INTERNATIONAL CENTRE *for* THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

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TOPOLOGICAL ASPECTS OF STRONG CORRELATIONS AND GAUGE THEORIES (ONLINE)

Efforts Towards the Quantum Simulation of Non-Abelian Gauge Theories

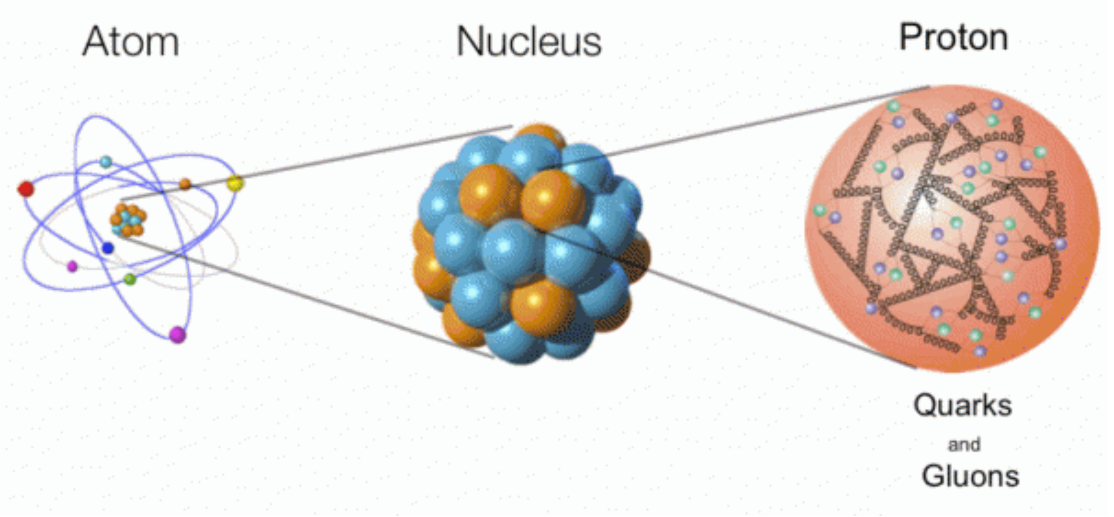
Indrakshi Raychowdhury

BITS-Pilani, K K Birla Goa Campus

10 September, 2021

Background:

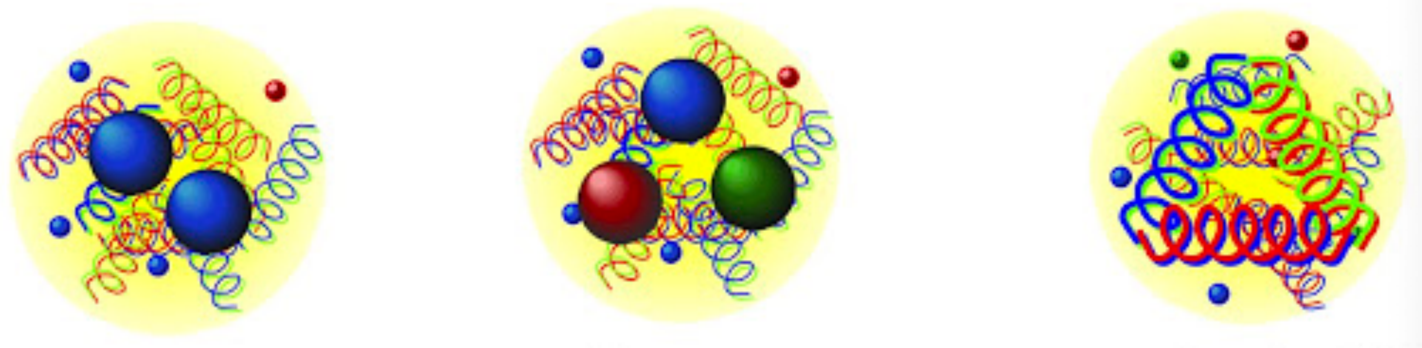
One of the four fundamental interaction of nature:
Strong Interaction



Quantum Chromodynamics

Quark Confinement?

Color-singlet composite particles:



LATTICE QCD

Largest Supercomputers

- LGT: Gauge theory formulated on space-time lattice (Wilson'74).
- Non-perturbative calculations using classical computational techniques: an extremely useful tool for nuclear and particle physics.
- Certain inaccessible regime: SIGN PROBLEM.
- Real time dynamics is non-trivial to compute/simulate in Euclidean formalism-Monte Carlo simulation.
- **Quantum Simulation is expected to be useful!**

The Pi0 cluster at Fermilab.

The 12s cluster at JLab.

and many more..

GOAL

Quantum simulating or quantum computing for Lattice QCD

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Classical Computation Era

Change of Paradigm

Quantum Computation Era

Our role

- Identifying the physics problem that would benefit from quantum computation
- Reformulating the problem suitable for quantum computation
- Propose and construct analog quantum simulators
- Constructing quantum algorithms for the particular problems

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Lattice gauge theory Computations

Classical Computation Era → ← Quantum Computation Era

- Requires different theoretical framework.
- Addressed different objectives
- Computational Methods are entirely different.

Ultimate goal: performing LATTICE-QCD calculations using Quantum Computer

Quantum Computing/Simulating QCD

Gauge theory,
SU(3) in 3+1 dimension

Too complicated to start with!

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Simpler, yet similar theories:

U(1) gauge theory: Quantum Electrodynamics (QED)

Yet, it provides a simple test bed for
novel analog quantum simulation proposals
as well as digital quantum simulation algorithms.

Hence, most popular!

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Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

Quantum-classical computation of Schwinger model dynamics using quantum computers

N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage

Towards analog quantum simulations of lattice gauge theories with trapped ions

Zohreh Davoudi,[1,2] Mohammad Hafezi,[3,4] Christopher Monroe,[3,5] Guido Pagano,[3,5,6] Alireza Seif,[3] and Andrew Shaw[1]

Quantum Algorithms for Simulating the Lattice Schwinger Model

Alexander F. Shaw[1,5], Pavel Lougovski[1], Jesse R. Stryker [2], and Nathan Wiebe[3,4]

and many more..

Experimental Demonstration:

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer 2016

Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

A scalable realization of local U(1) gauge invariance in cold atomic mixtures 2020

Alexander Mil[1,*], Torsten V. Zache[2], Apoorva Hegde[1], Andy Xia[1], Rohit P. Bhatt[1], Markus K. Oberthaler[1], Philipp Hauke[1,2,3], Jürgen Berges[2], Fred Jendrzejewski[1]

Observation of gauge invariance in a 71-site Bose–Hubbard quantum simulator 2020

Bing Yang, Hui Sun, Robert Ott, Han-Yi Wang, Torsten V. Zache, Jad C. Halimeh, Zhen-Sheng Yuan, Philipp Hauke & Jian-Wei Pan

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Talk by, Yuta Kikuchi

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$\mathbb{Z}_N$ gauge theory; $\mathbb{Z}_2$ gauge theory in 2+1 dimensions

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Experimental Demonstration:

Floquet approach to $\mathbb{Z}_2$ lattice gauge theories with ultracold atoms in optical lattices

Christian Schweizer, Fabian Grusdt, Moritz Berngruber, Luca Barbiero, Eugene Demler, Nathan Goldman, Immanuel Bloch & Monika Aidelsburger

2019

Realization of density-dependent Peierls phases to engineer quantized gauge fields coupled to ultracold matter

Frederik Görg, Kilian Sandholzer, Joaquín Minguzzi, Rémi Desbuquois, Michael Messer & Tilman Esslinger

2019

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Simplest, non-abelian gauge theory: SU(2) gauge theory

- **Captures all of the non-trivialities of actual QCD**

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Proposals for analog simulation:

PRL **110,** 125304 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Cold-Atom Quantum Simulator for SU(2) Yang-Mills Lattice Gauge Theory

Erez Zohar,[1] J. Ignacio Cirac,[2] and Benni Reznik[1]

PRL **110,** 125303 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Atomic Quantum Simulation of U(N) and SU(N) Non-Abelian Lattice Gauge Theories

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Constrained Dynamics via the Zeno Effect in Quantum Simulation: Implementing Non-Abelian Lattice Gauge Theories with Cold Atoms

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PRL **115,** 240502 (2015) PHYSICAL REVIEW LETTERS week ending 11 DECEMBER 2015

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A. Mezzacapo,[1,2] E. Rico,[1,3] C. Sabín,[4] I. L. Egusquiza,[5] L. Lamata,[1] and E. Solano[1,3]

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Cold Atom Quantum Simulator for String and Hadron Dynamics in Non-Abelian Lattice Gauge Theory

Raka Dasgupta[1] and Indrakshi Raychowdhury[2]

arXiv:2009.13969v1

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PHYSICAL REVIEW D **101,** 074512 (2020)

SU(2) non-Abelian gauge field theory in one dimension on digital quantum computers

Natalie Klco,[*] Martin J. Savage,[†] and Jesse R. Stryker[‡]

IQuS@UW-21-001

A Trailhead for Quantum Simulation of SU(3) Yang-Mills Lattice Gauge Theory in the Local Multiplet Basis

Anthony Ciavarella,[1,*] Natalie Klco,[2,†] and Martin J. Savage[1,‡]

SU(2) hadrons on a quantum computer

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PHYSICAL REVIEW D **104,** 034501 (2021)

SU(2) lattice gauge theory on a quantum annealer

Sarmed A Rahman, Randy Lewis, Emanuele Mendicelli, and Sarah Powell
Department of Physics and Astronomy, York University, Toronto, Ontario M3J 1P3, Canada

State of the art:

Not too many attempts!

Simplest, non-abelian gauge theory: **SU(2) gauge theory**

Why?

Proposals for analog simulation:

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Framework: Hamiltonian Formalism

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†

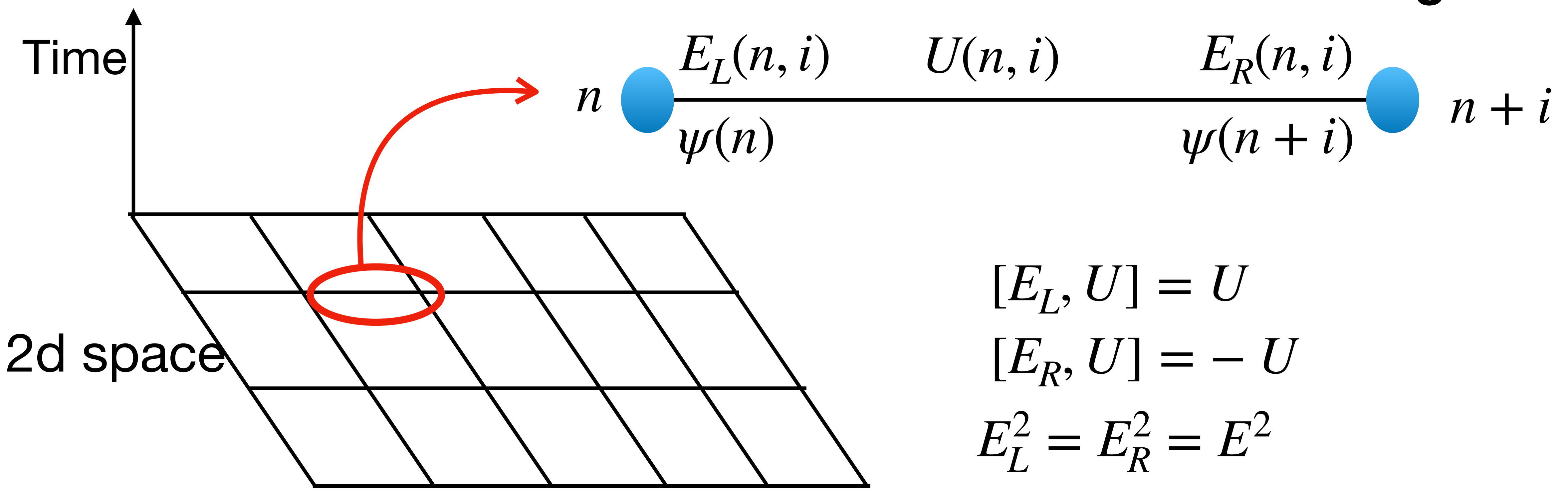
Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

Framework: Hamiltonian Formalism

Kogut-Susskind '74



$$[E_L, U] = U$$
$$[E_R, U] = -U$$
$$E_L^2 = E_R^2 = E^2$$

$$H = H_E + H_M + H_I + H_B$$

$$\frac{g^2 a}{2} \sum_{n,I} E^2(n,I)$$

$$m \sum (-1)^n \, \psi^\dagger(n)\psi(n)$$

Staggered fermion

$$\frac{1}{2a} \sum_{n,I} (-1)^n \, \psi^\dagger(n) U(n,I) \psi(n+I)$$

$$\frac{2a}{g^2} \sum_{plaquettes} \left[\mathrm{Tr} U_{plaquette} + h.c \right]$$

Framework: Hamiltonian Formalism

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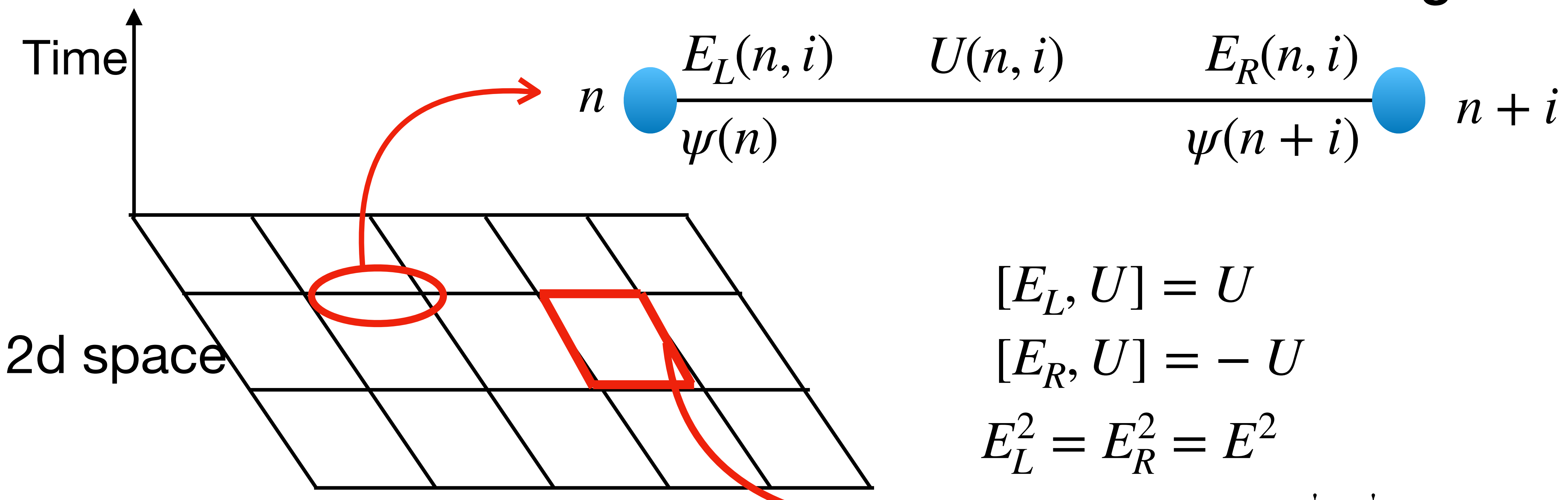

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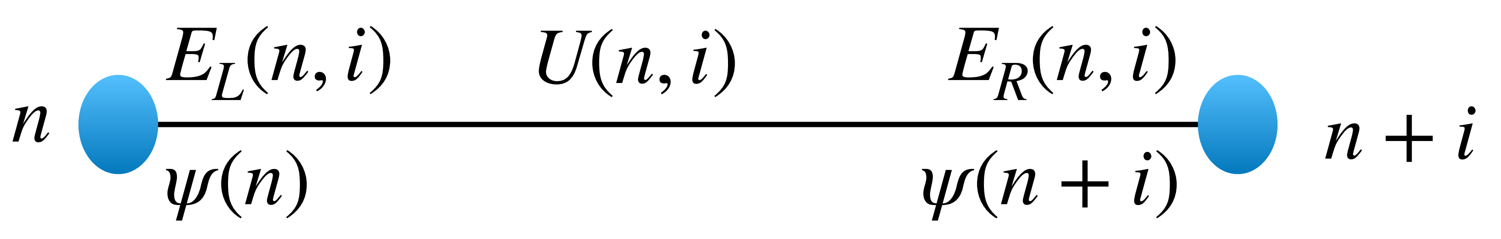

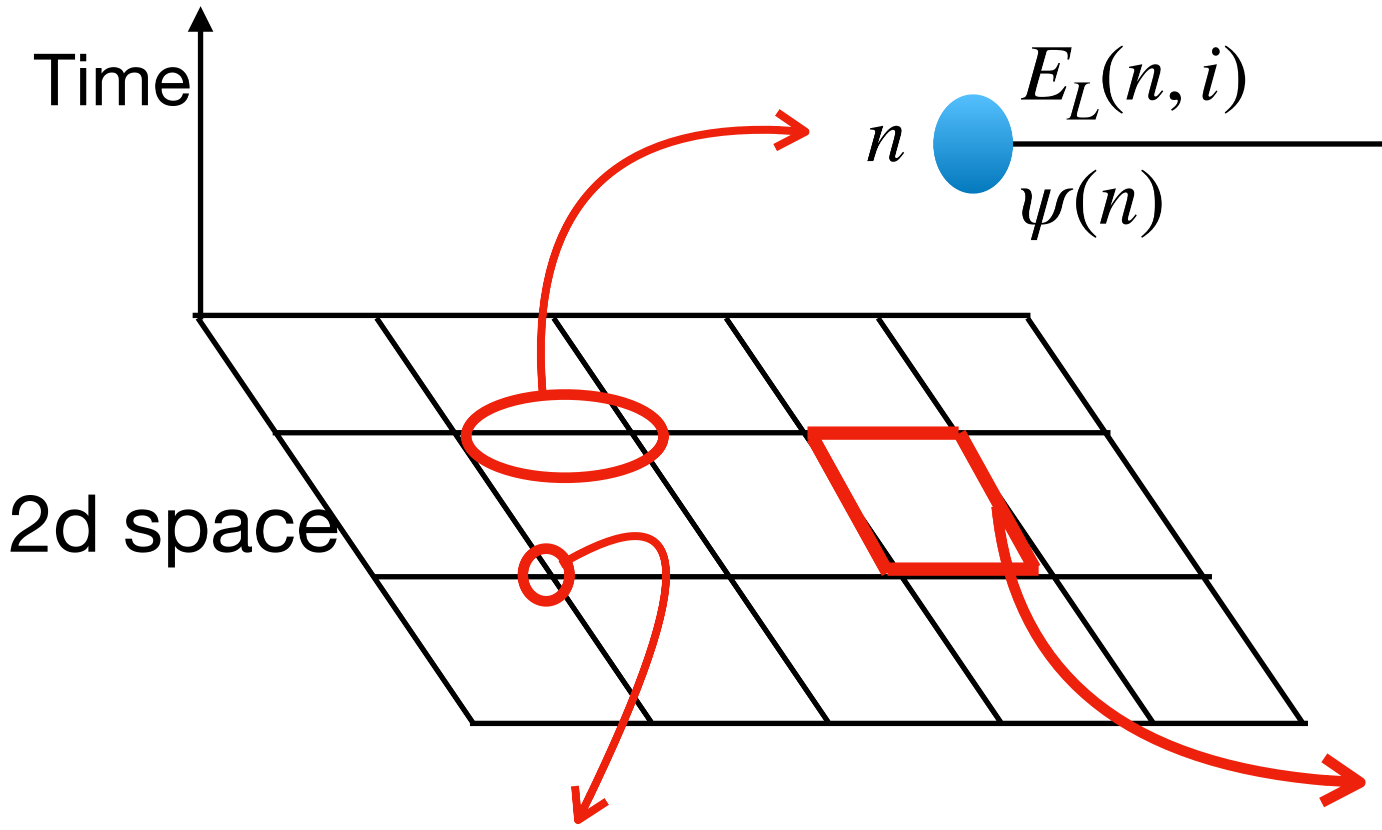

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Gauss' law constraint: $G(n)|\Psi_{phys}\rangle = 0$

$$G(n) = \sum_I \left[E_L(n,I) - E_R(n-I,I)\right] - \psi(n)^\dagger \psi(n)$$

$$H = H_E + H_M + H_I + H_B$$

$$H_E = \frac{g^2 a}{2} \sum_{n,I} E^2(n,I)$$

$$H_M = m \sum (-1)^n \, \psi^\dagger(n)\psi(n)$$

Staggered fermion

$$H_I = \frac{1}{2a} \sum_{n,I} (-1)^n \, \psi^\dagger(n) U(n,I) \psi(n+I)$$

$$H_B = \frac{2a}{g^2} \sum_{plaquettes} \left[\mathrm{Tr} U_{plaquette} + h.c\right]$$

Framework: Hamiltonian Formalism

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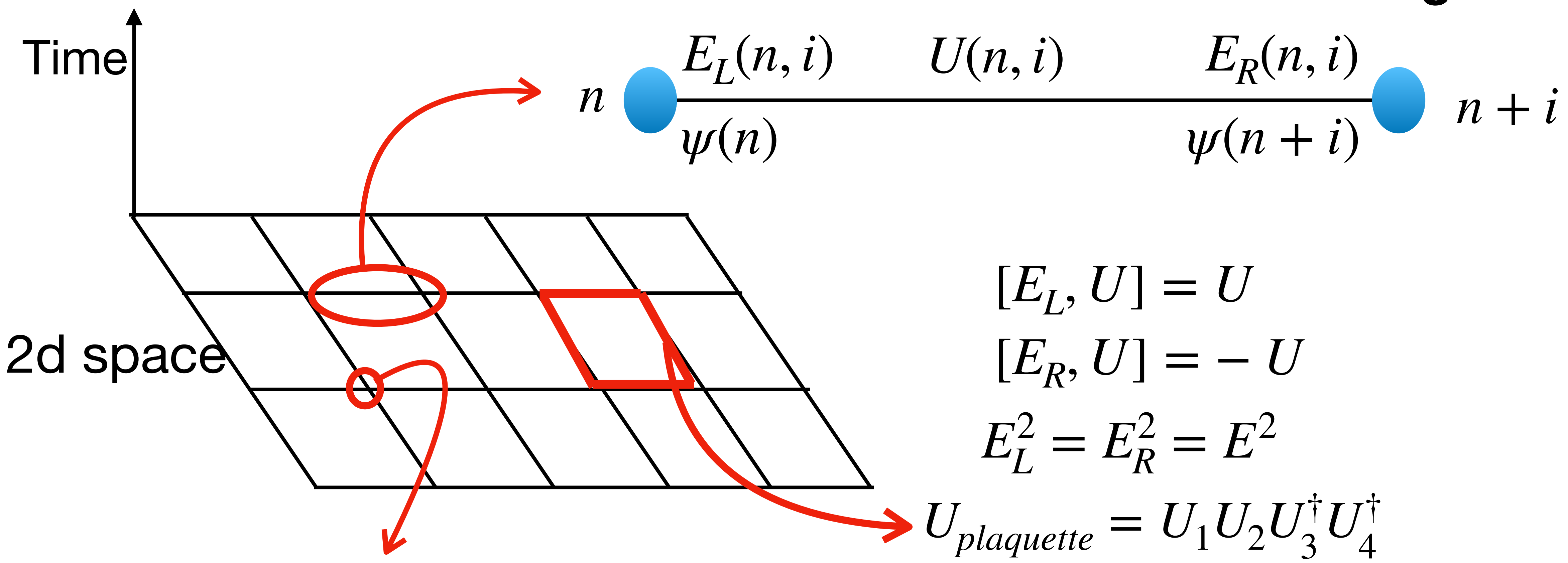

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$$U_{plaquette} = U_1 U_2 U_3^\dagger U_4^\dagger$$

Gauss' law constraint: $G(n)|\Psi_{phys}\rangle = 0$

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Strong Coupling Vacuum:
$(g \to \infty, a \to \text{finite})$

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Strong Coupling Basis:

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$$H = H_E + H_M + H_I + H_B$$

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$$m\sum (-1)^n\, \psi^\dagger(n)\psi(n)$$

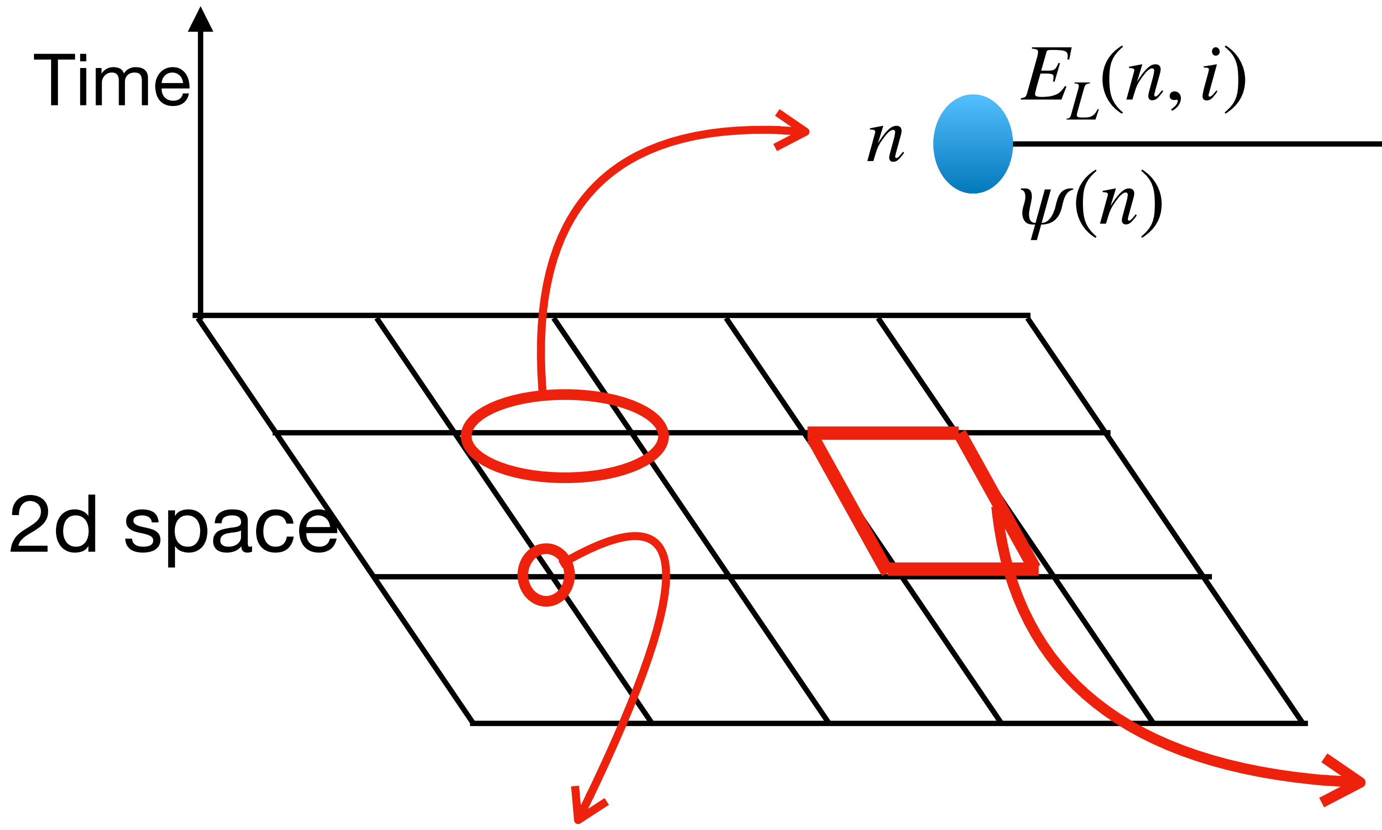
Staggered fermion

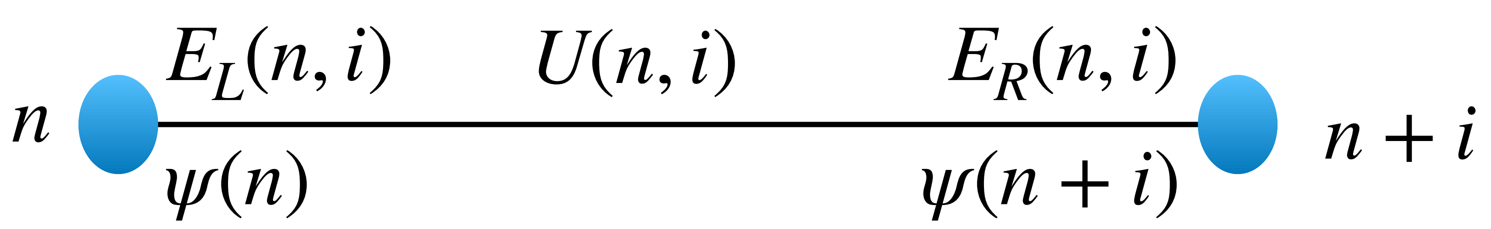
$$\frac{1}{2a}\sum_{n,I} (-1)^n\, \psi^\dagger(n) U(n,I) \psi(n+I)$$

$$\frac{2a}{g^2}\sum_{plaquettes}\left[\mathrm{Tr}U_{plaquette} + h.c\right]$$

Framework: Hamiltonian Formalism

Kogut-Susskind '74





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$$H_I: \quad \frac{1}{2a}\sum_{n,I} (-1)^n\, \psi^\dagger(n) U(n,I) \psi(n+I)$$

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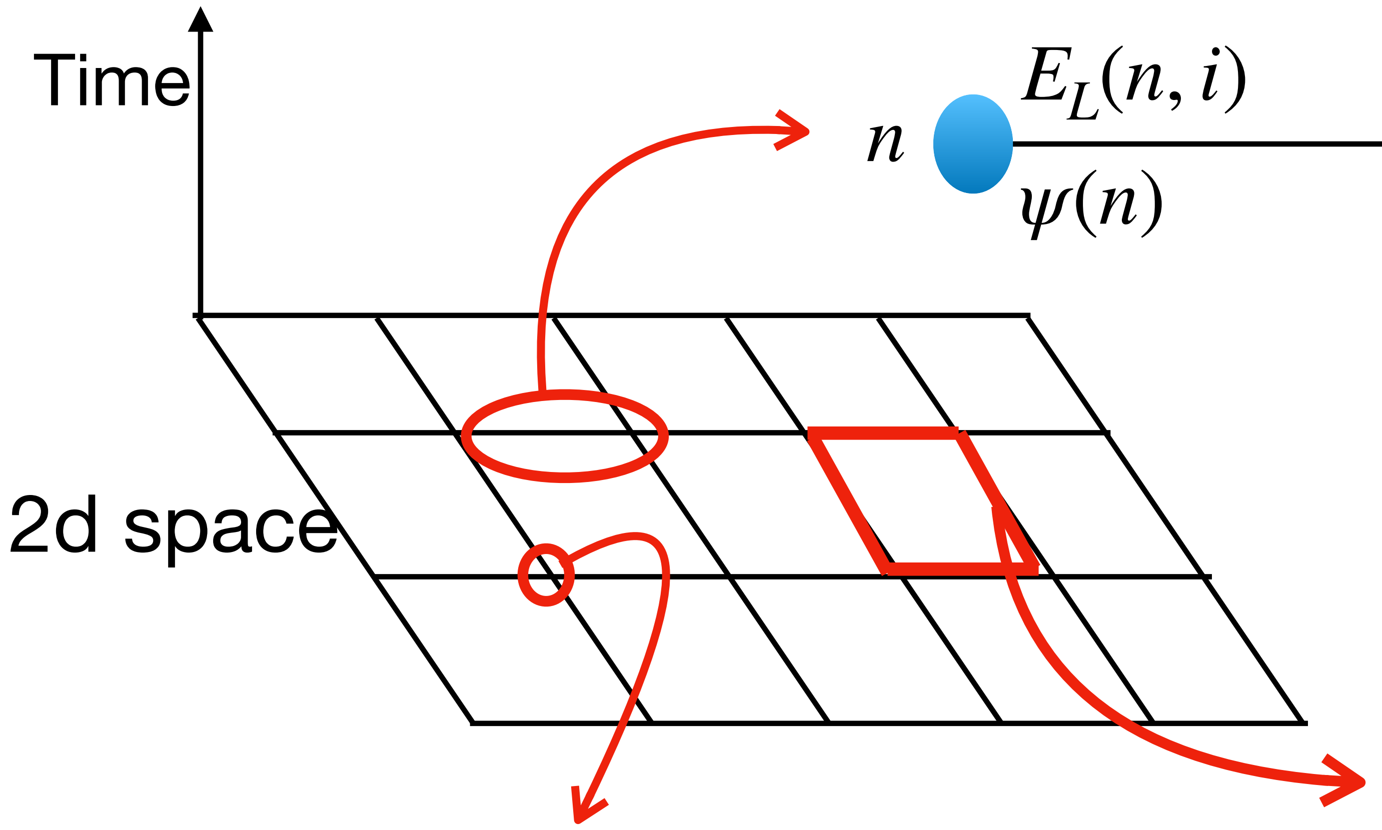

