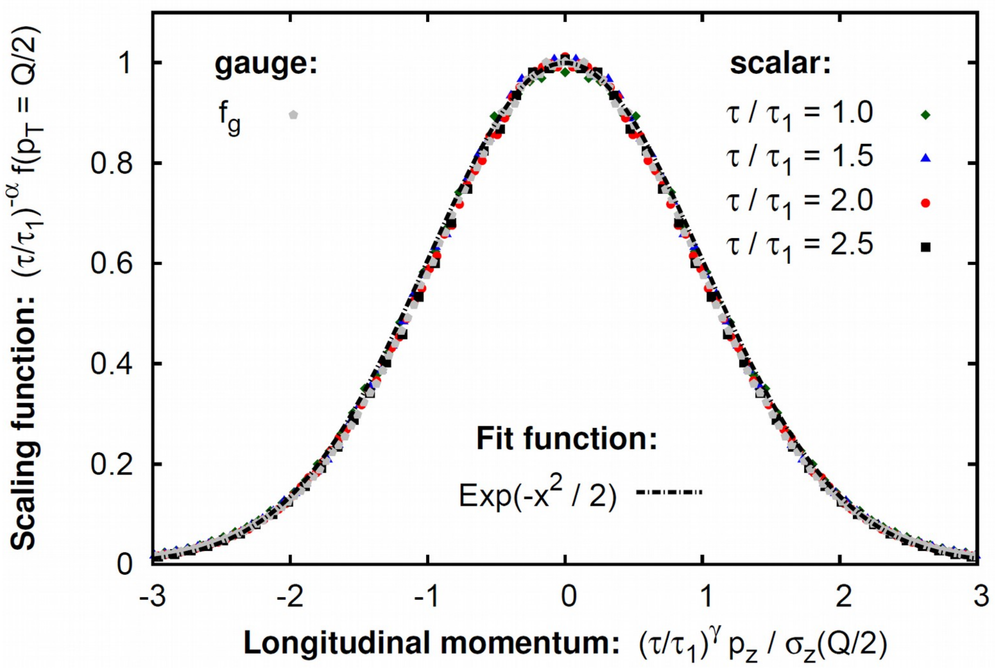
Same scaling

$$f_s(p),\ \alpha,\ \beta,\ \gamma$$

as in gauge theory!

Same attractor,
universality far from equilibrium

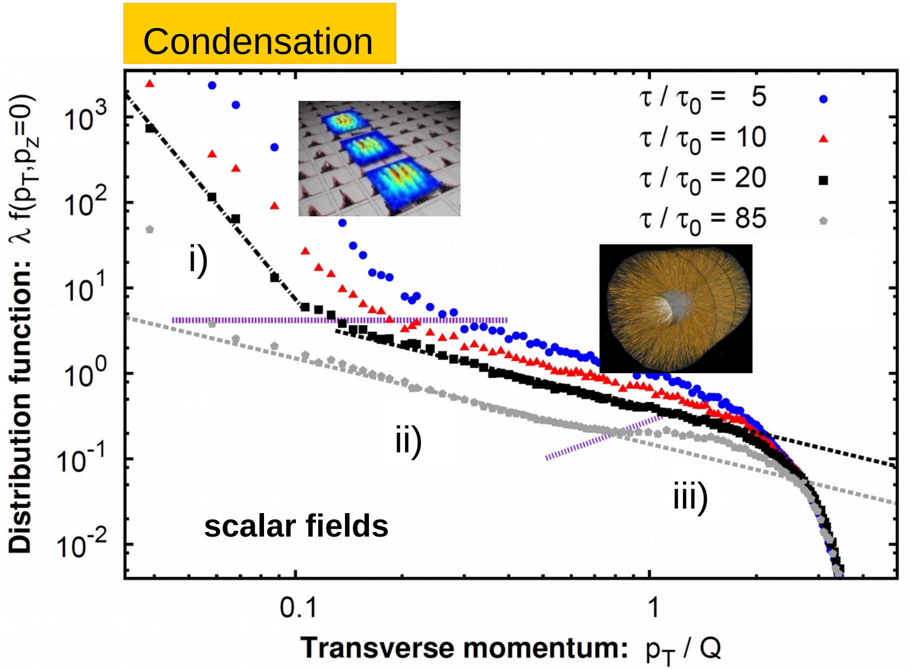
Rescaled distributions $(p_T = Q/2)$



Classical attractors in expanding scalar system

Link between the world's hottest and coldest matter

Berges, KB, Schlichting, Venugopalan, *PRD 92, 096006 (2015)*

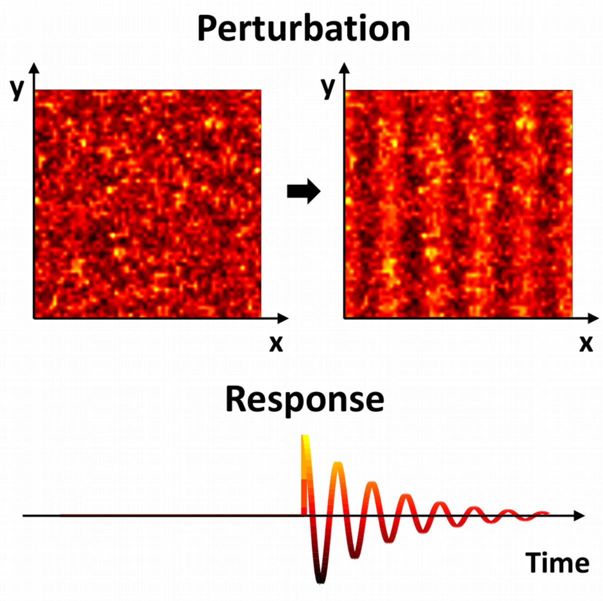


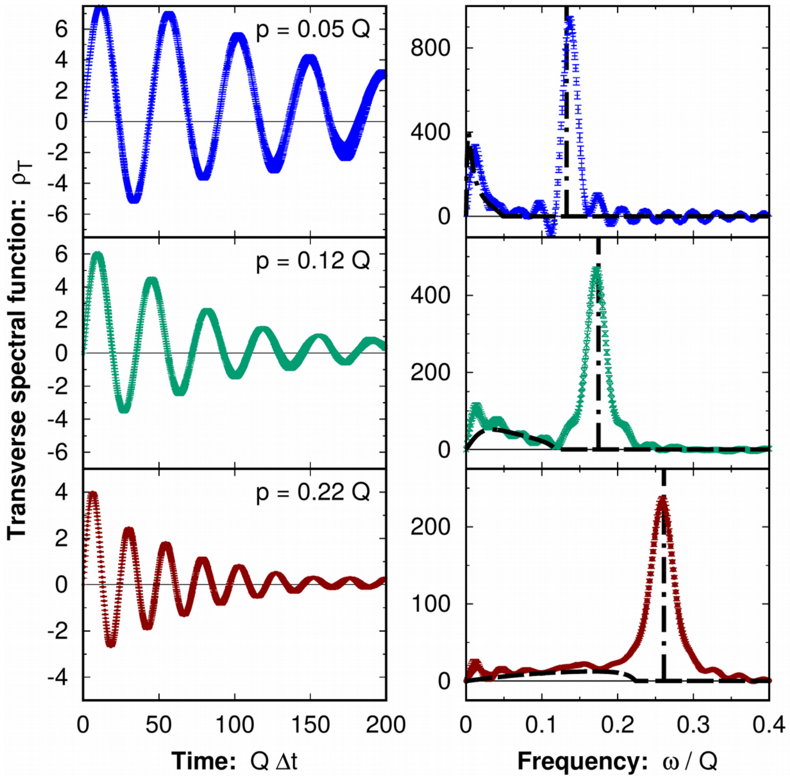
- Classical attractor in heavy-ion collisions emerges in p region ii)

- For classical infrared attractor i) in scalar systems, ultra-cold atoms:

  $\Rightarrow$ **See Backup**

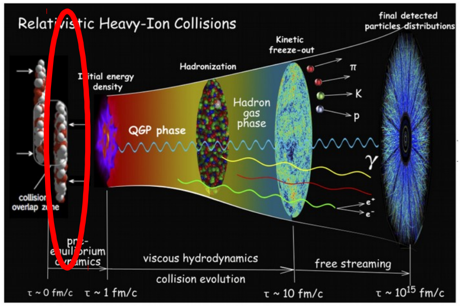
Spectral functions ρ





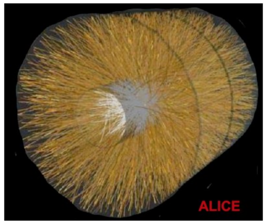
Spectral functions: some hard problems

- Initial stages in heavy-ion collisions :
  - ✔ Kinetic theory requires dominance of quasiparticles
  - ✔ What excitations are there? When is kinetic theory valid?

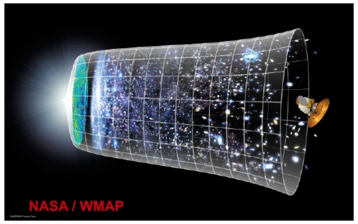


- Excitations spectrum of QCD, scalars out of equilibrium?

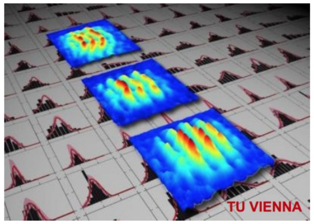
Heavy-ion collisions



Early Universe



Ultra-cold atoms



$\Rightarrow$ Knowledge of spectral functions or unequal time correlators needed!

- Perturbatively, hard loop (HTL) framework can be used to compute ρ if $m/\Lambda \ll 1$

  (For thermal case, HTL works for $g \sim m/T \ll 1$)

Spectral functions: hard-thermal loop (HTL) theory

Braaten, Pisarski (1990);
Blaizot, Iancu (2002); …

Perturbative formalism, usually applied to thermal equilibrium, high $T \gg T_c$ for $\omega, p \ll T$

Applicable out of equilibrium when:

- Hard scale Λ, soft modes m (mass), scale separation $m/\Lambda \ll 1$

  $\Rightarrow$ HTL can be applied to attractor in non-Abelian plasmas!

