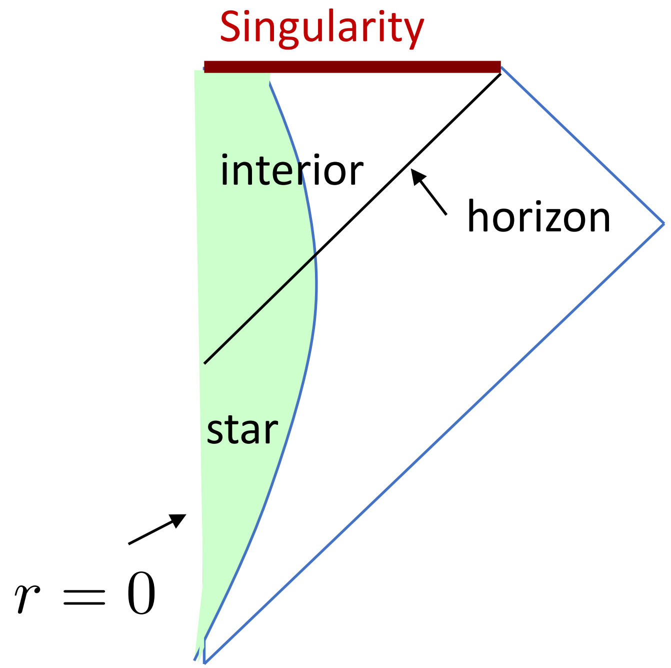
Lectures on quantum aspects of black holes

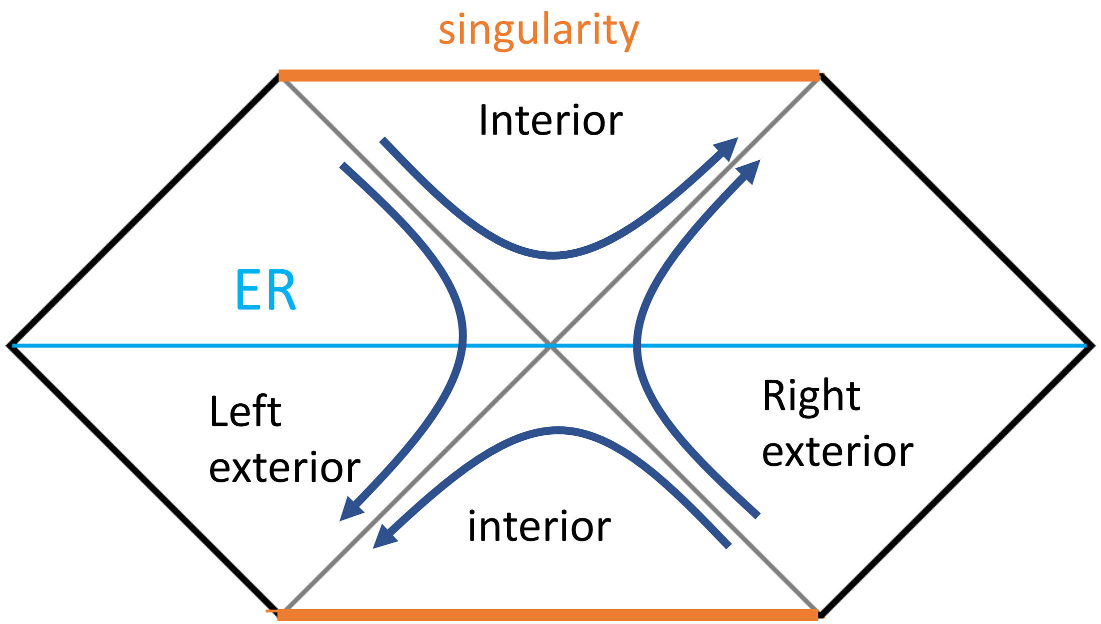
Lecture 2: chaos and the near horizon geometry.

Juan Maldacena

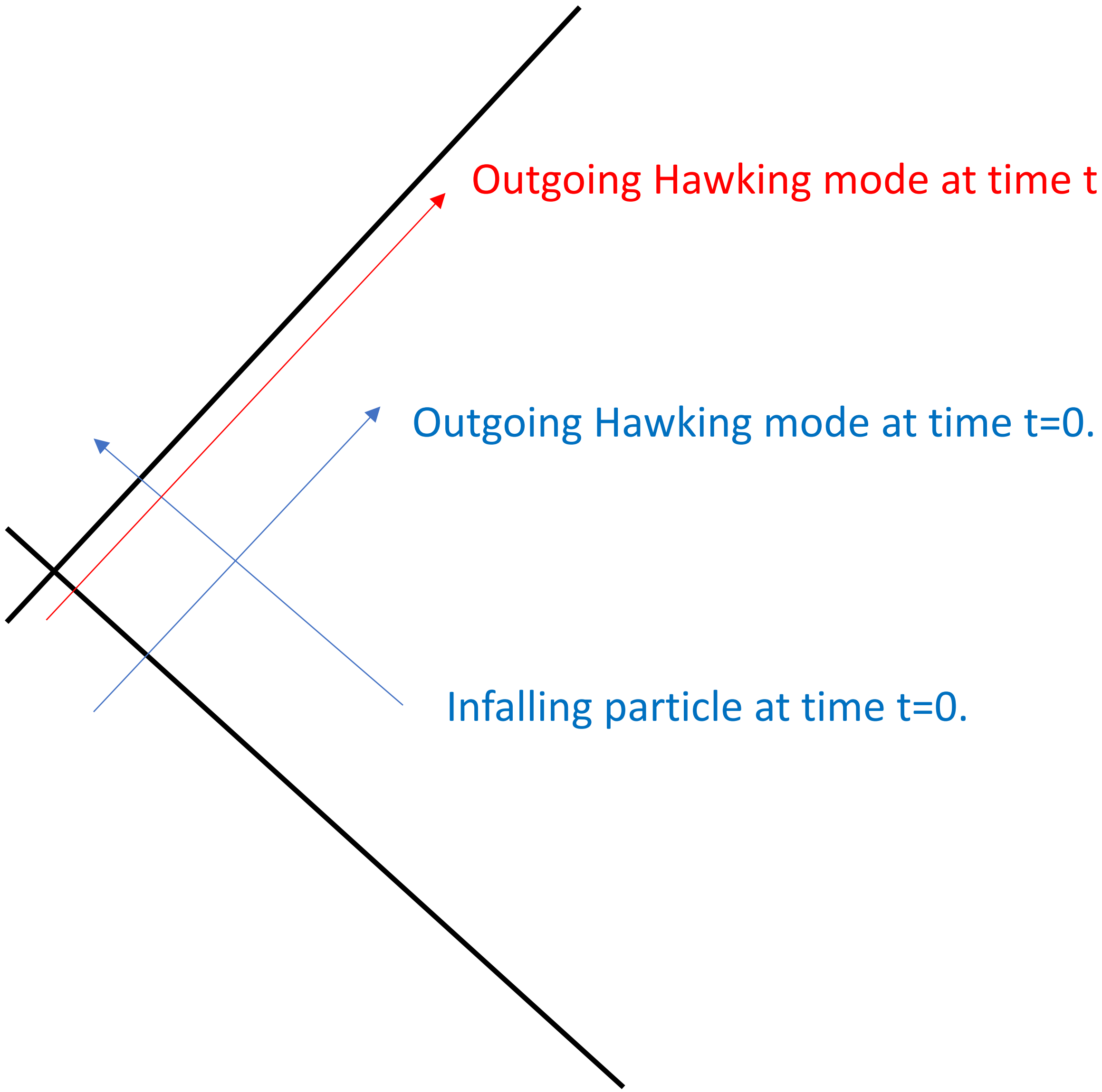
KAWS, January 2022

Time translation as boosts





We will study here one implication of this



Relatively small interaction between an infalling particle at time t=0 and the outgoing mode at that time.

However, if we consider the outgoing Hawking mode at time t, and we trace it back in time, then it is very highly boosted relative to the infalling particle at time t=0.

Center of mass energy grows as this factor:

$$e^{\tau} = e^{\kappa t} = e^{\frac{2\pi t}{\beta}} = e^{2\pi T t}$$

κ = surface gravity or acceleration

Large gravitational interactions

$$G_N s \propto G_N e^{\tau}$$

Is this relevant ?

- If we have the usual vacuum, we know that an infalling particle does not feel anything special when it crosses the horizon.
- The later Hawking mode was not ``created'' yet…
- Modes have a time delay, but the vacuum is invariant under translations, so one mode is replaced by another, and we do not see any local changes.
- Spherically symmetric waves → time delay is independent of the angular directions "y". No tidal forces.
- It does not look observable…

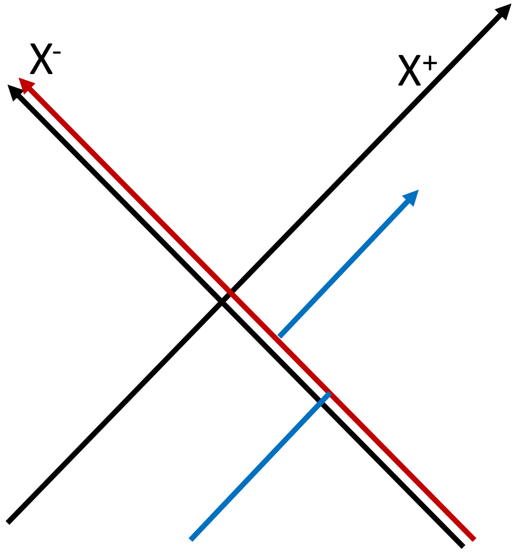
Gravitational interactions between highly boosted particles in flat space.

- We can look at the gravitational field of a fast moving particle

$$ds^2 = ds^2_{flat} + G_N(-P_+)\frac{1}{|y|^{D-4}}\delta(x^+)(dx^+)^2 \ , \qquad ds^2_{flat} = -dx^+dx^- + d\vec{y}^2$$

$$p_+ = -i\partial_{x^+} \ , \qquad \psi \sim e^{ip_+x^+ + p_-x^- + p_ix^i}$$

(no metric)



Define:

$$\tilde{x}^- = x^- - G_N(-P_-)\frac{1}{|y|^{D-4}}\theta(x^+)$$

$$ds^2 = -dx^+d\tilde{x}^- + d\vec{y}^2 + \cdots$$

$\tilde{x}^-$ is continuous → jump for x^-

Time delay !

- <u>The main effect is a time delay.</u>

- Other effects → small angle deflection. Involves first derivative in the y directions.

- Tidal forces, proportional to the second derivative in the y directions, R_{+i+j} . It can be relevant for strings, or other extended objects. A string can get excited by crossing the shock wave. Graviton → massive string state.

Horowitz -Steif

- There is an interaction between the infalling particles and the outgoing particles.

- This interaction is mainly a time delay.

- It grows at the relative boost increases

We will now stop talking about black holes and talk about quantum manybody systems.

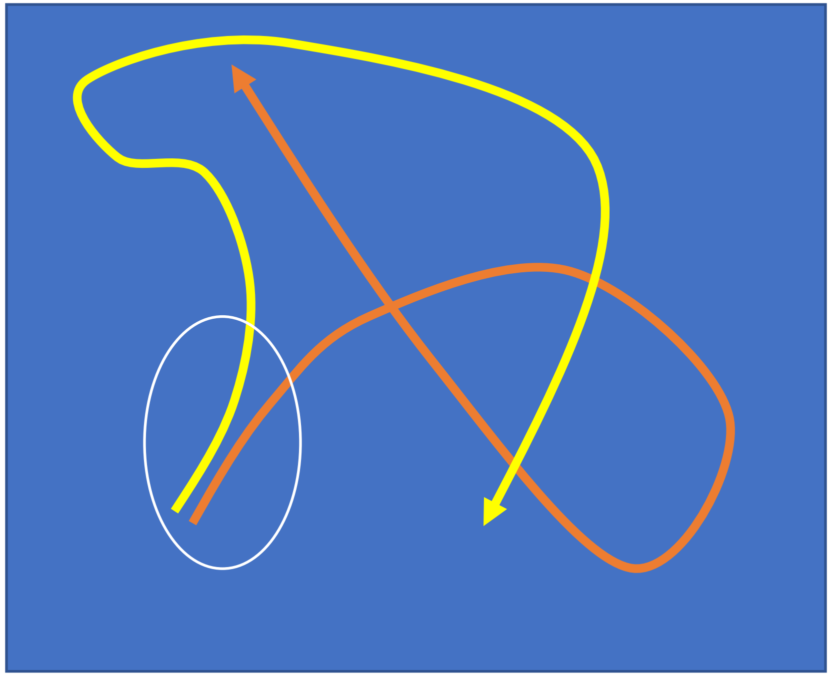
In particular, chaos in many body systems.

Chaos in classical systems

- Similar initial conditions → rather different outcomes

Initially → exponential deviation

At later times → far away in phase space. Two generic points



- If we look at a thermal state, we do not see any of this because a uniform density in phase space goes to a uniform density in phase space.
- → not visible in simple expectation values in a thermal state.

