Flavour Physics at Hadron Colliders

Lecture II: Unitarity Triangle metrology and CPV measurements

Guy Wilkinson
University of Oxford

Future Flavours, ICTS
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Lecture-II outline

- CKM matrix and the Unitarity Triangle(s)
- CKM metrology at the LHC
  - B_d and B_s mixing measurements
  - $|V_{ub}/V_{cb}|$
  - Measuring β and the challenges of time-dependent CPV measurements at a hadron machine
  - The long road to a precise determination of γ in $B \to DK$
  - The quest for φ_s: CPV violation in $B_s \to J/\psi\varphi$ and friends
- Conclusions and outlook

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There are *many more* CPV measurements being performed at the LHC than these !

Unitarity Triangles(s)

The CKM matrix must be unitarity: $V_{\mathrm{CKM}}^{\dagger} V_{\mathrm{CKM}} = V_{\mathrm{CKM}} V_{\mathrm{CKM}}^{\dagger} = 1$

This imposes various constraints, including $\sum_k V_{ik} V_{jk}^* = 0$ where $i \neq j$.

The are 6 such independent relations, which can be represented as **unitarity triangles** in the complex plane. Experimentally, the most interesting is:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

As the sides are of similar length, & its parameters can be studied in B^0, B^+ decays.

Another, relevant for B^0_s physics is:

$$V_{us} V_{ub}^* + V_{cs} V_{cb}^* + V_{ts} V_{tb}^* = 0$$

Note that the area of all triangles is the same = ½ J, the Jarlskog invariant.

$$J = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin\delta \approx 3 \times 10^{-5}$$

[Jarlskog, PRL 55 (1985) 1039]

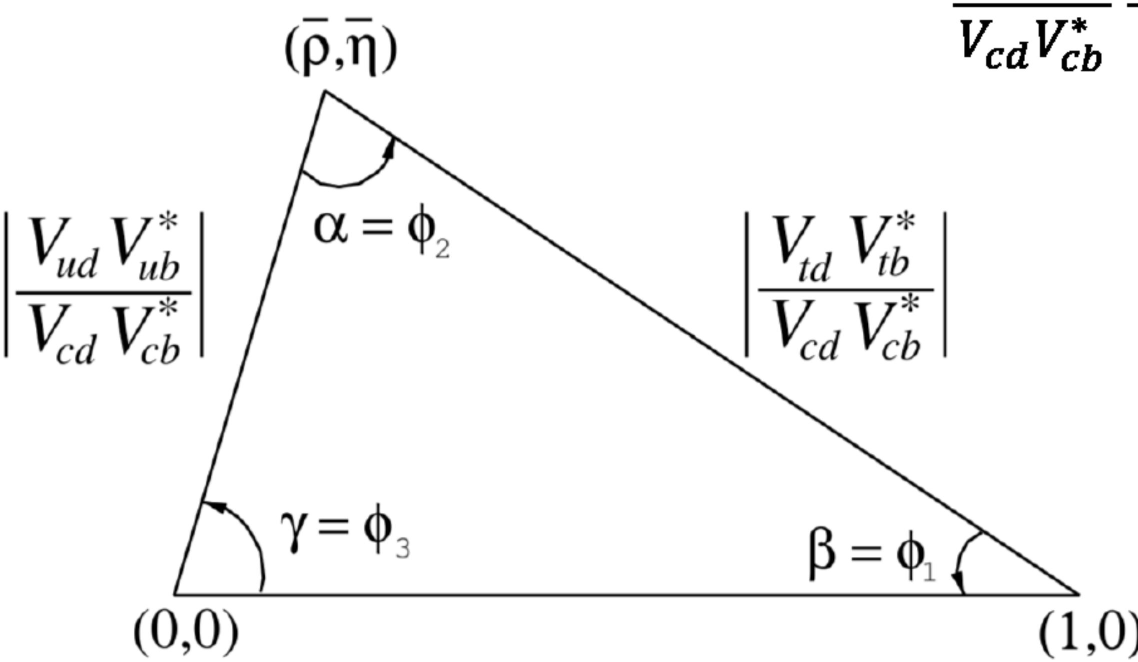
'The' Unitarity Triangle

Three complex vectors sum to zero

→ triangle in Argand plane

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$



Expressions for angles:

$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ub}V_{cb}^*}\right]$$

$$\beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

Upper vertex: $\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$

$\bar{\rho} = \rho(1 - \lambda^2/2 + \cdots)$ $\bar{\eta} = \eta(1 - \lambda^2/2 + \cdots)$

(φ_2, φ_1 & φ_3 alternative notation)

'The' Unitarity Triangle

Three complex vectors sum to zero

→ triangle in Argand plane

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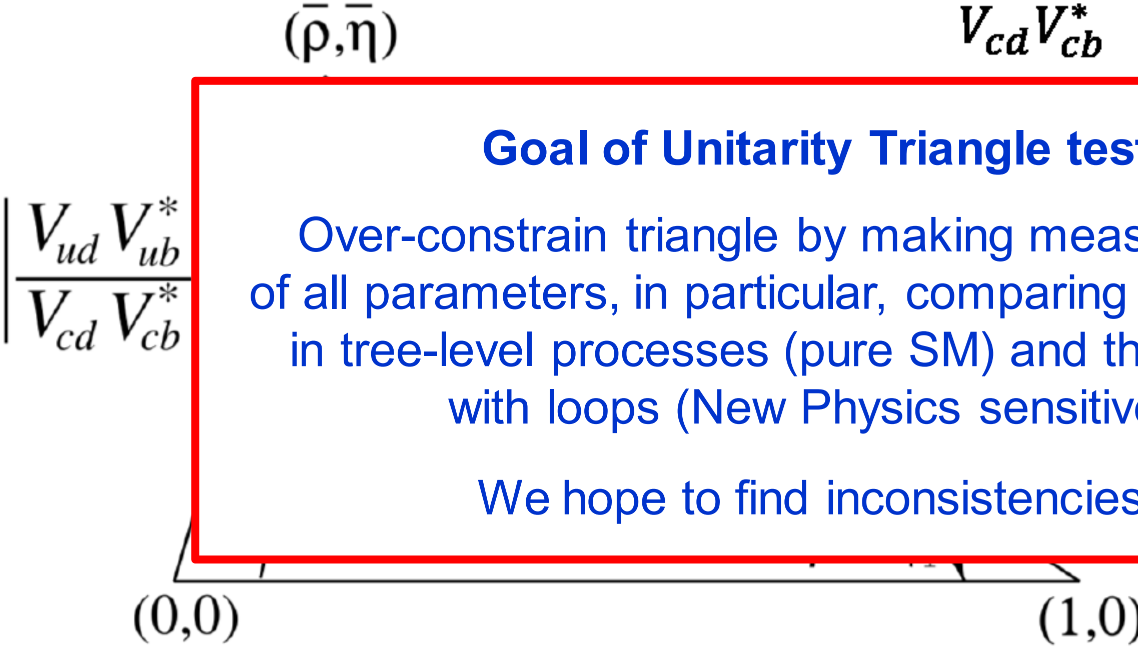


Goal of Unitarity Triangle tests

Over-constrain triangle by making measurements of all parameters, in particular, comparing those made in tree-level processes (pure SM) and those made with loops (New Physics sensitive).

We hope to find inconsistencies !

for angles:

$$\left[-\frac{V_{td}V_{tb}^*}{V_{ub}V_{cb}^*}\right]$$

$$\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

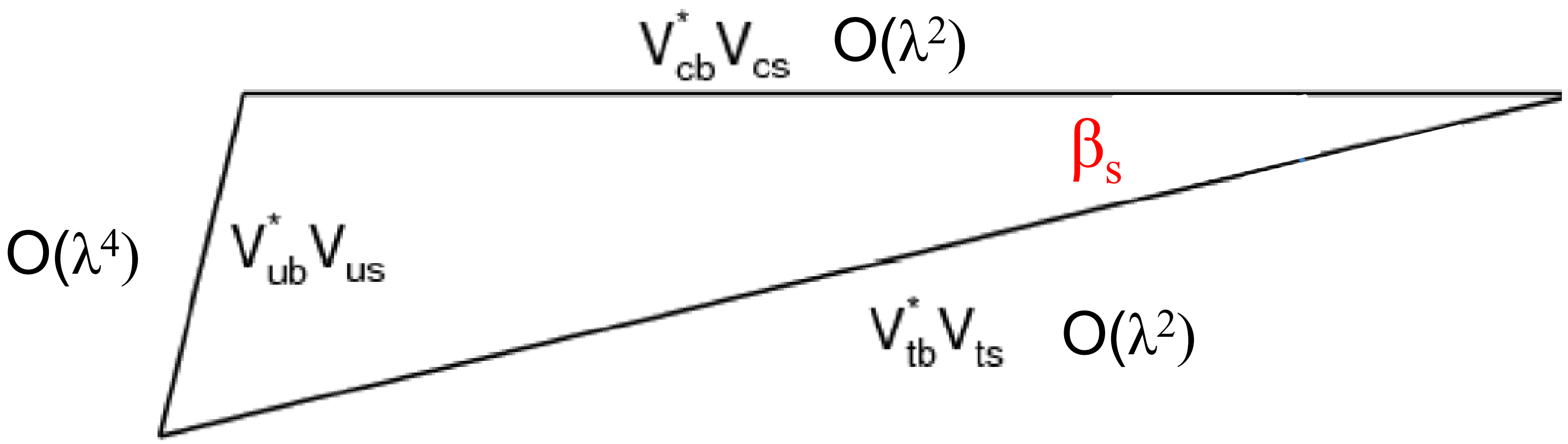
Upper vertex: $\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$

$\bar{\rho} = \rho(1-\lambda^2/2+\cdots)$ $\bar{\eta} = \eta(1-\lambda^2/2+\cdots)$

(φ_2, φ_1 & φ_3 alternative notation)

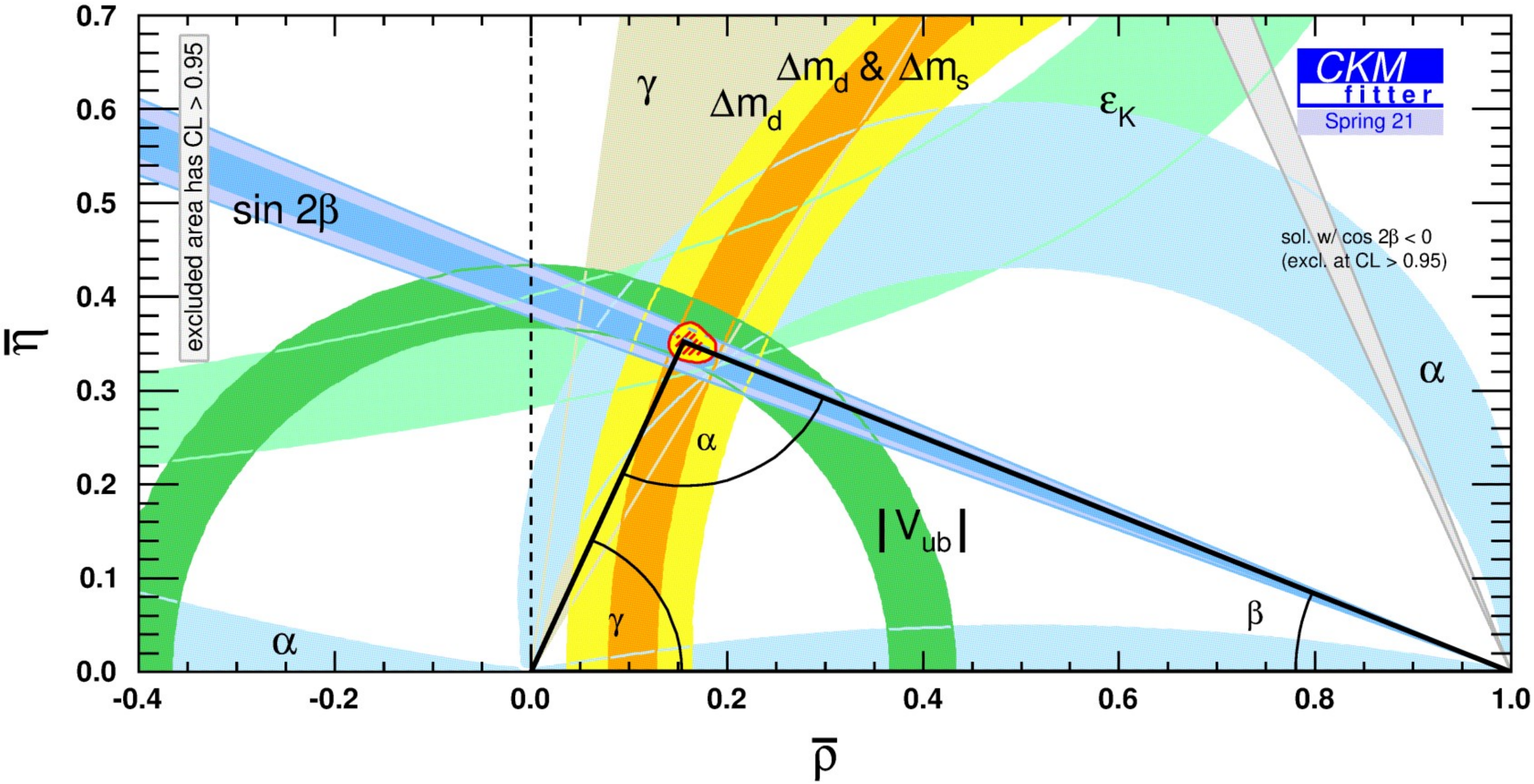
The B^0_s Unitarity Triangle

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$



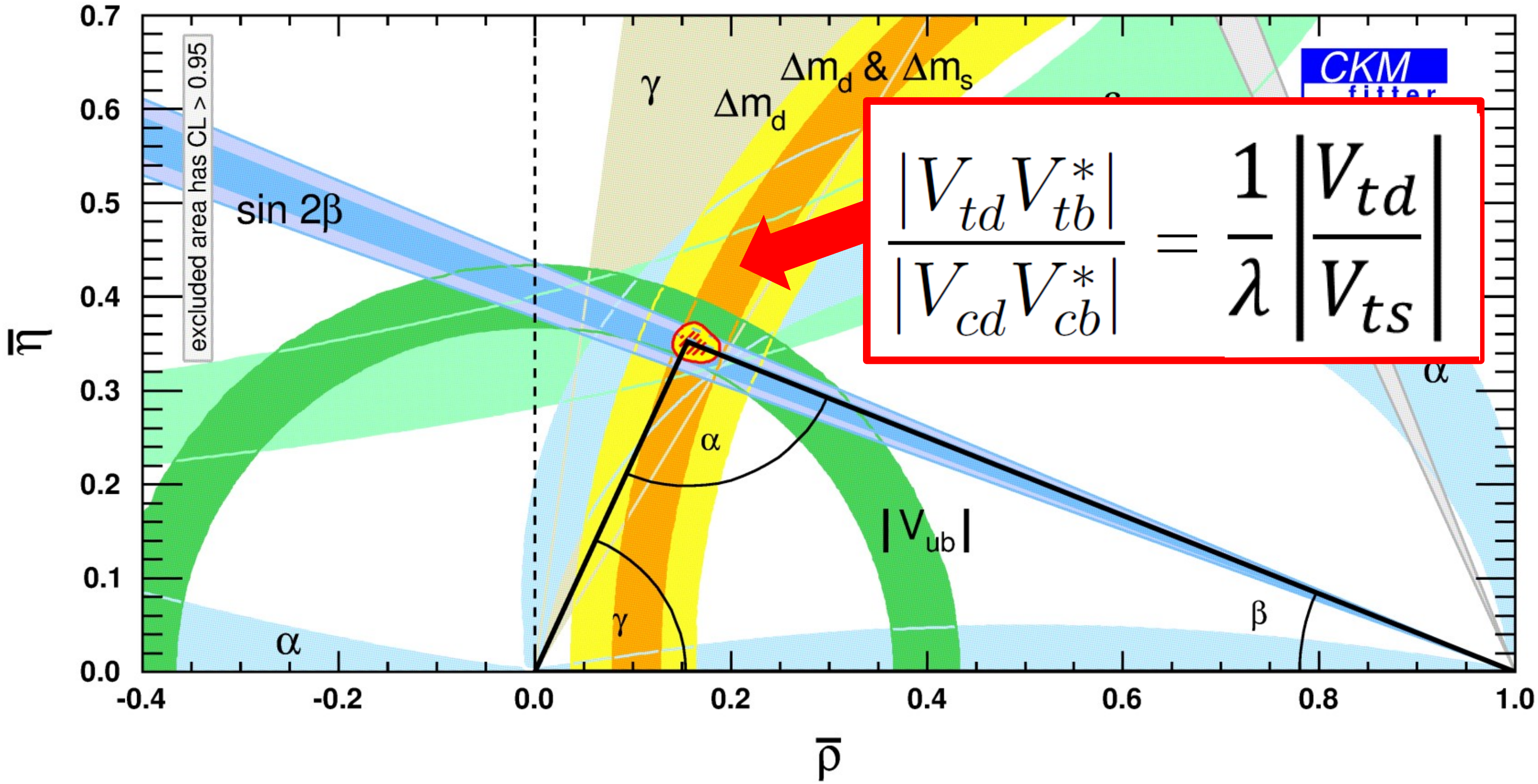
The B^0_s triangle is very squashed, & contains a small angle β_s (= $-\varphi_s/2$ – see later).

The Unitarity Triangle – CKM metrology at hadron machines



[CKMfitter, 2021]

The Unitarity Triangle – CKM metrology at hadron machines



$$\frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{ts}}\right|$$

[CKMfitter, 2021]

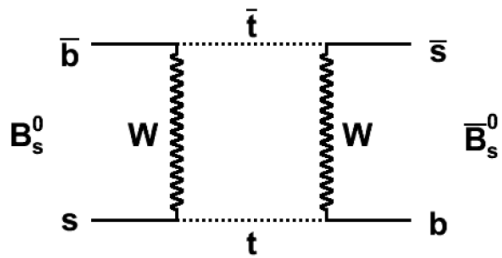
Length of side opposite γ is given by ratio of B^0 & B^0_s mixing freq.s & lattice QCD.

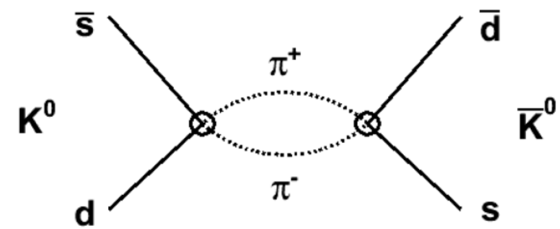
Neutral-meson mixing

Mixing is critical for much of following discussion, so warrants a recap of essentials.

Phenomenon occurs for K^0, D^0, B^0 and B^0_s systems. Physically caused by either and/or

Virtual, Short-range (box diagrams)





On-shell, long-range (common intermediate states)

Physical states are superposition of flavour eigenstates

Subscripts indicate Short or Long lived (see K^0 system); sometimes Heavy or Light used, or 1, 2.

$$B^0_{S,L} = pB^0 \pm q\overline{B^0}$$

p & *q* are complex and $|p|^2+|q|^2=1$

If CP is conserved the physical states = CP eigenstates, which means $\left|\frac{q}{p}\right| = 1$.

Known not to be the case in the K^0 system, where $\varepsilon = \frac{p-q}{q-p} \approx 2\times10^{-3}$ and the SM calculations indicate small, but finite, breaking in other systems too.

Mass and width splittings between physical states:

$$\Delta m = m_L - m_S$$ set by short-range effects

$$\Delta\Gamma = \Gamma_S - \Gamma_L$$ set by long-range effects

Neutral-meson mixing

There is a wide range in the sizes of the mixing parameters across the four systems, which has significant practical consequences for measurements.

	$\Delta m / \Gamma$		$\Delta\Gamma/2\Gamma$	
K^0	Large	~500	Maximal	~1
D^0	Small	0.39 ± 0.11 %	Small	$0.65 \pm 0.06\%$
B^0	Medium	0.769 ± 0.004	Small	$(20 \pm 5) \times 10^{-4}$
B^0_s	Large	26.81 ± 0.08	Medium	0.0675 ± 0.004

Refs: PDG, HFLAV and [Lenz & Nierste, JHEP 0706 (2007) 072]

Size of mixing effects is highly sensitive to SM parameters (CKM elements, GIM mechanism, quark masses…) and could easily be perturbed by New Physics. Indeed, mixing can be used to set severe bounds (~10^3 TeV) on most general forms of New Physics models (see *e.g.* Nir arXiv:1605.00433).

Neutral-meson mixing

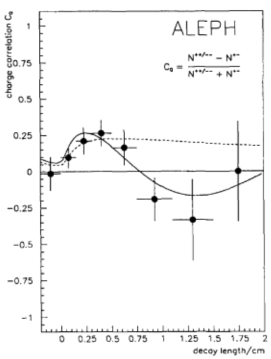
Mixing leads to an oscillation of probability to observe meson in either flavour eigenstate with proper time, *e.g.* if at t=0 we have a B^0, then at later time t:

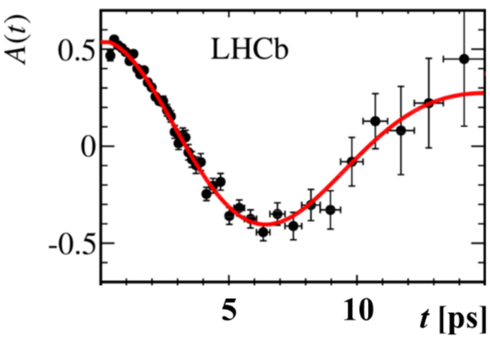
Prob. to decay as $\frac{\overline{B^0}}{B^0} \propto e^{-\Gamma_d t}(1 \mp \cos \Delta m_d\, t)$

Time-integrated B-oscillations were first observed by UA1 [PLB 186 (1987) 247] & ARGUS [PLB 192 (1987) 245]. B^0 (B^0_s) oscillations first resolved by ALEPH (CDF).