- Details of $f(t,p)$ “hidden” in few parameters

$$m^2 \approx 2N_c \int \frac{\mathrm{d}^3p}{(2\pi)^3} \frac{g^2 f(t,p)}{p}$$

Our strategy:

- Compute spectral function non-perturbatively using lattice simulations
- Use over-occupied systems, compare to HTL expressions

Spectral functions: HTL self-energies at LO

Polarization tensor: $(x = \omega/p)$

$$\Pi_T(x) = m^2\, x \left(x + (1-x^2)\frac{1}{2}\ln\frac{x+1}{x-1} \right)$$

$$\Pi_L(x) = -2m^2 \left(1 - x\,\frac{1}{2}\ln\frac{x+1}{x-1} \right)$$

Self-energies obtain imaginary part for

$$|x| < 1 \quad \Leftrightarrow \quad |\omega| < p$$

$\Rightarrow$ Landau cut / damping!

Retarded propagator: (for $A_0 = 0$ gauge)

$$G_T(\omega,p) = \frac{-1}{\omega^2 - p^2 - \Pi_T(\omega/p)}$$

$$G_L(\omega,p) = \frac{p^2}{\omega^2}\,\frac{-1}{p^2 - \Pi_L(\omega/p)}$$

Spectral function:

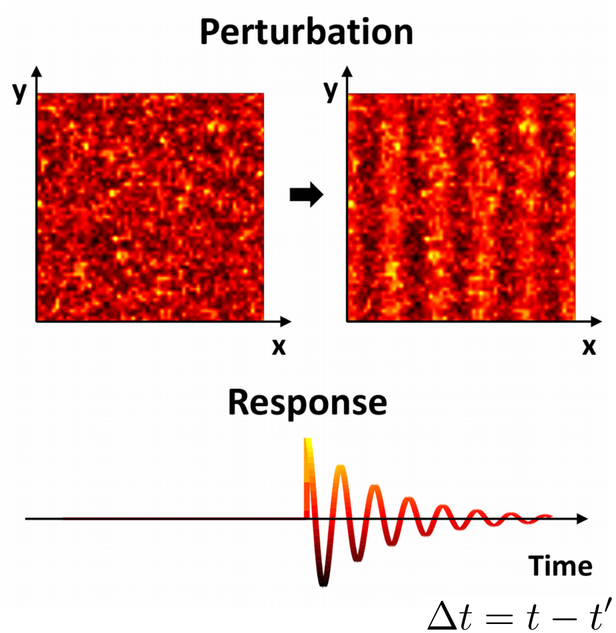
$$\rho_{T/L}(\omega,p) = 2\,\mathrm{Im}\,G_{T/L}(\omega,p)$$

The mass computed within HTL; no „free“ parameters left!

Spectral functions: non-perturbative computation

Classical-statistical SU(N) simulations + linear response

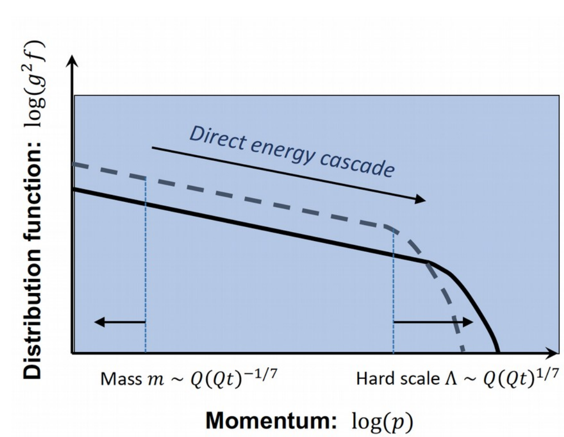
KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018), Editors' Suggestion



- Perturb (classical) field $A_j(t, \mathbf{x}),\ E^j(t, \mathbf{x})$
- Follow linearized EOM: Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- Response: $G_R(t, t', p) = \theta(t - t')\rho(t, t', p)$

Similar methods for scalars:

Aarts (2001); Pineiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Pineiro Orioli (2020)



Applied here to classical attractor in 3+1D gluon plasmas

$(m/\Lambda \ll 1)$

Spectral functions: highly occupied gluons

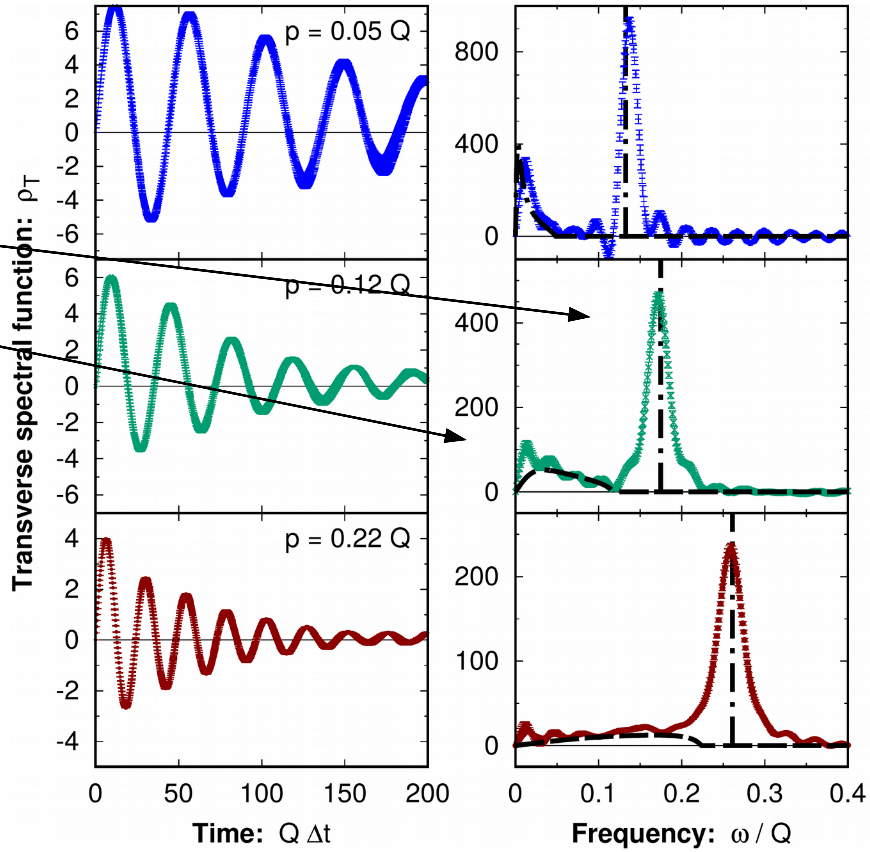
KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)

Transverse spectral function ρ_T (here for *SU(2)*)

$$\rho(t,t',\mathbf{x}-\mathbf{x}') \sim \langle [A(t,\mathbf{x}), A(t',\mathbf{x}')] \rangle$$

ρ_T as function of $\Delta t = t - t'$ (left) or frequency ω (right) at late time $t, t' \gg \Delta t$

- Lorentzian peaks:
  existence of quasi-particles
- for $|\omega| \leq p$: *Landau damping*
- black dashed lines:
  Perturbative expectation (HTL at LO)



✔ Good agreement with HTL at LO!

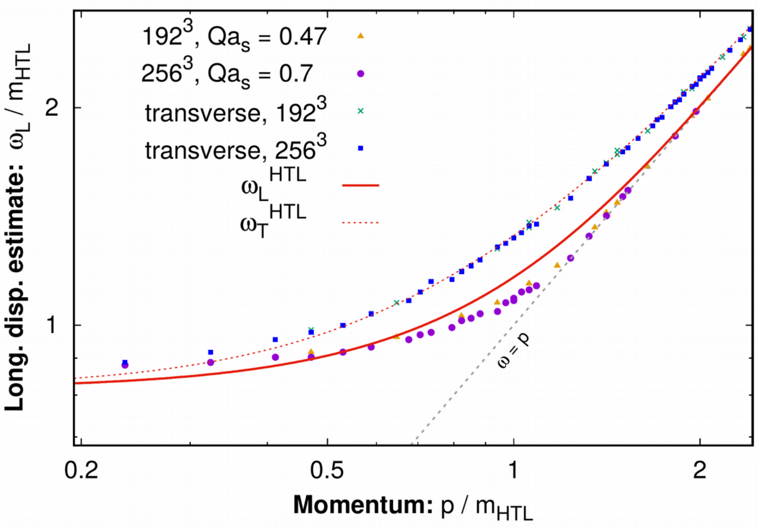
✔ System dominated by quasiparticles with narrow width (beyond HTL at LO)!