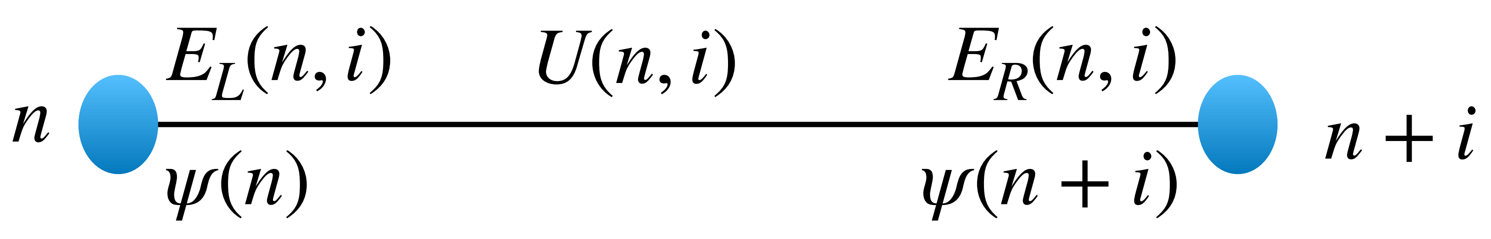

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Framework: Hamiltonian Formalism

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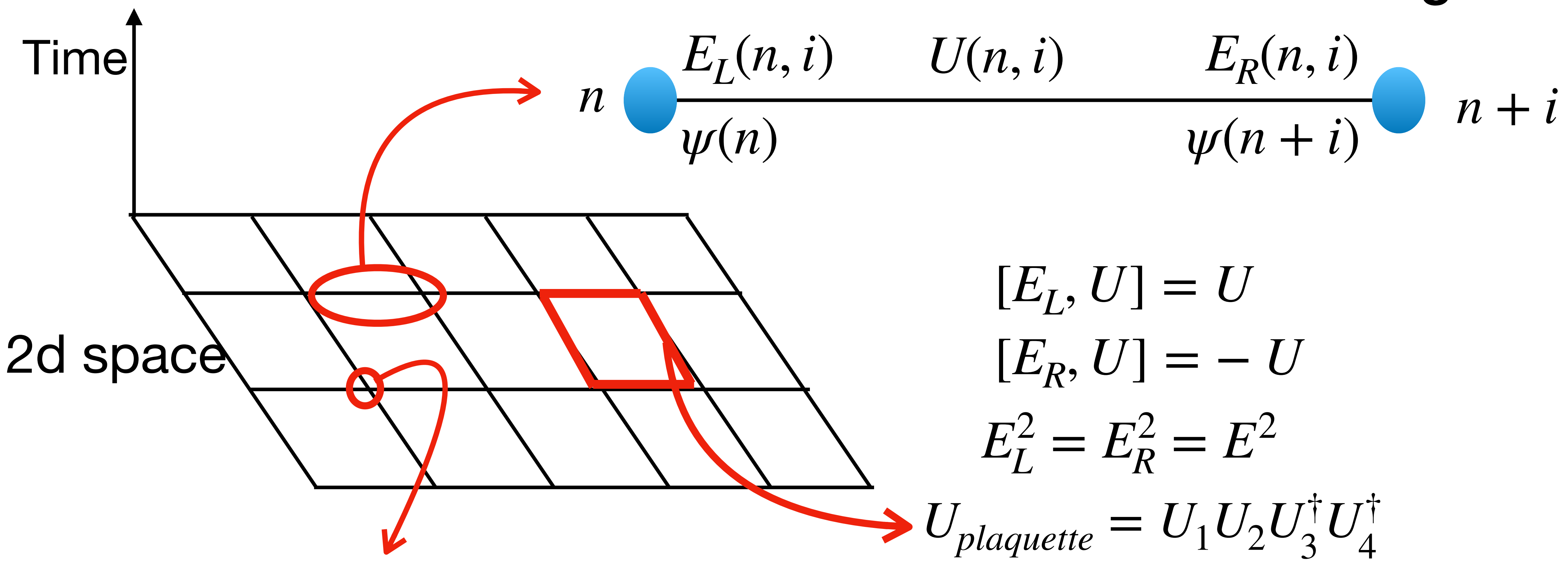

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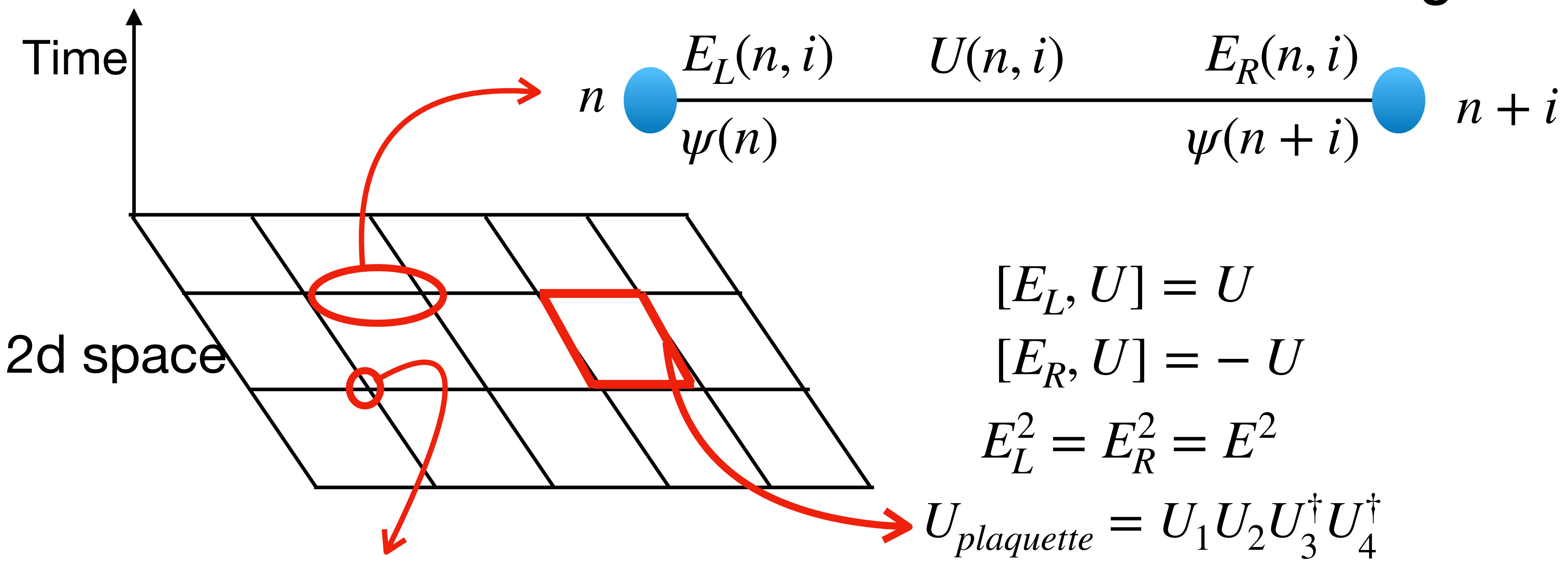

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SU(3): $a = 1,2,3,\ldots,8.$

Efforts in simulating KS Hamiltonian

IOP Publishing
Reports on Progress in Phy
Rep. Prog. Phys. 79 (2016) 014401 (30pp)
doi:10.1088/0034-4885/79/1/01

Report on Progress

Quantum simulations of lattice gauge theories using ultracold atoms in optical lattices

Erez Zohar[1], **J Ignacio Cirac**[1] **and Benni Reznik**[2]

[1] Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straß e 1, 85748 Garching, Germany
[2] School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel-Aviv 69978, Israel

PRL 110, 125304 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Cold-Atom Quantum Simulator for SU(2) Yang-Mills Lattice Gauge Theory

Erez Zohar,[1] J. Ignacio Cirac,[2] and Benni Reznik[1]

Uses prepotential formalism for gauge fields

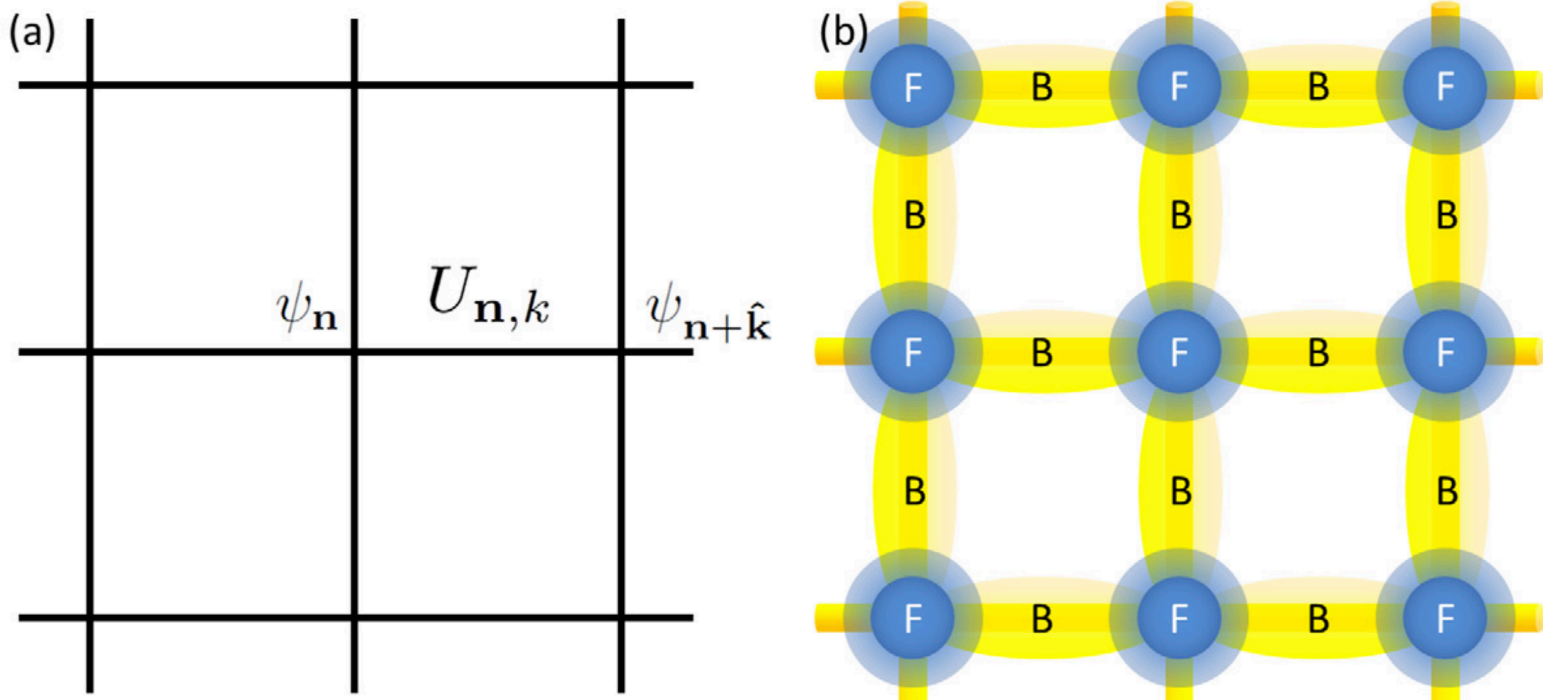


Figure 5. (a) The lattice structure. The gauge degrees of freedom occupy the links, whereas the fermionic matter the vertices. This structure is kept in the atomic quantum simulator whose schematic plot is presented in (b): the circles and ellipses represent the minima of the optical potentials, Bosonic (B, yellow) on the links, and Fermionic (F, blue) on the vertices.

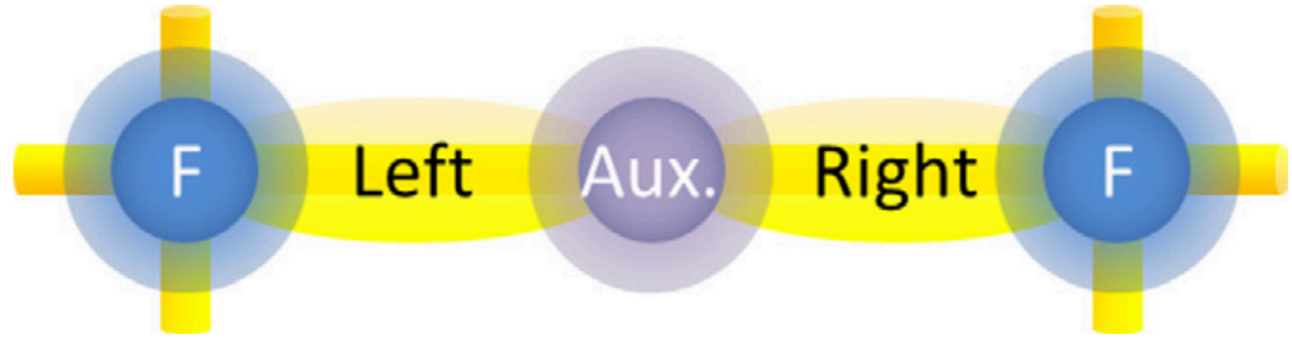


Figure 10. Structure of the non-Abelian lattice atomic simulator in the prepotential method: the simulated link is decomposed into two pieces, tied together by an auxiliary fermion.

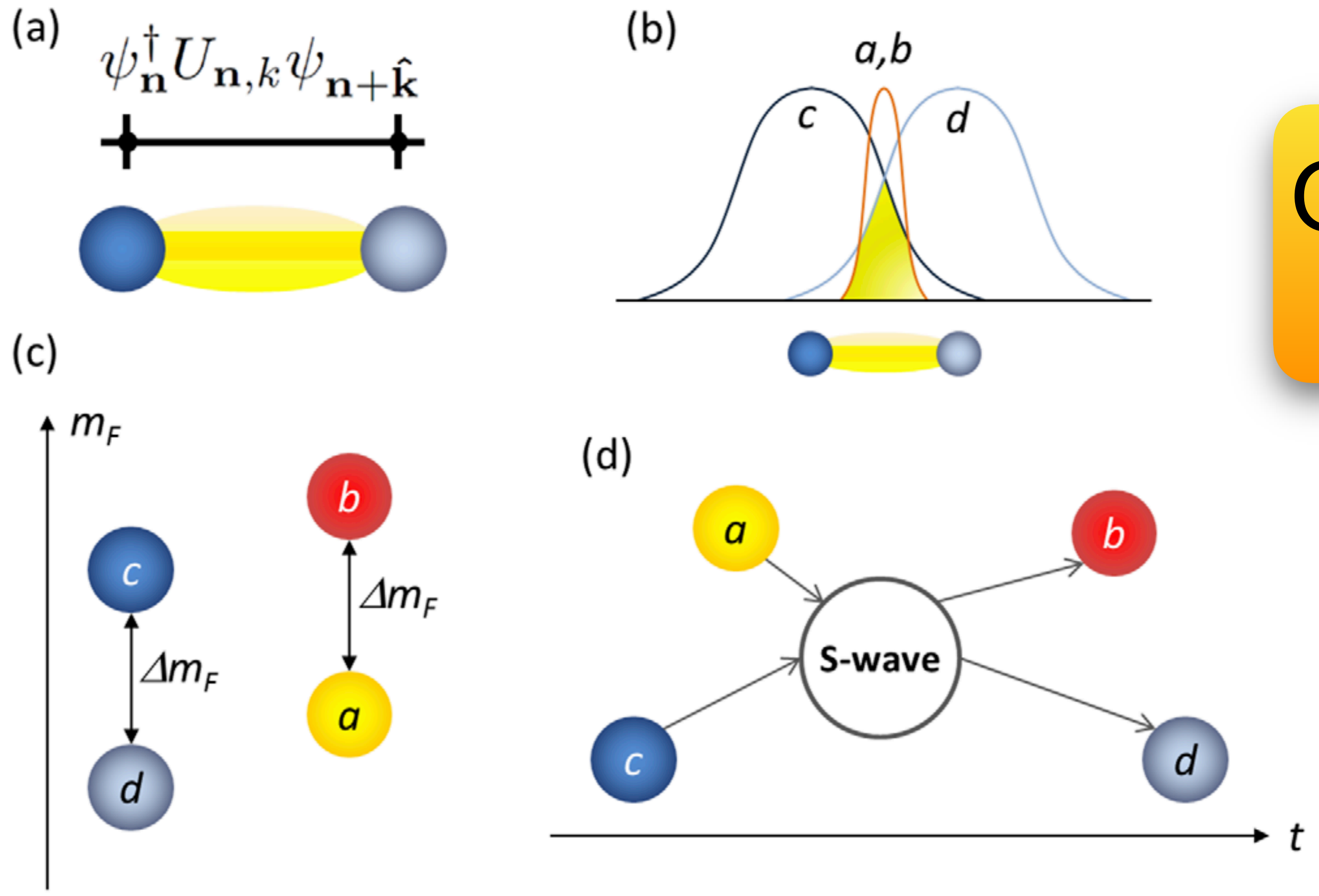


Figure 9. 'Exact gauge invariance'—the Schwinger model example. (a) The desired interactions (9) are obtained as the *F* preserving boson–fermion scattering processes, utilising the overlap of bosonic and fermionic Wannier functions on the link (b). By appropriately choosing the hyperfine levels representing the bosons and the fermions (c), only the gauge invariant interactions (128) of (113) are possible (d).

Gauge invariance ≡ conservation of angular momentum in a scattering event

Plaquette term is simulated at the 4th order effective Hamiltonian of the simulating system

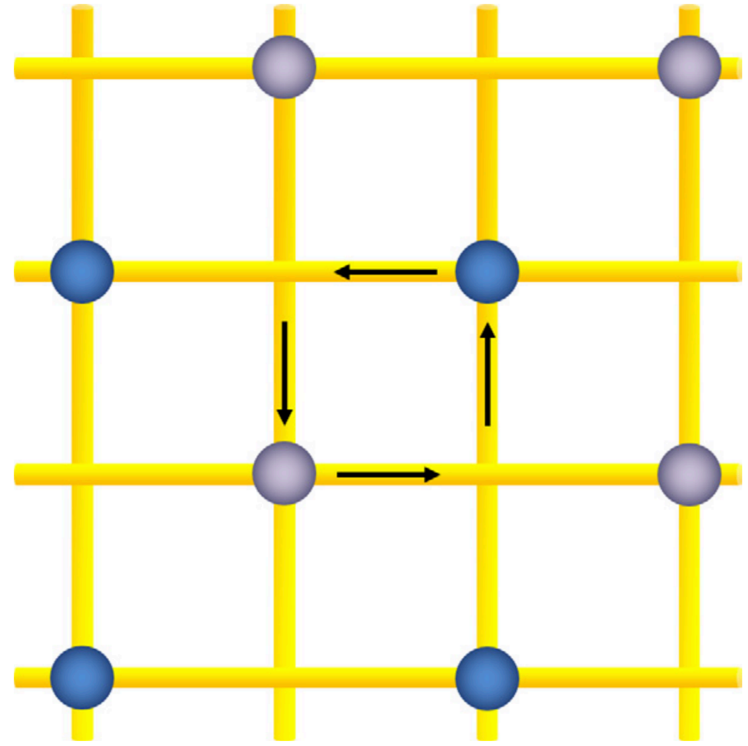


Figure 13. The loop method: heavy, ancillary fermions (of two types, represented by the colour) are constrained energetically to occupy certain vertices. Virtual loops of such fermions (as in the middle plaquette), carrying the gauge degrees of freedom along with them, generate the plaquette terms. For further details refer to [121].

Efforts in simulating QLM

PRL 110, 125303 (2013) PHYSICAL REVIEW LETTERS week ending 22 MARCH 2013

Atomic Quantum Simulation of U(N) and SU(N) Non-Abelian Lattice Gauge Theories

D. Banerjee,[1] M. Bögli,[1] M. Dalmonte,[2] E. Rico,[2,3] P. Stebler,[1] U.-J. Wiese,[1] and P. Zoller[2,3]

[1]*Albert Einstein Center, Institute for Theoretical Physics, Bern University, CH-3012 Bern, Switzerland*

[2]*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria*

[3]*Institute for Theoretical Physics, Innsbruck University, A-6020 Innsbruck, Austria*

(Received 12 November 2012; published 21 March 2013)

Using ultracold alkaline-earth atoms in optical lattices, we construct a quantum simulator for U(N) and SU(N) lattice gauge theories with fermionic matter based on quantum link models. These systems share qualitative features with QCD, including chiral symmetry breaking and restoration at nonzero temperature or baryon density. Unlike classical simulations, a quantum simulator does not suffer from sign problems and can address the corresponding chiral dynamics in real time.

DOI: 10.1103/PhysRevLett.110.125303 PACS numbers: 67.85.−d, 11.15.Ha, 37.10.Vz, 75.10.Jm

Quantum Link Model

gauge theory with finite dimensional Hilbert space

Qualitative proposal for the simulation

Not implemented experimentally

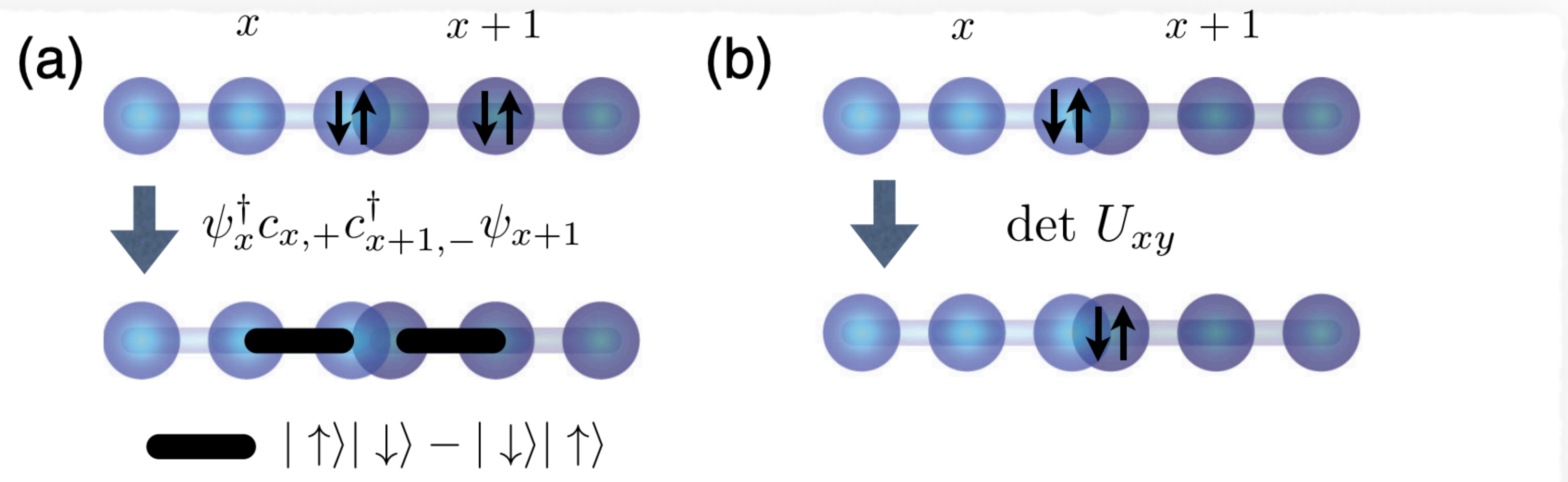
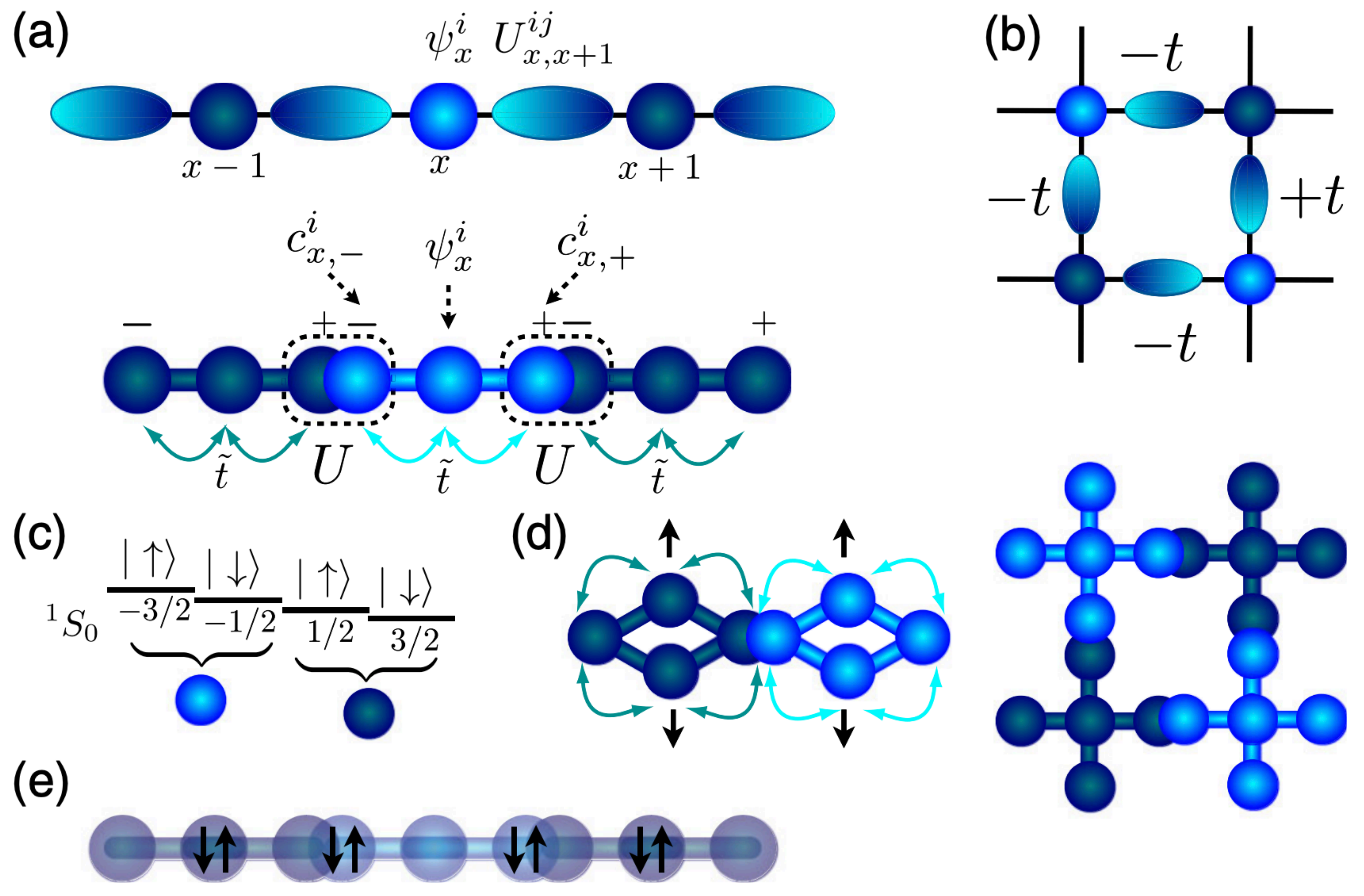


FIG. 2 (color online). Dynamical processes in U(2) QLMs with $\mathcal{N} = 2$. (a) Matter-gauge interaction as correlated hopping of quarks and rishons. Starting with a configuration of site singlets, the matter-gauge interaction converts them into nearest-neighbor singlets, keeping the rishon number per link constant. (b) The determinant term corresponds to two-body hopping of both rishons on the link.



- Gauge invariant initial state,
- Hamiltonian is gauge invariant,
- Imperfections may take the system away from gauge invariance gauge ~10%

Efforts in simulating QLM

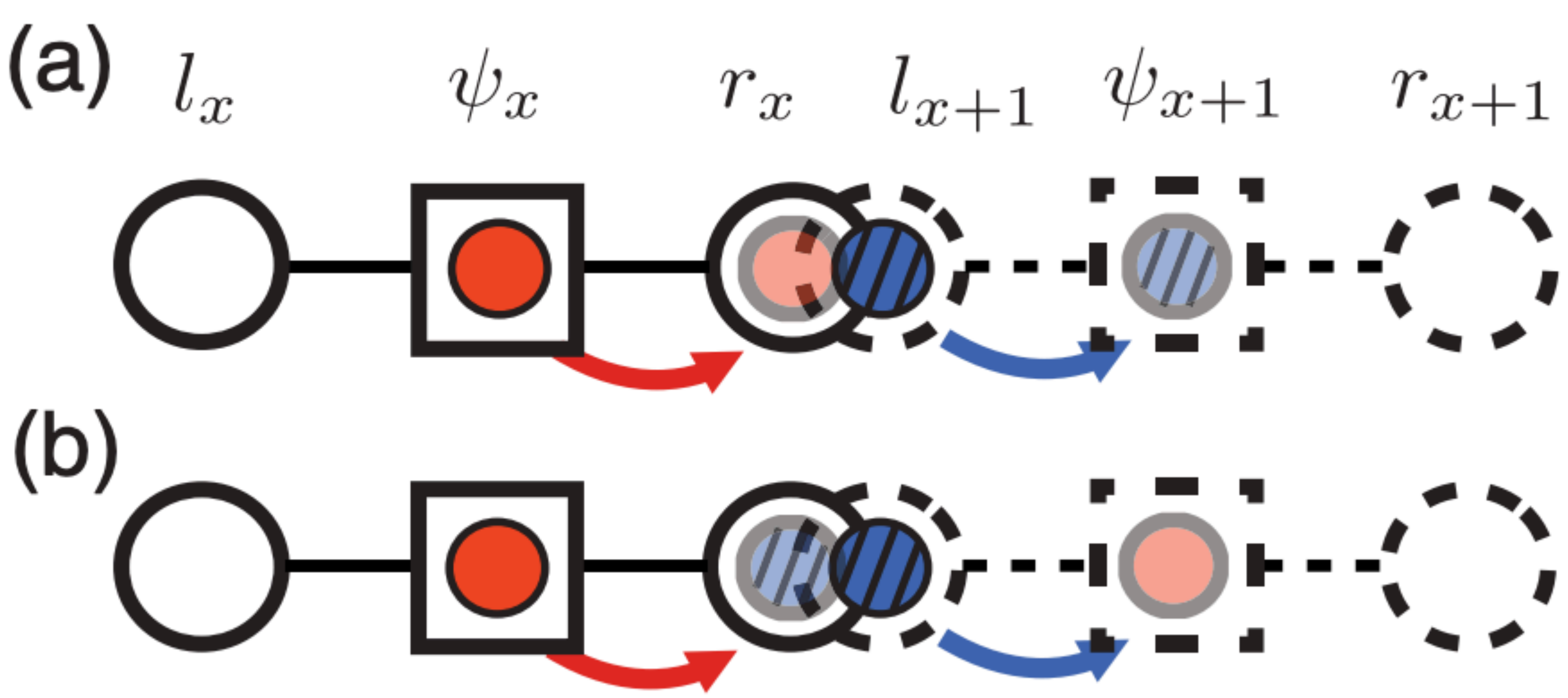
PRL 112, 120406 (2014) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2014

Constrained Dynamics via the Zeno Effect in Quantum Simulation: Implementing Non-Abelian Lattice Gauge Theories with Cold Atoms

K. Stannigel,[1] P. Hauke,[1,*] D. Marcos,[1] M. Hafezi,[2] S. Diehl,[1,3] M. Dalmonte,[1,3,†] and P. Zoller[1,3]
[1]Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, 6020 Innsbruck, Austria
[2]Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA
[3]Institute for Theoretical Physics, University of Innsbruck, 6020 Innsbruck, Austria
(Received 2 August 2013; published 26 March 2014)

We show how engineered classical noise can be used to generate constrained Hamiltonian dynamics in atomic quantum simulators of many-body systems, taking advantage of the continuous Zeno effect. After discussing the general theoretical framework, we focus on applications in the context of lattice gauge theories, where imposing exotic, quasilocal constraints is usually challenging. We demonstrate the effectiveness of the scheme for both Abelian and non-Abelian gauge theories, and discuss how engineering dissipative constraints substitutes complicated, nonlocal interaction patterns by global coupling to laser fields.