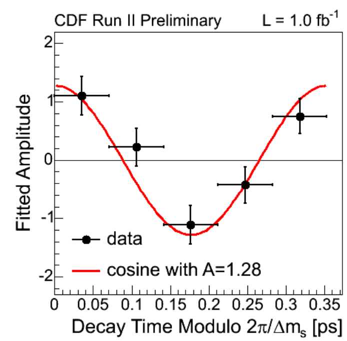
B^0 discovery → state-of-the-art B^0_s discovery → state-of-the-art

[PLB 313 (1993) 498]

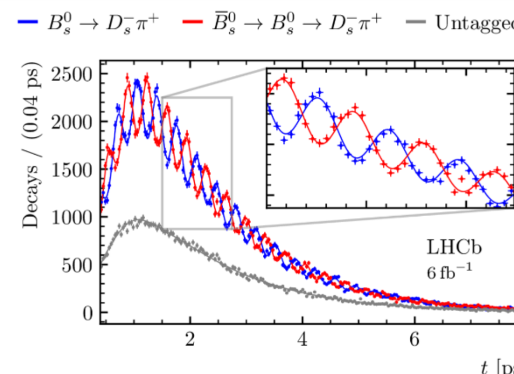




[EPJC 76 (2016) 412]



[PRL 97 (2006) 242003]

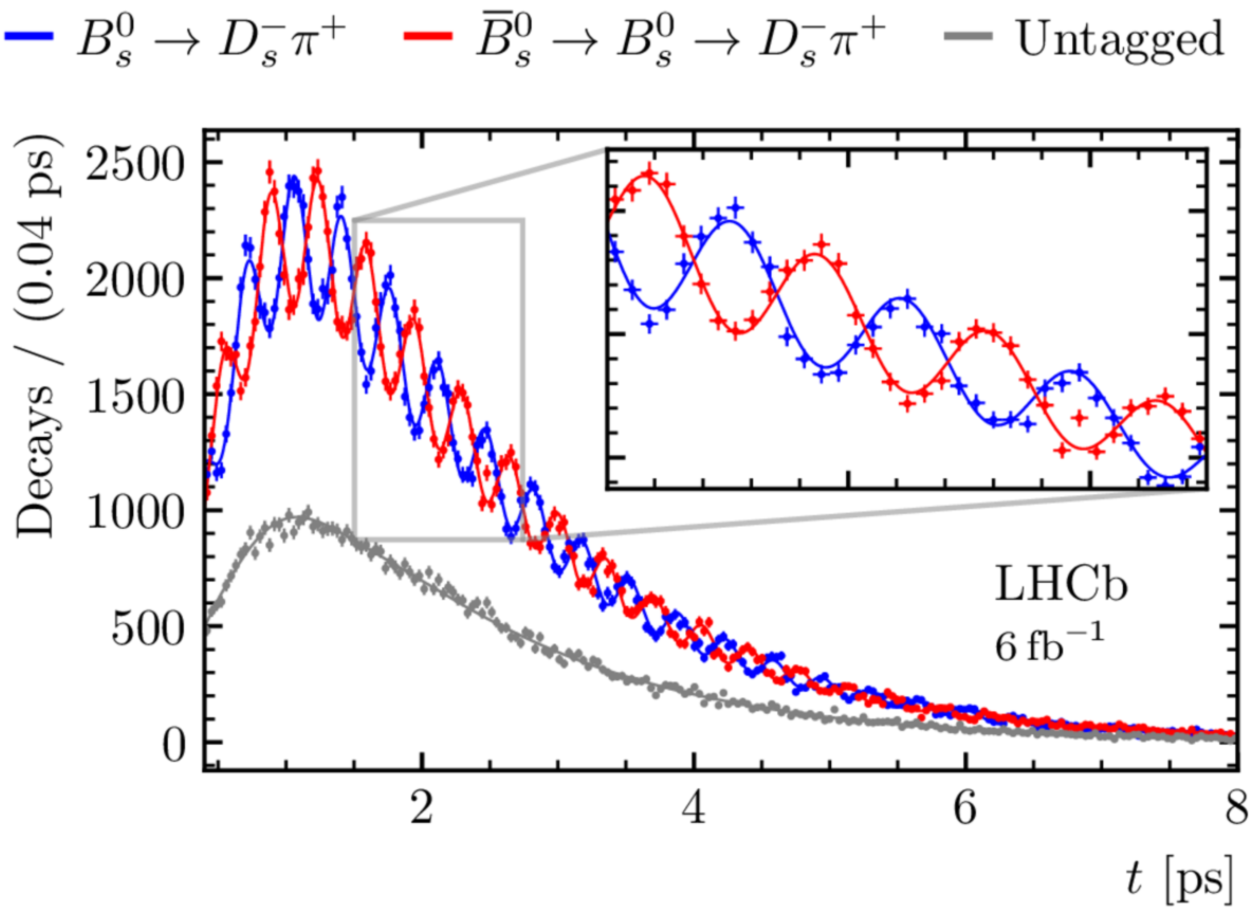


[Nature Phys. 18 (2022) 1]

State-of-the-art measurements in both B^0 and B^0_s systems are from LHCb.

B^0_s mixing – a closer look at that plot

B^0_s-mixing studies impossible at B-factories, due to E_{CM} & frequency of oscillations.

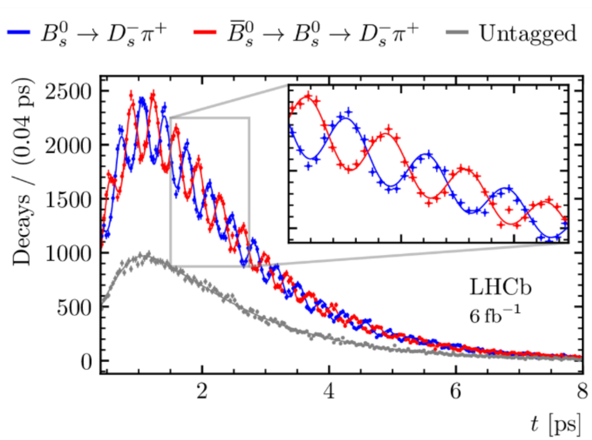


[Nature Phys. 18 (2022) 1]

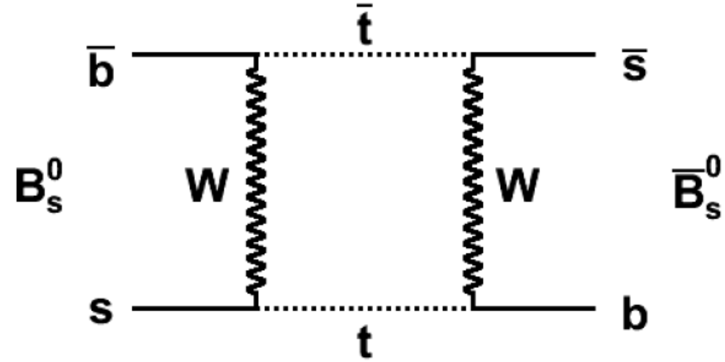
B^0_s studies are only possible at hadron machines (and at FCC-ee, but that's another story). Require significant boost and excellent proper-time resolution.

$B^0_{(s)}$-$\bar{B}^0_{(s)}$ mixing – accessing CKM elements

In B^0 and B^0_s systems, mixing driven by $\Delta m_{d(s)}$ and is calculable in SM.



Nature Phys. 18 (2022) 1



Depends on CKM elements in box & factors that can be calculated in lattice QCD.

For B^0_s case ⟶

$$\Delta m_s = \frac{G_F^2}{6\pi^2} m_{B_s} m_W^2 \eta_B S_0(x_t) f_{B_s}^2 B_s \left|V_{ts} V_{tb}^*\right|^2$$

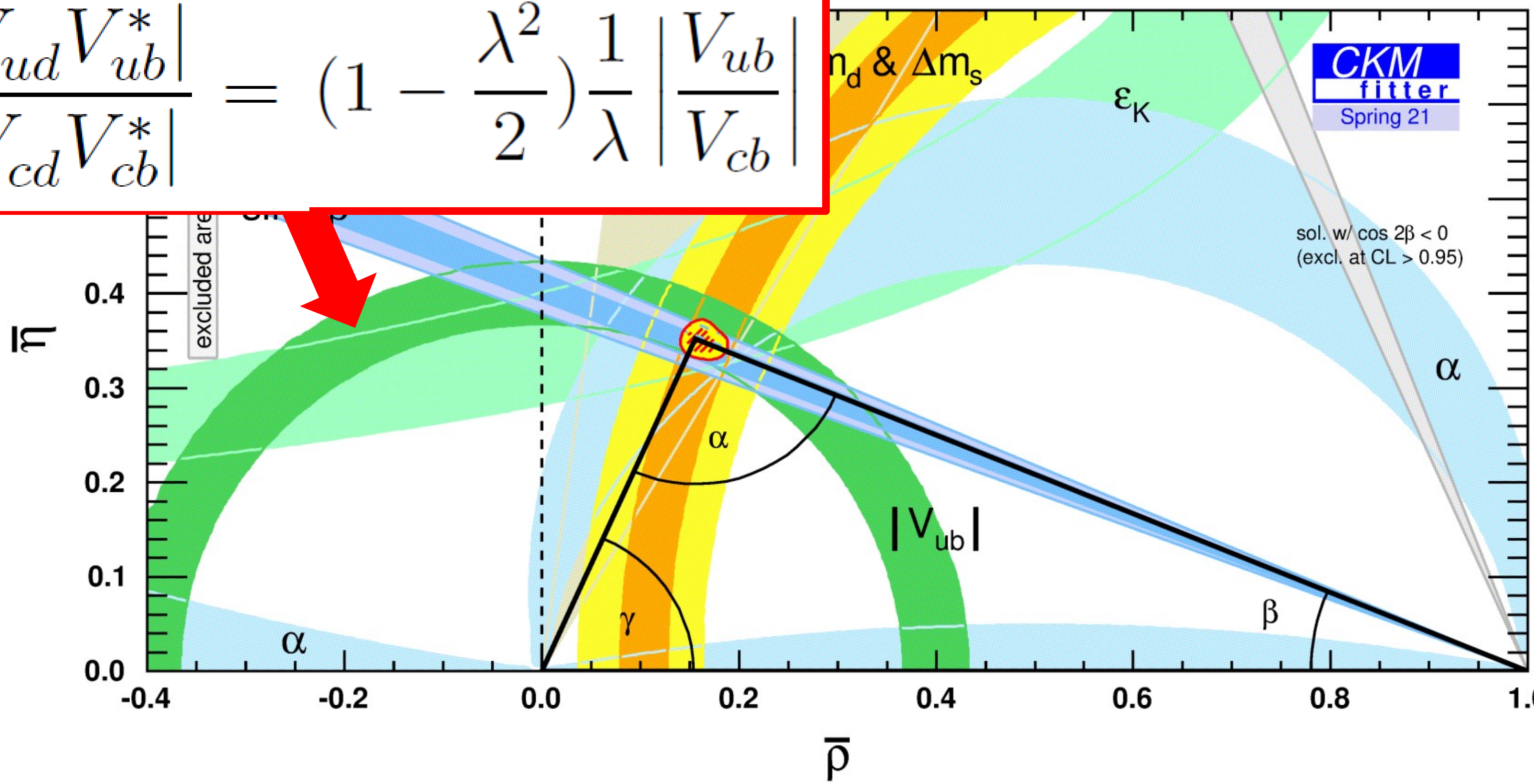
Equivalent expression for B^0 mixing, involving V_{td}. Ratio of frequencies is then

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{Bd}}{m_{Bs}} \xi_{\Delta m}^{-2} \frac{\left|V_{td}\right|^2}{\left|V_{ts}\right|^2}$$

$\zeta_{\Delta m}$, being a ratio of QCD factors of value close to 1 can be calculated to a few % in lattice QCD, hence giving access to $|V_{td}|/|V_{ts}|$. Experimental inputs dominated by LHCb, but it is lattice inputs that limit precision.

The Unitarity Triangle - CKM metrology at hadron machines

$$\frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = (1 - \frac{\lambda^2}{2})\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$



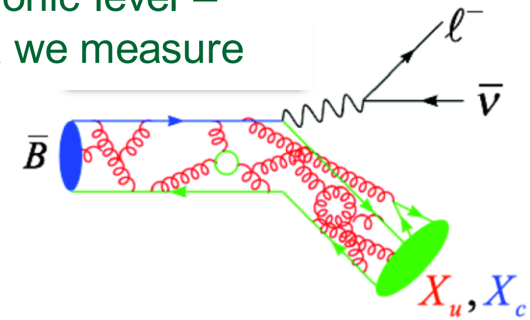
[CKMfitter, 2021]

Length of side opposite β is given by measuring $|V_{ub}|/|V_{cb}|$ from ratio b→u / b→c.

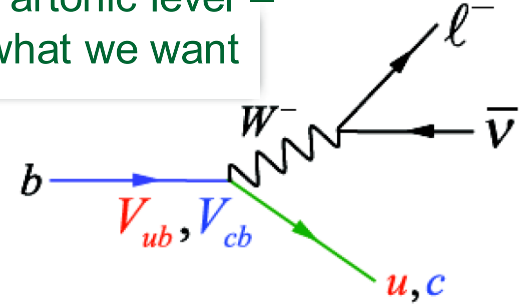
Measuring $|V_{ub}|/|V_{cb}|$

We can measure the ratio of b→ulν to b→clν processes at hadron level, but then must use theory or lattice QCD to correct back to quark level.

Hadronic level – what we measure



Partonic level – what we want



Two broad strategies followed:

- Inclusive $b \to X_u l\nu$, using *e.g.* endpoint of p_l spectrum to isolate signal from $b \to X_c l\nu$

$$|V_{ub}| = (4.25 \pm 0.30) \times 10^{-3}$$ [2019 PDG review]

- Exclusive, *e.g.* $B \to \pi l\nu$. But then need calculation of hadronic form factor.

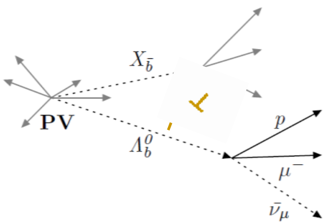
$$|V_{ub}| = (3.70 \pm 0.16) \times 10^{-3}$$ [2019 PDG review]

There is tension between these two numbers at the ~2 σ level, and a similar but worse issue with $|V_{cb}|$, which means that caution is needed when using results in UT.

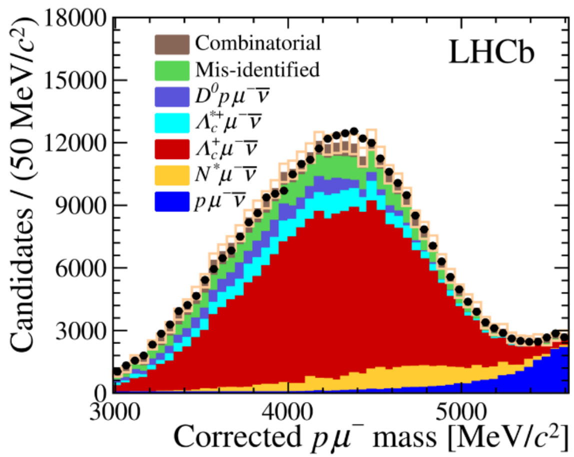
Semileptonic studies at the LHC

The e^+e^- environment is a natural laboratory for these studies, as the neutrino and backgrounds make life much more challenging at the LHC (inclusive measurements Impossible). But there are ways in which the LHC can make a unique contribution.

e.g. measurement of $|V_{ub}|$ from $\Lambda_b \to p\mu\upsilon$ decays, or more correctly $|V_{ub}|/|V_{cb}|$ through normalising the rate of these decays to those of $\Lambda_b \to \Lambda_c\mu\upsilon$ [Nature Phys. 11 (2015) 743] .



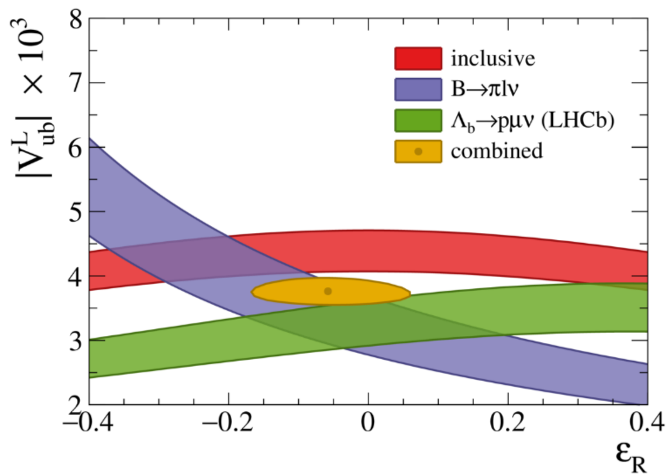
$$m_{corr} = \sqrt{m_{p\mu}^2 + p_\perp^2} + p_\perp$$



$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004 \pm 0.004$$

expt. lattice

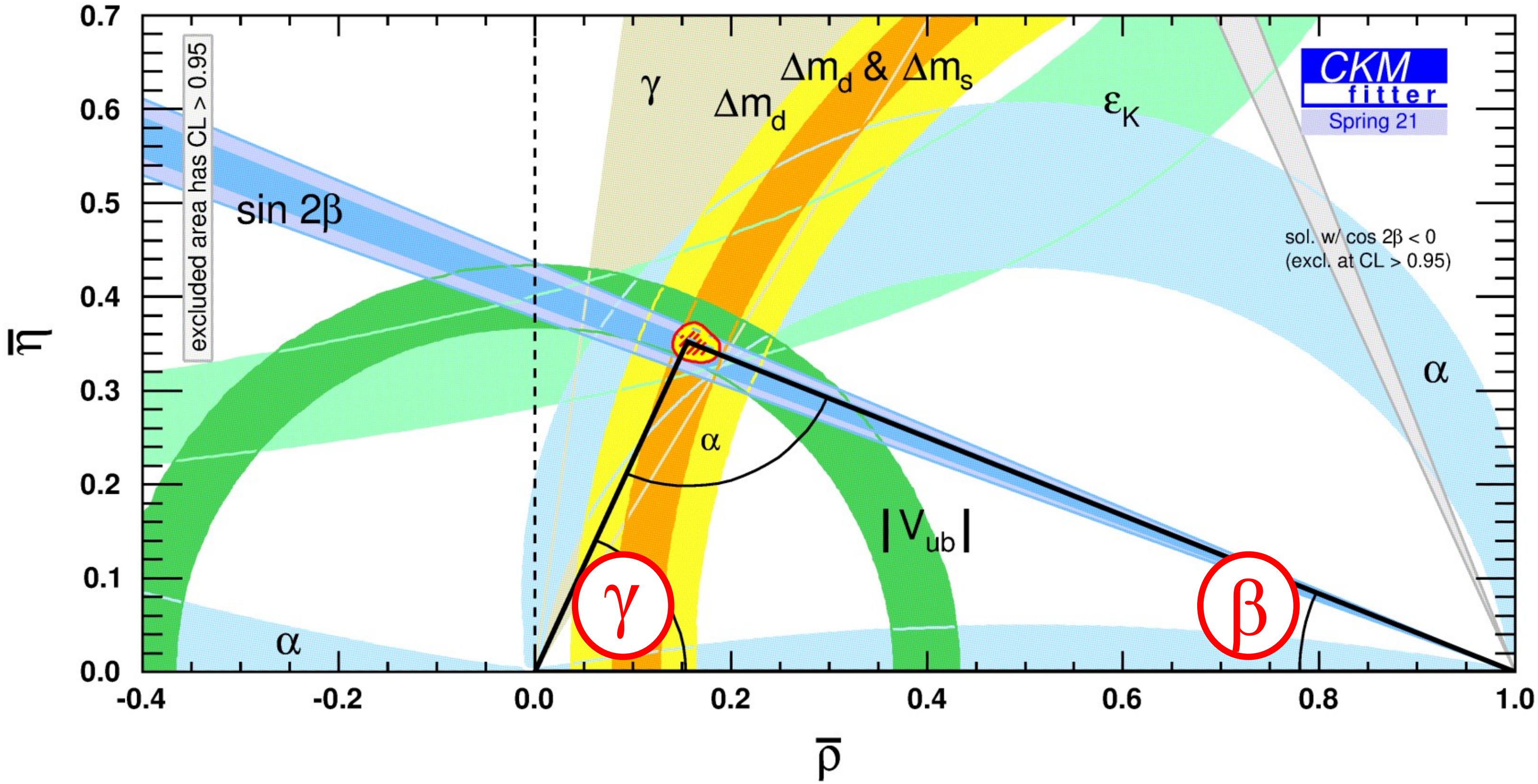
Very valuable complementary measurement as spin of Λ_b and proton brings additional info.



Helpful in *e.g.* excluding right-handed coupling invoked to explain inclusive vs exclusive tension.

