jvirto@ub.edu

FLAVOR PHYSICS BSM

J. VIRTO, U. Barcelona & ICCUB (APRIL 2022)

1. REMINDER: SM & FLAVOR SYMMETRY
2. THE STANDARD MODEL AS AN EFT
3. FLAVOR IN THE SMEFT
4. BELOW THE EW SCALE
5. WEAK DECAYS AS BSM PROBES

[6. BSM MODELS & MATCHING CALCULATIONS]

Note: We will focus on the grand area of "Heavy" New Physics. This is rather general but it is only "half" of the story. The issue of light BSM physics very weakly coupled to the SM is equally interesting and it is also related to FLAVOR PHYSICS.

1. BRIEF REMINDER : SM & FLAVOR SYMMETRY

The Standard Model SM is defined by:

- RENORMALIZABLE(*) QFT with
- GAUGE SYMMETRY : $SU(3) \times SU(2) \times U(1)$
- FIELD CONTENT (+REPS) : $(Q, U_R, D_R, L, E_R) \times 3 + H$
- SSB : $\langle H^\dagger H \rangle = v^2/2 \neq 0$.

with the following reps:

	Q	U_R	D_R	L	E_R	H
$SU(3)_c$	3	3	3	1	1	1
$SU(2)_L$	2	1	1	2	1	2
$U(1)_Y$	$1/6$	$2/3$	$-1/3$	$-1/2$	-1	$1/2$

Note: Remembering hypercharges:

- $Y_H = 1/2$
- $Y_{\psi_R} = Q_{\psi_R}$
- For ψ_L do $\underset{}{\bar{Q}}\, \underset{1/2}{H}\, \underset{-1/3}{D_R}$ and $\bar{L}\, \underset{1/2}{H}\, \underset{-1}{E_R}$

The procedure is the following:

1) Write a Lorentz-invariant, Gauge-invariant Lagrangian with all available fields, with all possible terms up to canonical dim ≤ 4, and give vev to the Higgs field

$$\Rightarrow \quad \mathcal{L}_{SM} = \mathcal{L}_{D\leq 4} .^{(*)}$$

2) Renormalize this Lagrangian and fix all renormalized couplings (Lagrangian parameters) from a set of experimental measurements according to some renormalization scheme.

3) Use the theory!

- Calculate observables ("predictions")
- Test experimental measurement
- Learn about the fundamental principles.

The level of success of this program up to now is the LEGACY OF THE 20th CENTURY.

Step 1 gives:

$$\mathcal{L}_{SM} = \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_F + \mathcal{L}_Y$$

with:

$$\mathcal{L}_G = -\frac{1}{4} \sum_{F=G,W,B} F^a_{\mu\nu} F^{a,\mu\nu}$$

$$\mathcal{L}_S = (D_\mu H)^\dagger (D^\mu H)$$

$$\mathcal{L}_F = \sum_{\psi,ij} \delta_{ij} \bar{\psi}_i (i\not{D}) \psi_j = \sum_{\psi,i} \bar{\psi}_i (i\not{D}) \psi_i$$

Always possible

where: $\psi_i = \{ L^i, E^i_R, Q^i, U^i_R, D^i_R \}$; $L = \begin{pmatrix} N_L \\ E_L \end{pmatrix}$; $Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$

$i = \{1,2,3\}$ (generation)

The blue part of the SM Lagrangian has a $U(3)^5$ symmetry under which:

$$\psi^i \longrightarrow U^{ij}_\psi \psi^j \quad , \quad U_\psi \in U(3)$$

This $U(3)^5$ symmetry is called the

FLAVOR SYMMETRY.

Note: $\mathcal{U}(3)^5 = \mathcal{U}(1)^5 \times SU(3)^5$

Separate Global phase transformations for $\{L, E_R, Q, U_R, D_R\}$

The $\mathcal{U}(3)^5$ Flavor symmetry is broken (partially) by $\mathcal{L}_Y$:

$$\mathcal{L}_Y = -y_d^{ij}\, \bar{Q}^i \cdot H\, D_R^j - y_u^{ij}\, \bar{Q}^i \cdot \tilde{H}\, U_R^j$$

$$\tilde{H} = i\sigma^2 H^*$$

$$- y_e^{ij}\, \bar{L}^i \cdot H\, E^j + h.c.$$

Using the $\mathcal{U}(3)^5$ symmetry of $\mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_F$ one can always write (EXERCISE)

$$y_e^{ij} = \mathrm{diag}(y_e, y_\mu, y_\tau)$$

$$y_u^{ij} = \mathrm{diag}(y_u, y_c, y_t)$$

$$y_d^{ij} = V_{CKM} \cdot \mathrm{diag}(y_d, y_s, y_b)$$

"CKM matrix"

V_{CKM} is a unitary 3×3 matrix parametrized by 3 angles + 1 phase.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

A convenient parametrization of the CKM Matrix is the Wolfenstain Parametrization:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 + & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)\,.$$

in terms of 4 (real) Wolfenstein Parameters,

$$\{\lambda,\ A,\ \rho,\ \eta\}$$

where $\lambda \simeq 0.22$ is the Cabibbo parameter, acting as an expansion parameter.

We'll extract the values of λ, A, ρ, η from experiment in the exercise session.

Once the Higgs acquires a vev:

$$H(x) = \frac{1}{\sqrt{2}}\, \mathcal{U}(x) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

then $\mathcal{L}_Y$ contains quadratic terms in fermion fields:

$$\mathcal{L}_{m_\psi} = -\sum_{\psi = E,D,U} M_\psi^{ij}\, \bar{\psi}_L^i\, \psi_R^j \quad + \ h.c.$$

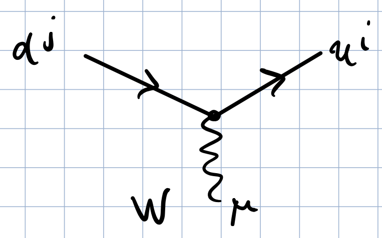
with

$$\boxed{M_\psi = \frac{v}{\sqrt{2}} y_\psi} \qquad (\#)$$

i.e:

$$M_e = M_E = \mathrm{diag}(m_e, m_\mu, m_\tau)$$
$$M_u = M_U = \mathrm{diag}(m_u, m_c, m_t)$$
$$M_D = V_{CKM} \times \mathrm{diag}(m_d, m_s, m_b) \equiv V_{CKM} \cdot M_d.$$

Rotating now the d_L^i fields separately from the u_L^i fields the matrix V_{CKM} can be removed from M_D, but then V_{CKM} appears in the W couplings:



$$= i \frac{g}{\sqrt{2}} V_{ij} \gamma_\mu P_L$$

Note: The relation (#) between the masses and the yukawa couplings is modified in the presence of BSM physics, as we will see in Chapter 3.

$\mathcal{L}_Y$ respects the subgroup (EXERCISE)

$$U(1)^5_{SM} = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_Y$$

where:

$U(1)_B$:
$$Q \to e^{i\alpha} Q$$
$$U_R \to e^{i\alpha} U_R$$
$$D_R \to e^{i\alpha} D_R$$

$U(1)_e$:
$$e_L \to e^{i\alpha} e_L$$
$$e_R \to e^{i\alpha} e_R$$

$U(1)_{\mu,\tau}$: analogous to $U(1)_e$

$U(1)_Y$: Hypercharge transformation (Gauged)

Comments:

1) the unbroken $U(1)^5_{SM}$ symmetry is an <u>ACCIDENTAL SYMMETRY</u> of the SM.

2) If $y_f^{ij} \ll 1$, then the <u>broken</u> flavor symmetry is an APPROXIMATE SYMMETRY. (E.g. $U(2)^5$)

2. THE STANDARD MODEL AS AN EFT

There is one axiom of the SM which was intro-duced only for the purpose of calculation: the axiom of RENORMALITZABILITY.