DOI: 10.1103/PhysRevLett.112.120406 PACS numbers: 03.65.Xp, 11.15.Ha, 37.10.Jk



The matter/gauge-field coupling corresponds to a simultaneous color-conserving tunneling of one fermion at site x to the link x, $x+1$ (red) and a rishon at x, $x+1$ to the site $x+1$ (blue, dashed).

- Implementing gauge symmetry
- Protecting gauge symmetry
- Scaling and imperfections

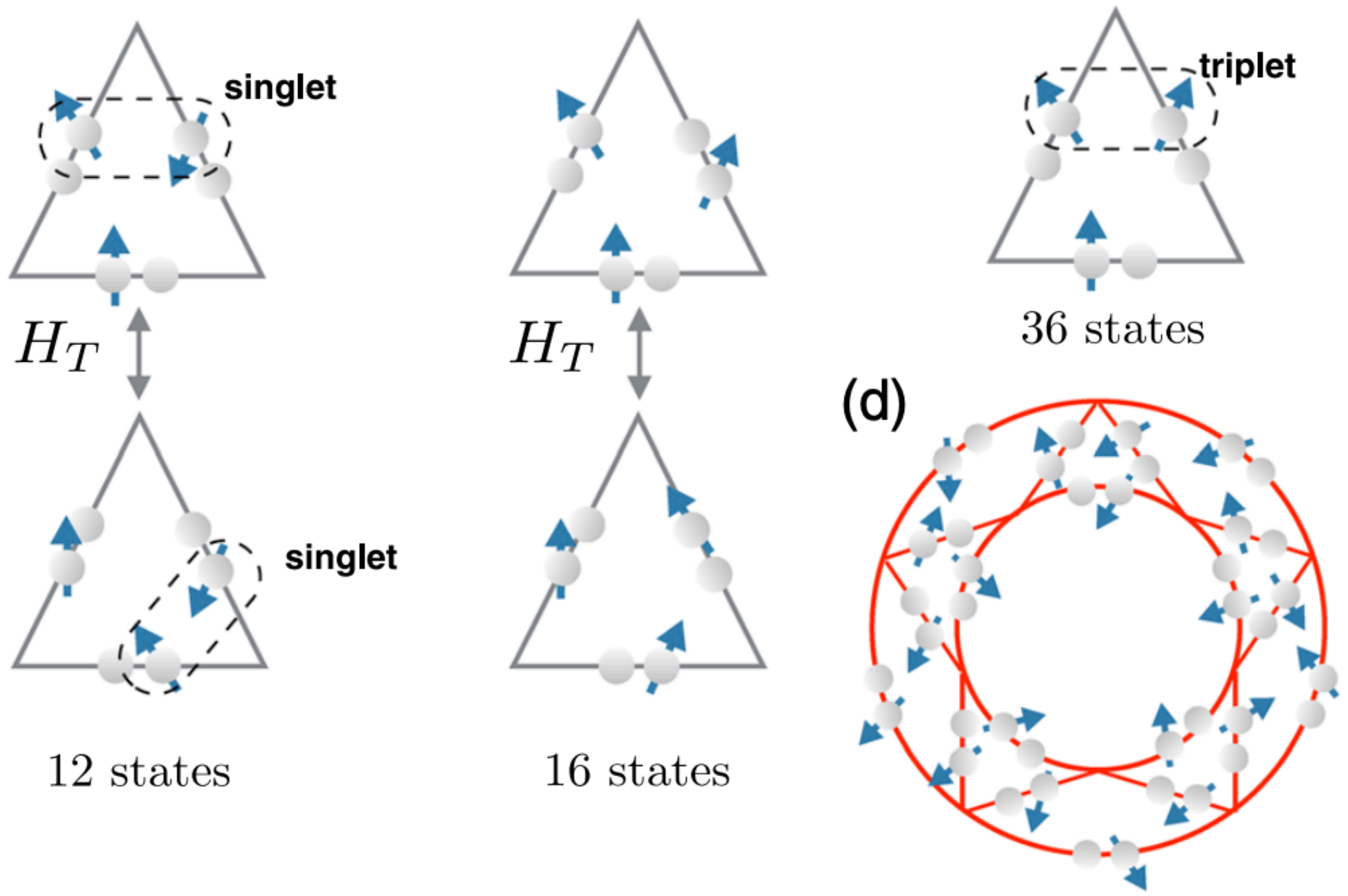
Other efforts:

PRL **115,** 240502 (2015) PHYSICAL REVIEW LETTERS week ending 11 DECEMBER 2015

Non-Abelian SU(2) Lattice Gauge Theories in Superconducting Circuits

A. Mezzacapo,[1,2] E. Rico,[1,3] C. Sabín,[4] I. L. Egusquiza,[5] L. Lamata,[1] and E. Solano[1,3]
[1]*Department of Physical Chemistry, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain*
[2]*IBM T.J. Watson Research Center, Yorktown Heights, New York 10598, USA*
[3]*IKERBASQUE, Basque Foundation for Science, Maria Diaz de Haro 3, 48013 Bilbao, Spain*
[4]*School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, United Kingdom*
[5]*Department of Theoretical Physics and History of Science, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain*
(Received 26 May 2015; published 9 December 2015)

We propose a digital quantum simulator of non-Abelian pure-gauge models with a superconducting circuit setup. Within the framework of quantum link models, we build a minimal instance of a pure SU(2) gauge theory, using triangular plaquettes involving geometric frustration. This realization is the least demanding, in terms of quantum simulation resources, of a non-Abelian gauge dynamics. We present two superconducting architectures that can host the quantum simulation, estimating the requirements needed to run possible experiments. The proposal establishes a path to the experimental simulation of non-Abelian physics with solid-state quantum platforms.



Other efforts:

ARTICLE

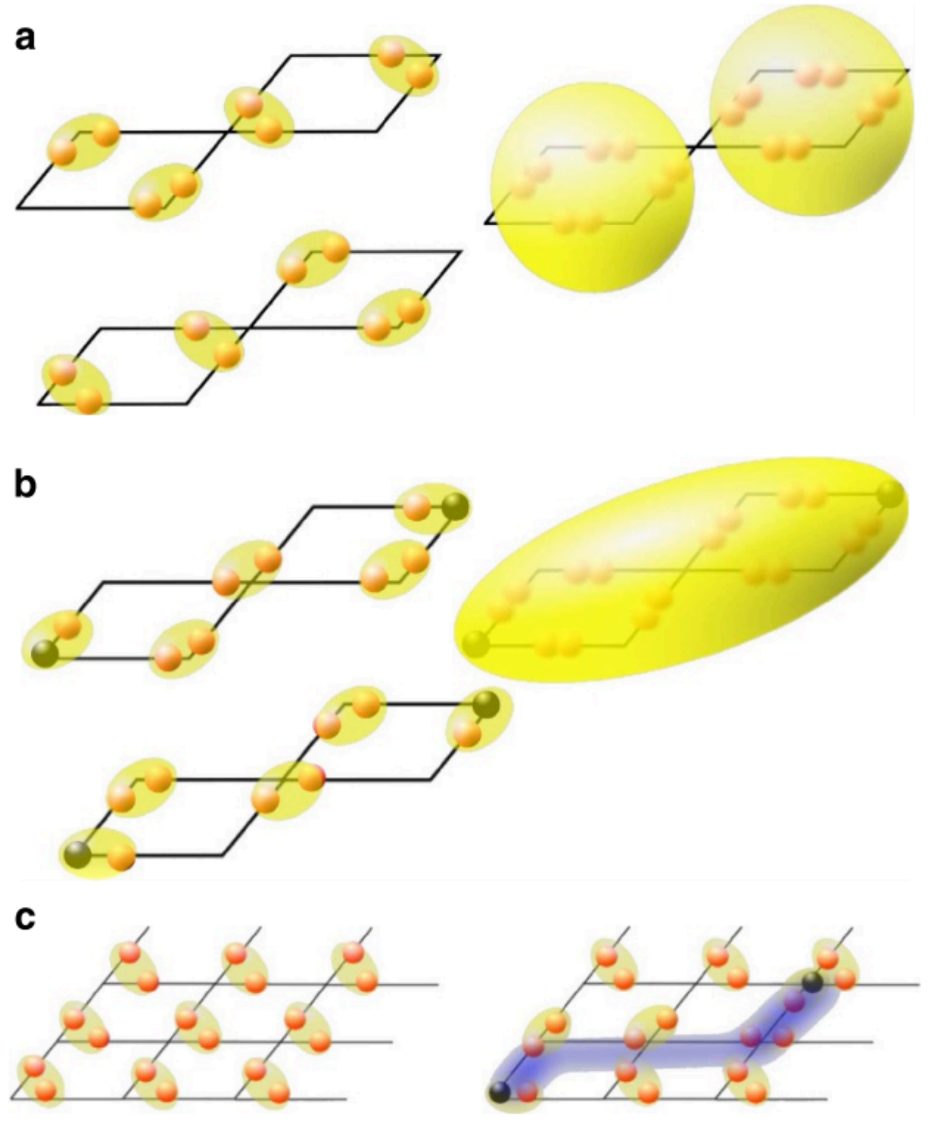
Received 28 Nov 2012 | Accepted 16 Sep 2013 | Published 28 Oct 2013

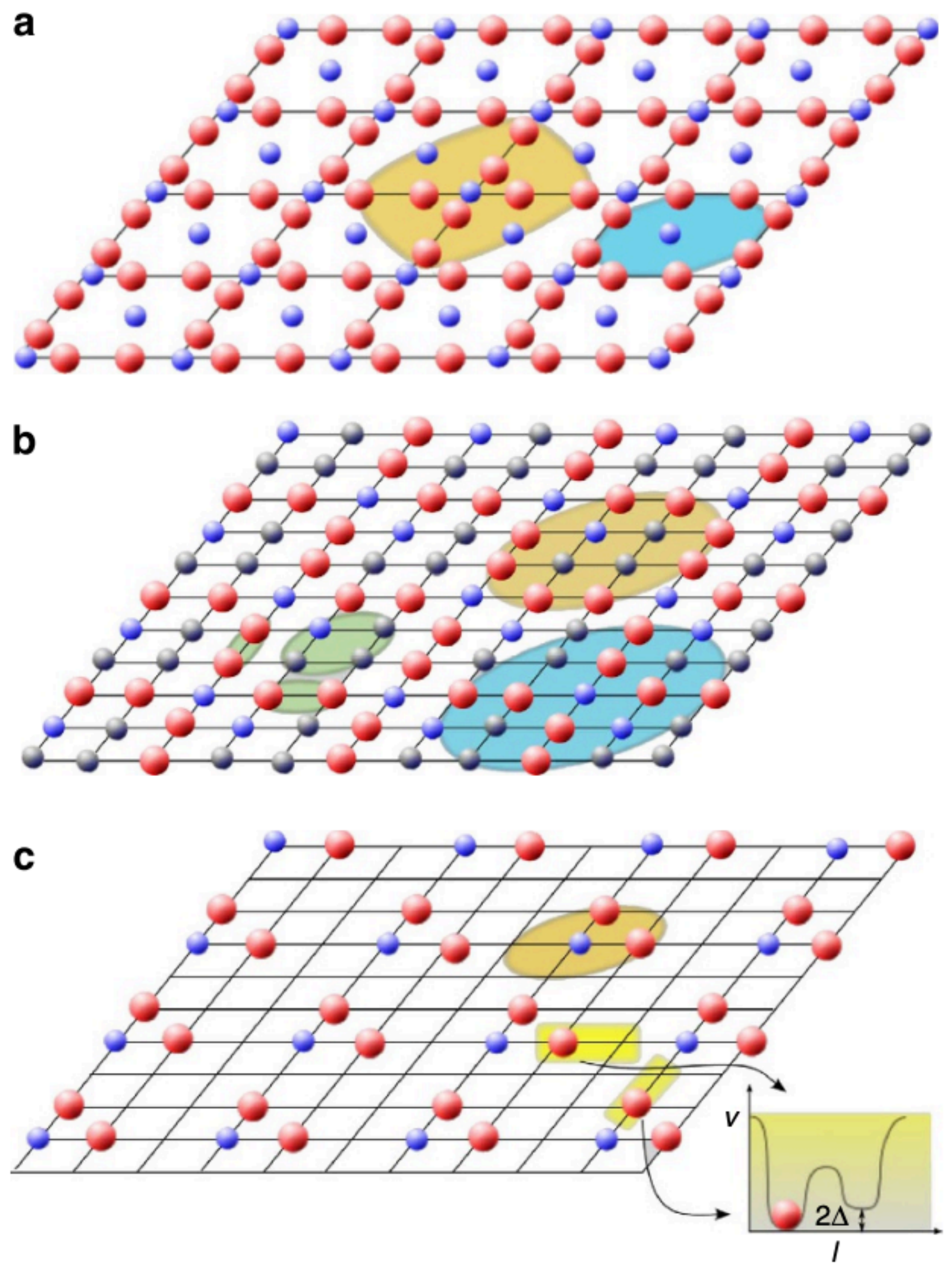
DOI: 10.1038/ncomms3615

Simulation of non-Abelian gauge theories with optical lattices

L. Tagliacozzo[1], A. Celi[1], P. Orland[2], M.W. Mitchell[1,3] & M. Lewenstein[1,3]

Many phenomena occurring in strongly correlated quantum systems still await conclusive explanations. The absence of isolated free quarks in nature is an example. It is attributed to quark confinement, whose origin is not yet understood. The phase diagram for nuclear matter at general temperatures and densities, studied in heavy-ion collisions, is not settled. Finally, we have no definitive theory of high-temperature superconductivity. Though we have theories that could underlie such physics, we lack the tools to determine the experimental consequences of these theories. Quantum simulators may provide such tools. Here we show how to engineer quantum simulators of non-Abelian lattice gauge theories. The systems we consider have several applications: they can be used to mimic quark confinement or to study dimer and valence-bond states (which may be relevant for high-temperature superconductors).





Preliminary proposals

Other efforts:

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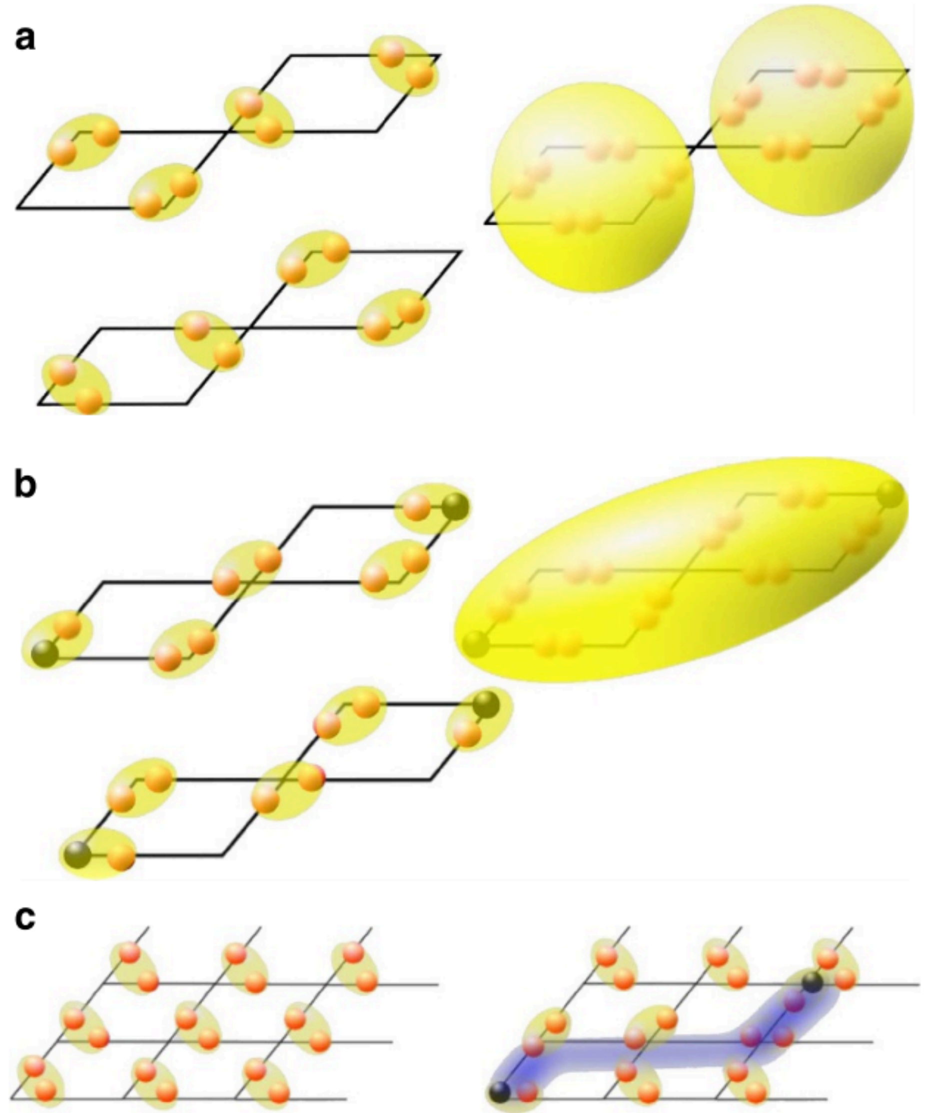
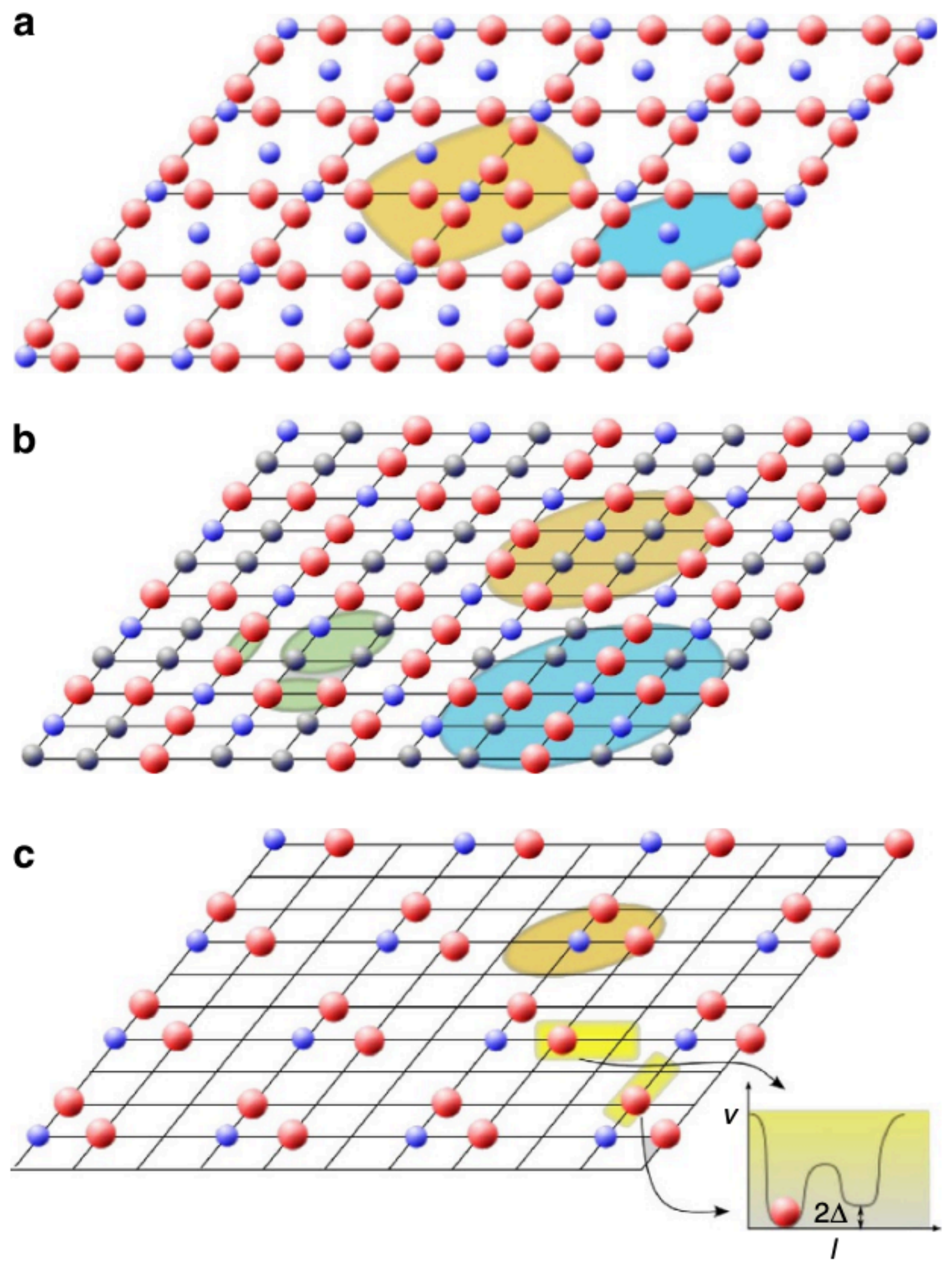




Preliminary proposals

Way out?

arXiv:2009.11802v1

UMD-PP-020-6

Search for Efficient Formulations for Hamiltonian Simulation of non-Abelian Lattice Gauge Theories

Zohreh Davoudi,[1,2] Indrakshi Raychowdhury,[1] and Andrew Shaw[1]

Removing staggered fermionic matter in $U(N)$ and $SU(N)$ lattice gauge theories

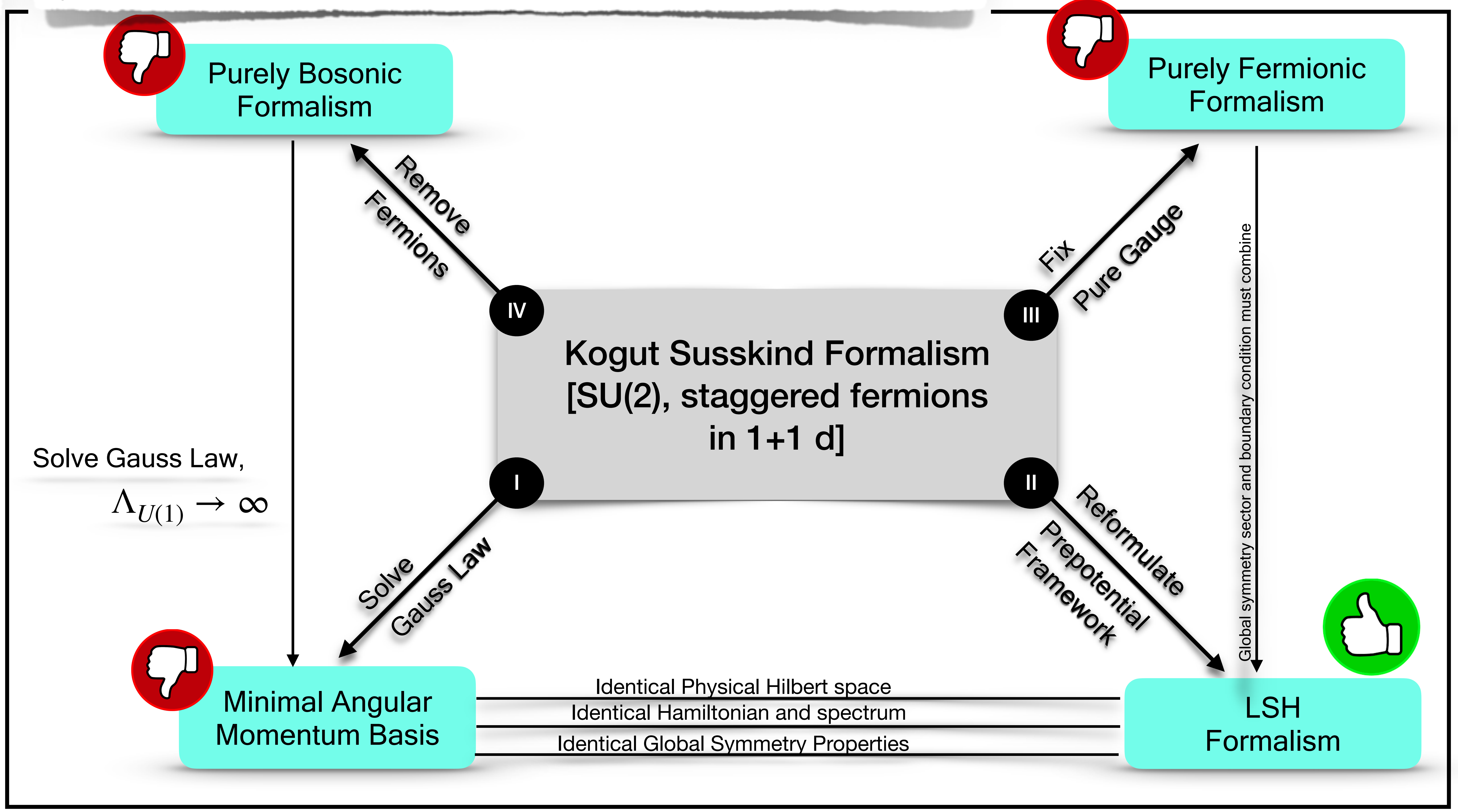
Erez Zohar and J. Ignacio Cirac

Phys. Rev. D **99**, 114511 – Published 28 June 2019

Collaborators:

Zohreh Davoudi, UMD

Andrew Shaw, UMD



Another (also most popular) candidate:
Quantum Link Model

QCD as a quantum link model

R. Brower, S. Chandrasekharan, and U.-J. Wiese

Phys. Rev. D **60**, 094502 – Published 27 September 1999

SU(2) rishon representation of gauge fields

Produces a different spectrum.

Prepotential Formulation of Gauge Theories

Collaborators:

Manu Mathur, SNBNCBS, India

Ramesh Anishetty, IMSc, India

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JPA 2009; JPA 2010; JMP 2009; JMP 2010; JMP 2011
***IR**, PhD Thesis, 2014;*
*Ramesh Anishetty, **IR**, PRD 2014;*
***IR**, arXiv: 1507.07305; EPJC 2019;*

Reformulation of the original Kogut-Susskind Formalism in terms of Schwinger bosons

Formulated for SU(2), SU(3) and arbitrary SU(N)

Formulated for any dimension

GAUGE INVARIANCE + PROLIFERATION OF LOOP DEGREES OF FREEDOM

Describes dynamics of only physical degrees of freedom

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(former grad student at INT), UMD

Reformulation of the original Kogut-Susskind Formalism in terms of Schwinger bosons

Formulated for SU(2), SU(3) and arbitrary SU(N)

Formulated for any dimension

GAUGE INVARIANCE + PROLIFERATION OF LOOP DEGREES OF FREEDOM

Describes dynamics of only physical degrees of freedom

Simplest case: SU(2) theory in 1+1 and 2+1 dimensions

Loop String Hadron (LSH) Formulation

PHYSICAL REVIEW D **101,** 114502 (2020)

Loop, string, and hadron dynamics in SU(2) Hamiltonian lattice gauge theories

Indrakshi Raychowdhury[1,*] and Jesse R. Stryker[2,†]

[1]*Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

[2]*Institute for Nuclear Theory, University of Washington, Seattle, Washington, DC 98195, USA*

(Received 24 December 2018; revised manuscript received 26 March 2020; accepted 4 May 2020; published 8 June 2020)

The question of how to efficiently formulate Hamiltonian gauge theories is experiencing renewed interest due to advances in building quantum simulation platforms. We introduce a reformulation of an SU(2) Hamiltonian lattice gauge theory—a loop-string-hadron (LSH) formulation—that describes dynamics directly in terms of its loop, string, and hadron degrees of freedom, while alleviating several disadvantages of quantum simulating the Kogut-Susskind formulation. This LSH formulation transcends the local loop formulation of $d+1$-dimensional lattice gauge theories by incorporating staggered quarks, furnishing the algebra of gauge-singlet operators, and being used to reconstruct dynamics between states that have Gauss's law built in to them. LSH operators are then factored into products of "normalized" ladder operators and diagonal matrices, priming them for classical or quantum information processing. Self-contained expressions of the Hamiltonian are given up to $d=3$. The LSH formalism makes little use of structures specific to SU(2), and its conceptual clarity makes it an attractive approach to apply to other non-Abelian groups like SU(3).