Sensitivity to initial conditions

$$(q_0^i, p_0^i) \longrightarrow (q^j(t), p^j(t))$$

$$(q_0, p_0 + \delta p_0) \longrightarrow (q(t), p(t)) + (\delta q(t), \delta p(t))$$

$$\delta q^j(t) = \frac{\partial q^j(t)}{\partial p_0^i} \delta p_0^i = \{q^j(t), q_0^i\} \delta p_0^i$$

$$(\Delta q^j)^2 = \{q^j(t), q_0^i\}^2 \sim e^{\lambda t}$$

$$\langle (\Delta q^j)^2 \rangle = \langle \{q^j(t), q_0^i\}^2 \rangle$$

Quantum mechanical system

Shenker, Stanford
Kitaev

$$\langle [V(t),W(0)]^2\rangle = \overbrace{\langle V(t)W(0)V(t)W(0)\rangle + \langle W(0)V(t)W(0)V(t)\rangle}^{\text{Out of time order correlator: OTOC}} \underbrace{- \langle V(t)W(0)W(0)V(t)\rangle - \langle W(0)V(t)V(t)W(0)\rangle}_{\text{time ordered}}$$

$$\langle [V(t),W(0)]^2\rangle \sim \frac{1}{N} e^{\lambda t}$$

V, W simple operators in the large N system (eg. Single trace operators, or operators acting on only one particle)

Initially, each is of order one and they cancel. The 1/N correction grows exponentially. The OTOC decays.

Out of time order correlator

Shenker, Stanford
Kitaev

$$\frac{\langle V(t)W(0)V(t)W(0)\rangle}{\langle V(t)V(t)\rangle\langle W(0)W(0)\rangle} = 1 + \underbrace{o(1/N)} - \frac{1}{N}e^{\lambda t}$$

Does not grow with time

1/N correction that grows with time.

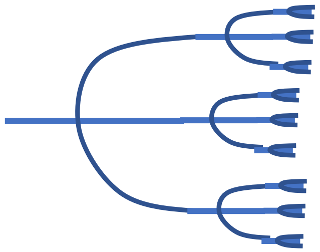
At longer times, $t > \frac{1}{\lambda}\log N$, the commutator is expected to be of order one. OTOC decays to zero.

Operator growth picture

Imagine a large set of qubits interacting via a k-local Hamiltonian (acts on k qubits at a time).

Large N gauge theories are of this type, where the interaction acts on three or four fields at a time.

$$V(0) \to [iH, V(0)] \to i^2[H,[H,V(0)]]$$



Number of different operators grows exponentially.

Until the operator V → consists of a sum of terms, each of which contains roughly all operators in the system. This happens at the "scrambling time".

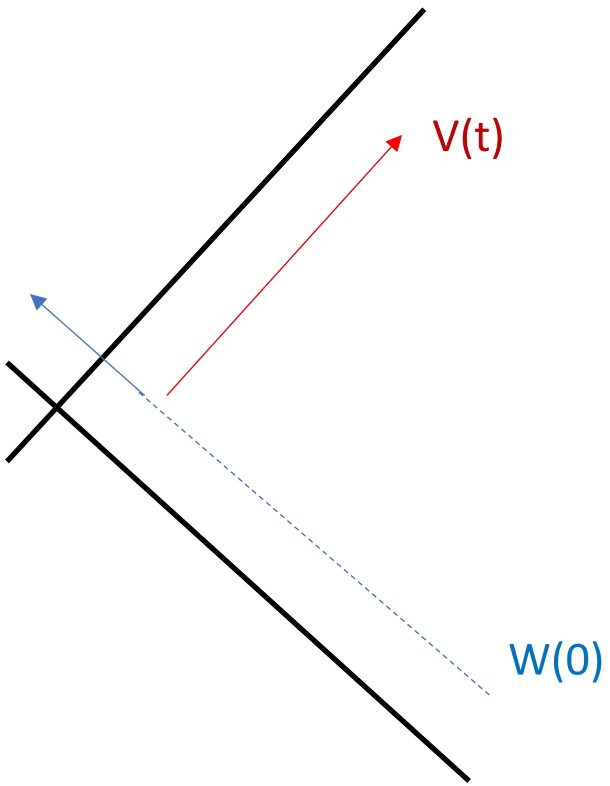
Stops growing at the scrambling time, but it continues to get more and more complex

OTOC as the overlap between two states

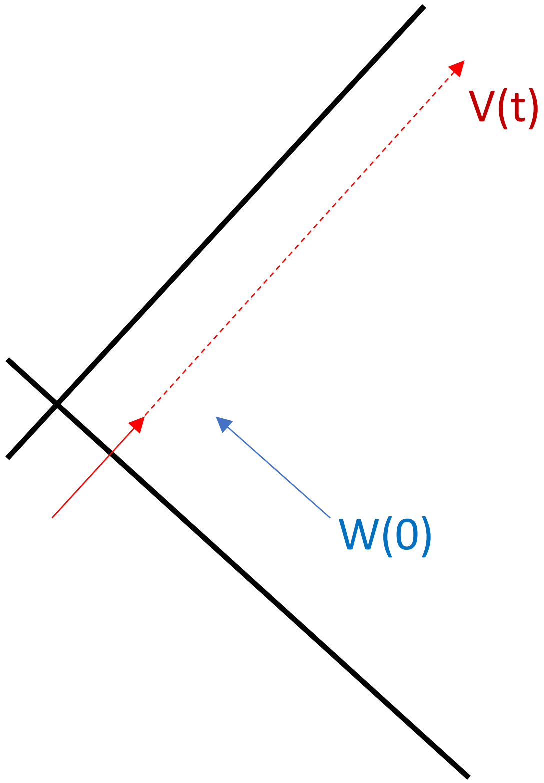
$$\langle V(t)W(0)V(t)W(0)\rangle = \langle\chi|\psi\rangle$$
$$|\psi\rangle = V(t)W(0)|TFD\rangle$$
$$|\chi\rangle = W(0)V(t)|TFD\rangle$$

Out of time order correlators in a black hole background

$$|\psi\rangle = V(t)W(0)|TFD\rangle$$

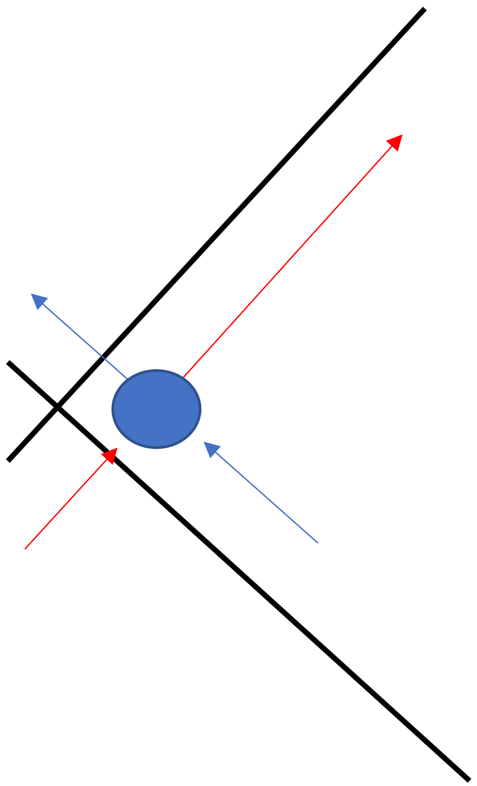


$$|\chi\rangle = W(0)V(t)|TFD\rangle$$



OTOC = Scattering amplitude

$$\langle V(t)W(0)V(t)W(0)\rangle = \langle \chi|\psi\rangle$$



For times less than the scrambling time

Exponential growth due to gravitational scattering

$$S = 1 + iG_N s + \cdots = 1 + iG_N e^{2\pi t/\beta} + \cdots$$

$$\lambda = \frac{2\pi}{\beta} = 2\pi T$$

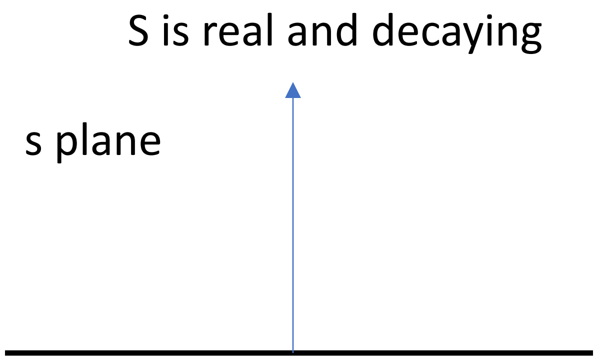
Shenker, Stanford
Kitaev

- It is possible to argue that for general many body systems, this is the maximal possible value.
- Let's give only a bulk rationale for this.

- Causality implies that the scattering S-matrix should be analytic in the upper half s-plane due to causality and also $|S|^2 \leq 1$

$$S = 1 + iG_N s + \cdots$$
$$S = 1 - G_N(-is)^a + \cdots \ , \qquad |S|^2 \leq 1 \longrightarrow a \leq 1$$

$$s = e^{2\pi t/\beta}$$



- Causality implies that the scattering S-matrix should be analytic in the upper half s-plane due to causality and also $|S|^2 \leq 1$

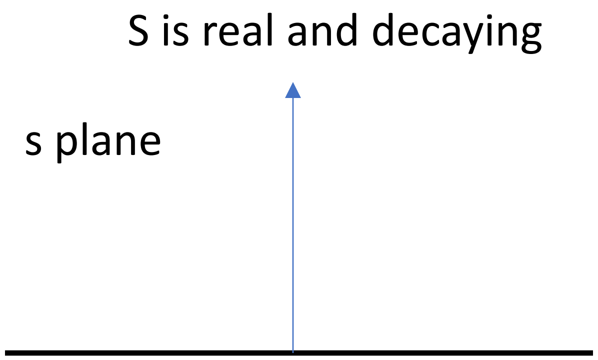
$$S = 1 + iG_N s + \cdots$$ gravity

Different exponent

$$S = 1 - G_N(-is)^a + \cdots = 1 - G_N s(\cos\frac{\pi a}{2} + i\sin\frac{\pi a}{2}) + \cdots$$

$$|S|^2 \leq 1 \longrightarrow a \leq 1$$

$$s = e^{\tau} = e^{\frac{2\pi t}{\beta}} \longrightarrow s^a = e^{\frac{2\pi a t}{\beta}} \quad \text{vs} \quad e^{\lambda t}$$



$$\lambda \leq \frac{2\pi}{\beta} = 2\pi\, T$$

Black holes are maximally chaotic

→ Quantum describing the black hole should be strongly coupled!

Weakly coupled systems have $\lambda \sim g^2$= coupling strength

These features of chaos also suggest that the dual quantum system has a k-local Hamiltonian.

Thinking about black holes → inspiration for a bound that applies to any many body quantum system

What looks like chaos in the quantum description → looks like classical (tree level) gravitational dynamics in the bulk

Shenker, Stanford

Non-maximal chaos and stringy corrections

- Gravity gives rise to maximal chaos.
- General quantum systems can have non-maximal chaos.
- What modification of gravity gives non-maximal chaos?
- Chaos is maximal due to the spin of the graviton → spin =2.
- String amplitudes can give rise to non-maximal chaos

$$\mathcal{A}_{\rm grav} \sim i G_N s^2/t \quad \longrightarrow \quad \mathcal{A}_{\rm string} \sim -G_N(-is)^{2+\alpha' t/2} F(t) \qquad t \propto -\frac{1}{R^2}$$

$$\lambda = \frac{2\pi}{\beta}\left(1 - \# \frac{\alpha'}{R^2} + \cdots\right)$$

Chaos in the S-matrix

Chaos in the S-matrix

- Imagine sending particles into a black hole and watching the ones coming out.
- This defines a ``black hole S-matrix''
- If we send an additional particle at time t=0, then the particles coming out at time t suffer a time delay relative to what we would have had if we had not sent the additional particle.

Polchinski

Longer times

- If the time delay is large → the amplitude becomes naively very small.
- However, the wavefunctions are not very sharply localized, so the particle we sent at t=0 has a tail that extends to larger times.

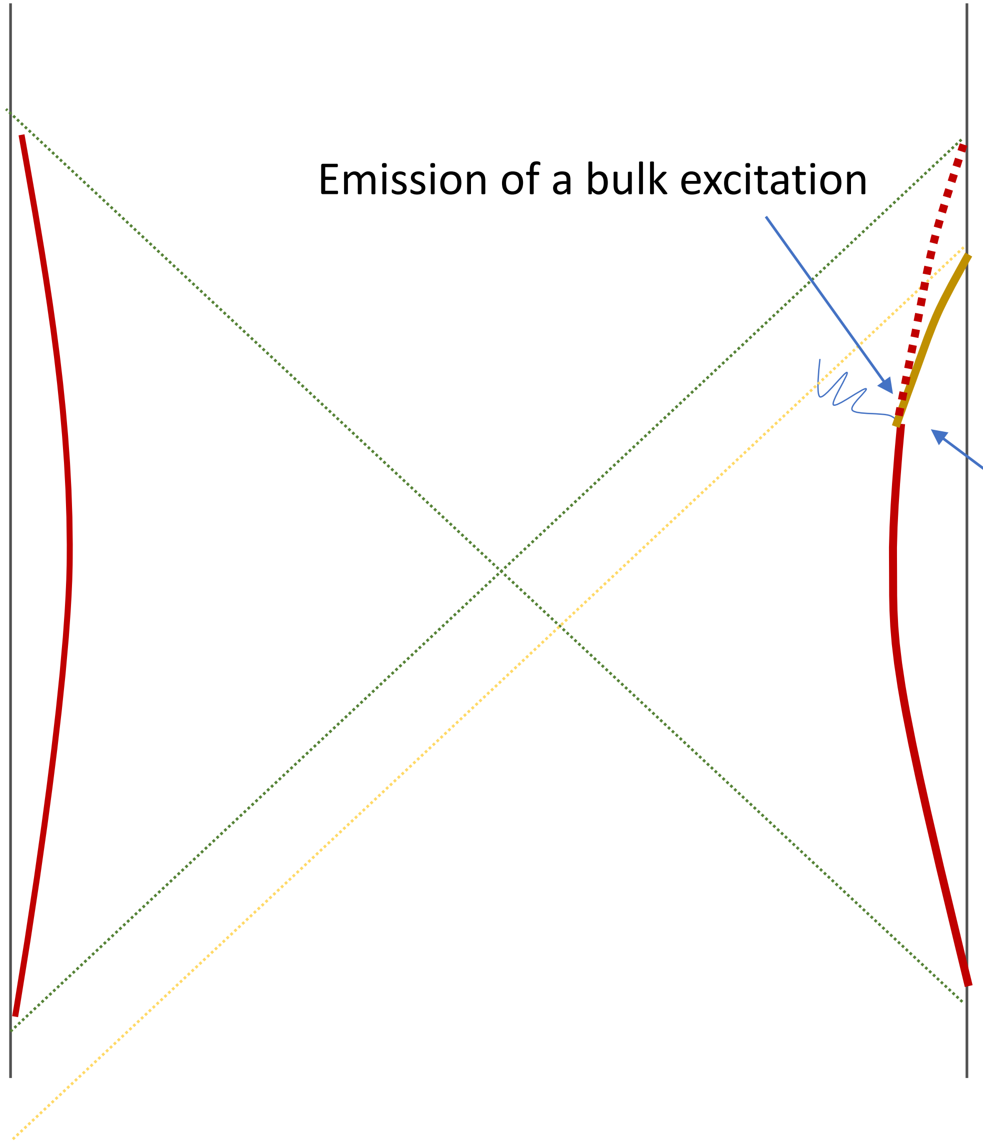
- OTOC at late times approaches zero, with a rate governed by the lowest quasinormal mode frequency associated to such fields,

$$OTOC \sim e^{-2\omega t} \,, \qquad \omega = \min\{\omega_V , \omega_W\}$$

A more explicit discussion for JT gravity

- Recall that all gravitational effects in JT gravity are given by the motion of the boundary particle.