Spectral functions: dispersion relations

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

- Extracted from peak position (for ω_L after subtracting HTL Landau cut)
- *Similar to HTL* predictions: $\omega_{T,L}^{\mathrm{HTL}}(p)$
- Deviations at small p, for finite m/Λ?
- "$\omega_L(p)$" deviates at $p \sim m$ because peak is smaller than Landau cut, harder to measure

$\omega_{T,L} \,/\, m_{\mathrm{HTL}}$



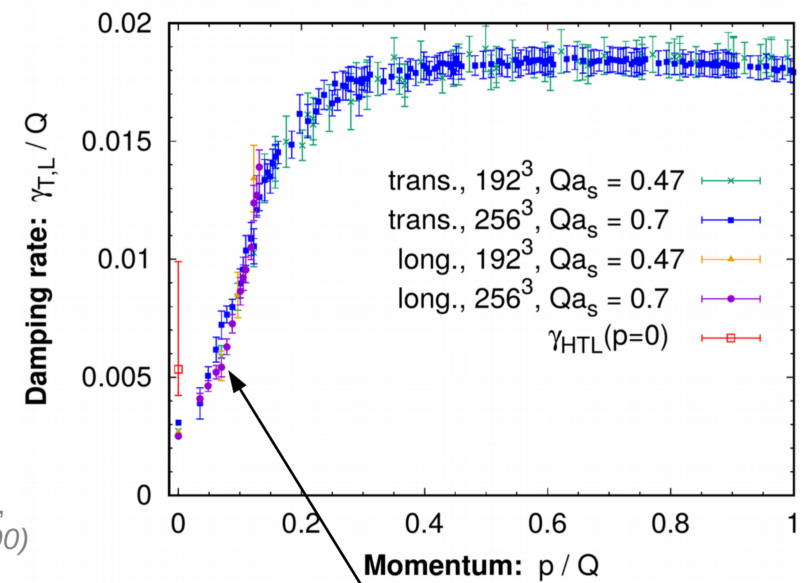
Spectral functions: extracted damping rates

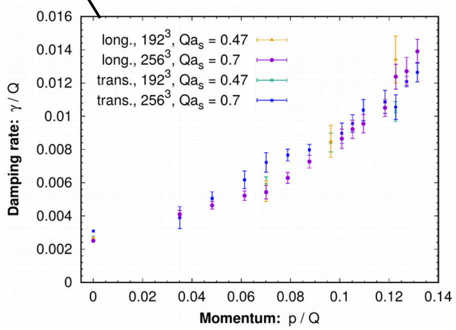
KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

- $\gamma_{T,L}(p)$ is $\mathcal{O}(g^2 Q)$ and *beyond HTL at LO*, it may contain non-perturbative contributions (*magnetic scale*)
- Here *first determination* of $\gamma_{T,L}(p)$!
- Extracted by fitting to a damped oscillator
- HTL prediction: $\gamma_{\mathrm{HTL}}(p=0)$ — Braaten, Pisarski, *PRD 42, 2156 (1990)*

  $\gamma_{\mathrm{HTL}}(p \to \infty)$: not included here, too large uncertainties due to necessary estimation of magnetic scale — Pisarski, *PRD 47, 5589 (1992)*

- "Isotropic" $\gamma_T \approx \gamma_L$ for $p \lesssim m$ while dispersions different $\omega_T > \omega_L$

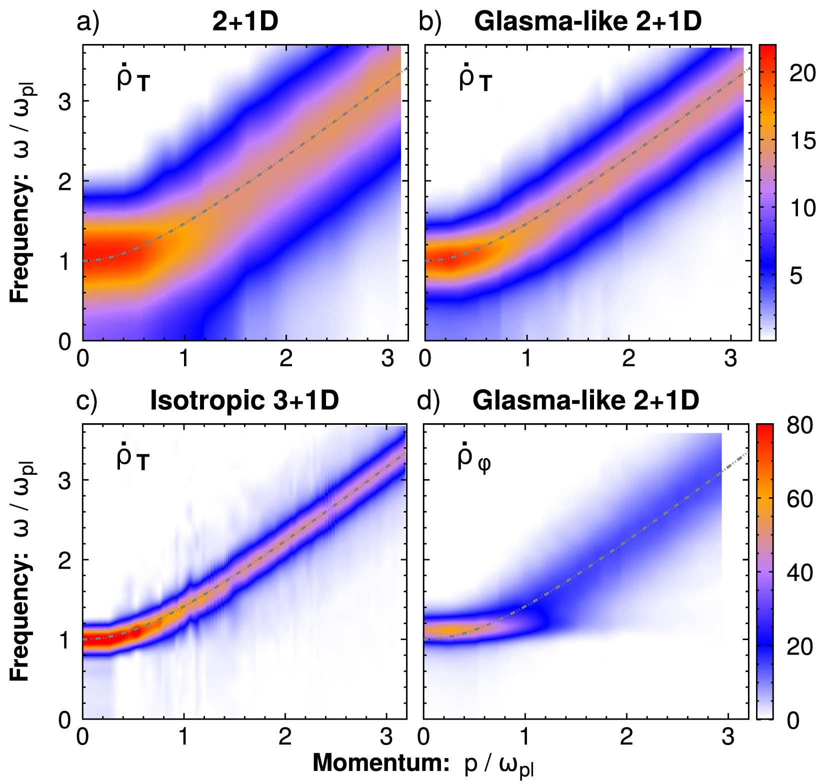




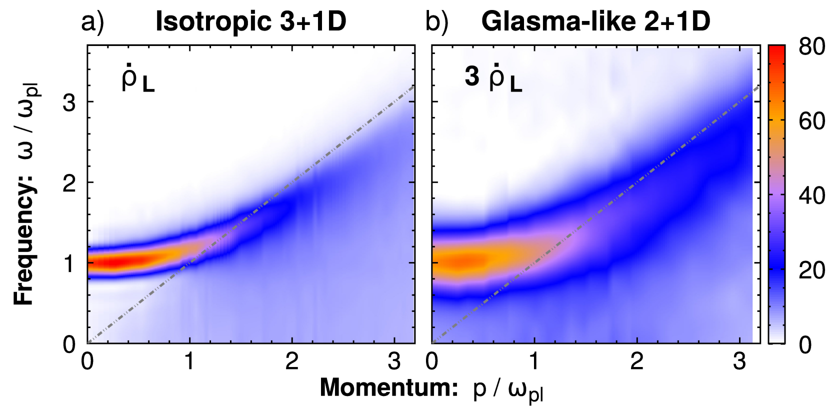
Spectral functions in gluonic 3+1D and 2+1D plasmas

KB, Kurkela, Lappi, Peuron, *in preparation*

Transverse



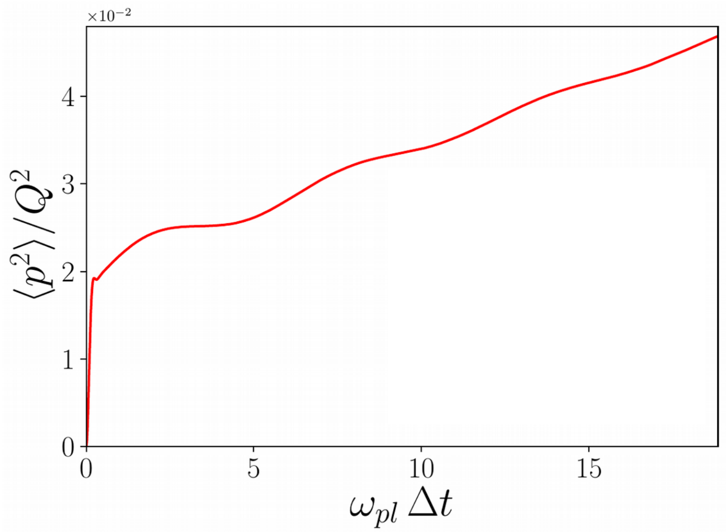
Longitudinal

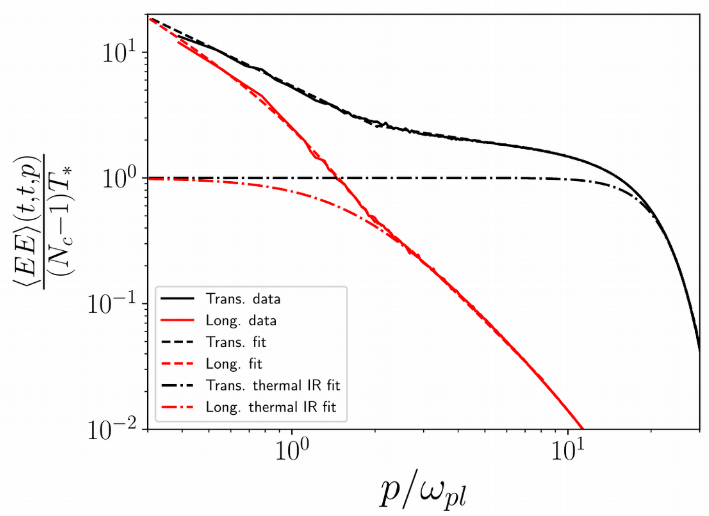


- ω_{pl} plasmon freq. (soft scale)
- Clear excitation peaks for all p
- Could kinetic description of initial stages be already valid early on?
- *But here:* over-occupied initial conditions, $P_L \geq 0$

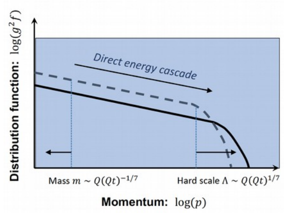
Spectral functions in scalar systems: KB, Pinerio Orioli, *PRD 101, 091902 (2020)* *(as Rapid Communication)* $\Rightarrow$ Backup

Heavy-quark diffusion & gluonic IR excess





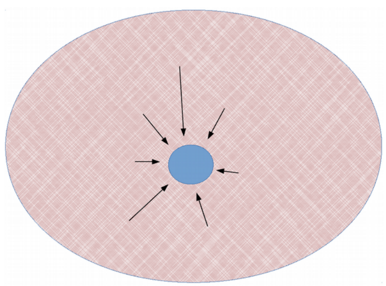
Heavy-quark diffusion: far from equilibrium