DOI: 10.1103/PhysRevD.101.114502

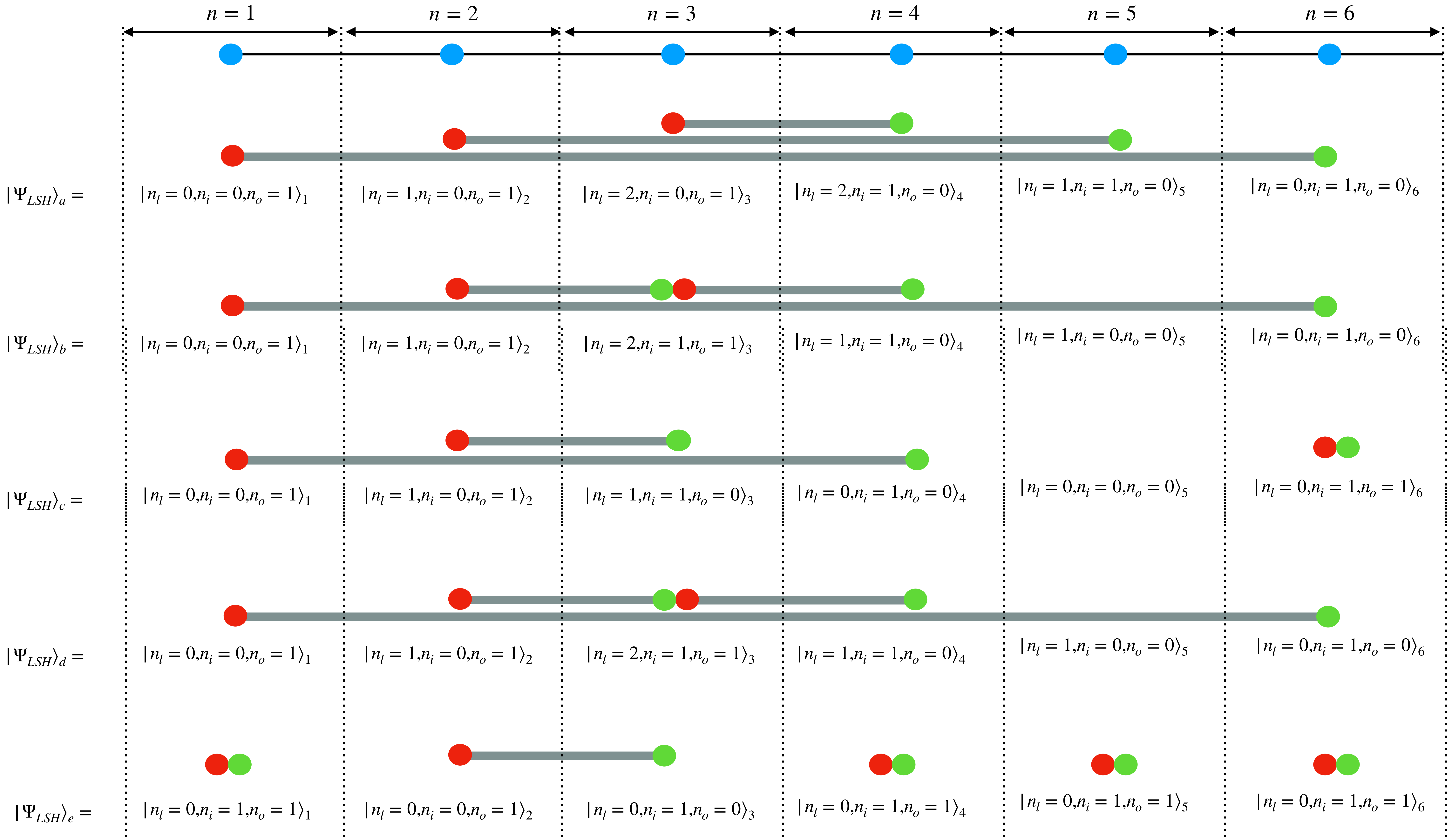
LSH Formulation: key ingredients

Local SU(2) invariant Hilbert space

Local constraint on each link: Abelian Gauss' law

Pictorially global LSH states in 1d

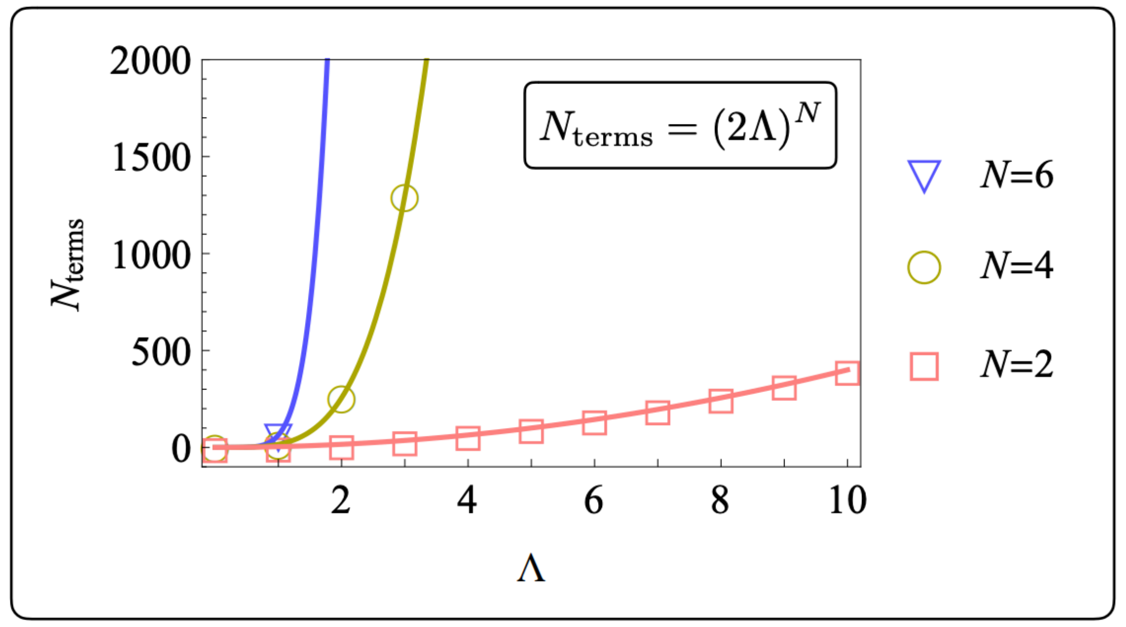
$$|\Psi_{LSH}\rangle = |n_l, n_i, n_o\rangle_1 \otimes |n_l, n_i, n_o\rangle_2 \otimes |n_l, n_i, n_o\rangle_3 \otimes |n_l, n_i, n_o\rangle_4 \otimes |n_l, n_i, n_o\rangle_5 \otimes |n_l, n_i, n_o\rangle_6$$



Practical benefits of working with LSH in 1d

- No need to impose Gauss law constraint: Significant reduction in the cost of Hilbert space generation.

- LSH basis is the physical basis: unlike KS, no nontrivial linear combinations of states are involved



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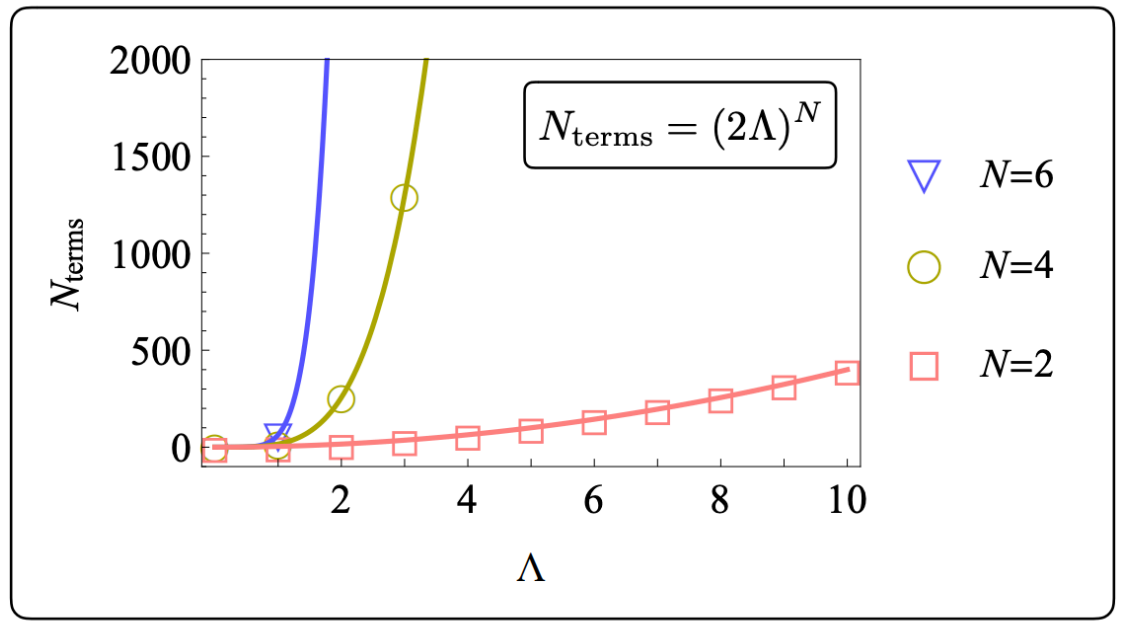

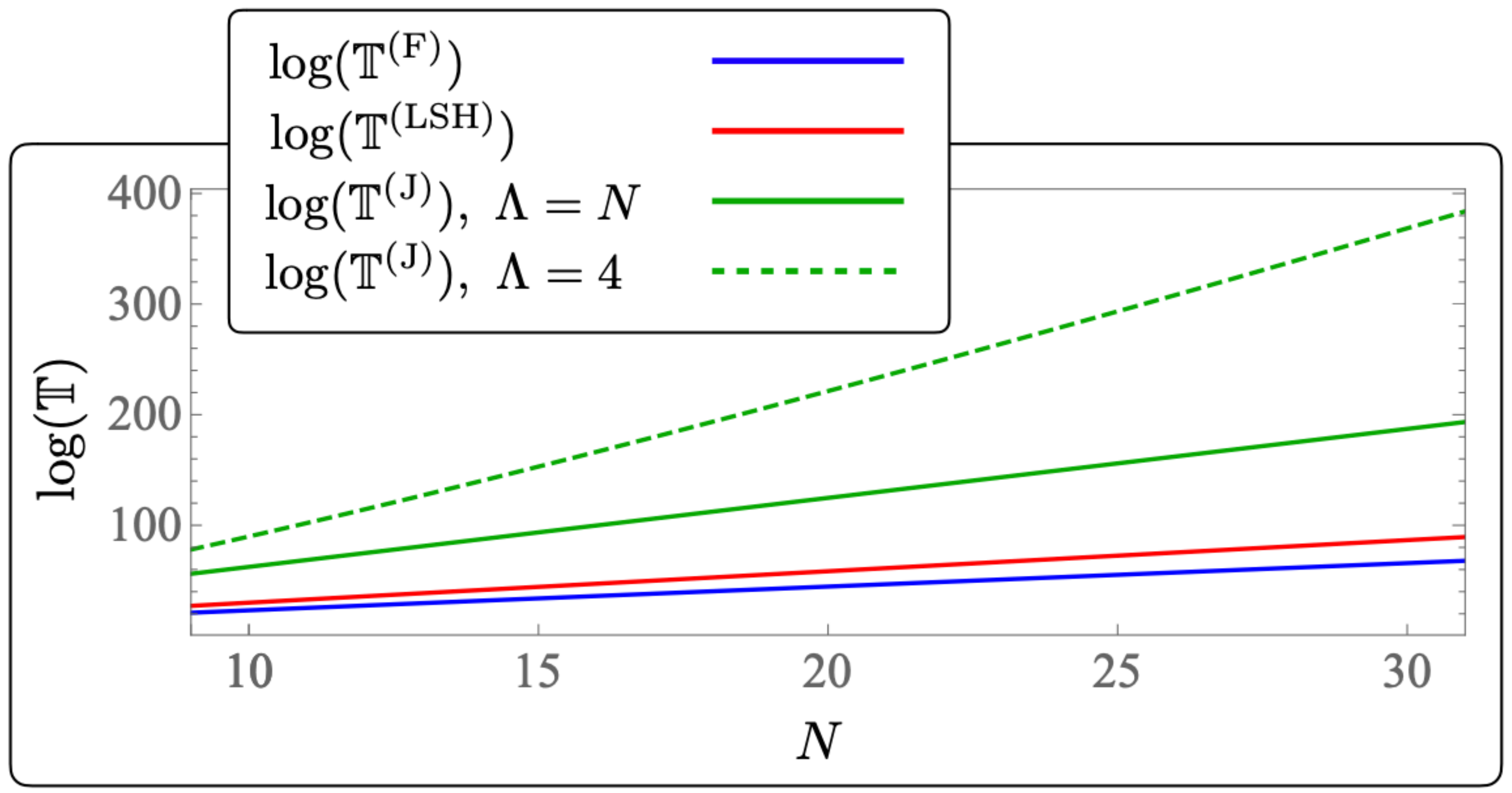

FIG. 14. The asymptotic cumulative cost of the three steps of given classical numerical algorithms for Hamiltonian simulation of the KS SU(2) LGT in 1+1 D with the fermionic formulation (F), LSH formulation, and the angular-momentum formulation in the physical sector (J), as a function of lattice size N for large N.

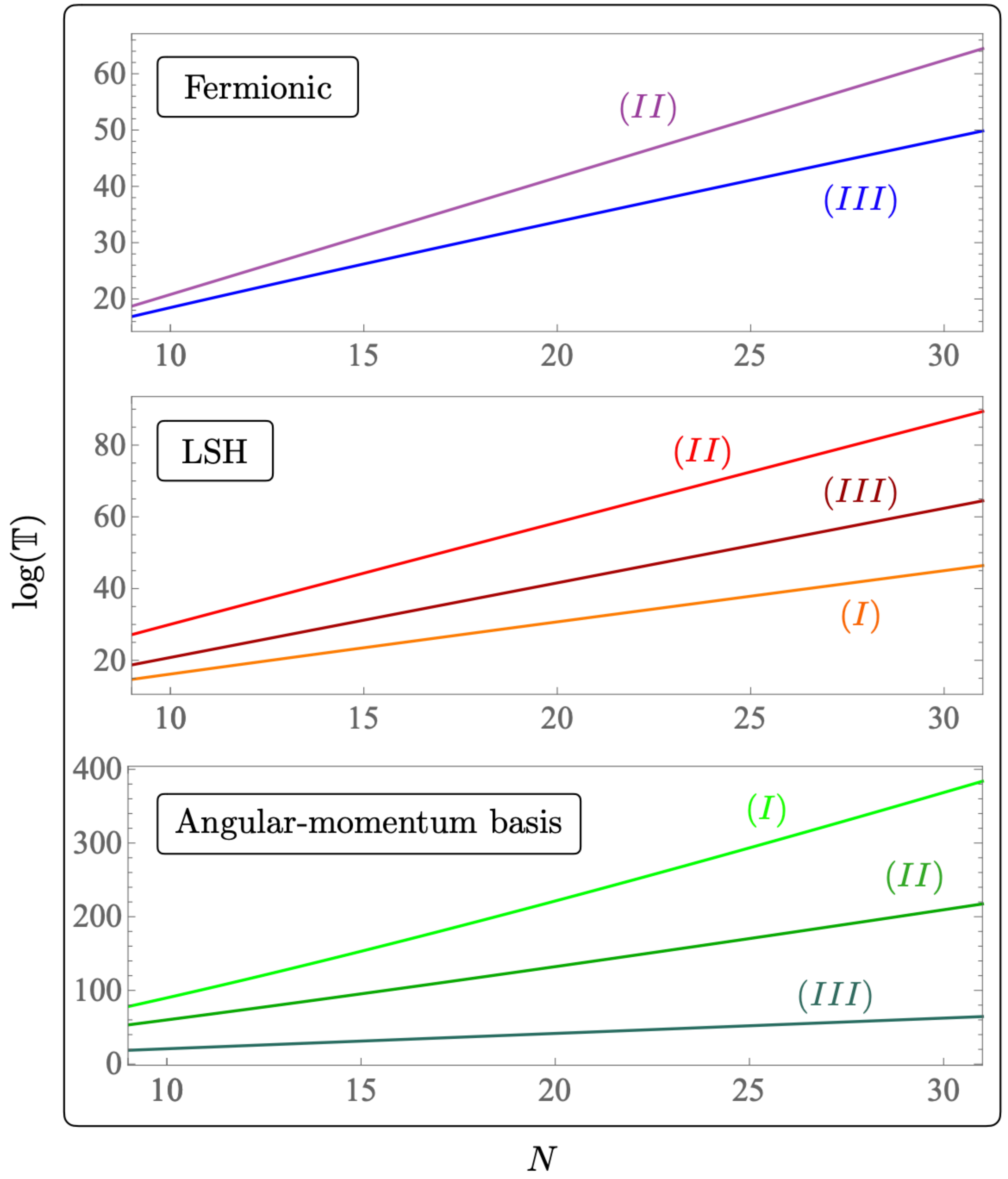

FIG. 15. The asymptotic cost of each step of given classical algorithms for Hamiltonian simulation of the KS SU(2) LGT in 1+1 D with the fermionic formulation (F), LSH formulation, and the angular-momentum basis in the physical sector (J), as a function of the lattice size N for large N. Step (I) refers to Hilbert-space construction, step (II) refers to Hamiltonian generation, and step (III) denotes observable computation assuming a generic scaling for sparse matrix manipulations, see the text. The step (I) for the fermionic formulation is of $\mathcal{O}(1)$ with the chosen algorithm and is not shown.

LSH Formulation: key ingredients

Local SU(2) invariant Hilbert space

$$|\, n_l, n_i, n_o \rangle$$

at matter gauge vertex in any dimension

$$|\, l_{12}, l_{23}, l_{31} \rangle$$

at pure gauge vertex in $d > 1$

Local constraint on each link: Abelian Gauss' law

LSH Formulation: key ingredients

Local SU(2) invariant Hilbert space

$$|\, n_l, n_i, n_o \rangle$$

at matter gauge vertex in any dimension

$$|\, l_{12}, l_{23}, l_{31} \rangle$$

at pure gauge vertex in $d > 1$

$$n_i, n_o \in \{0,\ 1\}$$

$$n_l, \{l_{ij}\} \in \{0,\ \infty\}$$

Local constraint on each link: Abelian Gauss' law

LSH Formulation: key ingredients

Local SU(2) invariant Hilbert space

$|n_l, n_i, n_o\rangle$

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at pure gauge vertex in $d > 1$

$n_i, n_o \in \{0,\ 1\}$

$n_l, \{l_{ij}\} \in \{0,\ \infty\}$

Impose a cut-off $\bar{j}$

$n_l, \{l_{ij}\} \in \{0,\ 2\bar{j}\}$

Local constraint on each link: Abelian Gauss' law

LSH Formulation: key ingredients

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$n_l, \{l_{ij}\} \in \{0,\ 2\bar{j}\}$

Local constraint on each link: Abelian Gauss' law

qq

$d = 1$

qg

$d = 2$

gg

LSH Formulation: key ingredients

Local SU(2) invariant Hilbert space

$|n_l, n_i, n_o\rangle$

at matter gauge vertex in any dimension

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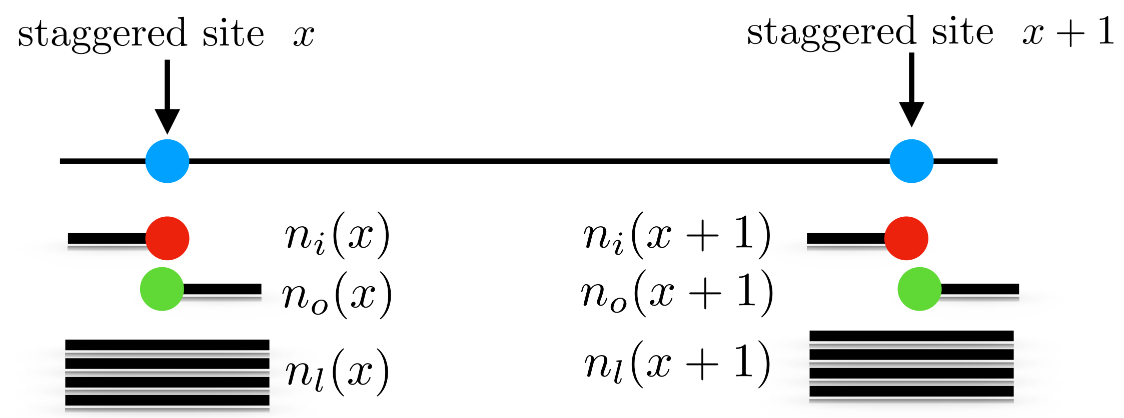
Impose a cut-off $\bar{j}$

$n_l, \{l_{ij}\} \in \{0,\ 2\bar{j}\}$

Local constraint on each link: Abelian Gauss' law

qq $\quad d = 1$

$$n_l + n_o(1 - n_i)\Big|_x = n_l + n_i(1 - n_o)\Big|_{x+1}$$



qg

$d = 2$

gg

LSH Formulation: key ingredients

Local SU(2) invariant Hilbert space

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at pure gauge vertex in $d > 1$

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$n_l, \{l_{ij}\} \in \{0,\ \infty\}$

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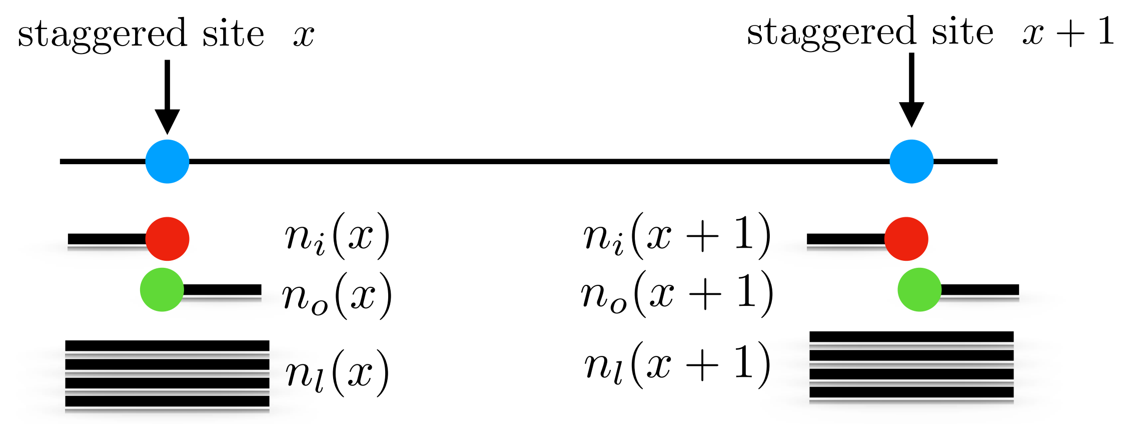
$n_l, \{l_{ij}\} \in \{0,\ 2\bar{j}\}$

Local constraint on each link: Abelian Gauss' law

qq

$d = 1$

$$n_l + n_o(1-n_i)\Big|_x = n_l + n_i(1-n_o)\Big|_{x+1}$$

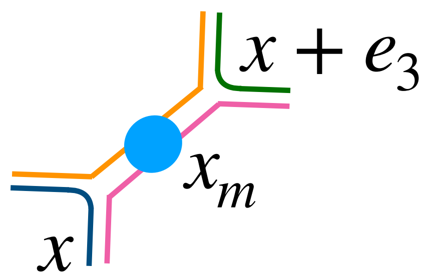


qg

$d = 2$

$$l_{23} + l_{31}\Big|_x = n_l + n_i(1-n_o)\Big|_{x_m}$$

$$l_{23} + l_{31}\Big|_{x+e_3} = n_l + n_o(1-n_i)\Big|_{x_m}$$



gg

LSH Formulation: key ingredients

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$|n_l, n_i, n_o\rangle$

at matter gauge vertex in any dimension

$|l_{12}, l_{23}, l_{31}\rangle$

at pure gauge vertex in $d > 1$

$n_i, n_o \in \{0,\ 1\}$

$n_l, \{l_{ij}\} \in \{0,\ \infty\}$

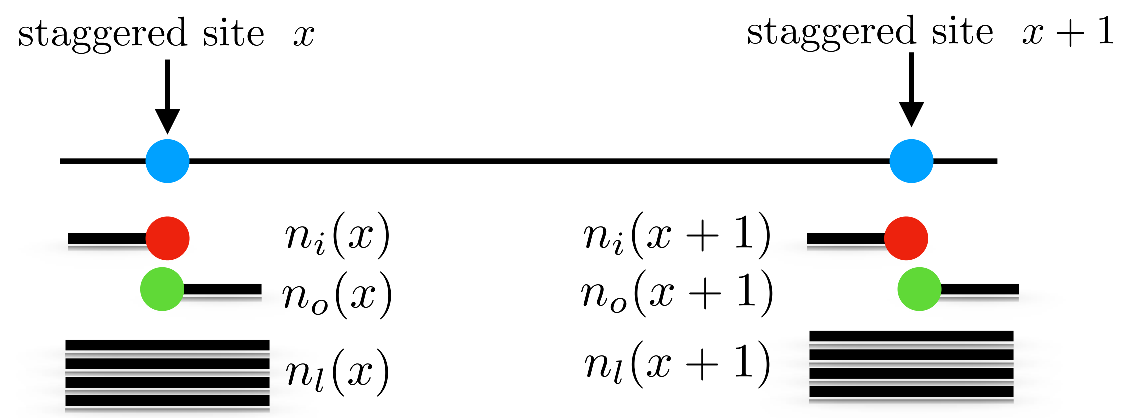
Impose a cut-off $\bar{j}$

$n_l, \{l_{ij}\} \in \{0,\ 2\bar{j}\}$

Local constraint on each link: Abelian Gauss' law

qq $d = 1$

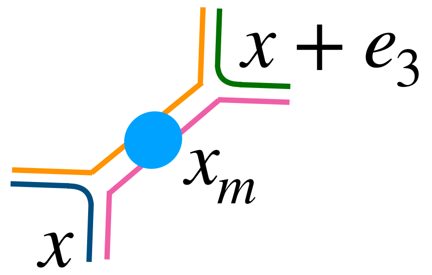
$$n_l + n_o(1 - n_i)\Big|_x = n_l + n_i(1 - n_o)\Big|_{x+1}$$



qg $d = 2$

$$l_{23} + l_{31}\Big|_x = n_l + n_i(1 - n_o)\Big|_{x_m}$$

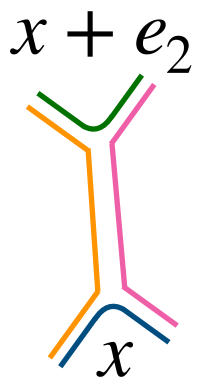
$$l_{23} + l_{31}\Big|_{x+e_3} = n_l + n_o(1 - n_i)\Big|_{x_m}$$



gg

$$l_{12} + l_{31}\Big|_x = l_{\bar{1}\bar{2}} + l_{\bar{3}\bar{1}}\Big|_{x+e_1}$$

$$l_{12} + l_{23}\Big|_x = l_{\bar{1}\bar{2}} + l_{\bar{2}\bar{3}}\Big|_{x+e_2}$$



LSH Formulation: key ingredients

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$$|n_l, n_i, n_o\rangle$$

at matter gauge vertex in any dimension

$$|l_{12}, l_{23}, l_{31}\rangle$$

at pure gauge vertex in $d > 1$

$$n_i, n_o \in \{0, 1\}$$

$$n_l, \{l_{ij}\} \in \{0, \infty\}$$

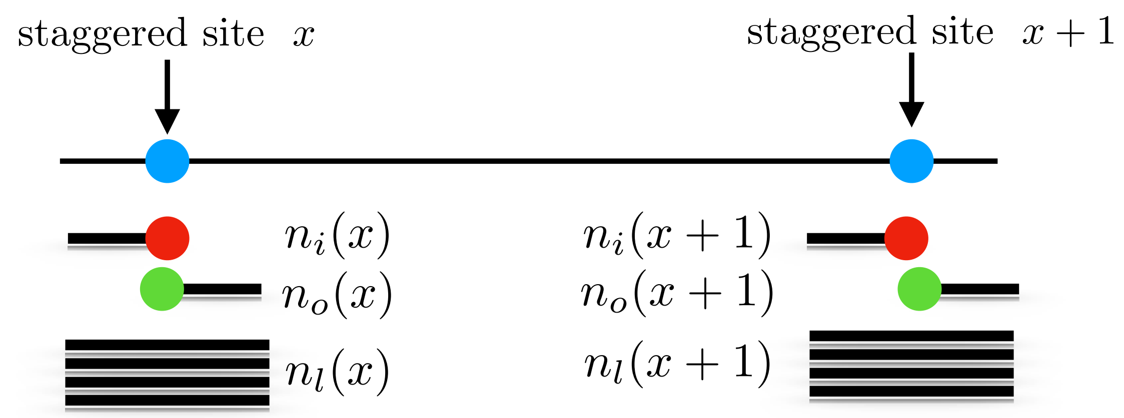
Impose a cut-off $\bar{j}$

$$n_l, \{l_{ij}\} \in \{0, 2\bar{j}\}$$

Local constraint on each link: Abelian Gauss' law

qq $\quad d = 1$

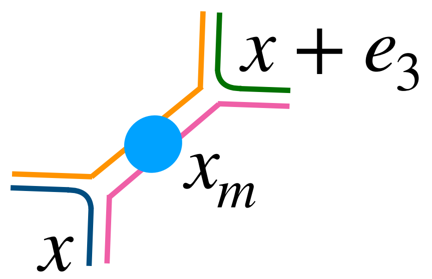
$$n_l + n_o(1 - n_i)\Big|_x = n_l + n_i(1 - n_o)\Big|_{x+1}$$



qg $\quad d = 2$

$$l_{23} + l_{31}\Big|_x = n_l + n_i(1 - n_o)\Big|_{x_m}$$

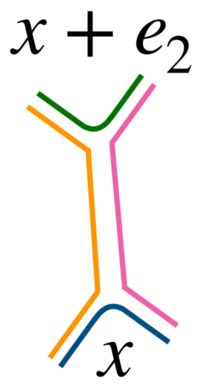
$$l_{23} + l_{31}\Big|_{x+e_3} = n_l + n_o(1 - n_i)\Big|_{x_m}$$



gg

$$l_{12} + l_{31}\Big|_x = l_{\bar{1}\bar{2}} + l_{\bar{3}\bar{1}}\Big|_{x+e_1}$$

$$l_{12} + l_{23}\Big|_x = l_{\bar{1}\bar{2}} + l_{\bar{2}\bar{3}}\Big|_{x+e_2}$$