Similarly, one can exploit B_s decays, *e.g.* $|V_{ub}|/|V_{cb}|$ in $B_s \to K\mu\upsilon$ [PRL 126 (2021) 081804].

The Unitarity Triangle: CP-violation measurements at hadron machines



[CKMfitter, 2021]

Now we will discuss the CPV measurements that access the angles β and γ.*

* Why not discuss α? Any α-related observable involves the same quark transitions as are probed in β and γ studies, so it is unlikely to tell us anything more. But improved measurements are always worthwhile !

Decays into CP eigenstates: $B^0 \to J/\psi K_S$

Potential for clean measurement of substantial CPV in B system first appreciated in early 1980s: [Carter and Sanda, PRD 23 (1981) 1567], [Bigi and Sanda, NPB 193 (1981) 85].

Incidentally, someone who was amongst the first to realise the potential of b-hadrons in CPV studies, and one responsible for a seminal paper, has since followed a very different career…

Obama-era U.S. defense secretary toasts the latest CP-violation results from LHCb

*

>800 citations

PHYSICAL REVIEW D VOLUME 23, NUMBER 7 1 APRIL 1981

CP violation in B-meson decays

Ashton B. Carter and A. I. Sanda
The Rockefeller University, New York, New York 10021
(Received 27 June 1980)

The pattern of CP violation in the bottom sector is discussed. We introduce general techniques to expose new CP-violating effects in the cascade decays of B mesons. In the Kobayashi-Maskawa (KM) model, the CP asymmetries so obtained range from 2–20 % for plausible values of the model parameters. This is to be compared with the small effects, of order 10^{-3}–10^{-4}, previously exhibited within this model. Effects of this size should be observable in upcoming experiments. Our approach stresses the on-shell transitions which make up the cascade decays of heavy mesons to ordinary hadrons, as opposed to the off-shell transitions which occur in the analogs of K^*-$\bar{K}^*$ mixing. The CP asymmetries generated by our techniques are of order $\sin\delta$, where δ is the KM phase angle, and thus represent the maximum effects obtainable in this model.

Decays into CP eigenstates: $B^0 \to J/\psi K_S$

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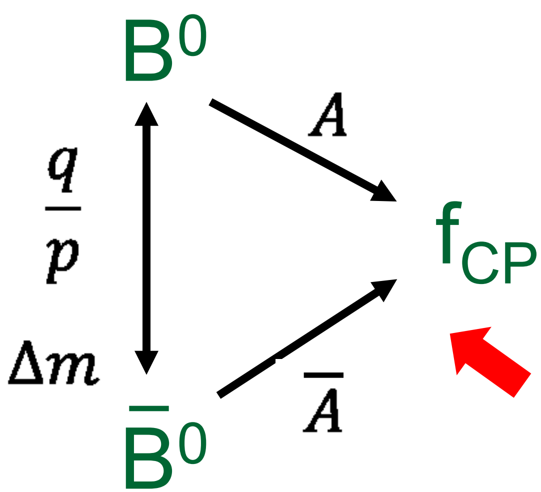
For meson that is B^0 or B^0bar at t=0, which decays into CP-eigenstate f_{CP} at time *t*:

$$\Gamma\left(B^0_{phys} \to f_{CP}(t)\right) \propto e^{-\Gamma t}\left(1 - \left(S \sin(\Delta m t) - C \cos(\Delta m t)\right)\right)$$
$$\Gamma\left(\overline{B}^0_{phys} \to f_{CP}(t)\right) \propto e^{-\Gamma t}\left(1 + \left(S \sin(\Delta m t) - C \cos(\Delta m t)\right)\right) \quad *$$



$$S = \frac{2\Im(\lambda_{CP})}{1 + |\lambda_{CP}^2|} \qquad C = \frac{1 - |\lambda_{CP}^2|}{1 + |\lambda_{CP}^2|} \qquad \lambda_{CP} = \frac{q}{p}\frac{\overline{A}}{A}$$

Key point: to observe a complex phase we need to have two (or more) interfering amplitudes, as here

* These expressions assumes width-splitting $\Delta\Gamma=0$, which is an excellent approximation in B^0 system.

Decays into CP eigenstates: $B^0 \to J/\psi K_S$

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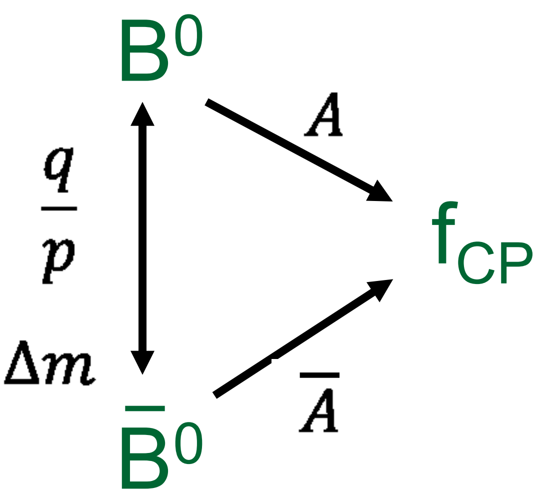

$$S = \frac{2\Im(\lambda_{CP})}{1 + |\lambda_{CP}^2|} \qquad C = \frac{1 - |\lambda_{CP}^2|}{1 + |\lambda_{CP}^2|} \qquad \lambda_{CP} = \frac{q}{p}\frac{\overline{A}}{A}$$

There are three ways that CP violation can appear:

CPV in the decay (or 'direct CPV').

$$|A| \neq |\overline{A}|$$

(This is also the only possibility that applies for charged hadron decays, for instance in the measurement of γ.)

* These expressions assumes width-splitting $\Delta\Gamma$=0, which is an excellent approximation in B^0 system.

Decays into CP eigenstates: $B^0 \rightarrow J/\psi K_S$

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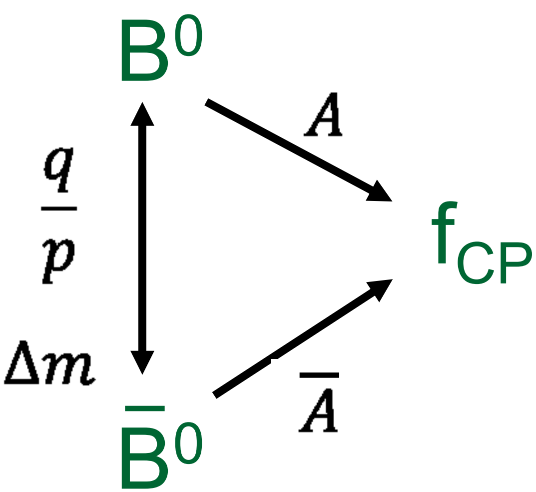

$$S = \frac{2\Im(\lambda_{CP})}{1+|\lambda_{CP}^2|} \qquad C = \frac{1-|\lambda_{CP}^2|}{1+|\lambda_{CP}^2|} \qquad \lambda_{CP} = \frac{q}{p}\frac{\overline{A}}{A}$$

There are three ways that CP violation can appear:

CPV in the mixing (one category of so-called 'indirect CPV').

Occurs if there are different ways to oscillate $B^0 \leftrightarrow B^0$bar. In SM very small.

$$\left|\frac{q}{p}\right| \neq 1$$

* These expressions assumes width-splitting $\Delta\Gamma=0$, which is an excellent approximation in B^0 system.

Decays into CP eigenstates: $B^0 \to J/\psi K_S$

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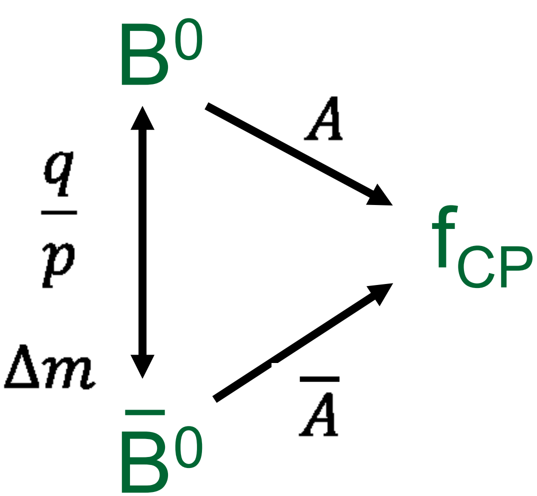


$$S = \frac{2\Im(\lambda_{CP})}{1+|\lambda_{CP}^2|} \qquad C = \frac{1-|\lambda_{CP}^2|}{1+|\lambda_{CP}^2|} \qquad \lambda_{CP} = \frac{q}{p}\frac{\overline{A}}{A}$$

There are three ways that CP violation can appear:

CPV in mixing-decay interference (also a category of 'indirect CPV', & the most relevant in the B^0B^0bar and $B^0_sB^0_s$bar systems).

$$\mathrm{Im}\lambda_{CP} \neq 0$$

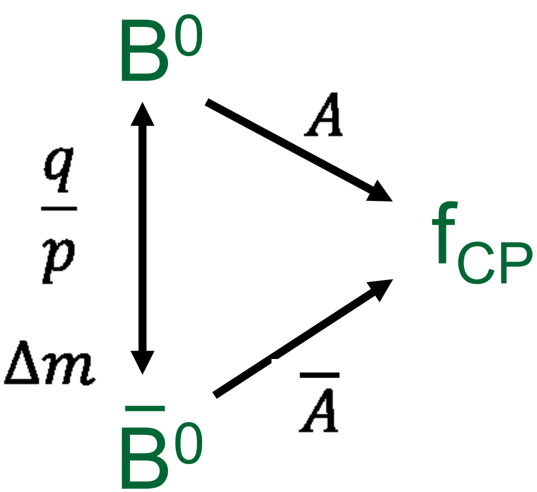
* These expressions assumes width-splitting ΔΓ=0, which is an excellent approximation in B^0 system.

Decays into CP eigenstates: $B^0 \rightarrow J/\psi K_S$

Potential for clean measurement of substantial CPV in B system first appreciated in early 1980s: [Carter and Sanda, PRD 23 (1981) 1567], [Bigi and Sanda, NPB 193 (1981) 85].

For meson that is B^0 or B^0bar at t=0, which decays into CP-eigenstate f_{CP} at time *t*:

$$\Gamma\left(B^0_{phys}\rightarrow f_{CP}(t)\right) \propto e^{-\Gamma t}\left(1-\left(S\sin(\Delta m t)-C\cos(\Delta m t)\right)\right)$$
$$\Gamma\left(\overline{B}^0_{phys}\rightarrow f_{CP}(t)\right) \propto e^{-\Gamma t}\left(1+\left(S\sin(\Delta m t)-C\cos(\Delta m t)\right)\right)$$ *



$$S=\frac{2\,\Im(\lambda_{CP})}{1+\left|\lambda_{CP}^2\right|} \qquad C=\frac{1-\left|\lambda_{CP}^2\right|}{1+\left|\lambda_{CP}^2\right|} \qquad \lambda_{CP}=\frac{q}{p}\frac{\overline{A}}{A}$$

Consider the classic case $B^0 \rightarrow J/\psi K_S$:

- Compared to the CPV signal we are expecting in B physics, we can treat K_S as a CP eigenstate.
- And in this decay C≈0, with no significant direct CPV (all the CPV comes from *mixing-decay interference*).

NB both these assumptions can be checked / corrected for.

* These expressions assumes width-splitting $\Delta\Gamma=0$, which is an excellent approximation in B^0 system.

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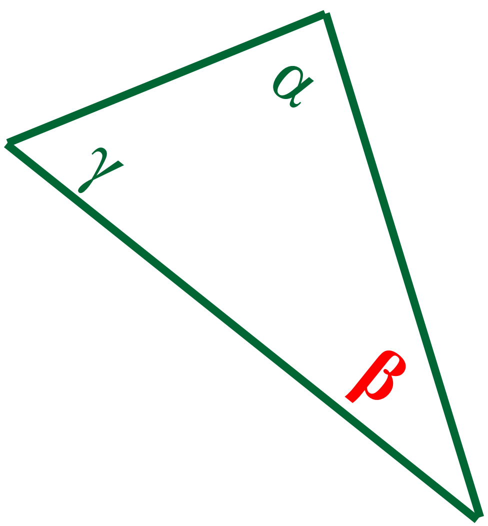

$$S = \frac{2\,\Im(\lambda_{CP})}{1+|\lambda^2_{CP}|} \qquad C = \frac{1-|\lambda^2_{CP}|}{1+|\lambda^2_{CP}|} \qquad \lambda_{CP} = \frac{q}{p}\frac{\overline{A}}{A}$$

Consider the classic case $B^0 \to J/\psi K_S$:

$$\lambda_{J/\psi K_S} = \frac{V^*_{tb}V_{td}V_{cb}V^*_{cs}}{V_{tb}V^*_{td}V^*_{cb}V_{cs}} = e^{i2\beta} \qquad \operatorname{Im}\lambda_{J/\psi K_S} = \sin 2\beta$$

* These expressions assumes width-splitting $\Delta\Gamma$=0, which is an excellent approximation in B^0 system.

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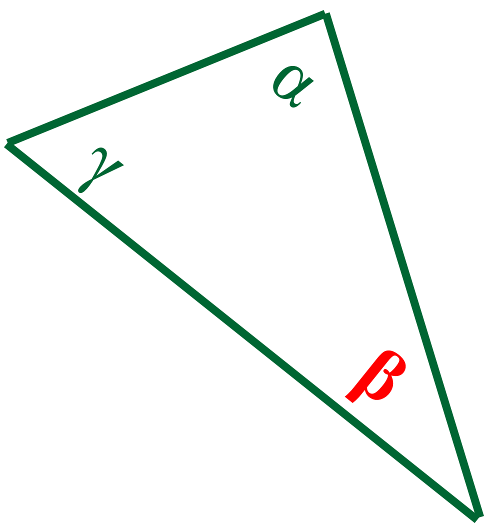

In practice we measure a *t*-dependent CP asymmetry:

$$a_{CP}(t) \equiv \frac{\Gamma(\overline{B}^0(t) \to J/\psi K^0_S) - \Gamma(B^0(t) \to J/\psi K^0_S)}{\Gamma(\overline{B}^0(t) \to J/\psi K^0_S) + \Gamma(B^0(t) \to J/\psi K^0_S)}$$
$$= \sin 2\beta \, \sin(\Delta m\, t)$$

This is theoretically ***clean*** !
(at least, at current precision)

* These expressions assumes width-splitting $\Delta\Gamma=0$, which is an excellent approximation in B^0 system.

Decays into CP eigenstates: $B^0 \to J/\psi K_S$

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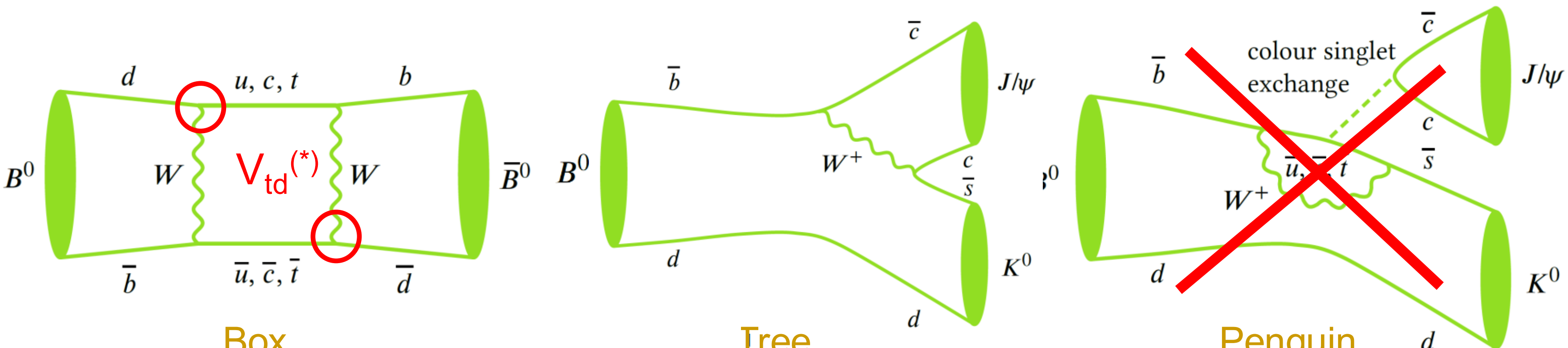
To reiterate, measurement probes interference between box and tree diagrams:



Box

Tree

Penguin
(suppressed)

Sensitive to any CP violating phases in either, but are only expected in the box. In the SM this comes from the phase difference associated with V_{td} , but could arise from other sources through New Physics. So possible $\sin 2\beta_{meas} \neq \sin 2\beta_{SM}$!

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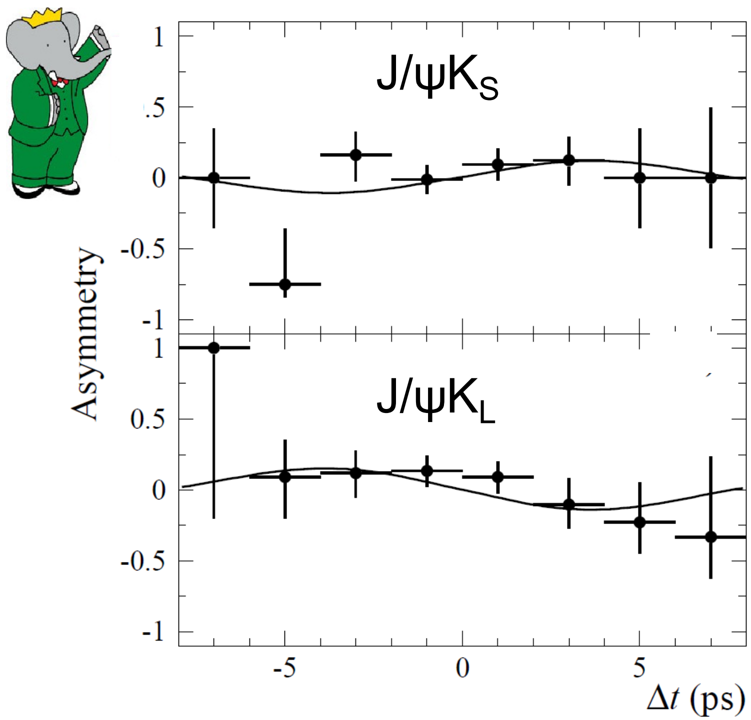
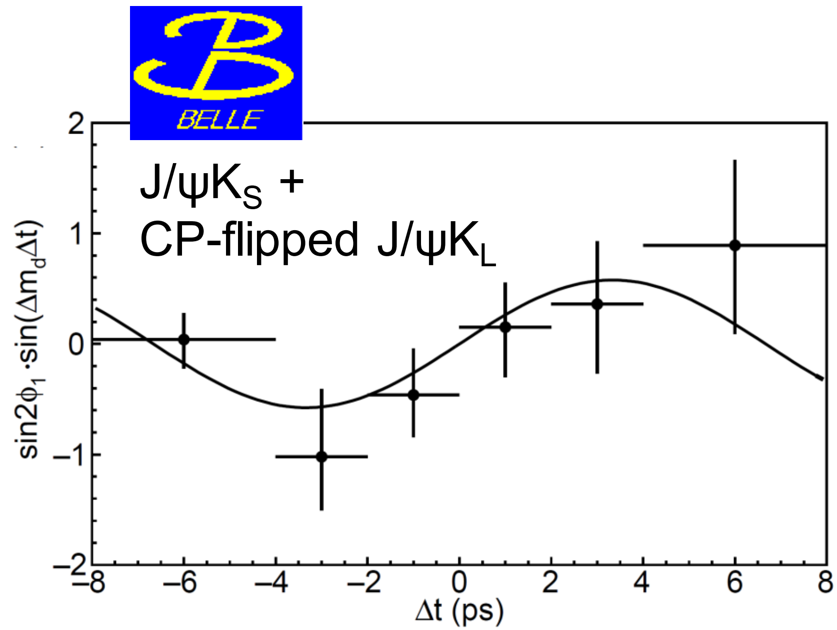
2001 – (the start of) a flavour odyssey

2008
Nobel
Prize

Modern flavour physics began at the B factories with the 2001 measurements of the CP-violating asymmetry in $B^0 \to J/\psi K^0$ decays that give unitarity triangle angle β.



[BaBar, PRL 86 (2001) 2515]



[Belle, PRL 86 (2001) 2509]

These studies, when improved with larger samples, confirmed the CKM paradigm as the dominant mechanism of CP violation in nature (→ 2008 Nobel Prize), and also opened up a rich and wide spectrum of complementary measurements.