KB, Kurkela, Lappi, Peuron, *JHEP 09, 077 (2020)*

Heavy quark in QGP

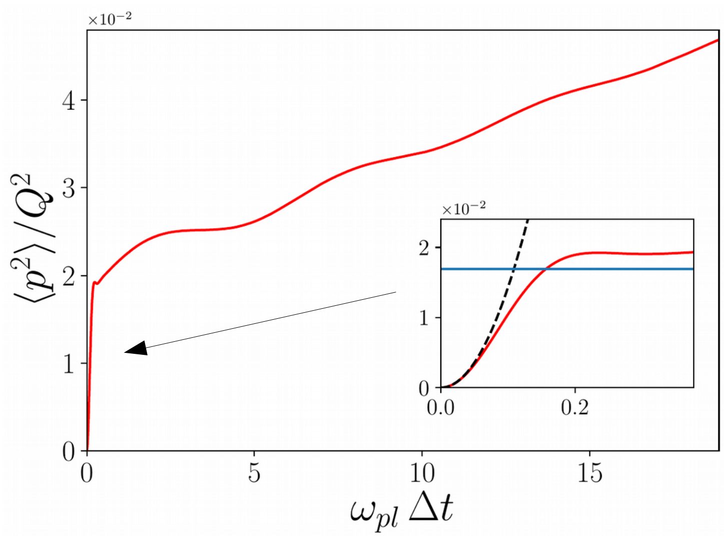
experiences “kicks” from the medium



Momentum broadening

from chromo-electric force, gauge-invariant observable

$$\langle p^2(t,\Delta t)\rangle$$
$$= \frac{g^2}{N_c}\int_t^{t+\Delta t}\mathrm{d}t'\int_t^{t+\Delta t}\mathrm{d}t''\,\mathrm{Tr}\,\langle E_i(t')U_0(t',t'')E_i(t'')U_0(t'',t')\rangle$$



Observed features

i. rapid initial growth with $\Delta t \approx 2\pi/\Lambda$

ii. damped oscillations with period $\Delta t \approx 2\pi/\omega_{\rm pl}$

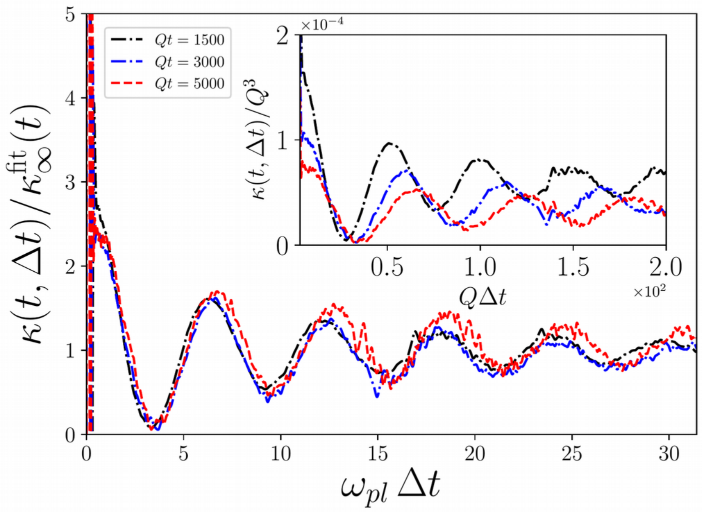
iii. overall approx. linear growth for $1/\Lambda \ll \Delta t \ll t$

Heavy-quark diffusion coefficient far from equilibrium

KB, Kurkela, Lappi, Peuron, *JHEP 09, 077 (2020)*

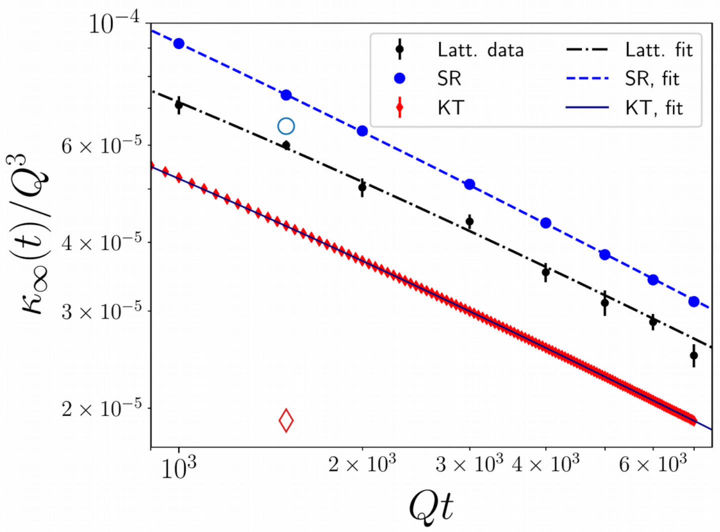
Self-similarity in *t*

$$3\kappa\,(t,\Delta t) = \frac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t,\Delta t)\rangle$$



Evolution of heavy-quark diff. coeff.

$$\kappa\,(t,\Delta t) \to \kappa_\infty(t)$$



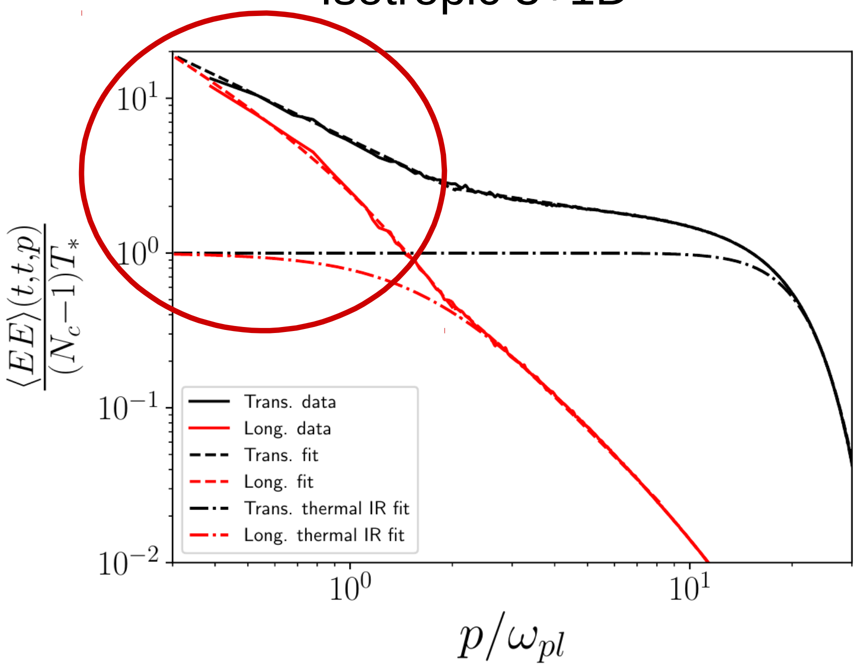
- Far-from-equilibrium transport coefficient larger than thermal value $\kappa_\infty(t) \gg \kappa_{\mathrm{therm}}$
- SR: "Spectral reconstruction", use spectral function in analytical computation
- KT: "Kinetic theory", use known expressions for analyt. comp.

Moore, Teaney (2005);
Caron-Huot, Moore (2008)

Excess of gluons at low p: IR enhancement

KB, Kurkela, Lappi, Peuron, *JHEP 09, 077 (2020)*

Isotropic 3+1D



Seen before in:

Kurkela, Moore (2012); KB, Kurkela, Lappi, Peuron (2018)

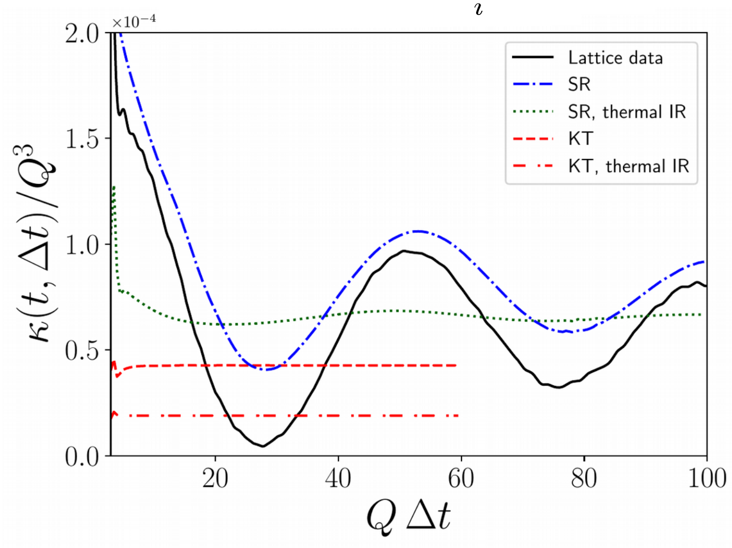
- Excess of gluons for $p \lesssim m \sim \omega_{\mathrm{pl}}$ compared to HTL predictions
- Similarly in Glasma-like systems (for scalar contribution) KB, Kurkela, Lappi, Peuron, *PRD 100, 094022 (2019)*
- This correlator gauge-fixed, is excess visible in gauge-invariant observables?
- Yes, open symbols for κ_∞ on prev. page: there $\langle EE\rangle$ without IR enhancement used

Heavy-quark diffusion: IR enhancement is observable

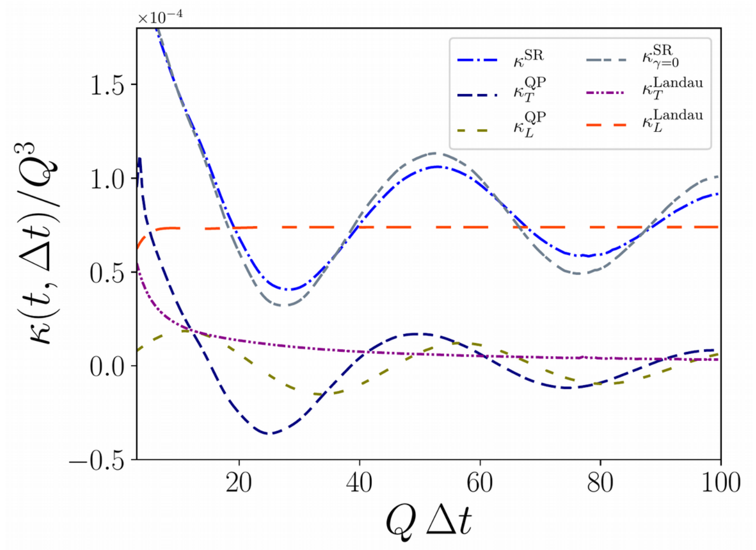
KB, Kurkela, Lappi, Peuron, *JHEP 09, 077 (2020)*

Diffusion coefficient $3\kappa\,(t,\Delta t) = \dfrac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t,\Delta t)\rangle$