Dynamics



Emission of a bulk excitation

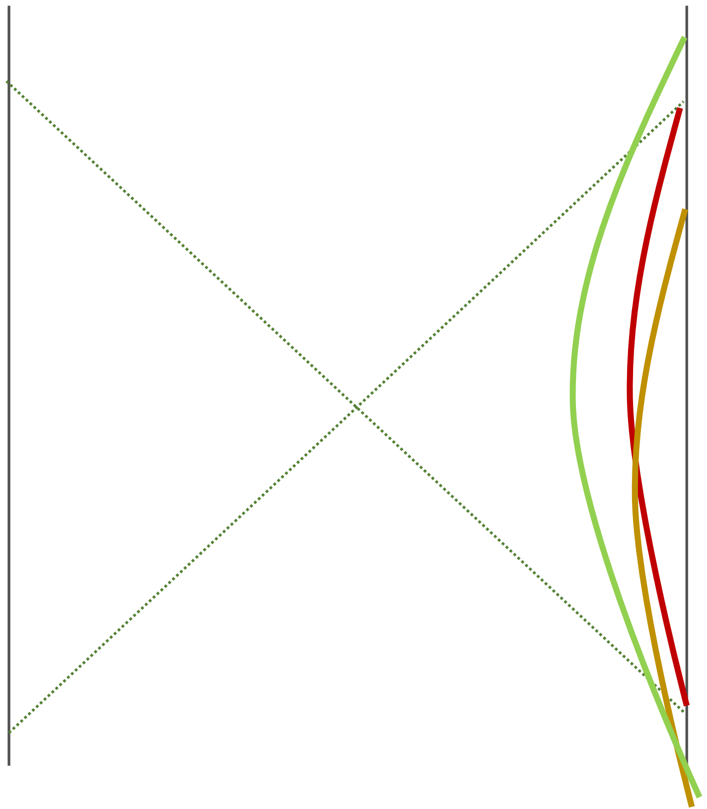
New position of the horizon

The boundary trajectory gets
a “kick” determined by local
energy momentum conservation.

Trajectories diverge exponentially → classical
chaos of motion in AdS space (similar to hyperbolic
space).

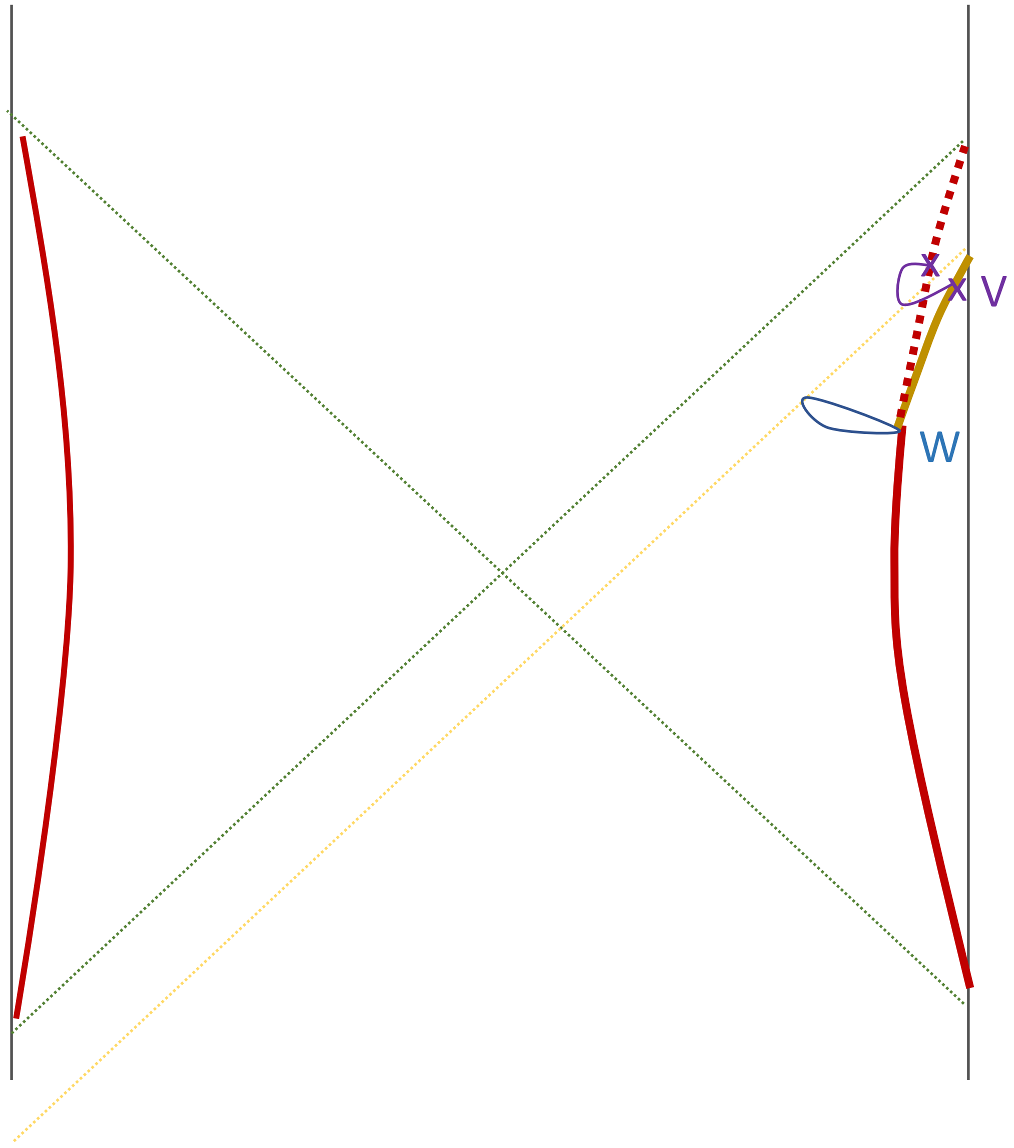
To detect it we need to compute an OTOC correlator.

- The various solutions for the boundary particle are specified by 4 parameters:
  - 1- The energy
  - 2- Moving the boundary particle trajectory with the action of the SL(2) charges
  - 1- Moving the boundary time relative to AdS_2 time along the trajectory.



The exponential growth is purely geometric, resulting from properties of the isometries of AdS_2

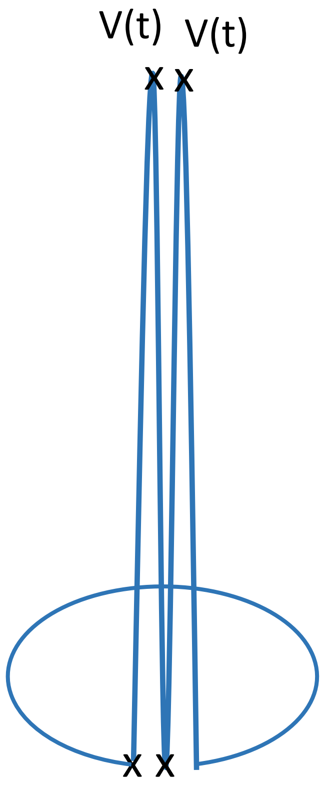
OTOC

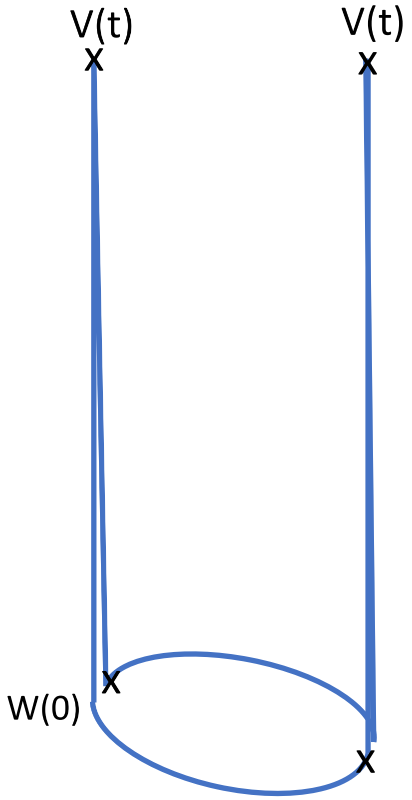


To detect it we need to compute an OTOC correlator.

We compute $\langle W(0)\ V(t)\ W(0)\ V(t) \rangle$

Two sided correlators





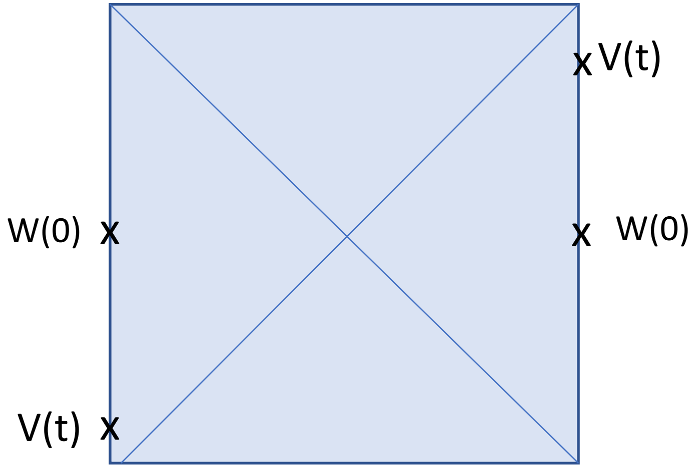
$$C_1(t) = Tr[e^{-\beta H} W V(t) W V(t)]$$

$$C_2(t) = Tr[e^{-\beta/2 H} W V(t) e^{-\beta/2 H} W V(t)] = \langle TFD | W_L(0) V_L(t) V_R(t) W_R(0) | TFD \rangle$$

$$F(t) = Tr[e^{-\beta/4 H} W e^{-\beta/4 H} V(t) e^{-\beta/4 H} W e^{-\beta/4 H} V(t)] = F(t)^*$$

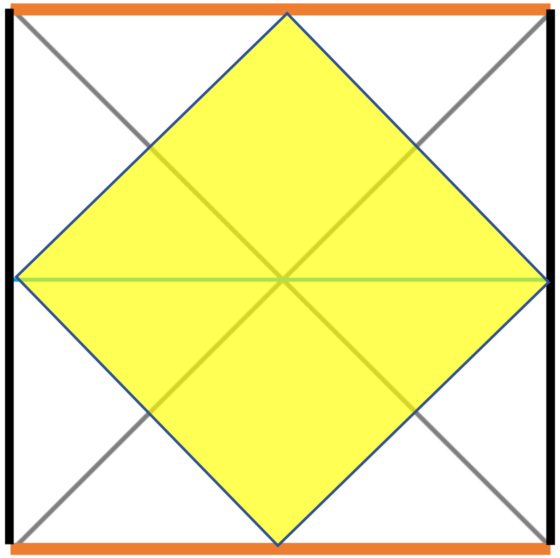
$$C_2(t) = F(t - i\beta/4)$$

$s = e^{2\pi t/\beta} \Rightarrow t \to t - i\beta/4$ leads to $s \to -is$



Traversable wormholes

Evolving the TFD

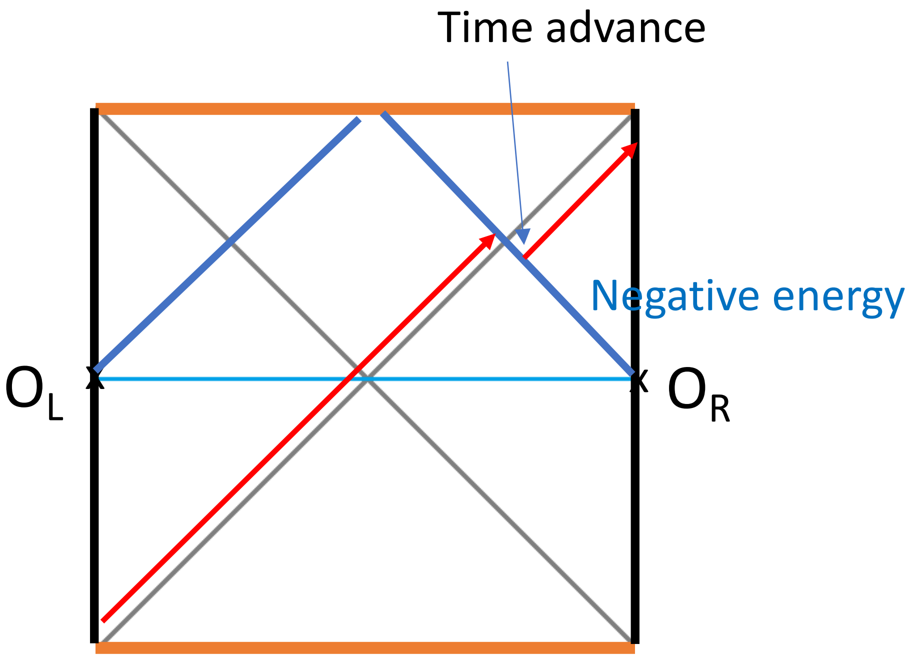


TFD = “Wheeler-de-Witt patch” = spacelike separated points.

