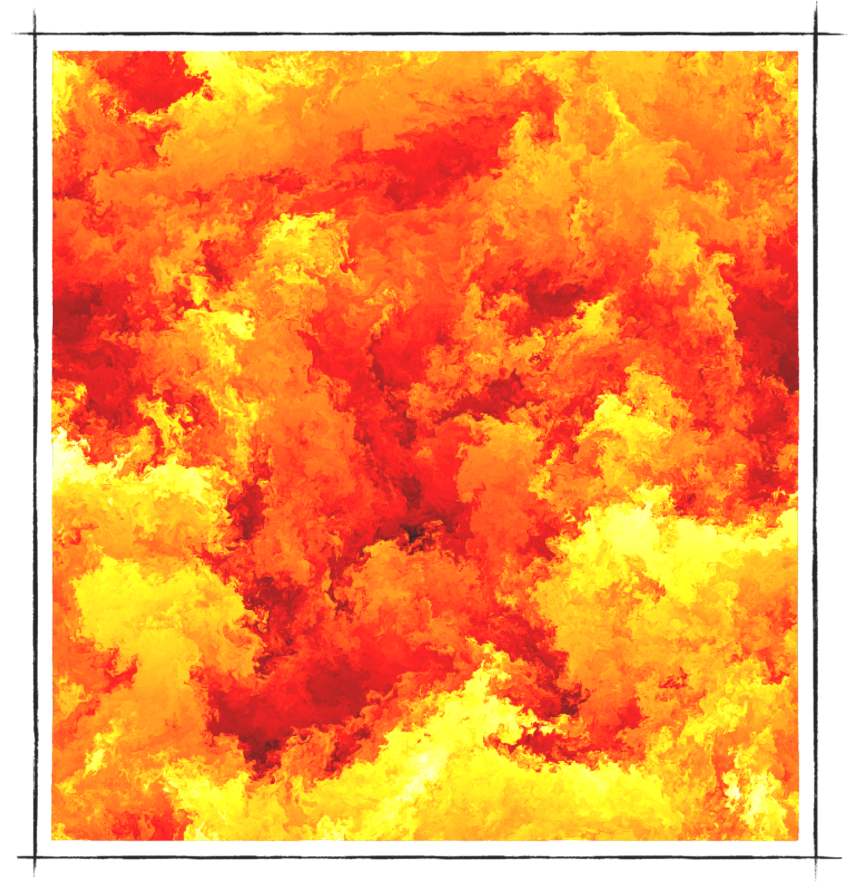
Turbulent dissipation along tracers

How to bridge dissipative anomaly and Lagrangian time irreversibility?



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Turbulence vs. Euler inviscid dynamics

Incompressible Navier–Stokes equation

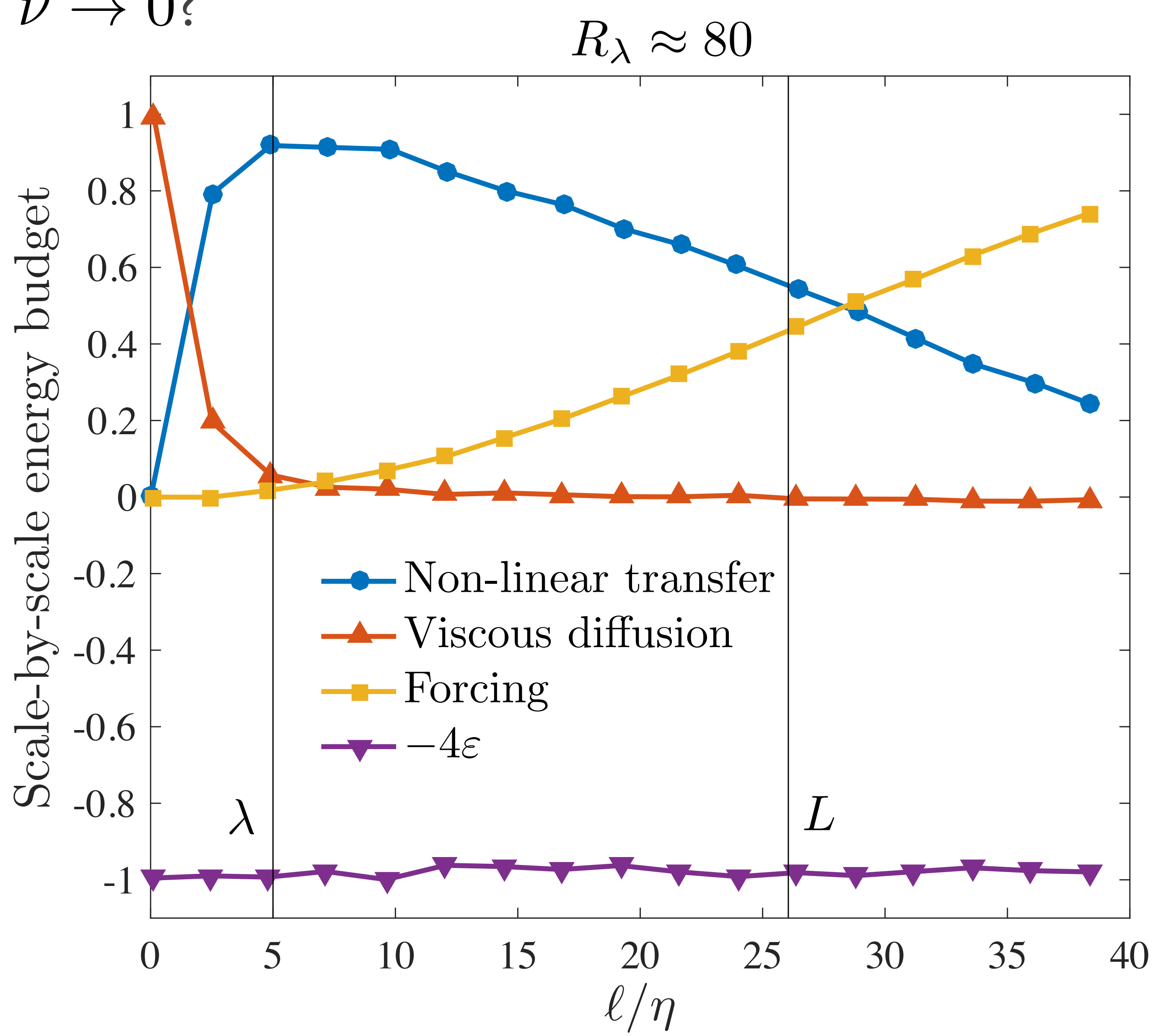
$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho_f} \nabla p + \underbrace{\nu \nabla^2 \boldsymbol{u}}_{\text{negligible when } \nu \to 0?} + \boldsymbol{f}, \;\; \nabla \cdot \boldsymbol{u} = 0$$

Inviscid dynamics gives a good handle on the inertial scales

- Scale-by-scale energy budget

Eyink 1994; Hill 2002; Casciola et al. 2003; Danaila et al. 2012; Yasuda & Vassilicos 2017

- Large-Eddy Simulations: only fluxes are important

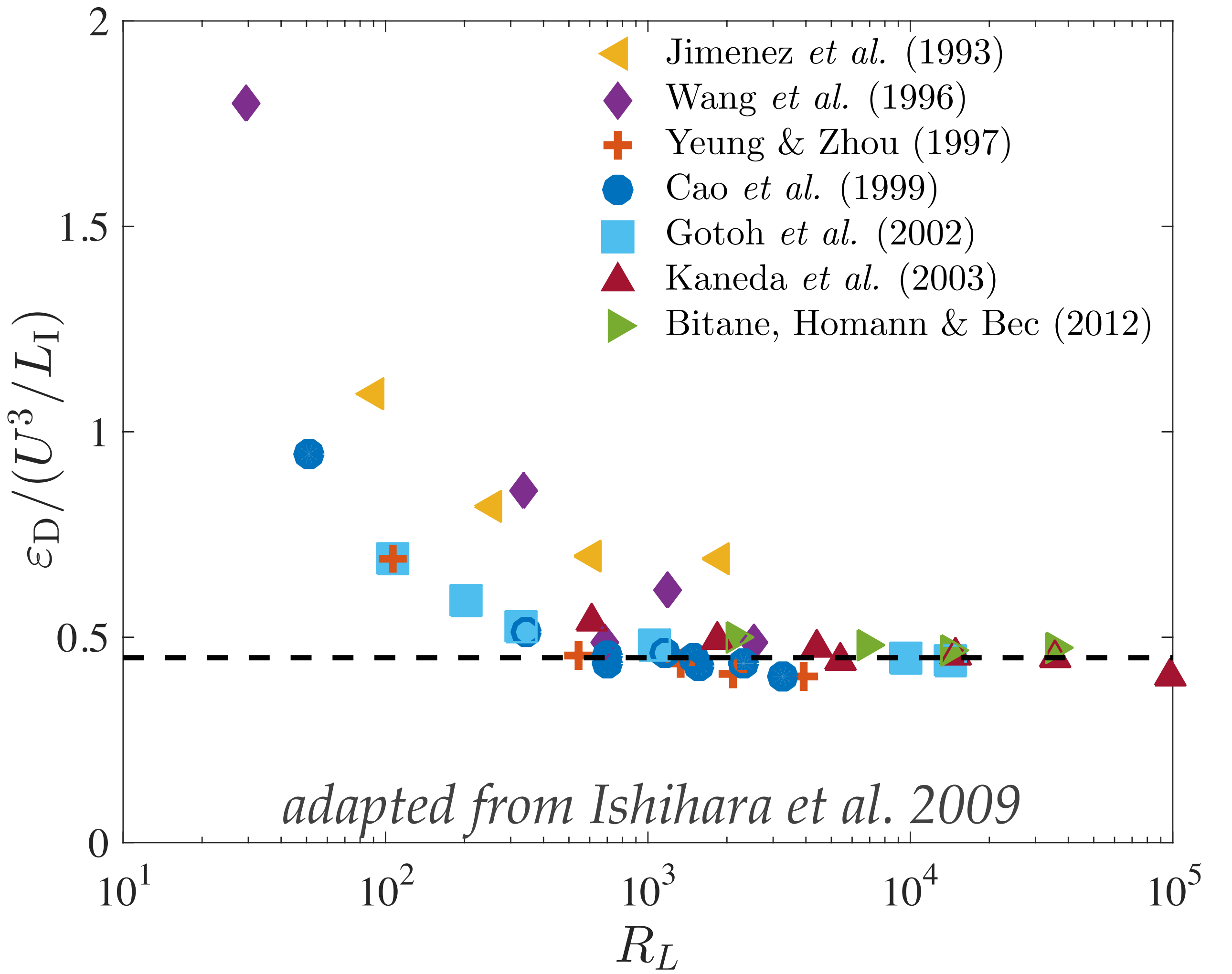


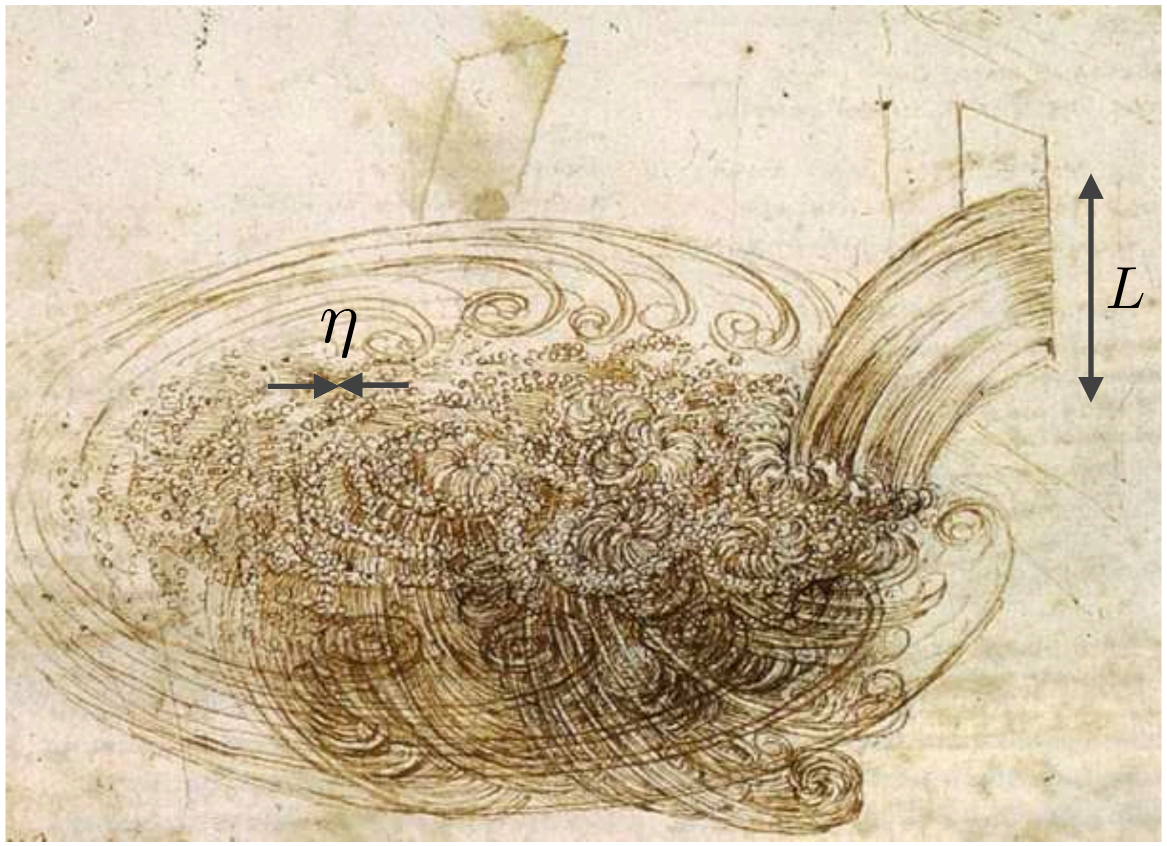
To what extent can Euler equation be used to describe turbulent flows?