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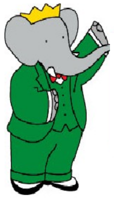

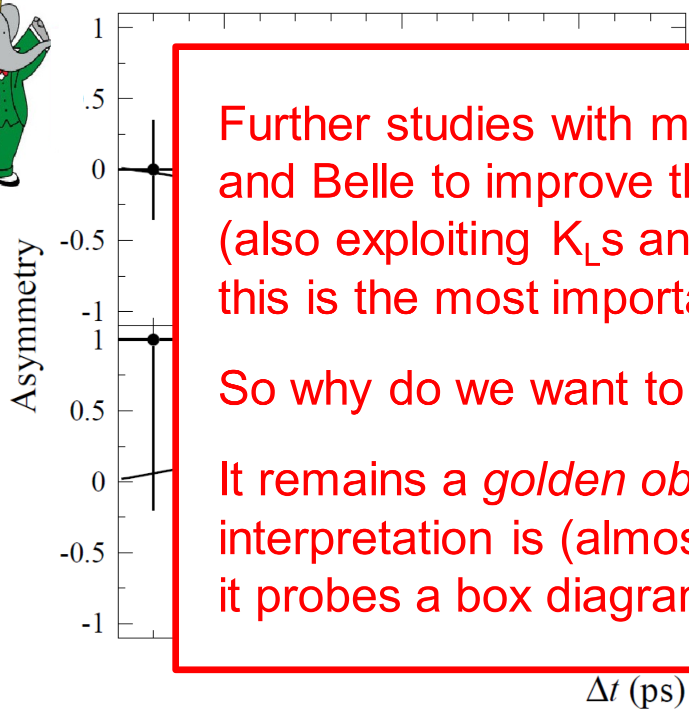

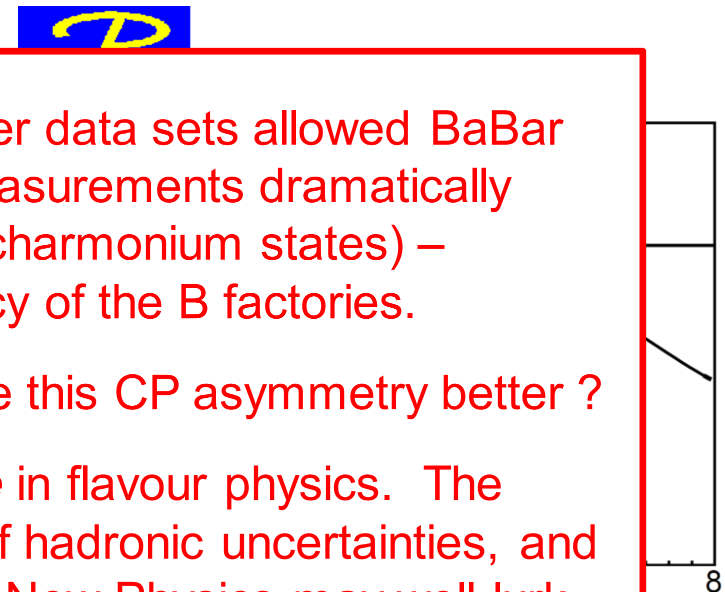

Further studies with much larger data sets allowed BaBar and Belle to improve these measurements dramatically (also exploiting K_Ls and other charmonium states) – this is the most important legacy of the B factories.

So why do we want to measure this CP asymmetry better ?

It remains a *golden observable* in flavour physics. The interpretation is (almost) free of hadronic uncertainties, and it probes a box diagram where New Physics may well lurk.

[BaBar, PRL 86 (2001) 2515] [Belle, PRL 86 (2001) 2509]

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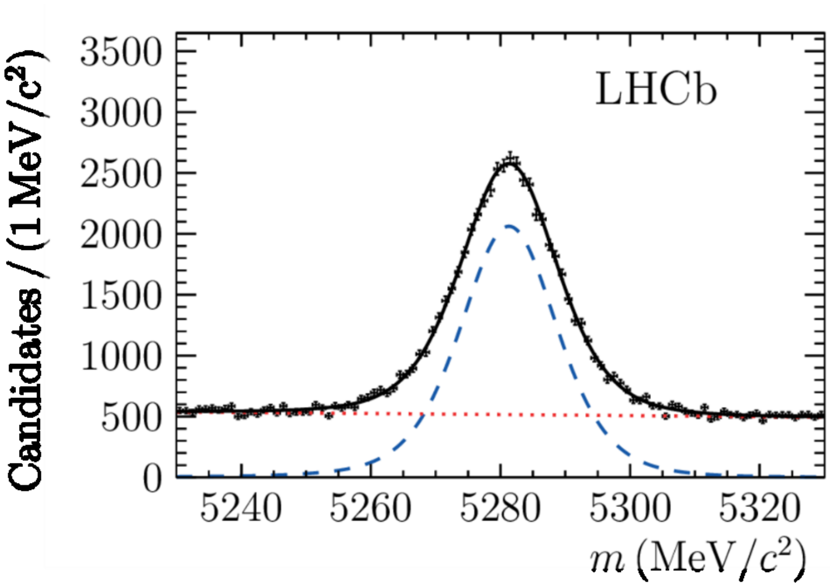
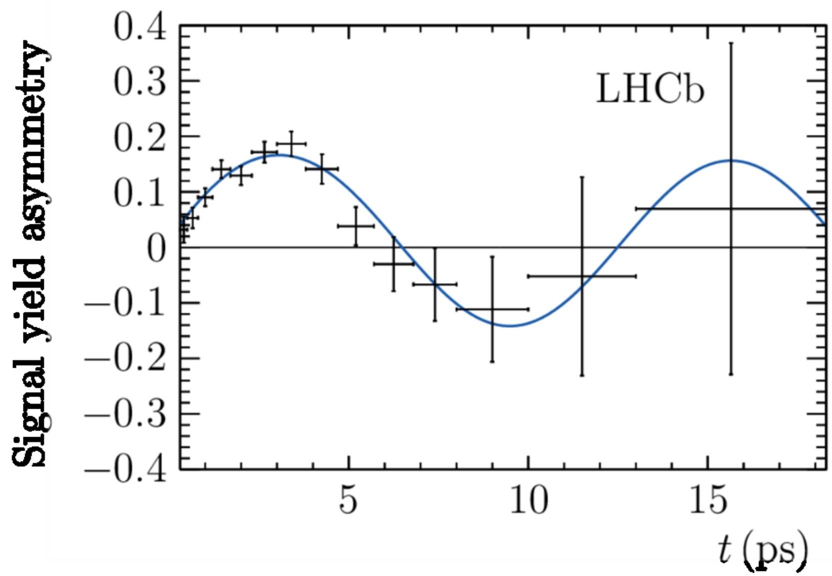
$B^0 \to J/\psi K_S$: LHCb comes to the party

LHCb has measured sin2β through the CP asymmetry in $B^0 \to J/\psi K_S$ decays, with Run 1 data alone, using $J/\psi \to \mu\mu$ [PRL 115 (2015) 031601] (there have been subsequent measurements involving $J\psi \to ee$ and also the $\psi(2S)$ [JHEP 11 (2017) 170] .



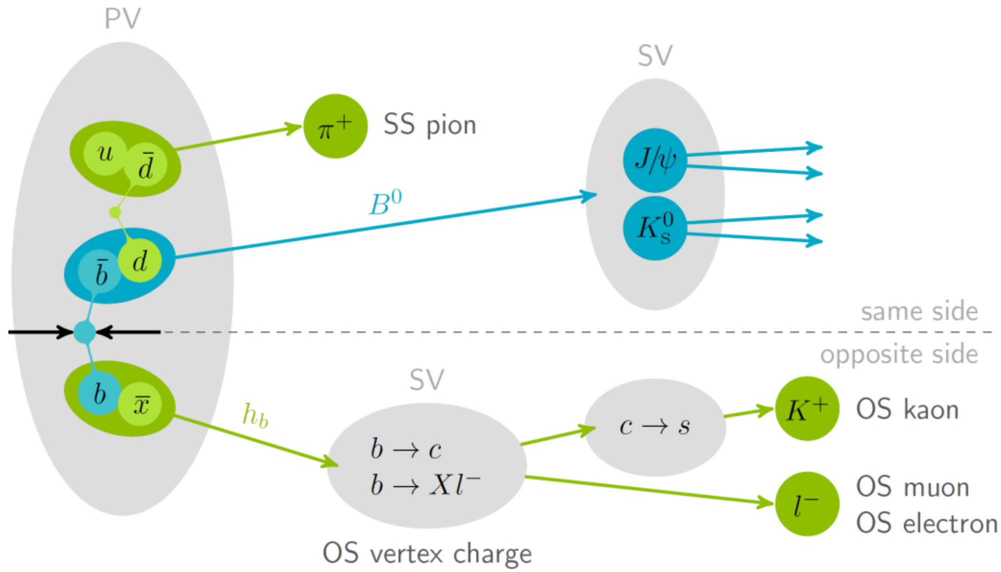


$$\sin(2\beta) = 0.746 \pm 0.030$$

Very similar precision to the B-factory measurements. But why not better, given that the sample is much larger (LHCb: 114k, BaBar ~10k [PRD 79 (2009) 072009]) ?

Flavour tagging at a hadron collider

Measurement demands we know whether decaying meson was B^0 or B^0bar at birth. This requires *flavour tagging* *. Look at either decay products of the other b-hadron ('opposite sign') or for fragmentation products associated with signal B ('same sign').



Flavour tag decision can be wrong, either through misidentification of mixing of OS b-hadron. This leads to *dilution* of asymmetry, and reduces effective signal statistics by a large factor (up to x ~1/30) at hadron collider experiments.

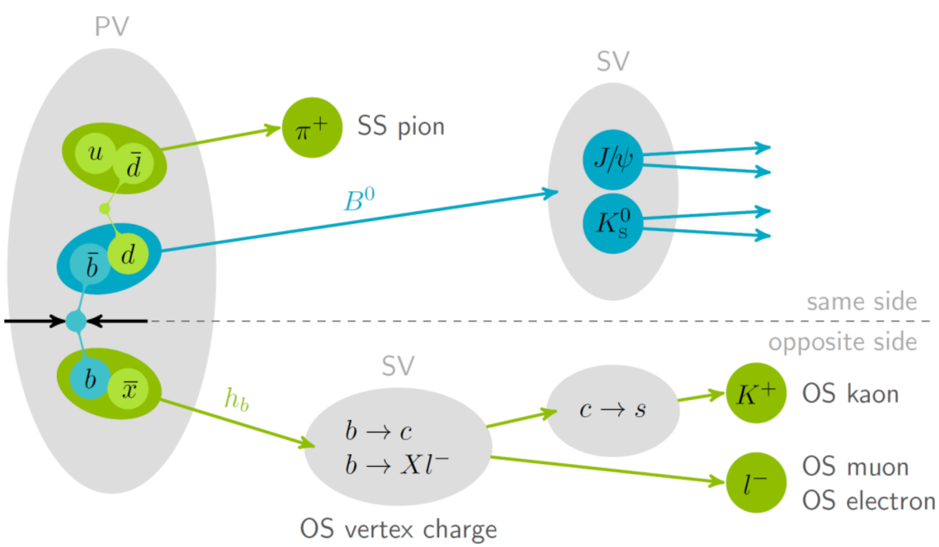
For *t* variable in asymmetry, we need to know proper time between birth & death of signal B, which at LHC is related to distance between primary and decay vertices.

* NB in high-p_T physics the term 'flavour tagging' means something different, typically 'is this jet b-like or c-like ?'.

Flavour tagging at a hadron collider

Effective tagging efficiency for a single tag given by $\varepsilon_{tag}(1-2\omega_{tag})^2$ with ε_{tag} the tagging efficiency, ω_{tag} the mistag probability.

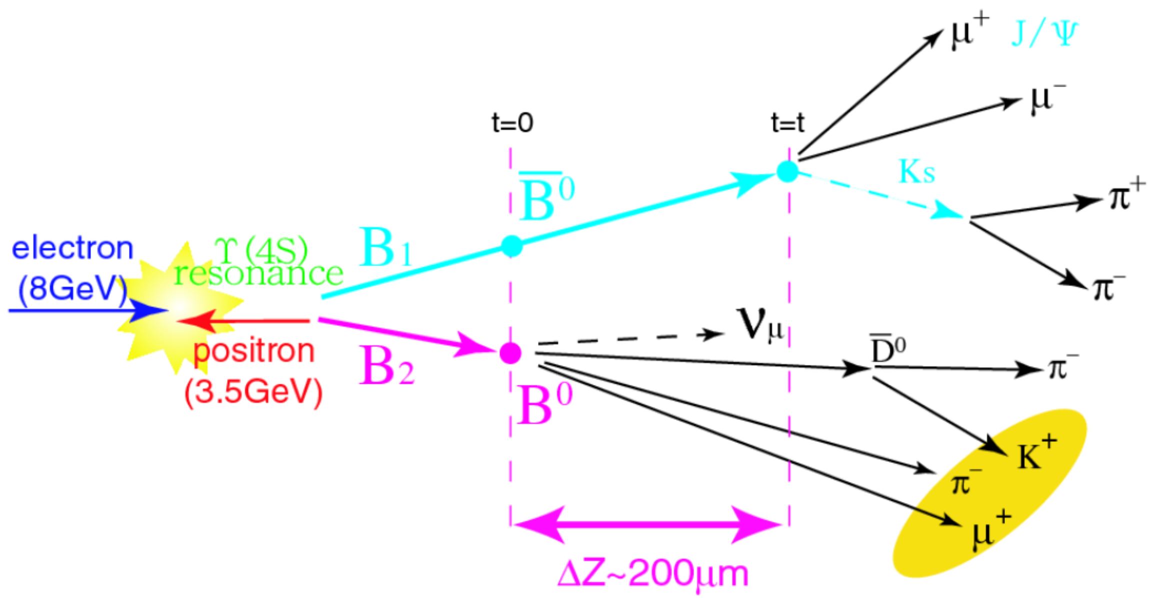
In practice such a quantity is formed for the ensemble of tags used in the analysis and gives a parameter that defines the proportion of events that, if perfectly tagged, would contribute to the measurement. Varies with meson type, how event is triggered, and with understanding of data set. Example values from LHCb studies.



Analysis	Effective tagging efficiency
Run 1 $B^0 \to J/\psi K_S$ [PRL 115 (2015) 031601]	**3.02 %**
Run 2 $B_s \to J/\psi KK$ [EPJC 79 (2019) 706]	**4.73 ± 0.34 %**
Run 2 $B_s \to D_s\pi$ [Nature Phys. 18 (2022) 1]	**6.10 ± 0.15 %**

Flavour tagging at the Y(4S)

Life is easier for BaBar/Belle and Belle-II At the Y(4S) one has no fragmentation particles and production of coherent B^0-B^0bar system → (i) No same sign tag (bad), (ii) many fewer mistags (very good), (iii) no mixing until one B decays (very good).

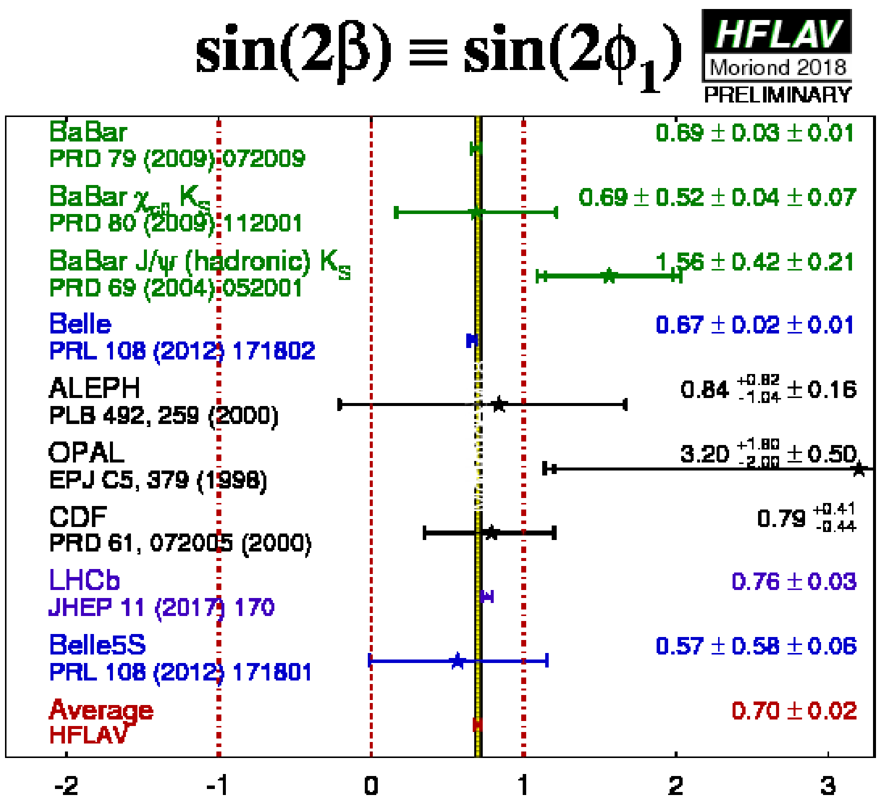


The dilution is less than at LHC, and reduces effective signal statistics by only ~1/3.

Why do B-factories have asymmetric beam energies? For coherent system what matters is the time-difference Δt *between* the two B decays. At the Y(4S) the mesons are produced at rest, & so it is necessary to *boost* system to measure Δt.

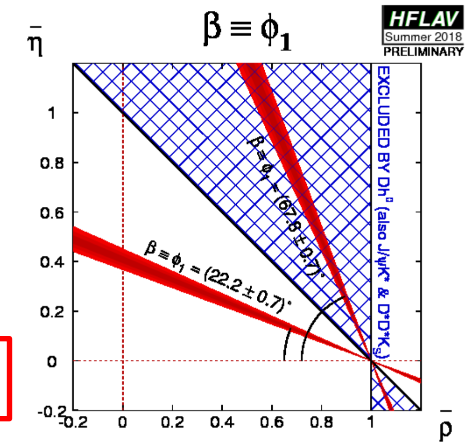
sin2β: current status and impact of the LHC

Global state of play:

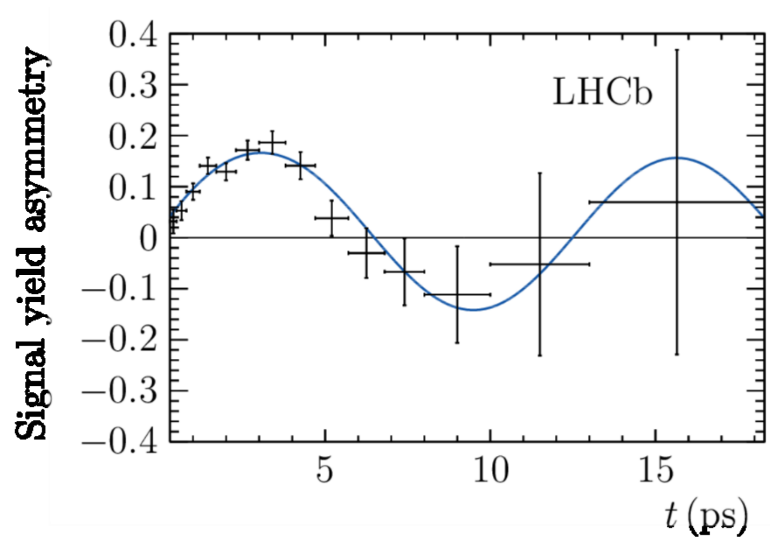


Both solutions for β shown in UT plane. ➡

$$\beta = (22.2 \pm 0.7)^{o}$$



LHCb run 1 $J/\psi K_S$ result has similar precision to B factories



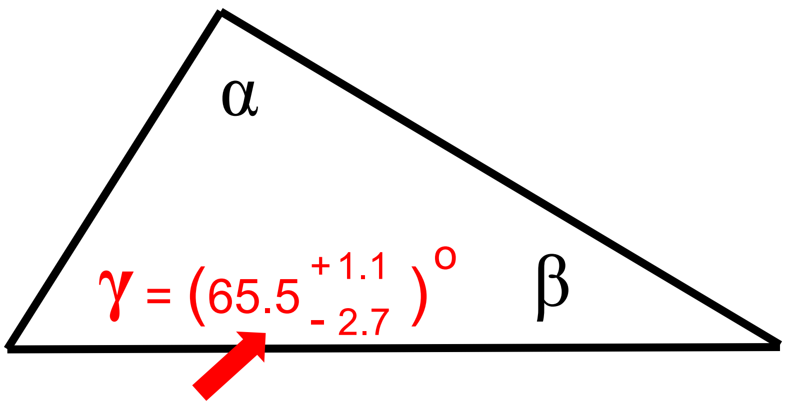
[LHCb, PRL 115 (2015) 031601]

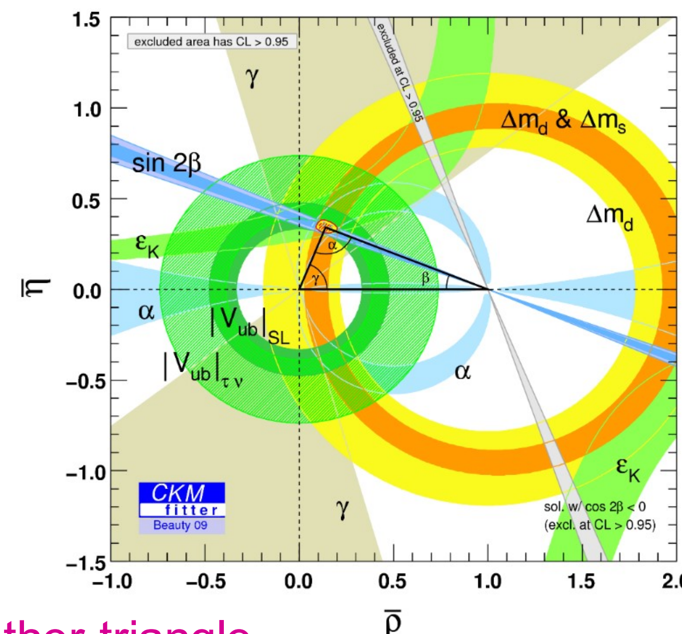
Improvements expected soon with LHCb Run 2 result (+ Run 3 data, plus Belle II…)

The long march: towards a precise determination of the UT angle γ

A particular responsibility for flavour physics at the LHC is to improve our knowledge of the angle γ.

