Dynamical Manifestations of Many-Body Quantum Chaos and Self-Averaging

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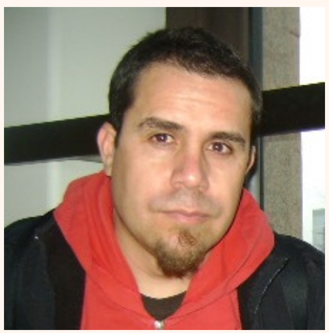
Jonathan Torres-Herrera



Mauro Schiulaz

Talía Lezama

Adway Kumar Das

Isaías Vallejo

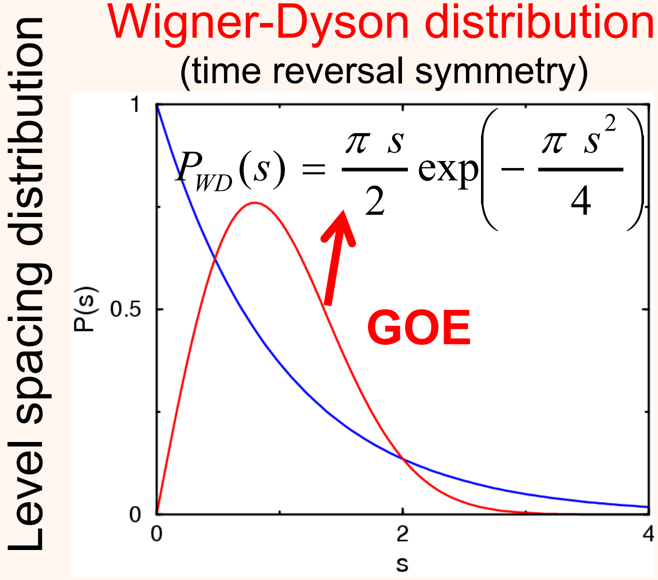
Patrick Pinney

Proposal for many-body quantum chaos detection
arXiv:2401.01401

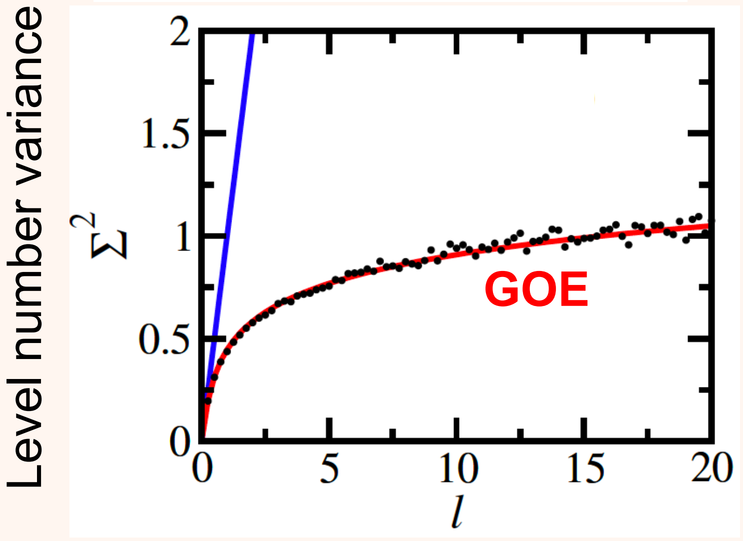
Quantum Chaos = Spectral Correlations

Quantum Chaos: Spectral correlations as in full random matrices

Wigner-Dyson distribution
(time reversal symmetry)



$\Sigma^2(l) \equiv \langle [N(l,\epsilon)]^2 \rangle - \langle N(l,\epsilon) \rangle^2$

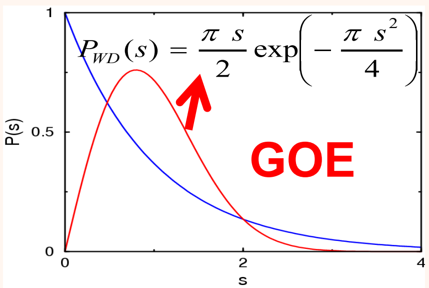


Level Statistics and the Spectral Form Factor

Quantum Chaos: Spectral correlations as in full random matrices

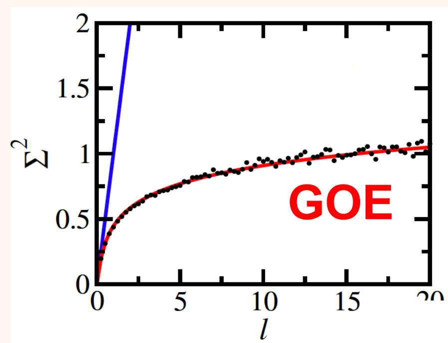
Wigner-Dyson distribution

Level spacing distribution



$$\Sigma^2(l) \equiv \langle [N(l,\epsilon)]^2 \rangle - \langle N(l,\epsilon) \rangle^2$$

Level number variance



GOE

D=5

D=15

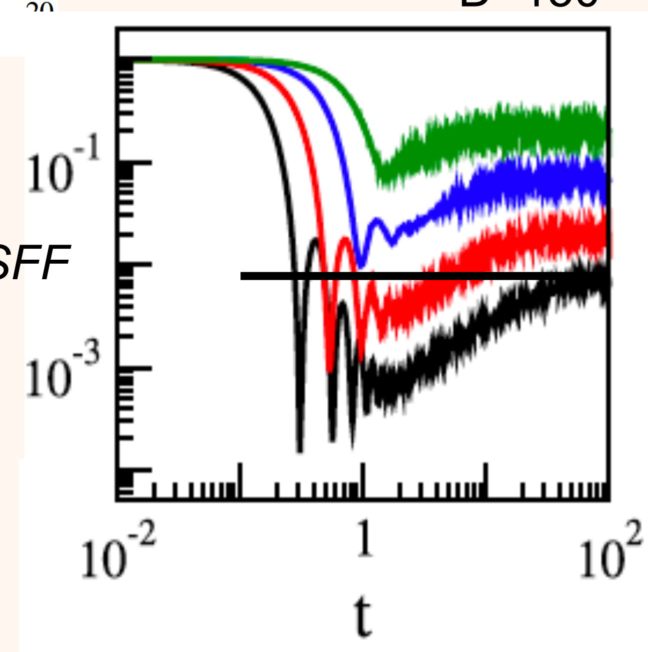
D=50

D=150

Spectral form factor

$$\mathrm{SFF}(t) = \frac{1}{D^2} \left\langle \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} \right\rangle$$

SFF



Fourier transform of the two-point spectral correlation function

Slope-dip-ramp-plateau structure

Lea F. Santos, UConn

Advantages of Spectral Form Factor

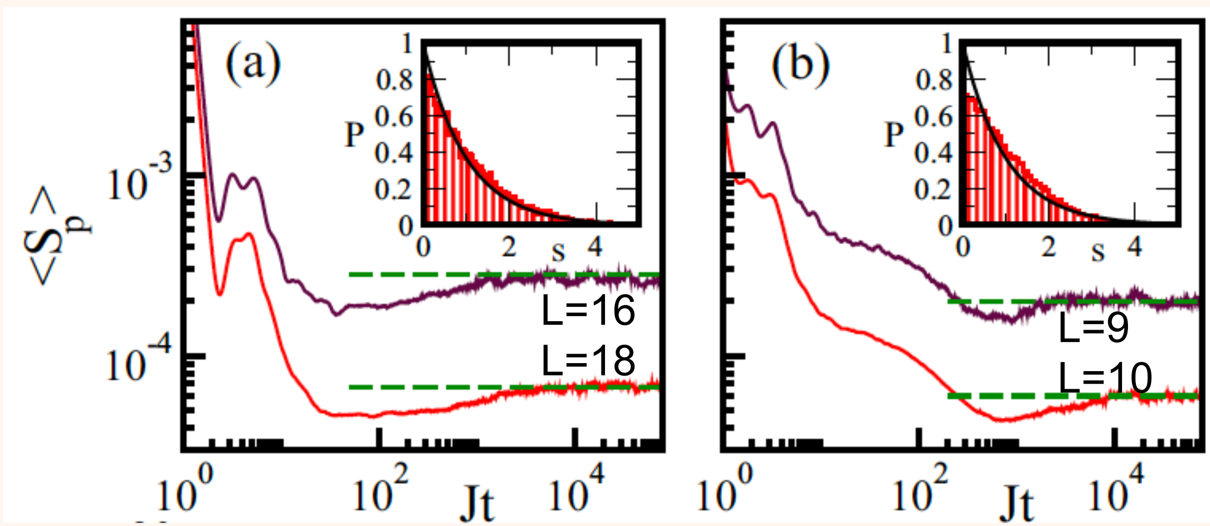
Short- and long-range correlation

No need for unfolding

Detect chaos despite symmetries

Spin-1/2 model

Spin-1 model



Slope-dip-ramp-plateau structure

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

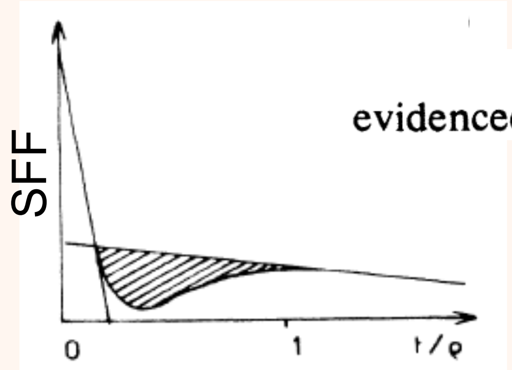
Correlation Hole

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Fourier Transform: A Tool to Measure Statistical Level Properties in Very Complex Spectra

Luc Leviandier, Maurice Lombardi, Rémi Jost, and Jean Paul Pique
Laboratoire de Spectrométrie Physique, Université Scientifique et Médicale de Grenoble, 38402 Saint Martin d'Hères, France, and Service National des Champs Intenses, Centre National de la Recherche Scientifique, 38042 Grenoble Cedex, France
(Received 27 November 1985)

We show that the Fourier transform of very complex spectra gives a sound measurement of long-range statistical properties of levels even in cases of badly resolved, poorly correlated spectra. Examples of nuclear energy levels, highly excited acetylene vibrational levels, and singlet-triplet anticrossing spectra in methylglyoxal are displayed.



The presence of level correlation is thus evidenced by a "correlation hole"

Correlation hole

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Fourier Transform: A Tool to Measure Statistical Level Properties in Very Complex Spectra

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(Received 27 November 1985)

VOLUME 58, NUMBER 5 PHYSICAL REVIEW LETTERS 2 FEBRUARY 1987

Chaos and Dynamics on 0.5–300-ps Time Scales in Vibrationally Excited Acetylene: Fourier Transform of Stimulated-Emission Pumping Spectrum

J. P. Pique,[a] Y. Chen, R. W. Field, and J. L. Kinsey
Department of Chemistry and George Harrison Spectroscopy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 27 October 1986)

Chemical Physics 146 (1990) 21–38
North-Holland

Correlations in anticrossing spectra and scattering theory. Analytical aspects

T. Guhr and H.A. Weidenmüller
Max-Planck-Institut für Kernphysik, 6900 Heidelberg, FRG

Received 12 December 1989

Experimental results of anticrossing spectroscopy in molecules, in particular the correlation hole, are discussed in a theoretical model. The laser measurements are modelled in terms of the scattering matrix formalism originally developed for compound nucleus scattering. Random matrix theory is used in the framework of this model. The correlation hole is analytically derived for small singlet–triplet coupling. In the case of the data on methylglyoxal this limit is realistic if the spectrum is indeed a superposition of several pure sequences as one can conclude from the analysis of the measurements.