Imagine adding to the SM the dim-6 operator

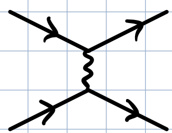
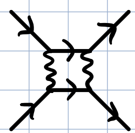
$$Q^{VLL}_{1122} = (\bar{e}_L \gamma_\alpha e_L)(\bar{\mu}_L \gamma^\alpha \mu_L)$$

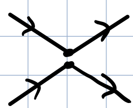
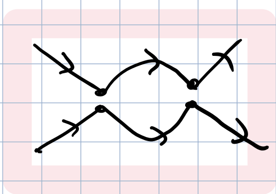
such that

has dim = $1/M^2$

$$\mathcal{L} = \mathcal{L}_{SM} + C^{VLL}_{1122}\, Q^{VLL}_{1122}$$

and calculate the amplitude for $e^-\mu^- \to e^-\mu^-$

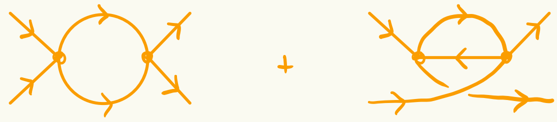
$$i\mathcal{A}(e^-\mu^- \to e^-\mu^-) =$$  $+$  $+ \dots$

$+$  $+$  $+ \dots$

Needs counterterm of order $1/M^4$

$\Rightarrow$ Needs counterterm of dim=8 operator!

$$\left(\text{e.g. } C^{(8)} \cdot [\bar{e}_L \gamma_\alpha \partial_\beta e_L][\bar{\mu}_L \gamma^\alpha \partial^\beta \mu_L]\right)$$ (*)

(*) 

$$= \frac{1}{\varepsilon}\cdot\frac{2}{3}(3s-u)\;\; \bar{u}(p_1')\gamma_\mu P_L u(p_1)\cdot \bar{u}(p_2')\gamma^\mu P_L u(p_2) \;+\; \text{finite}$$

This divergence can be cancelled by a counterterm of the dim = 8 operator:

$$O^{(8)} = 3\,(\bar{e}_L \gamma_\mu P_L \partial_\nu e_L)(\bar{\mu}_L \gamma^\mu P_L \partial^\nu \mu_L)$$
$$-\,(\bar{e}_L \gamma_\mu P_L \partial_\nu e_L)(\partial^\nu \bar{\mu}_L \gamma^\mu P_L \mu_L)$$

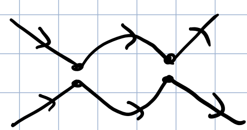
(see notebook)

The solution is to adopt a POWER COUNTING:

$$C^{VLL}_{1122} \sim 1/\Lambda^2 \quad ; \quad C^{(8)} \sim 1/\Lambda^4$$

for $\Lambda \gg E$, $\mathcal{A} = 1 + E^2/\Lambda^2 + E^4/\Lambda^4 + \dots$

$\Rightarrow$ We work to fixed order in $E/\Lambda \ll 1$.

To $O(E^2/\Lambda^2)$ the contribution  is of higher order and can be dropped. Then it is consistent to add only dim-6 operators.

$\Rightarrow$ this EFT can be renormalized.

Thus, removing the renormalitzability axiom:

$$\mathcal{L} = \underbrace{\mathcal{L}^{(D\leq 4)}}_{\mathcal{L}_{SM}} + \frac{1}{\Lambda^{D-4}} \mathcal{L}^{(D>4)}$$

"cut-off" scale

$\leftrightarrow$ Power counting parameter

At energies $E \ll \Lambda$:

$$\text{Observable} = \text{Observable}(SM) + c_5 \, E/\Lambda + c_6 \, E^2/\Lambda^2 + \cdots$$

The fact that $E \ll \Lambda$ for all energies E probed so far explains why the SM works so well !!!

The current consensus is that

$$\boxed{SM = EFT = SMEFT}$$

The SMEFT contains: (e.g. 1008.4884)

1 dim = 5 operator (Weinberg's op.)

59+4 dim = 6 operators.

$\times$ Flavor

$= \left| \begin{array}{l} 12 \ \text{dim}=5 \ \text{ops} \\ 2499 + 546 \ \ \text{dim}=6 \ \ \text{ops} \end{array} \right.$

(see notebook)

B-conserving B-violating

Examples:

1) Dim-5 operators:

$$Q_{\ell\ell HH} = (\tilde{H}^{+}\ell)^{T}\, C\, (\tilde{H}^{+}\ell) \qquad \left(\text{and} \quad Q_{\ell\ell HH}^{+}\right)$$

x Flavor:

$$Q_{\ell\ell HH}^{ij} = (\tilde{H}^{+}\ell_i)^{T}\, C\, (\tilde{H}^{+}\ell_j)$$

$$i,j = \{1,2,3\}$$

But: $\quad Q_{\ell\ell HH}^{ij} = Q_{\ell\ell HH}^{ji}$

$$\Rightarrow \; 6 \text{ indep. ops} \quad (+\, 6 \text{ h.c.}) = \underline{12}$$

2) Dim-6 B-violating op:

$$Q_{duue} = \varepsilon_{\alpha\beta\sigma}\, (d_\alpha^T C u_\beta)(u_\sigma^T C e) \qquad [+\text{ h.c.}]$$

x Flavor:

$$Q_{duue}^{ijkl} = \varepsilon_{\alpha\beta\sigma}\, (d_{\alpha i}^T C u_{\beta j})(u_{\sigma k}^T C e_l)$$

$$i,j,k,l = \{1,2,3\}$$

$$\Rightarrow \; 81 \text{ indep ops} \quad (+\, 81 \text{ h.c.}) = \underline{162}$$

The SMEFT Lagrangian is

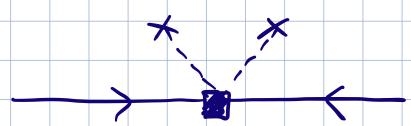
$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i C_i^{(5)} Q_i^{(5)} + \sum_i C_i^{(6)} Q_i^{(6)} + \cdots$$

where $$C_i^{(d)} \sim c/\Lambda^{d-4} .$$

The renormalizable SM has $C_i^{(d)} = 0$. Thus in order to probe this idea we need to measure non-zero values for some $C_i^{(d)}$.

In a sense we already have evidence of $C_i^{(5)}$:

$$Q_{\ell\ell HH} = (\tilde{H}^\dagger \ell)^T C (\tilde{H}^\dagger \ell) \xrightarrow[\langle H \rangle = v/\sqrt{2}]{} \frac{1}{2} v^2 \nu^T C \nu + \cdots$$



which is a Majorana mass for the neutrino:

$$M_\nu \sim c \frac{v^2}{\Lambda}$$

The smallness of M_ν implies $\Lambda \sim 10^{14}$ GeV.

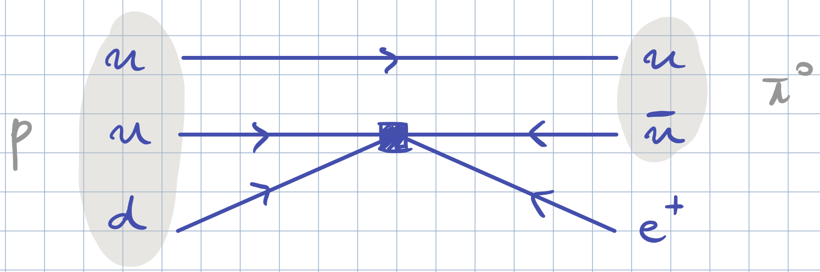
(with assumptions)

Note that Q_{eeHH} breaks Lepton number, which is an ACCIDENTAL SYMMETRY of $\mathcal{L}(D\leq 4)$.

Thus it seems that in order to test $\mathcal{L}_{D>4}$ it is a good idea to look at lepton number violating observables. These are obvious null tests of the SM.