At LHC turn-on γ uncertainty was >20°.





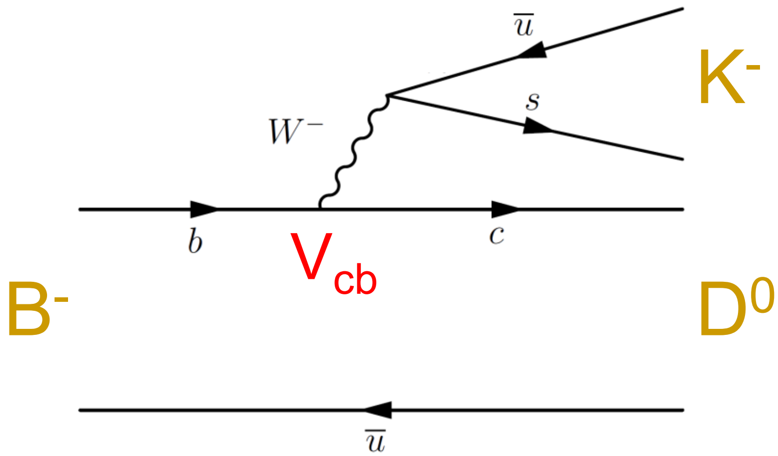
[CKMfitter, 2009]

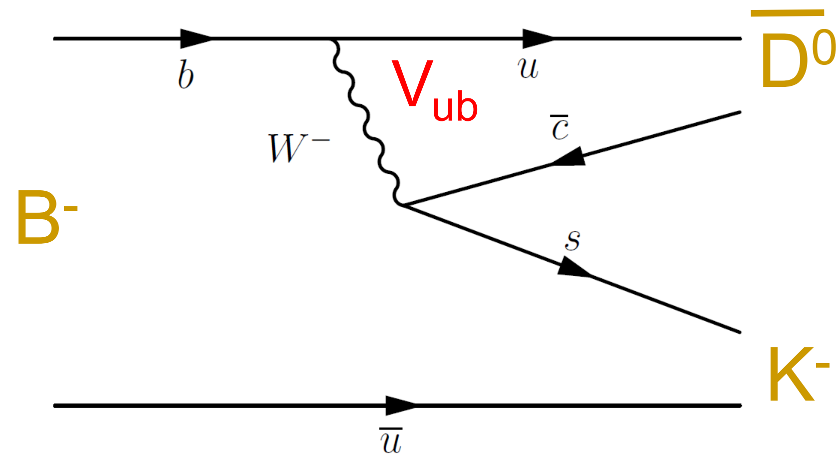
The predicted value of γ [CKMfitter, 2021] in context of SM is known very well from other triangle parameters (& will be known even better as experiment & lattice QCD improve).

A key task of flavour physics is to match this precision in a direct measurement !

The long march: towards a precise determination of the UT angle γ

This angle is special – it can be measured at tree-level through B→DK decays.





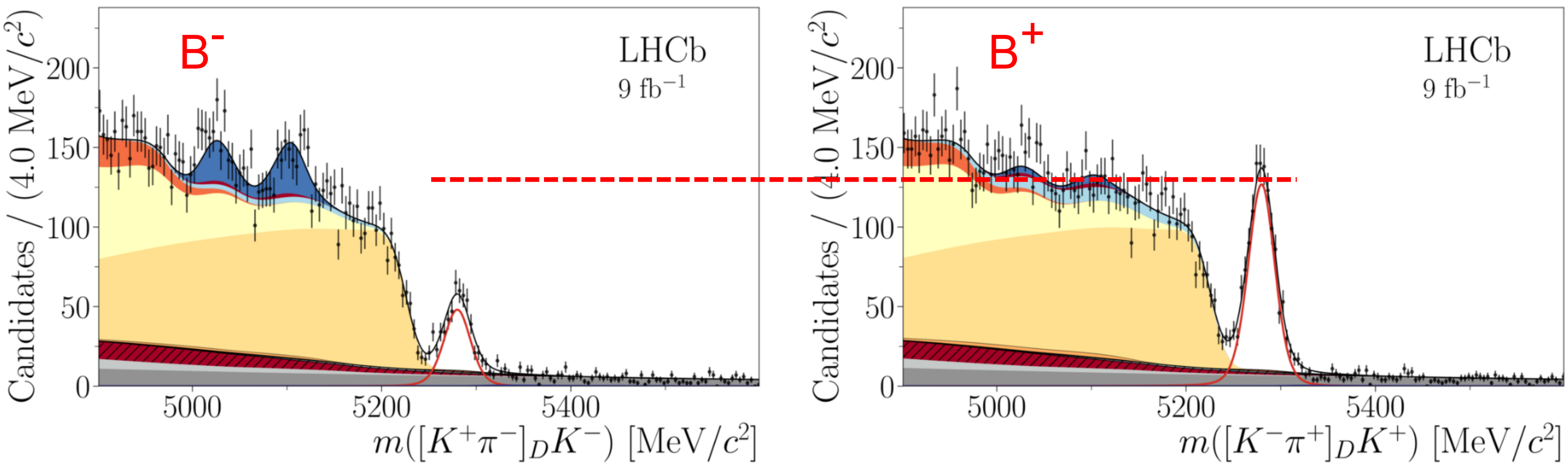
If we reconstruct D^0 and $\overline{D^0}$ in a state accessible to both, Interference occurs & decay rates become sensitive to relative phase between V_{cb} and V_{ub}, which is γ.

There are QCD nuisance parameters involved, but sufficient observables can be measured to determine these without any assumption. Theoretically ultra clean !

Tree level means New Physics unlikely to perturb measured value from the γ of the SM (*c.f.* β) , hence measurement provides 'SM benchmark' for other tests !

The Unitarity Triangle: measuring γ

To access these interference effects means looking for rather suppressed decays, *e.g.* this $B^- \to DK^-$ decay, with $D \to K^+\pi^-$ (and B^+ conjugate case): visible BR $\sim 10^{-8}$, Hence out of reach to previous generation of flavour physics experiments.



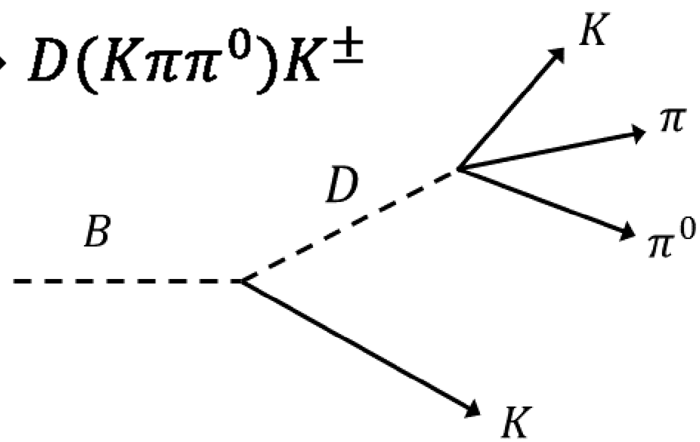
JHEP 04 (2021) 081

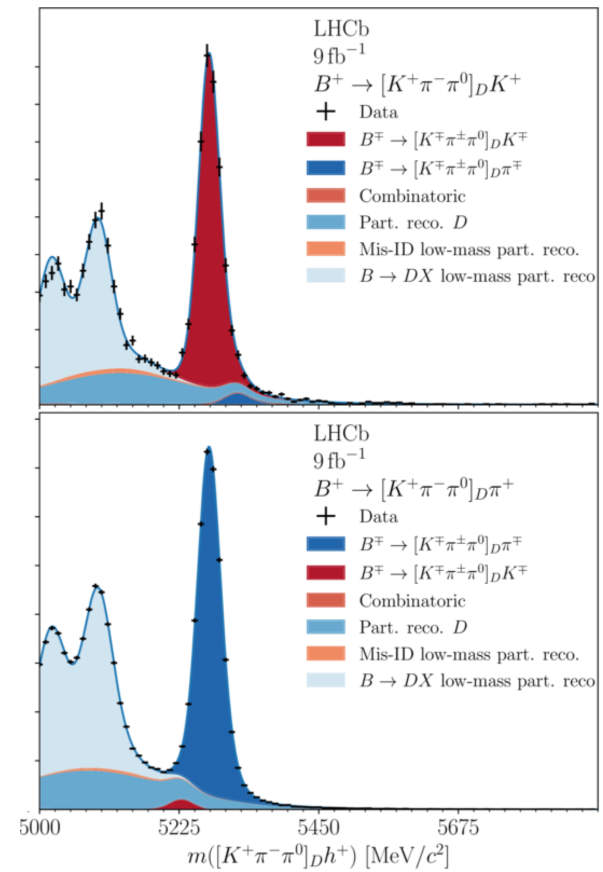
Very significant CP violation observed, that can be cleanly related to the phase γ.

Measuring γ at LHCb: remarkably clean signals

Despite the high multiplicity environment, the signals are remarkably clean, even in very challenging modes involving a π^0 [arXiv:2112.10617]. The flight distance of the B & D mesons suppresses combinatoric background from prompt charged tracks.

$$B^{\pm} \rightarrow D(K\pi\pi^0)K^{\pm}$$





Furthermore, the RICH detector does an excellent job in separating the B→DK mode (top plot) from the order-of-magnitude more abundant B→Dπ mode (bottom plot).

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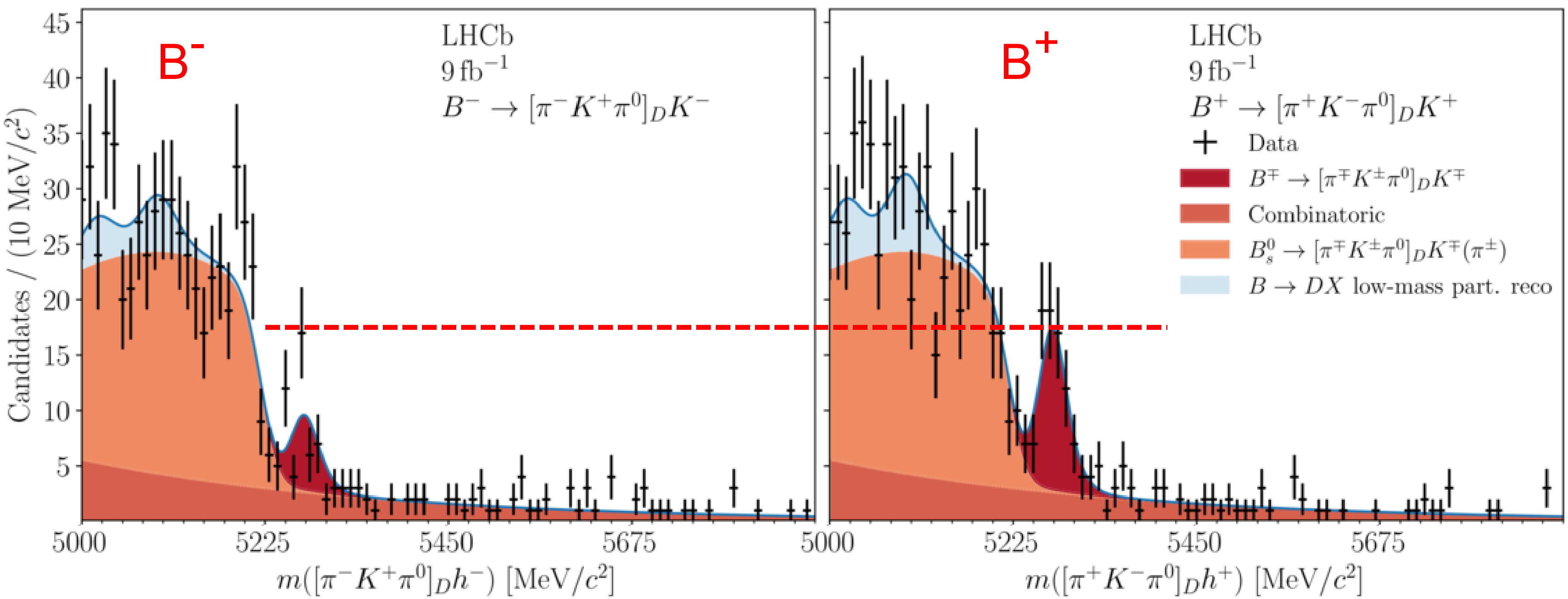
Thus, even in $B^{\pm} \rightarrow D(K\pi\pi^0)K^{\pm}$ the suppressed mode can be seen, together with its CP-violating asymmetry - again, this was not accessible at BaBar / Belle.

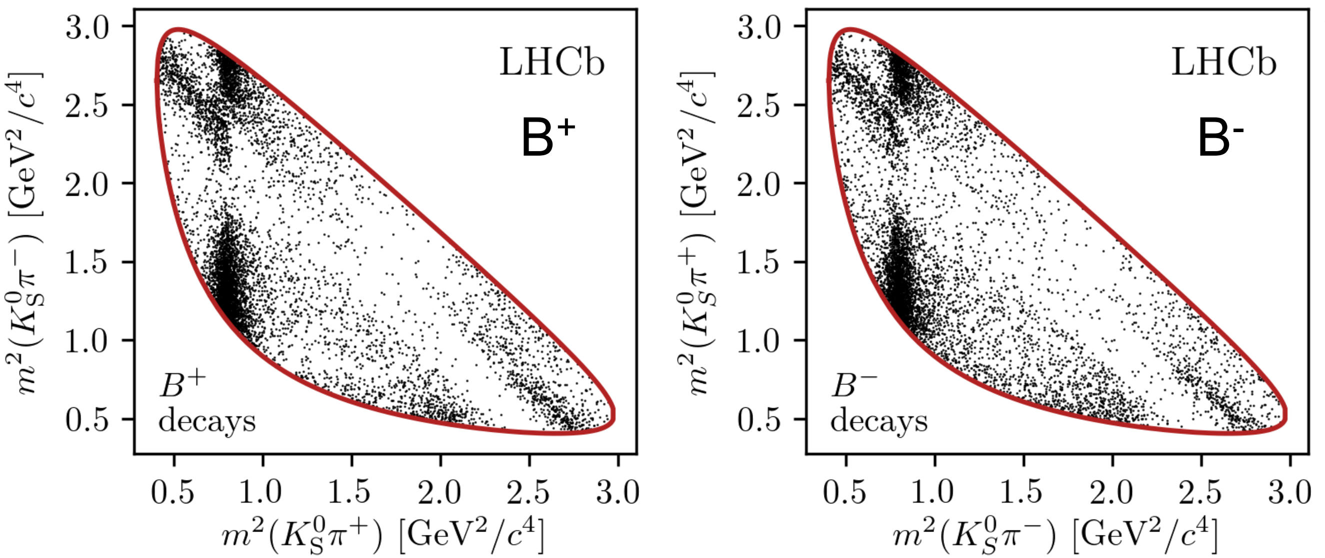


γ measurement at LHCb with B→DK decays: D→$K_S\pi\pi$ (and K_SKK)

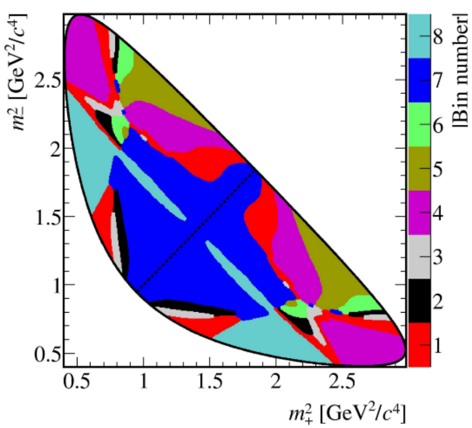
[JHEP 02 (2021) 169]

A powerful sub-set of B→DK analyses is when the D decays into a multibody final state, of which $K_S\pi\pi$ is the most prominent example. Variation of D strong phase over Dalitz space leads to corresponding variation in interference and CP violation.

Analysis of ~12,500 decays from Run 1 and Run 2 data



Study yields in bins of Dalitz space, chosen for optimal sensitivity.



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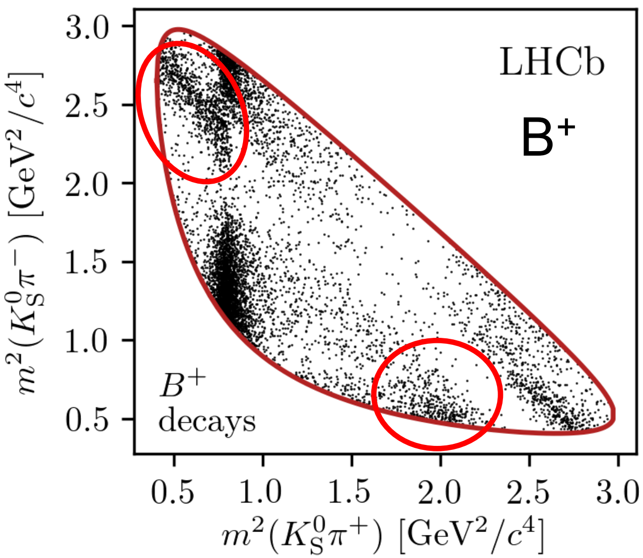

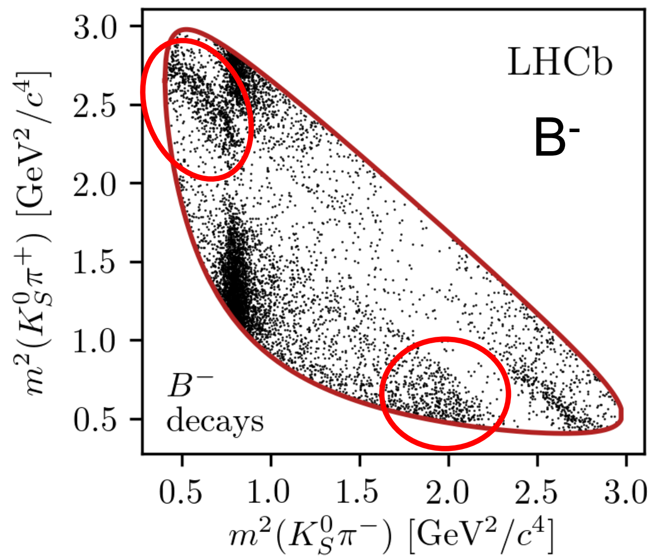

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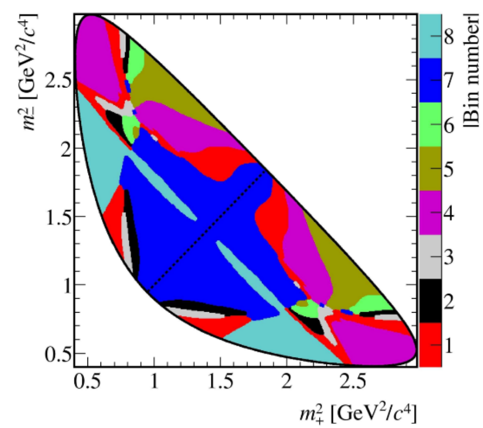
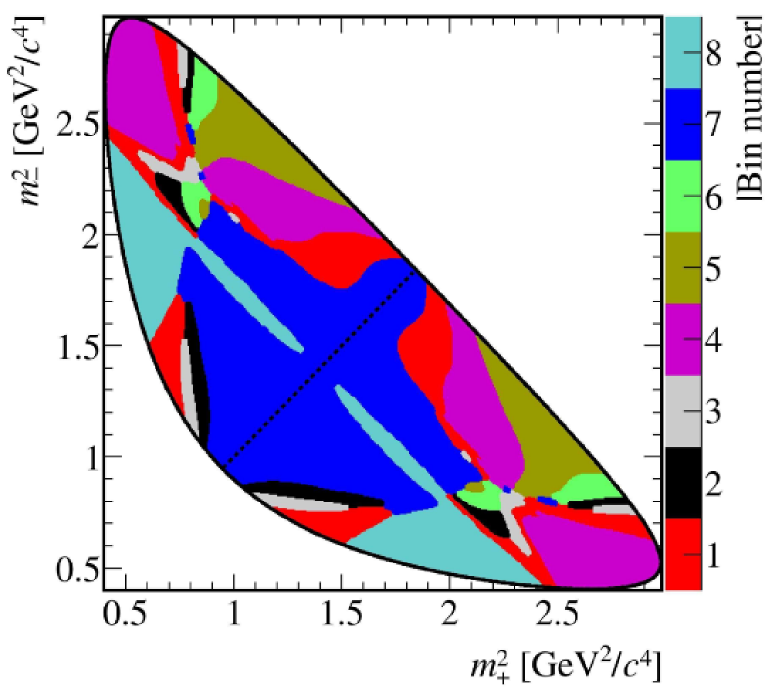


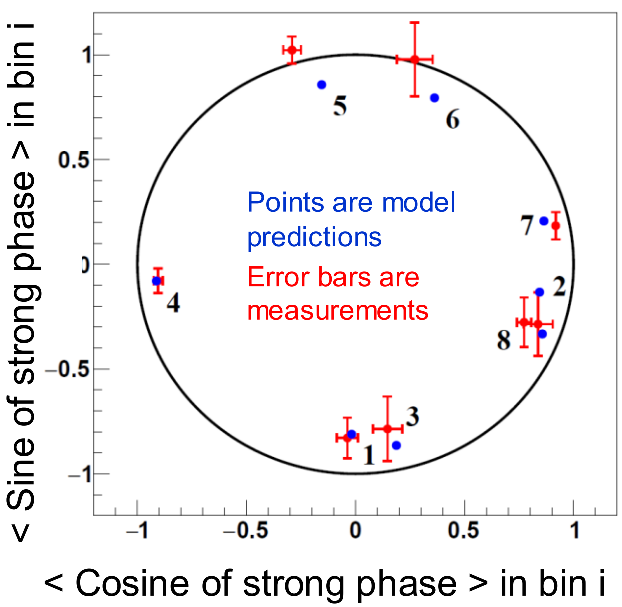
CP asymmetries visible by eye, but quantitative analysis requires external input...