PHYSICAL REVIEW E, VOLUME 65, 026214

Signatures of the correlation hole in total and partial cross sections

T. Gorin* and T. H. Seligman
Centro de Ciencias Fisicas, University of Mexico (UNAM), CP 62210 Cuernavaca, Mexico
(Received 3 August 2001; published 24 January 2002)

In a complex scattering system with few open channels, say a quantum dot with leads, the correlation properties of the poles of the scattering matrix are most directly related to the internal dynamics of the system. We may ask how to extract these properties from an analysis of cross sections. In general this is very difficult, if we leave the domain of isolated resonances. We propose to consider the cross correlation function of two different elastic or total cross sections. For these we can show numerically and to some extent also analytically a significant dependence on the correlations between the scattering poles. The difference between uncorrelated and strongly correlated poles is clearly visible, even for strongly overlapping resonances.

studies which have attempted to discern the difference in **quantum dynamics** of systems with regular versus irregular spectra, several have focused on the **survival probability**

PHYSICAL REVIEW A VOLUME 46, NUMBER 8 15 OCTOBER 1992

Spectral autocorrelation function in the statistical theory of energy levels

Y. Alhassid
Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecti and the A.W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut

R. D. Levine
The Fritz Haber Research Center for Molecular Dynamics, The Hebrew University, Jerusalem 91
(Received 11 October 1991; revised manuscript received 5 May 1992)

VOLUME 67, NUMBER 10 PHYSICAL REVIEW LETTERS 2 SEPTEMBER 1991

Time-Dependent Manifestations of Quantum Chaos

Joshua Wilkie and Paul Brumer
Chemical Physics Theory Group, Department of Chemistry, University of Toronto, Toronto, Ontario, Canada M5S 1A1
(Received 11 April 1991)

REVIEW: Dynamical Manifestations of Many-Body Quantum Chaos

Survival Probability: $\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}^{ini} e^{-iE_{\alpha} t} |\alpha\rangle$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_{\alpha}^{ini}|^2 |C_{\beta}^{ini}|^2 e^{-i(E_{\alpha} - E_{\beta})t} \rangle$$

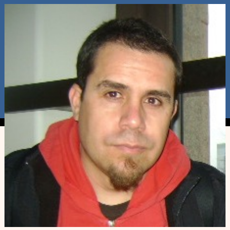
Quench dynamics $C_{\alpha}^{ini} = \langle \alpha | \Psi(0) \rangle$

$$\Psi(0) = \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow$$

$$\mathrm{SFF}(t) = \frac{1}{D^2} \left\langle \sum_{\alpha,\beta} e^{i(E_{\alpha} - E_{\beta})t} \right\rangle$$

ICTS 2024

REVIEW: Dynamical Manifestations of Many-Body Quantum Chaos



Survival Probability: $\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

Many-body localization

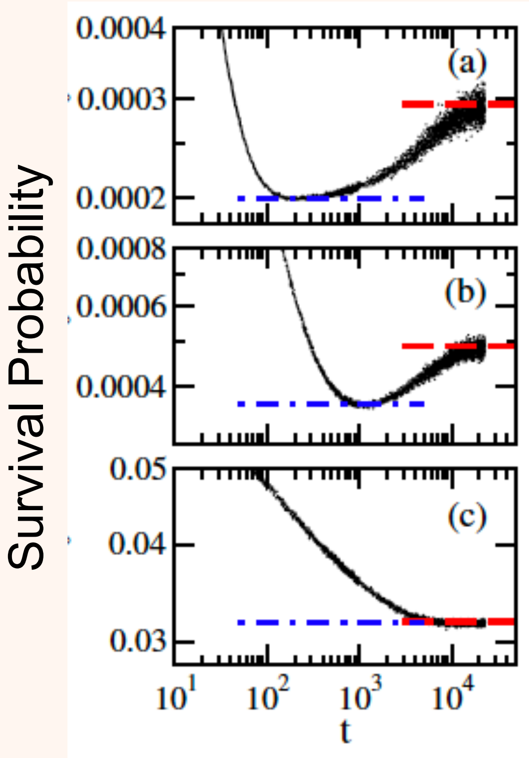
Ann. Phys. **529**, 1600284 (2017)

CORRELATION HOLE

$$\langle SP(t)\rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t}\rangle$$

Quench dynamics $C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$

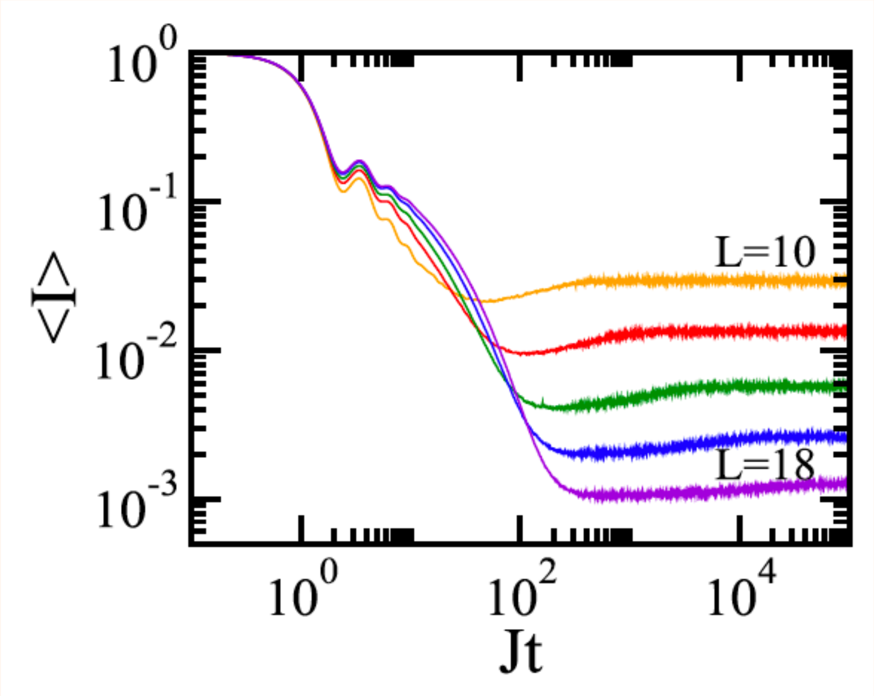
$$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$



Torres-Herrera & LFS
Ann. Phys. (Berlin) **529**, 1600284 (2017)
Philos. Trans. R. Soc. London Soc. A **375**, 20160434 (2017)

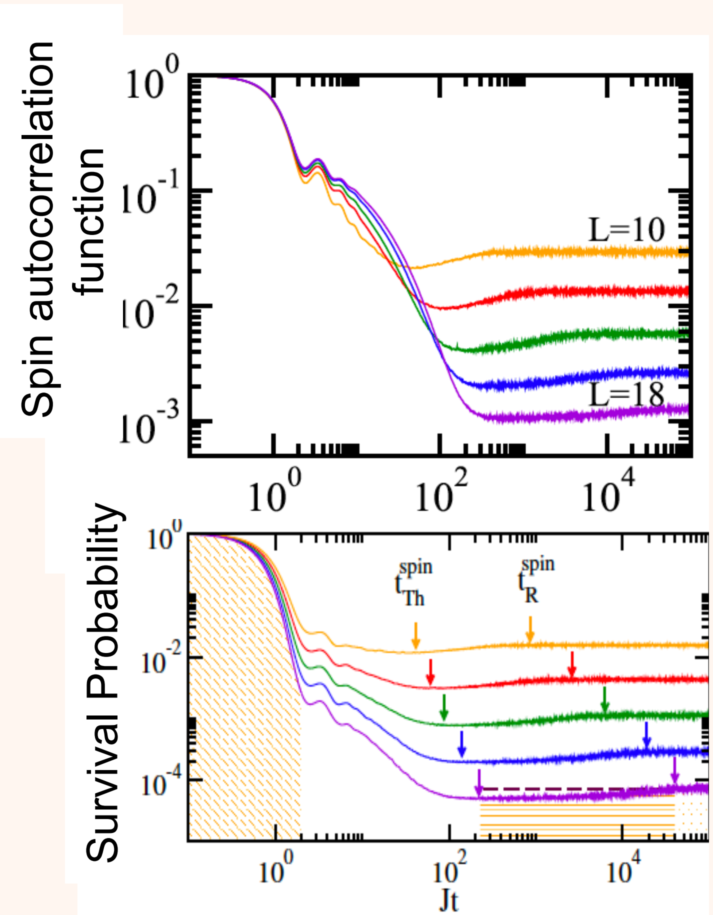
REVIEW: Dynamical Manifestations of Many-Body Quantum Chaos

Spin autocorrelation function: $$I(t) = \frac{1}{L}\sum_{k=1}^{L} \langle \Psi(0)|\sigma_k^z e^{iHt} \sigma_k^z e^{-iHt}|\Psi(0)\rangle$$



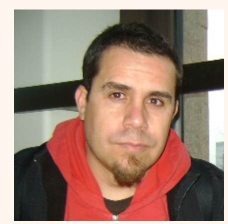
Torres-Herrera, García-García & LFS
PRB **97**, 060303(R) (2018)

REVIEW: Dynamical Manifestations of Many-Body Quantum Chaos



$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$

Torres-Herrera, García-García & LFS
PRB **97**, 060303(R) (2018)



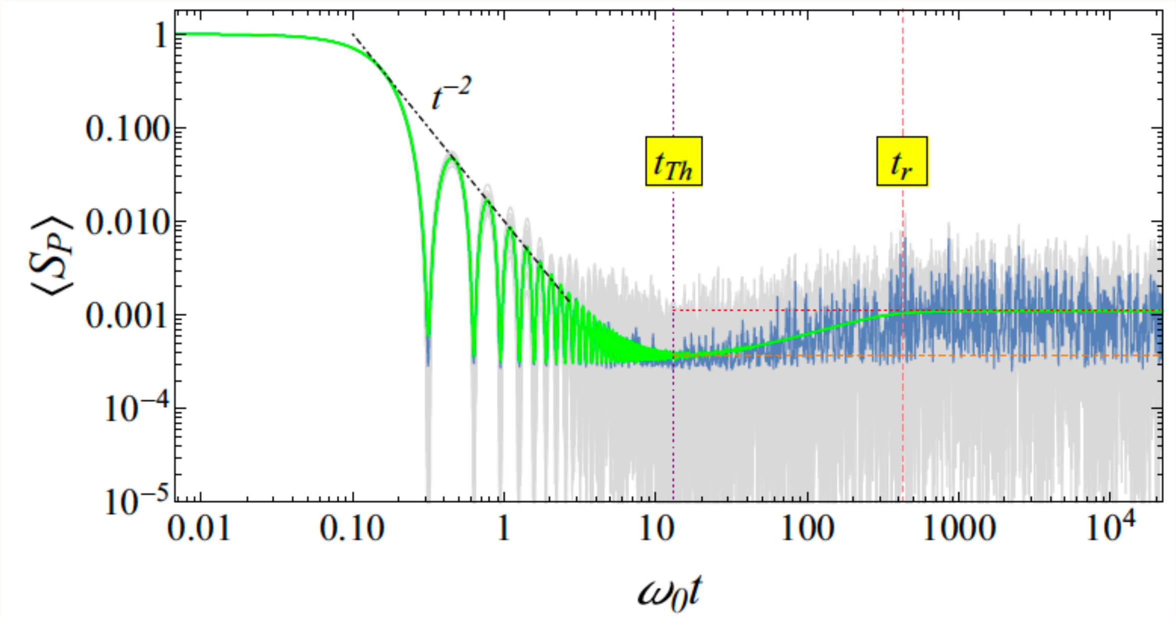
Schiulaz, Torres-Herrera & LFS,
PRB **99**, 174313 (2019)

$$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$$

D: dimension of the Hilbert space

Autocorrelation functions (survival probability, spin autocorrelation function) take an **exponentially long time** in system size to **thermalize.**

Lack of Self-Averaging



Dicke model
PRE **100**, 012218 (2019)

Chaotic regime:

PRB **101**, 174312 (2020)

Analytical: SP is nowhere self-averaging

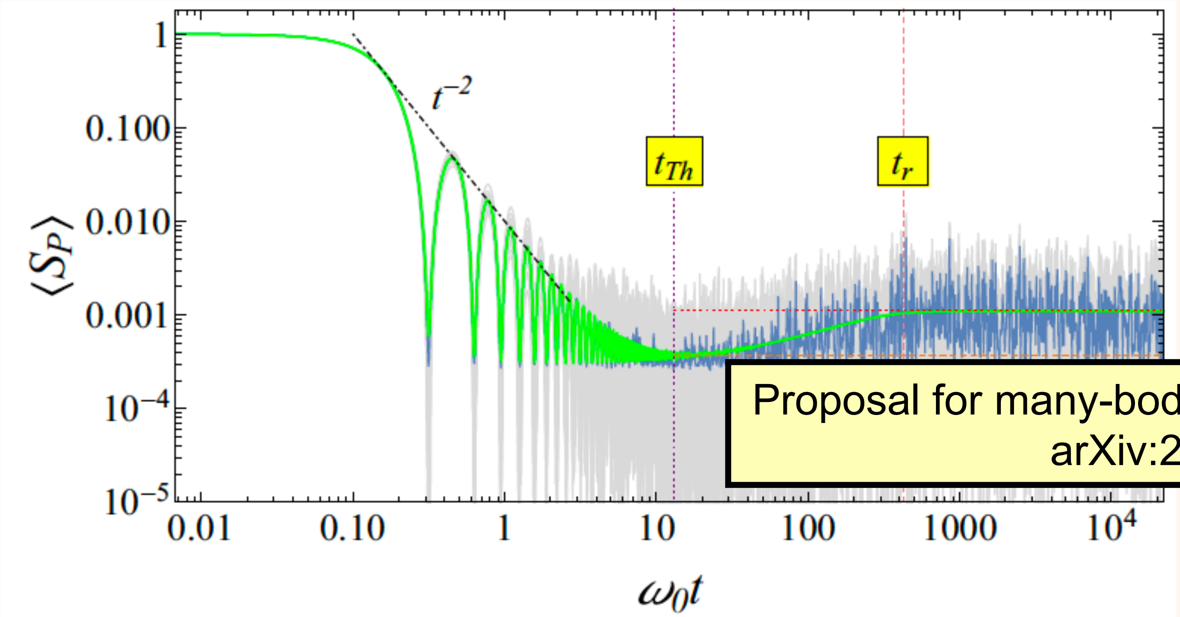
Large disorder:

PRB **102**, 094310 (2020)

Fluctuations increase as the disorder increases

PRR **3,** L032030 (2021)
PRE **102**, 062126 (2020)

Self-Averaging in Open Systems



Dicke model
PRE **100**, 012218 (2019)

Proposal for many-body quantum chaos detection
arXiv:2401.01401

Chaotic regime:

PRB **101**, 174312 (2020)

Analytical: SP is nowhere self-averaging

Large disorder:

PRB **102**, 094310 (2020)

Fluctuations increase as the disorder increases

How to circumvent the problem?