Total $\kappa\,(t,\Delta t) = \sum_i \kappa_i\,(t,\Delta t)$



Components $\kappa_i\,(t,\Delta t)$

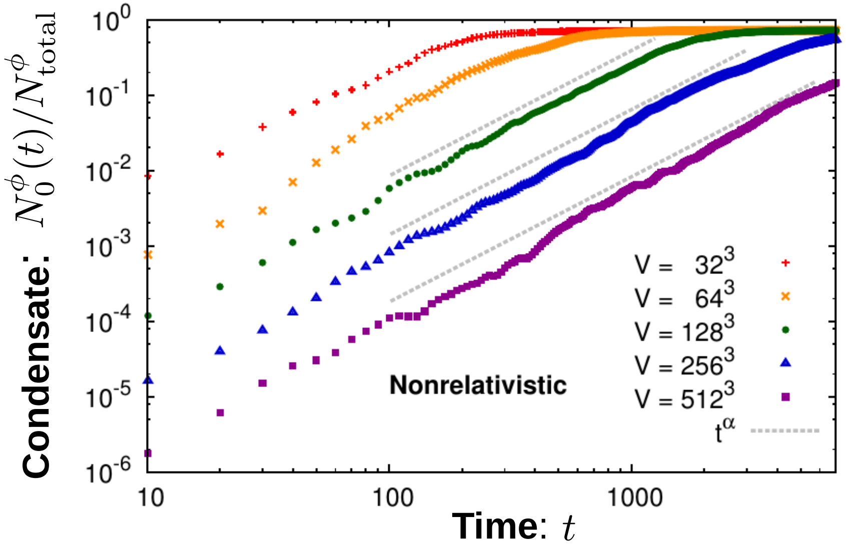


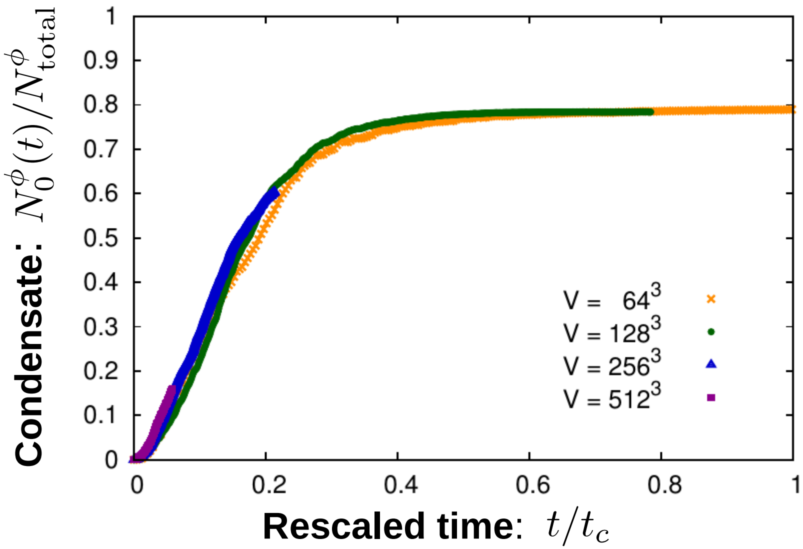
- Oscillations with plasmon freq. due to QP excitations, sign of IR enhancement
- Heavy quarks, quarkonia, jets may reveal IR dynamics of non-equilibrium QGP

Bose-Einstein condensation in scalars: an IR process

Non-relativistic, O(N) relativistic show this

Pinerio Orioli, KB, Berges,
PRD 92, 025041 (2015)





- $\frac{N_0^\phi(t)}{N_{\rm total}^\phi} = \frac{1}{V}\int_0^L d^dx\, \frac{\langle\{\phi(t,x),\phi^\dagger(t,0)\}\rangle}{\langle\{\phi(t,0),\phi^\dagger(t,0)\}\rangle}$ depends on condensation time $t_c \sim L^{1/\beta}$
- A far-from-equilibrium mechanism for condensation of particles

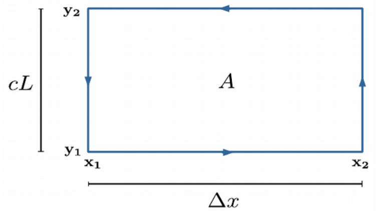
Berges, Sexty,
PRL 108, 161601 (2012)

- Universal exponent $\beta = 0.55 \pm 0.03$

(Origin for scalars is a classical IR attractor, see **Backup**)

Gauge-invariant condensation: a similar IR phenomenon

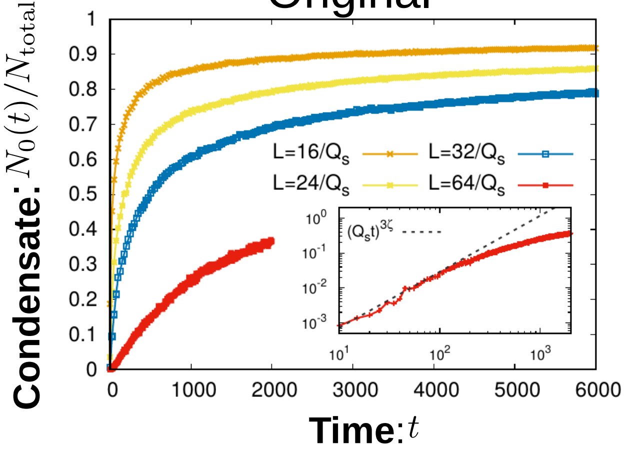
Berges, KB, Mace, Pawlowski, *PRD 102, 034014 (2020)*



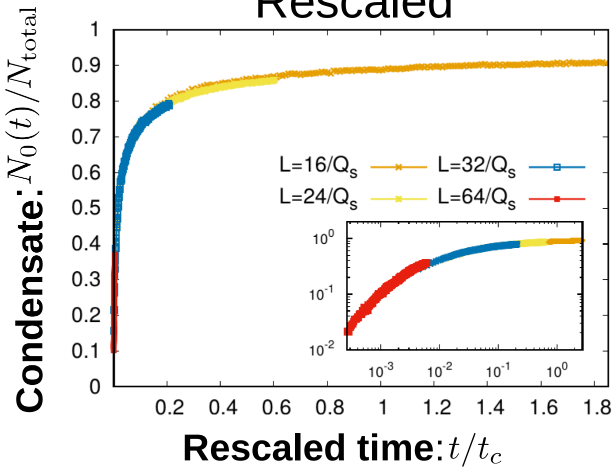
(Spatial) Wilson loop: $W(\Delta x, cL, t) = \frac{1}{N_c} \mathrm{Tr}\, \mathcal{P} e^{-i\, g \int_{C[\Delta x, cL]} \mathcal{A}_i(z,t)\, dz_i}$

Condensate fraction: $\frac{N_0(t, cL)}{N_{\mathrm{total}}} \equiv \frac{1}{V_c} \int_0^{cL} d^d \Delta x \, \langle W(\Delta x, cL, t) \rangle$

Original



Rescaled



Condensation time: $t_c \sim L^{2/\zeta}$ Exponent: $\zeta = 0.54 \pm 0.04\,(\mathrm{stat.}) \pm 0.05\,(\mathrm{sys.})$

- Self-similar evolution of Wilson loops: *Mace, Schlichting, Venugopalan, PRD 93, 074036 (2016); Berges, Mace Schlichting, PRL 118, 192005 (2017)*
- Relation to IR excess? $\langle W \rangle$ related to correlator of scalar fields?

Conclusion

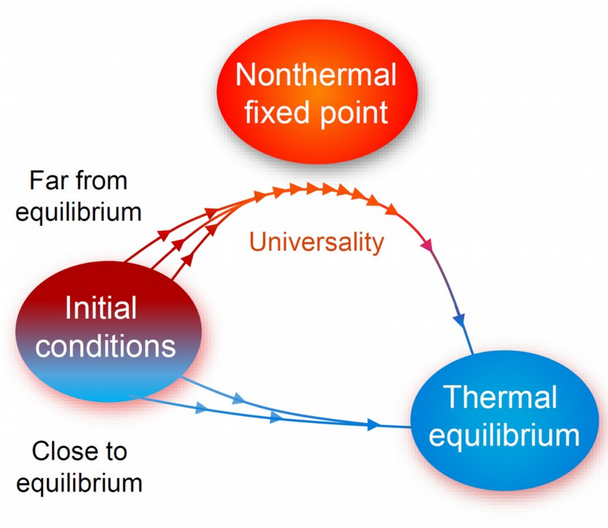
Over-occupied gluonic plasmas at initial stages in heavy-ion collisions:

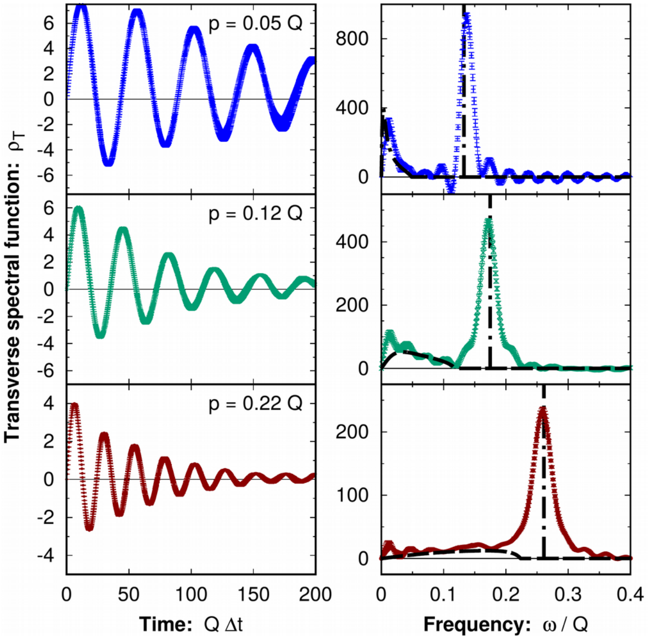
- ✔ Classical attractor can emerges at initial stages, universality class with scalars
- ✔ Spectral functions disclose relevant dynamical and quasiparticle properties
- ✔ Transport coefficients can be large, can display non-pert. properties
- ✔ Fascinating IR phenomena can arise

Outlook

- What are the spectral functions of (expanding) fermions and gluons at initial stages?
  Could kinetic description of initial stages be already valid early on?
- Can transport coefficients act as probes of initial stages by encoding specific pre-equilibrium signatures?
- What is the origin for IR phenomena of gluons, how do they affect the dynamics?