The turbulent dissipative anomaly

Limit of infinite Reynolds numbers:
$\nu \to 0$ with ε_{I} and L fixed

$$\varepsilon_{\mathrm{D}} = \frac{\nu}{2}\left\langle \|\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathsf{T}}\|^2 \right\rangle \to \mathrm{const}$$





A finite dissipation is ensured by increasingly singular structures

Breaks the time reversibility of the Euler equation

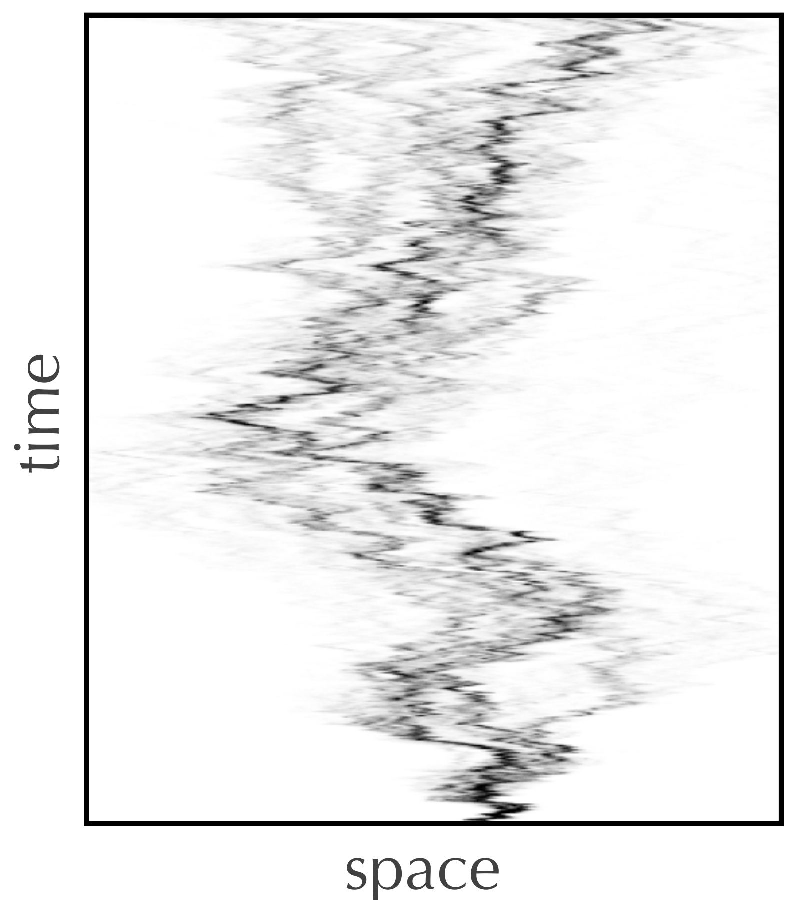
Singular (weak) solutions

Onsager's criterion: $|\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{\ell})-\boldsymbol{u}(\boldsymbol{x})| \sim \ell^h$ with $h \leq 1/3$

Constantin et al. 1994; Eyink 1994; Duchon & Robert 2000; Buckmaster 2015; Isett 2017

⇒ However not enough to select physically admissible solution

Lagrangian trajectories $\partial_t \boldsymbol{X}(t;\boldsymbol{x}_0) = \boldsymbol{u}(\boldsymbol{X}(t;\boldsymbol{x}_0),t)$ ⟵ non-Lipschitz



The Lagrangian flow is **non-unique**

⇒ largely explains anomalous dissipation and intermittency of advected passive scalars

Bernard et al. 1998; Pumir et al. 2000; Falkovich et al. 2001; Sawford 2001; Eyink 2008

Can Lagrangian statistics provide additional insight on admissibility?

Direct numerical simulations

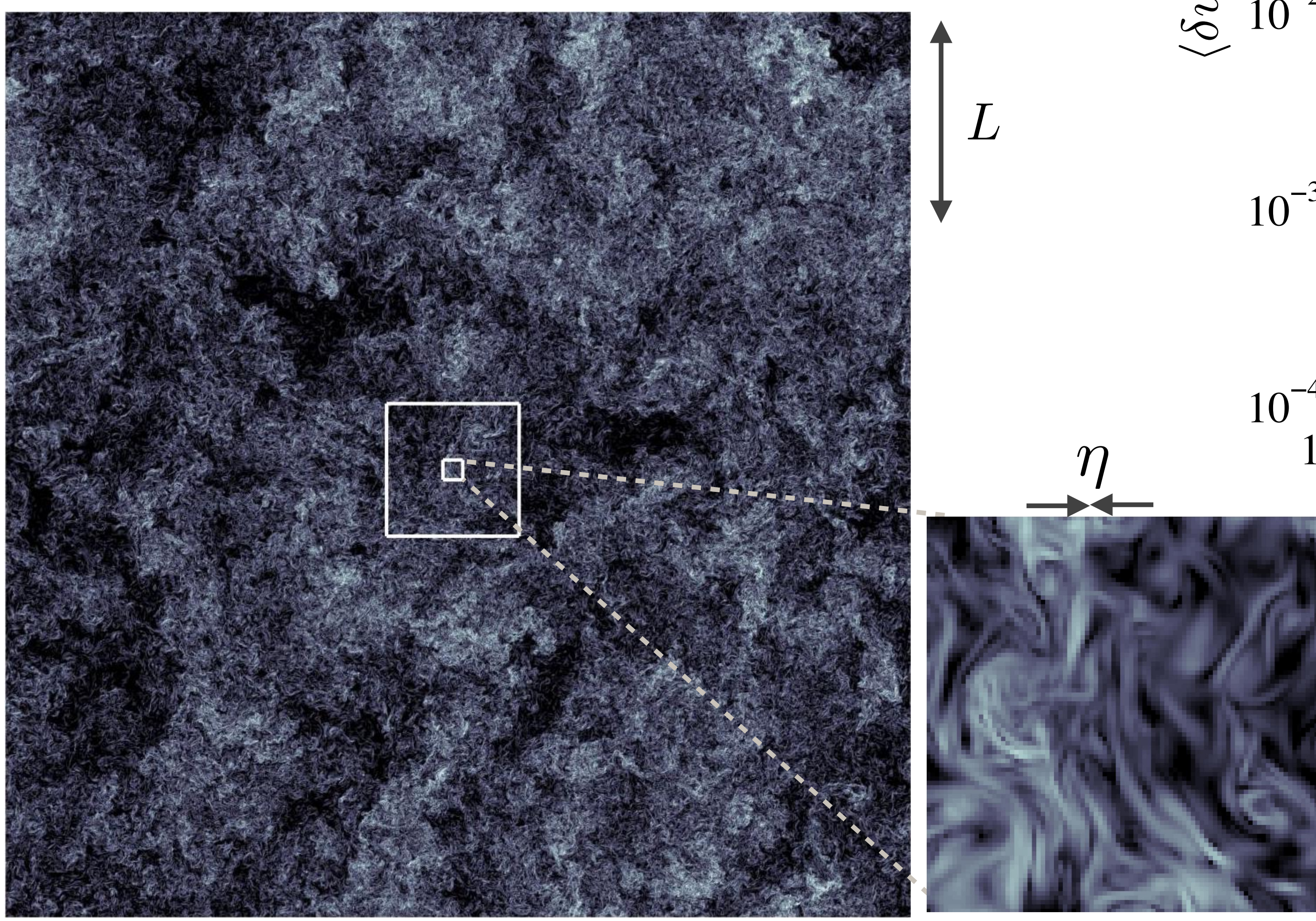
- ***LATU:*** ***LA***grangian ***TU***rbulence: MPI spectral solver *(Homann et al. 2007)*

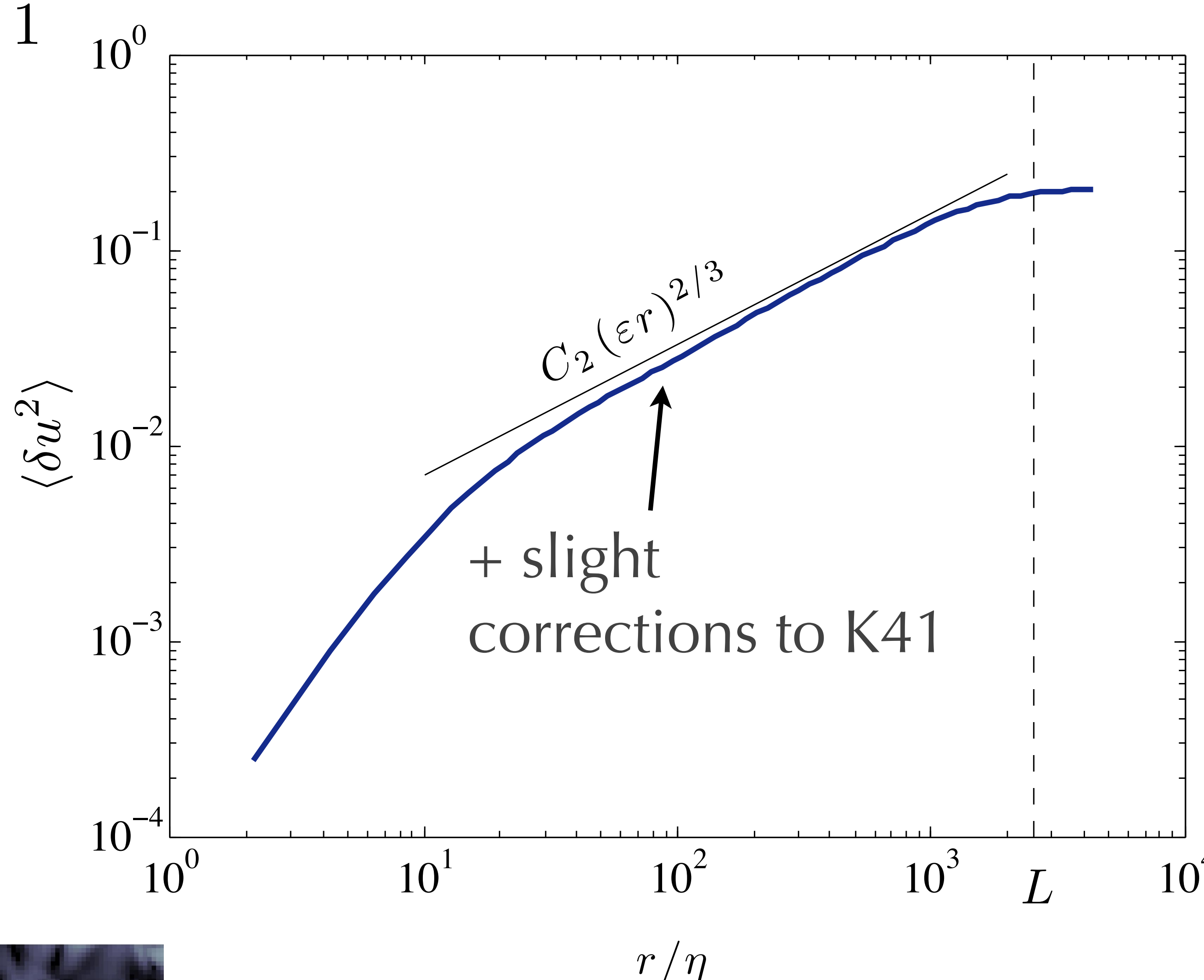
$\nu =$	N^3	Δt	R_λ
10^{-5}	4096^3	6×10^{-4}	730

$k_{\max}\,\eta \approx 1$

+1 billion tracer trajectories

$$\varepsilon_{\rm loc} = \frac{\nu}{2}\|\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathsf{T}}\|^2$$