Another example are BARYON NUMBER VIOLATING observables, such as proton decay. This can be mediated by e.g. the previous dim=6 op:

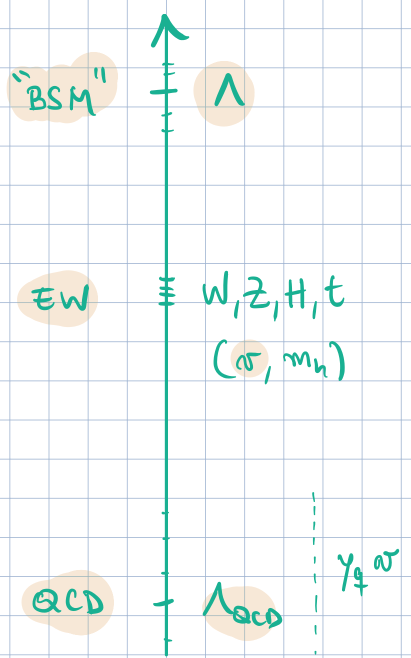
$$Q_{duue} = \varepsilon_{\alpha\beta\sigma}\,(d_\alpha^T\, C\, u_\beta)(u_\sigma^T\, C\, e)$$



Currently $\tau_p \gtrsim 10^{30}$ yrs $\Rightarrow$ $\Lambda \gtrsim 10^{15}$ GeV

Note that the scale Λ related to neutrino mass needs not be the same as the scale Λ related to p-decay.

The "cut-off" scale Λ is some scale related to some new physics at high energy:



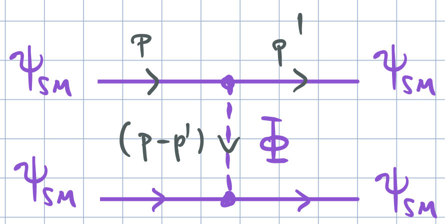
$$\mathcal{L} = \overbrace{\mathcal{L}_{QCD} + \mathcal{L}_{QED} + \mathcal{L}_{EW}}^{\mathcal{L}_{SM}} + \mathcal{L}_{\Lambda}$$

renorm.

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \mathcal{L}^{D>4}$$

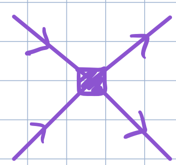
$$\mathcal{L}_{WET} = \mathcal{L}_{LEFT} \longrightarrow \text{Chapter 4.}$$

E.g.



$$= \left(\bar{\psi}_{SM}\psi_{SM}\right) \cdot \frac{i\, g^2}{t - M_\Phi^2} \cdot \left(\bar{\psi}_{SM}\psi_{SM}\right)$$

$$= -i \frac{g^2}{m_\phi^2}\left[1 + \frac{t}{m_\phi^2} + \frac{t^2}{m_\phi^4} + \cdots\right] =$$



$$\mathcal{L}_{eff} = -\frac{g^2}{m_\phi^2}\left(\bar{\psi}_{SM}\psi_{SM}\right)^2 + \cdots$$

$$\Rightarrow \mathcal{L}^{D>4} = \sum_i C_i(\mu)\, Q_i(\mu)$$

Depends on IR scales

Depends on UV scales.

The "Wilson coefficients" $C_i \sim c_i/\Lambda^{d-4}$ are just some couplings that can be calculated if we know the UV theory (Ch. 6).

Note: The SMEFT has to be renormalized at the given order in the $1/\Lambda$ expansion. This means the Wilson coefficients $C_i(\mu)$ and the operators $O_i(\mu)$ depend on the renormalization scale μ. The dependence of $C_i(\mu)$ (or $O_i(\mu)$) with μ is described by the BETA FUNCTIONS or ANOMALOUS DIMENSIONS of the SMEFT operators, in the form of an RGE, e.g:

$$\boxed{\frac{d}{d\log\mu} C_i(\mu) = \beta_i(\vec{C})}$$

and leads also to operator mixing.

We will not discuss this for now.

(See: 1308.2627; 1310.4838; 1312.2014)

DIMENSION-6 OPS IN THE SMEFT

$(\varphi \equiv H)$ [From 1704.04504. Original: 1008.4884]

dim	class	# operators	quantum numbers
5	Dimension-five	1	$\Delta L = 2$
6	X^3	4	
6	φ^6	1	
6	$\varphi^4 D^2$	2	
6	$X^2\varphi^2$	8	
6	$\psi^2\varphi^3$	3	
6	$\psi^2 X\varphi$	8	
6	$\psi^2\varphi^2 D$	8	
6	$(\bar{L}L)(\bar{L}L)$	5	
6	$(\bar{R}R)(\bar{R}R)$	7	
6	$(\bar{L}L)(\bar{R}R)$	8	
6	$(\bar{L}R)(\bar{L}R)$	4	
6	$(\bar{L}R)(\bar{R}L)$	1	
6	Baryon-number-violating	4	$\Delta B = \Delta L = 1$

× Flavor

PURELY BOSONIC

X^3		$X^2\varphi^2$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{\widetilde W}$	$\epsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$
φ^6		$Q_{\varphi \widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$
Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{\varphi \widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$
$\varphi^4 D^2$		$Q_{\varphi \widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$
$Q_{\varphi\Box}$	$(\varphi^\dagger \varphi)\Box(\varphi^\dagger \varphi)$	$Q_{\varphi \widetilde W B}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$
$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu \varphi\right)^* \left(\varphi^\dagger D_\mu \varphi\right)$		

2-FERMION

$\psi^2\varphi^3$		$\psi^2\varphi^2 D$	
$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar q u \widetilde\varphi)$	$Q^{(1)}_{\varphi\ell}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar \ell \gamma^\mu \ell)$
$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar q d \varphi)$	$Q^{(3)}_{\varphi\ell}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar \ell \tau^I \gamma^\mu \ell)$
$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar \ell e \varphi)$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e \gamma^\mu e)$
$\psi^2 X\varphi$		$Q^{(1)}_{\varphi q}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q \gamma^\mu q)$
Q_{eW}	$(\bar \ell \sigma^{\mu\nu} e)\, \tau^I \varphi W^I_{\mu\nu}$	$Q^{(3)}_{\varphi q}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar q \tau^I \gamma^\mu q)$
Q_{eB}	$(\bar \ell \sigma^{\mu\nu} e)\, \varphi B_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u \gamma^\mu u)$
Q_{uG}	$(\bar q \sigma^{\mu\nu} T^A u)\, \widetilde\varphi G^A_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d \gamma^\mu d)$
Q_{uW}	$(\bar q \sigma^{\mu\nu} u)\, \tau^I \widetilde\varphi W^I_{\mu\nu}$	$Q_{\varphi ud}$	$(\widetilde\varphi^\dagger i D_\mu \varphi)(\bar u \gamma^\mu d)$
Q_{uB}	$(\bar q \sigma^{\mu\nu} u)\, \widetilde\varphi B_{\mu\nu}$		
Q_{dG}	$(\bar q \sigma^{\mu\nu} T^A d)\, \varphi G^A_{\mu\nu}$		
Q_{dW}	$(\bar q \sigma^{\mu\nu} d)\, \tau^I \varphi W^I_{\mu\nu}$		
Q_{dB}	$(\bar q \sigma^{\mu\nu} d)\, \varphi B_{\mu\nu}$		