PRA **108**, 062201 (2023)

Open the system!

PRR **3,** L032030 (2021)
PRE **102**, 062126 (2020)

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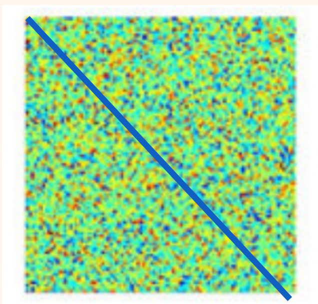
REVIEW:

Dynamical Manifestations
of
Many-Body Quantum Chaos

GOE Random Matrices

$$H = H_0 + V$$

H_0 Diagonal matrix,
Gaussian real random numbers



V Off-diagonal matrix,
Gaussian real random numbers

Strong level repulsion

Matrix of eigenvectors

Each column is an eigenvector.

$$\begin{matrix} & \boldsymbol{\alpha_1} & \boldsymbol{\alpha_2} & \boldsymbol{\alpha_3} & \boldsymbol{\alpha_4} \\ & C_1^1 & C_2^1 & C_3^1 & C_4^1 \\ \boldsymbol{\Psi(0)=} & C_1^2 & C_2^2 & C_3^2 & C_4^2 \\ & C_1^3 & C_2^3 & C_3^3 & C_4^3 \\ & C_1^4 & C_2^4 & C_3^4 & C_4^4 \end{matrix}$$

Eigenstates of full random matrices are **random vectors**
Components are random numbers from a Gaussian distribution

Normalization: $\sum_{n=1}^{Dim} |C_n^\alpha|^2 = 1$

Survival Probability for GOE

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle$$

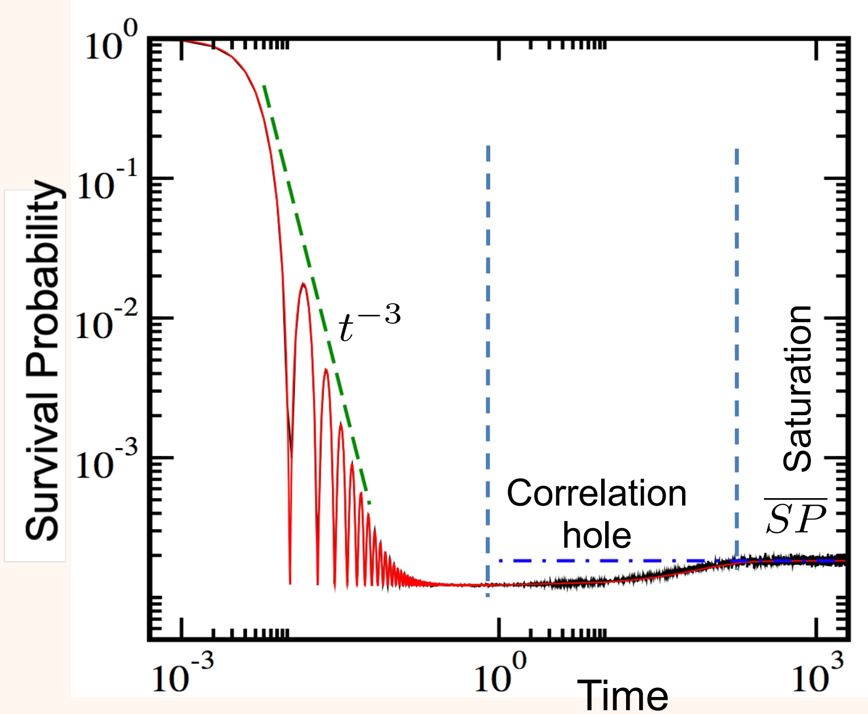
$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$

$$\overline{SP} = 3/D$$

two-level form factor

$$b_2(t) = \begin{cases} 1 - 2t + t\ln(1+2t), & t \le 1 \\ t\ln\left(\frac{2t+1}{2t-1}\right) - 1, & t > 1 \end{cases}$$

Correlation hole = ramp

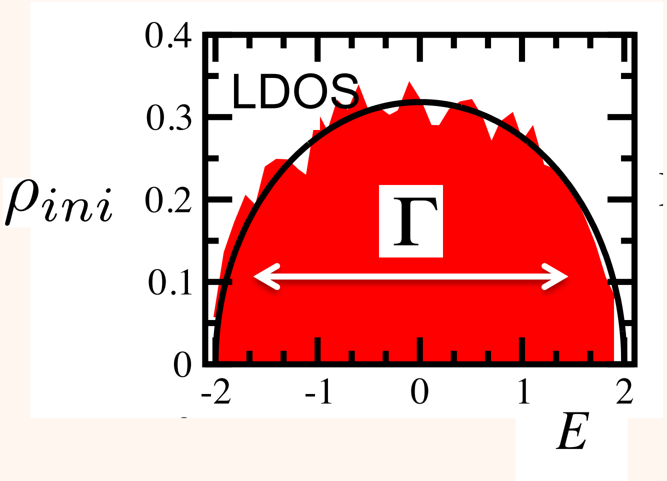


PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Survival Probability and LDOS

$$\left|\langle \Psi(0) \mid \Psi(t) \rangle\right|^2$$

$$SP(t) = \left|\langle \Psi(0) \mid \Psi(t) \rangle\right|^2 = \left|\sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 e^{-iE_{\alpha}t}\right|^2 \cong \left|\int \rho_{ini}(E) e^{-iEt}\, dE\right|^2$$



$$\boldsymbol{\rho}_{ini}(E) = \sum_{\alpha} \left|C_{\alpha}^{ini}\right|^2 \boldsymbol{\delta}(E - E_{\alpha})$$

Energy distribution of the initial state

Fourier transform

$$\frac{\left|\mathcal{J}_1(2\Gamma t)\right|^2}{\Gamma^2 t^2}$$

Power-law decay

$$\frac{\left|\mathcal{J}_1(2\Gamma t)\right|^2}{\Gamma^2 t^2} \to t^{-3}$$

Survival Probability for GOE

$$|\langle \Psi(0) | \Psi(t) \rangle|^2$$

$$\langle SP(t)\rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle$$

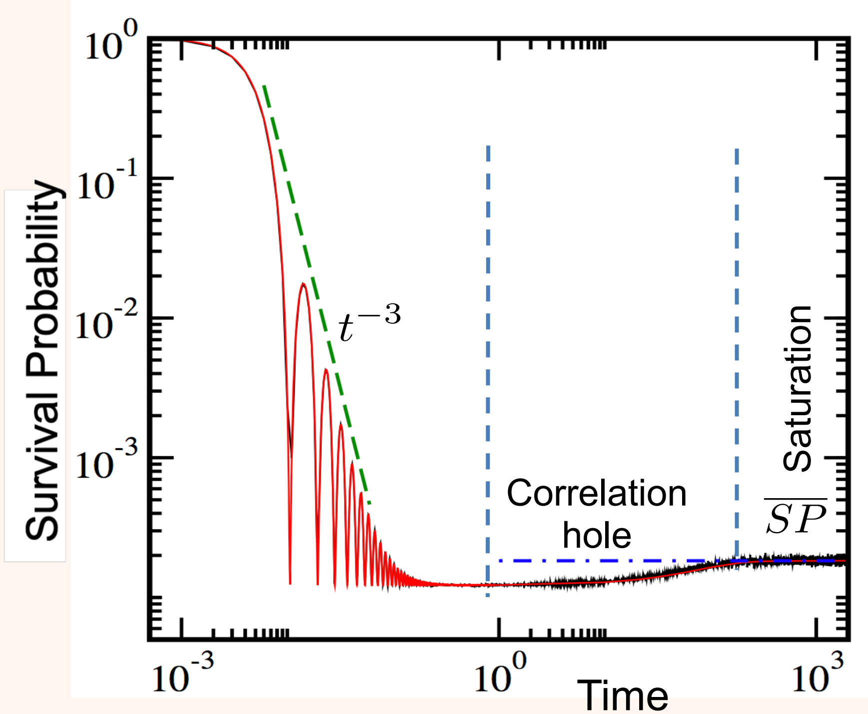
$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$

$$\overline{SP} = 3/D$$

two-level form factor

$$b_2(t) = \begin{cases} 1 - 2t + t\ln(1+2t), & t \le 1 \\ t\ln\left(\frac{2t+1}{2t-1}\right) - 1, & t > 1 \end{cases}$$

Correlation hole = ramp

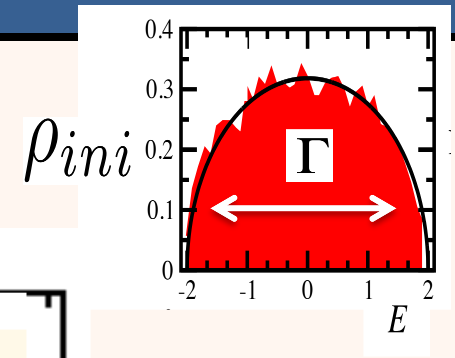


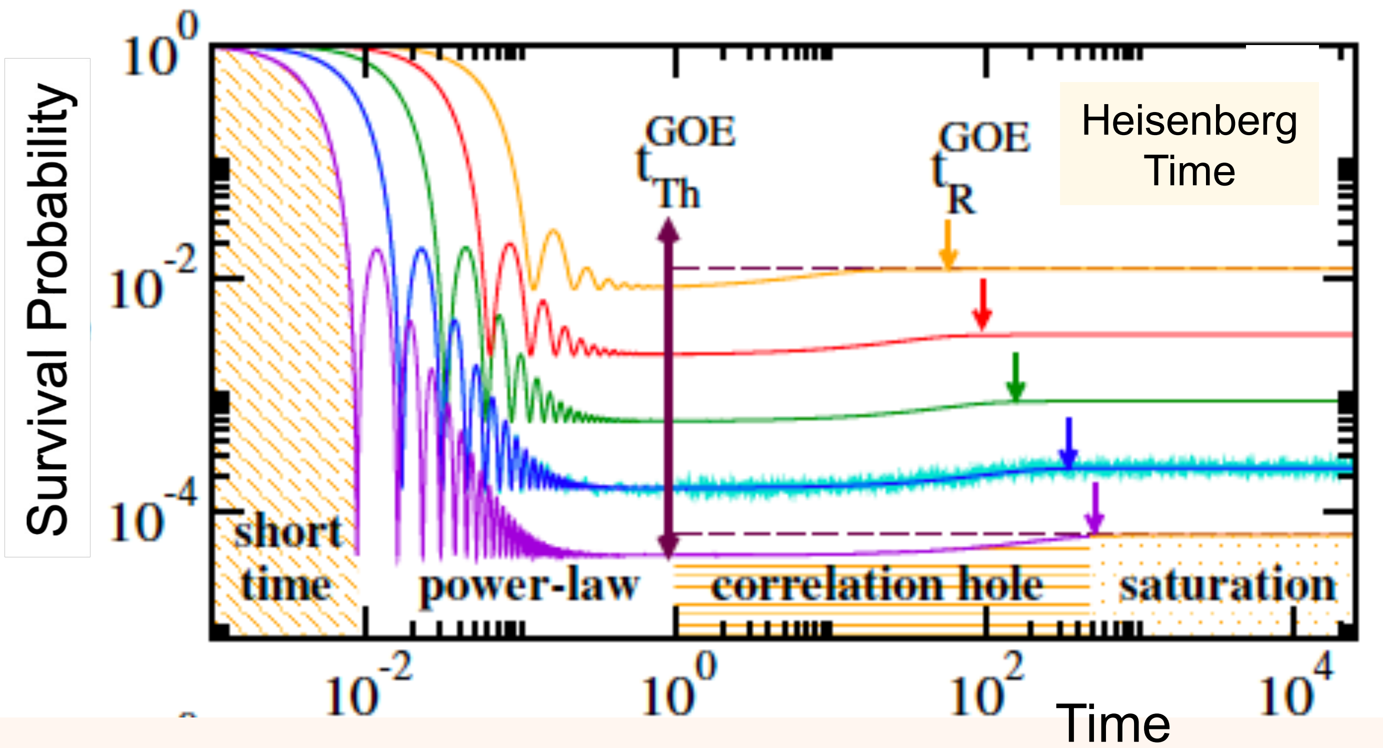
PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Timescale for the correlation hole in GOE

$$t_{Th} = \left(\frac{3}{\pi}\right)^{1/4}$$

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$





Dim=
252
924
3432
12870
48620

PRB **99**, 174313 (2019)