Dissipation vs. dispersion

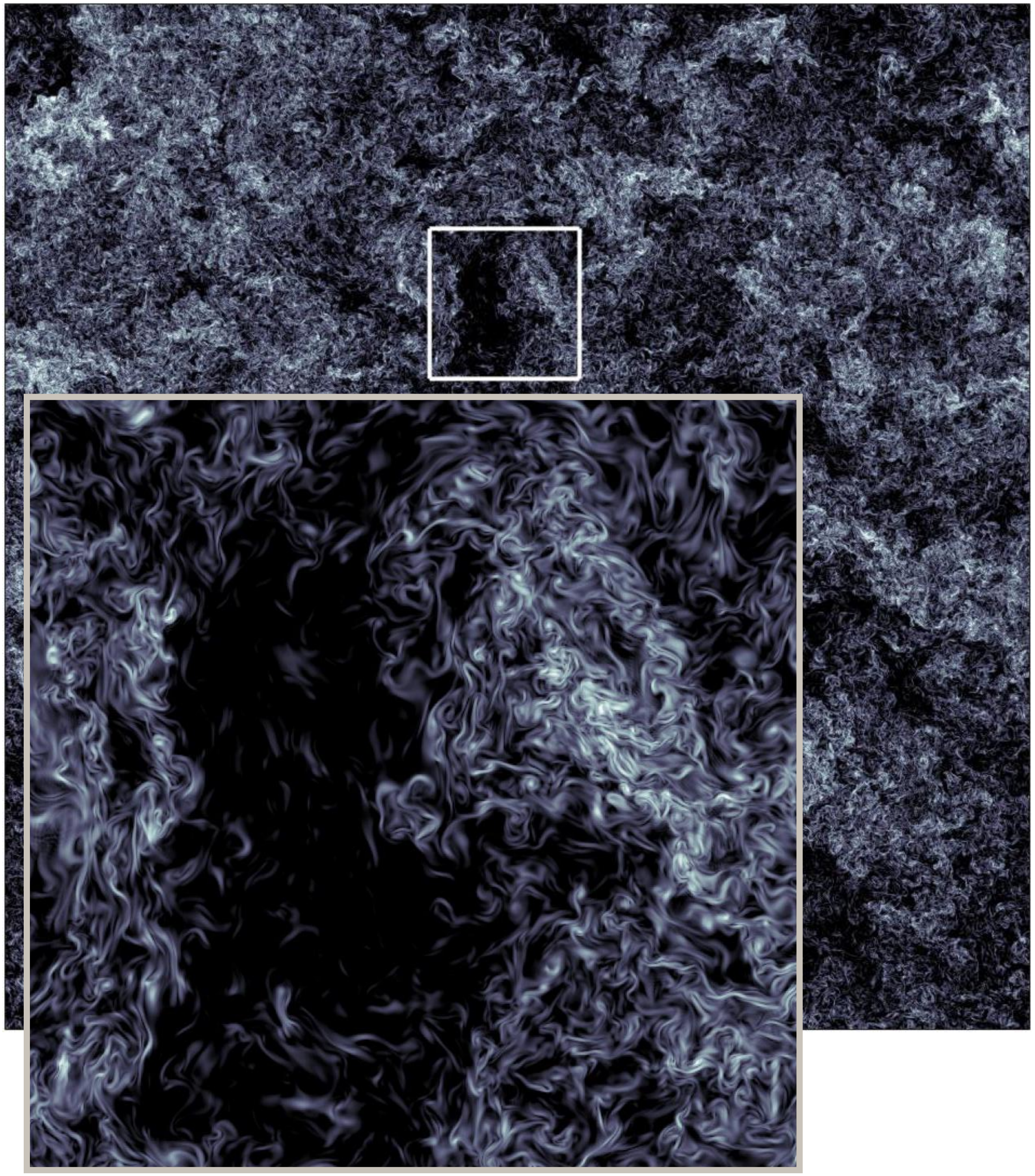
Local dissipation

$$\varepsilon_{\mathrm{loc}} = \frac{\nu}{2}\|\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathsf{T}}\|^2$$

$R_\lambda \approx 730$

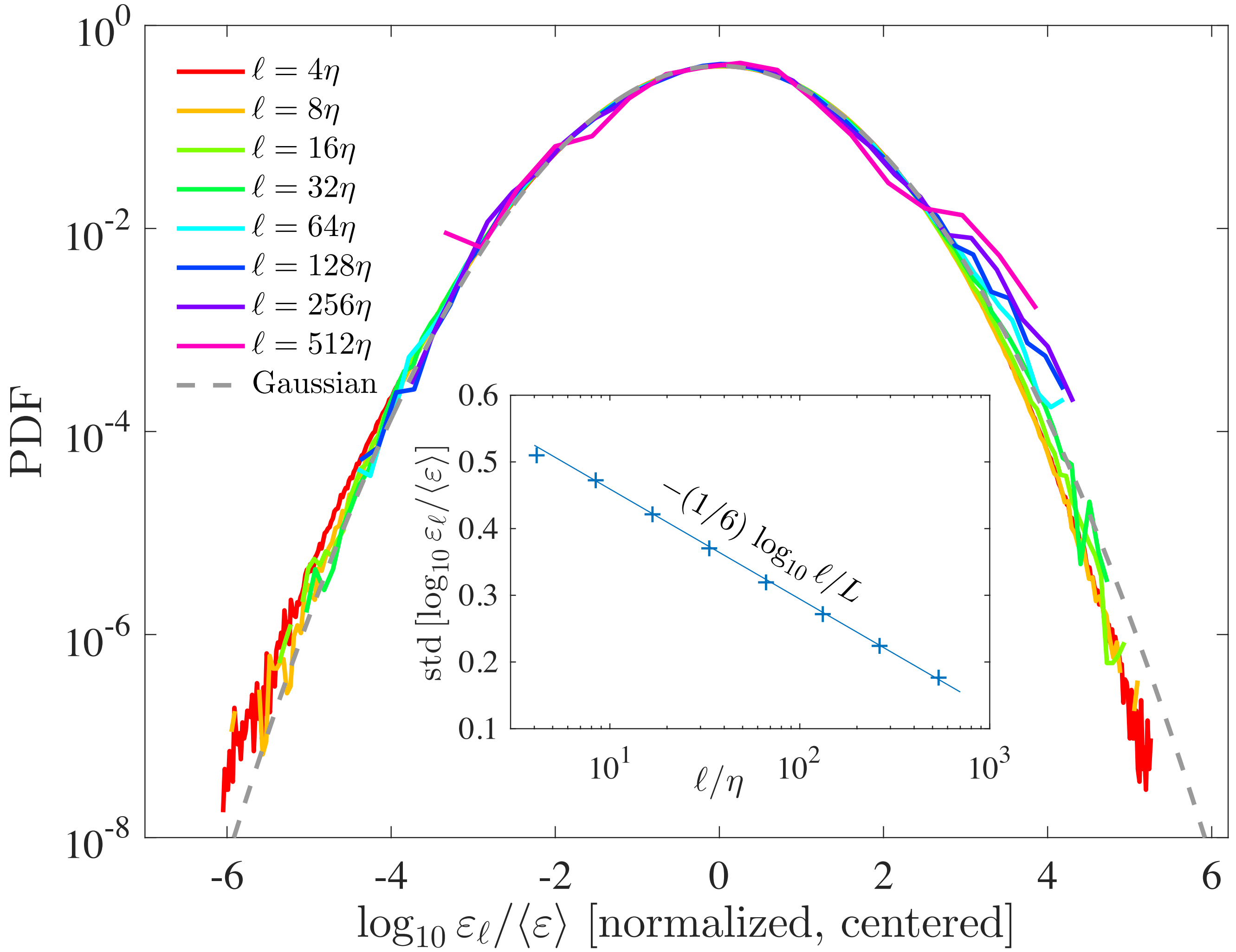
Distance travelled by tracers

$$|\boldsymbol{X}(t, \boldsymbol{x}_0) - \boldsymbol{X}(t-\tau, \boldsymbol{x}_0)| \quad \tau \approx \tau_L$$



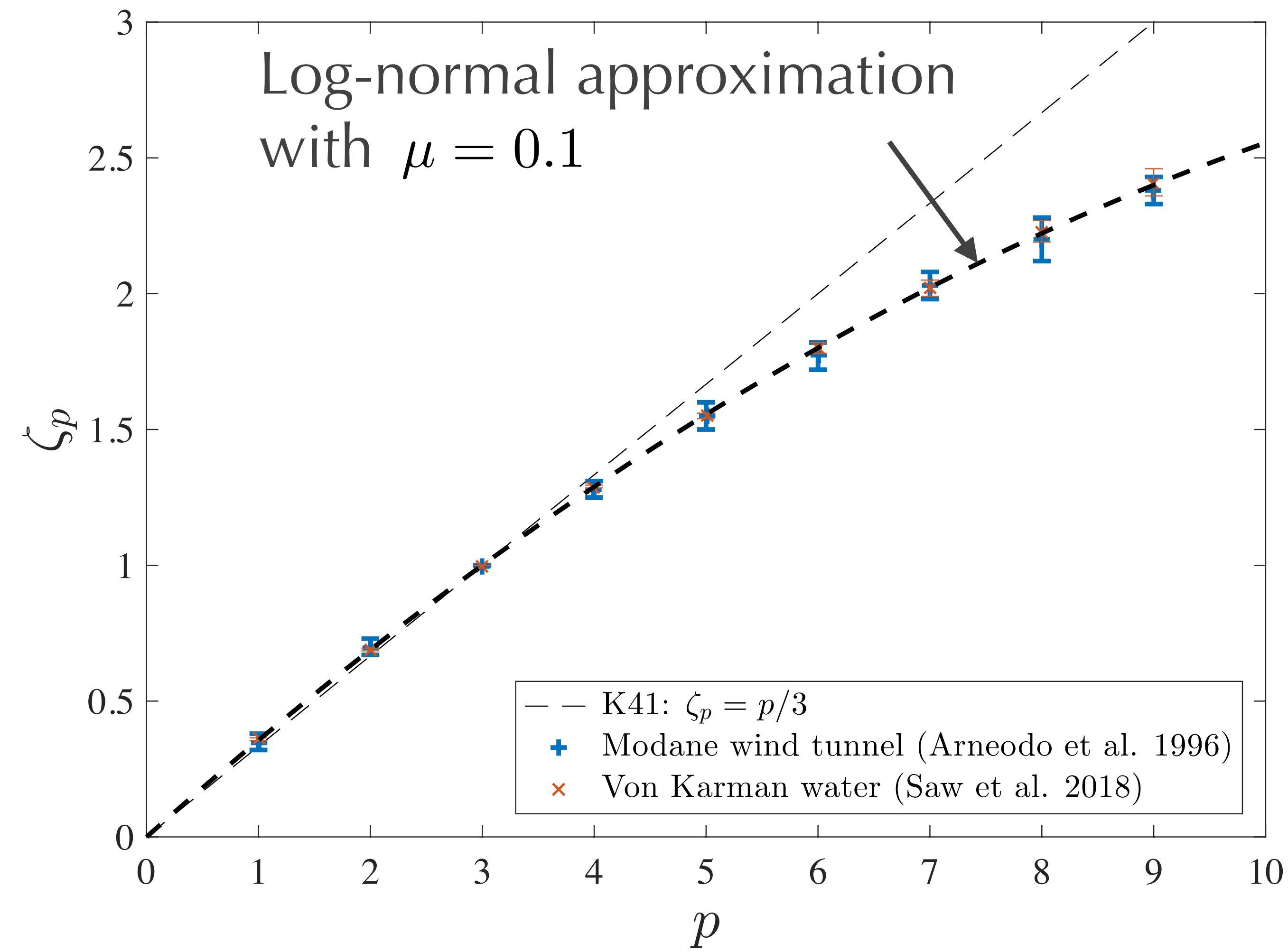
Euleriandissipation

$$\varepsilon_{\rm loc} = \frac{\nu}{2}\|\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathsf T}\|^2 \quad \Rightarrow \text{coarse-graining} \quad \varepsilon_\ell = \frac{1}{|\mathcal{B}_\ell|}\int_{\mathcal{B}_\ell} \varepsilon_{\rm loc}(\boldsymbol{x},t)\,\mathrm{d}^3x$$