4- FERMION

$(\bar{L}L)(\bar{L}L)$		$(\bar{L}L)(\bar{R}R)$	
$Q_{\ell\ell}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{\ell}\gamma^\mu \ell)$	$Q_{\ell e}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{e}\gamma^\mu e)$
$Q_{qq}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu q)$	$Q_{\ell u}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{u}\gamma^\mu u)$
$Q_{qq}^{(3)}$	$(\bar{q}\gamma_\mu \tau^I q)(\bar{q}\gamma^\mu \tau^I q)$	$Q_{\ell d}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{d}\gamma^\mu d)$
$Q_{\ell q}^{(1)}$	$(\bar{\ell}\gamma_\mu \ell)(\bar{q}\gamma^\mu q)$	Q_{qe}	$(\bar{q}\gamma_\mu q)(\bar{e}\gamma^\mu e)$
$Q_{\ell q}^{(3)}$	$(\bar{\ell}\gamma_\mu \tau^I \ell)(\bar{q}\gamma^\mu \tau^I q)$	$Q_{qu}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{u}\gamma^\mu u)$
$(\bar{R}R)(\bar{R}R)$		$Q_{qu}^{(8)}$	$(\bar{q}\gamma_\mu T^A q)(\bar{u}\gamma^\mu T^A u)$
Q_{ee}	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$	$Q_{qd}^{(1)}$	$(\bar{q}\gamma_\mu q)(\bar{d}\gamma^\mu d)$
Q_{uu}	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$	$Q_{qd}^{(8)}$	$(\bar{q}\gamma_\mu T^A q)(\bar{d}\gamma^\mu T^A d)$
Q_{dd}	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$	$(\bar{L}R)(\bar{R}L)$	
Q_{eu}	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$	$Q_{\ell edq}$	$(\bar{\ell}^j e)(\bar{d} q^j)$
Q_{ed}	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$	$(\bar{L}R)(\bar{L}R)$	
$Q_{ud}^{(1)}$	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$	$Q_{quqd}^{(1)}$	$(\bar{q}^j u)\epsilon_{jk}(\bar{q}^k d)$
$Q_{ud}^{(8)}$	$(\bar{u}\gamma_\mu T^A u)(\bar{d}\gamma^\mu T^A d)$	$Q_{quqd}^{(8)}$	$(\bar{q}^j T^A u)\epsilon_{jk}(\bar{q}^k T^A d)$
		$Q_{\ell equ}^{(1)}$	$(\bar{\ell}^j e)\epsilon_{jk}(\bar{q}^k u)$
		$Q_{\ell equ}^{(3)}$	$(\bar{\ell}^j \sigma_{\mu\nu} e)\epsilon_{jk}(\bar{q}^k \sigma^{\mu\nu} u)$

Baryon-number-violating	
$Q_{duq\ell}$	$(d^T C u)(q^T C \ell)$
Q_{qque}	$(q^T C q)(u^T C e)$
$Q_{qqq\ell}$	$\epsilon_{il}\epsilon_{jk}(q_i^T C q_j)(q_k^T C \ell_l)$
Q_{duue}	$(d^T C u)(u^T C e)$

3. FLAVOR IN THE SMEFT

Let's start by looking at fermion masses and the CKM matrix within the setting of the SMEFT.

The relevant terms in the Lagrangian that will lead to fermion mass terms after SSB include of course the Yukawas, but now there are additional contributions. At dim = 6 we have:

$$Q_{uH}^{ij} = (H^\dagger H)(\bar{q}^i u^j \tilde{H}) \longrightarrow \frac{v^3}{2\sqrt{2}} \bar{u}_L^i u_R^j$$

$$Q_{dH}^{ij} = (H^\dagger H)(\bar{q}^i d^j H) \longrightarrow \frac{v^3}{2\sqrt{2}} \bar{d}_L^i d_R^j$$

$$Q_{eH}^{ij} = (H^\dagger H)(\bar{l}^i e^j H) \longrightarrow \frac{v^3}{2\sqrt{2}} \bar{e}_L^i e_R^j$$

Exercise: Check this and convince yourself that no other $d=6$ SMEFT ops contribute to $\mathcal{L}_{m_\psi}$.

Thus in the SMEFT at $d=6$ we have:

$$\mathcal{L}_{m_\psi} = - \sum_{\psi = E,D,U} M_\psi^{ij} \bar{\psi}_L^i \psi_R^j + h.c.$$

with now:

$$\boxed{M_\psi = \frac{v}{\sqrt{2}}\left[y_\psi - \frac{v^2}{2} C_{\psi H} \right]} \qquad (*)$$

and thus the relationship among fermion masses and Yukawas is modified.

One can still use the flavor symmetry to write:

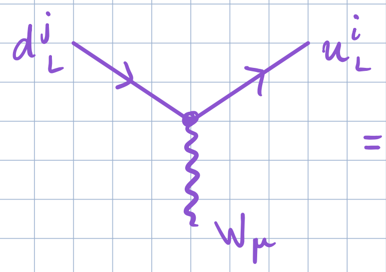
$$M_e = M_E = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$
$$M_u = M_U = \mathrm{diag}\,(m_u, m_c, m_t)$$
$$M_D = V_{CKM} \times \mathrm{diag}\,(m_d, m_s, m_b) \equiv V_{CKM} \cdot M_d .$$

but now the Yukawa couplings do <u>not</u> follow this pattern anymore.

The matrix V_{CKM} is still called the CKM matrix, although it is affected by $d=6$ operators. It is still a <u>unitary 3×3 matrix</u>, and still parametrized by 4 real parameters (including one phase).
We can still use the Wolfenstein parametrization.

But contrary to the SM, the structure of charged currents is <u>not</u> uniquely determined by the

CKM matrix, but it is also affected by the presence of dim=6 operators, e.g.:



$$= -i\,\frac{g}{\sqrt{2}}\left[V_{ij} + v^2\left[C_{Hq}^{(3)}\right]_{ij}\right]\gamma_\mu \qquad (\#)$$

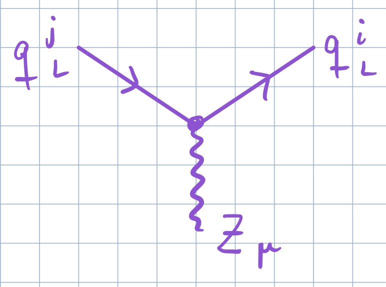
<u>Exercise:</u> Prove this and check if there are further contributions from other dim-6 ops in the SMEFT.

In eqs. (*) and (#) the quantity v refers to the Higgs vev in the presence of dim-6 ops:

$$v = \left(1 + \frac{3 C_H v^2}{8\lambda}\right) v_{SM}$$

where C_H is the coefficient of $O_H = (H^\dagger H)^3$.

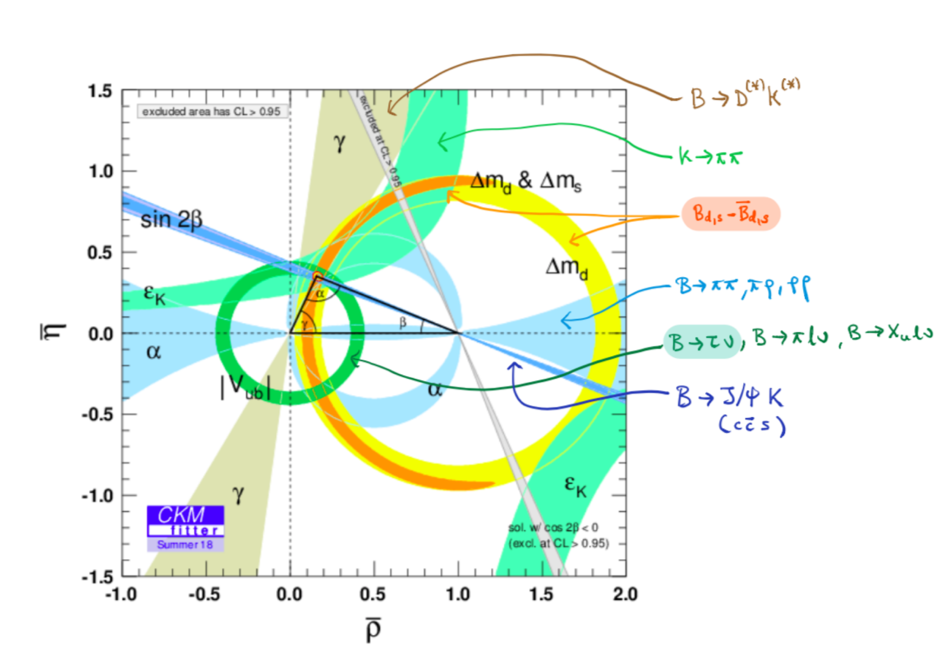
In the presence of dim-6 ops there are many new contributions that lead to ΔF transitions, e.g.