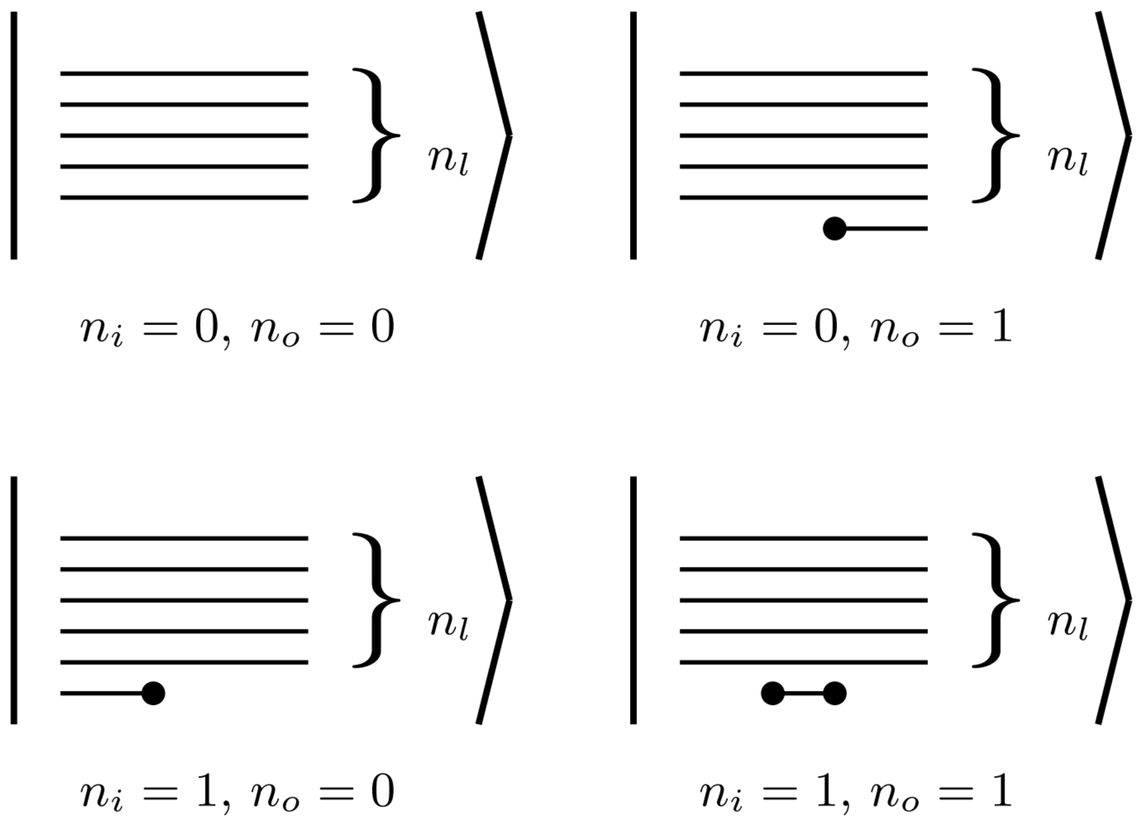


$d > 2$ **straightforward generalization**

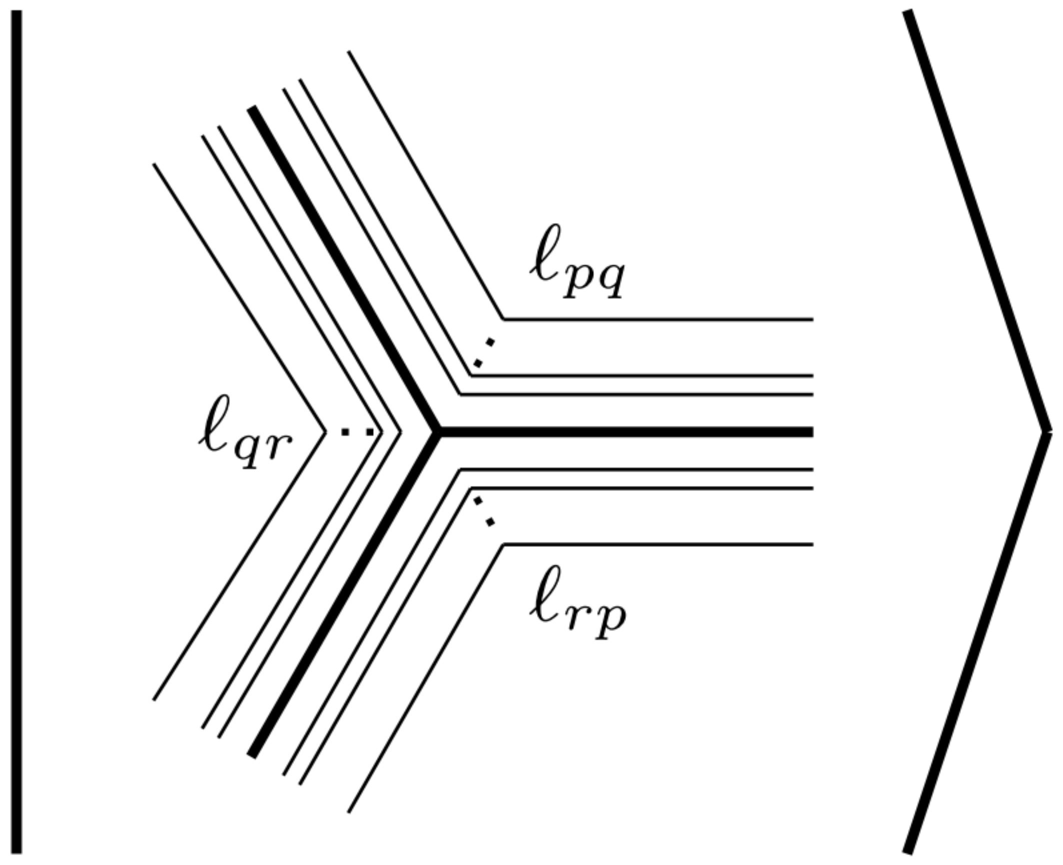
Digitization of LSH Hilbert space

Binary Representation of Loop Quantum Numbers

Matter site



Gluonic site



$\sim |\ell_{pq}, \ell_{qr}, \ell_{rp}\rangle$

(i) $N+1$ qubits per quark site loop number n_ℓ,
(ii) N qubits per gluonic site loop number ℓ_{ij},
where

$$N = \lceil \log_2(\bar{j}+1) \rceil .$$

The quark occupancy numbers require no truncation.

$$n_\ell = \sum_{m=0}^{N} 2^m n_{\ell,m} \quad (n_{\ell,m} = 0, 1),$$

$$\ell_{ij} = \sum_{m=0}^{N-1} 2^m \ell_{ij,m} \quad (\ell_{ij,m} = 0, 1)$$

$$|n_\ell\rangle = \bigotimes_{m=0}^{N} |n_{\ell,m}\rangle ,$$

$$|\ell_{ij}\rangle = \bigotimes_{m=0}^{N-1} |\ell_{ij,m}\rangle .$$

Application of LSH: Digital Quantum Computation

PHYSICAL REVIEW RESEARCH **2**, 033039 (2020)

Solving Gauss's law on digital quantum computers with loop-string-hadron digitization

Indrakshi Raychowdhury[*]
Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, Maryland 20742, USA

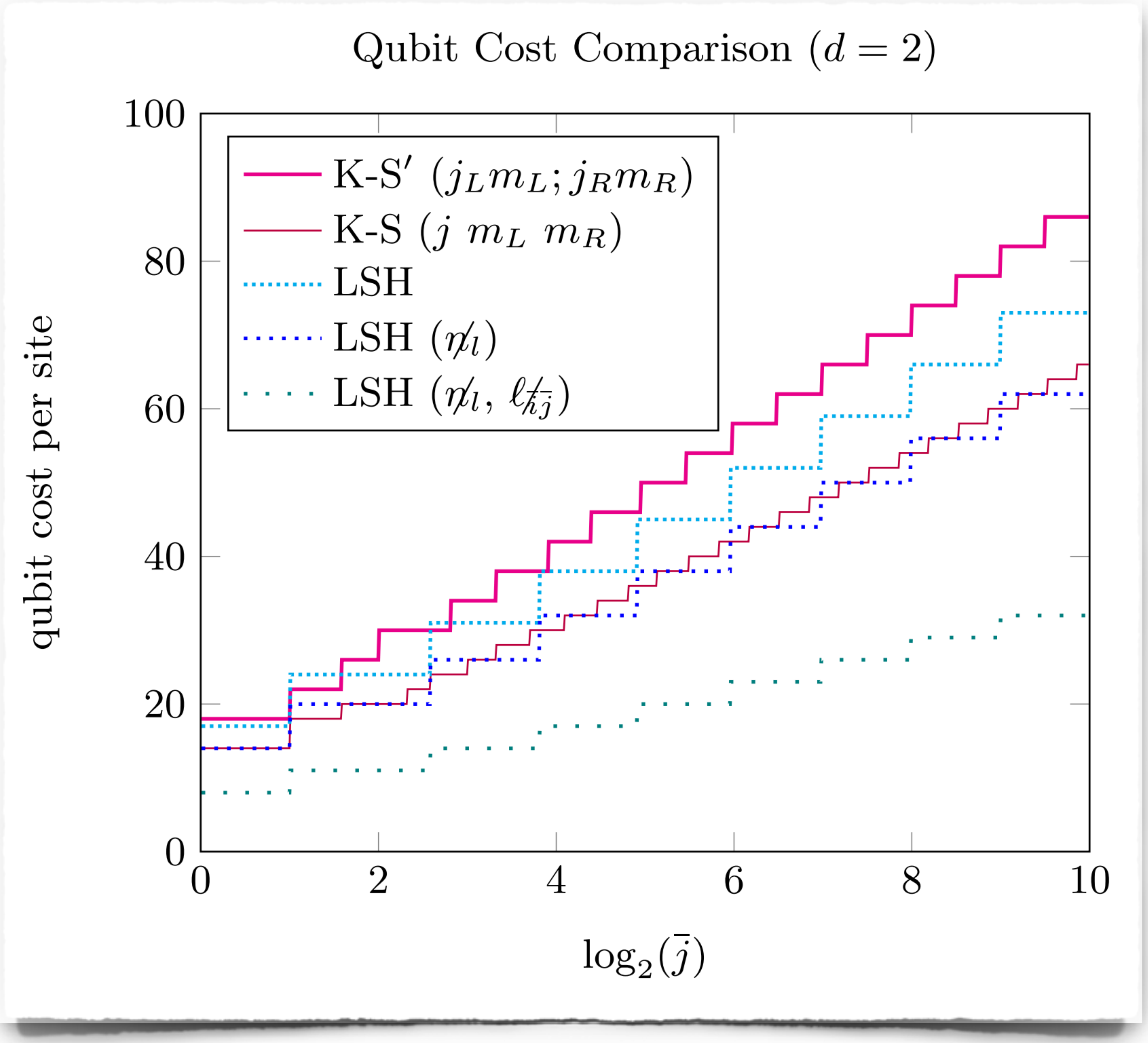
Jesse R. Stryker[†]
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA

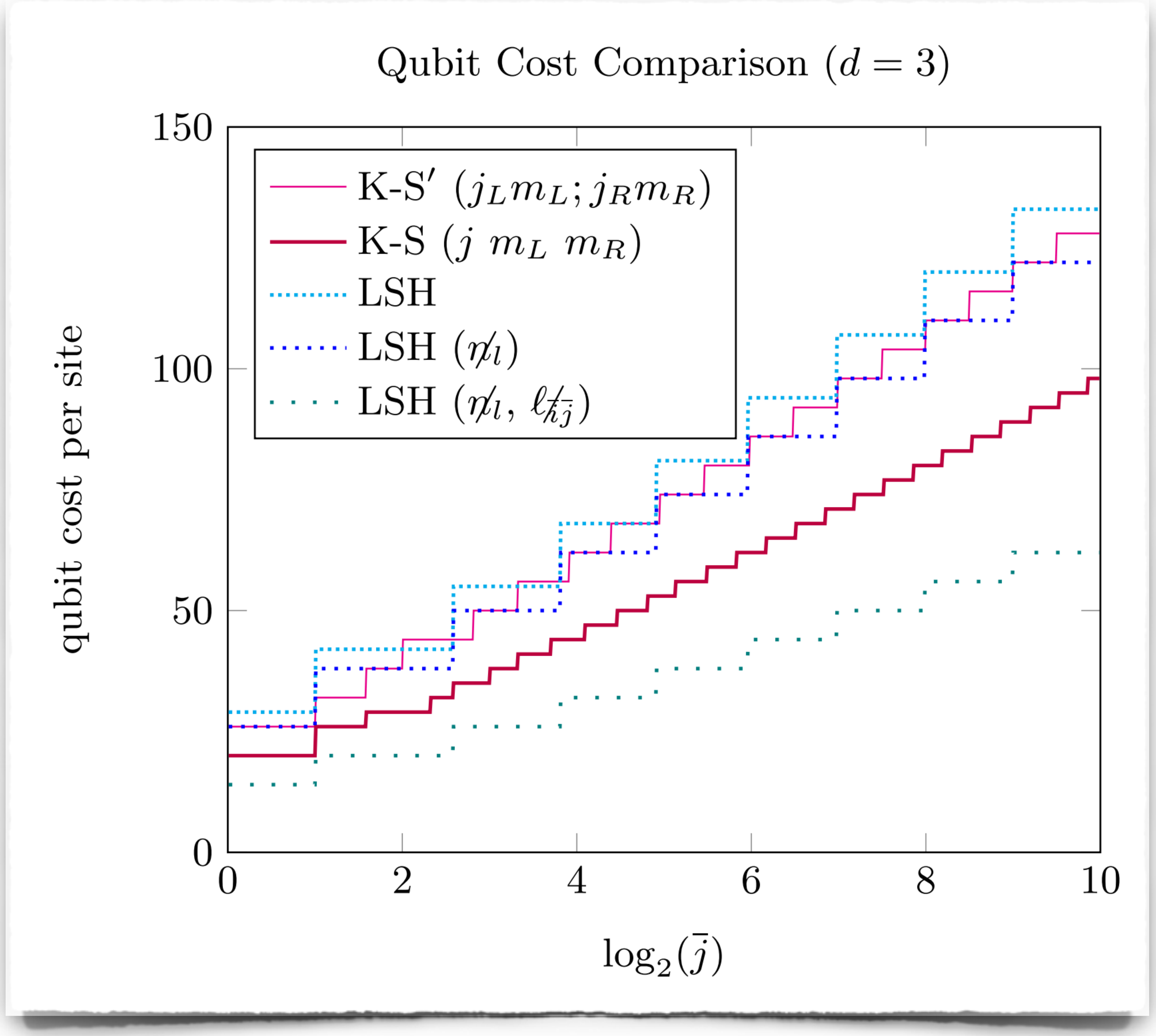
(Received 22 April 2020; accepted 4 June 2020; published 9 July 2020)

We show that using the loop-string-hadron (LSH) formulation of SU(2) lattice gauge theory (I. Raychowdhury and J. R. Stryker, Phys. Rev. D **101**, 114502 (2020)) as a basis for digital quantum computation easily solves an important problem of fundamental interest: implementing gauge invariance (or Gauss's law) exactly. We first discuss the structure of the LSH Hilbert space in d spatial dimensions, its truncation, and its digitization with qubits. Error detection and mitigation in gauge theory simulations would benefit from physicality "oracles," so we decompose circuits that flag gauge-invariant wave functions. We then analyze the logical qubit costs and entangling gate counts involved with the protocols. The LSH basis could save or cost more qubits than a Kogut-Susskind-type representation basis, depending on how the bases are digitized as well as the spatial dimension. The numerous other clear benefits encourage future studies into applying this framework.

DOI: 10.1103/PhysRevResearch.2.033039

Qubit Cost Analysis

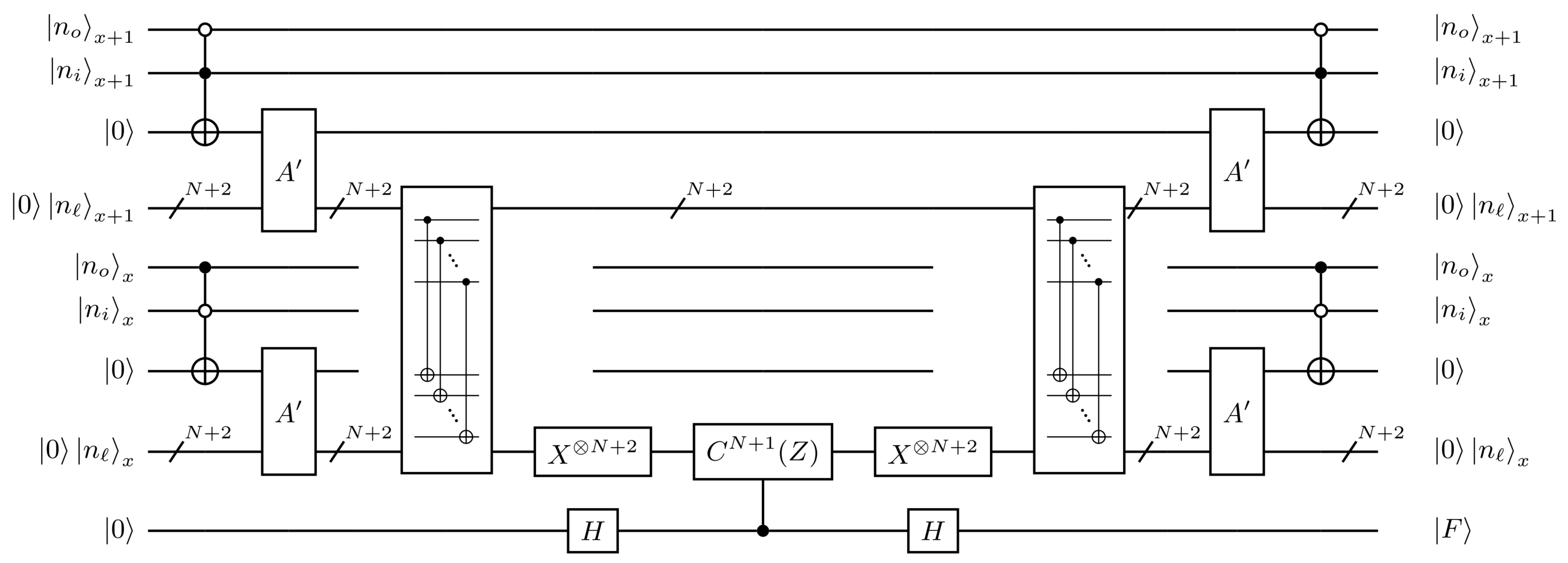




Oracle to check the Abelian Gauss law

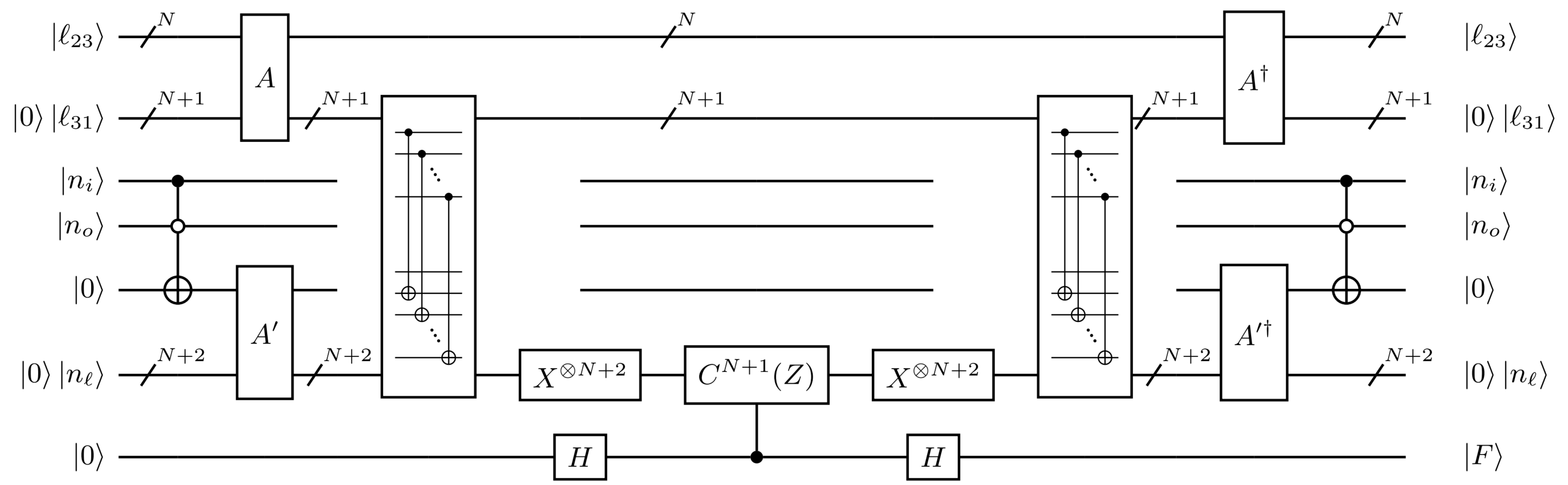
1d

qq

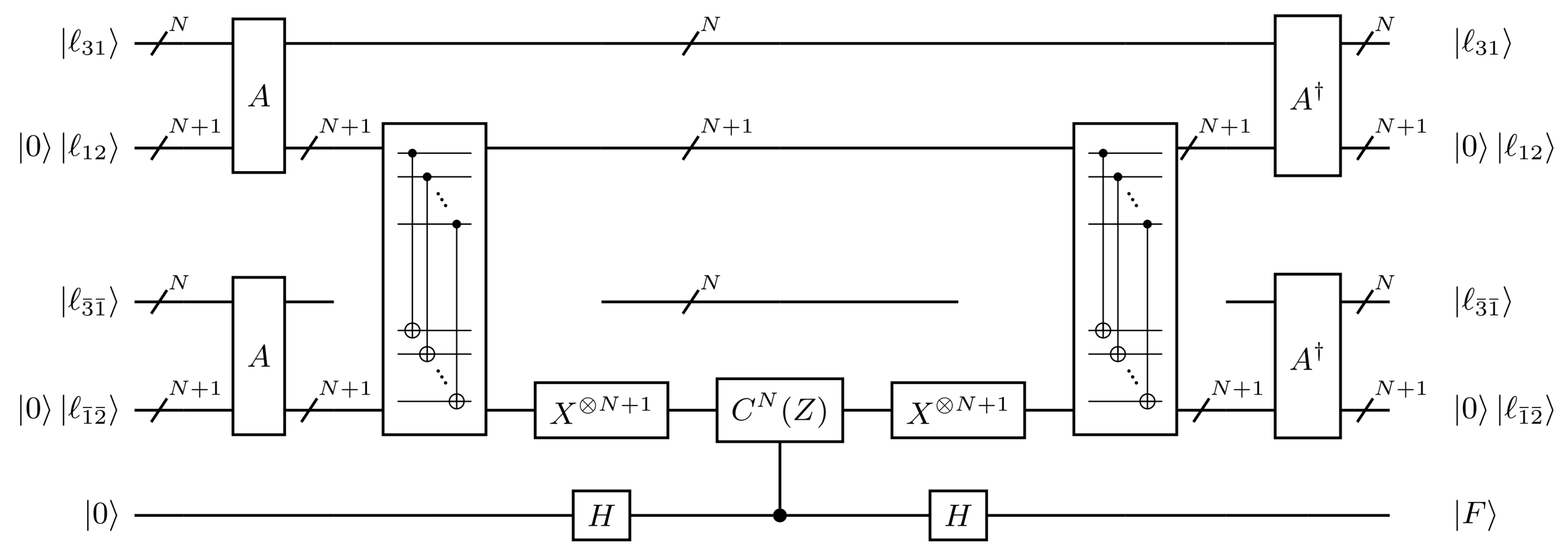


d >1

qg



gg



Recent proposal for Analog Quantum Simulation

UMD-PP-020-8

Cold Atom Quantum Simulator for String and Hadron Dynamics in Non-Abelian Lattice Gauge Theory

Raka Dasgupta[1] and Indrakshi Raychowdhury[2]

[1] *Dept. of Physics, University of Calcutta, 92 A. P. C. Road, Kolkata-700009, India**

[2] *Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, MD 20742, USA*†

(Dated: September 29, 2020)

We propose an analog quantum simulator for simulating real time dynamics of $(1+1)$-d non-Abelian gauge theory well within the existing capacity of ultracold atom experiments. The scheme calls for the realization of a two-state ultracold fermionic system in a 1-dimensional bipartite lattice, and the observation of subsequent tunneling dynamics. Being based on novel loop string hadron formalism of SU(2) lattice gauge theory, this simulation technique is completely SU(2) invariant and simulates accurate dynamics of physical phenomena such as string breaking and/or pair production. The scheme is scalable, and particularly effective in simulating the theory in weak coupling regime, and also bulk limit of the theory in strong coupling regime up to certain approximations. This paper also presents a numerical benchmark comparison of exact spectrum and real time dynamics of lattice gauge theory to that of the atomic Hamiltonian with experimentally realizable range of parameters.

arXiv:2009.13969

Application: Analog Quantum computation

Simulated system	Simulating system
SU(2) gauge theory coupled to staggered fermions in 1+1 dimension described by Kogut-Susskind Hamiltonian	A system of ultracold fermions trapped in optical lattices described by the Hamiltonian of ionic Hubbard model

Application: Analog Quantum computation

Simulated system

SU(2) gauge theory
coupled to staggered
fermions
in 1+1 dimension
described by
Kogut-Susskind
Hamiltonian

↓

LSH Hamiltonian

Simulating system

A system of ultracold
fermions trapped in
optical lattices
described by the
Hamiltonian of
ionic Hubbard model

→

Manifestly gauge invariant simulation scheme

Application: Analog Quantum computation

Simulated system		Simulating system
SU(2) gauge theory coupled to staggered fermions in 1+1 dimension described by Kogut-Susskind Hamiltonian		A system of ultracold fermions trapped in optical lattices described by the Hamiltonian of ionic Hubbard model
↓		
LSH Hamiltonian	→	**Manifestly gauge invariant simulation scheme**

Simulates string and hadron dynamics at weak and intermediate coupling regime

Application: Analog Quantum computation

Experimental Demonstration: minor modification/combination of

PRL **115,** 115303 (2015) PHYSICAL REVIEW LETTERS week ending 11 SEPTEMBER 2015

Exploring Competing Density Order in the Ionic Hubbard Model with Ultracold Fermions

Michael Messer,[1] Rémi Desbuquois,[1] Thomas Uehlinger,[1] Gregor Jotzu,[1] Sebastian Huber,[2] Daniel Greif,[1] and Tilman Esslinger[1]
[1]*Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland*
[2]*Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland*
(Received 18 March 2015; revised manuscript received 1 July 2015; published 9 September 2015)

We realize and study the ionic Hubbard model using an interacting two-component gas of fermionic atoms loaded into an optical lattice. The bipartite lattice has a honeycomb geometry with a staggered energy offset that explicitly breaks the inversion symmetry. Distinct density-ordered phases are identified using noise correlation measurements of the atomic momentum distribution. For weak interactions the geometry induces a charge density wave. For strong repulsive interactions we detect a strong suppression of doubly occupied sites, as expected for a Mott insulating state, and the externally broken inversion symmetry is not visible anymore in the density distribution. The local density distributions in different configurations are characterized by measuring the number of doubly occupied lattice sites as a function of interaction and energy offset. We further probe the excitations of the system using direction dependent modulation spectroscopy and discover a complex spectrum, which we compare with a theoretical model.

DOI: 10.1103/PhysRevLett.115.115303 PACS numbers: 67.85.Lm, 71.10.Fd, 71.30.+h, 73.22.Pr

PHYSICAL REVIEW LETTERS **121,** 130402 (2018)

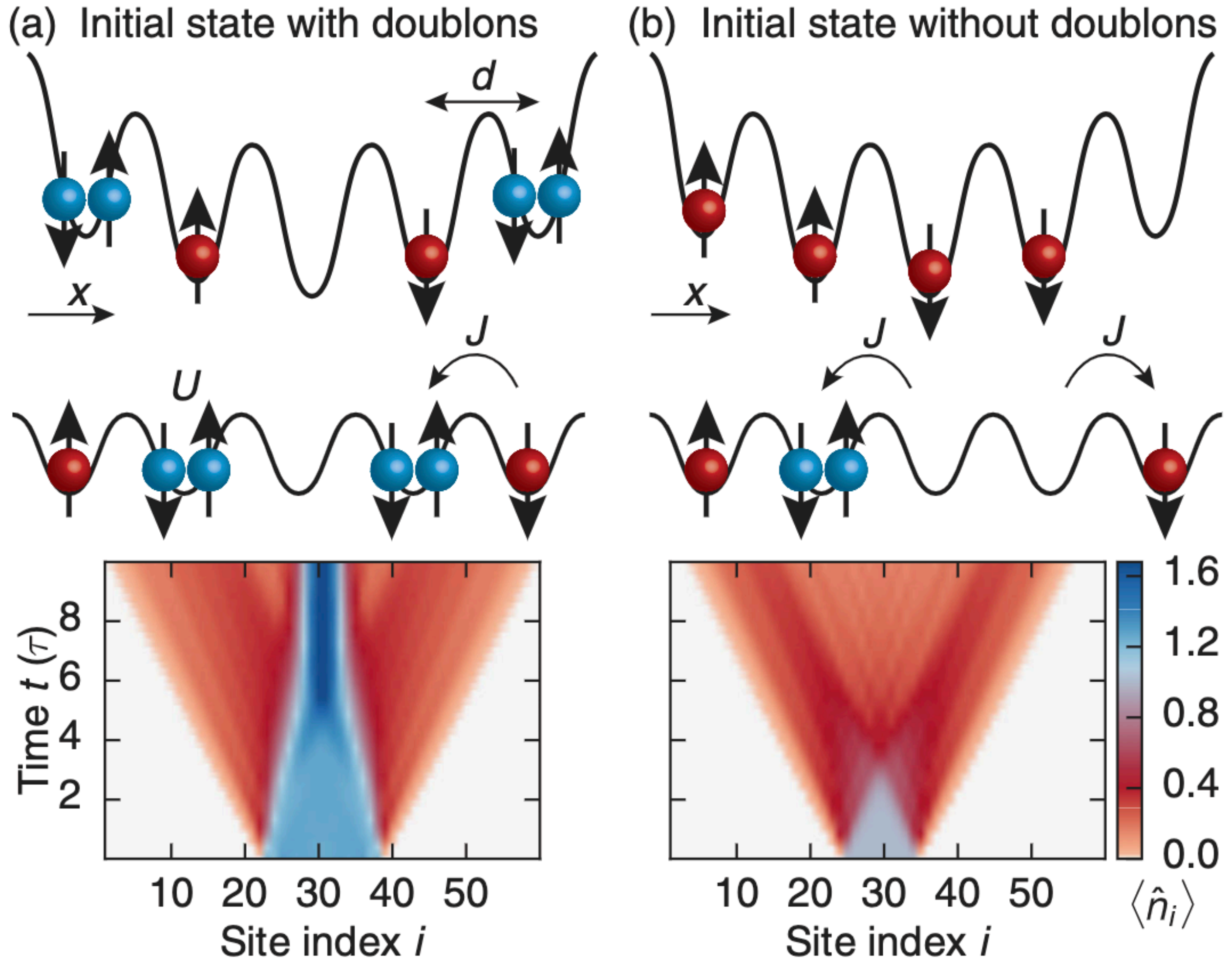
Nonequilibrium Mass Transport in the 1D Fermi-Hubbard Model

S. Scherg,[1,2] T. Kohlert,[1,2] J. Herbrych,[3,4] J. Stolpp,[1,5,6] P. Bordia,[1,2] U. Schneider,[1,2,7] F. Heidrich-Meisner,[5] I. Bloch,[1,2] and M. Aidelsburger[1,2,*]
[1]*Fakultät für Physik, Ludwig-Maximilians-Universität München, Munich, Germany*
[2]*Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[3]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
aterials Science and Technology Division Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA
[5]*Institute for Theoretical Physics, Georg-August-Universität Göttingen, 37077 Göttingen, Germany*
ld Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, 80333 Munich, Germany
[7]*Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, United Kingdom*

(Received 20 June 2018; revised manuscript received 21 August 2018; published 25 September 2018)

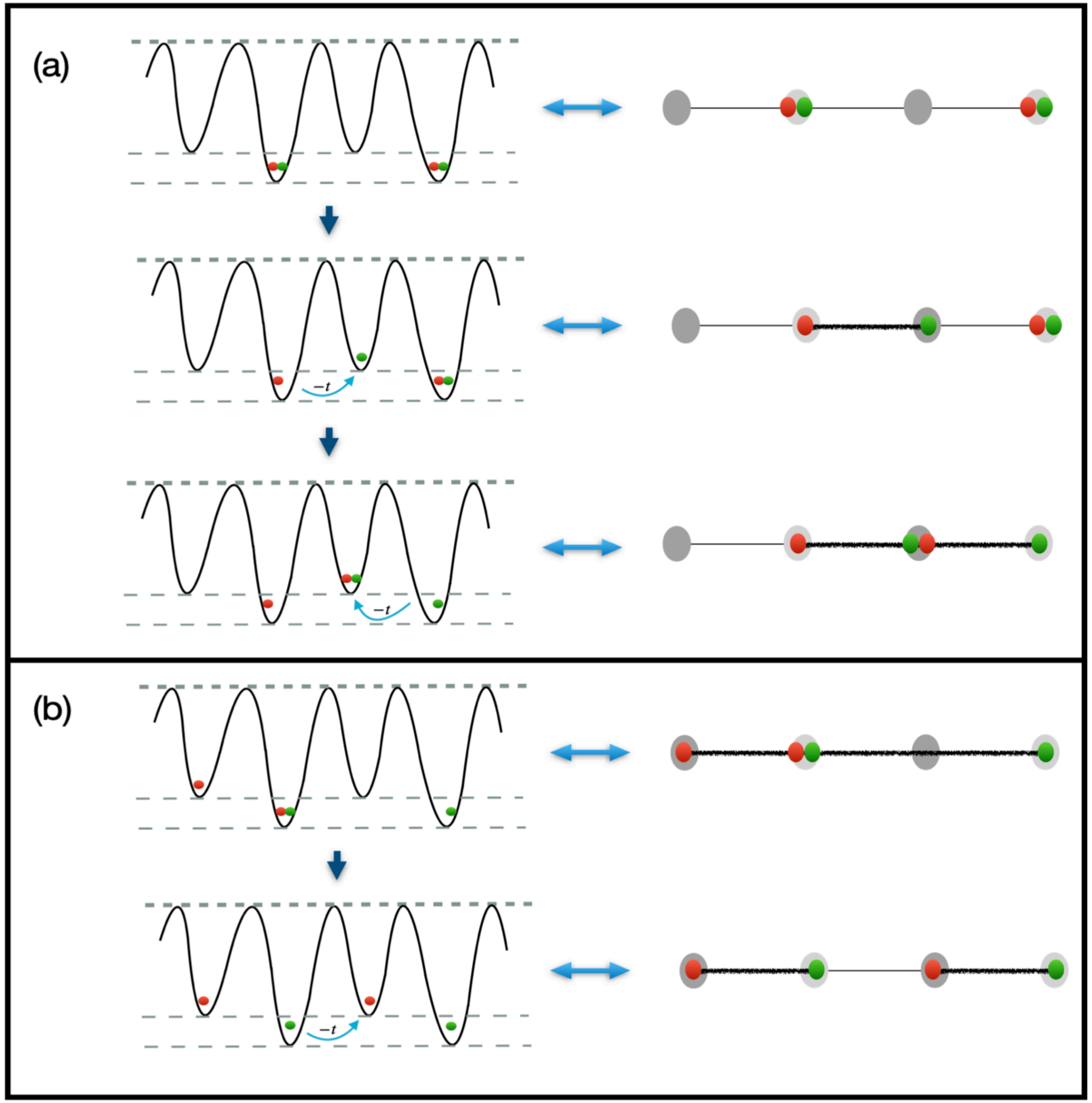
We experimentally and numerically investigate the sudden expansion of fermions in a homogeneous one-dimensional optical lattice. For initial states with an appreciable amount of doublons, we observe a dynamical phase separation between rapidly expanding singlons and slow doublons remaining in the trap center, realizing the key aspect of fermionic quantum distillation in the strongly interacting limit. For initial states without doublons, we find a reduced interaction dependence of the asymptotic expansion speed compared to bosons, which is explained by the interaction energy produced in the quench.