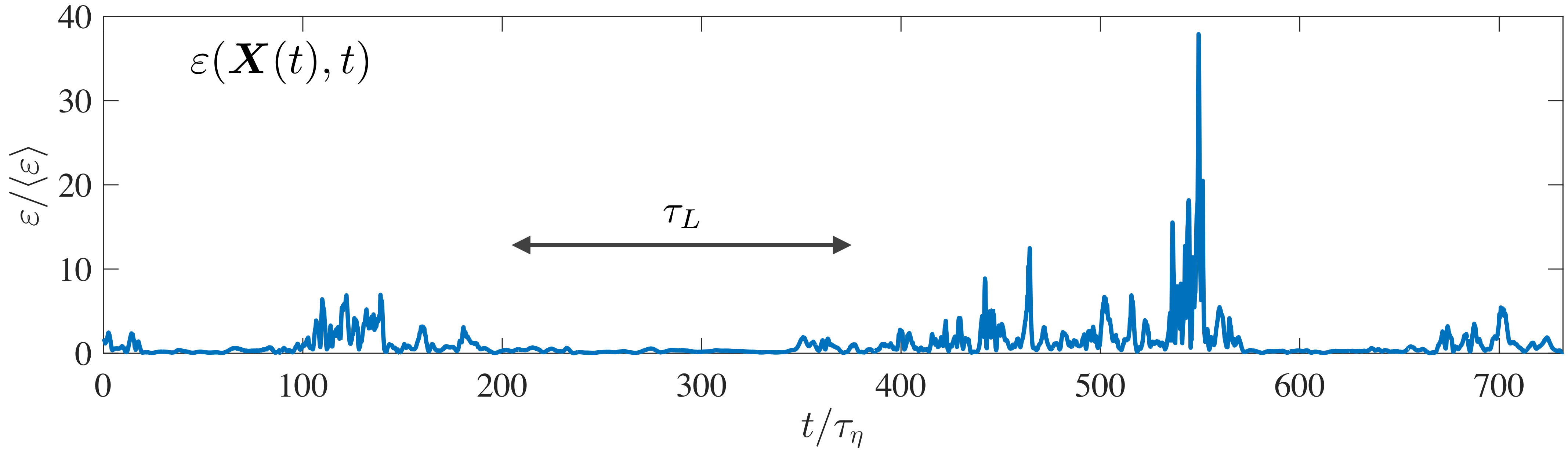
rather clean scaling/distribution

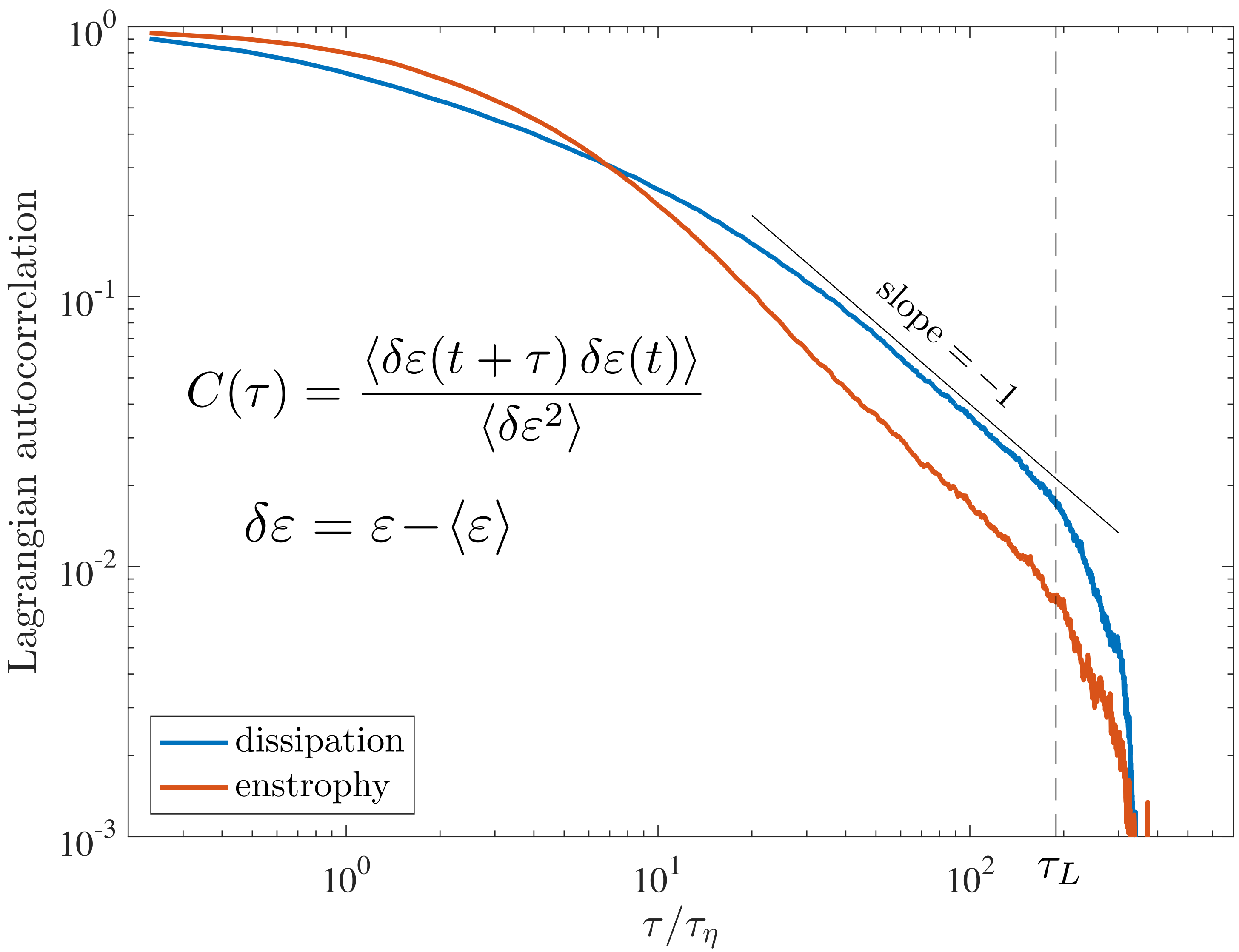
+ good link to velocity exponents



Lagrangian dissipation

Along tracers





$$C(\tau) = \frac{\langle \delta\varepsilon(t+\tau)\,\delta\varepsilon(t)\rangle}{\langle \delta\varepsilon^2\rangle}$$

$$\delta\varepsilon = \varepsilon - \langle\varepsilon\rangle$$

Almost long-range !

(law of large numbers, central-limit, large deviations???)

A given fluctuation might influence the whole dispersion process.

Turbulent pair dispersion

$$\boldsymbol{R}(t) = \boldsymbol{X}(t;\boldsymbol{x}_0^{(1)}) - \boldsymbol{X}(t;\boldsymbol{x}_0^{(2)})$$

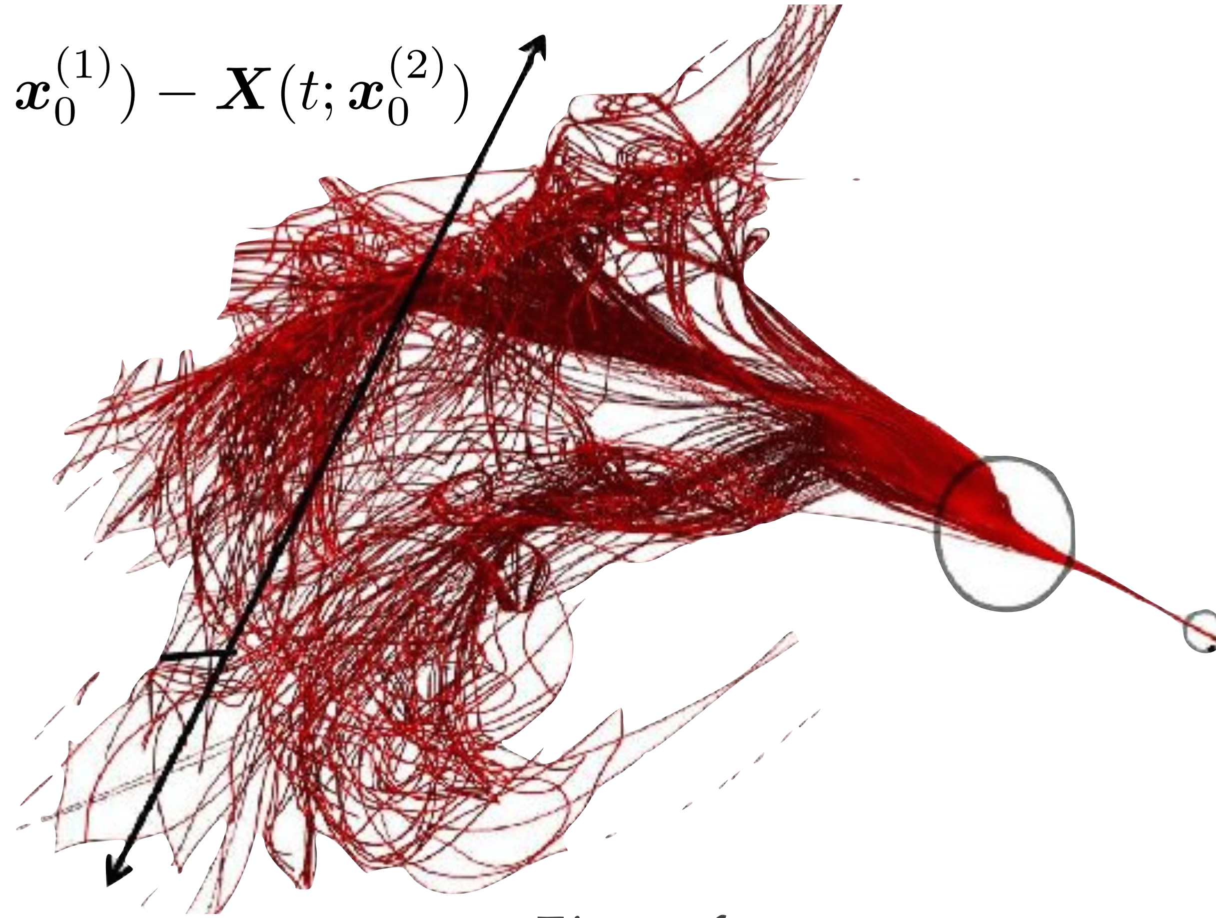
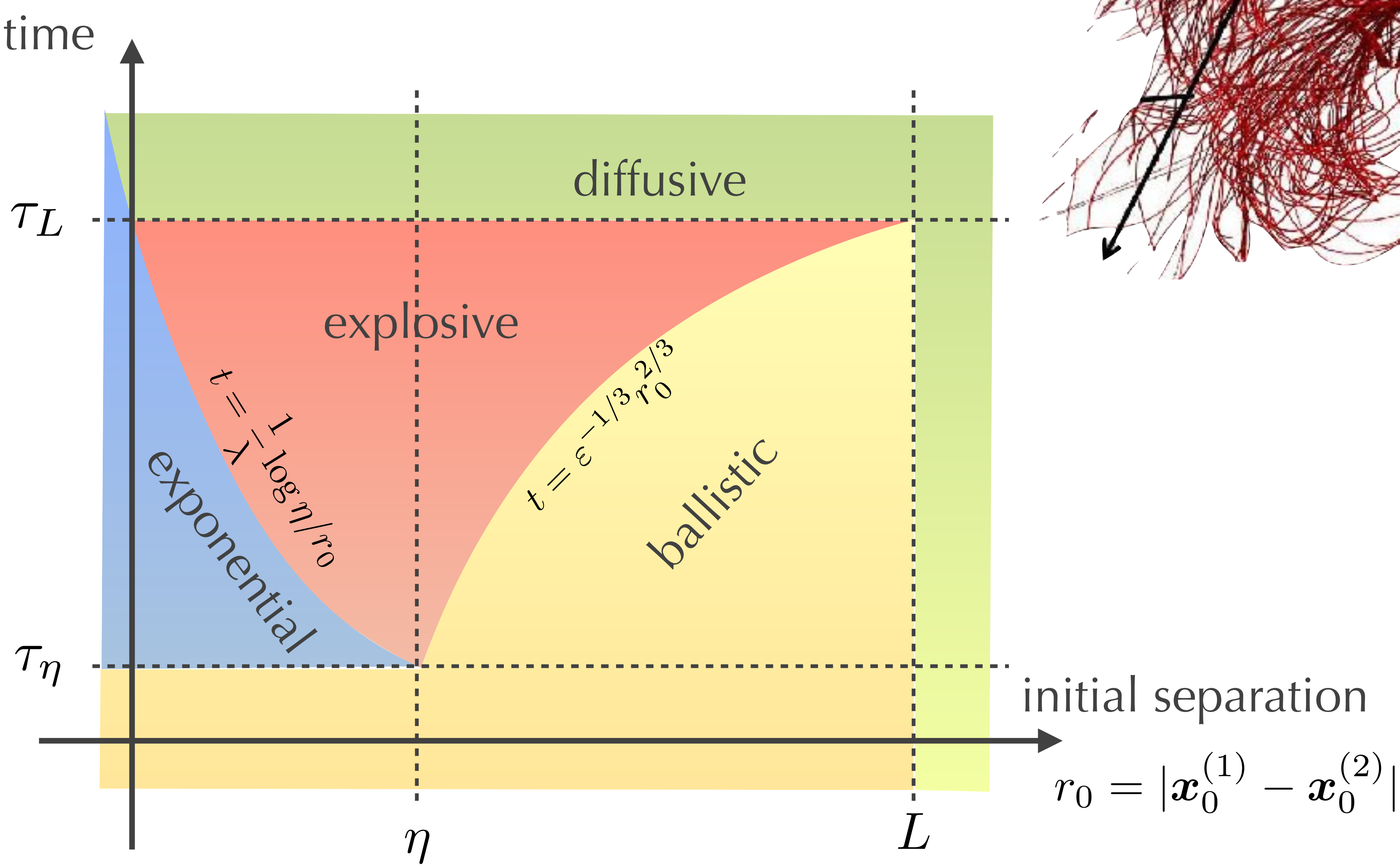