Measuring γ – a synergy of experiments

In order to make sense of these *CP* asymmetries, we need to know how the CP-conserving strong phase between D & Dbar varies over the Dalitz plot.

This information can be measured in bins on the Dalitz plot from quantum-correlated ψ(3770)→DDbar events, available at BESIII [PRD 101 (2020) 112002].





BESIII data (here combined with older CLEO results) adequate for current LHCb sample sizes.

LHCb Upgrade data & Belle II will require improved measurements from BES III !

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These strong-phase measurements are an excellent example of synergy between HEP facilities !

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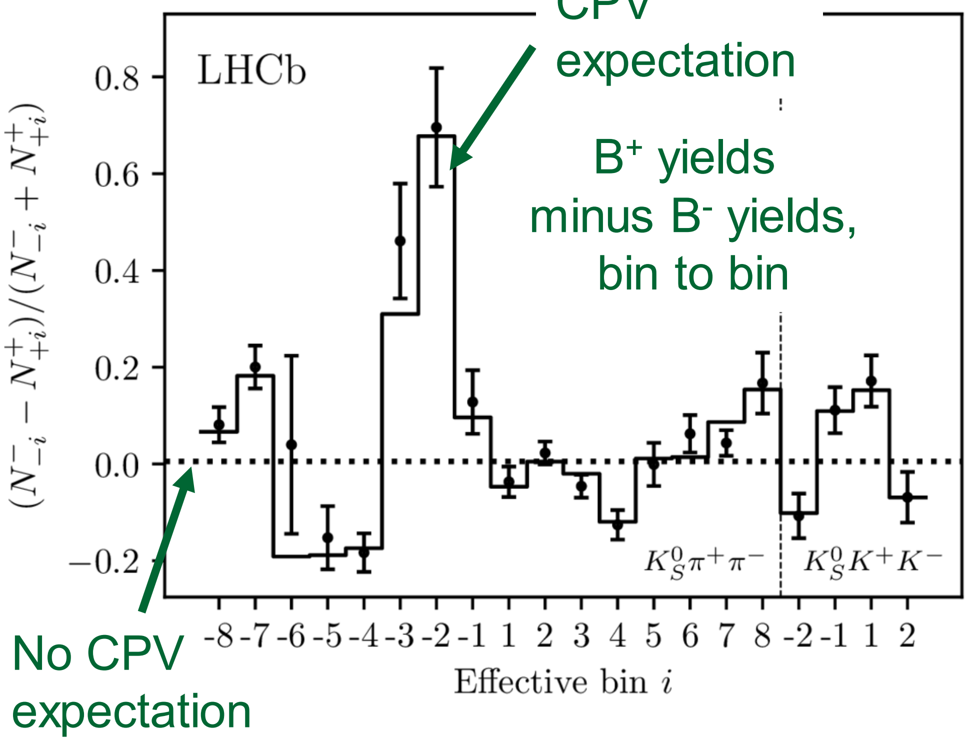


Gives a result of:

$$\gamma = (68.7^{+5.2}_{-5.1})^\circ$$

which is the single most precise determination of γ.

This, and ensemble of other LHCb results (but not yet including new B→D(Kππ^0)K results) gives

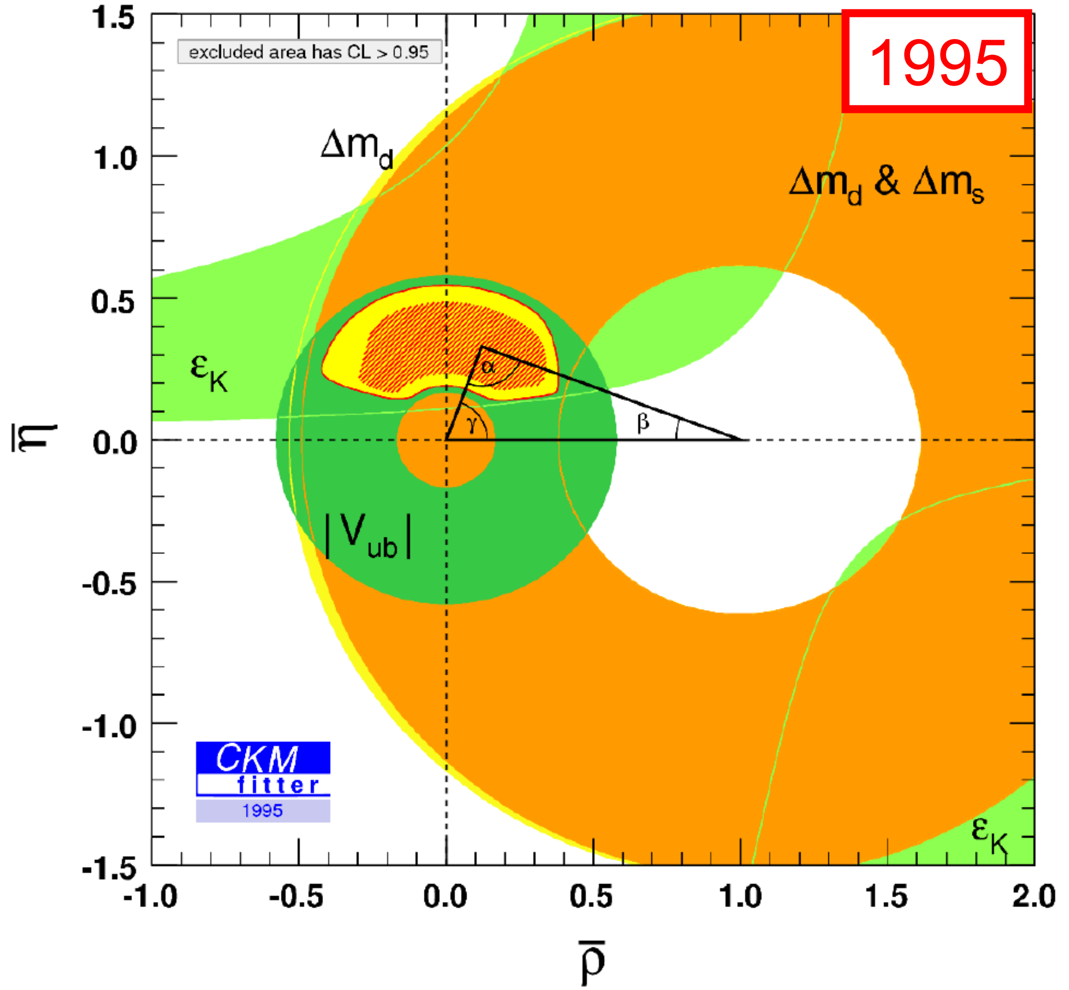
$$\gamma = (65.4^{+3.8}_{-4.2})^\circ$$

[JHEP 12 (2021) 141]

Final LHCb Run 1 + 2 result should have a precision of 2-3 degrees.

In agreement with indirect prediction but not yet as precise → need more data !

Unitarity Triangle: ~25 years of progress



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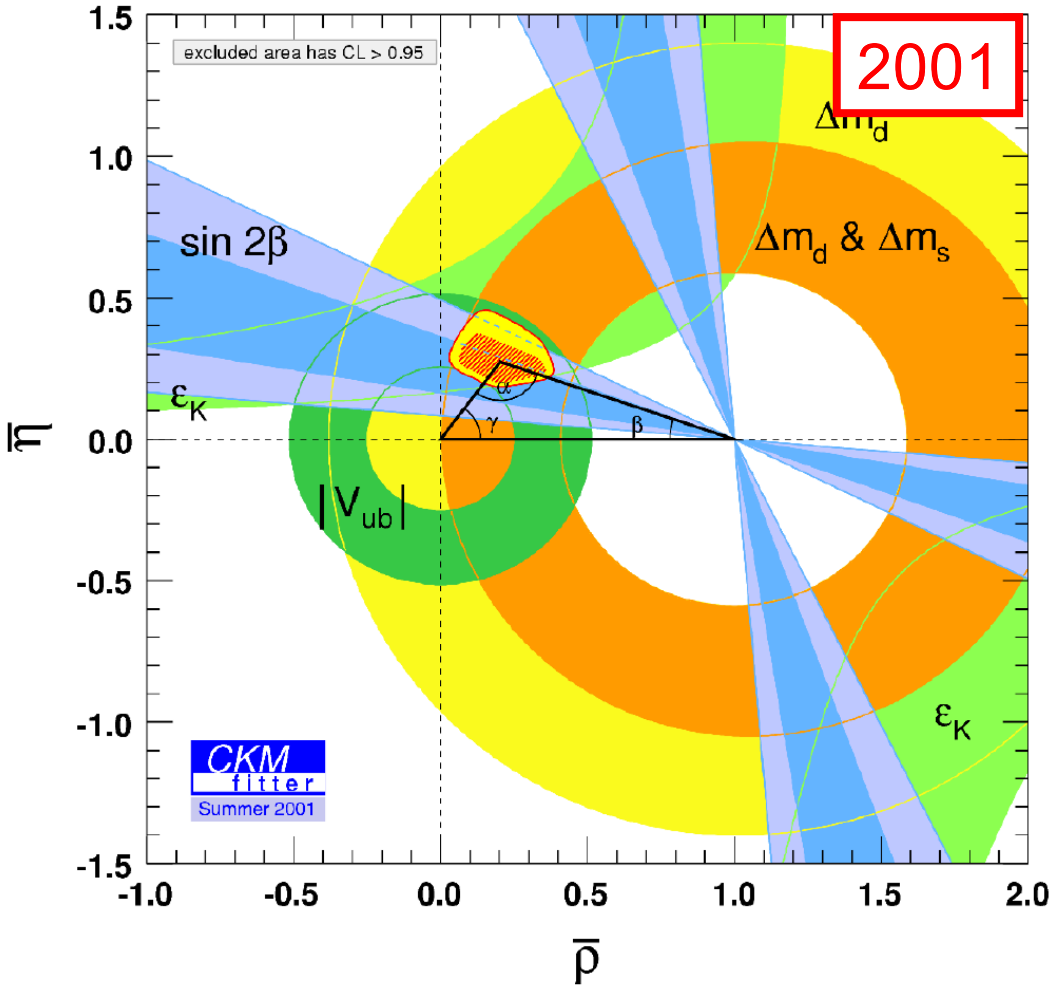

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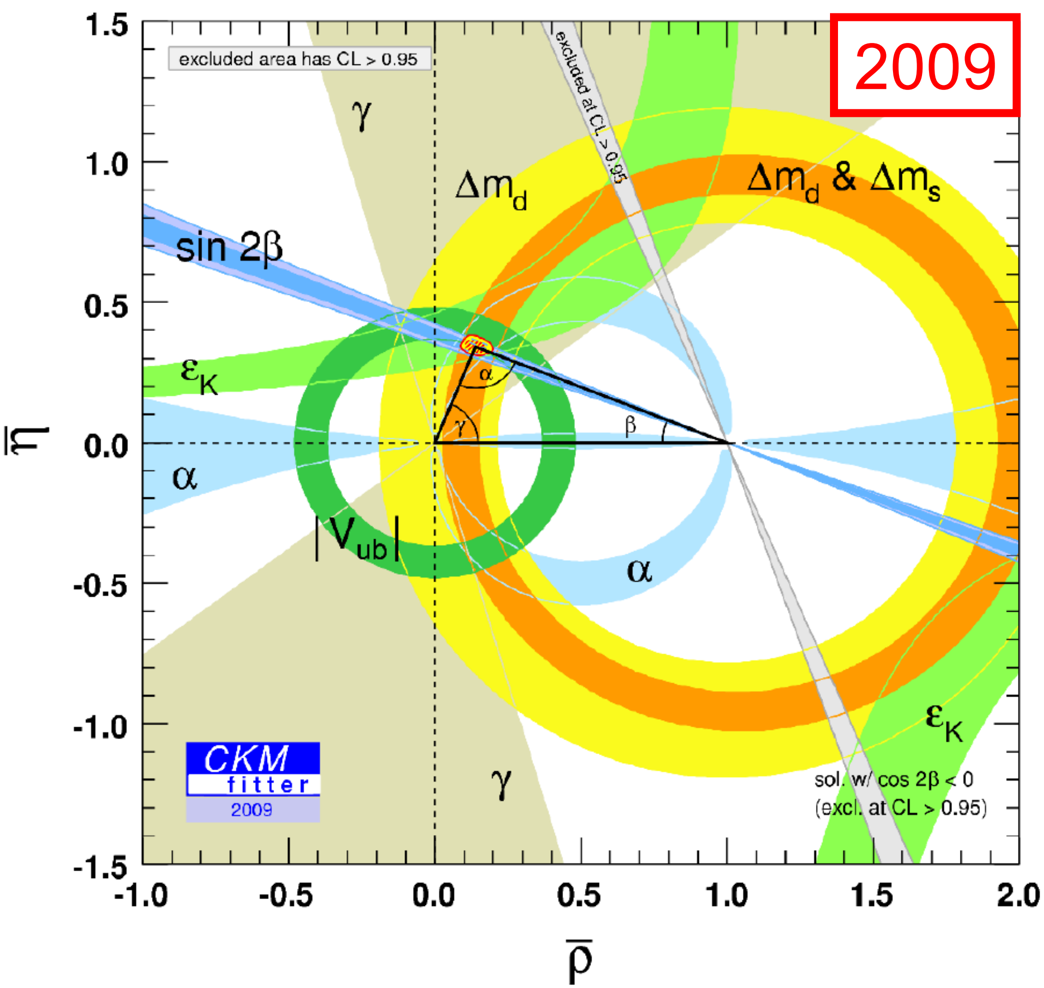

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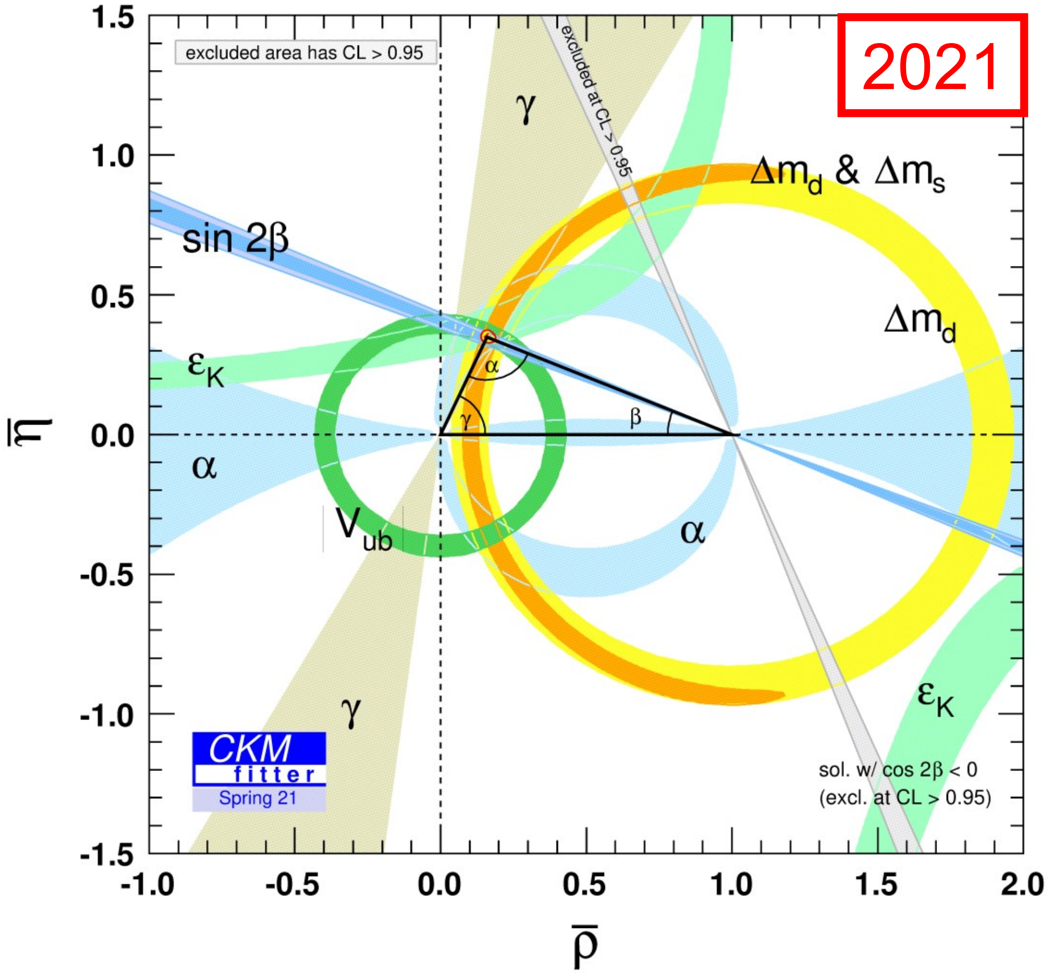
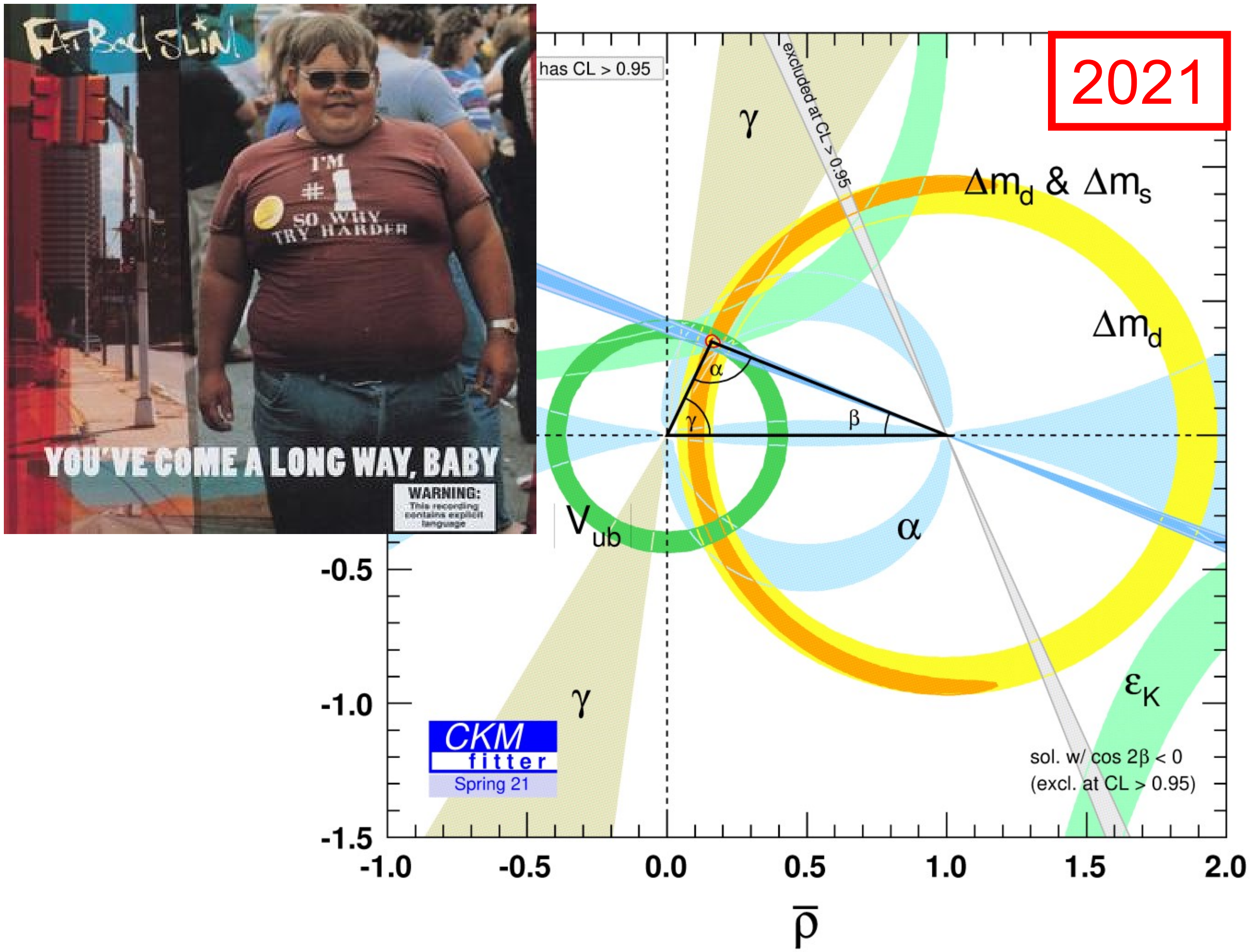