Physical Model

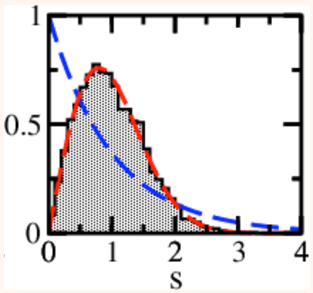
Disordered Spin Model

Random numbers
$h_n \in [-h, h]$

$$H_0 = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} \sigma_n^z \sigma_{n+1}^z$$

$$\Psi(0) = \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow$$

$$V = \sum_{n=1}^{L} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

P


Chaotic h=0.5J

PRA **69**, 042304 (2004)

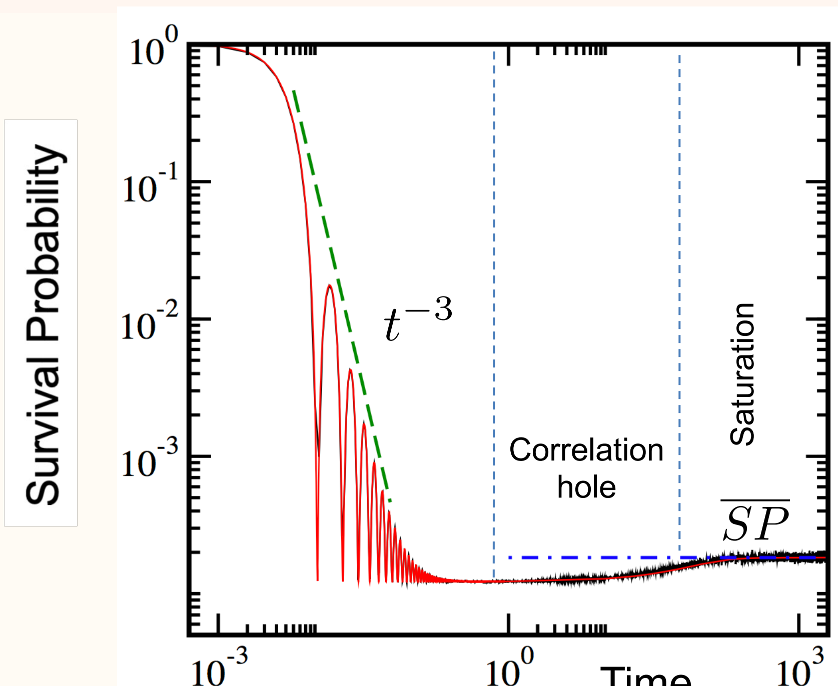
Survival Probability

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

GOE Full Random Matrices

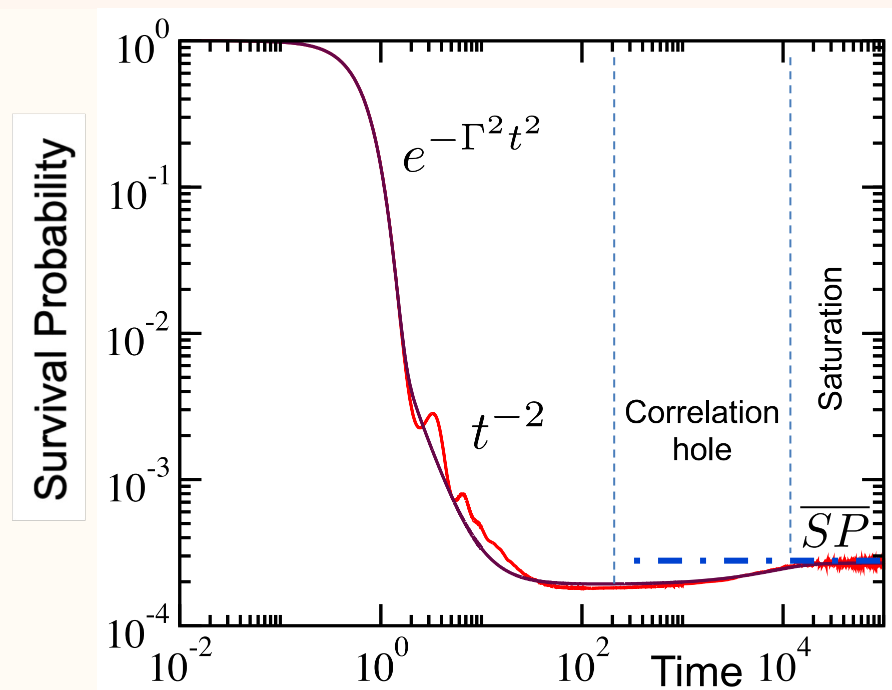
$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$



Disordered Spin Model

$$\frac{1-\overline{SP}}{(D-1)}\left[\frac{De^{-\Gamma^2 t^2}}{\mathcal{N}^2}\mathcal{F}(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

$$\mathcal{F}(t) = \left|\text{erf}\left(\frac{E_{\text{max}} + it\Gamma^2}{\sqrt{2}\Gamma}\right) - \text{erf}\left(\frac{E_{\text{min}} + it\Gamma^2}{\sqrt{2}\Gamma}\right)\right|^2$$



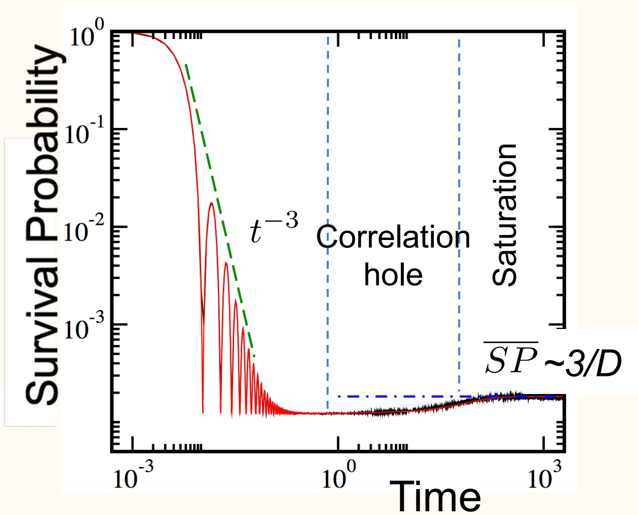
Survival Probability $\left|\langle\Psi(0)|\Psi(t)\rangle\right|^2$

GOE Full Random Matrices

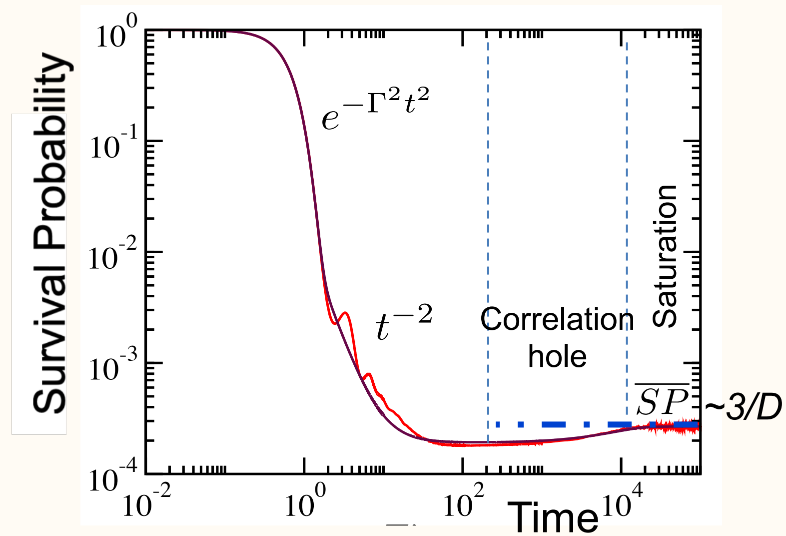
Realistic Many-Body Model

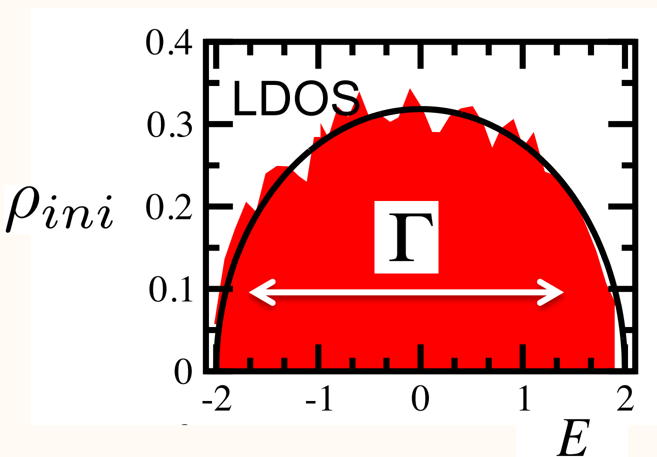
$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2}-b_2\left(\frac{\Gamma t}{2D}\right)\right]+\overline{SP}$$

$$\frac{1-\overline{SP}}{(D-1)}\left[\frac{De^{-\Gamma^2 t^2}}{\mathcal{N}^2}\mathcal{F}(t)-b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right]+\overline{SP}$$

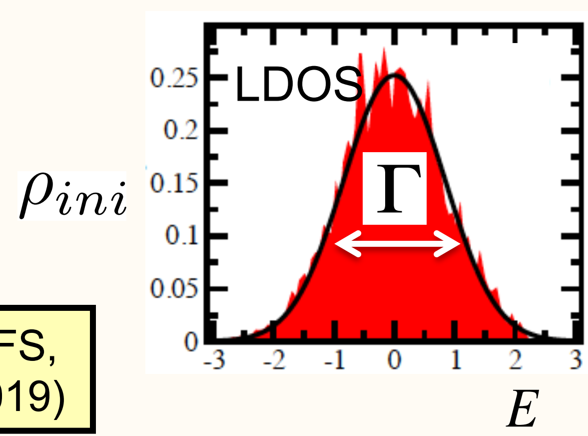


$1/\Gamma$



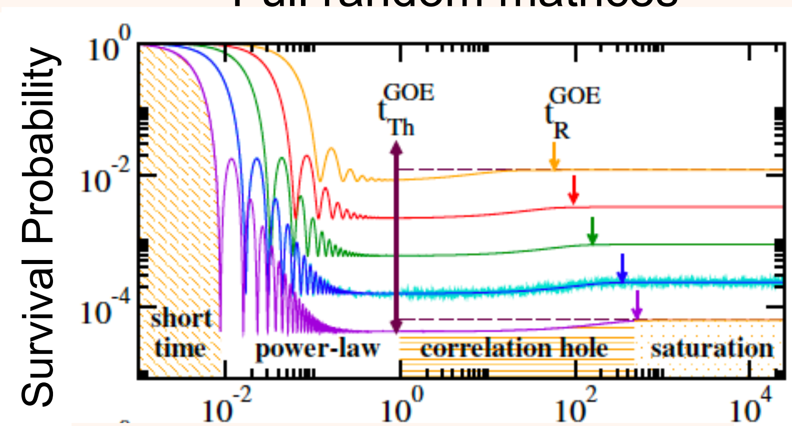


Schiulaz, Torres & LFS, PRB **99**, 174313 (2019)