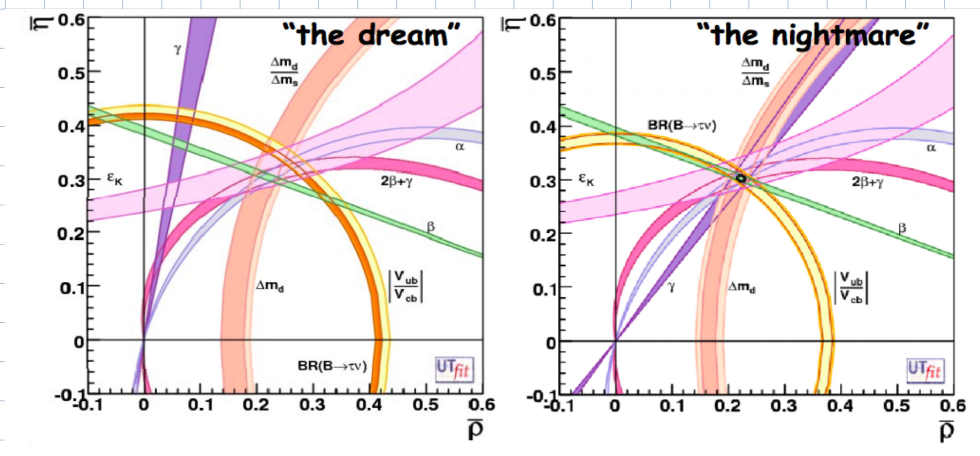
$$\propto \left[C_{Hq}^{(1)}\right]_{ij}, \quad \left[C_{Hq}^{(3)}\right]_{ij}$$

($\equiv$ anomalous Z couplings)

All this has many implications. One of them is that the global (SM) determination of the CKM parameters will not work if some SMEFT dim = 6 coefficients are $\neq 0$.



Two typical situations may be the following:



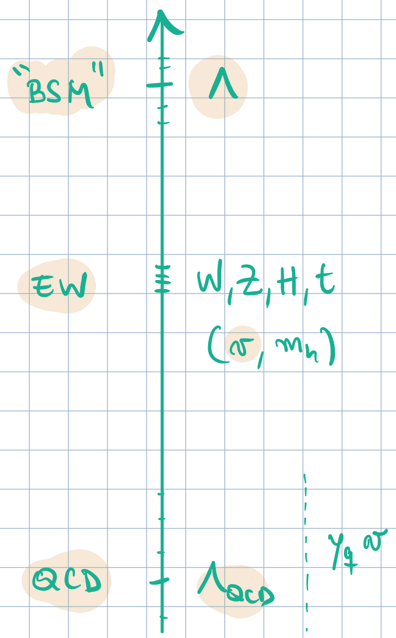
We'll see an example in the Exercise session.

[The general case is discussed in 1812.08163]

4. BELOW THE EW SCALE

[E.g. hep-ph/9512380 ; 1704.05672 ; 1709.04486]

In experiments with $E \ll \Lambda_{EW} \sim v$ we can perform the same type of EFT expansion as before, but with EW-scale particles integrated out:

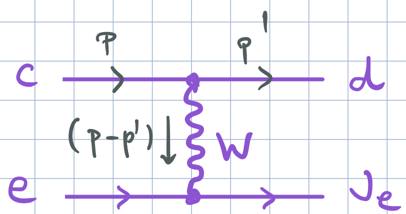


$$\mathcal{L}_{SMEFT} = \overbrace{\mathcal{L}_{QCD} + \mathcal{L}_{QED} + \mathcal{L}_{EW}}^{\mathcal{L}_{SM}} + \mathcal{L}^{D>4}$$

$$\boxed{\mathcal{L}_{\underset{(LEFT)}{WET}} = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \mathcal{L}^{D>4}_{WET}}$$

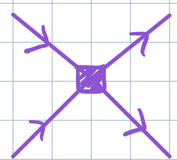
[WET ≡ WEAK EFFECTIVE THEORY
LEFT ≡ LOW ENERGY EFT]

E.g.



$$= (\bar{d}\gamma_\mu P_L c) \cdot \frac{i\, g^2/2}{t - M_W^2} \cdot (\bar{\nu}_e \gamma^\mu P_L e)$$

$$= -i \frac{g^2}{2 m_W^2}\left[1 + \frac{t}{m_W^2} + \frac{t^2}{m_W^4} + \dots\right] \langle O_{eff} \rangle =$$



$$\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}} (\bar{d}\gamma_\mu P_L c)(\bar{\nu}_e \gamma^\mu e) + \dots$$

where $G_F \equiv \frac{g^2}{4\sqrt{2}\, m_W^2}$ is FERMI'S CONSTANT.

Thus:

$$\boxed{\mathcal{L}_{WET/LEFT} = \mathcal{L}_{QCD} + \mathcal{L}_{QED} + \sum_i L_i\, \mathcal{O}_i}$$

without top quark

"Wilson" coefficients

Operators build from light fields and invariant under $SU(3)_c \times U(1)_{em}$.

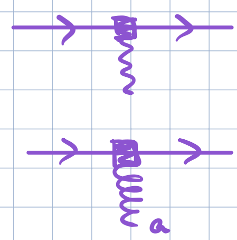
Some WET/LEFT operators:

dim-5:

$$\mathcal{O}_{e\gamma}^{11} = \bar{e}_L\, \sigma^{\mu\nu} e_R\, F_{\mu\nu}$$

$$\mathcal{O}_{d\gamma}^{23} = \bar{s}_L\, \sigma^{\mu\nu} b_R\, F_{\mu\nu}$$

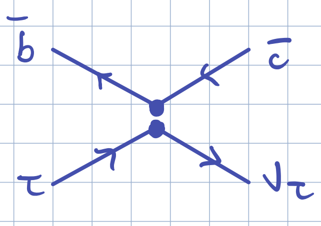
$$\mathcal{O}_{dG}^{13} = \bar{d}_L\, \sigma^{\mu\nu} T^a b_R\, G^a_{\mu\nu}$$

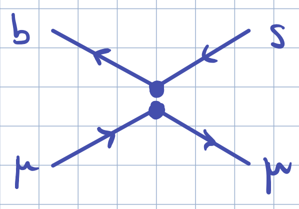


dim-6:

$$\left[\mathcal{O}^{V,LL}_{\nu edu}\right]_{3332} = (\bar{J}_\mu \gamma^\alpha \mu)(\bar{b}_L \gamma_\alpha c_L)$$

$$\left[\mathcal{O}^{V,RR}_{ed}\right]_{2232} = (\bar{\mu}_R \gamma^\alpha \mu_R)(\bar{b}_R \gamma_\alpha s_R)$$





The number of independent operators is even larger than the case of SMEFT (e.g. 1709.04486)

$$\left.\begin{array}{l} 6 \text{ dim-5 ops} \quad (\text{dipole}) \\ 73+16 \text{ dim-6 ops} \end{array}\right\} \times \text{Flavor}$$

$$= \left| \begin{array}{l} 75 \text{ dim-5 ops} \\ \underbrace{6115}_{\text{B-conserving}} + \underbrace{1032}_{\text{B-violating}} \text{ dim-6 ops} \end{array}\right.$$

(see notebook)

We can also calculate the Wilson coefficients if we know the UV theory. In this case, the WCs are non-zero in the pure SM case. E.g.:

$$[L_{\nu edu}]_{3332} = -\frac{2}{v^2} V_{cb}^* + 2 V_{ib}^* \left[C_{lq}^{(3)}\right]_{33i2} - 2 V_{ib}^* \left[C_{Hq}^{(3)}\right]^*_{2i} - 2 V_{cb}^* \left[C_{He}^{(3)}\right]_{33}$$

Exercise: Prove this and show that there are no further contributions from other dim-6 ops in the SMEFT.

Some important comments:

→ QCD+QED is the Low-E EFT of the SM (EFT)

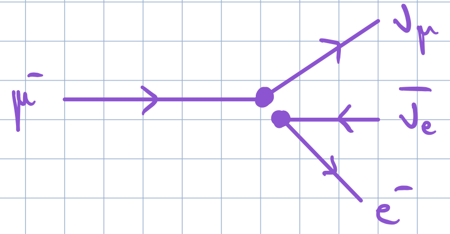
→ the cut-off is $\Lambda \sim \Lambda_{EW} \sim v$ (EW is "NP")

→ The full FLAVOUR Group is ACCIDENTAL SYM.