Figure from
Scatamacchia et al. 2013



Dissipative-range separations

Infinitesimal separation $|\boldsymbol{R}(t)| \ll \eta$ described by the tangent dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{R}(t) = \mathbb{A}(\boldsymbol{X}(t),t)\,\boldsymbol{R}(t) \quad \text{with} \quad A_{ij} = \partial u_i/\partial x_j$$

$$\boldsymbol{R}(t) = \mathbb{J}_{0,t}\,\boldsymbol{R}(0) \quad \text{with} \quad \mathbb{J}_{0,t} = \nabla_{x_0}\boldsymbol{X}(t;\boldsymbol{x}_0) \quad \textbf{Jacobian matrix}$$

$$\mathbb{J}_{0,t}^{\mathsf{T}}\mathbb{J}_{0,t} = \mathbb{Q}_{0,t}^{\mathsf{T}} \begin{bmatrix} \mathrm{e}^{2t\rho_1} & 0 & 0 \\ 0 & \mathrm{e}^{2t\rho_2} & 0 \\ 0 & 0 & \mathrm{e}^{2t\rho_3} \end{bmatrix} \mathbb{Q}_{0,t}$$

stretching rates
(finite-time Lyapunov exponents)

$$\rho_1(t) \geq \rho_2(t) \geq \rho_3(t) \qquad \rho_1+\rho_2+\rho_3 = 0$$

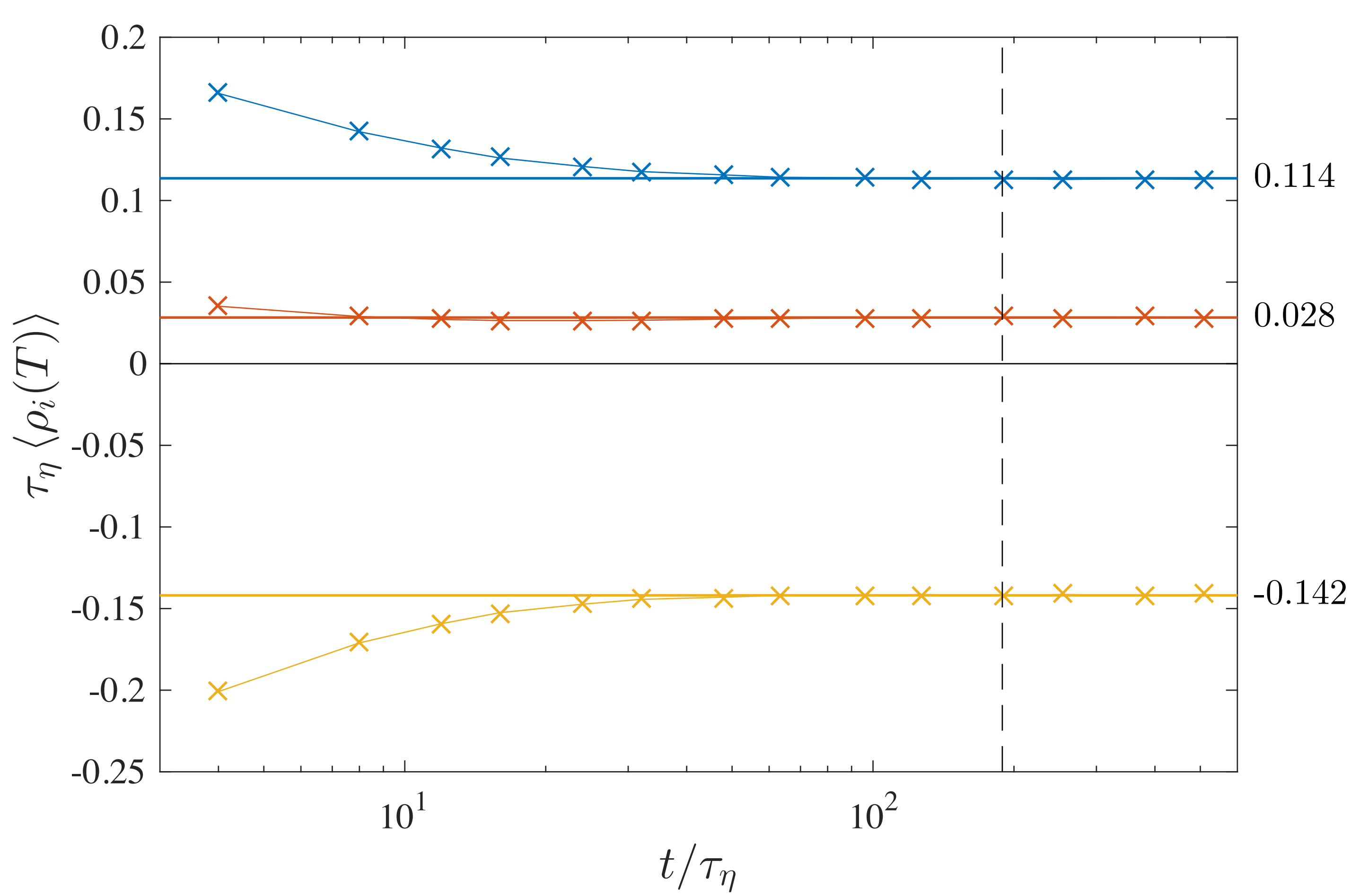
Multiplicative ergodic theorem

$$\lim_{t\to\infty} \rho_i(t) = \lambda_i \quad \textbf{Lyapunov exponents}$$

Oseledets 1968

Irreversibility:

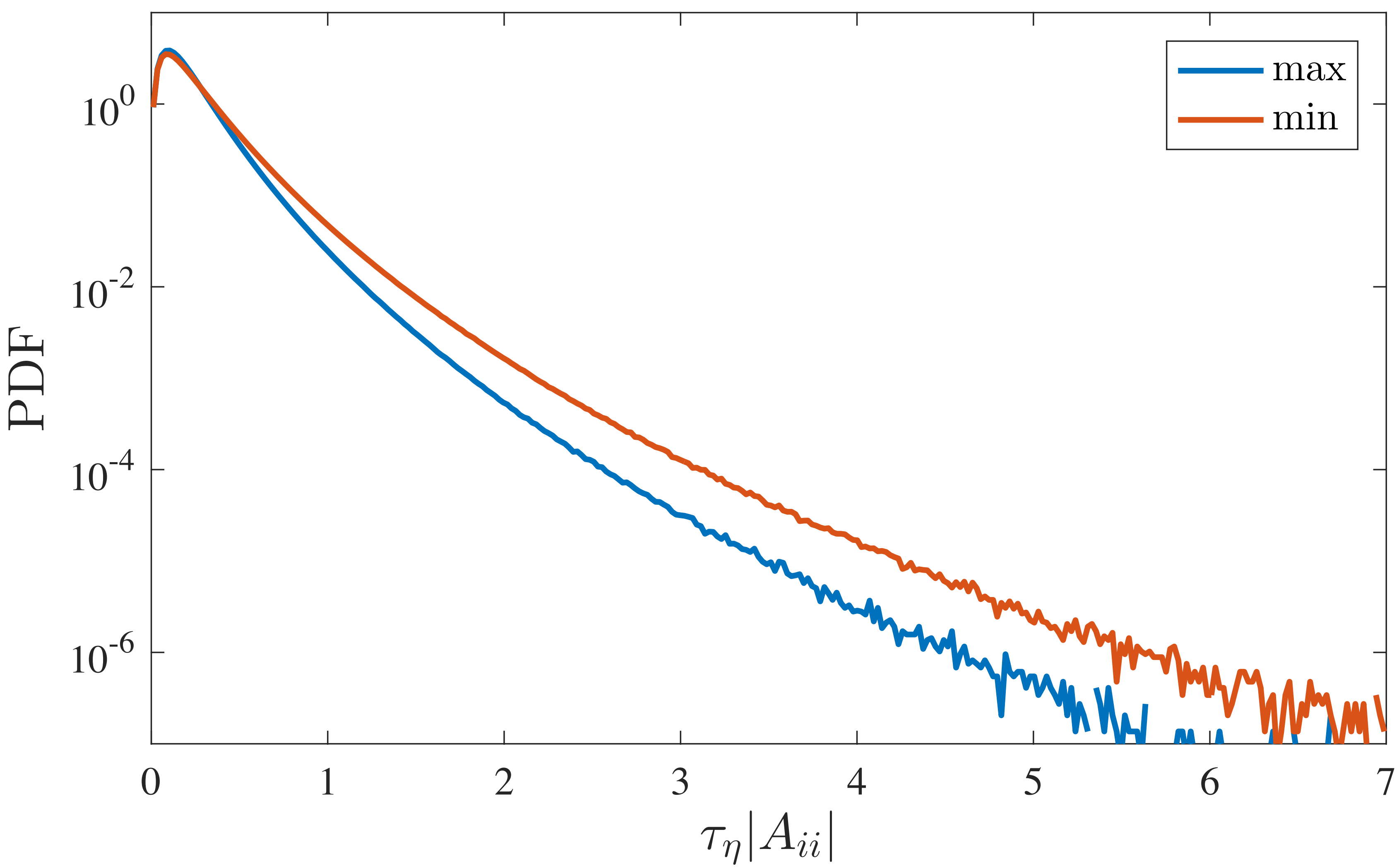
$$\lambda_3 \neq -\lambda_1 \qquad \lambda_2 > 0$$



Irreversible Lyapunov exponents

Asymmetry seems to mainly originate from **asymmetric gradients**:

compare $a_{\max} = \max\limits_{|\boldsymbol{n}|=1} \boldsymbol{n}^{\mathsf{T}} \mathbb{A}\, \boldsymbol{n}$ and $a_{\min} = \min\limits_{|\boldsymbol{n}|=1} \boldsymbol{n}^{\mathsf{T}} \mathbb{A}\, \boldsymbol{n}$



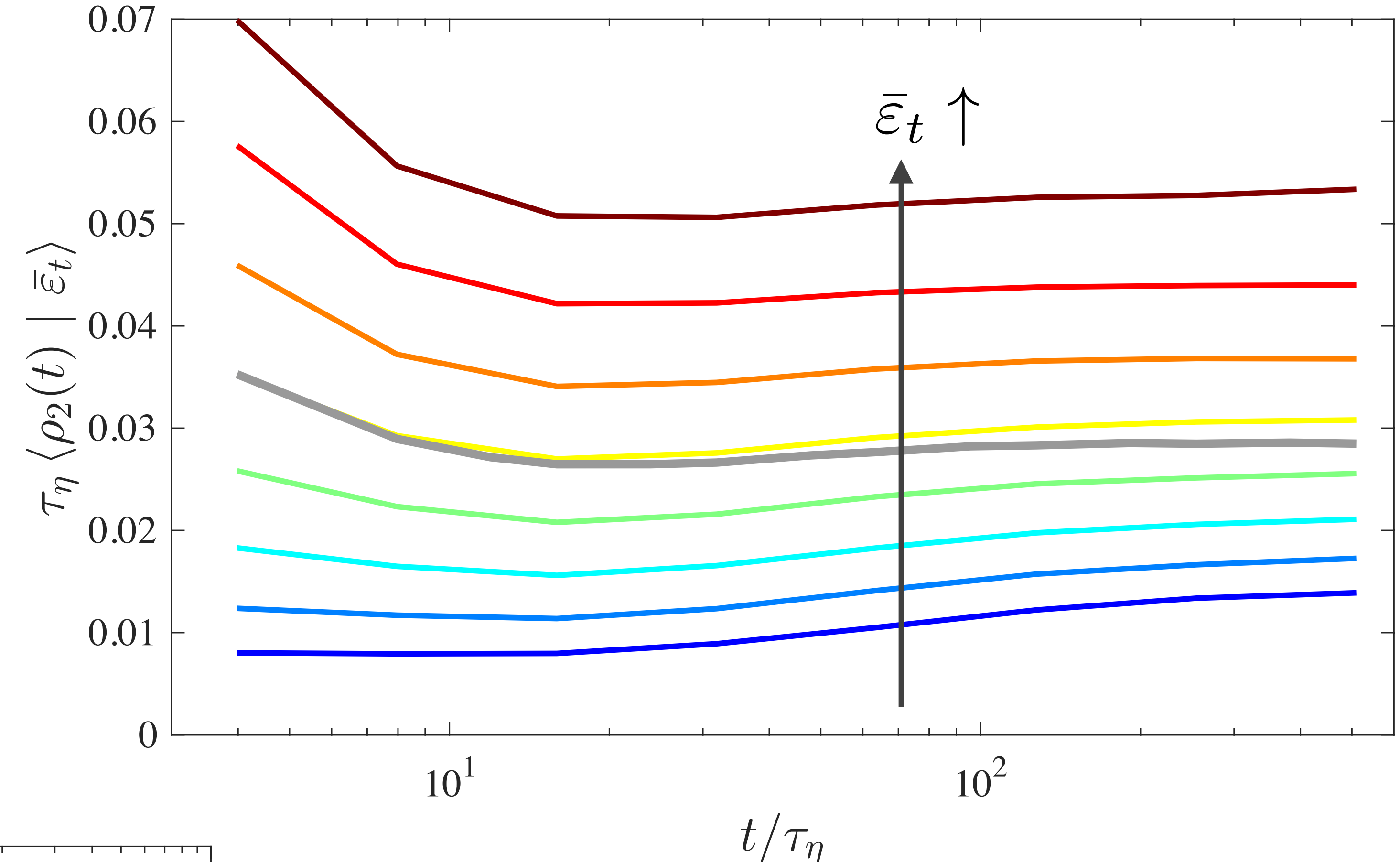
$$\frac{\langle a_{\max}\rangle}{\langle a_{\min}\rangle} \approx -0.8 \approx \frac{\lambda_1}{\lambda_3}$$