Thank you for your attention!





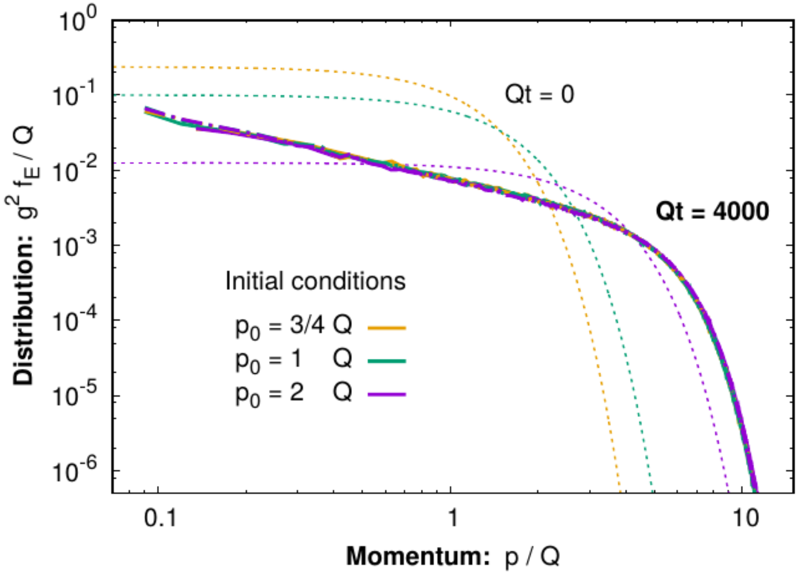
BACKUP SLIDES

Classical attractors: independence of initial conditions

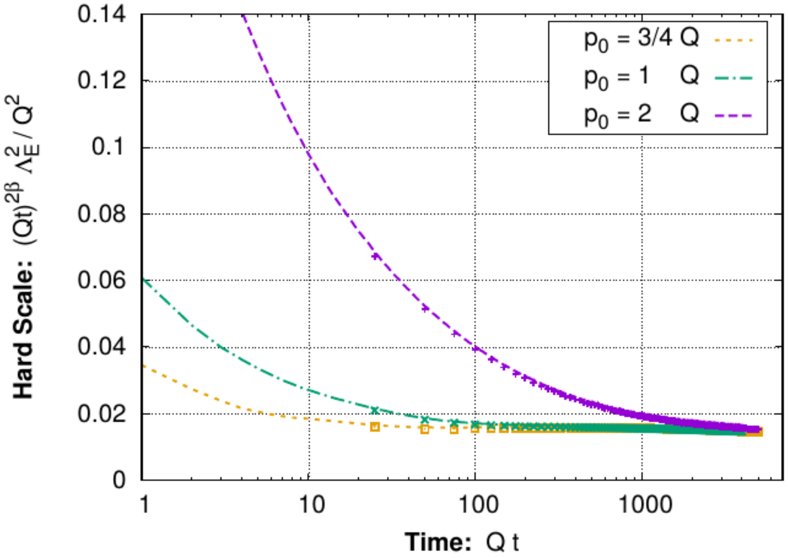
(at the example of 2+1D Glasma-like systems)

KB, Kurkela, Lappi, Peuron, *PRD 100, 094022 (2019)*

Distribution function



Hard momentum scale



Similarly for scalars: Berges, KB, Schlichting, Venugopalan, *JHEP 05, 054 (2014)*

Classical attractors: comparison to phase transitions (PT)

Scaling region (of a nonthermal fixed point)

$$\boxed{f(t,p) = t^{\alpha} f_s(t^{\beta} p)}$$

Self-similar evolution of distribution function f

with scaling behavior of typical scales $\langle p \rangle \sim t^{-\beta}\,,\; f(\langle p \rangle) \sim \frac{1}{g^2}\, t^{\alpha}$

Classification: universality classes far from equilibrium

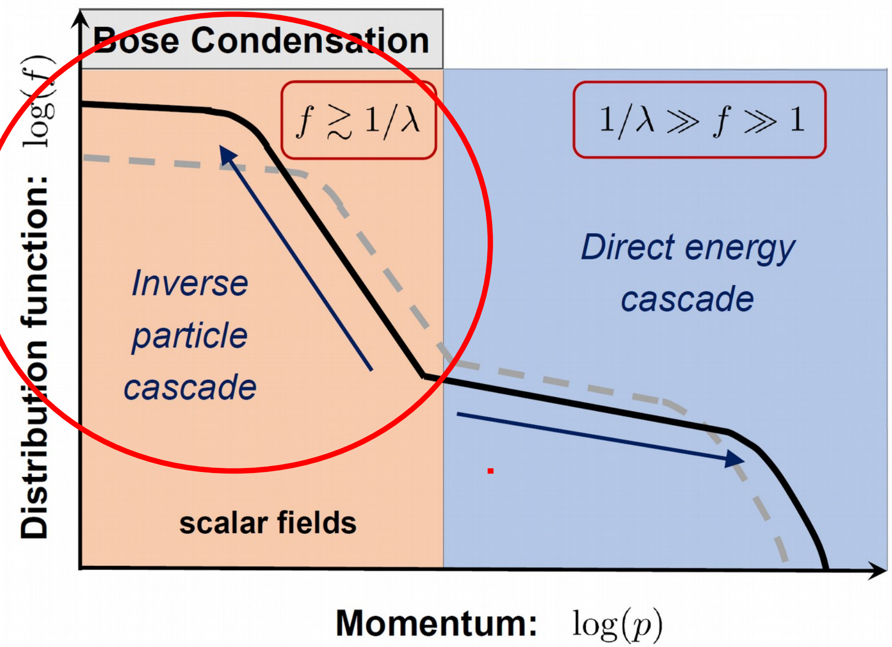
Via scaling exponents α, β and the scaling function $f_S(x)$

NTFP	Close to 2nd order PT
Time scale t	Inverse reduced temp. T_c / $(T-T_c)$
Self-similar evolution	Critical slowing down, power laws
Scaling exponents & function	Critical exponents & surface

Universality in scalar systems and condensation

- **Relativistic** O(N)-symmetric scalars:
  (inflaton, axions, dark matter, …)
- **Nonrelativistic** scalars
  (for ultra-cold atom experiments, …)
- Experimental observation (cold atoms)

  Prüfer et al., *Nature 563, 217 (2018);*
  Erne et al., *Nature 563, 225 (2018);*
  Glidden et al., arXiv:200601118



$$S_{\mathrm{O(N)}} = \int d^4x \left[\frac{1}{2} \left(\partial_t \phi_a\right)^2 - \frac{1}{2} \left(\nabla \phi_a\right)^2 - \frac{1}{2} m^2 \phi_a \phi_a - \frac{\lambda}{24N} \left(\phi_a \phi_a\right)^2 \right]$$

$$S_{\mathrm{nonrel}} = \int d^4x \left[\frac{i}{2} \left(\psi^* \partial_t \psi - \psi \partial_t \psi^*\right) - \frac{1}{2m} |\nabla \psi|^2 - \frac{g}{2} |\psi|^4 \right]$$

Classical IR attractor in scalar systems

Self-similar evolution

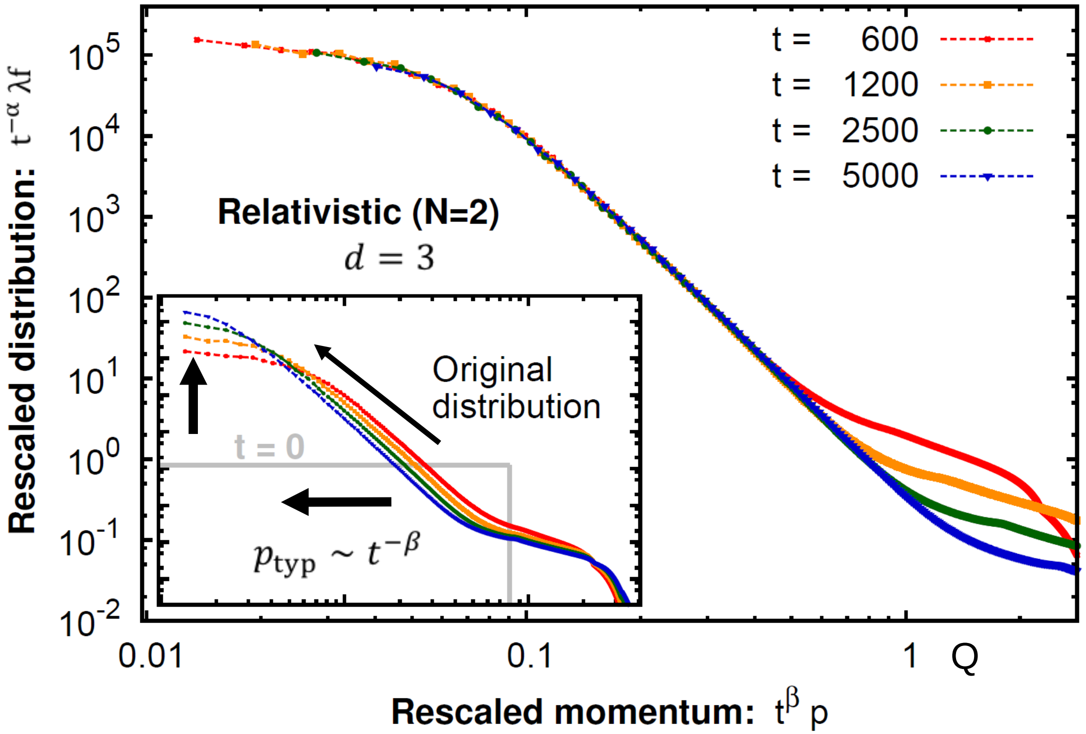
Piñeiro Orioli, KB, Berges,
PRD 92, 025041 (2015)