The full geometry = decoupled evolution.

Making the wormhole traversable

Gao, Jafferis, Wall



Simplest modification to the evolution:

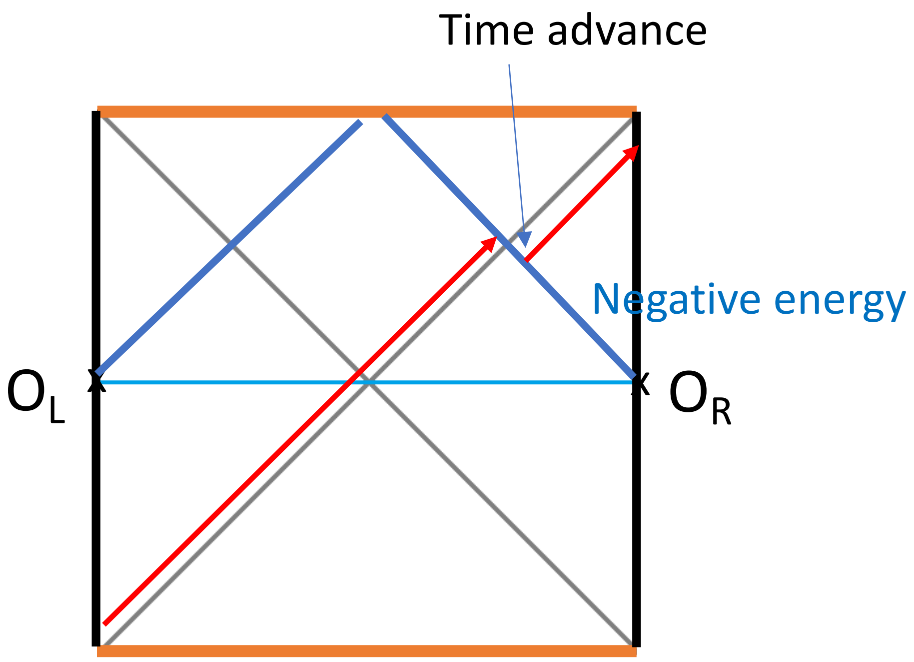
Add an operator that couples the two sides

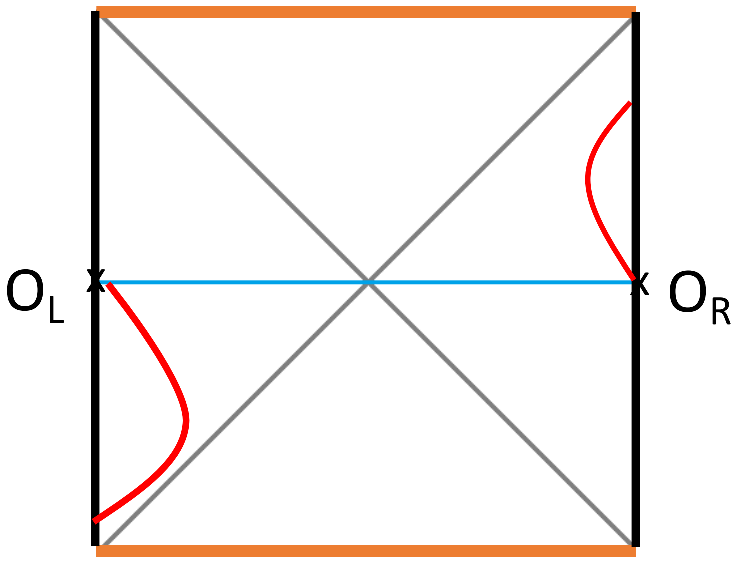
$$gO_L(0)O_R(0)$$

For a suitable choice of the sign of g, this produces a negative energy shock wave.
This produces a time advance.

Makes the wormhole traversable!

- Given that we have coupled the two sides, it is not surprising that a signal goes between the left and the right.
- What is interesting is <u>how</u> the signal goes.



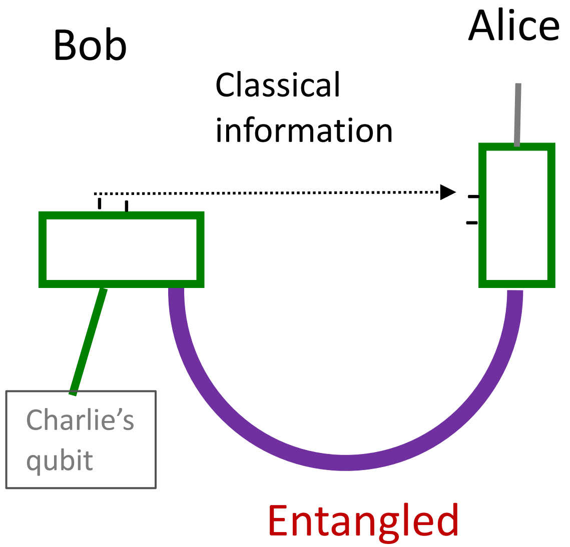


What one might have naively expected

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

Quantum teleportation

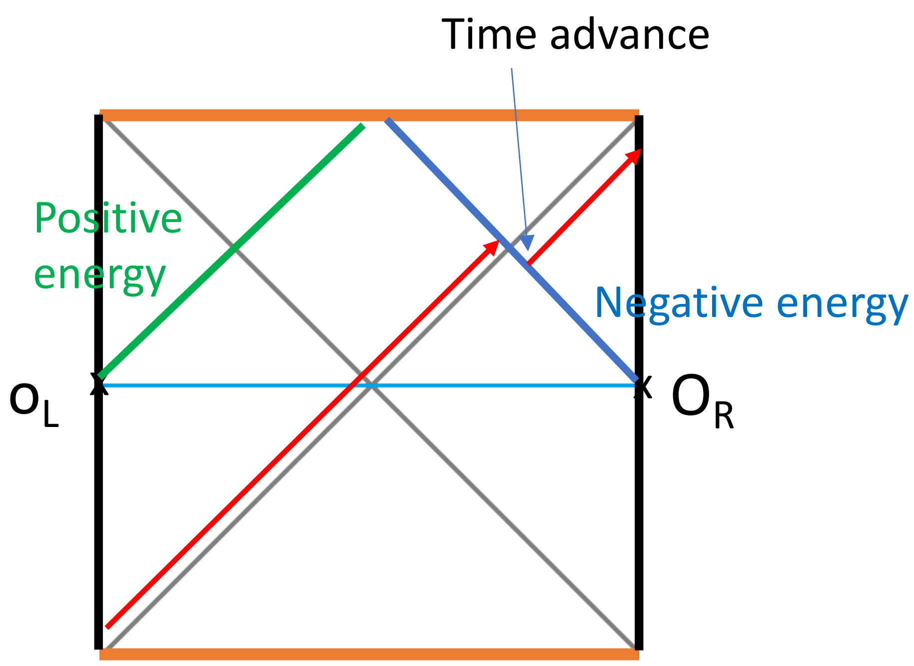
- Bob and Alice share an entangled pair of qubits.
- Charlie gives Bob a qubit and he wants to send it to Alice.
- Bob mixes Charlies qubit with his share of the entangled pair (unitary operation)
- Bob does a joint measurement of both qubits.
- Sends the the result to Alice as classical information
- Alice does an operation on his qubit that depends on Bob's result.
- Alice gets the qubit.



- Resources needed to send a qubit: One entangled qubit and 2 bits of classical information.

Would you like to be teleported ?

Quantum teleportation



Simplest modification to the evolution:

Measure the left operator → classical eigenvalue o_L

$$g o_L(0) O_R(0)$$

For a suitable choice of the sign of g, this produces a negative energy shock wave.
This produces a time advance.

Makes the wormhole traversable!

We only need to send classical information from the left to the right side.

Quantum gravity in the lab

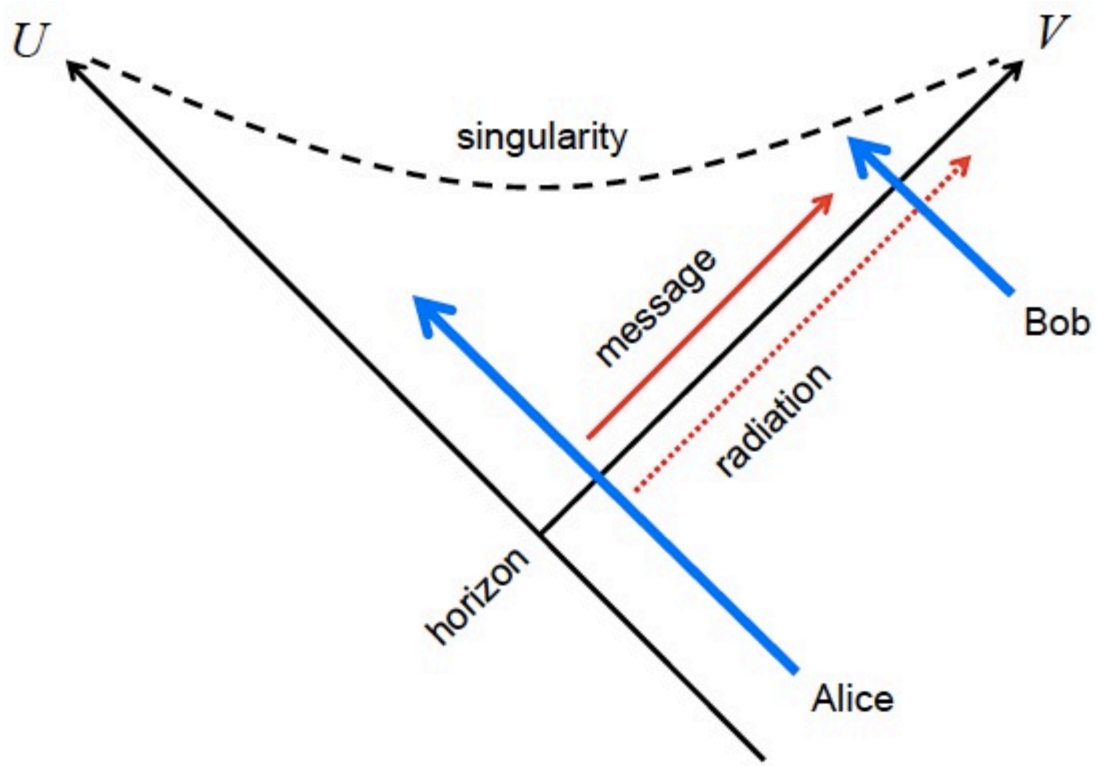
- People are devising simple enough systems that display this type of quantum teleportation.
- You would send just qubits.
- It is inspired by the connection to gravity.

Schuster, Kobrin, Gao, Cong, Khabiboulline, Linke, Lukin, Monroe, Yosihda, Yao

Nezami, Lin, Brown, Ghharibyan, Leichenauer, Salton, Susskind, Swingle, Walter

Application to the Hayden Preskill problem

Figure from the paper of Preskill and Hayden.



Suppose that Bob has been observing the black hole for a long time, and has a quantum system maximally entangled with it.