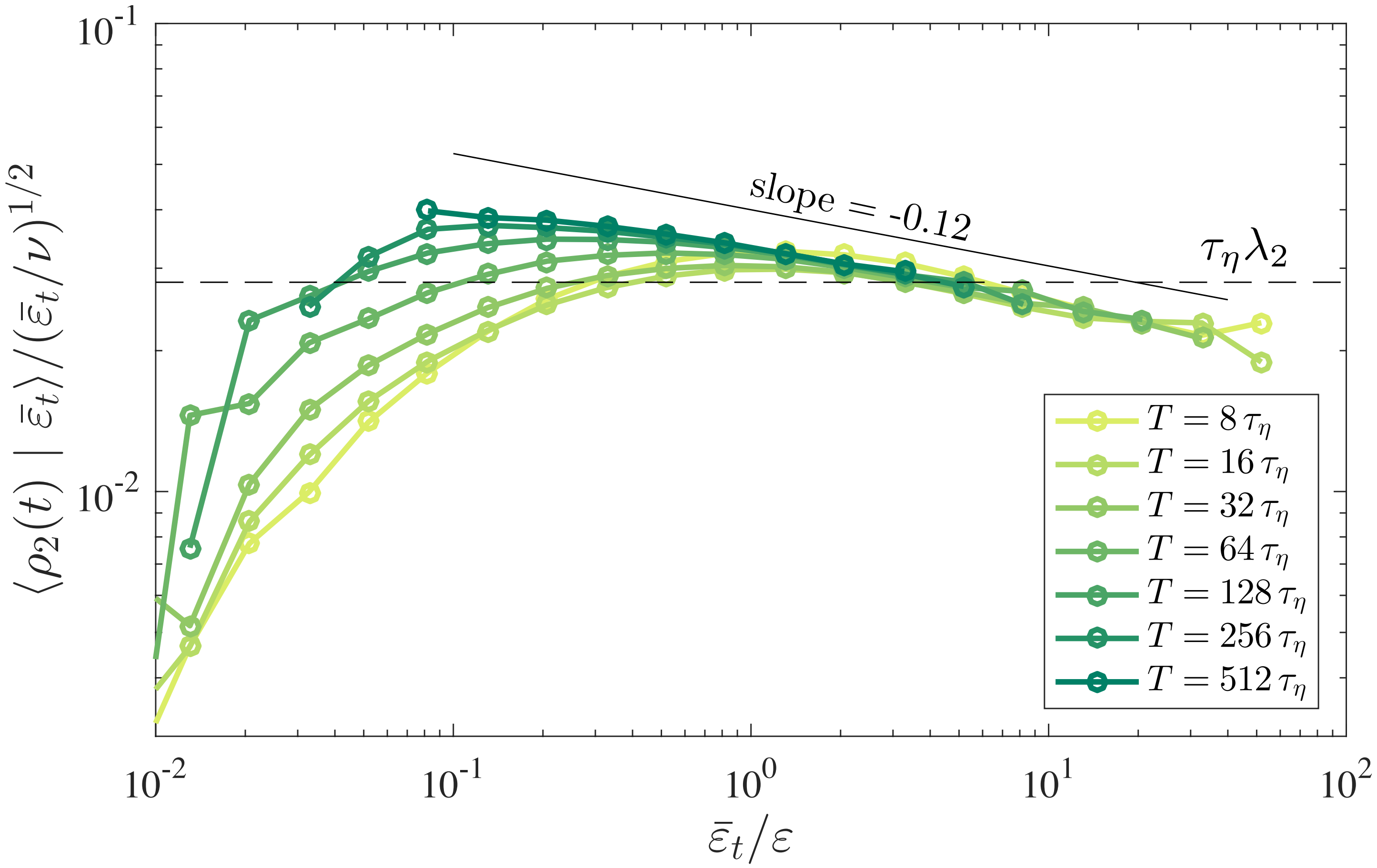
Link to dissipation

Stretching rates conditioned on

$$\bar{\varepsilon}_t = \frac{1}{t}\int_0^t \varepsilon_{\mathrm{loc}}(\boldsymbol{X}(s), s)\,\mathrm{d}s$$

higher asymmetries associated to stronger dissipative events





$$\langle \rho_2(t) \mid \bar{\varepsilon}_t \rangle \sim \frac{1}{\tau_\eta(\bar{\varepsilon}_t)} \left(\frac{\bar{\varepsilon}_t}{\varepsilon}\right)^{\alpha}$$

with $\alpha \approx -0.12$

Relation between this scaling and intermittency?

Central-limit / large deviations

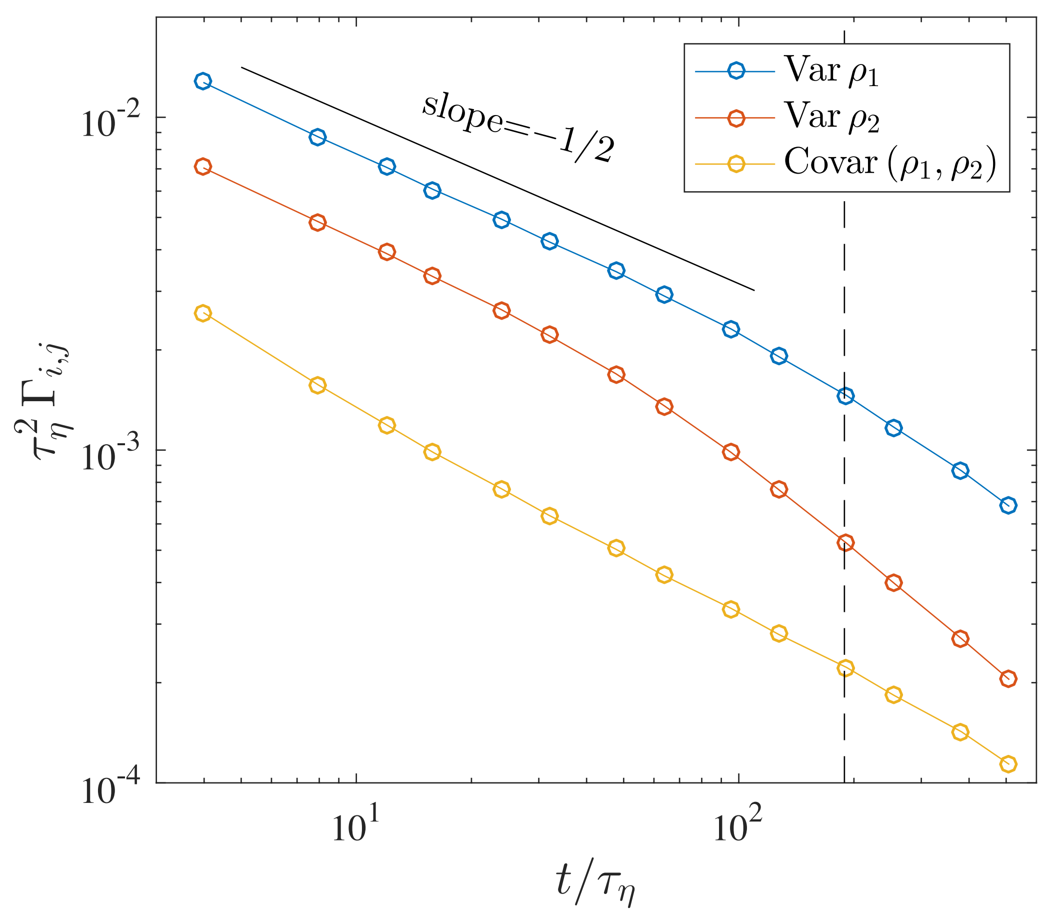
Expected for sufficiently short-correlated gradients:

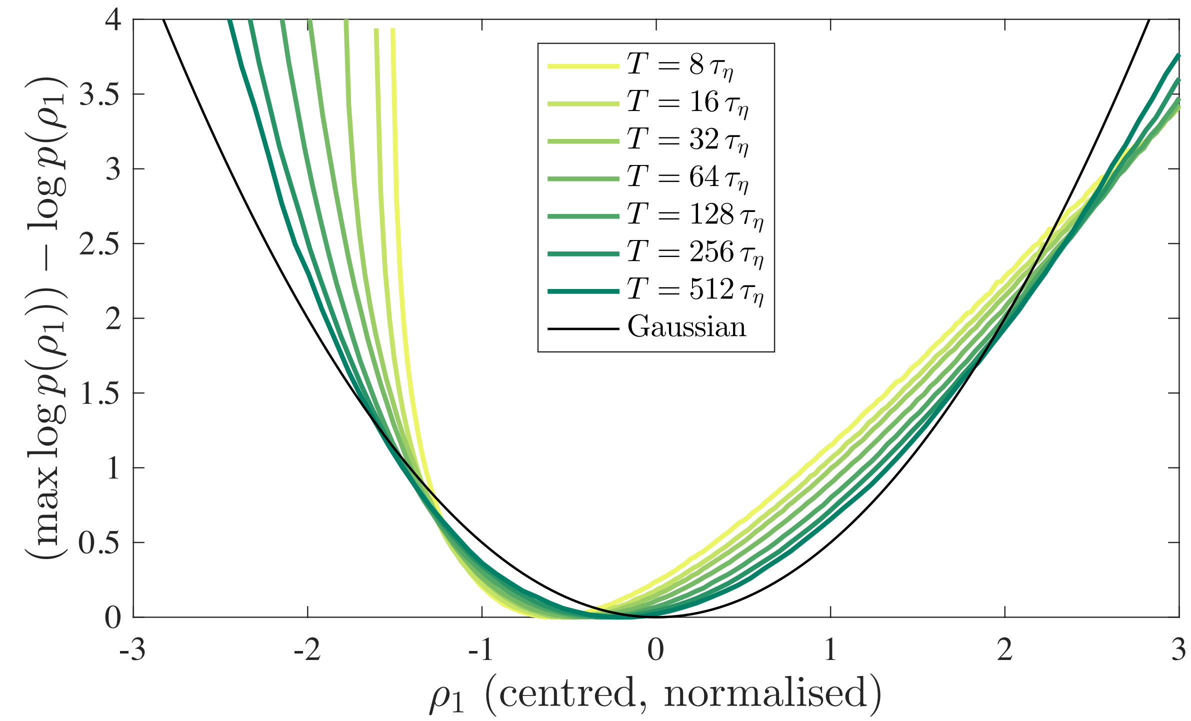
$$p(\rho_1, \rho_2, \rho_3; t) \sim \delta(\rho_1 + \rho_2 + \rho_3)\, \mathrm{e}^{-t\,\mathcal{H}(\rho_1,\rho_2)}$$