Self-similarity $f(t,p) = t^{\alpha} f_s(t^{\beta} p)$

Universal scaling exponents $\boxed{\alpha \approx \frac{d}{2}\,, \;\beta \approx \frac{1}{2}}$

$$\int d^d p\, f(t,p) \approx const$$ (Particle number conserved)

$$F(t, \mathbf{x}-\mathbf{x}') = \frac{1}{2}\left\langle \left\{ \hat{\phi}(t,\mathbf{x}), \hat{\phi}(t,\mathbf{x}') \right\} \right\rangle_C$$
$$f(t,p) = \sqrt{F(t,p)\,\ddot{F}(t,p)} \approx \omega(t,p)\,F(t,p)$$



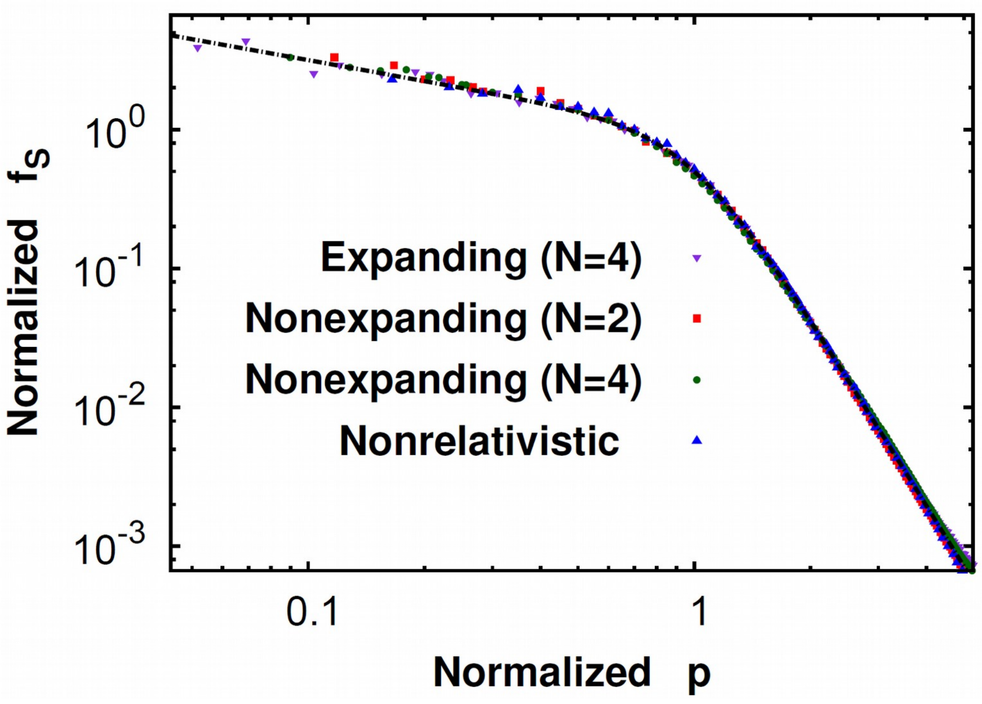
Berges, Rothkopf, Schmidt (2008); Piñeiro Orioli, KB, Berges (2015); Berges, KB, Schlichting, Venugopalan (2015); Moore (2016); Karl, Gasenzer (2016); Walz, KB, Berges (2017); Berges, KB, Chatrchyan, Jäckel (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018); Schmied, Mikheev, Gasenzer (2018) ...

Universality far from equilibrium

A single broad far-from-equilibrium universality class?

Pinerio Orioli, KB, Berges, *PRD 92, 025041 (2015)*
Berges, KB, Schlichting, Venugopalan, *PRD 92, 096006 (2015)*

Later more!



Self-similar evolution

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

Application: Nonthermal fixed points

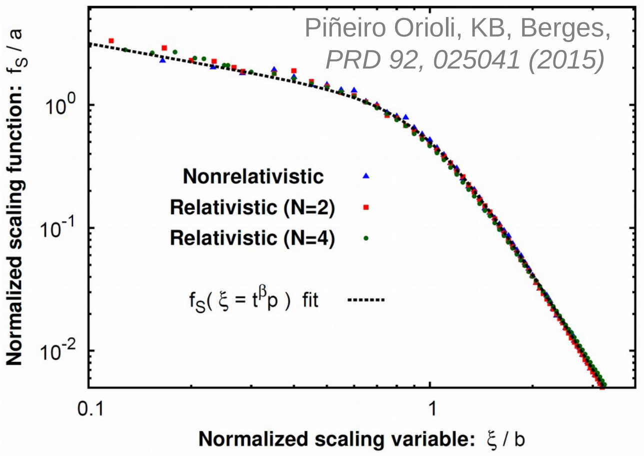
Self-similar attractor

- *Same α, β and scaling function*

$$\boxed{\lambda f_S \simeq \frac{a}{(|\mathbf{p}|/b)^{\kappa_<} + (|\mathbf{p}|/b)^{\kappa_>}}}$$

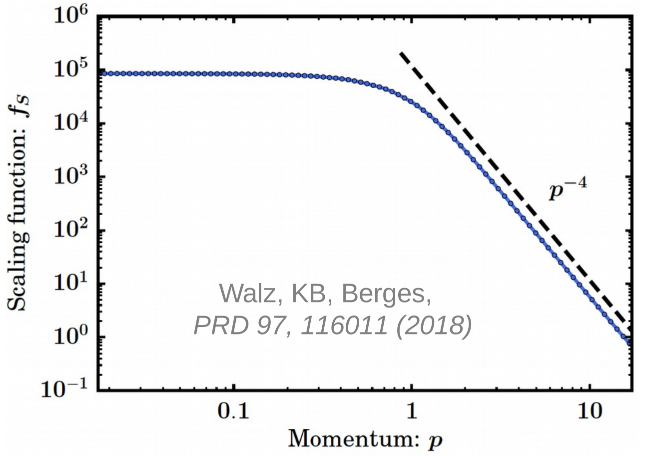
with $\kappa_< \simeq 0 - 0.5$ and $\kappa_> \simeq 4 - 4.5$

Across relativistic (different N), nonrelativistic

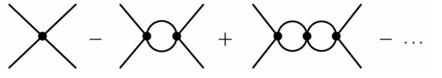


- New *large-N kinetic theory* describes it quantitatively, shows that $\kappa_< \to 0$, $\kappa_> \to 4$

Piñeiro Orioli, KB, Berges (2015); Walz, KB, Berges (2017); Chantesana, Piñeiro Orioli, Gasenzer (2018)



- (Systematically derived in $1/N$, resums vertex)



Application: Nonthermal fixed points

Experimental observation in a spinor gas (ultra-cold atoms)

Prüfer et al., *Nature 563, 217 (2018)*

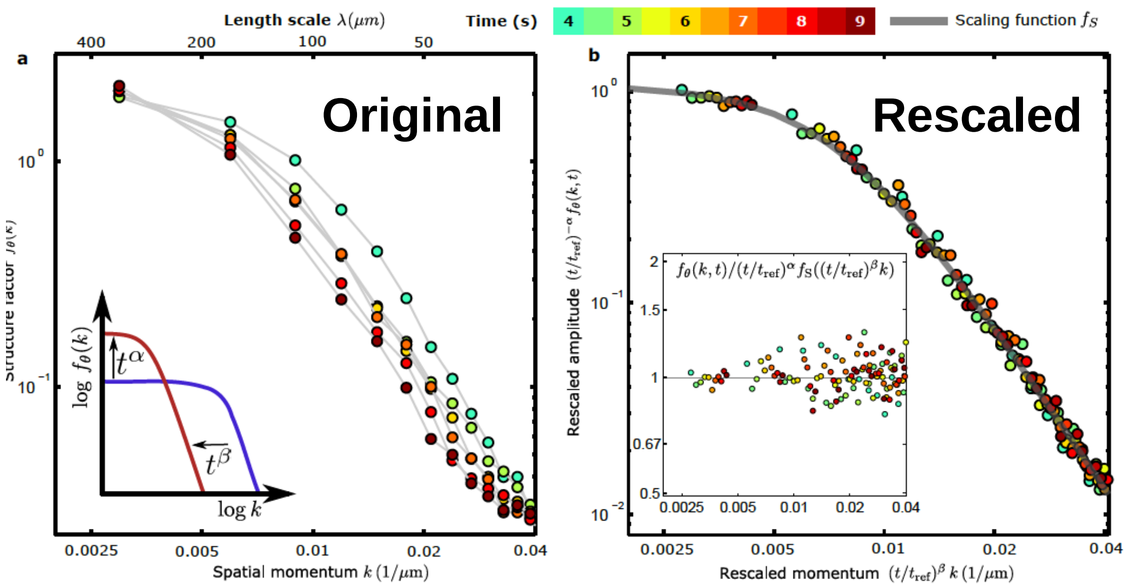
Self-similar evolution

$$f(t,p) = t^{\alpha} f_s(t^{\beta} p)$$

Observation also

$$\alpha \approx \frac{d}{2}, \qquad \beta \approx \frac{1}{2}$$

(for effectively $d \simeq 1$)