Pair creation dynamics in lattice gauge theory



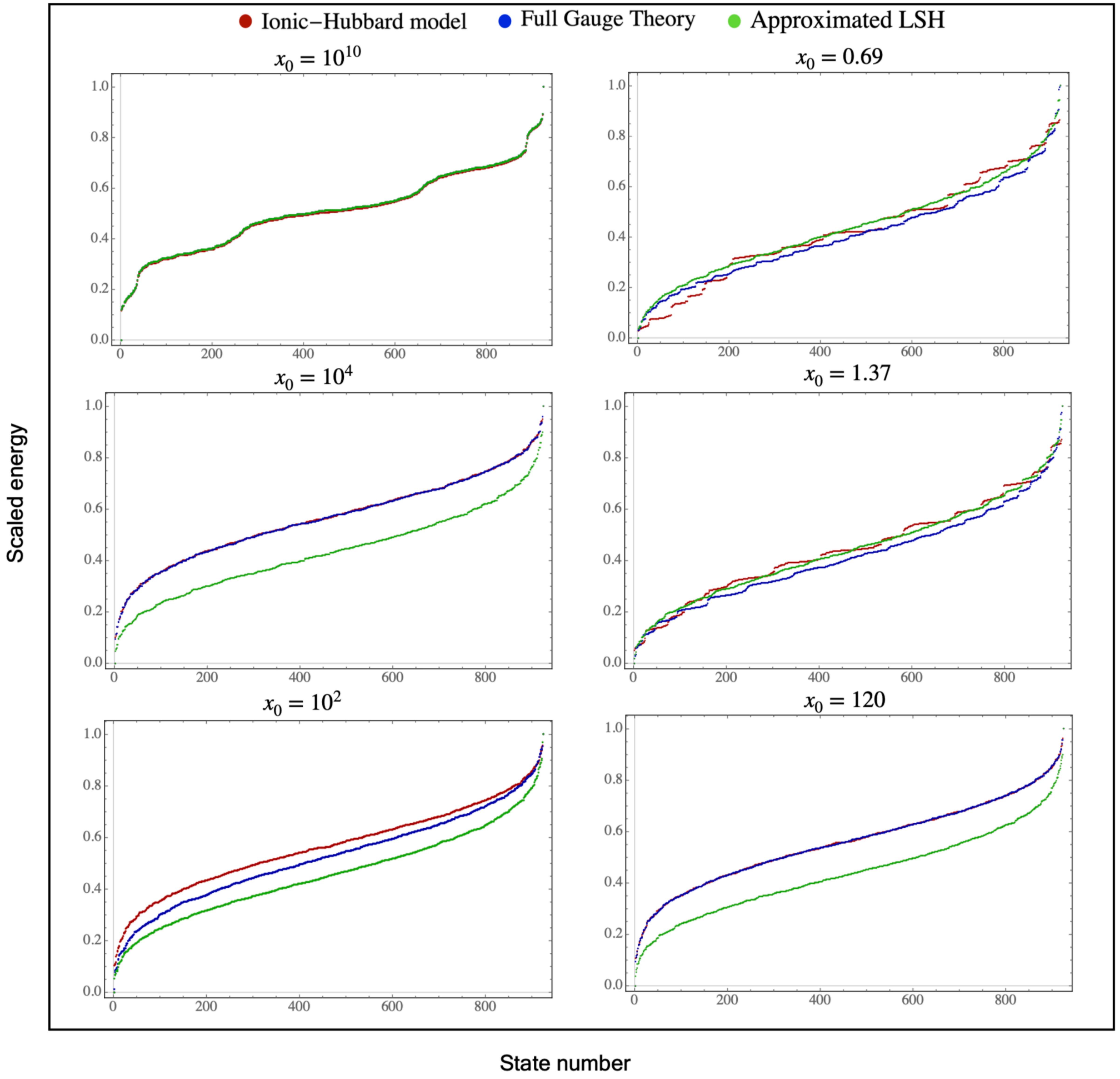
Recent Analog Simulation Proposal

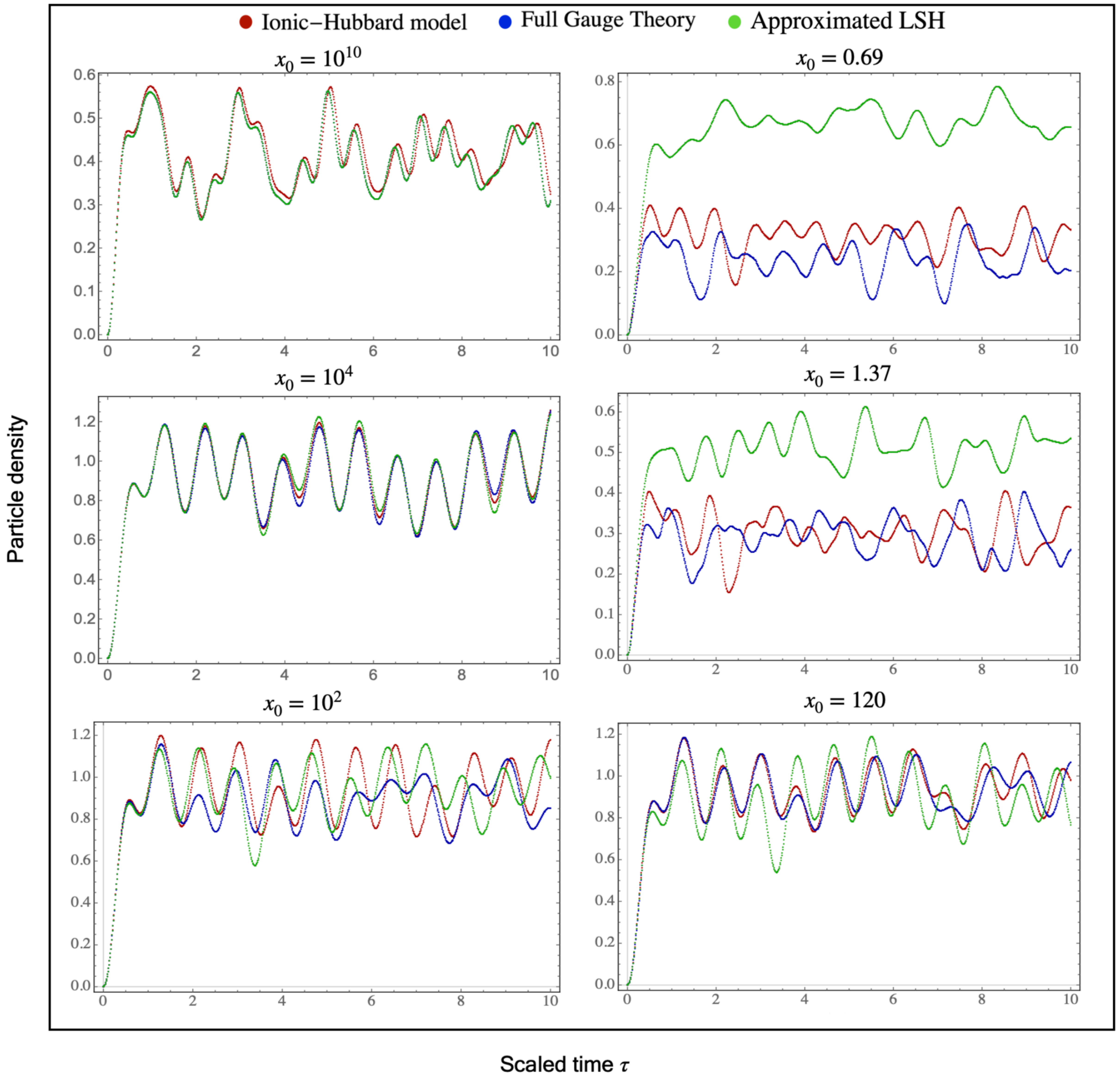
Simulated Dynamics: cartoon



Recent Analog Simulation Proposal

Numerical Comparison: Exact diagonalization for small lattice





$$x_0 = \frac{1}{g^2 a^2}$$

$$\tau_{\text{exp}} = \frac{\hbar \tau_{\text{atomic}}}{t} \equiv \frac{\tau_{\text{atomic}}}{1.5716} \text{ ms}$$

$$\Rightarrow \equiv \frac{2a\tau_{\text{gauge}}}{1.5716} \text{ms.}$$

Other recent efforts: Digital quantum computing for SU(2)

PHYSICAL REVIEW D **101**, 074512 (2020)

SU(2) non-Abelian gauge field theory in one dimension on digital quantum computers

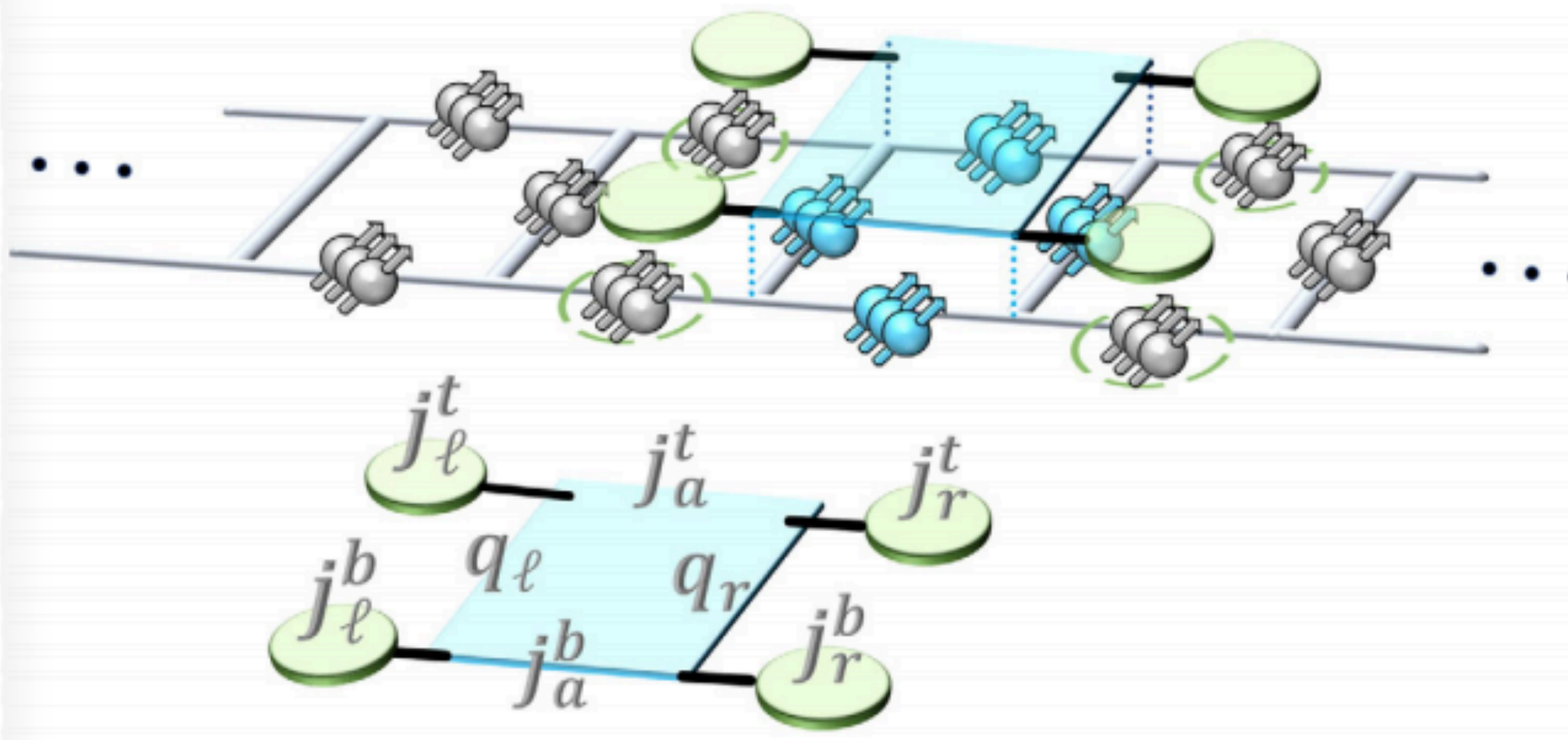
Natalie Klco,* Martin J. Savage,† and Jesse R. Stryker‡
Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA

(Received 4 September 2019; revised manuscript received 24 December 2019; accepted 20 March 2020; published 21 April 2020)

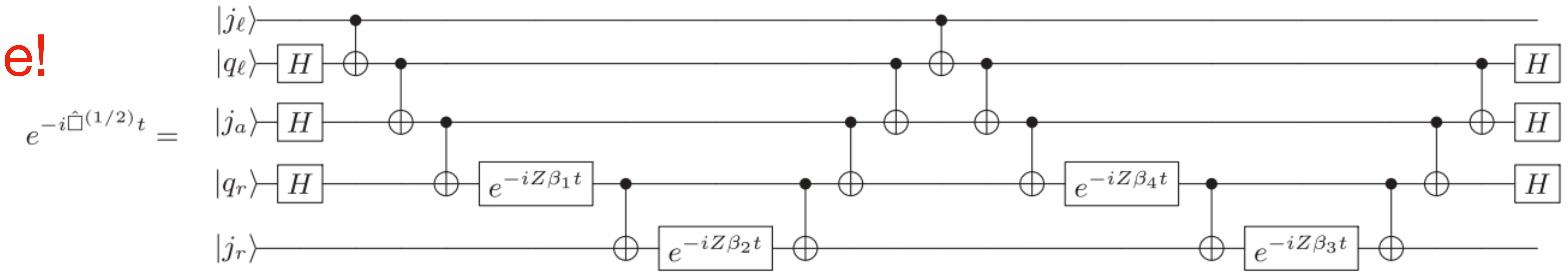
A dynamical quantum simulation of SU(2) non-Abelian gauge field theory on a digital quantum computer is presented. This was enabled on current quantum hardware by introducing a mapping of the field onto a register of qubits that utilizes local gauge symmetry while preserving local constraints on the fields, reducing the dimensionality of the calculation. Controlled plaquette operators and gauge-variant completions in the unphysical part of the Hilbert space were designed and used to implement time evolution. The new techniques developed in this work generalize to quantum simulations of higher dimensional gauge field theories.

DOI: 10.1103/PhysRevD.101.074512

$$\hat{U}_{\alpha\beta}|j,a,b\rangle = \sum_{\oplus J}\sqrt{\frac{\dim(j)}{\dim(J)}}|J,a+\alpha,b+\beta\rangle \times \left\langle j,a,\frac{1}{2},\alpha\middle|J,a+\alpha\right\rangle\left\langle j,b,\frac{1}{2},\beta\middle|J,b+\beta\right\rangle$$



Not a scalable scheme!



Other recent efforts: Digital quantum computing for SU(2)

SU(2) hadrons on a quantum computer

Yasar Atas *,[1,2,†] Jinglei Zhang *,[1,2,‡] Randy Lewis,[3] Amin Jahanpour,[1,2] Jan F. Haase,[1,2,§] and Christine A. Muschik[1,2,4]

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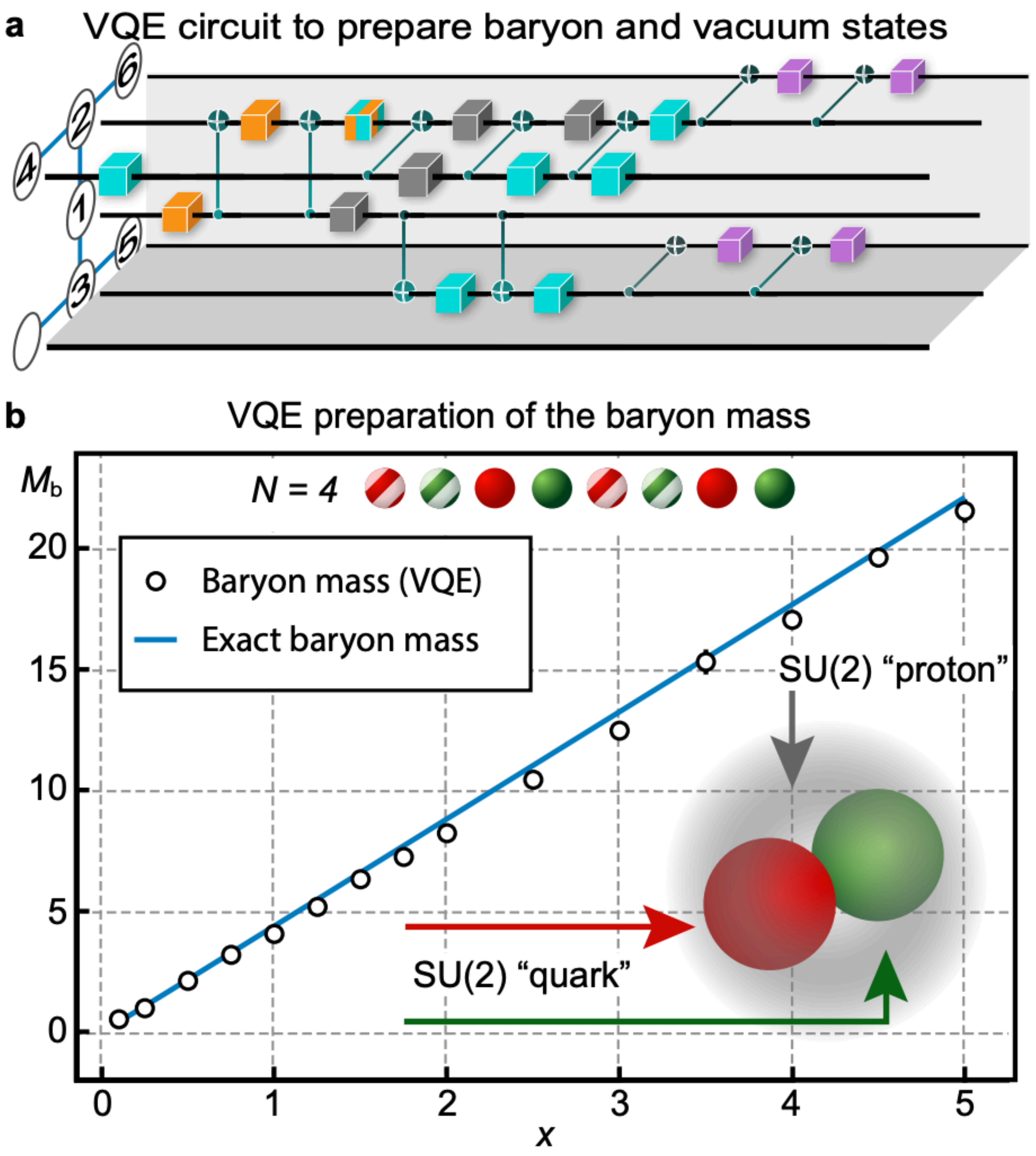
(Dated: March 3, 2021)

We realize, for the first time, a non-Abelian gauge theory with both gauge and matter fields on a quantum computer. This enables the observation of hadrons and the calculation of their associated masses. The SU(2) gauge group considered here represents an important first step towards ultimately studying quantum chromodynamics, the theory that describes the properties of protons, neutrons and other hadrons. Quantum computers are able to create important new opportunities for ongoing essential research on gauge theories by providing simulations that are unattainable on classical computers. Our calculations on an IBM superconducting platform utilize a variational quantum eigensolver to study both meson and baryon states, hadrons which have never been seen in a non-Abelian simulation on a quantum computer. We develop a resource-efficient approach that not only allows the implementation of a full SU(2) gauge theory on present-day quantum hardware, but further lays out the premises for future quantum simulations that will address currently unanswered questions in particle and nuclear physics.



Not generalisable to higher dimensions!

Other recent efforts: Digital quantum computing for SU(2)

PHYSICAL REVIEW D **104**, 034501 (2021)

SU(2) lattice gauge theory on a quantum annealer

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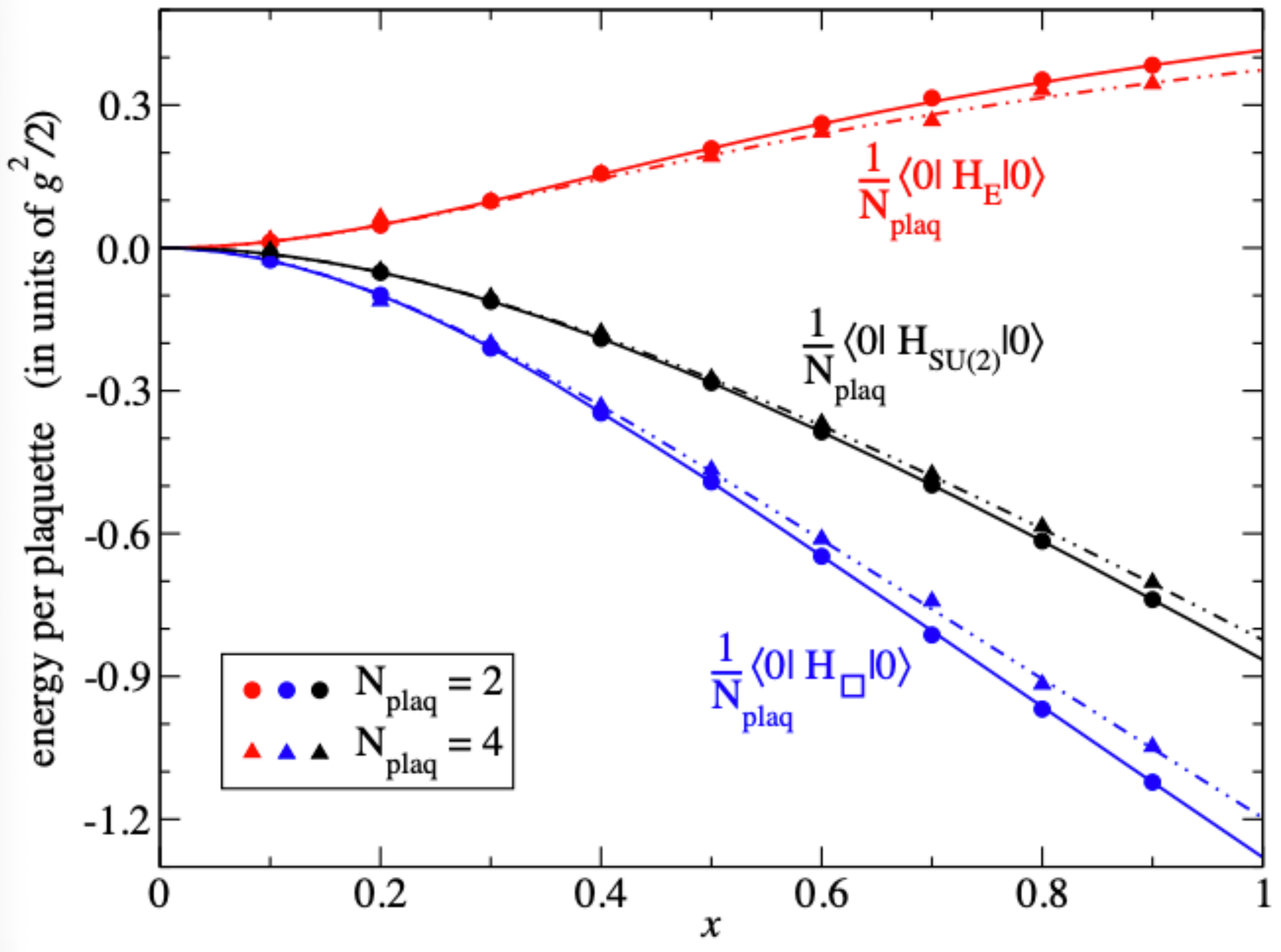
Lattice gauge theory is an essential tool for strongly interacting non-Abelian fields, such as those in quantum chromodynamics where lattice results have been of central importance for several decades. Recent studies suggest that quantum computers could extend the reach of lattice gauge theory in dramatic ways, but the usefulness of quantum annealing hardware for lattice gauge theory has not yet been explored. In this work, we implement SU(2) pure gauge theory on a quantum annealer for lattices comprising a few plaquettes in a row with a periodic boundary condition. These plaquettes are in two spatial dimensions and calculations use the Hamiltonian formulation where time is not discretized. Numerical results are obtained from calculations on D-Wave Advantage hardware for eigenvalues, eigenvectors, vacuum expectation values, and time evolution. The success of this initial exploration indicates that the quantum annealer might become a useful hardware platform for some aspects of lattice gauge theories.



Alternative for gate based quantum computing: D-wave annealer
Promising as the data points are more precise than NISQ hardwares

Most promising recent effort: SU(3) gauge theory on a quantum computer

PHYSICAL REVIEW D **103,** 094501 (2021)

Editors' Suggestion

Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis

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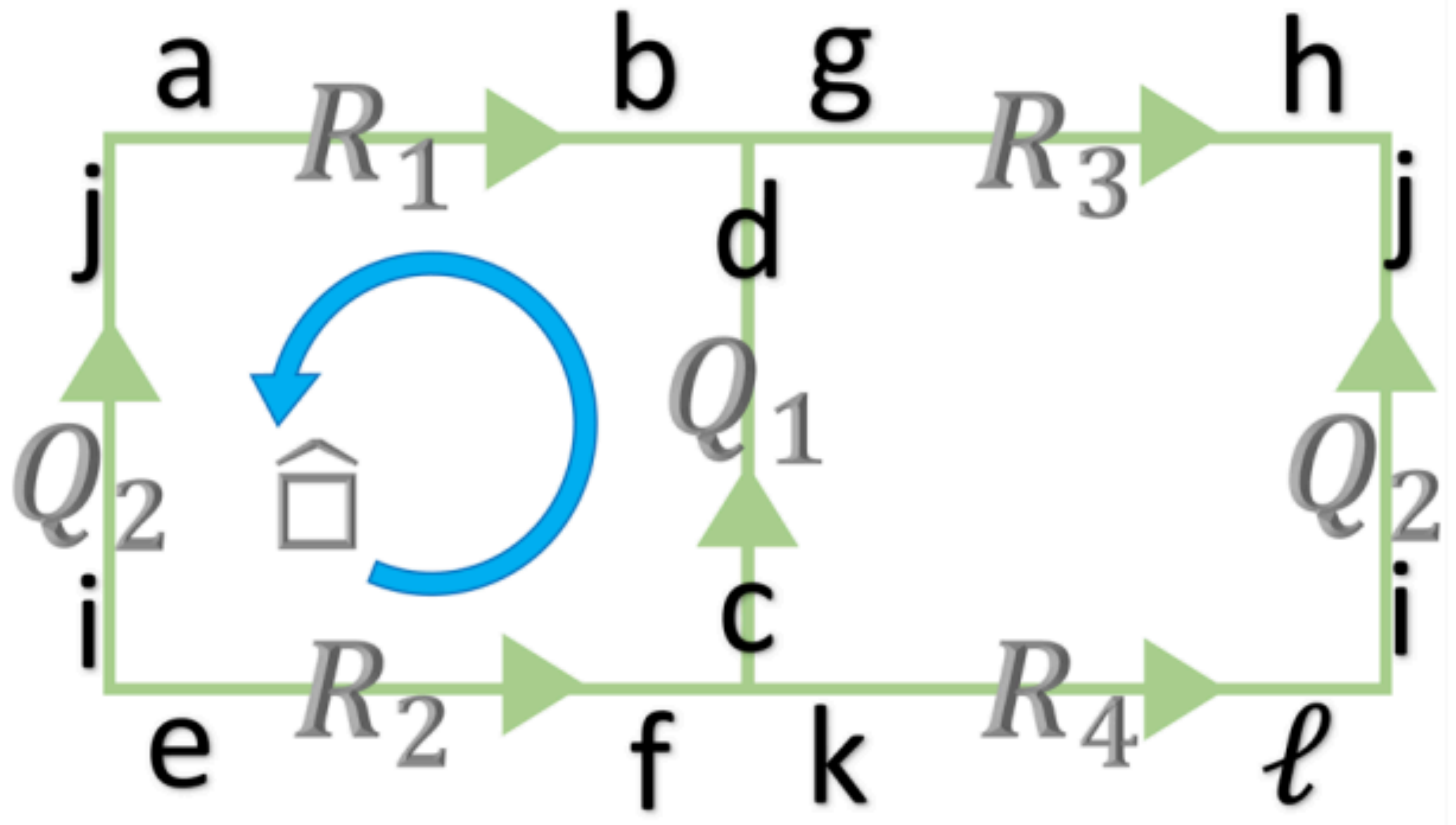
Maintaining local interactions in the quantum simulation of gauge field theories relegates most states in the Hilbert space to be unphysical—theoretically benign, but experimentally difficult to avoid. Reformulations of the gauge fields can modify the ratio of physical to gauge-variant states often through classically preprocessing the Hilbert space and modifying the representation of the field on qubit degrees of freedom. This paper considers the implications of representing SU(3) Yang-Mills gauge theory on a lattice of irreducible representations in both a global basis of projected global quantum numbers and a local basis in which controlled-plaquette operators support efficient time evolution. Classically integrating over the internal gauge space at each vertex (e.g., color isospin and color hypercharge) significantly reduces both the qubit requirements and the dimensionality of the unphysical Hilbert space. Initiating tuning procedures that may inform future calculations at scale, the time evolution of one and two plaquettes are implemented on one of IBM's superconducting quantum devices, and early benchmark quantities are identified. The potential advantages of qudit environments, with either constrained two-dimensional hexagonal or one-dimensional nearest-neighbor internal state connectivity, are discussed for future large-scale calculations.