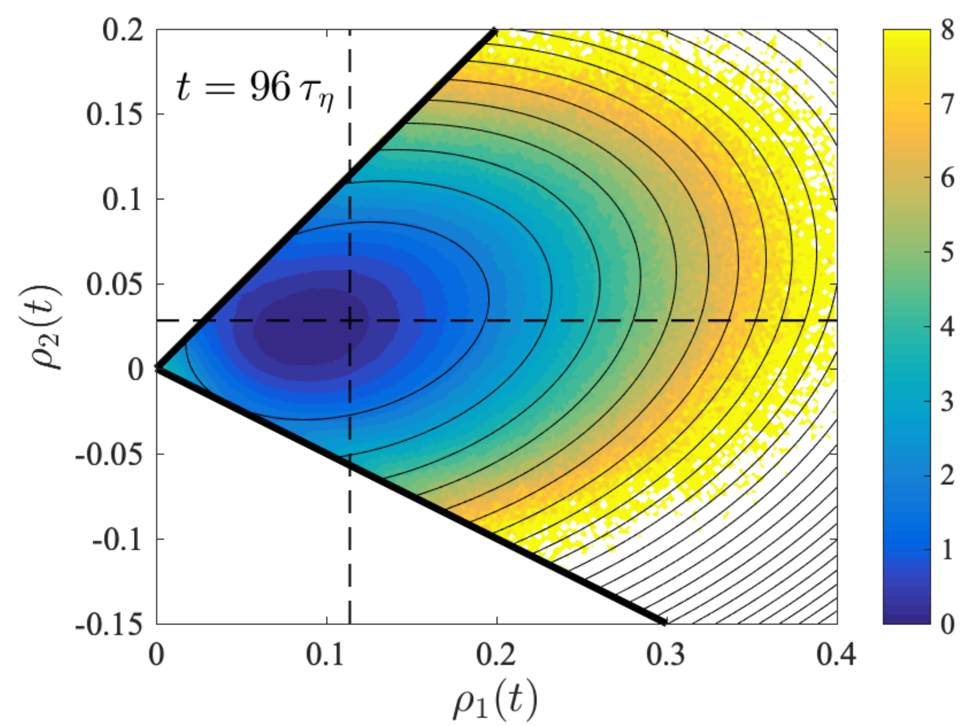
$\mathcal{H} \geq 0$ = convex rate function, with minimum at (λ_1, λ_2)

Central-limit theorem:

$$\mathcal{H}(\rho_1, \rho_2) \approx \frac{1}{2} \begin{pmatrix} \rho_1 - \lambda_1 \\ \rho_2 - \lambda_2 \end{pmatrix}^{\mathsf{T}} \Gamma^{-1} \begin{pmatrix} \rho_1 - \lambda_1 \\ \rho_2 - \lambda_2 \end{pmatrix}$$







No central-limit theorem?

Inertial-range separations

Irreversibility understood in the **ballistic** regime $t \ll t_0 \approx \varepsilon^{-1/3} r_0^{2/3}$

$$\left\langle |\boldsymbol{R}(t) - \boldsymbol{R}(0)|^2 \right\rangle_{r_0} = t^2\, S_2(r_0) + t^3 \underbrace{\langle \delta \boldsymbol{u} \cdot \delta \boldsymbol{a} \rangle}_{= -2\,\varepsilon} + \mathcal{O}(t^4)$$

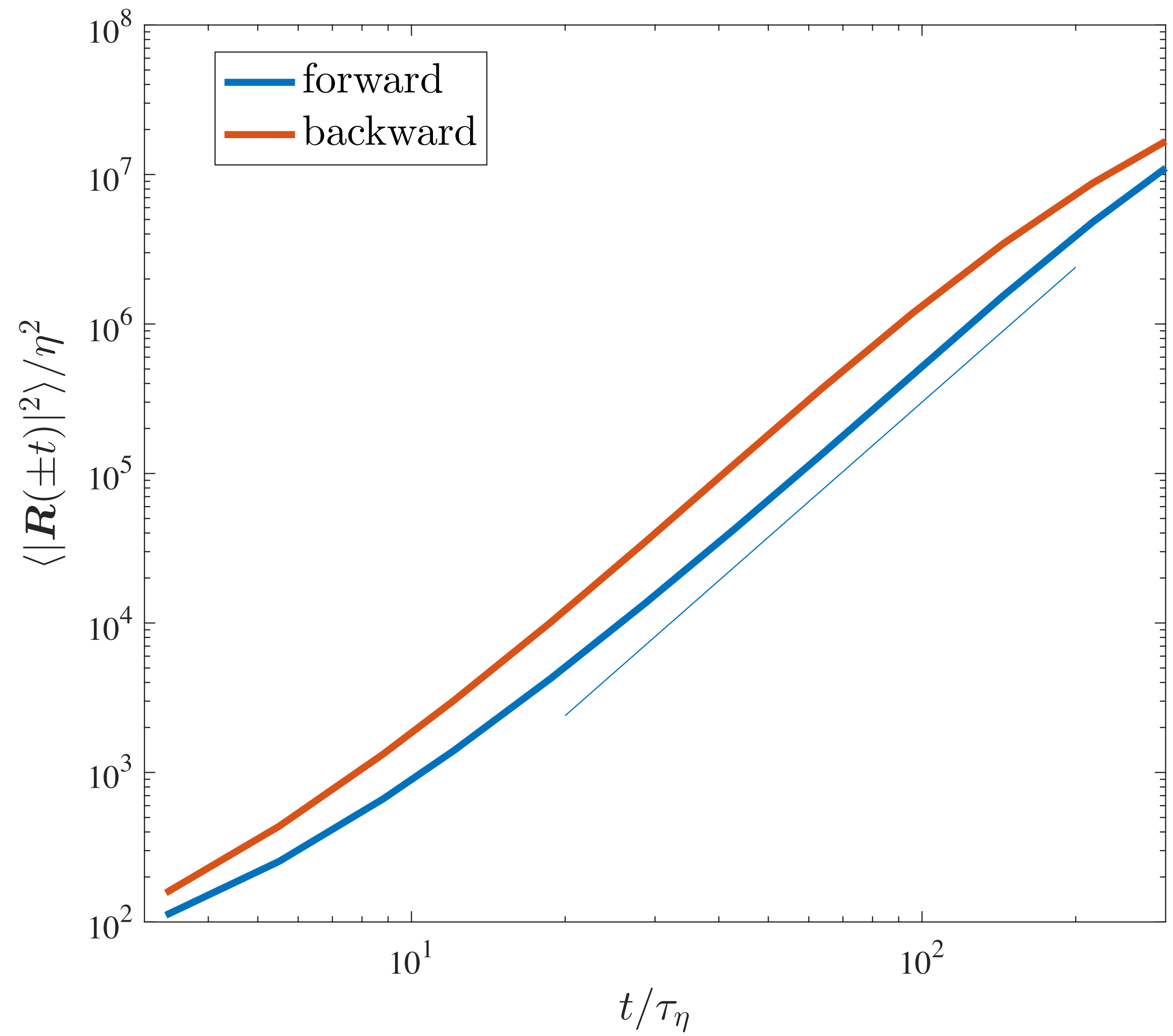
Jucha et al. 2014; Drivas 2018

$= -2\,\varepsilon$ breaks time reversibility

Richardson's **explosive** separation $t_0 \ll t \ll \tau_L$

$$\left\langle |\boldsymbol{R}(\pm t)|^2 \right\rangle_{r_0} = g_{\pm} \varepsilon\, t^3$$

$$g_+ / g_+ \approx 2$$

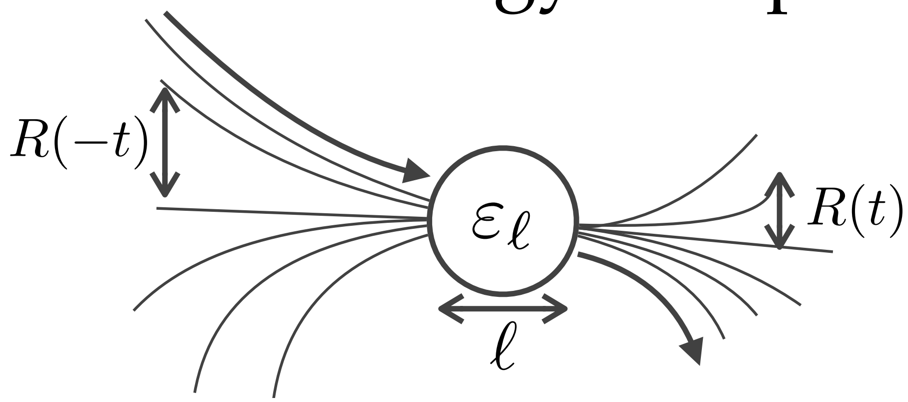


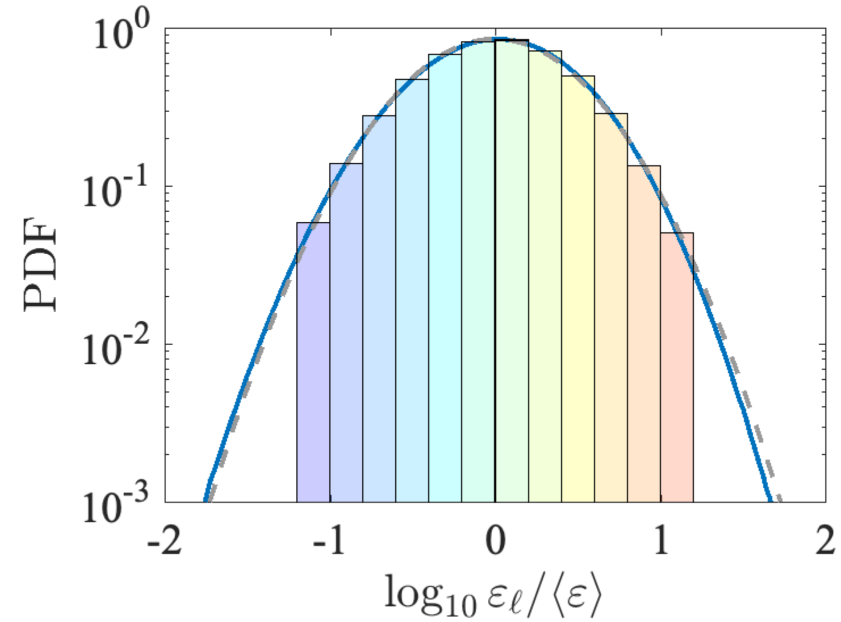
consistent with a negative skewness of longitudinal velocity increments but only understood at a qualitative level.

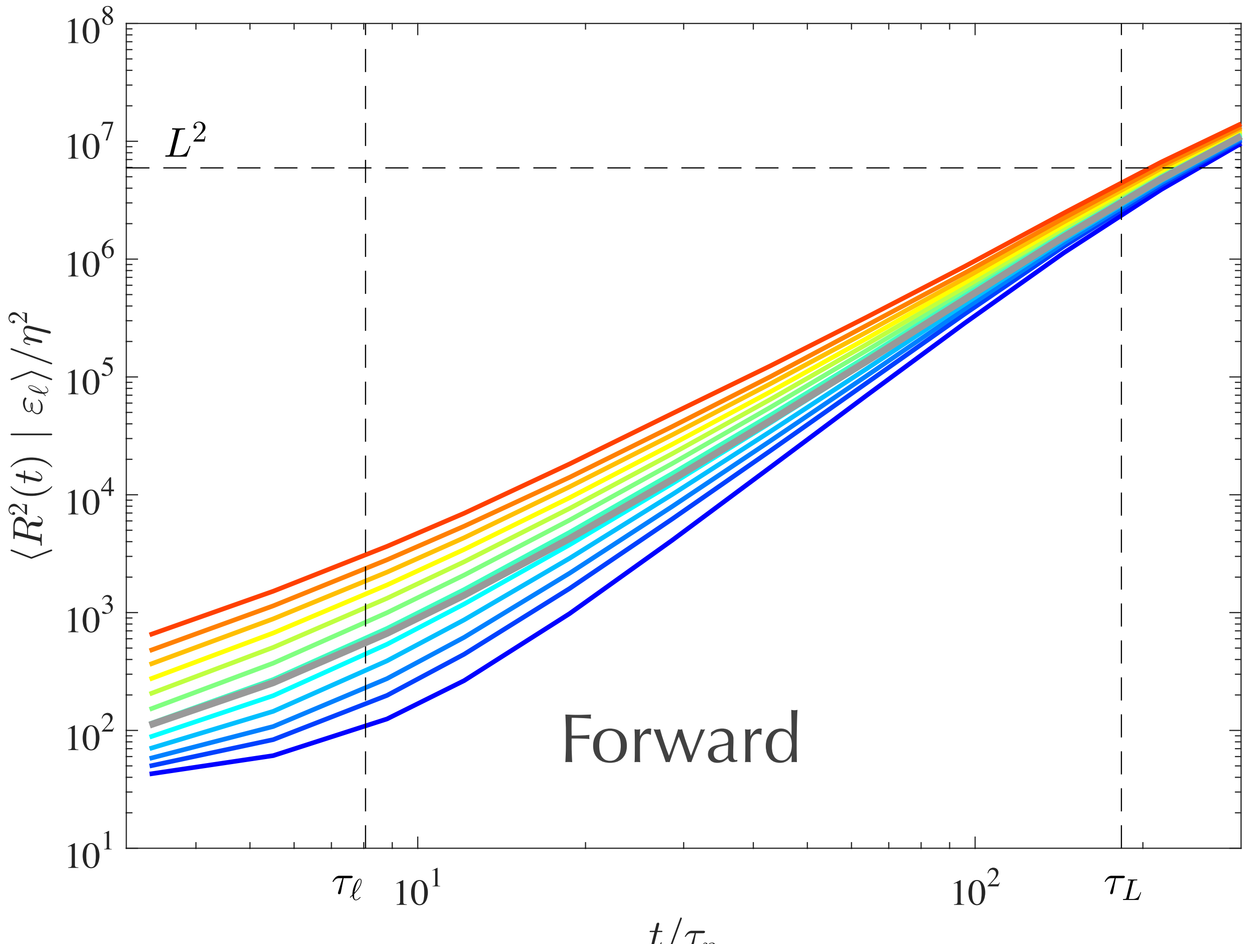
Berg et al. 2006; Bragg et al. 2014; Bourgoin 2015