Timescale for the correlation hole

Full random matrices



$$t_{Th} = \left(\frac{3}{\pi}\right)^{1/4}$$

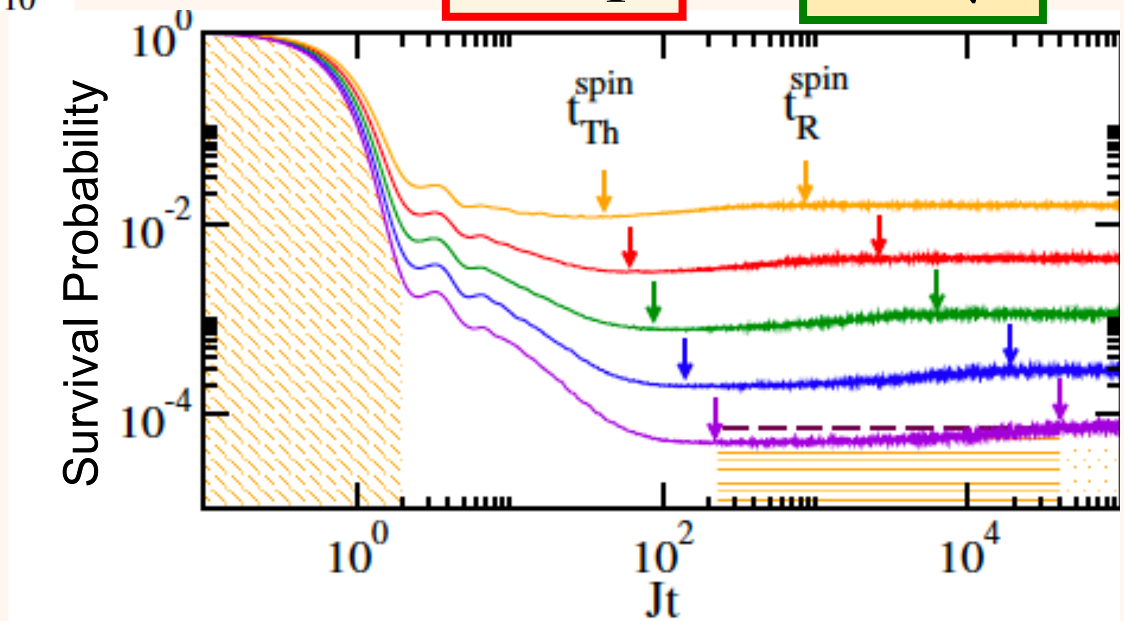
$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$

$$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

Disordered spin-1/2 model

$$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$$

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$



Relaxation time is exponentially long in L

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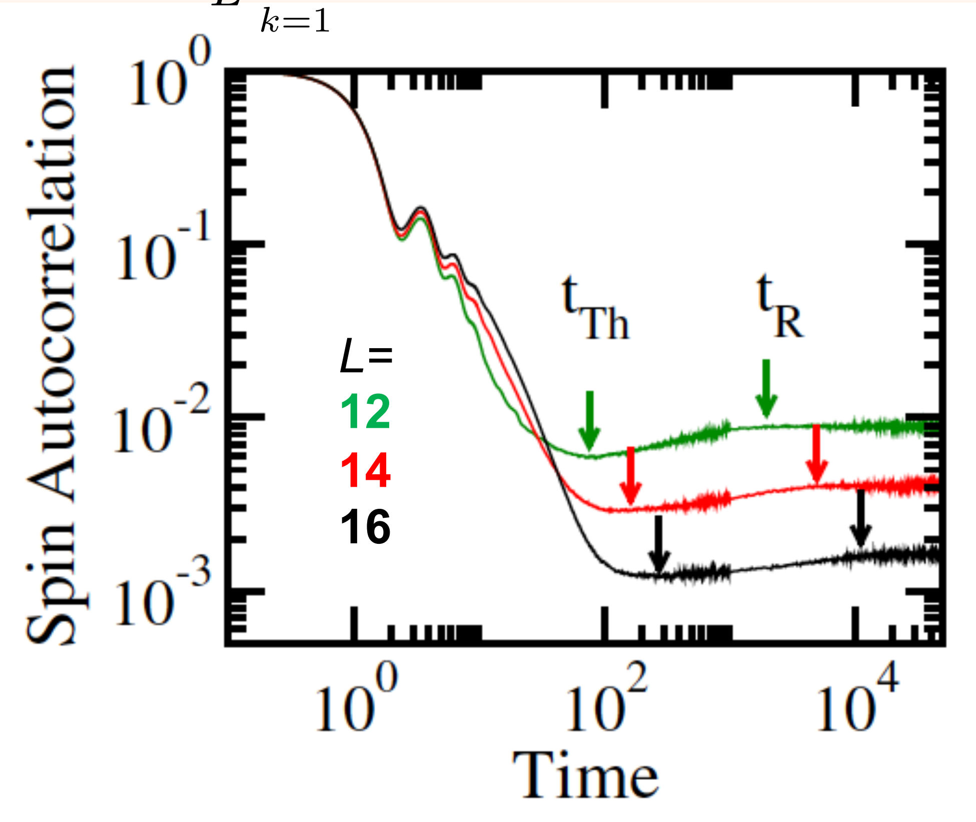
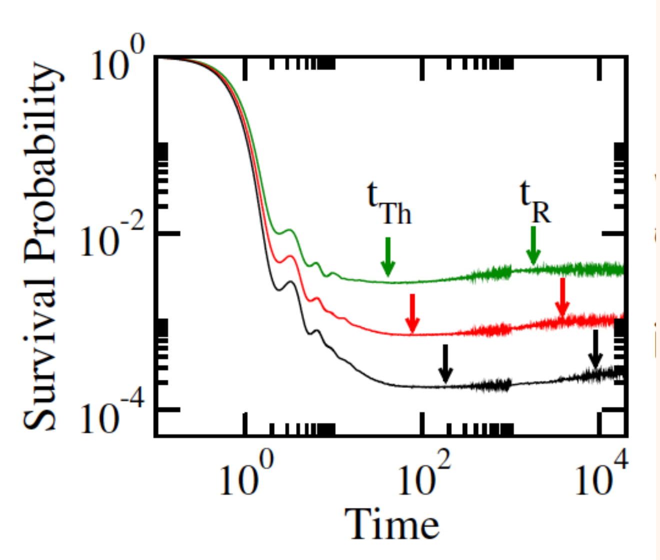
Spin Autocorrelation Function

Chaotic h=0.5J

$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$



Torres, García-García, LFS
PRB **97**, 060303 (R) (2018)

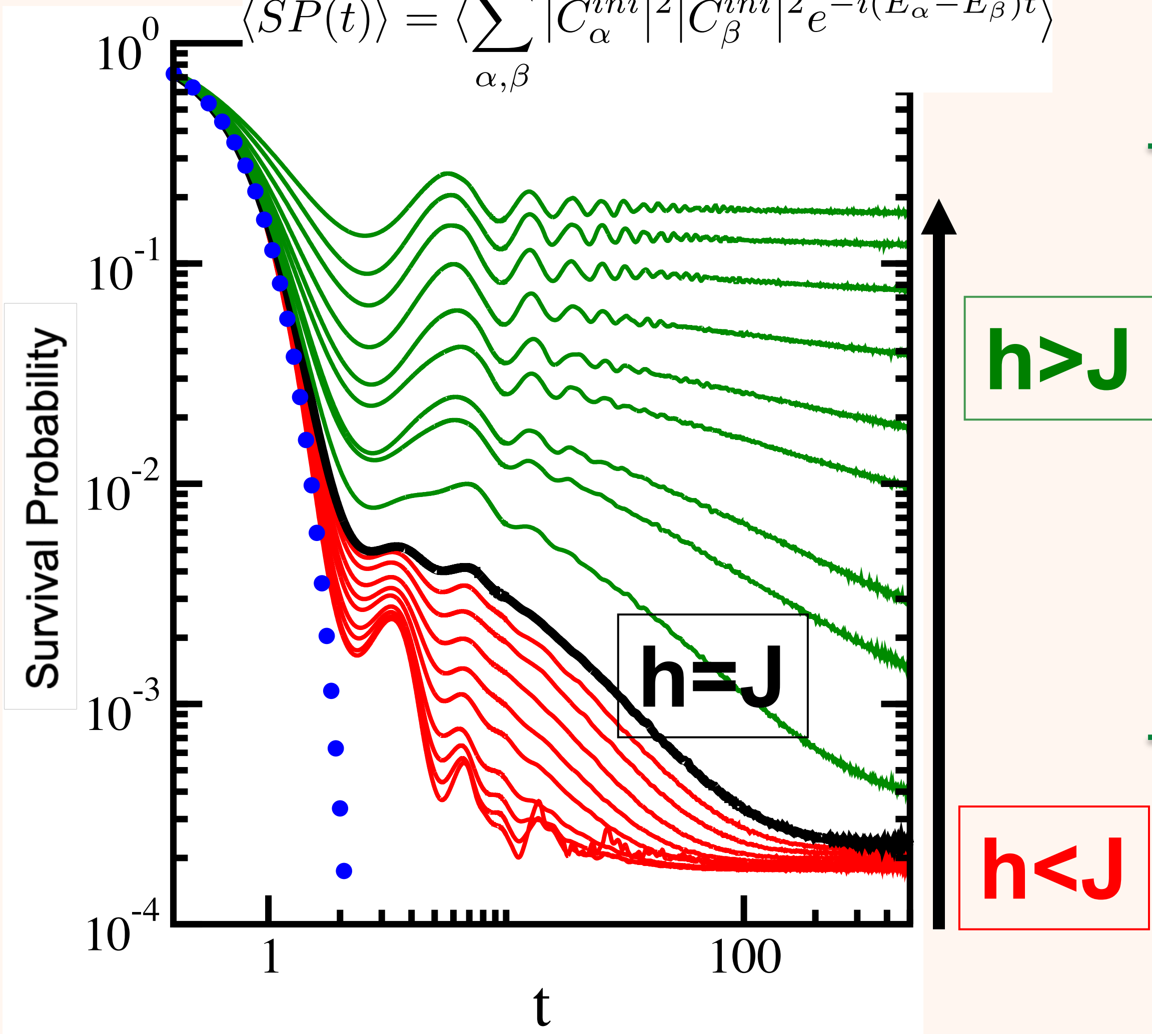


Schiulaz, Torres-Herrera, LFS,
PRB **99**, 174313 (2019)

Increasing the disorder strength

Power-law Decay

$$\langle SP(t)\rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t}\rangle$$



Power-law exponent coincides with the generalized dimension D_2

$t^{-\gamma}$ $\gamma < 1$

$PR \propto Dim^{D_2}$

Torres & LFS
PRB **92**, 01420 (2015)

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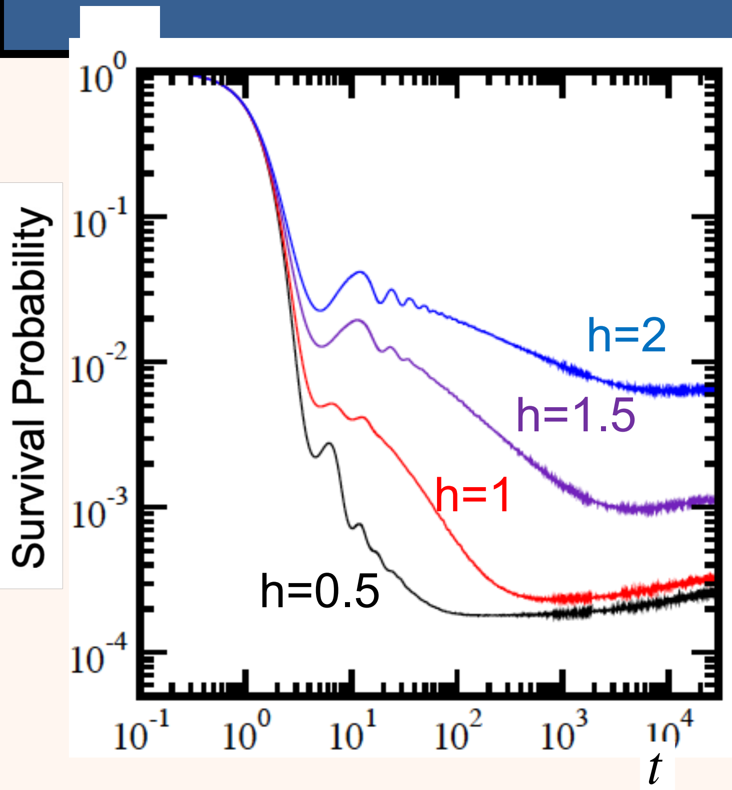
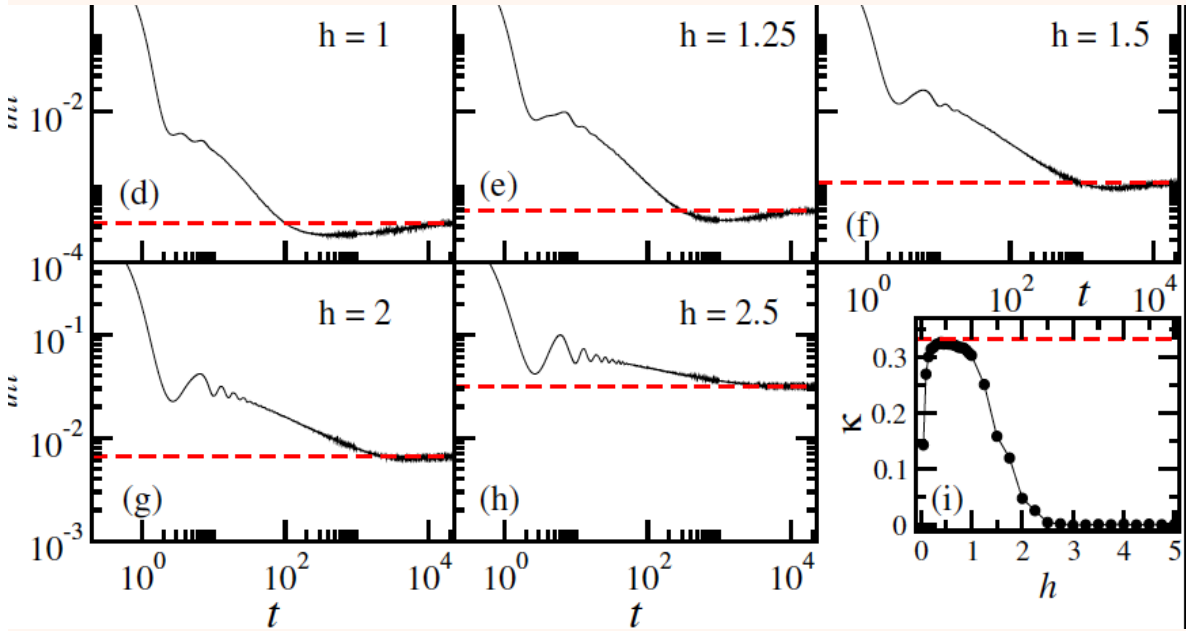
Correlation Hole and Disorder Strength





The hole becomes narrower and shallower.