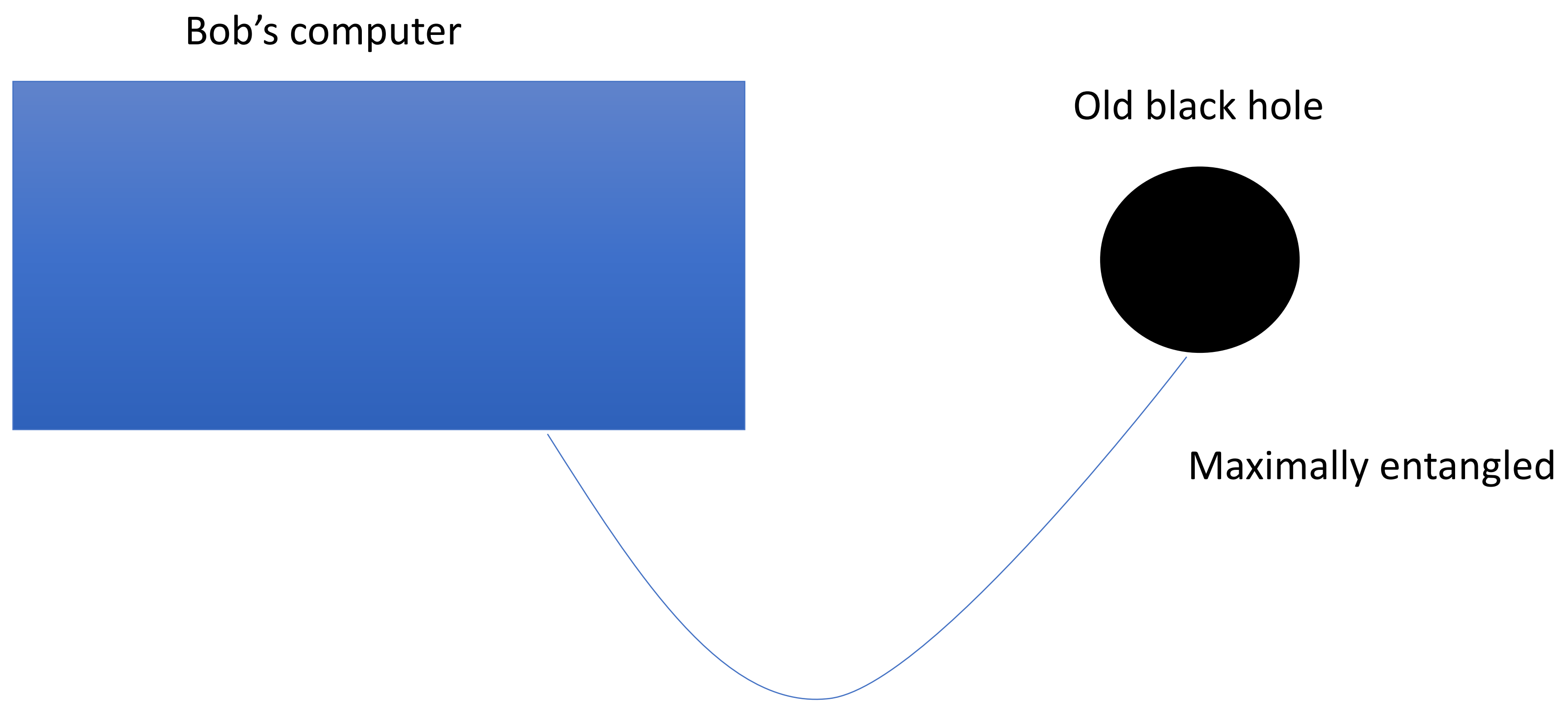
Alice drops in a single qubit message.

Bob waits for a scrambling time, and then collects a few qubits of the radiation.

Then Bob can recover the message.

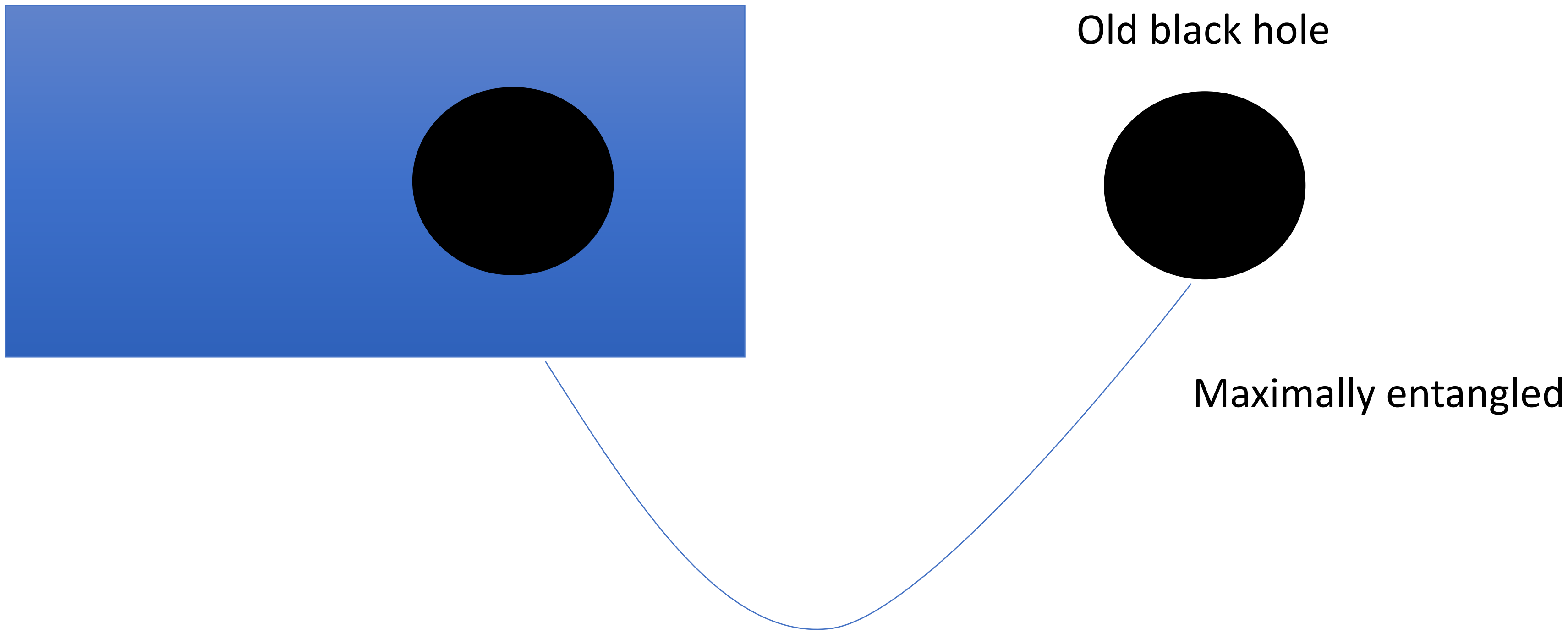
But then he can jump in and see another copy of the message.

Duplication of information in the geometry. In the black hole interior.


Bob's computer
Old black hole
Maximally entangled

Bob → produces a second black hole,
maximally entangled with the first.

(This is hard to do
Harlow Hayden)



Bob → produces a second black hole,
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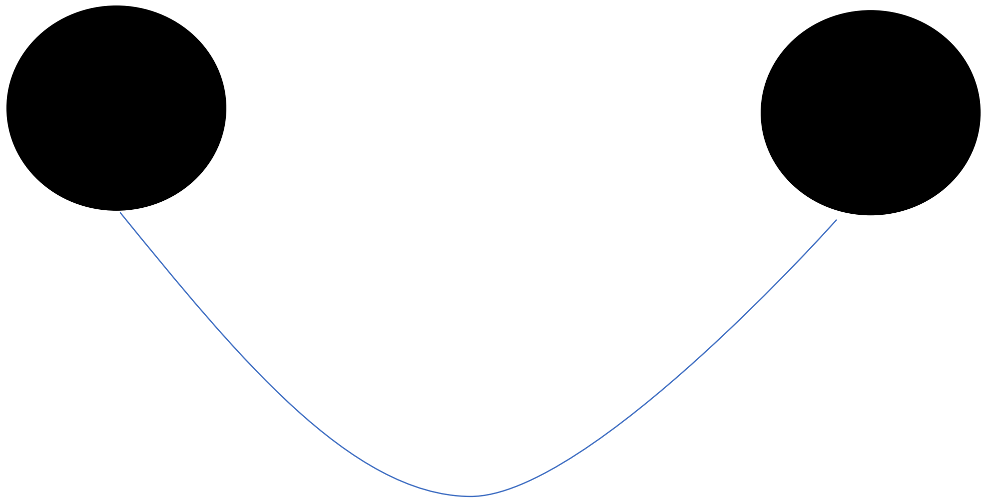
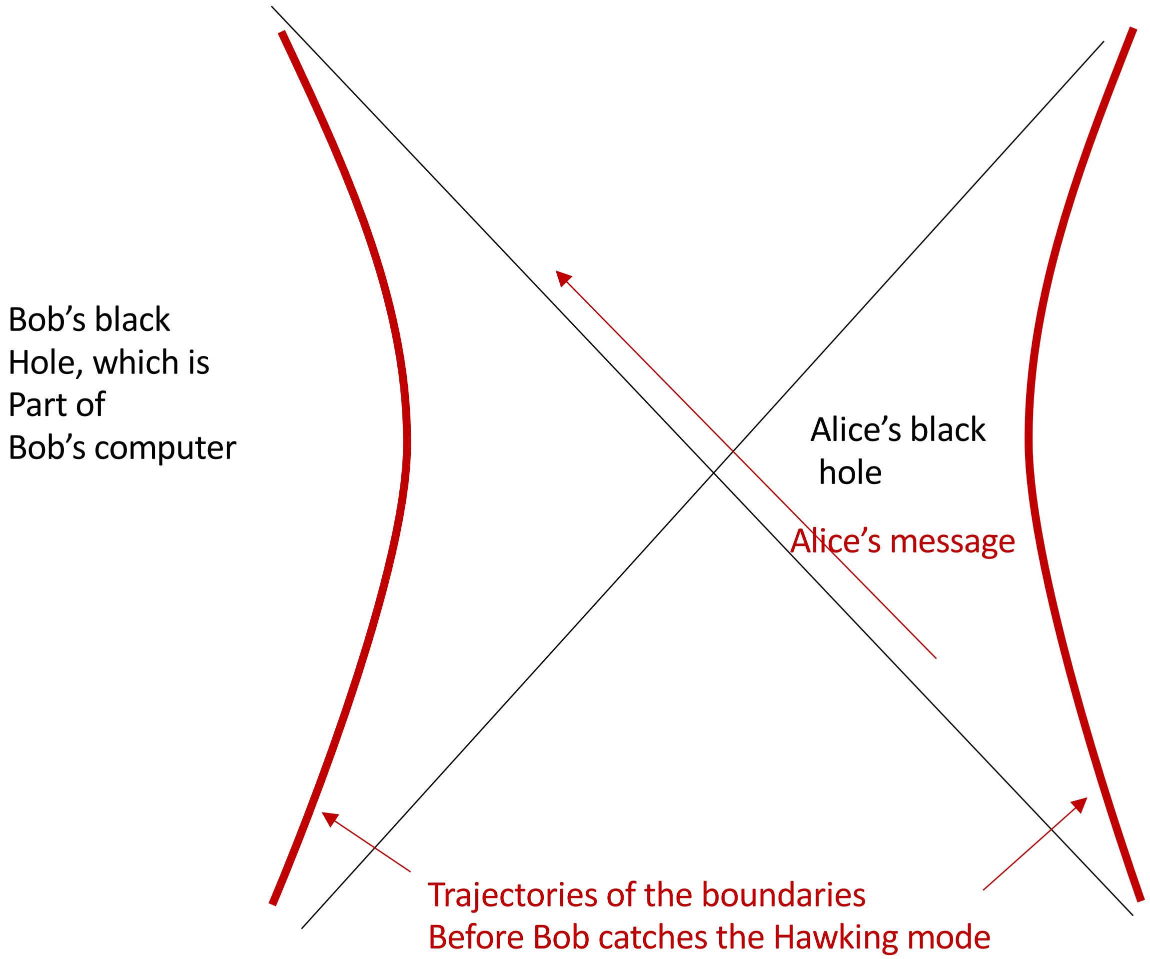
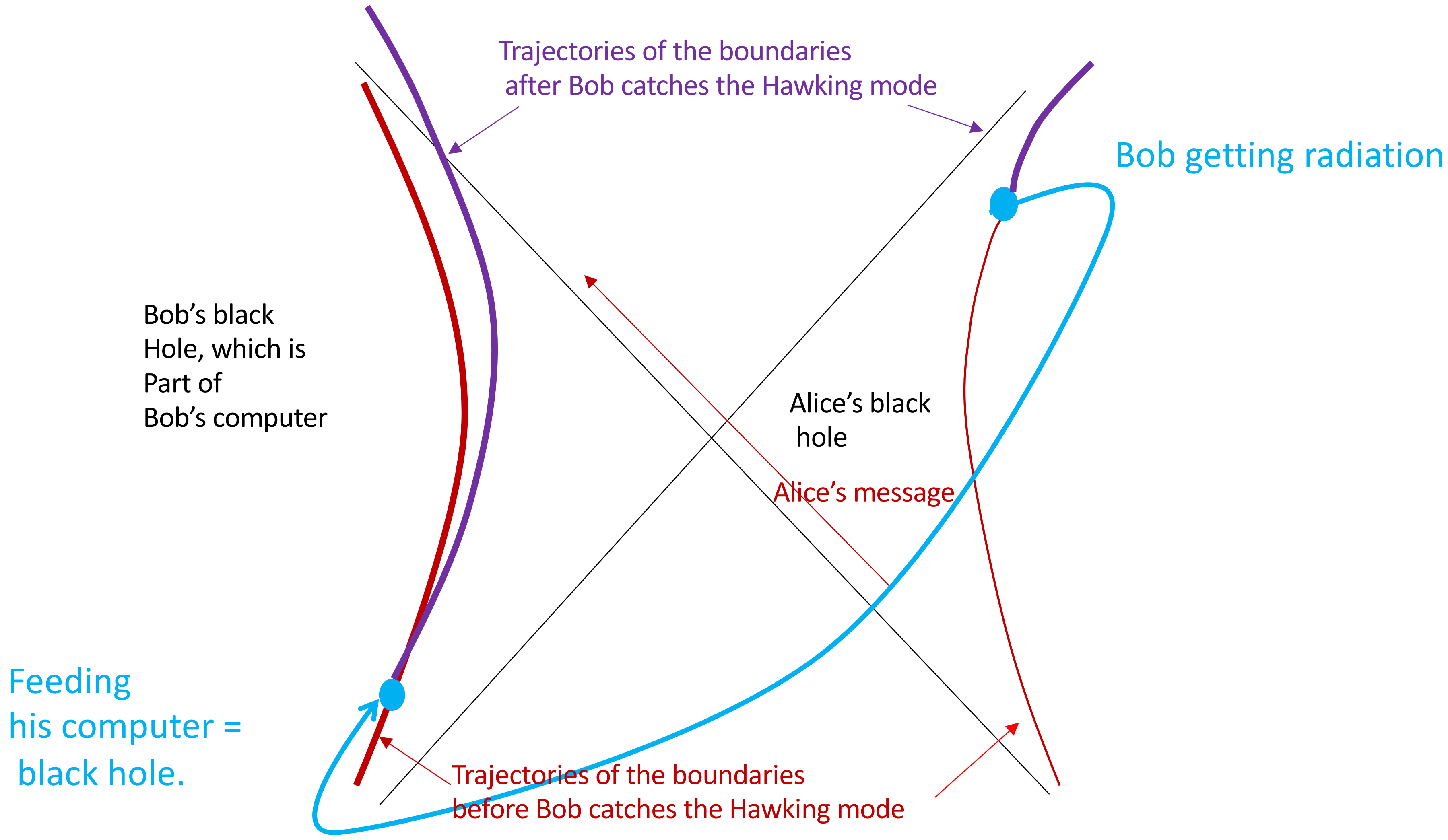