Example: Muon decay

The muon is stable in QCD + QED. Its decay amplitude arises from dim-6 ops in the LEFT, e.g.

$$\delta \mathcal{L} = \frac{c}{\Lambda^2} (\bar{\nu}_\mu \gamma^\alpha P_L \mu)(\bar{e} \gamma_\alpha P_L \nu_e)$$



At tree level (EXERCISE): $\Gamma_\mu \simeq \dfrac{m_\mu^5 \, c^2}{1536 \, \pi^3 \Lambda^4}$ (1)

Experimentally: $\tau_\mu^{exp} = 2.2 \times 10^{-6}$ s

$$\Rightarrow \quad \Gamma_\mu^{exp} = 3 \cdot 10^{-19} \text{ GeV} \qquad (2)$$

Comparing (1) and (2), with $c \sim 1$:

$$\Lambda \simeq 172 \text{ GeV} \simeq v/\sqrt{2} \quad \Rightarrow \quad \Lambda \simeq \Lambda_{EW} \ ! \quad ✓$$

$\Rightarrow$ We "discover" the EW scale, and measure it!

<u>Note:</u> the tree-level SM matching calculation gives:

$$c/\Lambda^2 = -g^2/2M_W^2 = -2/v^2 = -4G_F/\sqrt{2}$$

τ_μ is used to determine v (or G_F) :

$$v = 246.21965(6) \text{ GeV}$$
$$G_F = 1.1663787(5) \text{ GeV}^{-2}$$
(SM)

But this determination is modified if there is BSM physics:

$$v = 246.21965(6)\left(1 + \delta_v\right)$$

with

$$\delta_v = v^2\left([C_{He}^{(3)}]_{\mu\mu} + [C_{He}^{(3)}]_{ee} - \tfrac{1}{2}[C_{ll}]_{\mu e e \mu} - \tfrac{1}{2}[C_{ll}]_{e \mu \mu e}\right)$$

(See e.g. 1812.08163)

The β-decay and μ-decay phylosophy applies to all flavor transitions.

$$\boxed{\text{LOW-LYING HADRONS are STABLE at } D \leq 4}$$

$K^0 \sim \bar{s}d$; $K^+ \sim \bar{s}u$; $D^0 \sim \bar{u}c$; $D^+ \sim \bar{d}c$

$B^0 \sim \bar{b}d$; $B^+ \sim \bar{b}u$; $B_s^0 \sim \bar{b}s$; $\Lambda_b \sim udb$; ...

Exercise: Look at the PDG (Particle data group) particle listings and check that:

$$\tau_{mesons} \sim \tau_{baryons} \gtrsim 10^{-12}\, s \sim 10^{12}\, GeV^{-1}$$

$$\tau_{Resonances} \lesssim 10^{-23}\, s \sim 10\, GeV^{-1}$$

This fact has two consequences:

1. Definition of Hadrons

$$H = H_0 + H_{int} \quad ; \qquad e.g. \quad H_0 |B\rangle = m_B |B\rangle$$

QCD (+QED) $\quad$ D>4

2. Weak decays as probes of BSM vs EW. (Chapter 5).

5. WEAK DECAYS AS PROBES OF BSM vs EW

We have seen that studying transitions in hadrons that are stable under QCD we are directly probing flavor transitions, which can happen through either EW physics or BSM physics. And therefore weak decays of hadrons are excellent tests of EW physics and probes of New Physics.

We will go through 3 simple examples using the formalism that we have described in previous sections.

■ EXAMPLE 1 : $b \to c\, l\nu$

Consider the transition $b \to c\, l \nu$ mediated by the term in the eff. Lagrangian:

$$\mathcal{L}_{eff}^{(6)} = C_1 \, (\bar{c}\, \gamma_\mu P_L\, b)(\bar{l}\, \gamma^\mu P_L\, \nu)$$

(there are actually 5 independent $d=6$ operators of the $b \to c\, l\nu$ type. For this example one is enough.)

This quark-level transition is realized in decay observables such as

$$B \to D \ell \bar{\nu}\,,\; B \to D^* \ell \bar{\nu}\,,\; B_s \to D_s \ell \bar{\nu}\,,\; \Lambda_b \to \Lambda_c \ell \bar{\nu}\,, \dots$$

$$B \to X_c \ell \bar{\nu}\,, \; \dots$$

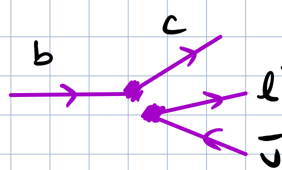
The theory predictions for all these observables will depend on the Wilson coefficient C_1.

What is C_1 in the SM and in BSM models? To answer this question we need to do a matching calculation.

We consider the partonic amplitude $\mathcal{A}(b \to c \ell \bar{\nu})$, and require the EFT amplitude to be equal to the (expanded) "full theory" amplitude.

We have (up to higher perturbative orders)

$$i \mathcal{A}_{EFT} =$$

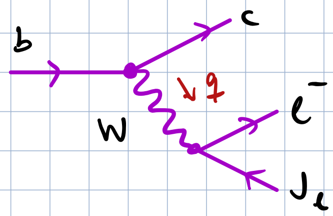


$$= i\, C_1 \left(\bar{u}_c \gamma_\mu P_L u_b\right)\left(\bar{u}_\ell \gamma^\mu P_L v_\nu\right)$$

$$\Rightarrow \quad \mathcal{A}_{EFT} = C_1\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_\ell \gamma^\mu P_L v_\nu$$

In the SM, the process $b \to c \ell \nu$ is mediated by W exchange:

$$q^2 = (p_b - p_c)^2 \leq (m_b - m_c)^2 \ll M_W^2$$

$i \mathcal{A}_{SM} =$ 

$+ \ldots$

$$= -\frac{g^2}{2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma_\nu P_L v_\nu \cdot \underbrace{\frac{-i}{q^2 - M_W^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}\right)}_{\frac{i}{M_W^2} g^{\mu\nu} + \mathcal{O}(q^2/M_W^2)}$$

$$+ \text{ higher perturbative orders}$$

$$= -i \frac{g^2}{2 M_W^2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma^\mu P_L v_\nu + \cdots$$

$$\Rightarrow \quad \mathcal{A}_{SM} = -\frac{g^2}{2 M_W^2} V_{cb}\, \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_e \gamma^\mu P_L v_\nu + \cdots$$

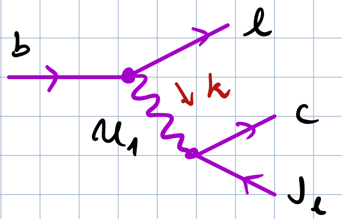
The LO SM matching condition is thus:

$$C_1^{SM} = -\frac{g^2 V_{cb}}{2 M_W^2}$$ Flavor suppression $\sim \lambda^2$

Let's consider now a new "vector leptoquark" U_1:

$$\mathcal{L}_{LQ} = g_b\, \bar{e}\, \not{U}_1 P_L b + g_c\, \bar{\nu}_e \not{U}_1 P_L c + h.c.$$
$$+ \frac{1}{2} M_{U_1}^2 U_1^\mu U_{1\mu}$$

The LO contribution to the $b \to c \ell \nu$ amplitude from U_1 exchange is given by:

$i\mathcal{A}_{U_1} =$  $+ \dots$

$$= -g_b g_c \left[\frac{i}{M_{U_1}^2} + \mathcal{O}\left(k^2/M_{U_1}^2\right) \right] \bar{u}_\ell \gamma_\mu P_L u_b \cdot \bar{u}_c \gamma^\mu P_L v_\nu + \dots$$

$$= -i \frac{g_b g_c}{M_{U_1}^2} \underbrace{\bar{u}_\ell \gamma_\mu P_L u_b \cdot \bar{u}_c \gamma^\mu P_L v_\nu}_{-\bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_\ell \gamma^\mu P_L v_\nu \quad (\text{Fierz})} + \dots$$

$$\Rightarrow \quad \mathcal{A}_{U_1} = \frac{g_b g_c}{M_{U_1}^2} \bar{u}_c \gamma_\mu P_L u_b \cdot \bar{u}_\ell \gamma^\mu P_L v_\nu$$

Thus, the LO contribution to the matching condition from U_1 exchange is:

$$C_1^{U_1} = \frac{g_b g_c}{M_{U_1}^2}$$

Combining everything:

$$\boxed{C_1 = -\frac{g^2 V_{cb}}{2 M_W^2} + \frac{g_b g_c}{M_{U_1}^2} + \dots = C_1^{SM} \left[1 - 2 \frac{g_b g_c}{g^2 V_{cb}} \cdot \frac{M_W^2}{M_{U_1}^2} + \dots \right]}$$

What does this mean?