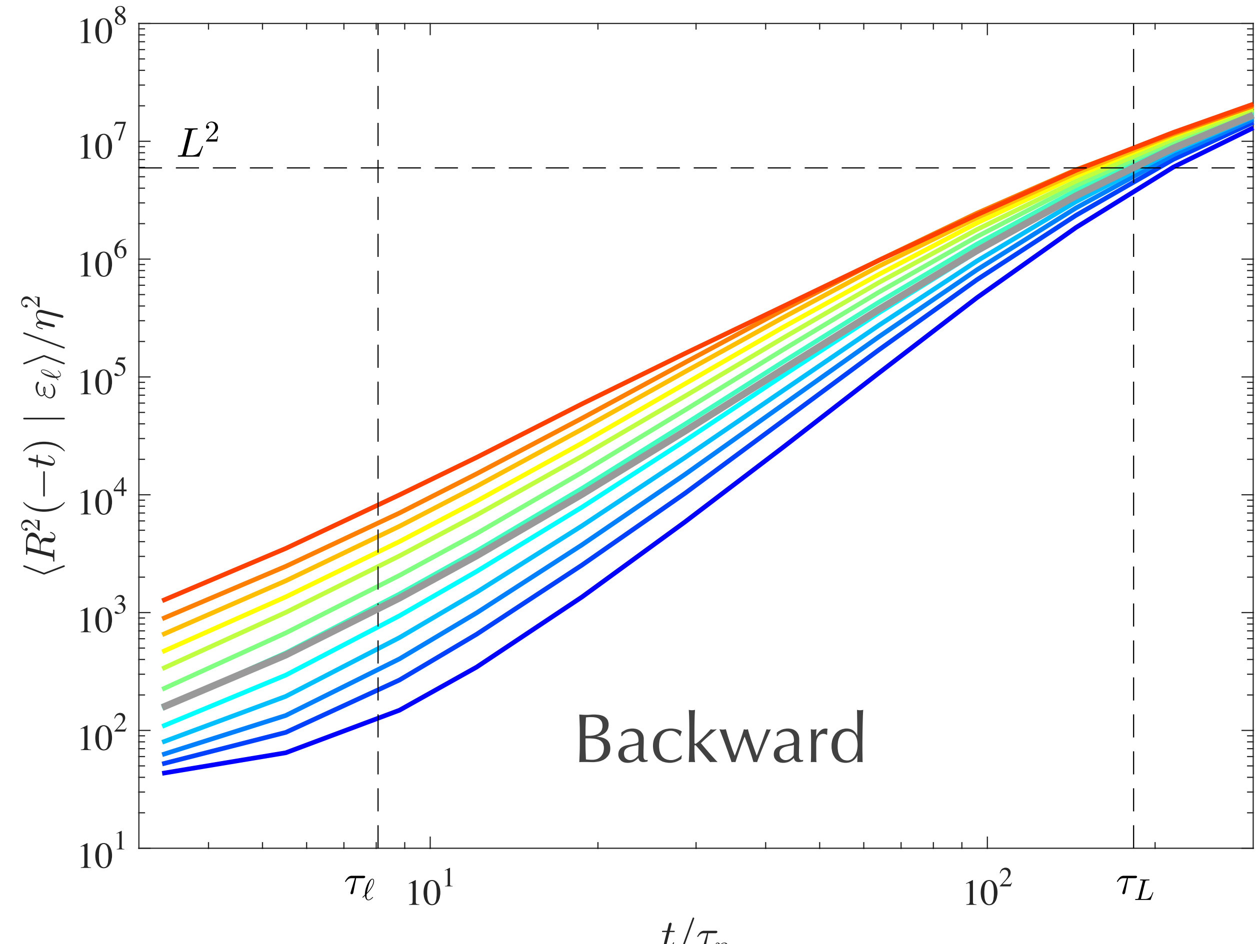
Conditional statistics

Pair separation conditioned on the local kinetic energy dissipation







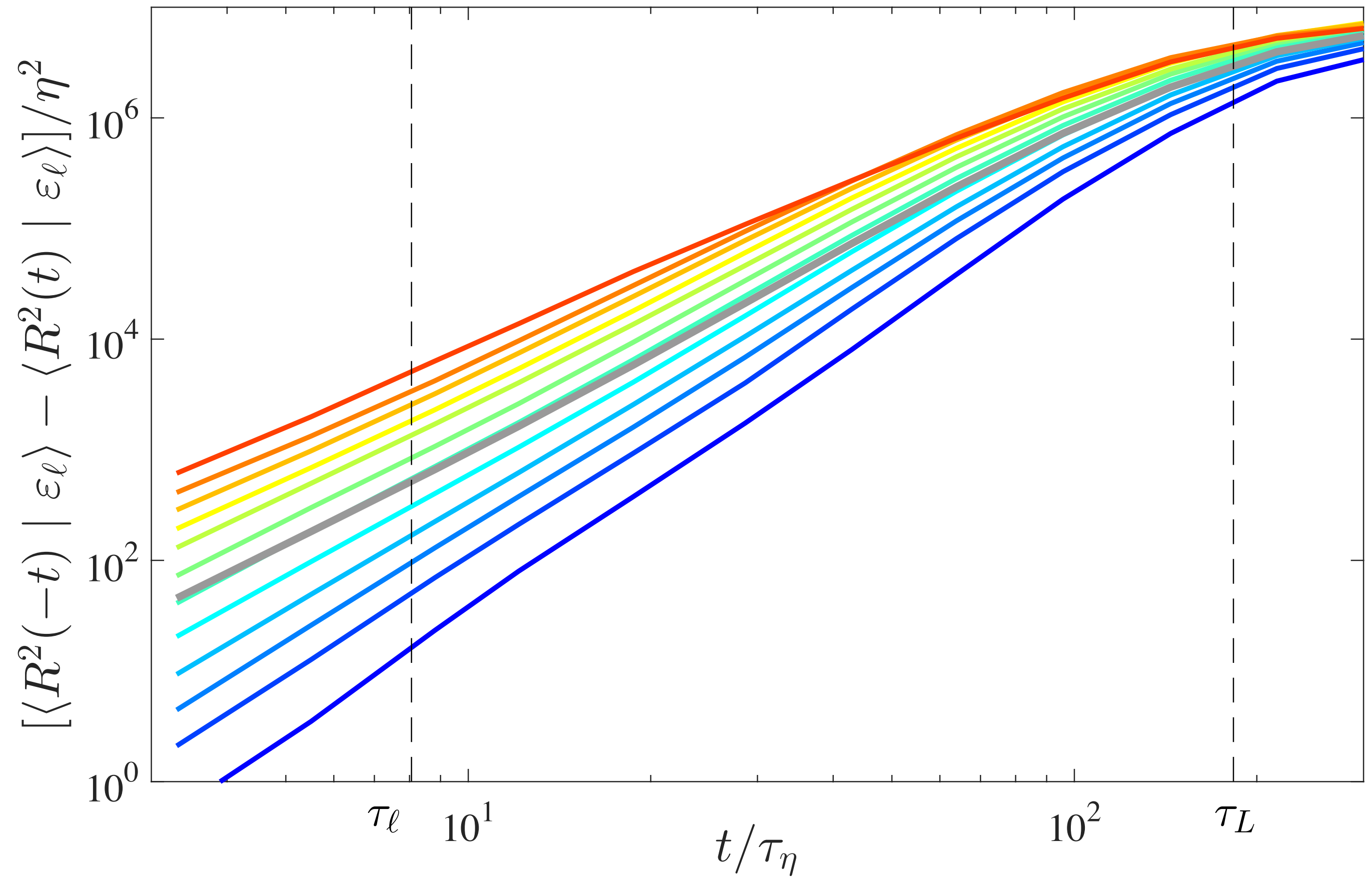


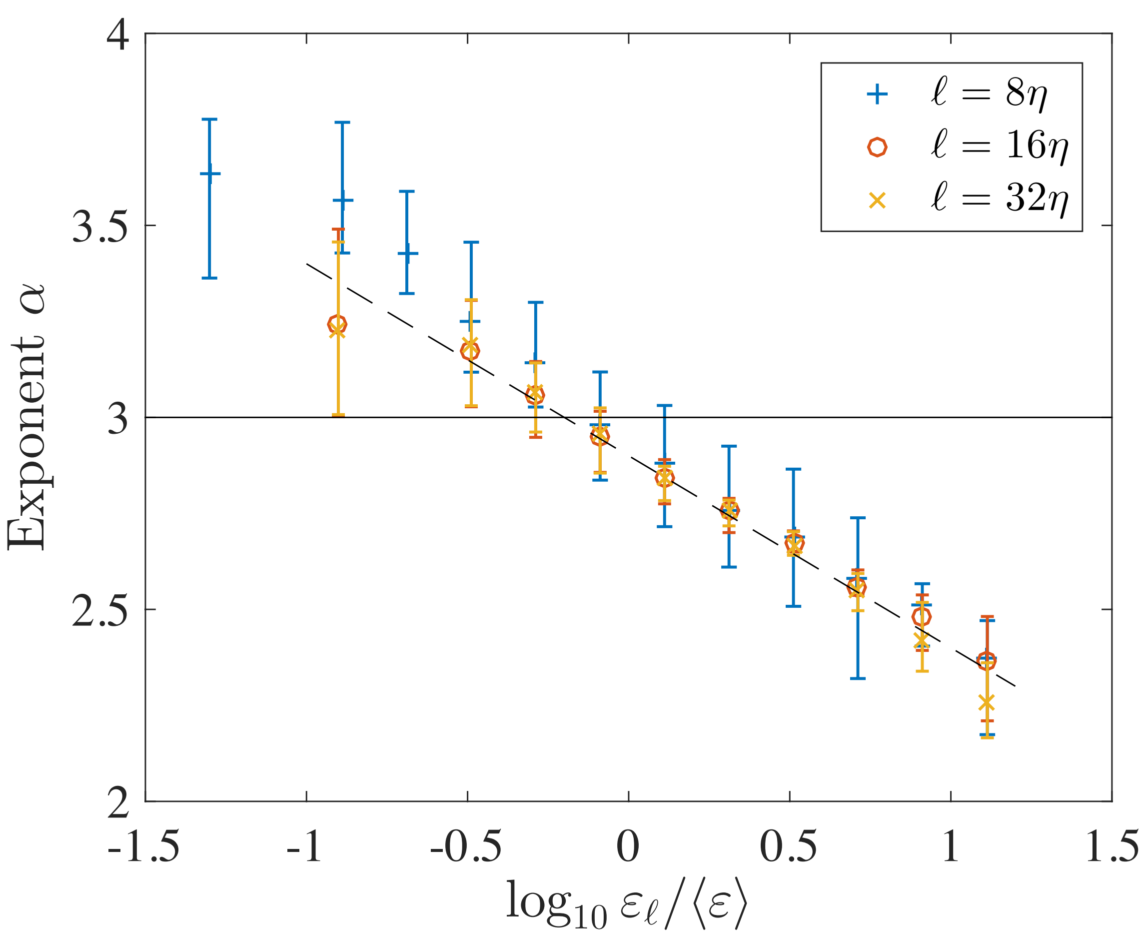
Stronger dispersion for stronger dissipative events

memory of initial/final turbulent state is kept

Irreversibility and dissipation

Fluctuations of dissipation alter the **nature** of relative dispersion!





$$\langle R^2(-t) \mid \varepsilon_\ell\rangle - \langle R^2(t) \mid \varepsilon_\ell\rangle \propto t^{\alpha(\varepsilon_\ell)}$$

Possible signature of intermittency on relative dispersion?

Summary

in the realm of turbulence research…

- **The Pandora-box effect**

  As expected, intricate relations between dissipation and Lagrangian time irreversibility

  The nature of dispersion seems itself affected by local fluctuations of dissipation.

  Partial lack of “ergodicity” for both dissipative-range and inertial-range separations due to long-range correlations of dissipation

- **The power-law effect**

  Several non trivial scaling arise

  Refined self-similarity? Possible interpretation from multifractal approaches or terms stemming from non-trivial cancellations (subleading)?

- **The still-a-lot-to-do effect**

  There is still a long way before interpreting observations in terms of conditions on physical admissibility.