Maximally entangled

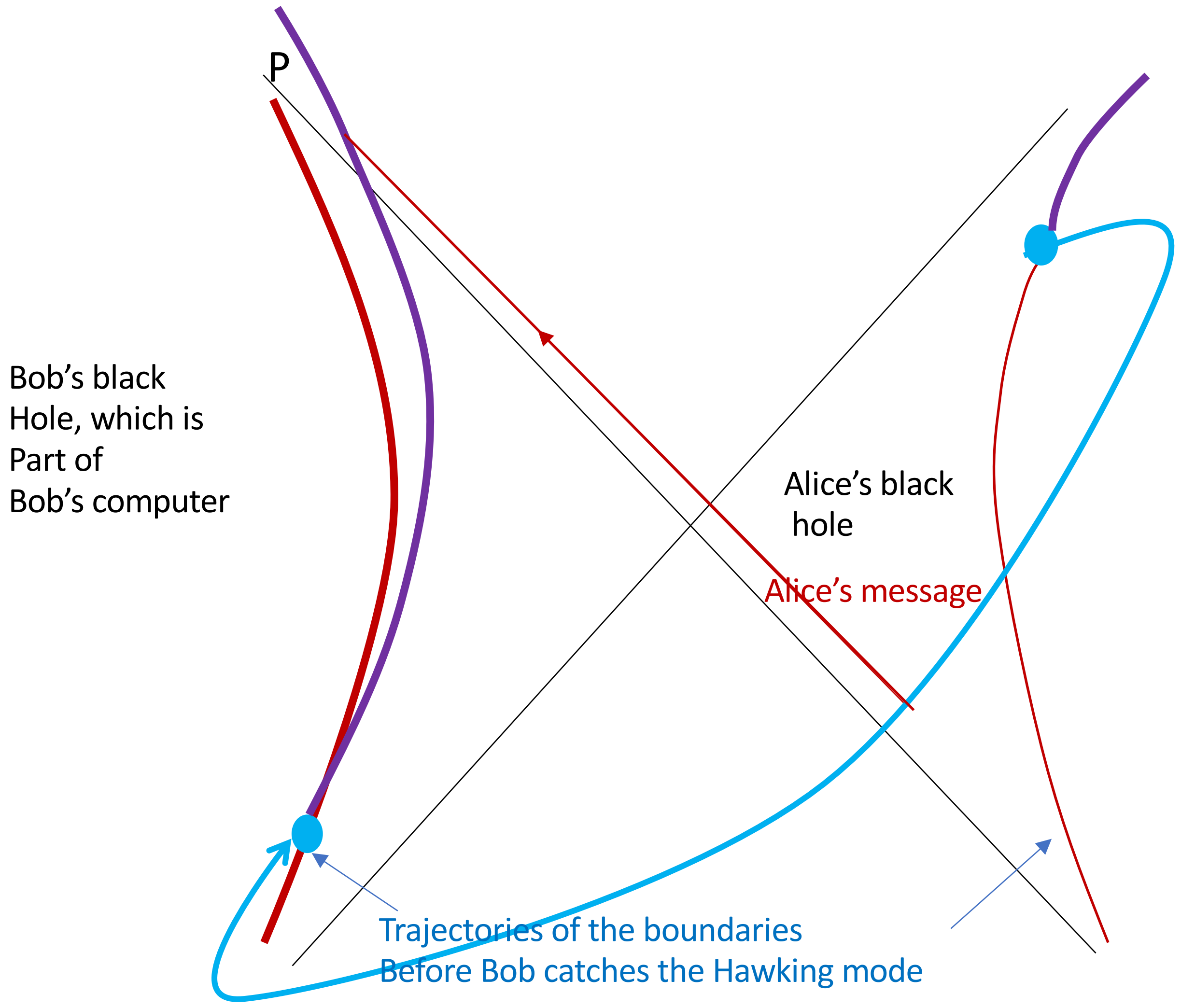
Say they are nearly AdS_2 black holes…


Bob's black
Hole, which is
Part of
Bob's computer
Alice's black
hole
Alice's message
Trajectories of the boundaries
Before Bob catches the Hawking mode

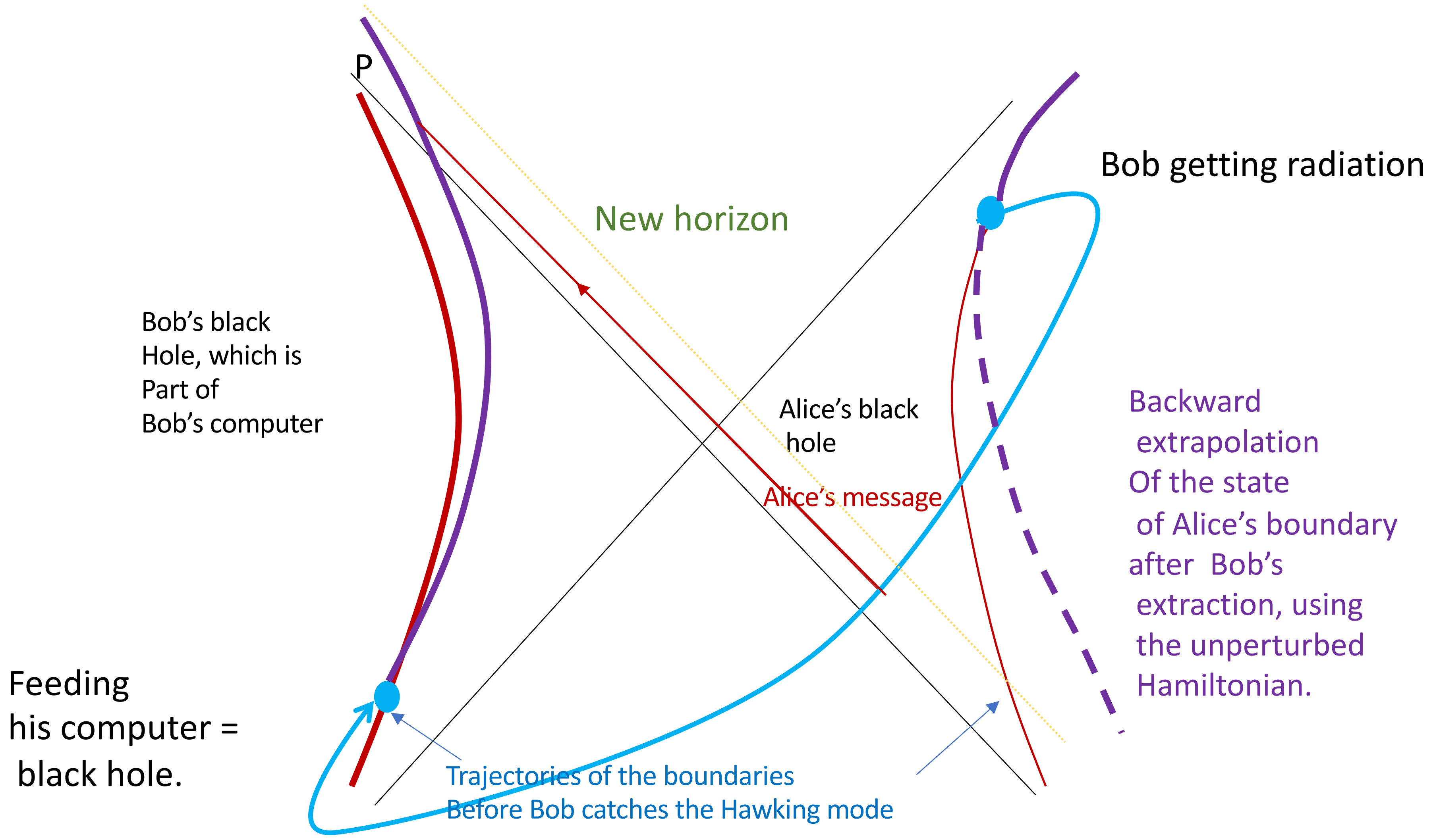
Bob gets some radiation and feeds it to his computer.



Bob now gets the message at P

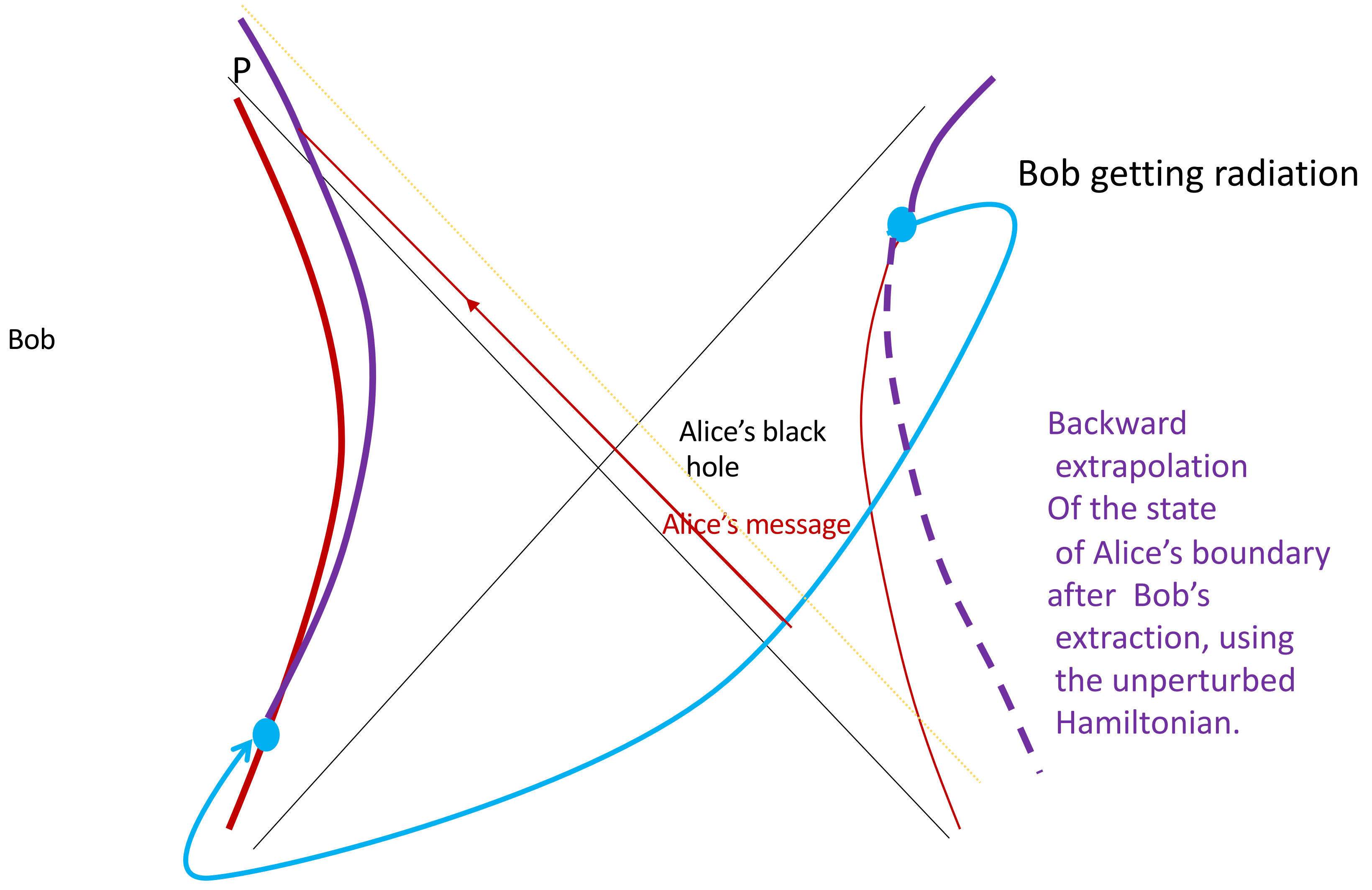


The message switched sides !