Imagine we measure some $b \to c l \nu$ observable and extract an experimental value for C_1 with a relative uncertainty σ, and consistent with the SM prediction:

$$C_1^{exp} = C_1^{SM} (1 \pm \sigma)$$

Then we conclude that

$$2 \frac{g_b g_c}{g^2 V_{cb}} \cdot \frac{M_W^2}{M_{U_1}^2} < \sigma \qquad (\text{e.g. @ } 68\% \text{ C.L.})$$

and therefore,

$$M_{U_1} > \sqrt{\frac{2 g_b g_c}{g^2 \sigma V_{cb}}} \, M_W .$$

Setting $g_b \sim g_c \sim g$

$V_{cb} \sim \lambda^2 \sim 0.04$

$\sigma \sim 10\%$

we find that $M_{U_1} \gtrsim 20 \, M_W \sim 2 \text{ TeV}.$

We probe the TeV scale!

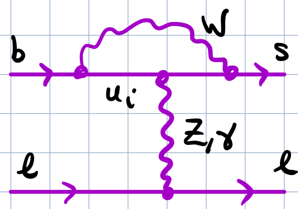
EXAMPLE 2 : $b \to s \ell^+ \ell^-$ (FCNCs)

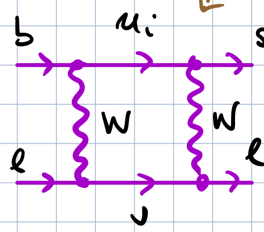
We consider now the FCNC transition $b \to s \ell^+ \ell^-$ which is mediated in the EFT by operators of the form (barring dipole ops)

$$C_i \, (\bar{s}\, \Gamma_1^{(i)} b)(\bar{\ell}\, \Gamma_2^{(i)} \ell)$$

for different Dirac structures $\Gamma_j^{(i)}$.

The matching coefficients C_i in the SM arise only at one loop : (c.f. Ch. 6.)

$C_i^{SM} \sim$ 

$+$ 

$+ \cdots$

$$\sim \frac{g^4}{M_W^2} \times \underbrace{\frac{1}{(4\pi)^2}}_{\text{LOOP}} \times \underbrace{V_{is}^* V_{ib}}_{\text{CKM}}$$

Notes:

1. It happens that $V_{us}^* V_{ub} \sim \lambda^4 \ll V_{cs}^* V_{cb} \sim V_{ts}^* V_{tb} \sim \lambda^2$
2. In the literature you will find:

$$\mathcal{L}_{eff} = \underbrace{\frac{4 G_F}{\sqrt{2}}}_{g^2/2M_W^2} V_{ts}^* V_{tb} \underbrace{\frac{\alpha}{4\pi}}_{e^2/(4\pi)^2} \Big(\underbrace{C_9}_{\simeq 4} O_9 + \underbrace{C_{10}}_{\simeq -4} O_{10} \Big)$$

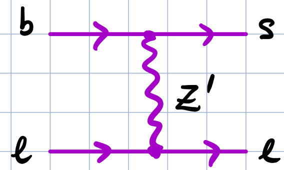
$O_{9(10)}$: $[\bar{s}\gamma_\mu P_L b][\bar{\ell}\gamma^\mu(\gamma_5)\ell]$

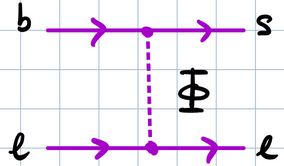
Let us consider two possible tree-level models:

$$\mathcal{L}_{Z'} = g_{bs}\, \bar{s} \not{Z}' P_L b + g_\ell\, \bar{\ell} \not{Z}' \ell + h.c. + \frac{1}{2} M_{Z'}^2 Z'_\mu Z'^\mu$$

$$\mathcal{L}_{\Phi} = g_b\, \Phi\, \bar{\ell} P_R b + g_s\, \Phi\, \bar{\ell} P_R s + h.c. + M_\Phi^2\, \Phi^\dagger \Phi$$

We have:

$C_i^{Z'} \sim$  $\sim \dfrac{g_{bs}\, g_\ell}{M_{Z'}^2}$

$C_i^{\Phi} \sim$  $\sim \dfrac{g_b\, g_s}{M_\Phi^2}$

Thus, assuming as before that C_i is measured compatible with the SM with a relative accuracy of $\sigma \sim 10\%$, and assuming

$$g_s \sim g_b \sim g_{sb} \sim g_\ell \sim g$$

we find that

Tens of TeV!

$$\boxed{M_{Z',\Phi} \gtrsim \frac{4\pi}{g\lambda\sqrt{\sigma}} M_W \simeq 20 \text{ TeV}}$$

[For numerics note that $G_F = \dfrac{g^2}{4\sqrt{2}\, M_W^2} = 1.17 \cdot 10^{-5}\ \text{GeV}^{-2}$]

We will consider specific bounds on C_{10} and to $M_{Z'}$ and M_ϕ from the measurement of the $B_s \to \mu^+\mu^-$ branching ratio in the Exercise session.

Comment: There are currently some tensions between experimental determination and SM prediction for C_9, C_{10} from $b \to s\mu\mu$ observables. These tensions are sometimes called the "Neutral-current B anomalies".

We can try to explain the anomalies by introducing either a Z' or a LQ ϕ.

This provides a measurement of the combinations (respectively)

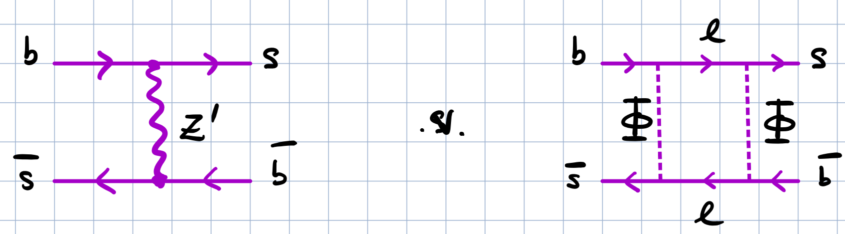
$$\frac{g_{bs}\, g_\ell}{M_{Z'}^2}, \quad \frac{g_b\, g_s}{M_\Phi^2} \neq 0$$

which need to be different from zero in order to address the anomalies.

However, these new particles contribute also to other processes that do not deviate from the SM.

An example is given by ΔM_s, which is related to the $b\bar{s} \to s\bar{b}$ amplitude.