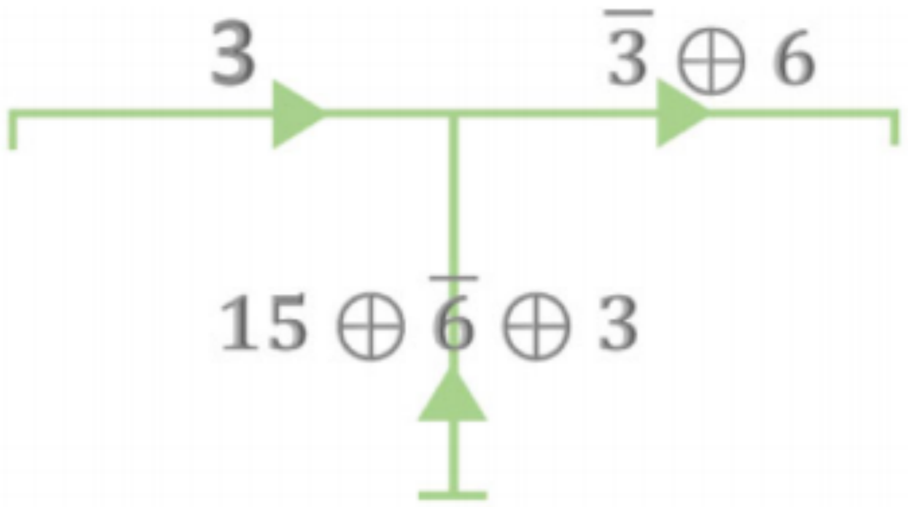
Most promising recent effort: SU(3) gauge theory on a quantum computer

Actual implementation:
Global basis for a 2 plaquette system
With minimal cut-off.



Local multiplet basis at each site:

$\hat{\Box}^\dagger |\mathbf{3}\rangle_{C_1} |\mathbf{8}\rangle_{Q_\ell} |\mathbf{3}\rangle_{R_t} \rightarrow$



$$\left\langle \begin{pmatrix} \mathbf{C}_1, \mathbf{R}'_t, \mathbf{C}_3 \\ \mathbf{Q}'_\ell, \mathbf{Q}'_r \\ \mathbf{C}_2, \mathbf{R}'_b, \mathbf{C}_4 \end{pmatrix} \middle| \hat{\Box} \middle| \begin{pmatrix} \mathbf{C}_1, \mathbf{R}_t, \mathbf{C}_3 \\ \mathbf{Q}_\ell, \mathbf{Q}_r \\ \mathbf{C}_2, \mathbf{R}_b, \mathbf{C}_4 \end{pmatrix} \right\rangle = \sqrt{\frac{\dim(\mathbf{R}_t)\dim(\mathbf{R}_b)}{\dim(\mathbf{R}'_t)\dim(\mathbf{R}'_b)\dim(\mathbf{Q}_\ell)\dim(\mathbf{Q}_r)\dim(\mathbf{Q}'_\ell)^3\dim(\mathbf{Q}'_r)^3}}$$

$$\begin{Bmatrix} \overline{\mathbf{R}}_t & \mathbf{C}_1 & \overline{\mathbf{Q}}_\ell \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \overline{\mathbf{R}}'_t & \mathbf{C}_1 & \overline{\mathbf{Q}}'_\ell \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_t & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}_r \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \mathbf{R}'_t & \overline{\mathbf{C}}_3 & \overline{\mathbf{Q}}'_r \end{Bmatrix} \begin{Bmatrix} \overline{\mathbf{R}}_b & \mathbf{C}_2 & \mathbf{Q}_\ell \\ \overline{\mathbf{3}} & \mathbf{1} & \overline{\mathbf{3}} \\ \overline{\mathbf{R}}'_b & \mathbf{C}_2 & \mathbf{Q}'_\ell \end{Bmatrix} \begin{Bmatrix} \mathbf{R}_b & \overline{\mathbf{C}}_4 & \mathbf{Q}_r \\ \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{R}'_b & \overline{\mathbf{C}}_4 & \mathbf{Q}'_r \end{Bmatrix}$$

Summary

Quantum computation/simulation of LGT demands for convenient framework and basis.

With the original Kogut-Susskind formalism: beyond Schwinger model is extremely difficult.

LSH formalism: takes care of most of the complications,
exactly equivalent to the original formulation,
transforms SU(2) LGT in any dimension equivalent to Schwinger model,

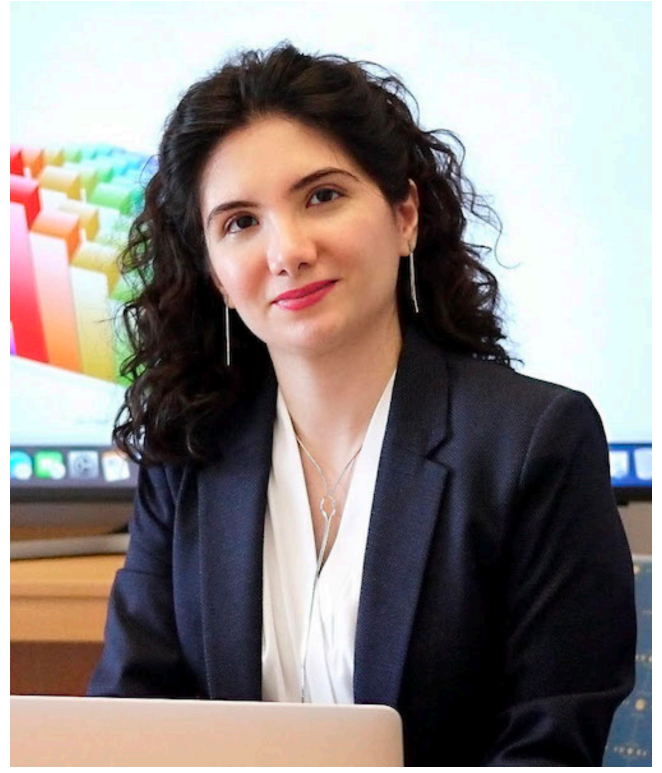
Immediate and straightforward applications of LSH both in analog and digital simulation has demonstrated profound advantages over any other framework

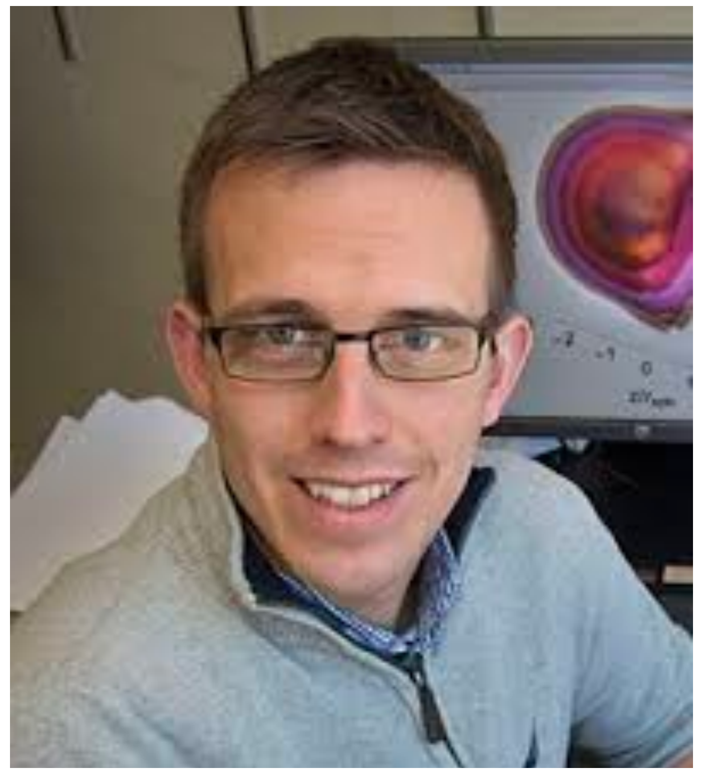
Future goal:

LSH formalism for SU(3) in 3+1 dimensions

—-—> expected to be useful for quantum computing QCD.

Current Collaborators


Zohreh Davoudi


Niklas Mueller


Jesse Stryker


Saurabh Kadam


Raka Dasgupta


Aniruddha Bapat


Alexey Gorshkov


Przemyslaw Bienias


Mohammad Hafezi


Victor Albert

Thank You

Back-up slides

Loop String Hadron (LSH) Formulation in 1 spatial dimension

$$\hat{E}^{\mathrm{a}}_{L/R} \equiv \hat{a}^\dagger(L/R) T^{\mathrm{a}} \hat{a}(L/R)$$

$$[\hat{E}^{\mathrm{a}}_L, \hat{E}^{\mathrm{b}}_L] = i\epsilon^{\mathrm{abc}} \hat{E}^{\mathrm{c}}_L,$$

$$[\hat{E}^{\mathrm{a}}_R, \hat{E}^{\mathrm{b}}_R] = i\epsilon^{\mathrm{abc}} \hat{E}^{\mathrm{c}}_R,$$

$$[\hat{E}^{\mathrm{a}}_L, \hat{E}^{\mathrm{b}}_R] = 0.$$

$$\hat{E}^2 \equiv \hat{E}^{\mathrm{a}}_L \hat{E}^{\mathrm{a}}_L = \hat{E}^{\mathrm{a}}_R \hat{E}^{\mathrm{a}}_R$$

$$\hat{N}_{L/R} = \hat{a}^\dagger(L/R) \cdot \hat{a}(L/R)$$

$$[\hat{E}^{\mathrm{a}}_L, \hat{U}] = -T^{\mathrm{a}} \hat{U},$$

$$[\hat{E}^{\mathrm{a}}_R, \hat{U}] = +\hat{U} T^{\mathrm{a}},$$

$$[\hat{U}_{\alpha\beta}, \hat{U}_{\gamma\delta}] = [\hat{U}_{\alpha\beta}, (\hat{U}_{\gamma\delta})^\dagger] = 0$$

$$\hat{U}_L \equiv \frac{1}{\sqrt{\hat{N}_L + 1}} \begin{pmatrix} \hat{a}^\dagger_2(L) & \hat{a}_1(L) \\ -\hat{a}^\dagger_1(L) & \hat{a}_2(L) \end{pmatrix},$$

$$\hat{U}_R \equiv \begin{pmatrix} \hat{a}^\dagger_1(R) & \hat{a}^\dagger_2(R) \\ -\hat{a}_2(R) & \hat{a}_1(R) \end{pmatrix} \frac{1}{\sqrt{\hat{N}_R + 1}}.$$

Abelian Gauss' Law

$$N_L(x) = N_R(x)$$

Local SU(2) Invariant Operators in 1d: loops-strings- hadrons

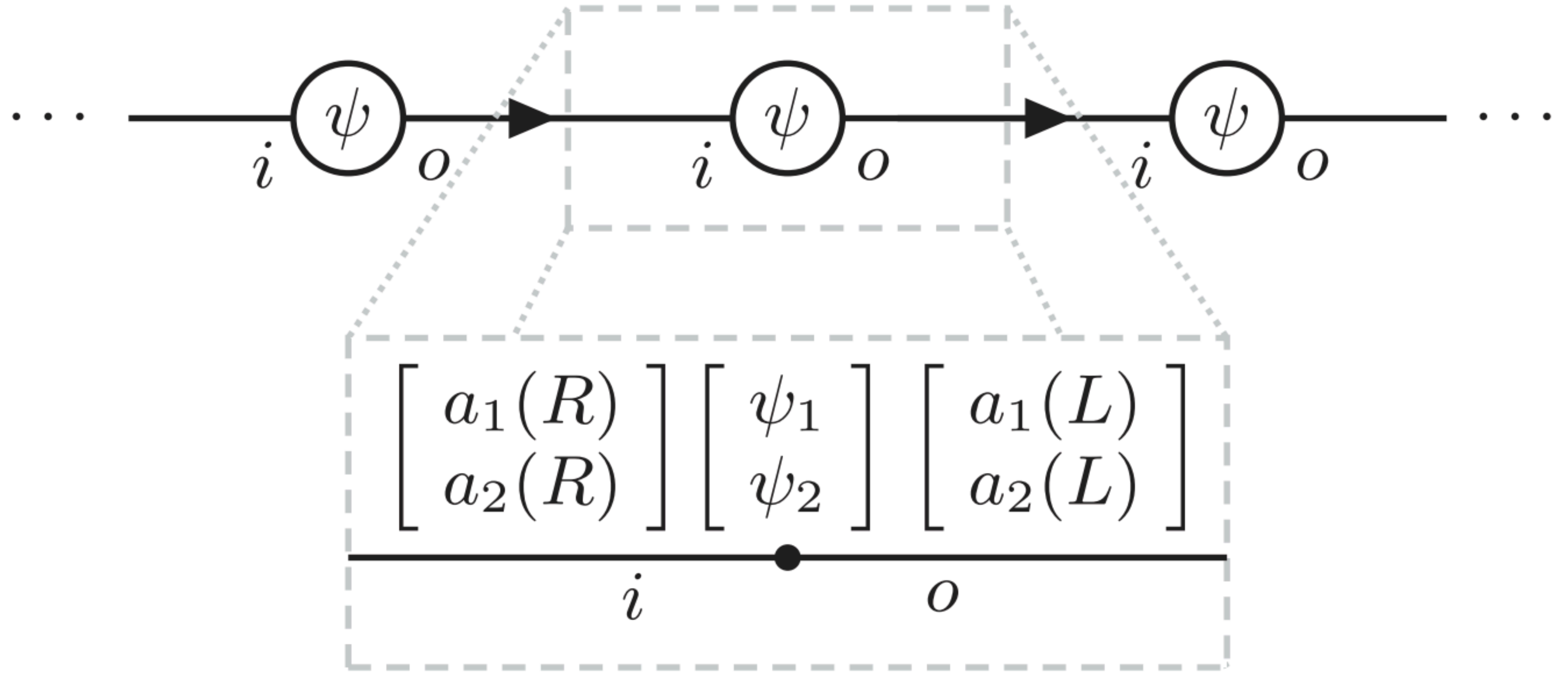
(i) *Pure gauge loop operators.*—$\mathcal{L}^{\sigma,\sigma'}$:

$$\mathcal{L}^{++} = a(R)^\dagger_\alpha a(L)^\dagger_\beta \epsilon_{\alpha\beta}$$

$$\mathcal{L}^{--} = a(R)_\alpha a(L)_\beta \epsilon_{\alpha\beta} = (\mathcal{L}^{++})^\dagger$$

$$\mathcal{L}^{+-} = a(R)^\dagger_\alpha a(L)_\beta \delta_{\alpha\beta}$$

$$\mathcal{L}^{-+} = a(R)_\alpha a(L)^\dagger_\beta \delta_{\alpha\beta} = (\mathcal{L}^{+-})^\dagger.$$



(ii) *Incoming string operators.*—$\mathcal{S}^{\sigma,\sigma'}_{\text{in}}$:

$$\mathcal{S}^{++}_{\text{in}} = a(R)^\dagger_\alpha \psi^\dagger_\beta \epsilon_{\alpha\beta}$$

$$\mathcal{S}^{--}_{\text{in}} = a(R)_\alpha \psi_\beta \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{in}})^\dagger$$

$$\mathcal{S}^{+-}_{\text{in}} = a(R)^\dagger_\alpha \psi_\beta \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{in}} = a(R)_\alpha \psi^\dagger_\beta \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{in}})^\dagger.$$

Outgoing string operators.—$\mathcal{S}^{\sigma,\sigma'}_{\text{out}}$:

$$\mathcal{S}^{++}_{\text{out}} = \psi^\dagger_\alpha a(L)^\dagger_\beta \epsilon_{\alpha\beta}$$

$$\mathcal{S}^{--}_{\text{out}} = \psi_\alpha a(L)_\beta \epsilon_{\alpha\beta} = (\mathcal{S}^{++}_{\text{out}})^\dagger$$

$$\mathcal{S}^{+-}_{\text{out}} = \psi^\dagger_\alpha a(L)_\beta \delta_{\alpha\beta}$$

$$\mathcal{S}^{-+}_{\text{out}} = \psi_\alpha a(L)^\dagger_\beta \delta_{\alpha\beta} = (\mathcal{S}^{+-}_{\text{out}})^\dagger.$$

Hadron operators.—$\mathcal{H}^{\sigma,\sigma}$:

$$\mathcal{H}^{++} = -\frac{1}{2!}\psi^\dagger_\alpha \psi^\dagger_\beta \epsilon_{\alpha\beta}$$

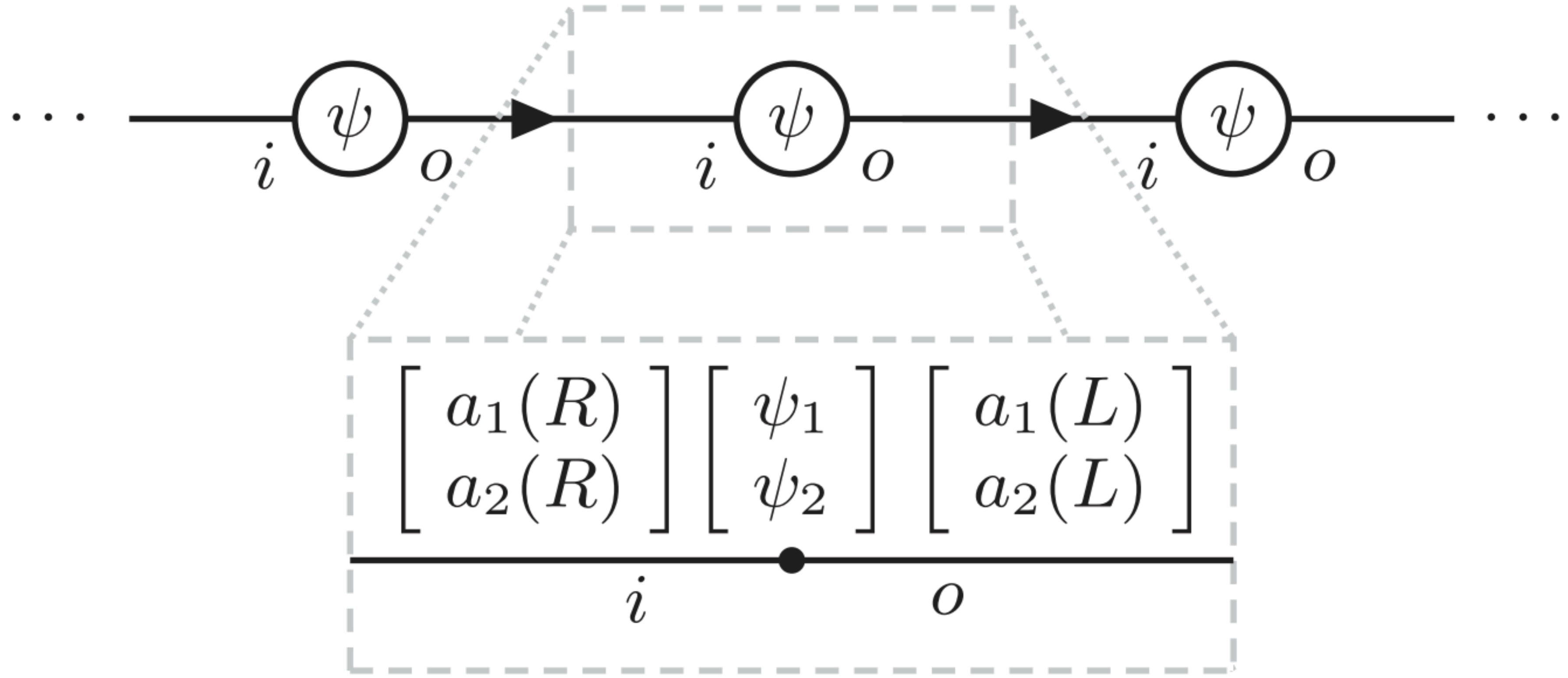
$$\mathcal{H}^{--} = \frac{1}{2!}\psi_\alpha \psi_\beta \epsilon_{\alpha\beta} = (\mathcal{H}^{++})^\dagger.$$

$$(1/2)\mathcal{L}^{--}(\mathcal{S}^{++}_{\text{in}})^{n_i}(\mathcal{S}^{++}_{\text{out}})^{n_o}|0\rangle = \delta_{n_i,1}\delta_{n_o,1}\mathcal{H}^{++}|0\rangle$$

Local SU(2) Invariant Operators in 1d: loops-strings- hadrons

(i) *Pure gauge loop operators.*—$\mathcal{L}^{\sigma,\sigma'}$:

Gauge singlets constructed out of left and right bosons



(ii) *Incoming string operators.*—$\mathcal{S}_{\text{in}}^{\sigma,\sigma'}$:

Gauge singlets constructed out of left bosons and fermions

Outgoing string operators.—$\mathcal{S}_{\text{out}}^{\sigma,\sigma'}$:

Gauge singlets constructed out of Right boson and fermions

Hadron operators.—$\mathcal{H}^{\sigma,\sigma}$:

Gauge singlets constructed out of two fermions

$$(1/2)\mathcal{L}^{--}(\mathcal{S}_{\text{in}}^{++})^{n_i}(\mathcal{S}_{\text{out}}^{++})^{n_o}|0\rangle = \delta_{n_i,1}\delta_{n_o,1}\mathcal{H}^{++}|0\rangle$$

LSH Formulation: local LSH basis

At each site define: $n_l(x), n_i(x), n_o(x)$,

$$|n_l, n_i, n_o\rangle = \left(\mathscr{L}^{++}\right)^{n_l} \left(\mathscr{S}_i^{++}\right)^{n_i} \left(\mathscr{S}_o^{++}\right)^{n_o} |0\rangle$$

$$\begin{aligned} &0 \le n_l(x) \le \infty, \\ &n_i(x) \in \{0,1\}, \\ &n_o(x) \in \{0,1\}. \end{aligned}$$

Abelian weaving along the link: $\hat{n}_l(x) + \hat{n}_o(x)(1 - \hat{n}_i(x)) = \hat{n}_l(x+1) + \hat{n}_i(x+1)(1 - \hat{n}_o(x+1))$

Hamiltonian, describing dynamics of loops, strings and hadrons.

$$H^{(\mathrm{LSH})} = H_I^{(\mathrm{LSH})} + H_E^{(\mathrm{LSH})} + H_M^{(\mathrm{LSH})}$$

$$H_I^{(\mathrm{LSH})} = \frac{1}{2a}\sum_n \Bigg\{ \frac{1}{\sqrt{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x)) + 1}} \times \left[\hat{S}_o^{++}(x)\hat{S}_i^{+-}(x+1) + \hat{S}_o^{+-}(x)\hat{S}_i^{--}(x+1)\right] \times \frac{1}{\sqrt{\hat{n}_l(x+1) + \hat{n}_i(x+1)(1-\hat{n}_o(x+1)) + 1}} + \mathrm{h.c.} \Bigg\},$$

$$H_E^{(\mathrm{LSH})} = \frac{g^2 a}{2}\sum_n \left[\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} \times \left(\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} + 1 \right) \right],$$

$$H_M^{(\mathrm{LSH})} = m \sum_n (-1)^x (\hat{n}_i(x) + \hat{n}_o(x)),$$

$$\begin{aligned}
\hat{S}_o^{++} &= \hat{\chi}_o^{+}(\lambda^+)^{\hat{n}_i}\sqrt{\hat{n}_l + 2 - \hat{n}_i},\\
\hat{S}_o^{--} &= \hat{\chi}_o^{-}(\lambda^-)^{\hat{n}_i}\sqrt{\hat{n}_l + 2(1 - \hat{n}_i)},\\
\hat{S}_o^{+-} &= \hat{\chi}_i^{+}(\lambda^-)^{1-\hat{n}_o}\sqrt{\hat{n}_l + 2\hat{n}_o},\\
\hat{S}_o^{-+} &= \hat{\chi}_i^{-}(\lambda^+)^{1-\hat{n}_o}\sqrt{\hat{n}_l + 1 + \hat{n}_o},
\end{aligned}$$

$$\begin{aligned}
\hat{S}_i^{+-} &= \hat{\chi}_o^{-}(\lambda^+)^{1-\hat{n}_i}\sqrt{\hat{n}_l + 1 + \hat{n}_i},\\
\hat{S}_i^{-+} &= \hat{\chi}_o^{+}(\lambda^-)^{1-\hat{n}_i}\sqrt{\hat{n}_l + 2\hat{n}_i},\\
\hat{S}_i^{--} &= \hat{\chi}_i^{-}(\lambda^-)^{\hat{n}_o}\sqrt{\hat{n}_l + 2(1 - \hat{n}_o)},\\
\hat{S}_i^{++} &= \hat{\chi}_i^{+}(\lambda^+)^{\hat{n}_o}\sqrt{\hat{n}_l + 2 - \hat{n}_o}.
\end{aligned}$$

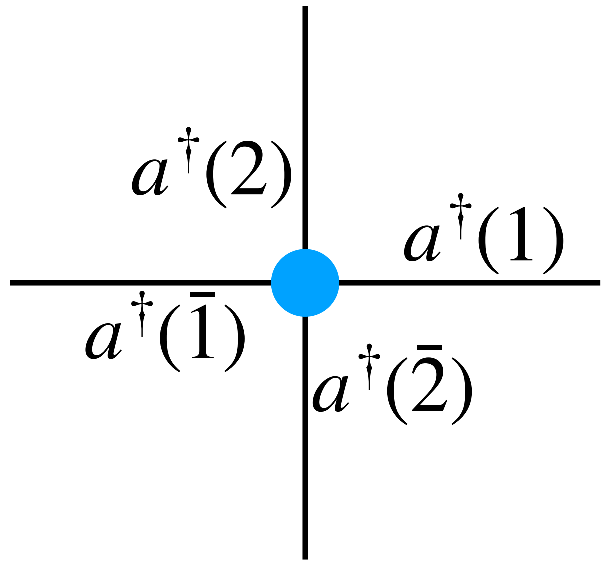
The strong-coupling vacuum of the LSH Hamiltonian is given by

$$\begin{aligned}
&n_l(x) = 0, \ \text{for all } x,\\
&n_i(x) = 0, \ n_o(x) = 0, \ \text{for } x \text{ even},\\
&n_i(x) = 1, \ n_o(x) = 1, \ \text{for } \ x \text{ odd}.
\end{aligned}$$

Hamiltonian, describing dynamics of loops, strings and hadrons: Identical spectrum to KS

LSH framework in $d > 1$

Prepotential Formulation for 2+1 d:

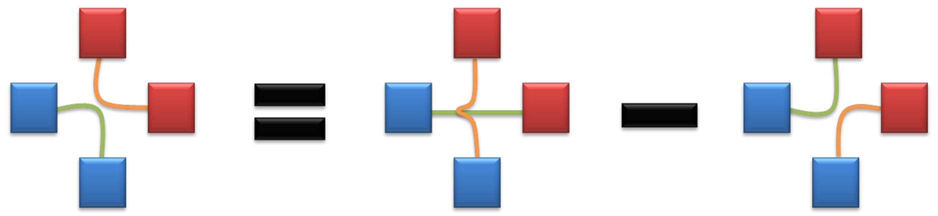


Local Loop Operator: $\mathscr{L}^{++}_{ij} = \epsilon^{\alpha\beta} a^\dagger_\alpha(i) a^\dagger_\beta(j)$

Pictorial representation:

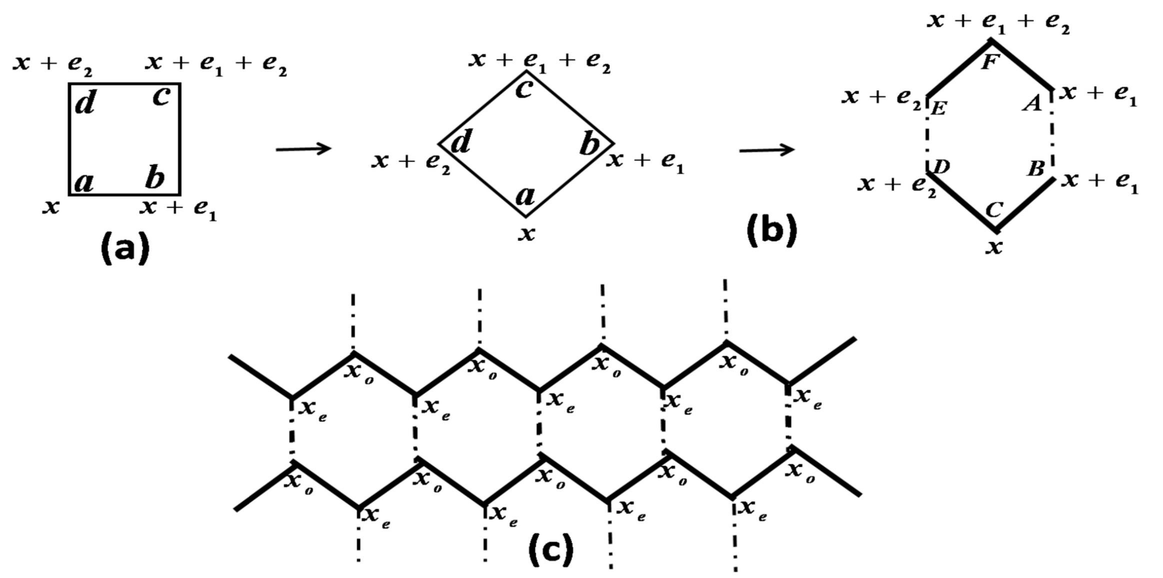
Overcomplete

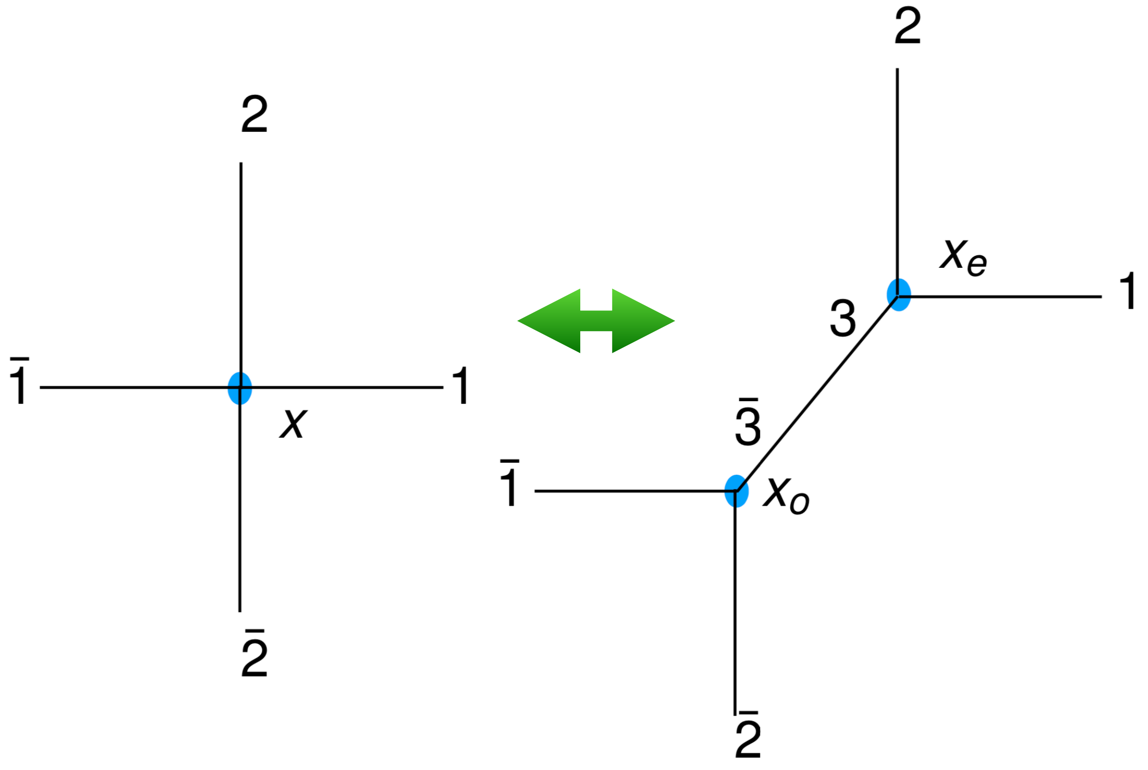
3 physical d.o.f = 6 (local loop quantum numbers in 2d)
- 2(Abelian Gauss' law constraint along 2 link directions)
-1 (**Mandelstam constraint**)