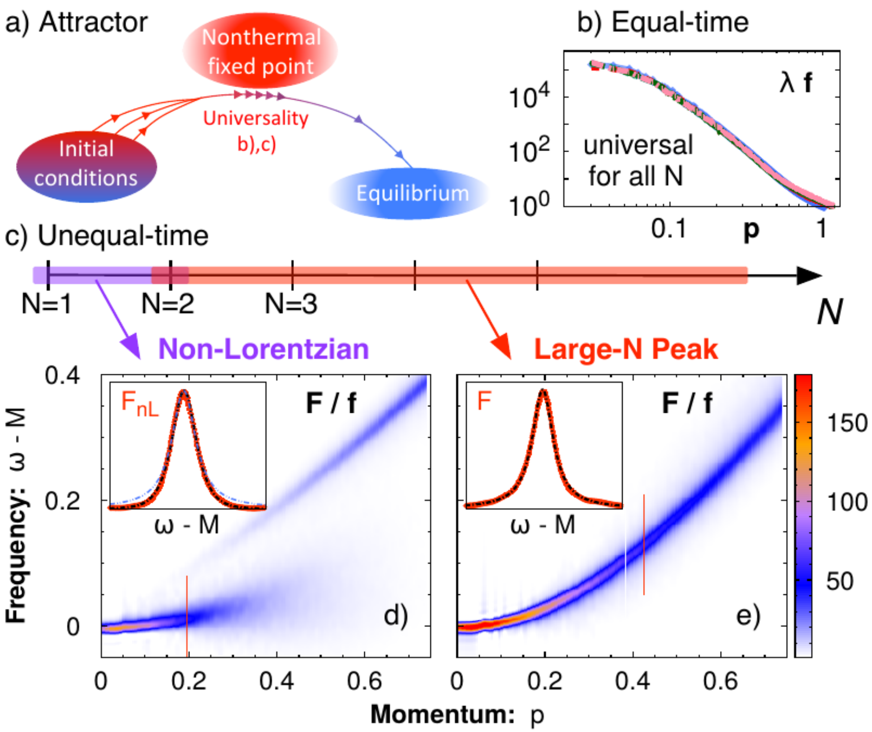
Same scaling exponents as found in: Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)*

Classical attractors in other experiments: Erne et al., *Nature 563, 225 (2018); Glidden et al., arXiv:200601118*

Spectral functions in scalar $\lambda(\phi_a\phi_a)^2$ theories: IR Classification of *O(N)* symmetric theories using F, ρ

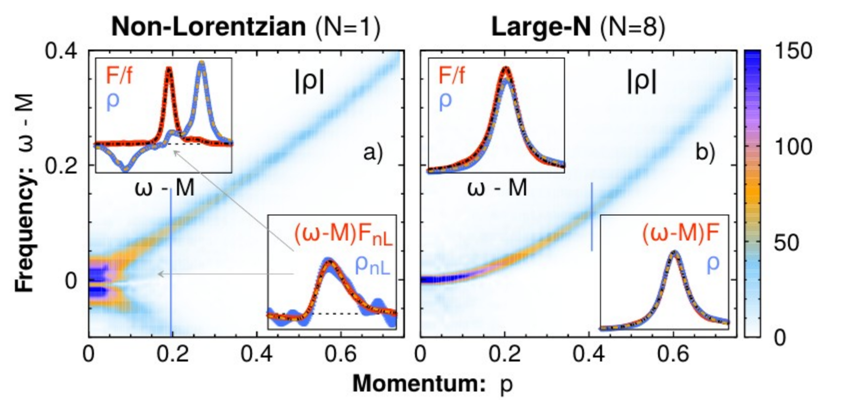
KB, Pinerio Orioli, *PRD 101, 091902 (2020)* *(as Rapid Communication)*

Problem: $f(t,p) = t^{\alpha} f_s(t^{\beta} p)$ universal for all N Piñeiro Orioli, KB, Berges, *PRD 92, 025041 (2015)* ; check with F, ρ



$$F(t,t',\mathbf{x}-\mathbf{x}') = \frac{1}{2N}\left\langle\left\{\hat{\phi}_a(t,\mathbf{x}), \hat{\phi}_a(t',\mathbf{x}')\right\}\right\rangle_C$$

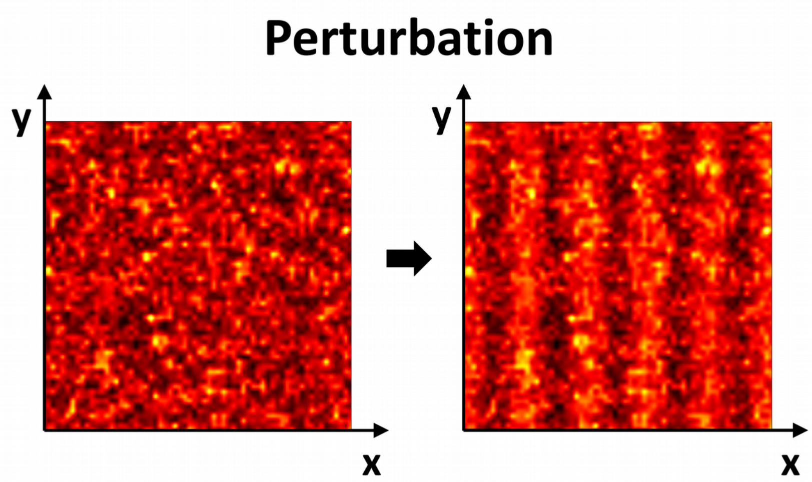
$$\rho(t,t',\mathbf{x}-\mathbf{x}') = \frac{1}{N}\left\langle\left[\hat{\phi}_a(t,\mathbf{x}), \hat{\phi}_a(t',\mathbf{x}')\right]\right\rangle$$



Spectral functions: method in more detail

Combine: Classical simulations, Linear response

KB, Kurkela, Lappi, Peuron,
PRD 98, 014006 (2018)



- Simulations for $SU(N_c)$ fields $E_i(t_0, \boldsymbol{x})$, $U_i(t_0, \boldsymbol{x}) \simeq e^{iga_sA_i(t_0,\boldsymbol{x})}$ on N_s^3 lattice ($A_0 = 0$)
- Instant j at time t', Coulomb gauge $\partial_j A_j = 0$
- Split $A_j(t, \boldsymbol{x}) \mapsto A_j(t, \boldsymbol{x}) + a_j(t, \boldsymbol{x})$
- Response in linear fluctuations a_j for $t > t'$: Solve (linear) EOM for a_j

Kurkela, Lappi, Peuron,
EUJC 76 (2016) 688

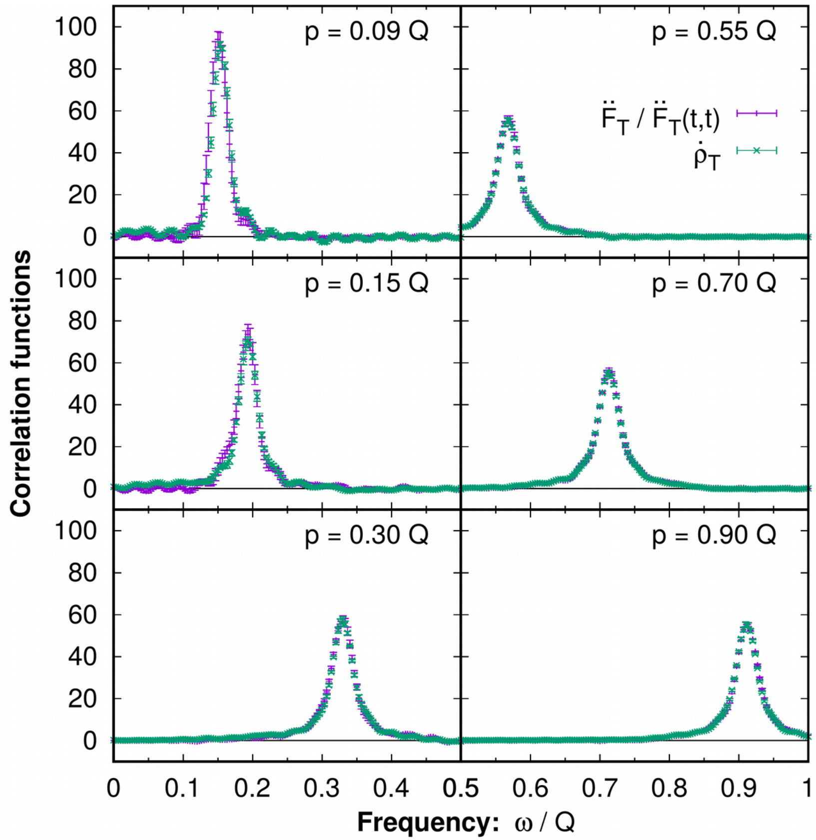
$\Delta t = t - t'$

- $\langle a_j(t, \boldsymbol{p}) \rangle = G_{R,jk}(t, t', \boldsymbol{p})\, j^k(t', \boldsymbol{p})$, obtain ret. propagator $G_{R,jk}$ from response
- Spectral function: $G_{R,jk} = \theta(t - t')\, \rho_{jk}$
- Distinguish polarizations (trans., long.)

Spectral functions – fluctuation dissipation relation

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*

Statistical correlation function F



Remarks: $\dot{\rho}_T = \partial_t \rho_T \,,\;\; E = \partial_t A$

(Classical) definition:

$$\ddot{F}^{jk}(t,\Delta t,p) = \frac{1}{(N_c^2-1)V}\langle E^j(t,p)E^{*,k}(t,p)\rangle$$

(As always: $\Delta t = t - t'$ Fourier transform to ω)

(Thermal) fluctuation-dissipation relation:

$$\ddot{F}_T(t,\omega,p)/T = \dot{\rho}_T(t,\omega,p)$$

Used to estimate thermal spectral function

Example: G. Aarts, *PLB 518, 315 (2001)*

We *extract* $\ddot{F}, \dot{\rho}$ *independently* and observe a generalized fluctuation-dissipation relation:

$$\boxed{\frac{\ddot{F}_{T,L}(t,\omega,p)}{\ddot{F}_{T,L}(t,\Delta t=0,p)} = \frac{\dot{\rho}_{T,L}(t,\omega,p)}{\dot{\rho}_{T,L}(t,\Delta t=0,p)}}$$