Time for the minimum of the hole (T_{Th}) gets postponed as the disorder (h) increases.

Torres-Herrera & LFS
Ann. Phys. (Berlin) **529**, 1600284 (2017)
Philos. Trans. R. Soc. London Soc. A **375**, 20160434 (2017)

L=16

Thouless Time and Disorder Strength

Thouless dimensionless conductance $\frac{t_R}{t_{Th}} \propto \frac{E_{th}}{MLS}$

SURVIVAL PROBABILITY

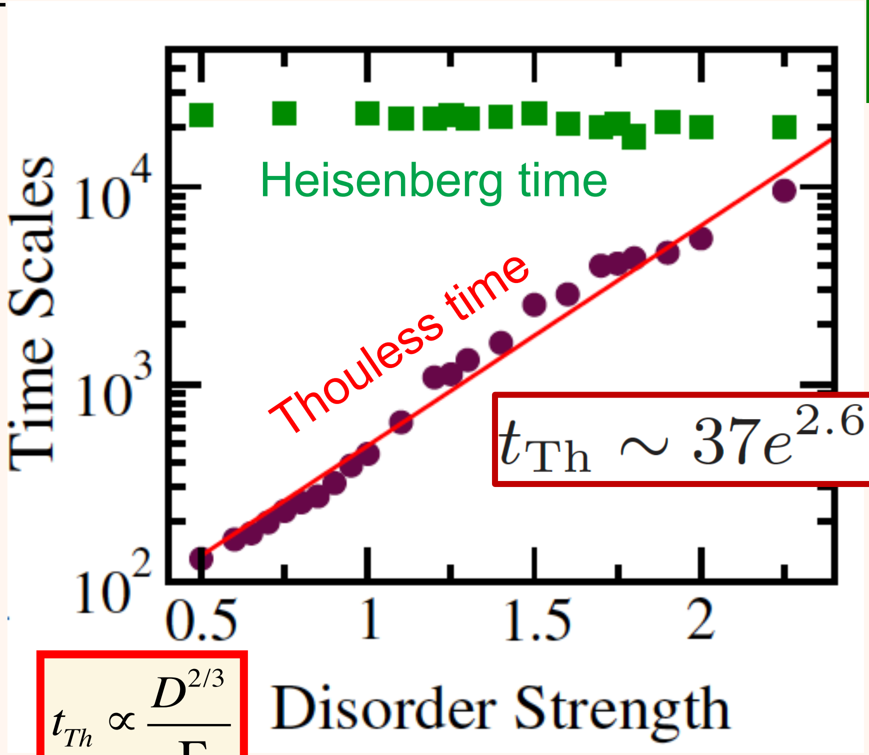
$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$

Delocalized (chaotic)

$$\frac{t_R}{t_{Th}} \propto e^{(L \ln 2)/3}$$

Toward localization

$$\frac{t_R}{t_{Th}} \to 1$$



$t_{\mathrm{Th}} \sim 37e^{2.6h}$

$t_{Th} \propto \frac{D^{2/3}}{\Gamma}$

Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Šuntajs, Bonča, Prosen, Vidmar PRE **102**, 062144 (2020)

Spin Autocorrelation Function and Disorder Strength

Thouless dimensionless conductance $\frac{t_R}{t_{Th}} \propto \frac{E_{th}}{MLS}$

Delocalized (chaotic)

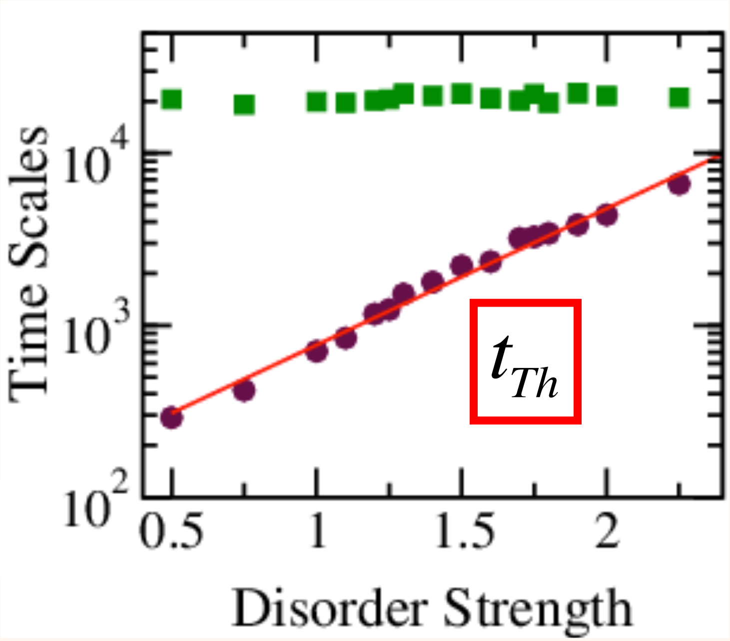
$$\frac{t_R}{t_{Th}} \propto e^{(L \ln 2)/3}$$

Toward localization

$$\frac{t_R}{t_{Th}} \to 1$$

SPIN AUTOCORRELATION

$$t_R \propto \frac{D}{\Gamma\sqrt{\delta}}$$



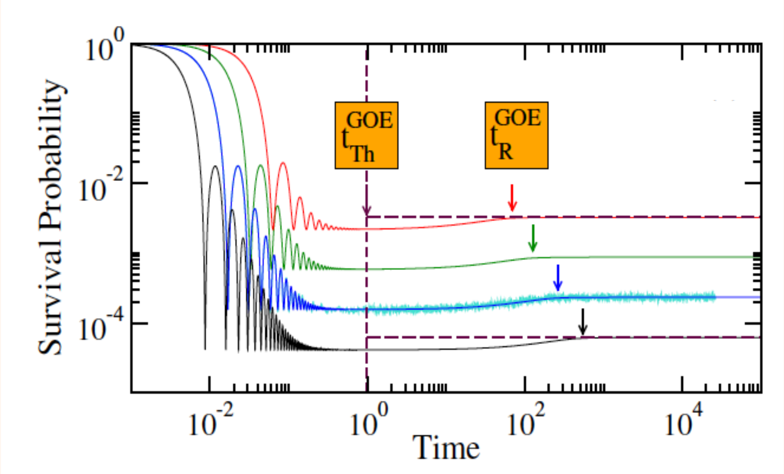
Schiulaz, Torres-Herrera & LFS, PRB **99**, 174313 (2019)

Correlation Hole and System Size

Survival probability for GOE matrices:

$$\overline{SP} = 3/D$$

$$SP_{min} = 2/D$$



Relative depth of the correlation hole:

$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

$$\kappa = 1/3$$

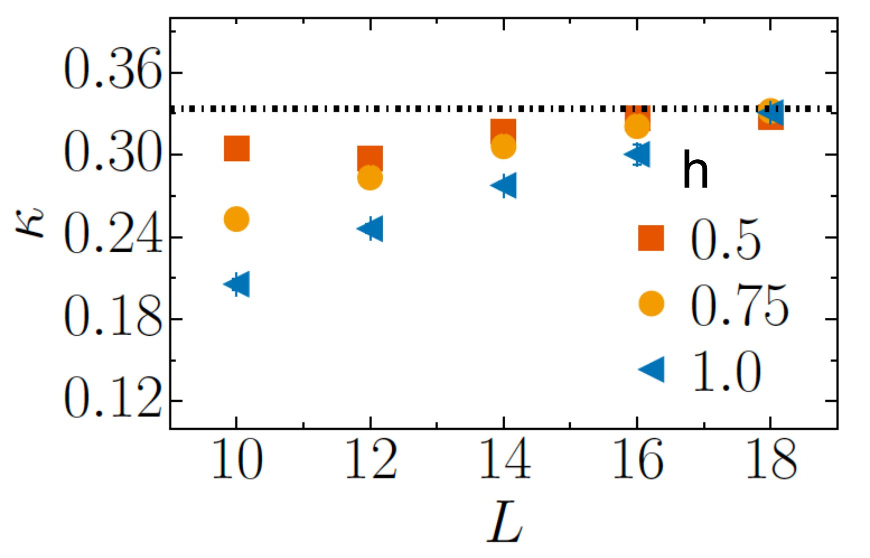
Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

ICTS 2024

Correlation Hole for the Survival Probability

Relative depth of the correlation hole:

$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$



<u>Survival probability</u> for realistic chaotic systems:

$$\kappa = 1/3$$

$h \leq J = 1$

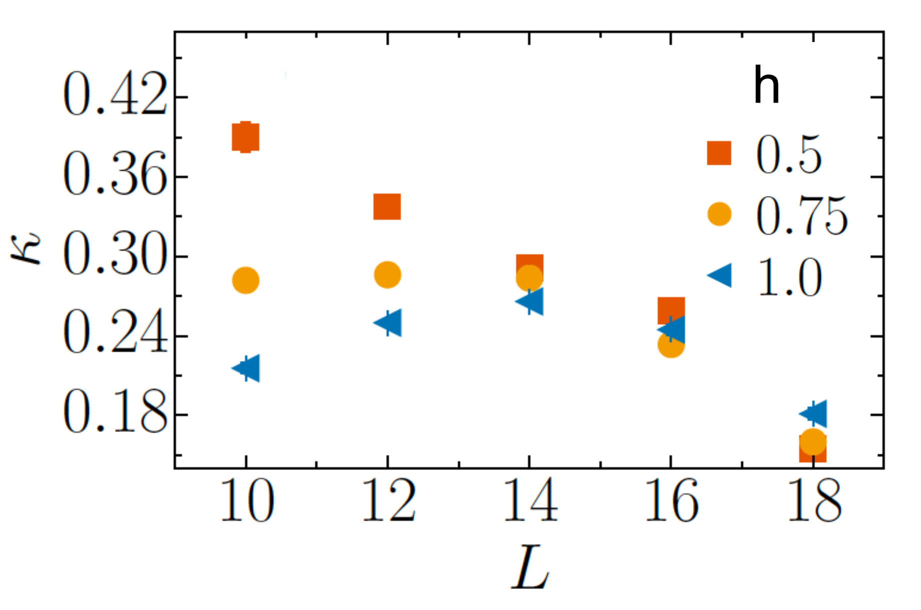
Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

Correlation Hole
for the Spin Autocorrelation Function

Relative depth of the correlation hole: $$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

<u>Spin autocorrelation function</u> for realistic chaotic systems:

$$h \leq J = 1$$

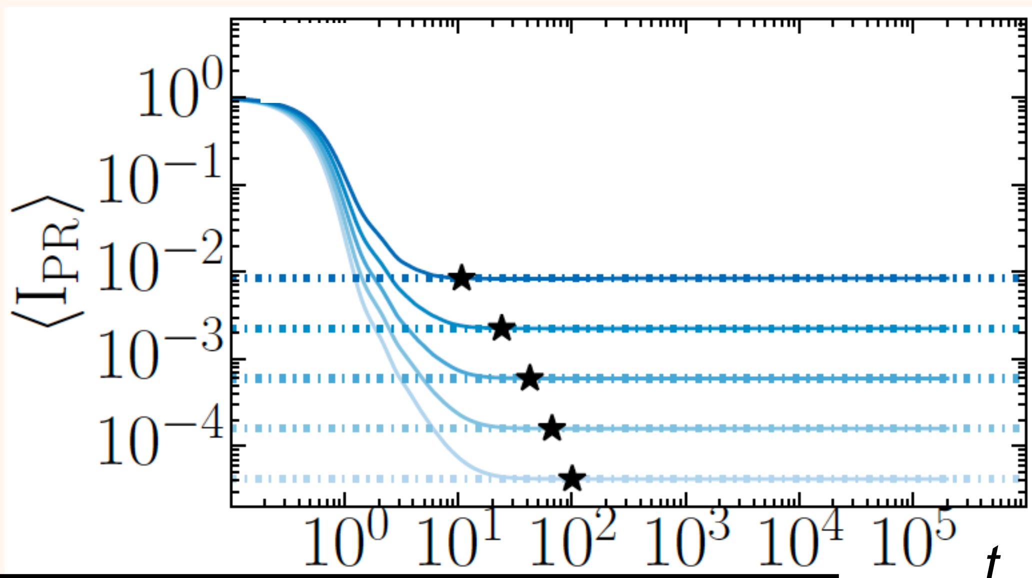


Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117(2021)