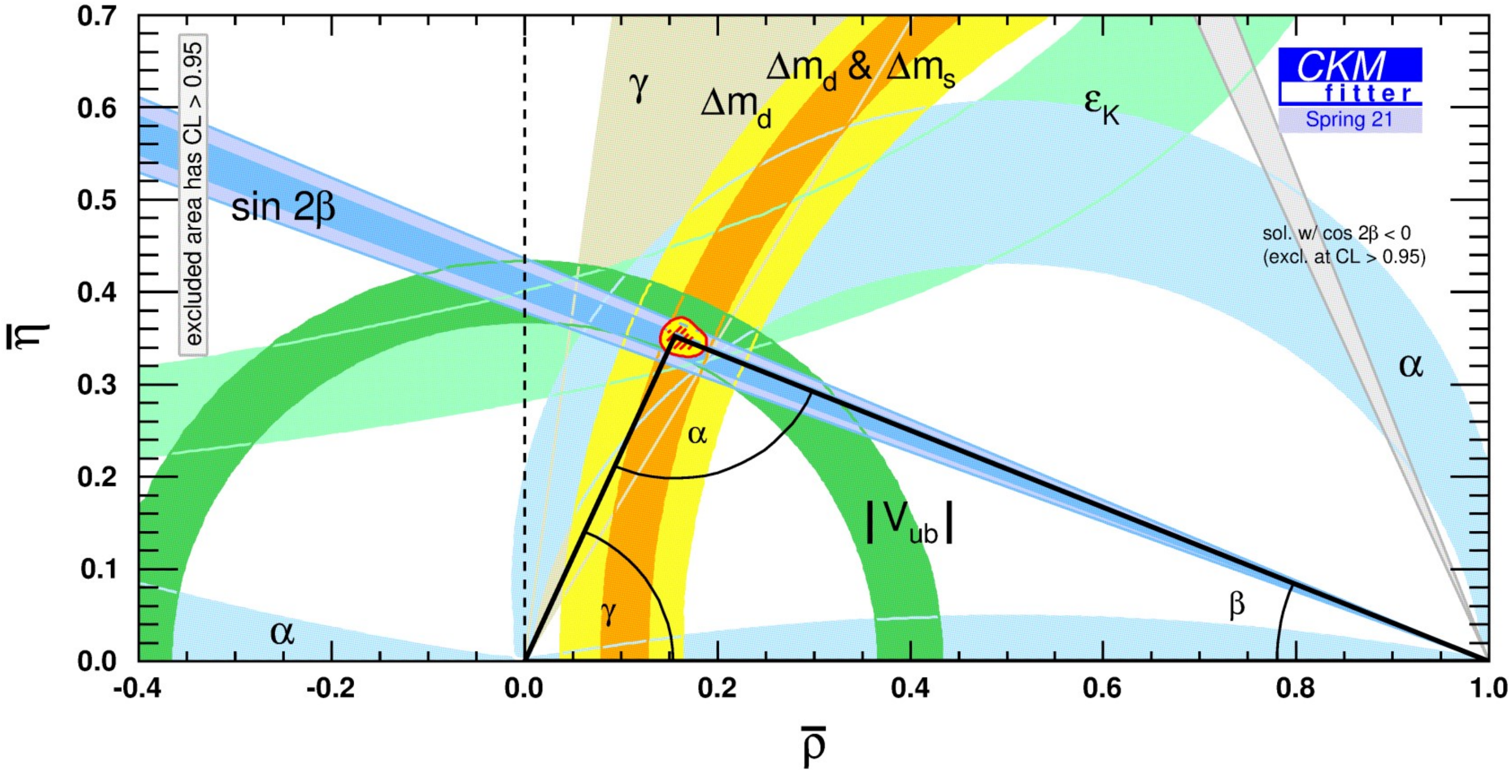
Unitarity Triangle: ~25 years of progress



Enormous improvements in precision, thanks to both experiment and theory (esp. lattice), with LHCb playing an increasingly important role – set to continue.

Overall consistency of the Unitarity Triangle

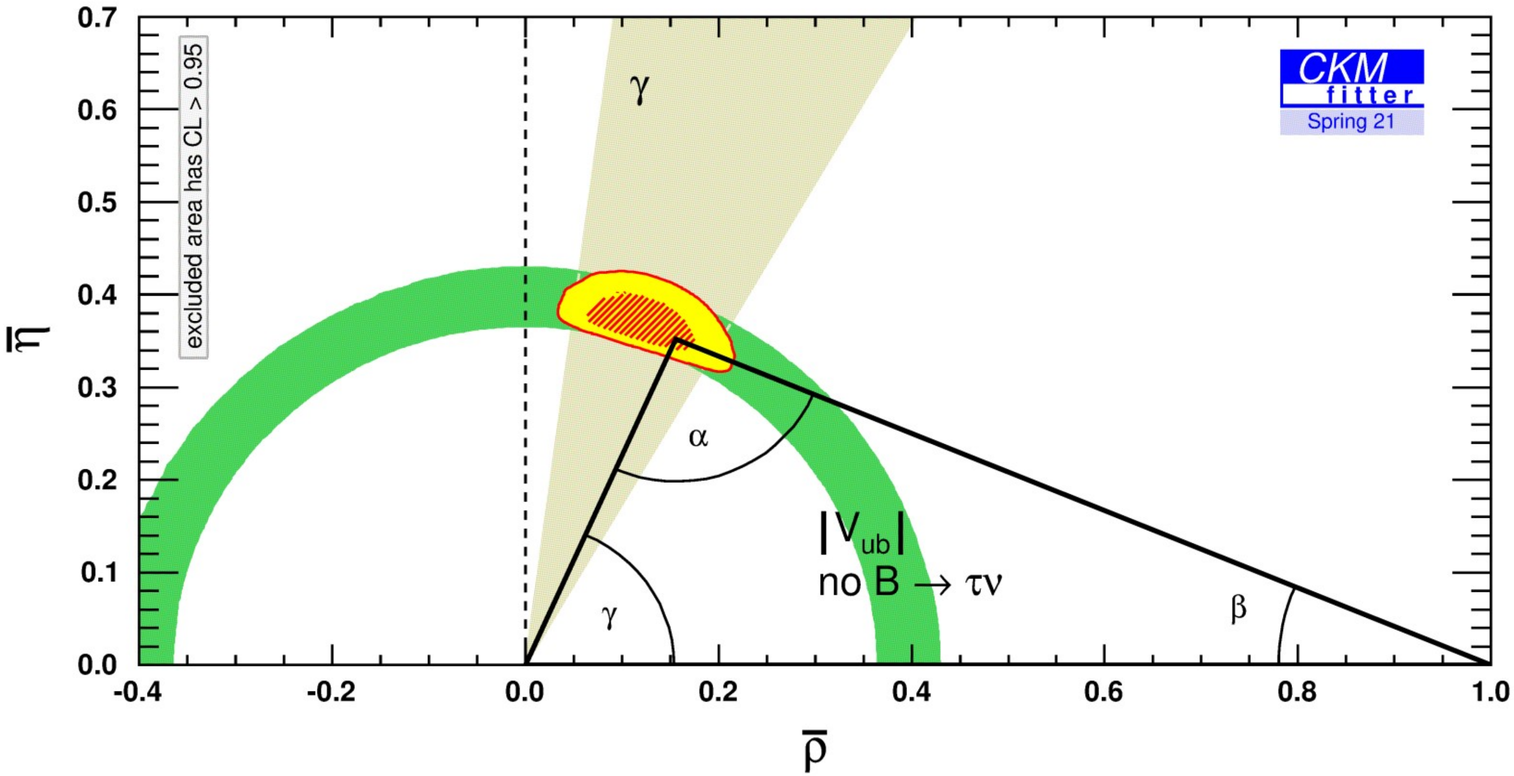
There is broad consistency between all current measurements of the UT. (But, a closer look can reveal intriguing tensions, *e.g.* [Blanke & Buras, EPJC 79 (2019) 159].)



The CKM paradigm is the dominant mechanism of CPV in nature, but it is certainly possible for New Physics to give ~10 % level effects. More measurements needed !

Unitarity Triangle: tree-level observables

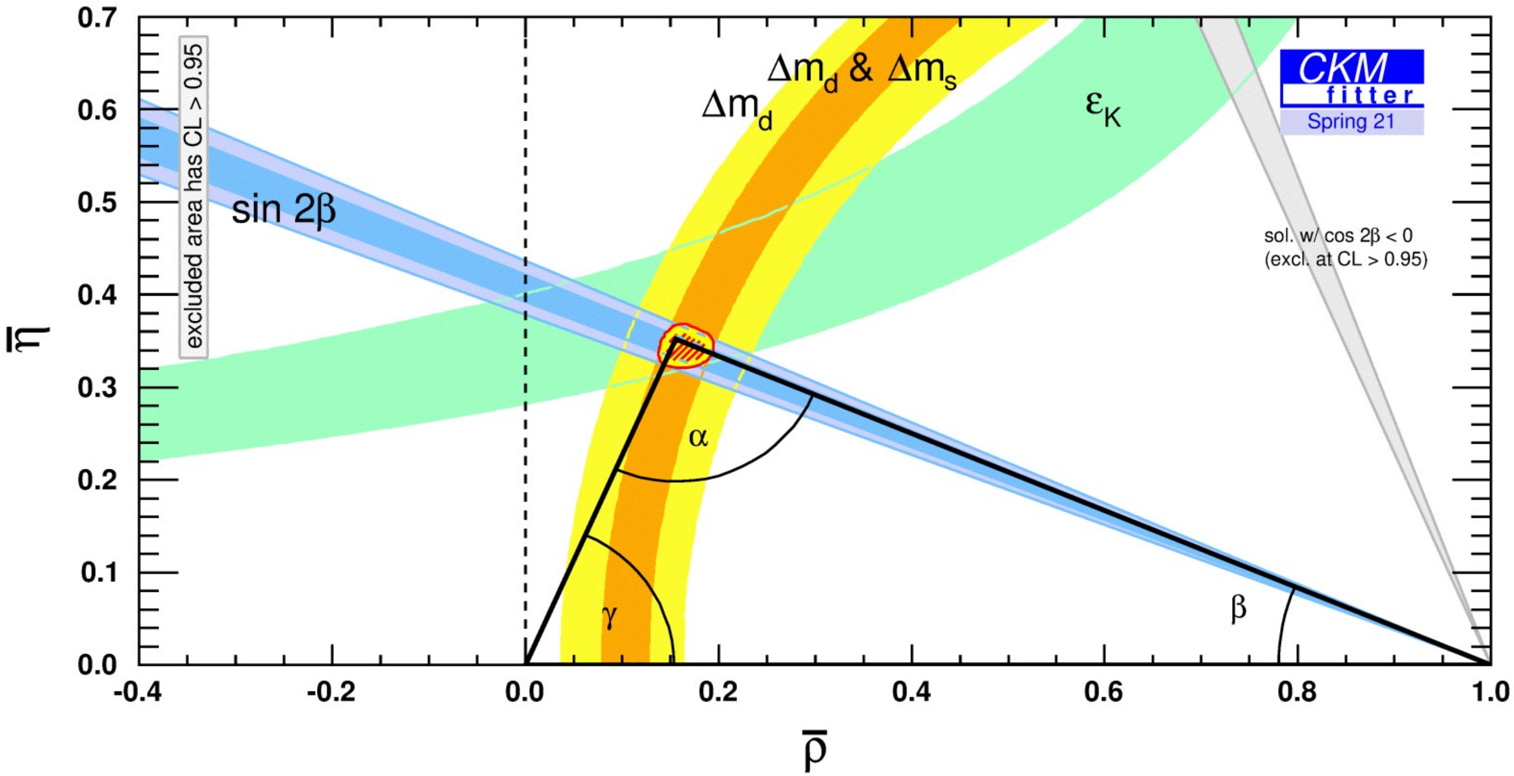
Unitarity Triangle formed from only tree-level quantities → assumed pure SM.



Tree observables are γ & the $|V_{ub}|/|V_{cb}|$ side, here showing exclusive measurement.

Unitarity Triangle: loop-level observables

Unitarity Triangle formed from only loop-level quantities → possibility of NP effects.

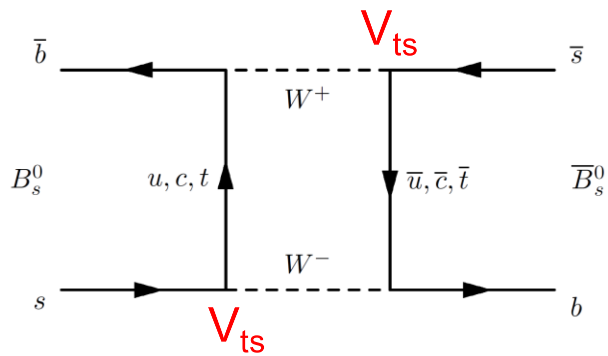


There is good consistency between the tree and loop measurements. There's a need to improve the precision of former to allow for a more sensitive comparison.

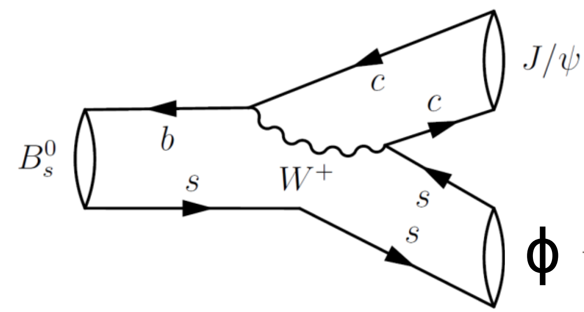
Indirect CPV in B_s system: φ_s

Measuring the CPV phase, φ_s, in B_s mixing-decay interference, *e.g.* with $B_s \to J/\Psi\Phi$, is **the B_s analogue of the sin2β measurement**. In the SM this phase is very small & precisely predicted. Box diagram offers tempting entry point for NP !

Once more interference between mixing…



…and decay



Now we probe CKM elements that are complex only at higher order

$$V_{\text{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\begin{pmatrix} -\frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6) & \mathcal{O}(\lambda^7) & 0 \\ \frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] + \mathcal{O}(\lambda^7) & -\frac{1}{8}\lambda^4(1+4A^2) + \mathcal{O}(\lambda^6) & \mathcal{O}(\lambda^8) \\ \frac{1}{2}A\lambda^5(\rho+i\eta) + \mathcal{O}(\lambda^7) & \boxed{\frac{1}{2}A\lambda^4(1-2(\rho+i\eta)) + \mathcal{O}(\lambda^6)} & -\frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6) \end{pmatrix}$$

$$\phi_s^{\text{SM}} \equiv -2\arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right) = -36.3^{+1.6}_{-1.5}\,\text{mrad}$$

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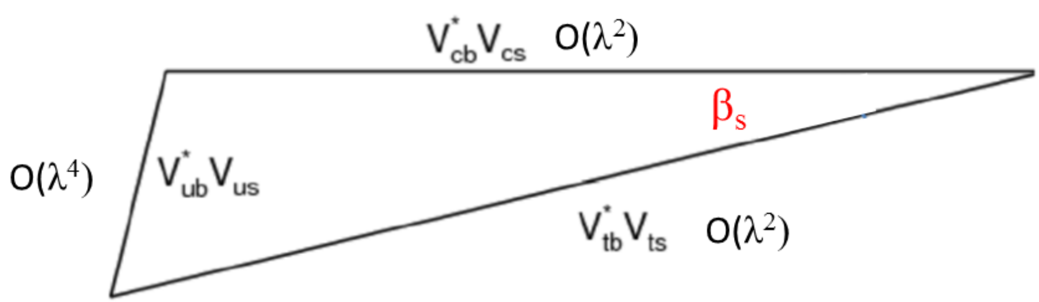
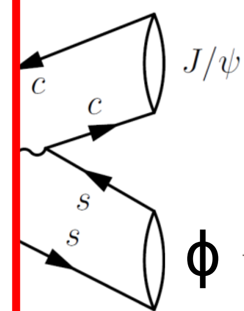
Once mo
interferen
between
mixing…

Recall the squashed B^0_s triangle:



In SM $\varphi_s = -2\beta_s$



Now we
elements
complex

$$\ldots \eta) \Big) + \mathcal{O}(\lambda^4)$$

$$\begin{pmatrix} \ldots & \ldots & 0 \\ \frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] + \mathcal{O}(\lambda^7) & -\frac{1}{8}\lambda^4(1+4A^2) + \mathcal{O}(\lambda^6) & \mathcal{O}(\lambda^8) \\ \frac{1}{2}A\lambda^5(\rho+i\eta) + \mathcal{O}(\lambda^7) & \boxed{\frac{1}{2}A\lambda^4(1-2(\rho+i\eta)) + \mathcal{O}(\lambda^6)} & -\frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6) \end{pmatrix}$$

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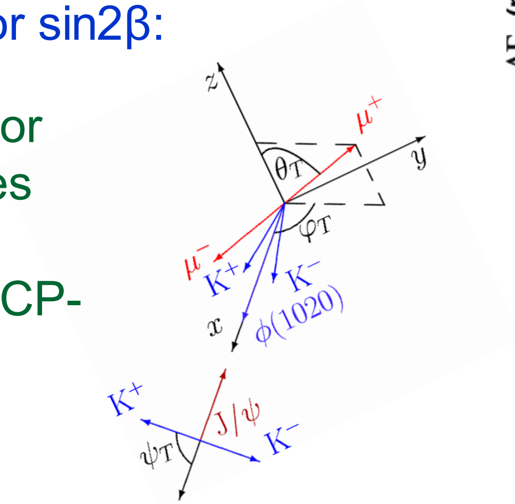
However the measurement is considerably trickier than is the case for sin2β:

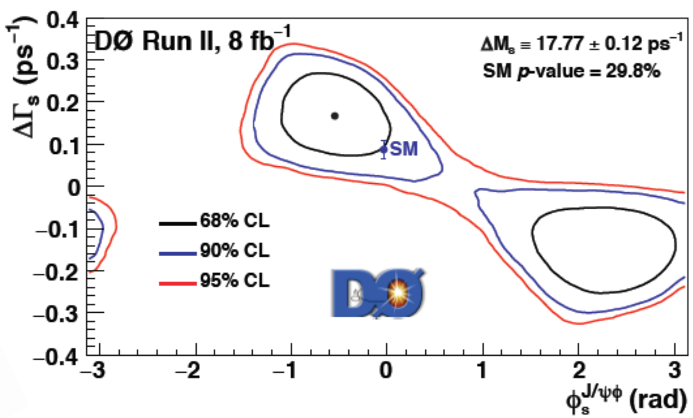
- $J/\Psi\varphi$ is a vector-vector final state, so requires angular analysis to separate out CP+ & CP-



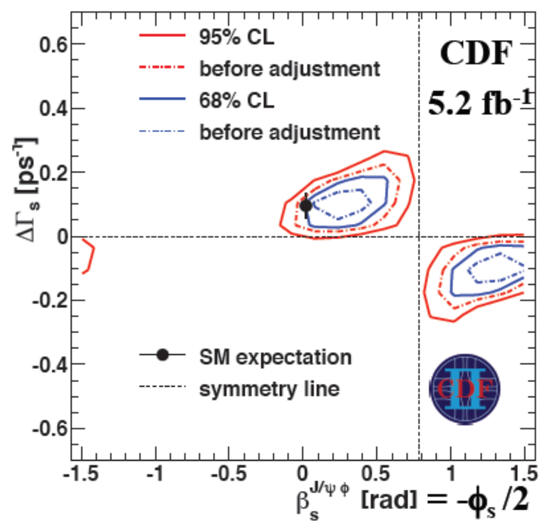
- Very fast oscillations ($\Delta m_s >> \Delta m_d$)

- Possibility of KK S-wave under φ

Heroic early analyses performed by Tevatron. Consistent results and mild (~1σ) tension with SM.



[PRD 85 (2012) 032006]



[PRD 85 (2012) 072002]

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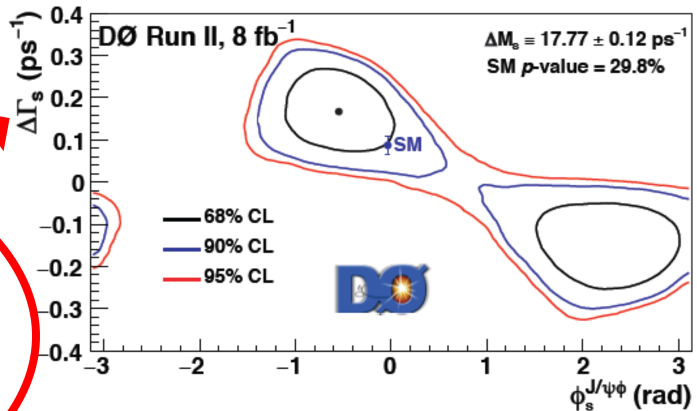
However the measurement is considerably trickier than is th… in2β:

One other detail: in contrast to the B^0 case, the width-splitting $\Delta\Gamma_s$ between the mass eigenstates Is here non-negligible (~0.1). When included in the formalism this brings additional handles to the analysis, & also provides an additional observable to be measured.

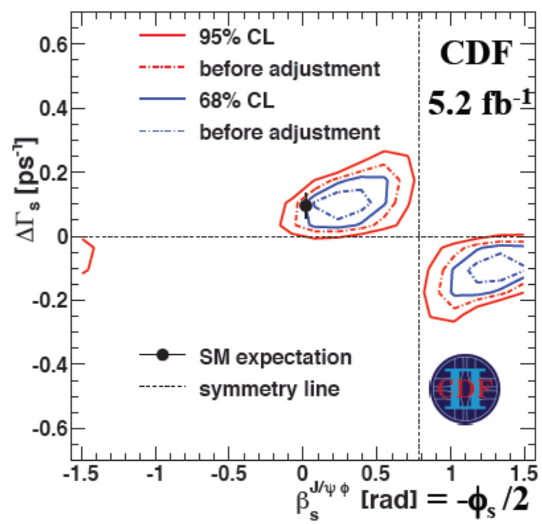
(ΔΓ…

- Possibility of KK S-wave under φ

Heroic early analyses performed by Tevatron. Consistent results and mild (~1σ) tension with SM.