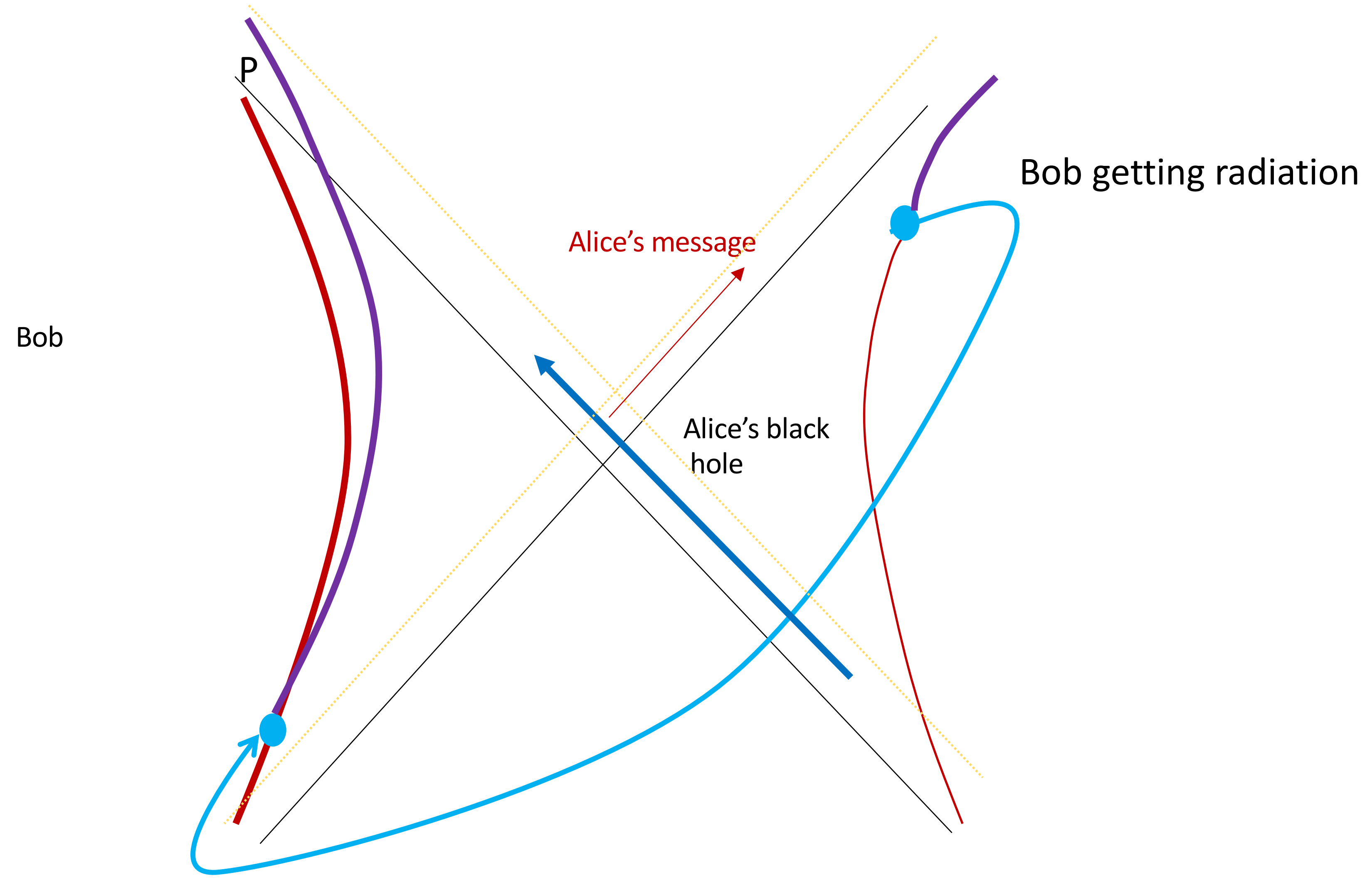
Before transfer: Alice has the message but Bob does not

After transfer: Bob has it but Alice does not !



Only one copy of the message throughout!

More like the HP figure

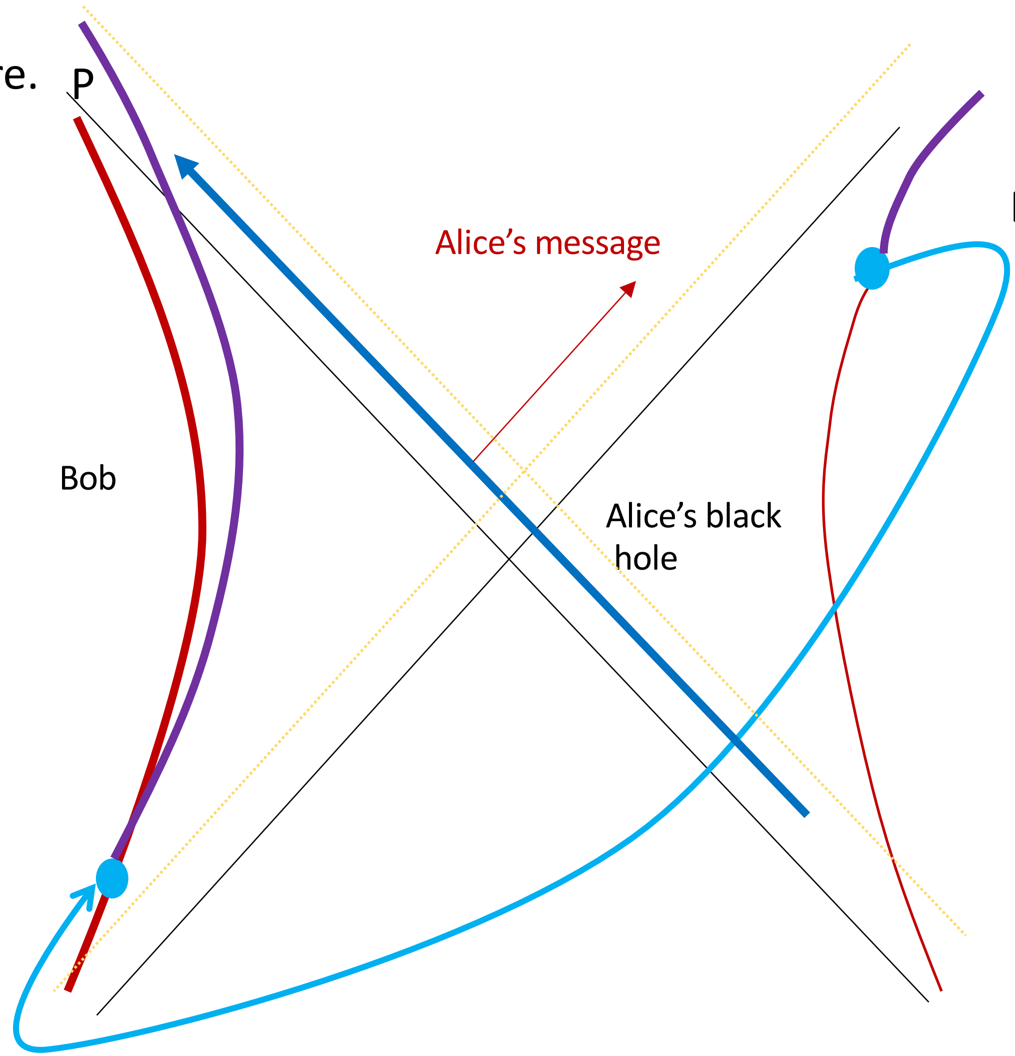


Now Bob cannot get the message !, it is still in Alice's possesion.

Now Alice send is later…

Bob will not get it here.

Bob gets the machinery that sent Alice’s message, but with no message.

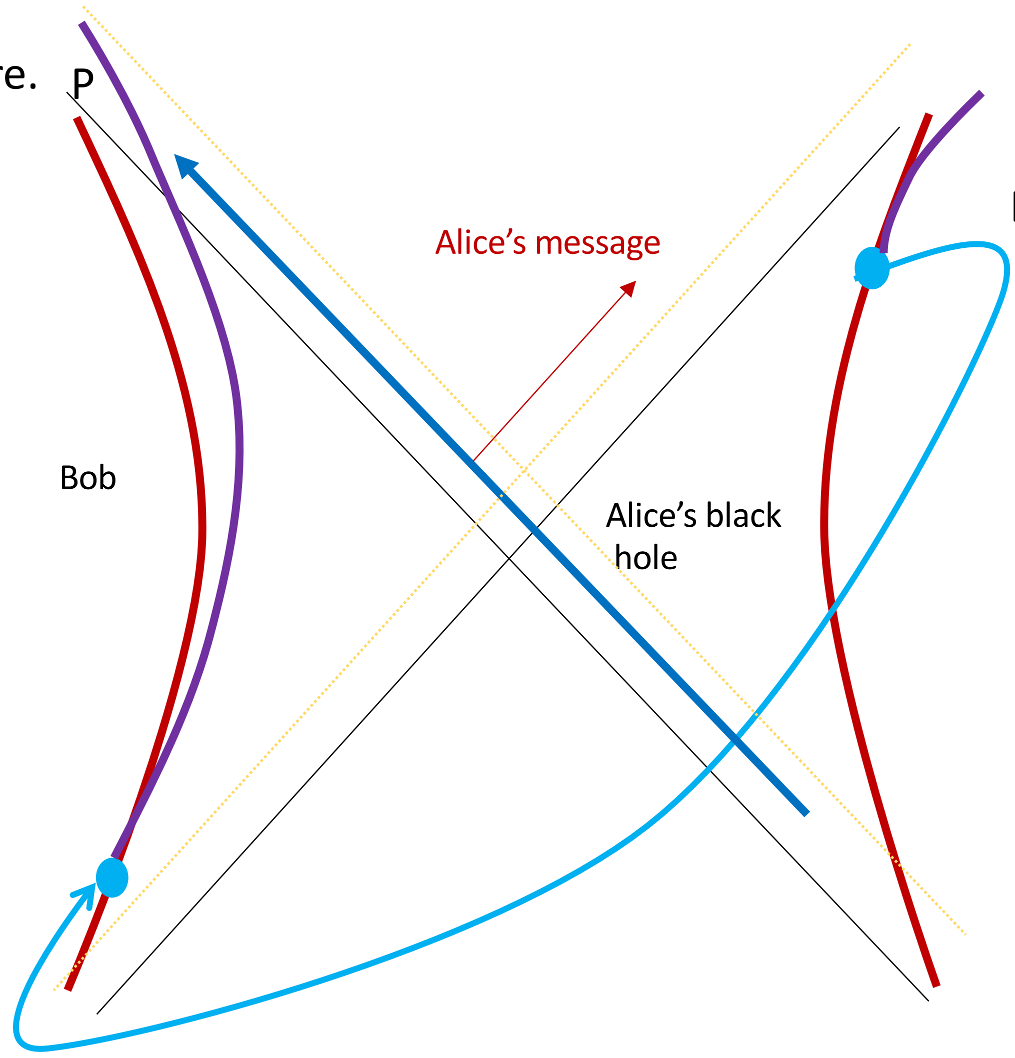


Bob getting radiation

Now Alice send is later…

Bob will not get it here.