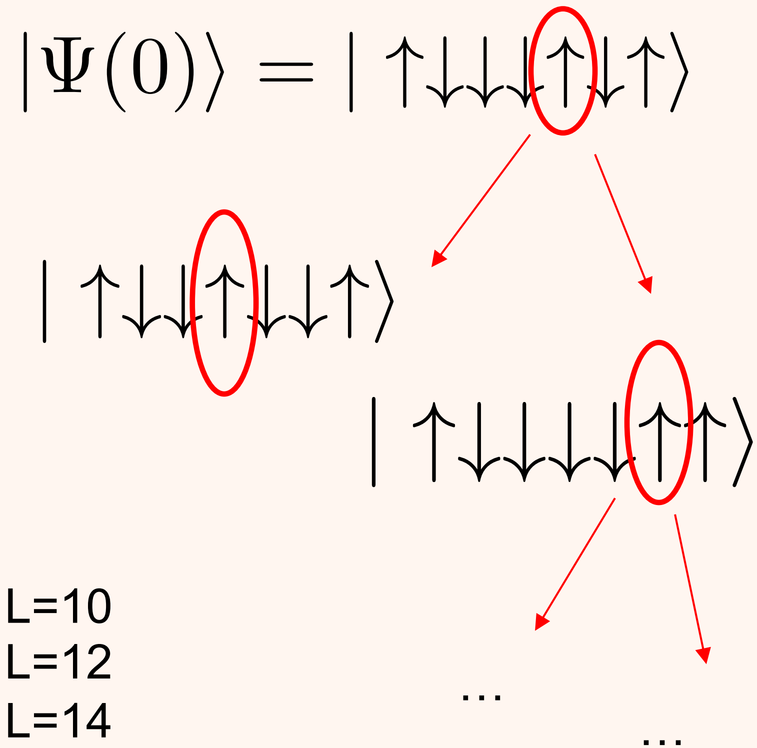
Inverse Participation Ratio

$$I_{PR}(t) = \sum_n |\langle \phi_n | \Psi(t) \rangle|^4$$

-Log(IPR) =
2nd-order Rényi entropy



L=10
L=12
L=14
L=16
L=18

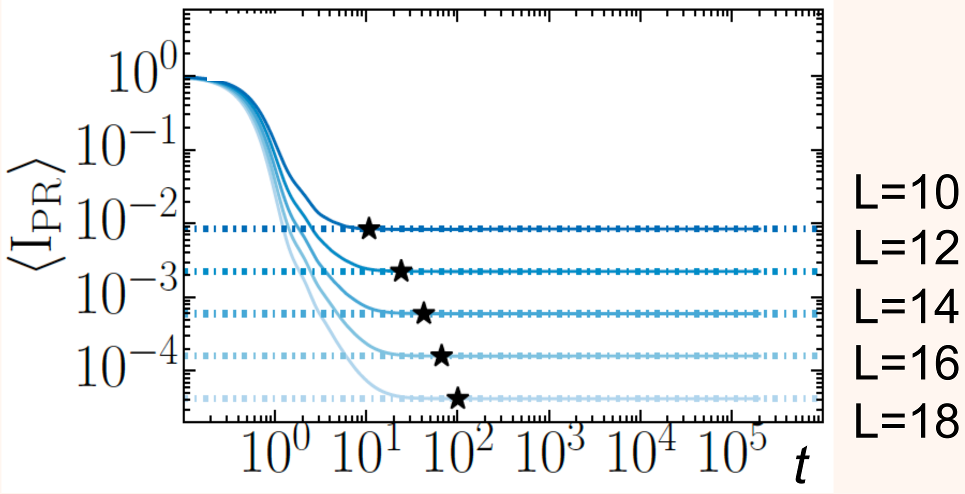


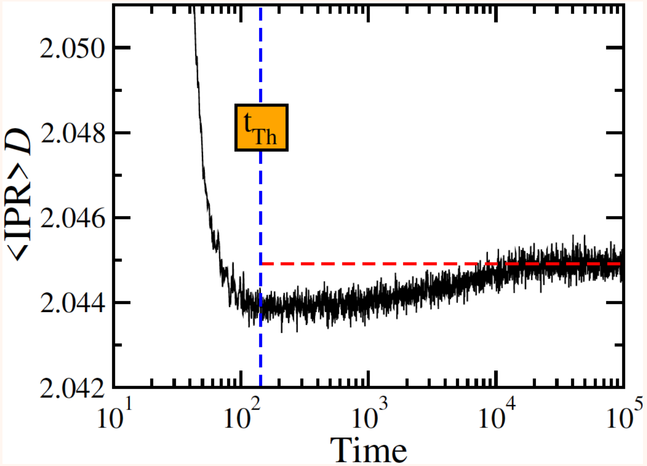
Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117(2021)

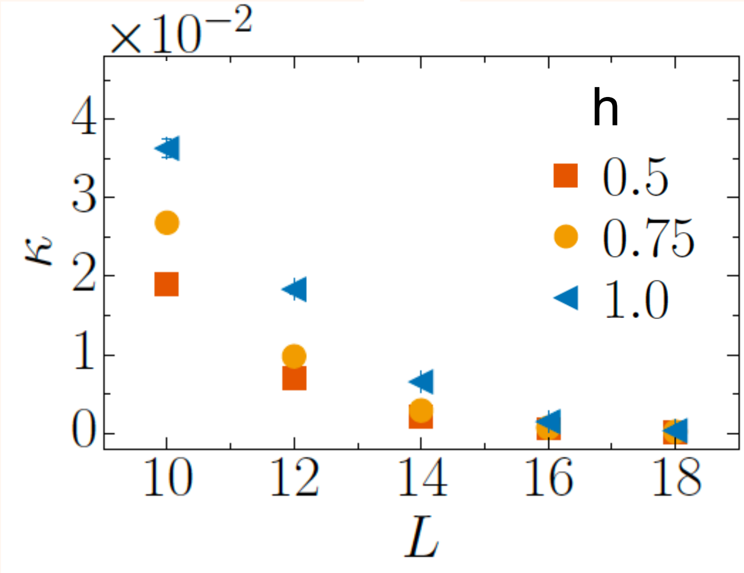
ICTS 2024

Inverse Participation Ratio

$$\mathrm{I_{PR}}(t) = \sum_n |\langle \phi_n | \Psi(t) \rangle|^4$$



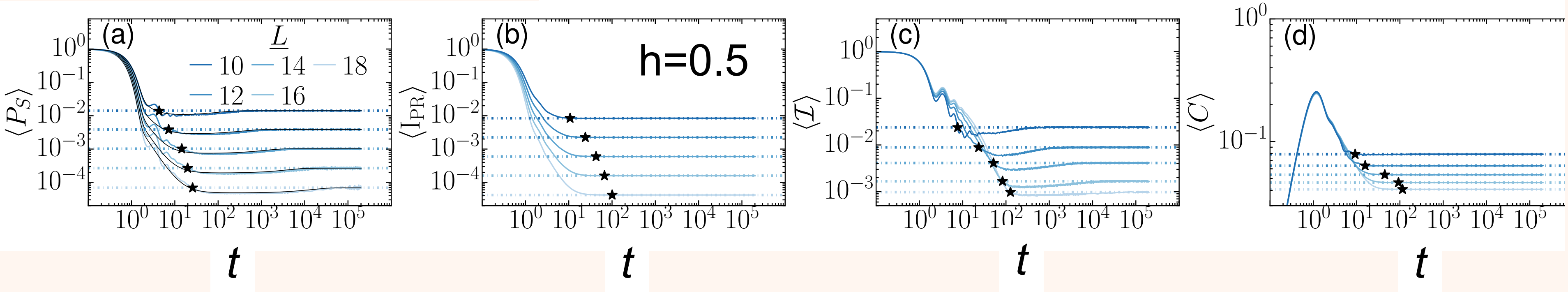




Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

Thermalization Time
Before the Correlation Hole

$$C(t) = \frac{4}{L} \sum_{i=1} \left[\langle \Psi(t) | \hat{S}_i^z \hat{S}_{i+1}^z | \Psi(t) \rangle - \langle \Psi(t) | \hat{S}_i^z | \Psi(t) \rangle \langle \Psi(t) | \hat{S}_{i+1}^z | \Psi(t) \rangle \right]$$



(deep chaotic region)
h=0.5

$$t^* \propto e^L$$

$$t^* \propto L^\gamma$$

$$\gamma > 3$$

Other L’s, models, quantities.
Quantum-classical parallel.

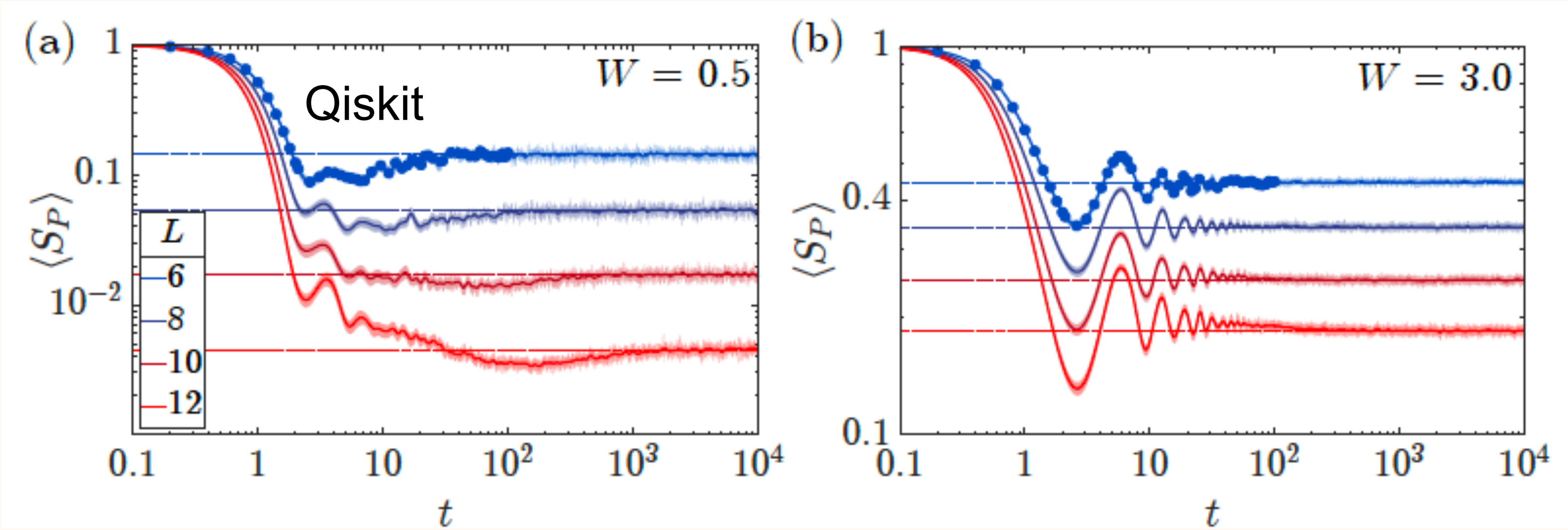
Lezama, Torres, Bernal, Bar Lev & LFS,
PRB **104**, 085117(2021)

Detecting the correlation hole

&

Ensuring self-averaging

Many-Body Quantum Chaos Detection



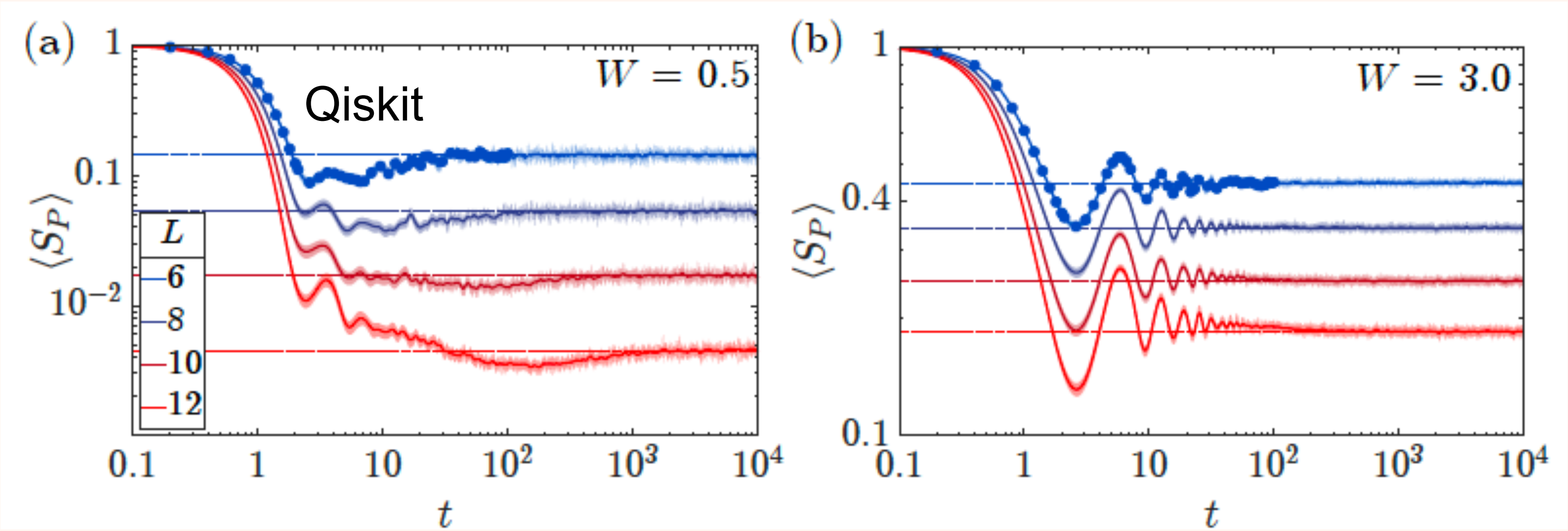
Spin-1/2 models:
Disordered Heisenberg model
Disordered long-range Ising model in a transverse field