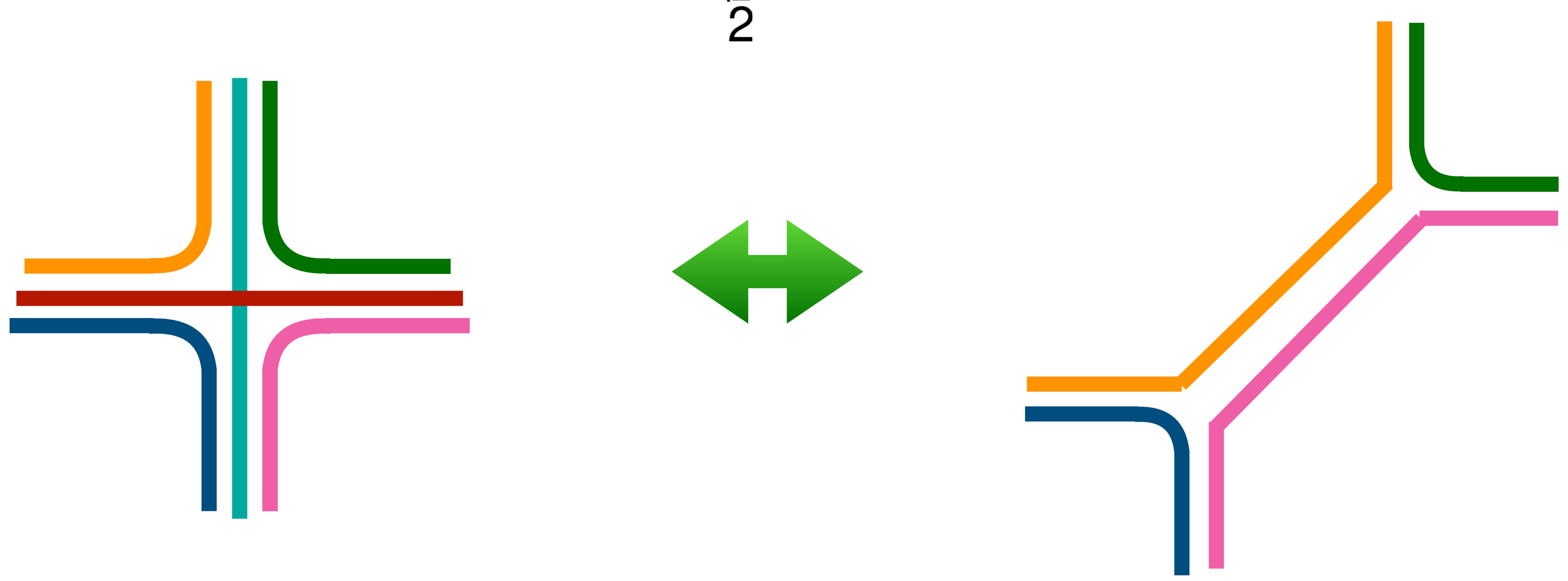


Non-linear constraints, become increasingly complicated with increasing dimension

Way out? Virtual point splitting scheme:



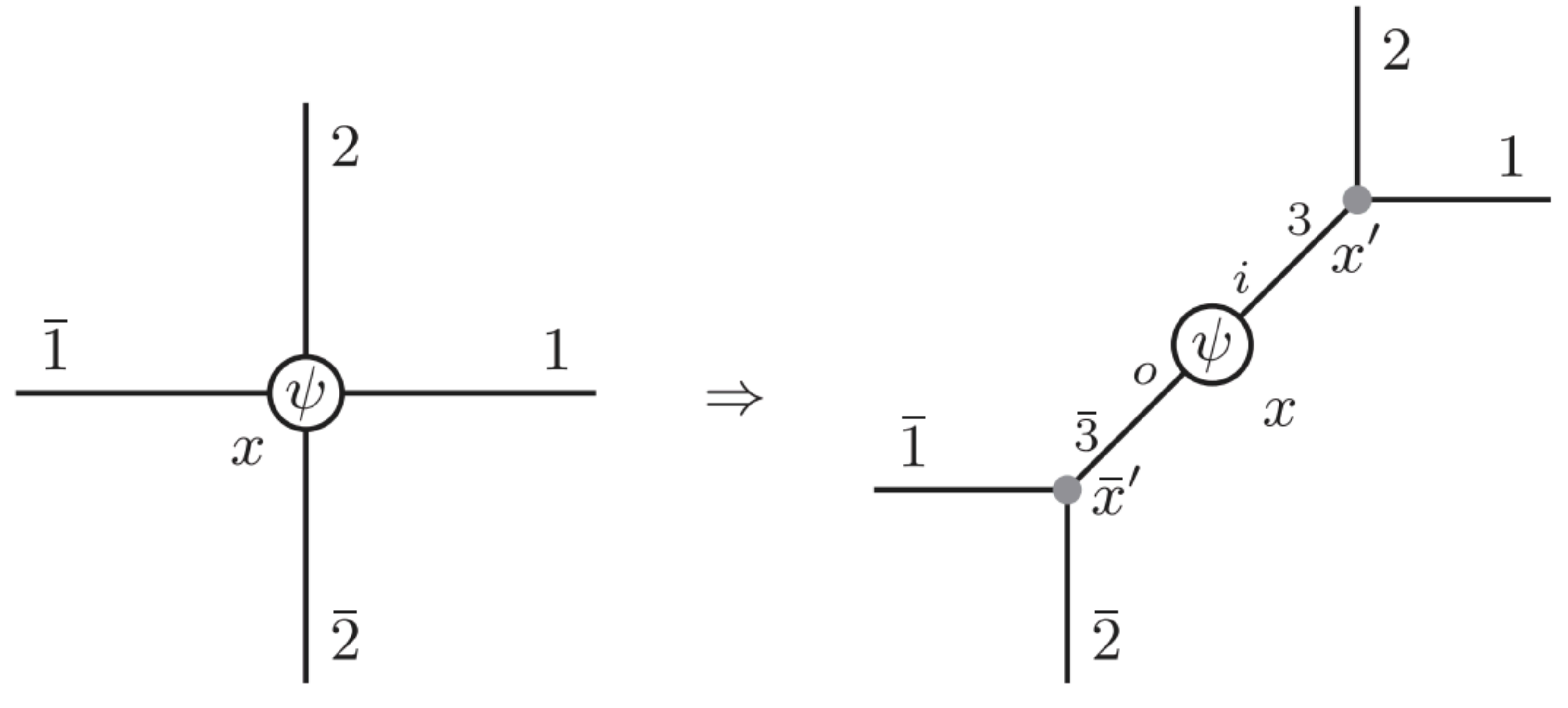




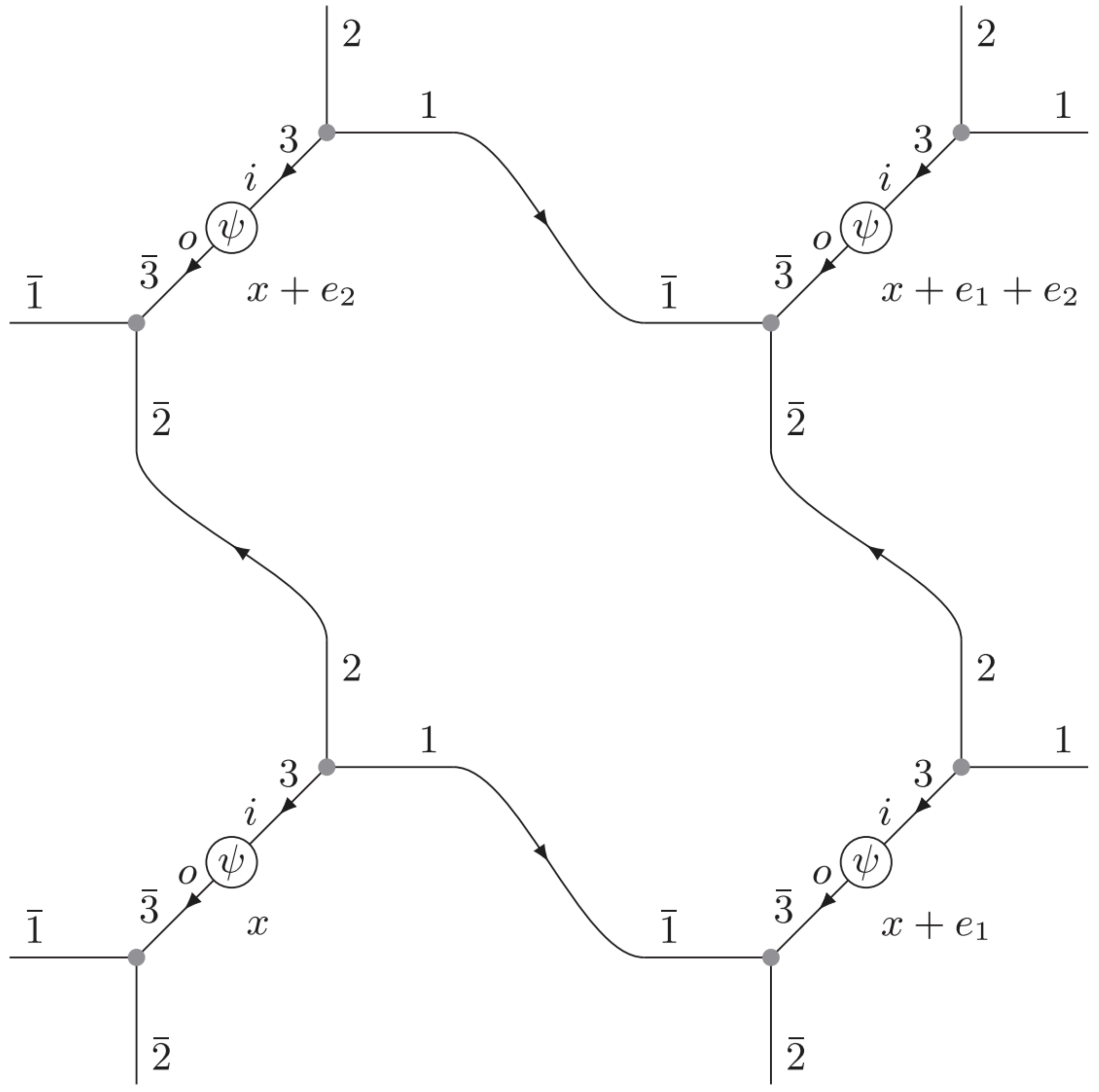
3 physical d.o.f = 2 x 3 (local loop quantum numbers in 2d)
- 3(Abelian Gauss' law constraint)
+ **0** (**Mandelstam constraint**)

Generalized for arbitrary dimension!

LSH Formalism: 2+1 d

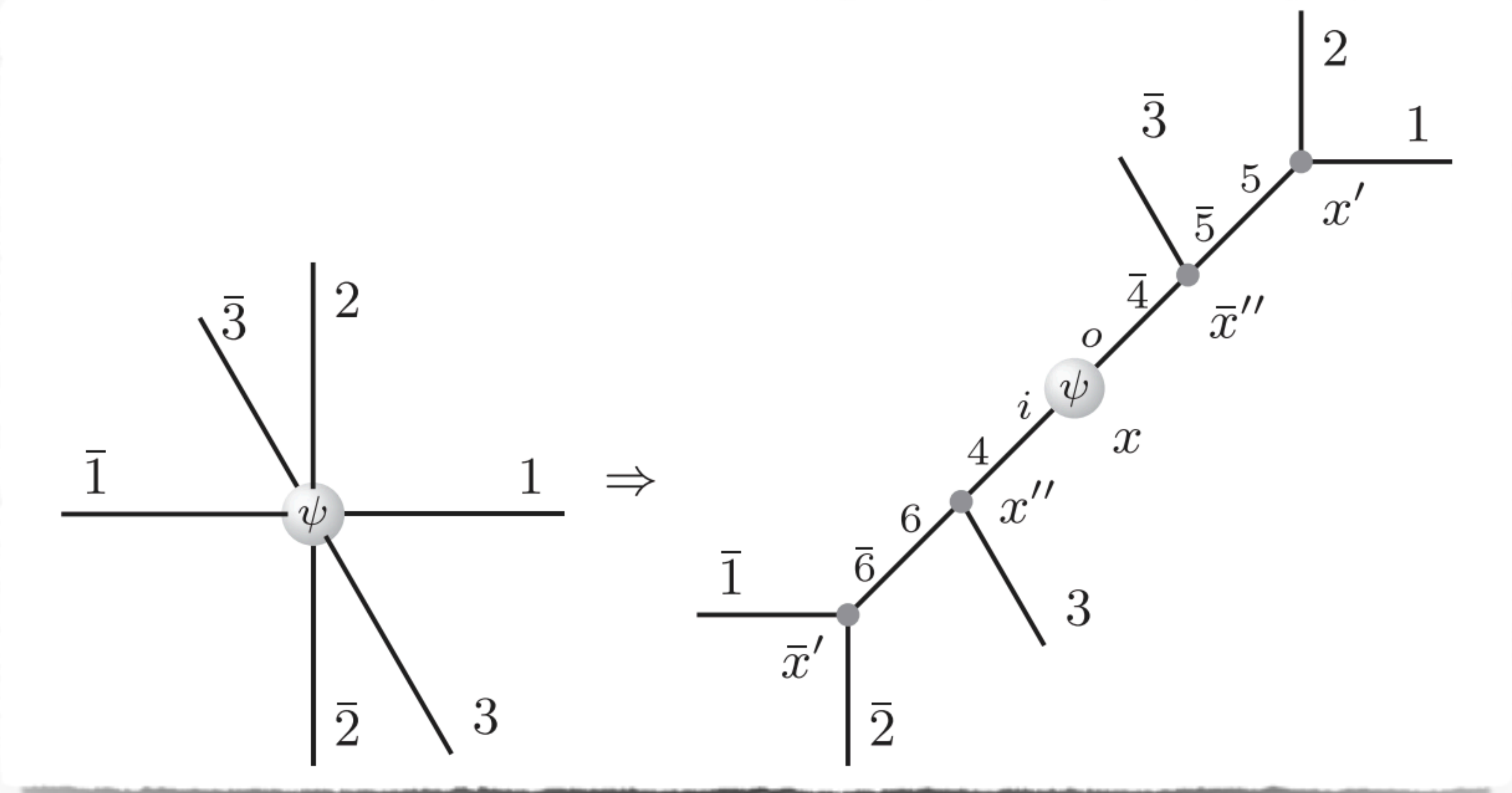


(a) (b)



Matter-Gauge interactions are same as in 1d

LSH Formalism: 3+1 d



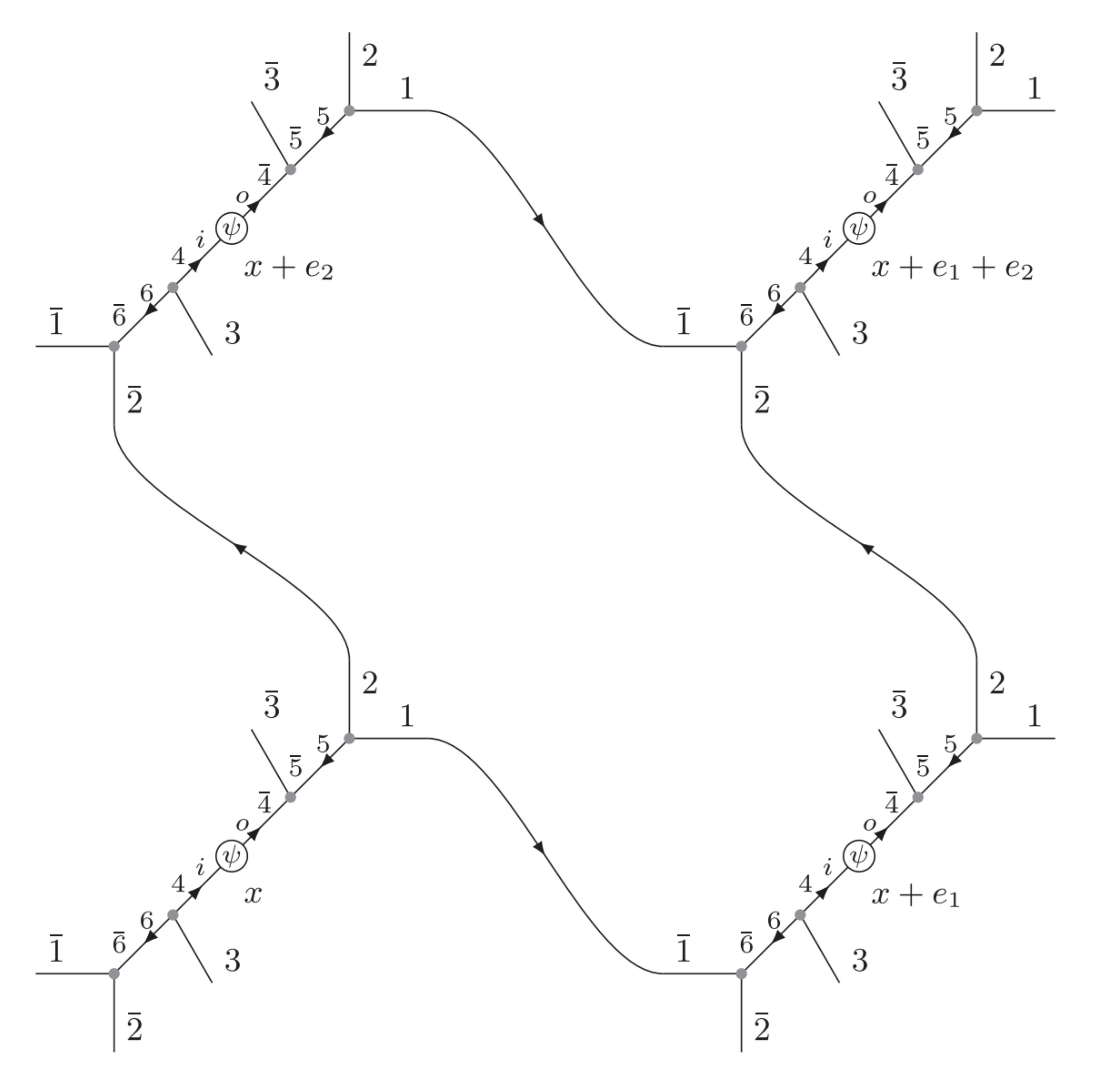


FIG. 7. Connectivity of a *xy*-plaquette in three dimensions.

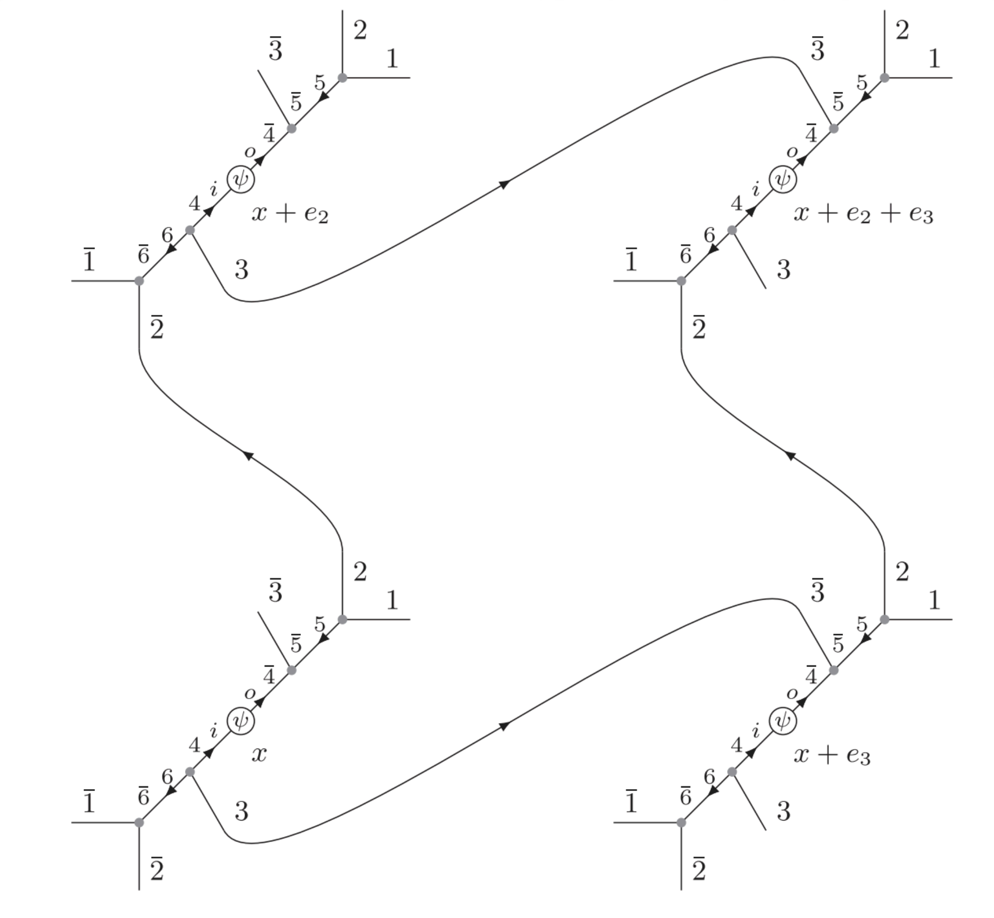


FIG. 8. Connectivity of a *yz*-plaquette in three dimensions.

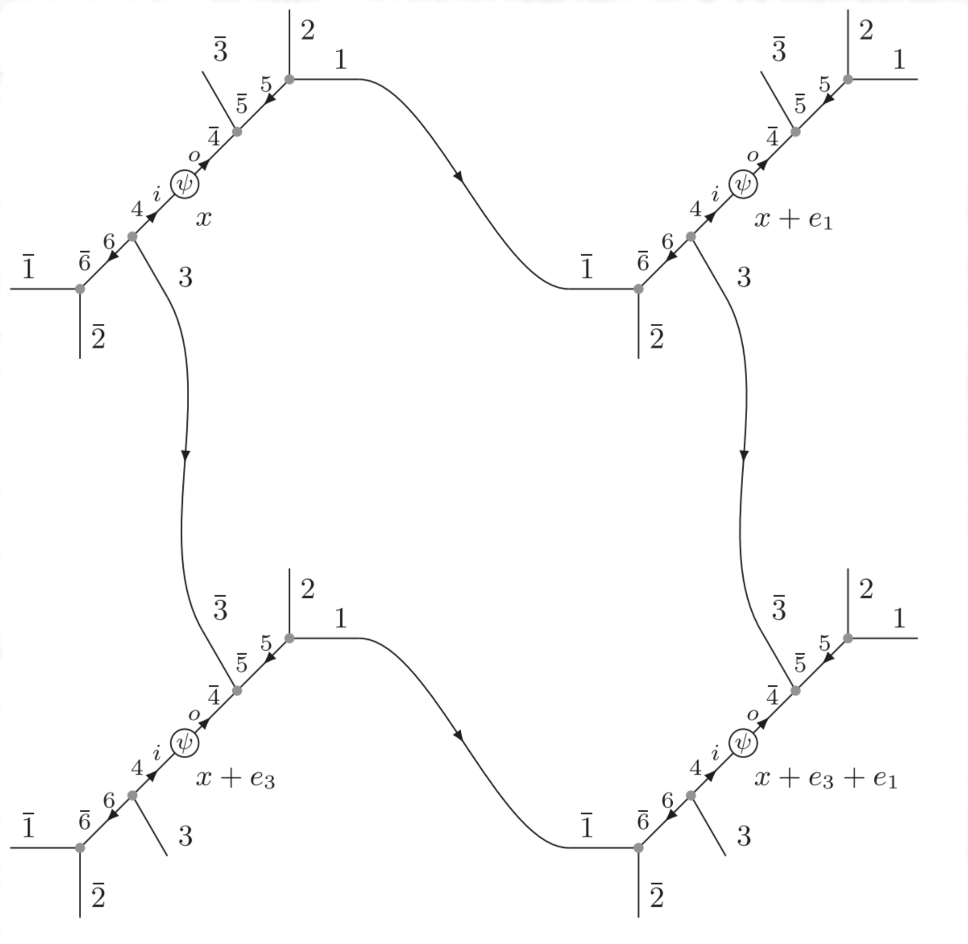


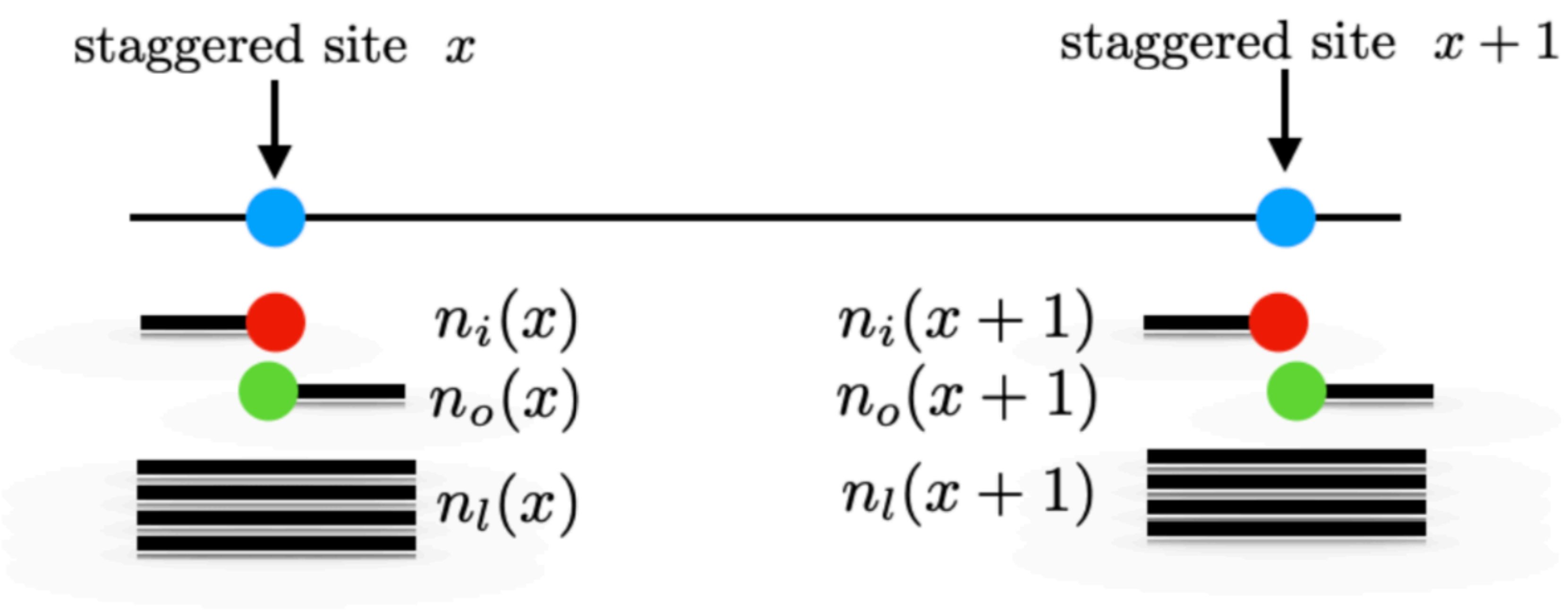
FIG. 9. Connectivity of a *zx*-plaquette in three dimensions.

- Matter-Gauge interactions are same as in 1+1d
- Pure gauge interactions are same as in 2+1d

Application: Analog Quantum computation

Mean Field Approximaton for LSH

In 1d, loop flux is not independent dynamical observable



Low energy sector of weak coupling limit: $g \to 0$ ➡ $n_l \gg 0$

$$H_E^{(\mathrm{LSH})} = \frac{g^2 a}{2} \sum_n \left[\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} \times \left(\frac{\hat{n}_l(x) + \hat{n}_o(x)(1-\hat{n}_i(x))}{2} + 1 \right) \right],$$

$$H_M^{(\mathrm{LSH})} = m \sum_n (-1)^x (\hat{n}_i(x) + \hat{n}_o(x)),$$

Approximation: $n_l(j) = l_i \equiv n_l \quad \forall j.$

Valid Approximation in weak coupling regime!

Application: Analog Quantum computation

Simulated Hamiltonian

$g \to 0$

$$H_E^{(\text{approx})} = \frac{g^2 a}{2} N h_E^0$$

Approximated electric term,
negligible contribution

$$H_M^{(\text{approx})} = m \sum_j (-1)^j (\hat{n}_i(j) + \hat{n}_o(j))$$

$$H_I^{(\text{approx})} = \frac{1}{2a} \sum_j \Big[\chi_o^+(j)\chi_o^-(j+1) + \chi_o^-(j)\chi_o^+(j+1) + \chi_i^+(j)\chi_i^-(j+1) + \chi_i^-(j)\chi_i^+(j+1)\Big]$$

Approximated hopping term,
good approximation,
significant contribution

$g \to \infty$

$$H_E^{(\text{mLSH})} = \frac{g^2 a}{2}\left[N\frac{n_l}{2}\left(\frac{n_l}{2}+1\right) + \frac{N}{4}\left(\frac{n_l}{2}+\frac{3}{4}\right)\right]$$

Approximated electric term + correction,
significant contribution

$$H_M^{(\text{approx})} = m \sum_j (-1)^j (\hat{n}_i(j) + \hat{n}_o(j))$$

$$H_I^{(\text{approx})} = \frac{1}{2a} \sum_j \Big[\chi_o^+(j)\chi_o^-(j+1) + \chi_o^-(j)\chi_o^+(j+1) + \chi_i^+(j)\chi_i^-(j+1) + \chi_i^-(j)\chi_i^+(j+1)\Big]$$

Approximated hopping term,
bad approximation,
negligible contribution