[PRD 85 (2012) 032006]

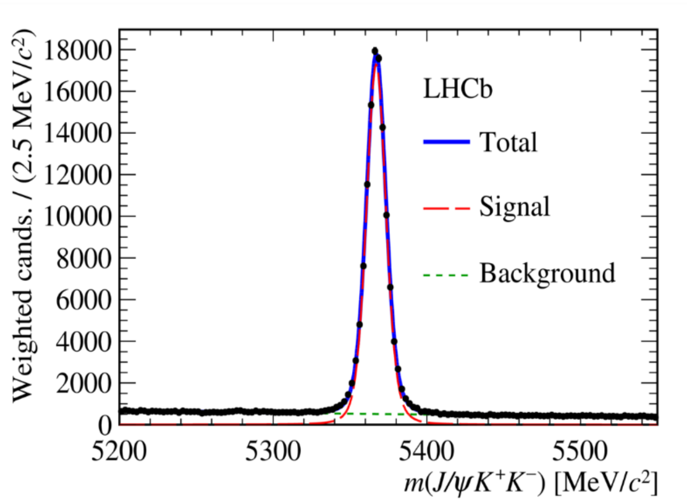


[PRD 85 (2012) 072002]

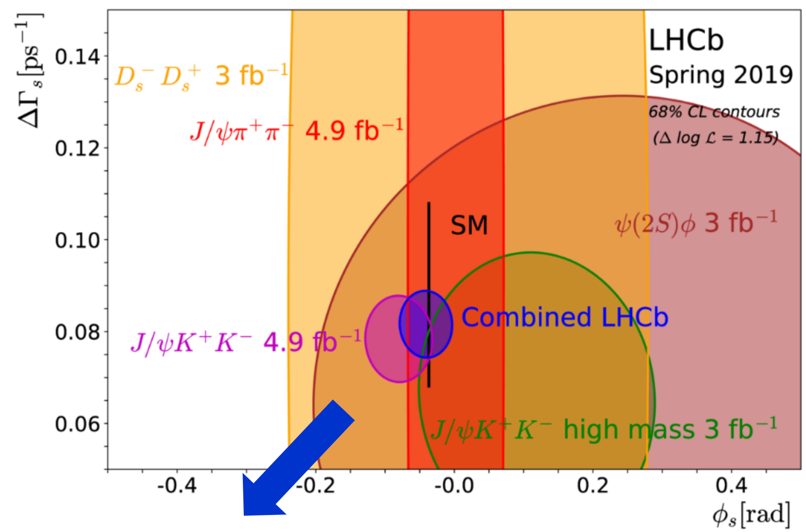
φ_s – impact of LHCb

LHC has been able to go far beyond the Tevatron measurements, thanks to much larger yields, and (in case of LHCb) excellent proper time resolution, & access to complementary modes beyond J/ψφ (*e.g.* $B_s \to J/\psi\pi\pi$ [PLB 797 (2019) 134789] .)

$B_s \to J/\psi\varphi$ signal peak in early Run 2 analysis (117k decays, in 1.9 fb^{-1} *c.f.* 6.5k at CDF).



Results for early Run 2 J/ψφ study, together with Run 1 measurements.



[EPJC 79 (2019) 706]

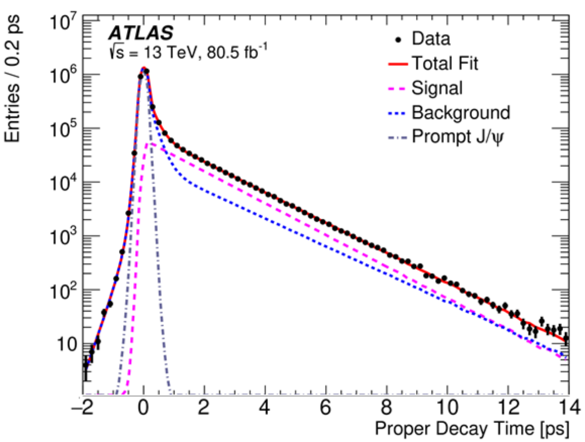
$$\phi_s = -0.041 \pm 0.025\,\text{rad} \qquad \Delta\Gamma_s = 0.0816 \pm 0.0048\,\text{ps}^{-1}$$

Measurement of φ_s at ATLAS and CMS

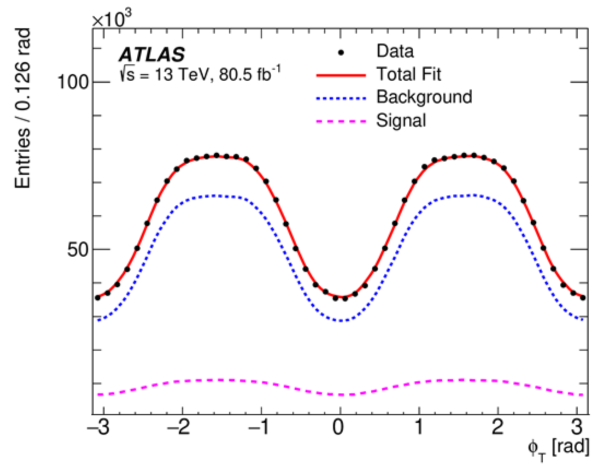
Measurement of φ_s is an key goal of the ATLAS and CMS flavour physics programme, enabled by excellent detector performance and $J/\Psi\to\mu\mu$ trigger.

e.g. ATLAS $B_s\to J/\Psi\varphi$ Run 2 analysis with 80 fb^{-1} [Eur. Phys. J. C 81 (2021) 342] :

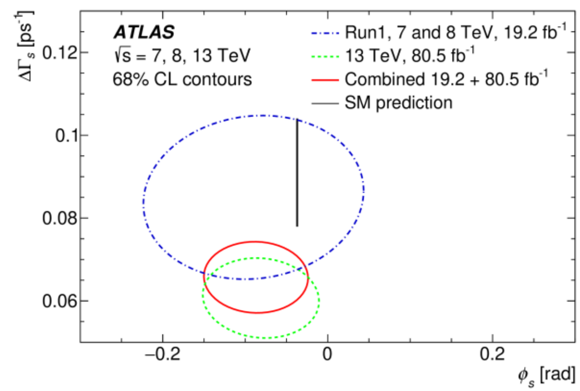
Proper decay time



Transversity angle φ_T



Results, including those of Run 1 [JHEP 08 (2016) 147]



Combining with Run 1 results [JHEP 08 (2016) 147]

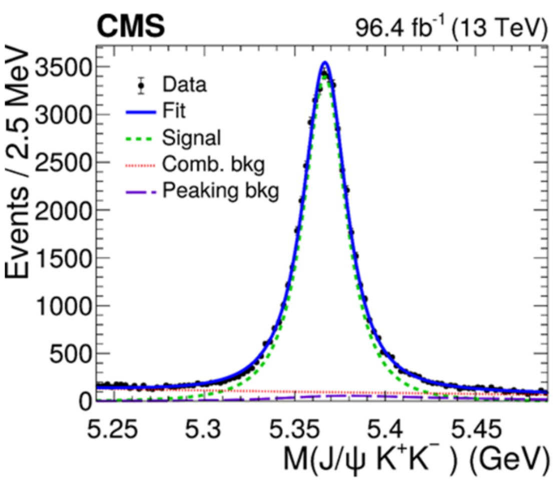
$$\phi_s = -0.087 \pm 0.036\ (\text{stat.}) \pm 0.021\ (\text{syst.})\ \text{rad}$$
$$\Delta\Gamma_s = 0.0657 \pm 0.0043\ (\text{stat.}) \pm 0.0037\ (\text{syst.})\ \text{ps}^{-1}$$

Measurement of φ_s at ATLAS and CMS

Measurement of φ_s is an key goal of the ATLAS and CMS flavour physics programme, enabled by excellent detector performance and $J/\Psi\to\mu\mu$ trigger.

e.g. CMS $B_s\to J/\Psi\varphi$ Run 2 analysis with 96 fb^{-1} [PLB 816 (2021) 136188]

Invariant mass



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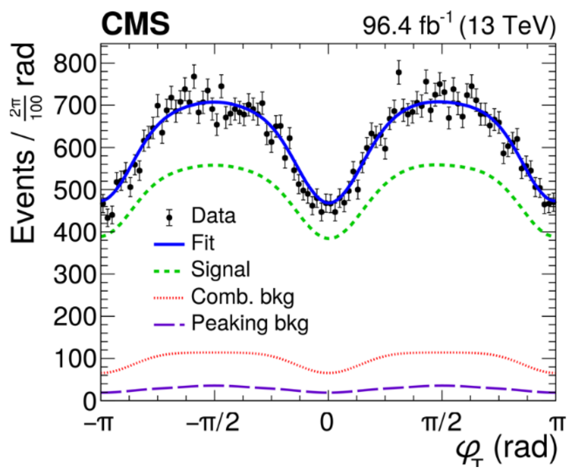
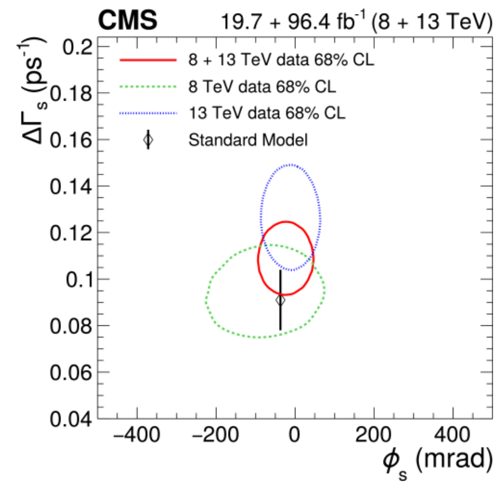


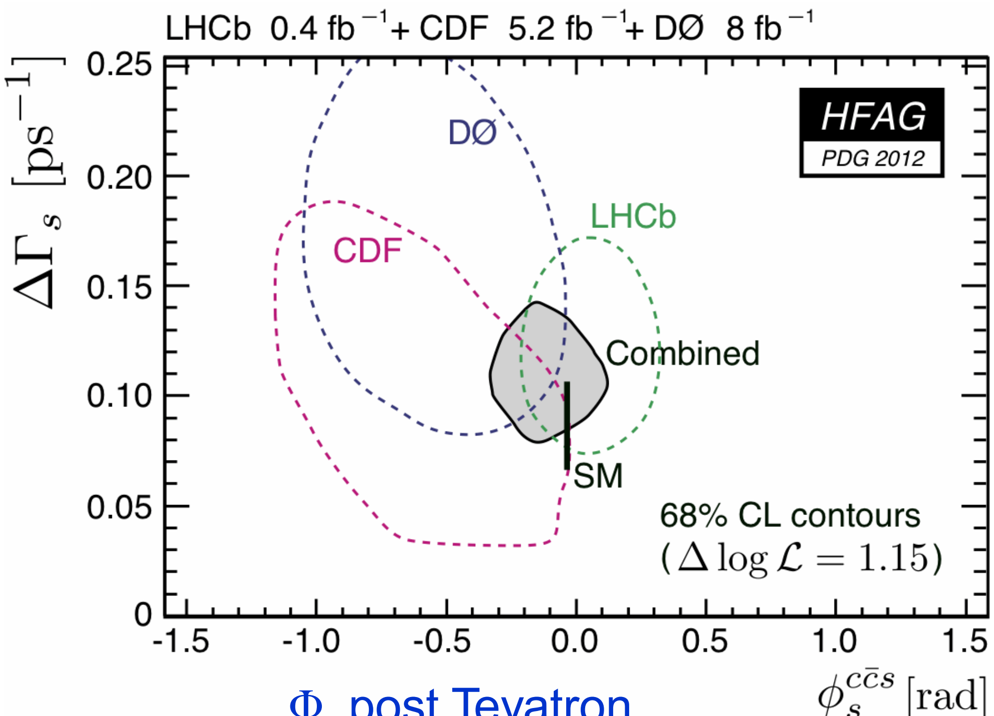
Result contours



Combining with Run 1 results [PLB 757 (2016) 97]

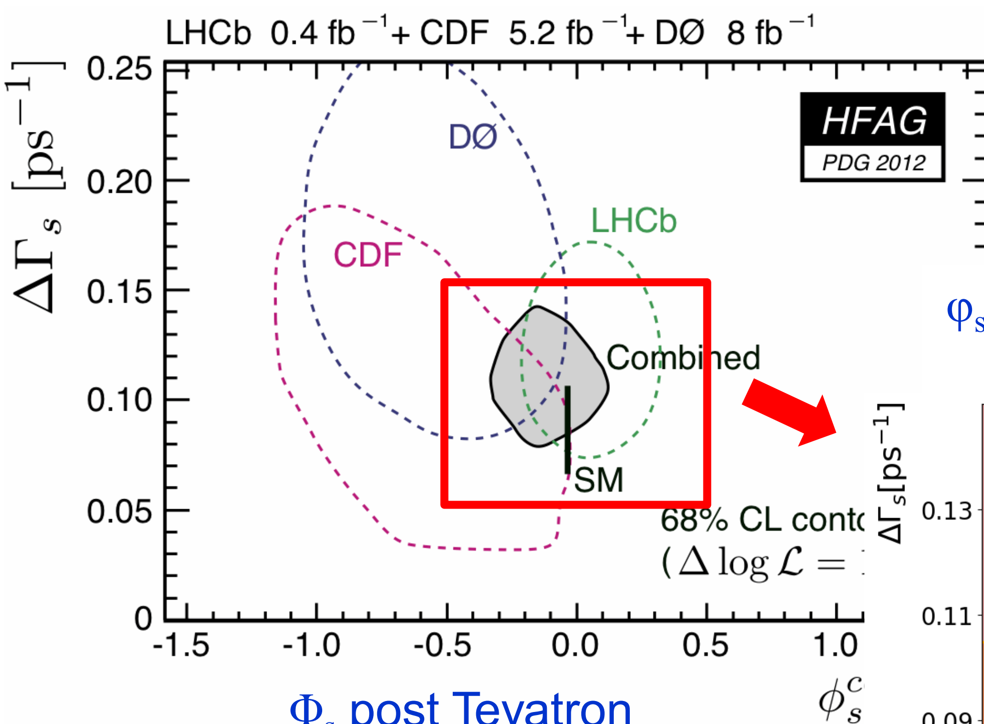
$$\phi_s = -21 \pm 44\,(\text{stat}) \pm 10\,(\text{syst})\,\text{mrad},$$
$$\Delta\Gamma_s = 0.1032 \pm 0.0095\,(\text{stat}) \pm 0.0048\,(\text{syst})\,\text{ps}^{-1}.$$

φ_s : the impact of the LHC



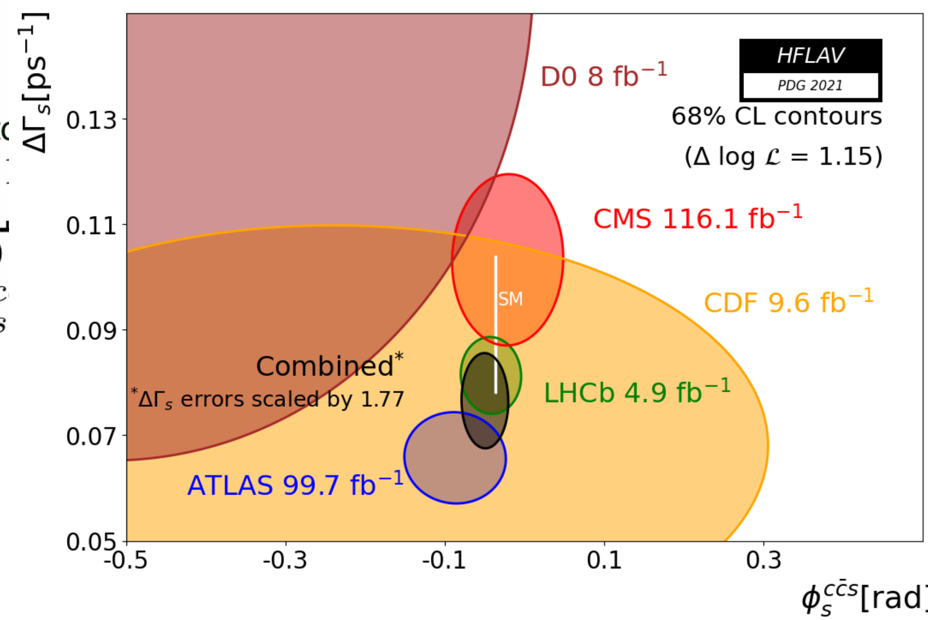
Φ_s post Tevatron
and early LHC data

φ_s : the impact of the LHC

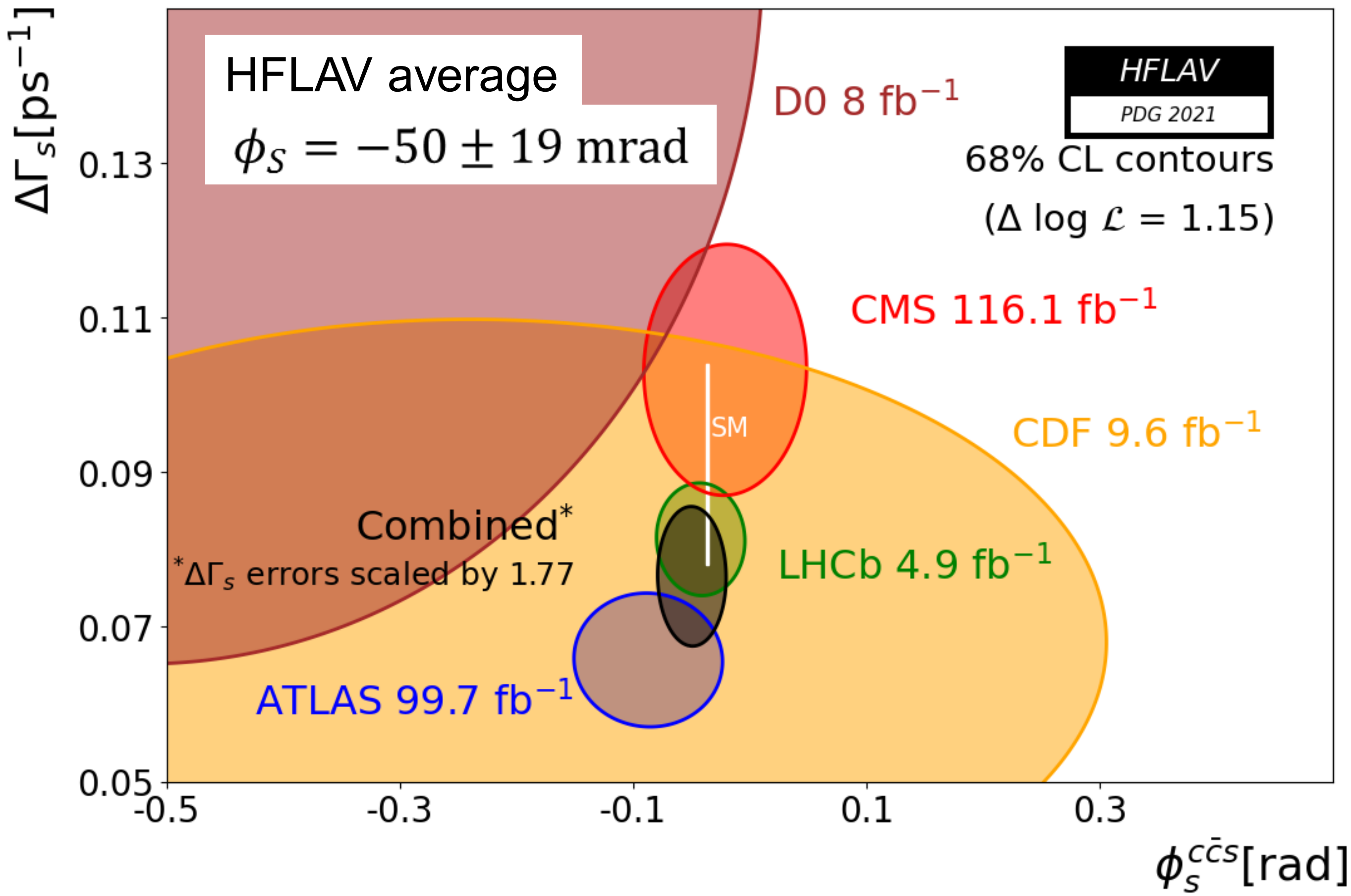


Φ_s post Tevatron
and early LHC data

φ_s current LHC including Run 2 ATLAS & CMS & some Run 2 LHCb data



φ_s : the current state of play



φ_s now measured with 19 mrad precision and so far compatible with SM.
Hint of non-zero value emerging – will be very interesting with Run 3 dataset !

Conclusions and outlook

The CKM matrix and CP violation lie at the heart of some of the deepest problems in modern physics.

The B factories showed us, triumphantly, that the CKM paradigm is correct at first order, but more precise tests are required. Indeed many observables are theoretically pristine and should be measured with the highest precision attainable.

Hadron colliders are ideally suited to this challenge, as shown by achievements in the measurement of β and, even more so, γ and φ_s. The prospects for improving these measurements are outstanding (see lecture IV).

Many, many other CPV studies out there (*e.g.* those of charmless B decays).