Bob can extract
that machinery



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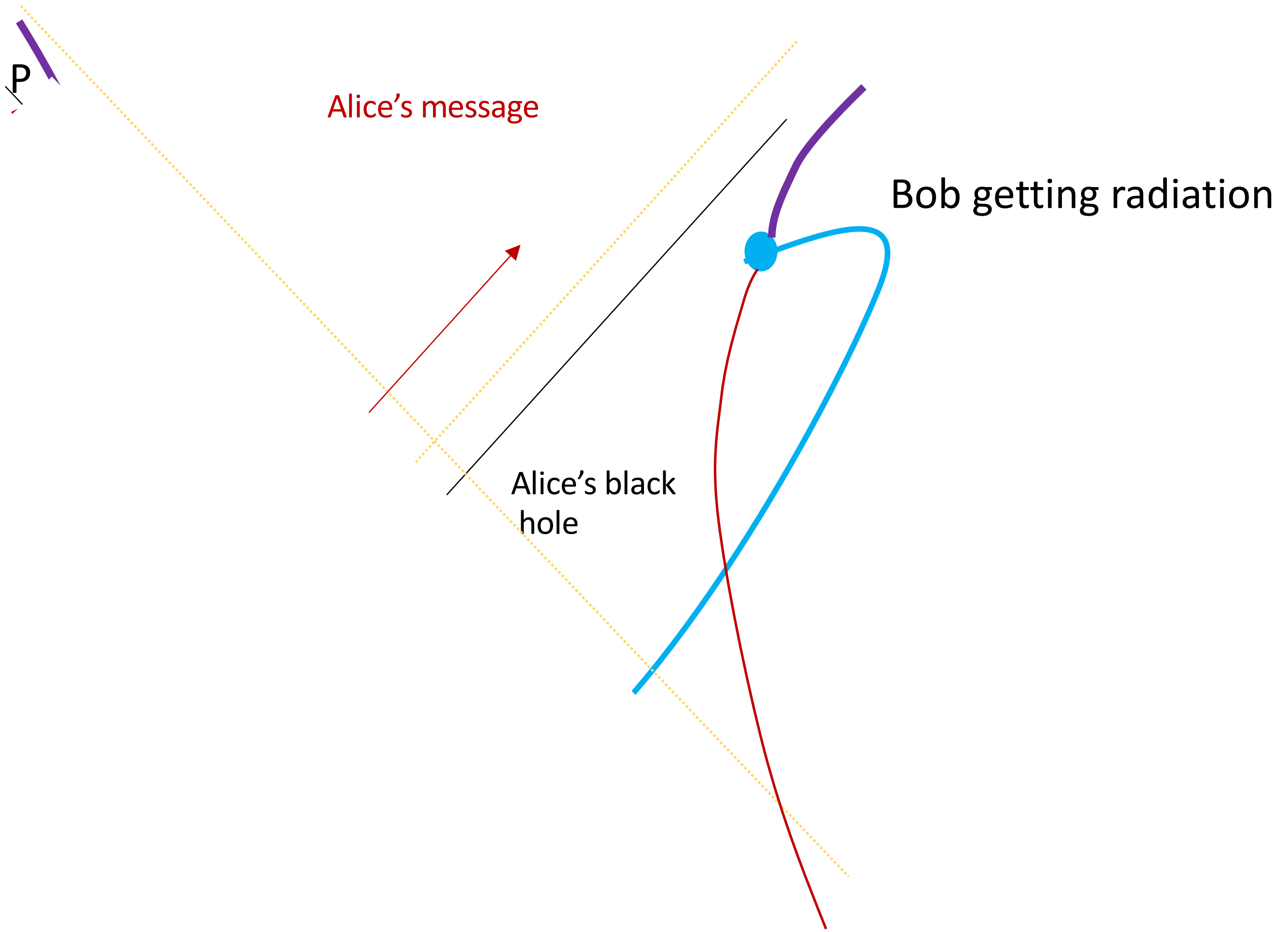
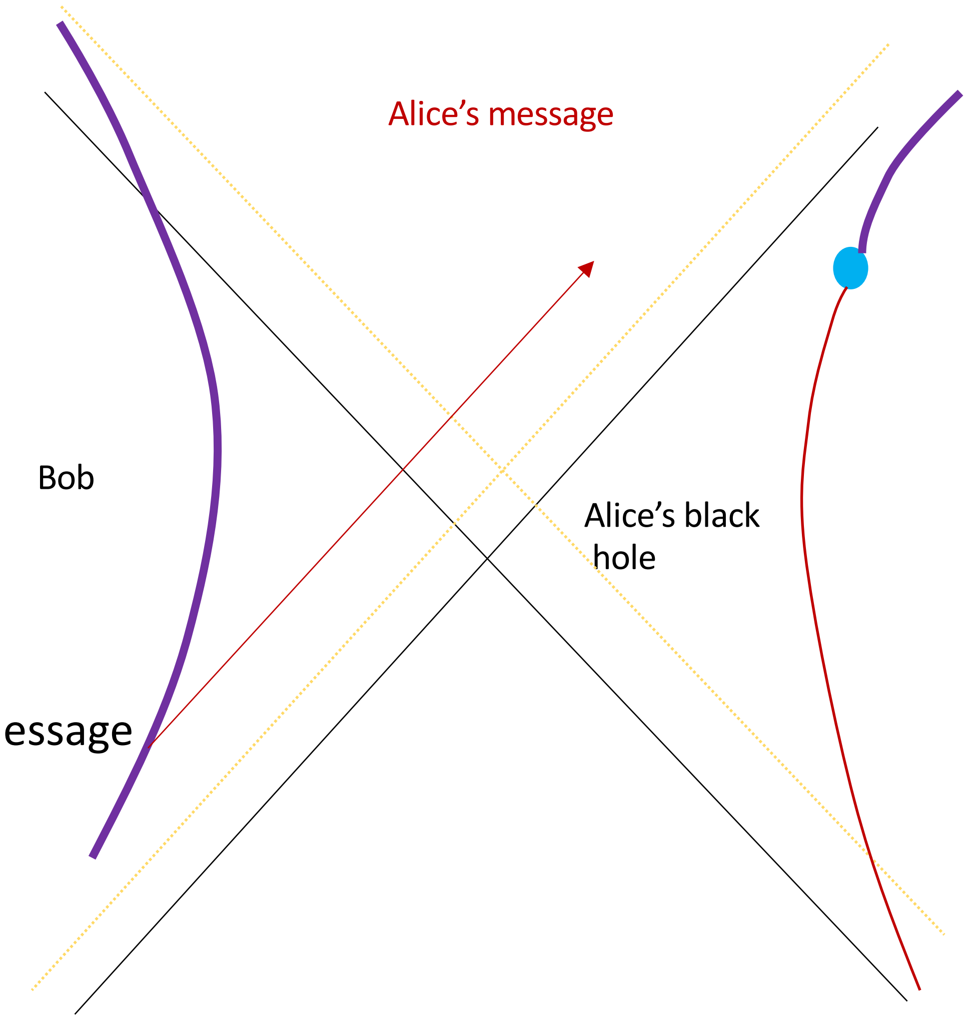


Now Alice send is later…



Bob can evolve his side backwards in time and recover the message here

- There are never two copies of the message.
- The message is snatched from the interior of the first black hole…
- When we do a complicated quantum computation, we should include the spacetime ``generated’’ by such a computation!

One other variant of the same basic traversable wormhole idea

Eternal traversable wormholes

JM & Qi

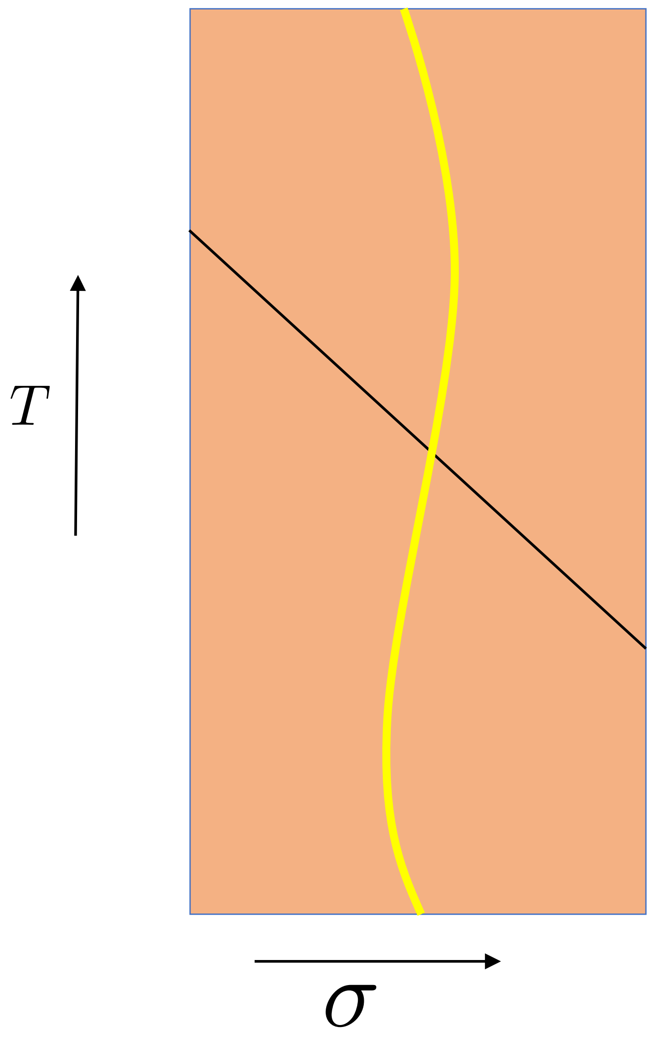
$$H = H_L + H_R + \mu \sum_i O_L^i O_R^i$$

Leave the interaction on.

It works nicely in Nearly-AdS_2

If OO is relevant → flows to a gapped system, whose ground state is close to the TFD of the decoupled system

AdS$_2$ - Global coordinates

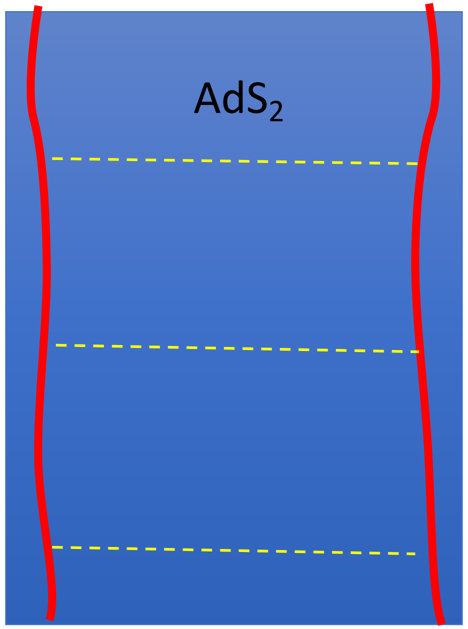


$$ds^2 = \frac{-dT^2 + d\sigma^2}{(\sin\sigma)^2}$$

- SL(2,R) isometries
- Two boundaries
- Causally connected
- Particle dynamics → oscillatory behavior → gapped spectrum
- Global coordinates

AdS_2 gravity +

Interaction



Consequences of the symmetries

- Spectrum = Part determined by the SL(2) symmetry + part coming from the boundary degree of freedom.

$$E = w_0 \left[m\sqrt{2(1-\Delta)} + \sum_i (n_i + \Delta_i) \right] \quad , \qquad m,\ n_i = \text{Integers}$$

Not determined by the symmetries, depends on μ

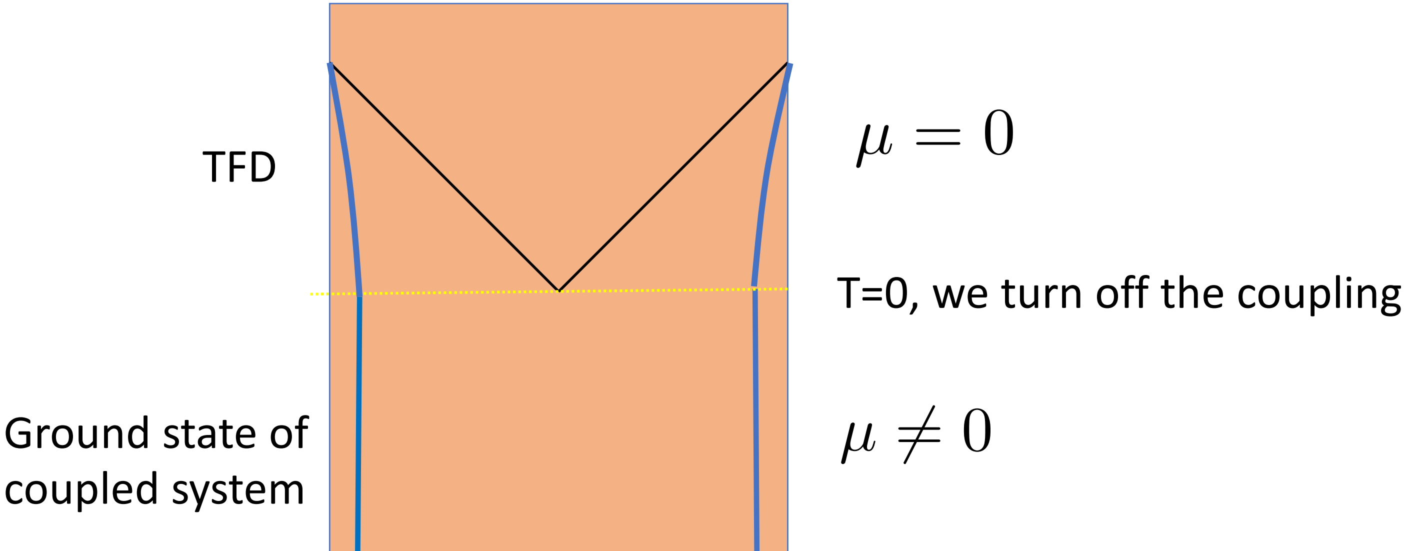
SL(2) representations. Bulk fields or conformal sector of the SYK model.

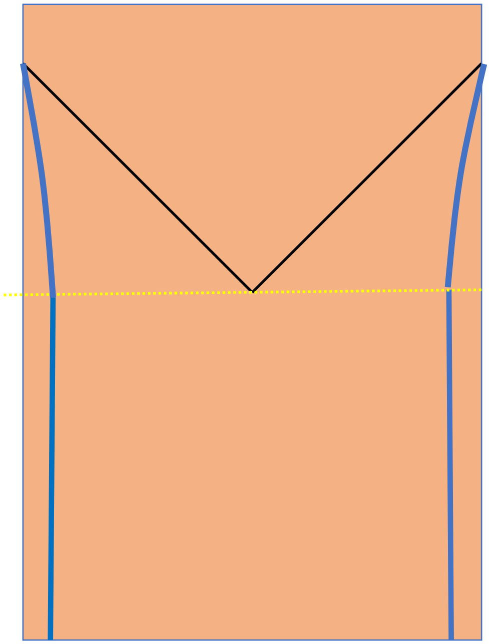
Motion of the boundary particles, of the Schwarzian action.

Making the TFD

- Create two SYK systems.
- Couple term. $\mu \neq 0$
- Couple them further to a heat sink and let them cool down to find its ground state.
- At t=0, turn off the left-right coupling. $\mu = 0$

- → Get a state that is close to the TFD.





$\mu = 0$

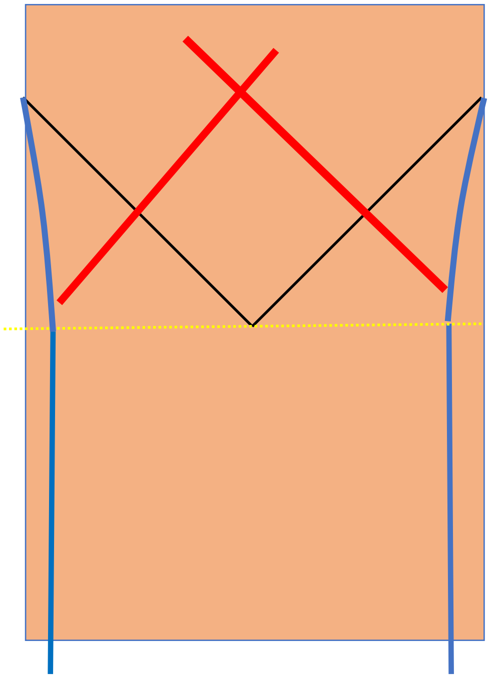
$\mu \neq 0$

Blackness (horizon) of the black hole depends on the evolution.
It is not a property of the state, but of how it evolves.

We could evolve the same state with the coupled Hamiltonian and we would not get a horizon.

The black hole is not a state but a state together with some particular evolution law.

Meeting behind the horizon



$\mu = 0$

$\mu \neq 0$

Send signals after we decouple.

They meet behind the horizon.

But we do not see any effect of their meeting if we look at only one side.

If we look at both sides?

We can choose to evolve with the coupled Hamiltonian → we would see the effects of their meeting.

End of lecture 2