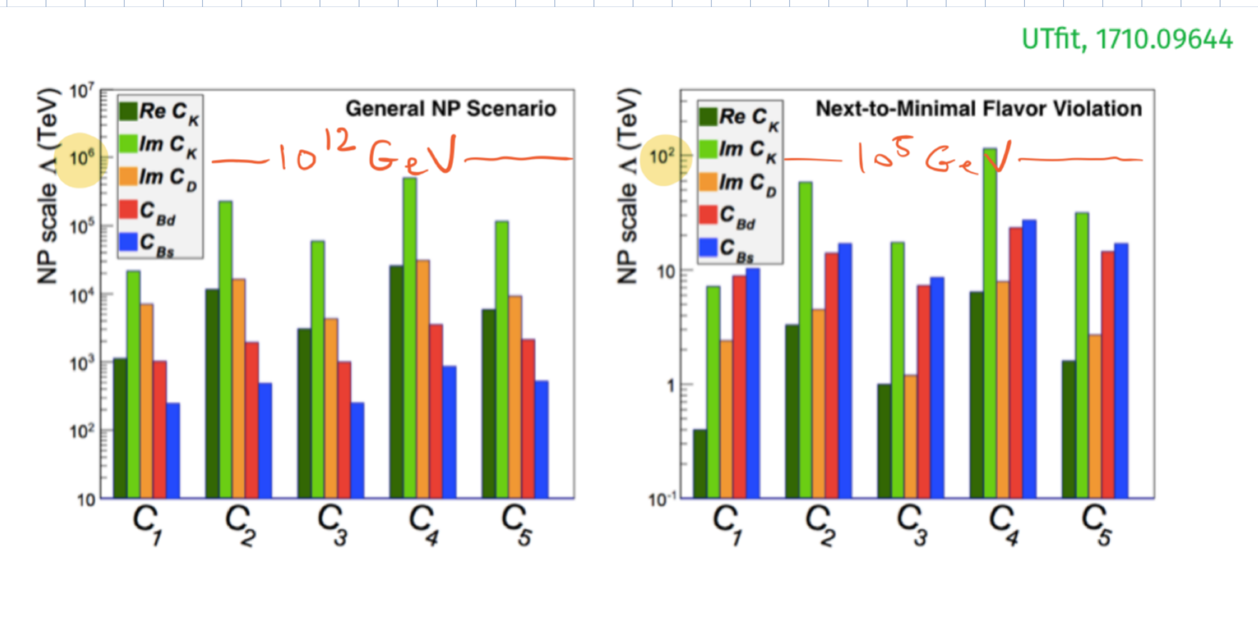
Note that this constraint is much more important for Z' than for ϕ, because:



This is one reason for which LQ models are so popular when addressing the B anomalies.

EXAMPLE 3 : Neutral meson mixing

(e.g. 0707.0636)



SUMMARY SO FAR

- SM = EFT (one less axiom)

  No problem (power counting)

  → $\Lambda \gg \Lambda_{EW}$ explains success of SM

  → Best tests : Accidental symmetries (p-decay , ν-masses)

$$\boxed{\mathcal{L} = \mathcal{L}^{D\leq 4} + \sum_i \frac{c_i}{\Lambda^{d-4}} O_i^{(d)}}$$

Measure to learn about UV

- Flavor = Accidental symmetry at $E \ll \Lambda_{EW}$

  ↳ Look at weak decay & mixing of "stable" hadrons (e.g. K, $D_{(s)}$, $B_{(s)}$, Λ_b ...) (Leptons, e.g. τ, equally interesting)

  ↳ FCNC suppression of EW physics allows to reach $\underbrace{\Lambda \gg \Lambda_{EW}}_{\text{Beyond the reach of direct production.}}$.

GLOBAL RESEARCH PROGRAM

$$g_i$$

1. Write relevant EFT $\mathcal{L} = \mathcal{L}_{D\leq 4} + \sum_i c_i O_i$

2. Choose suitable set of OBSERVABLES Θ_j

   2.1 Calculate $\Theta_j(g_i, c_i)$ (*) --> DIFFICULT !!!!

   2.2 Measure Θ_j^{exp}

   No info

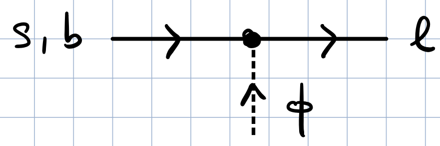
   2.3 Fit to determine $\{g_i^{exp}, c_i^{exp}\}$

3. Compare c_i^{exp} to models.

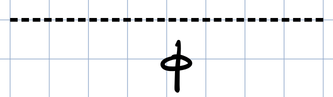
1/2 of HEP community works on some part of this Program

Example of 1-loop matching calculation and momentum regions.

We consider the scalar LQ model



$$= i\, g_{s,b}\, P_L$$



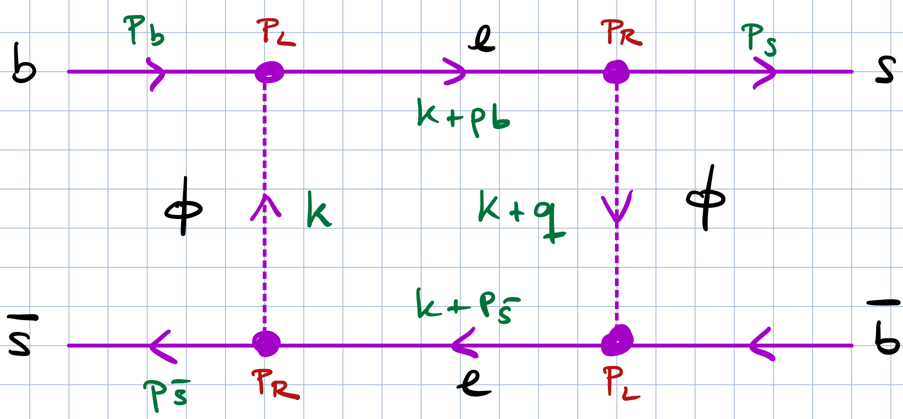
$$= \frac{i}{M_\phi^2}$$

We want to consider the (one-loop) matching to an operator of the type

$$(\bar{s}\,\Gamma\, b)(\bar{s}\,\Gamma'\, b)$$

contributing to $\bar{B}_s - B_s$ mixing.

One of the two LO diagrams contributing in the LQ model to the $b\bar{s} \to s\bar{b}$ amplitude:



$$q = p_b - p_s$$

The contribution to the amplitude from this diagram is:

$$i\mathcal{A} = G \int \frac{d^4k}{(2\pi)^4} \frac{\left[P_R\,(\not{k}+\not{p}_b)\,P_L\right] \otimes \left[P_R\,(\not{k}+\not{p}_s)\,P_L\right]}{\left[(k+p_b)^2 - m_e^2\right]\left[(k+p_s)^2 - m_e^2\right]\left[k^2 - M_\phi^2\right]\left[(k+q)^2 - M_\phi^2\right]}$$

$\equiv g_s^2\, g_b^2$

We are going to divide the integral into two regions:

- Region I $\quad k \sim M_\phi \gg m_b$
- Region II $\quad k \sim m_b \ll M_\phi$

Region I:

In this region we have:

- $\not{k} + \not{p}_b = \not{k}\left(1 + \mathcal{O}(m_b/M_\phi)\right)$
- $\not{k} + \not{p}_s = \not{k}\left(1 + \mathcal{O}(m_b/M_\phi)\right)$
- $\left[(k+p_b)^2 - m_e^2\right]\left[(k+p_s)^2 - m_e^2\right] = k^4\left(1 + \mathcal{O}(m_b^2/M_\phi^2)\right)$
- $\left[(k+q)^2 - M_\phi^2\right] = (k^2 - M_\phi^2)\left(1 + \mathcal{O}(m_b^2/M_\phi^2)\right)$

Therefore:

$$i\mathcal{A}_I = \underbrace{G\int \frac{d^4k}{(2\pi)^4}\frac{[\not{k}P_L]\otimes[\not{k}P_L]}{k^4\,(k^2-M_\phi^2)^2}}_{= -\frac{i}{64\pi^2}\frac{1}{M_\phi^2}[\gamma_\mu P_L]\otimes[\gamma^\mu P_L]}\left[1+\mathcal{O}(m_b^2/M_\phi^2)\right]$$

$$\Rightarrow \mathcal{A}_I = -\underbrace{\frac{g_s^2 g_b^2}{64\pi^2 M_\phi^2}}_{\hookrightarrow \text{(will lead to matching coeff.)}}\ \bar{u}_s\gamma_\mu P_L u_b \cdot \bar{v}_s\gamma^\mu P_L v_b$$

Region II:

In this region we have:

$$[k^2-m_\phi^2][(k+q)^2-M_\phi^2] = M_\phi^4\left(1+\mathcal{O}(m_b^2/M_\phi^2)\right)$$

the rest of the integrand has no M_ϕ dependence.

Thus:

$$\mathcal{A}_{II} \sim \frac{m_b^2}{M_\phi^4}$$

which is suppressed by a factor m_b^2/M_ϕ^4 and thus does not contribute to the amplitude at order $1/M_\phi^2$.

For an enlightening exposition of the method of regions, see

Ch. 2.1 of 1410.1892 (Becher et.al.)