Quantities:
Survival probability
Spin autocorrelation function

Proposal for many-body quantum chaos detection
arXiv:2401.01401

ICTS 2024

Many-Body Quantum Chaos Detection



Spin-1/2 models:
Disordered Heisenberg model
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Quantities:
Survival probability
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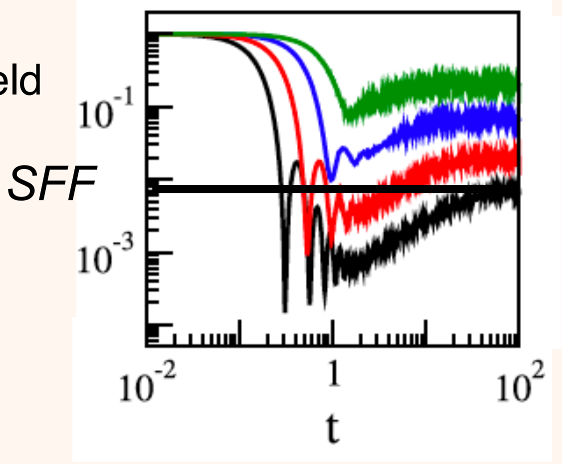
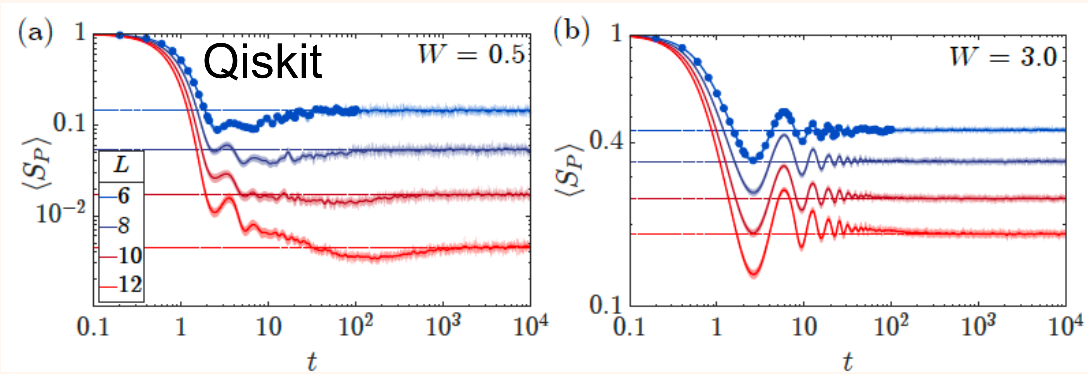


Proposal for many-body quantum chaos detection
arXiv:2401.01401

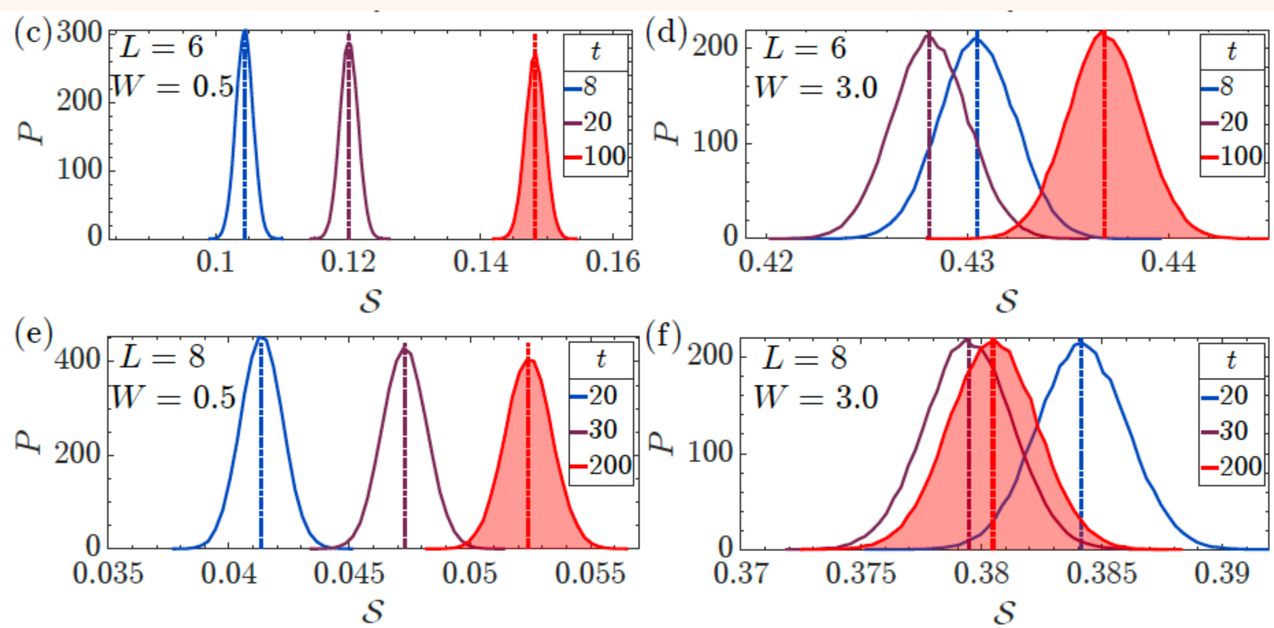
ICTS 2024

Many-Body Quantum Chaos Detection



Let the initial state evolve unitarily for a time t, then perform a projective measurement. For M number of shots, one gets the outcome 1 for M_1 number of times

$$\lim_{M\to\infty} \frac{M_1}{M} = S_P(t)$$



Proposal for many-body quantum chaos detection
arXiv:2401.01401

ICTS 2024

Self-Averaging

A quantity O is self-averaging when its relative variance goes to zero as the system size increases

$$\mathcal{R}_O(t) = \frac{\sigma_O^2(t)}{\langle O(t)\rangle^2} = \frac{\langle O^2(t)\rangle - \langle O(t)\rangle^2}{\langle O(t)\rangle^2}$$

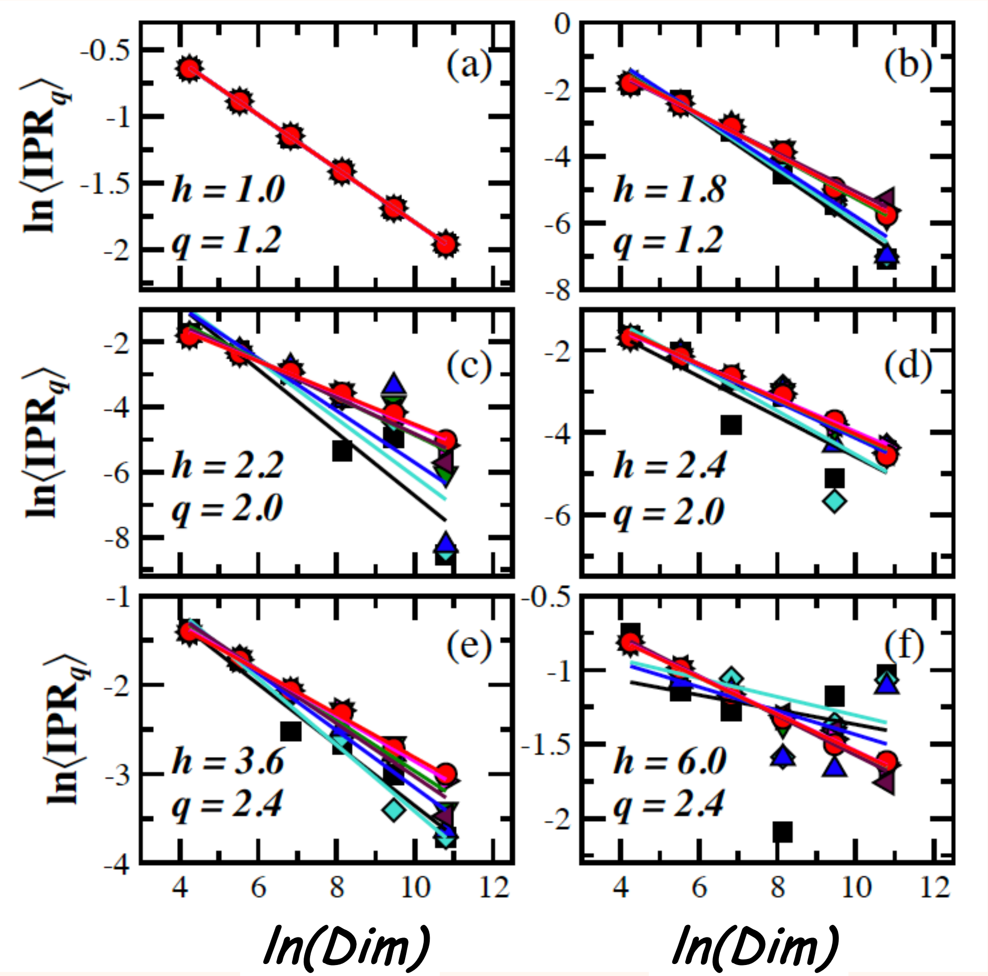
By increasing the system size, one can **reduce** the number of samples used in

- experiments
- statistical analysis.

If the system exhibits self-averaging, its physical properties are independent of the specific realization.

PRR **3,** L032030 (2021)
PRE **102**, 062126 (2020)
PRB **102**, 094310 (2020)
PRB **101**, 174312 (2020)

Lack of Self-Averaging



$$IPR_q^\alpha = \sum_k |C_k^\alpha|^{2q}$$

$$\langle IPR_q \rangle \propto Dim^{-(q-1)D_q}$$

10^2 (black squares)

5×10^2 (turquoise diamonds)

1×10^3 (blue up triangles)

5×10^3 (green down triangles)

1×10^4 (maroon left triangles)

2×10^4 (magenta right triangles)

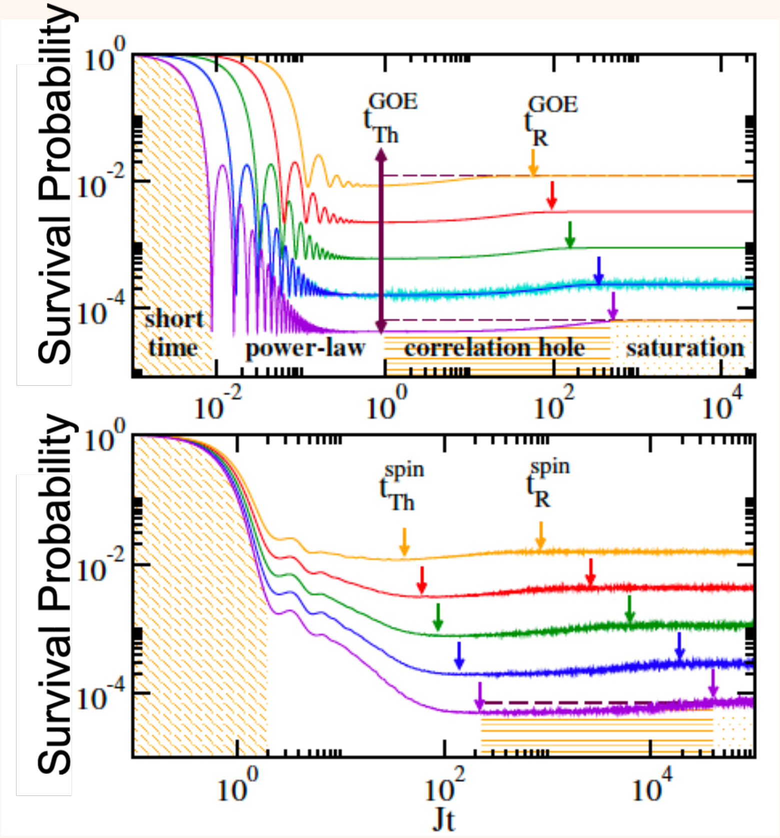
3×10^4 (red circles)

Solórzano, LFS, Torres-Herrera
PRR **3**, L032030 (2021)

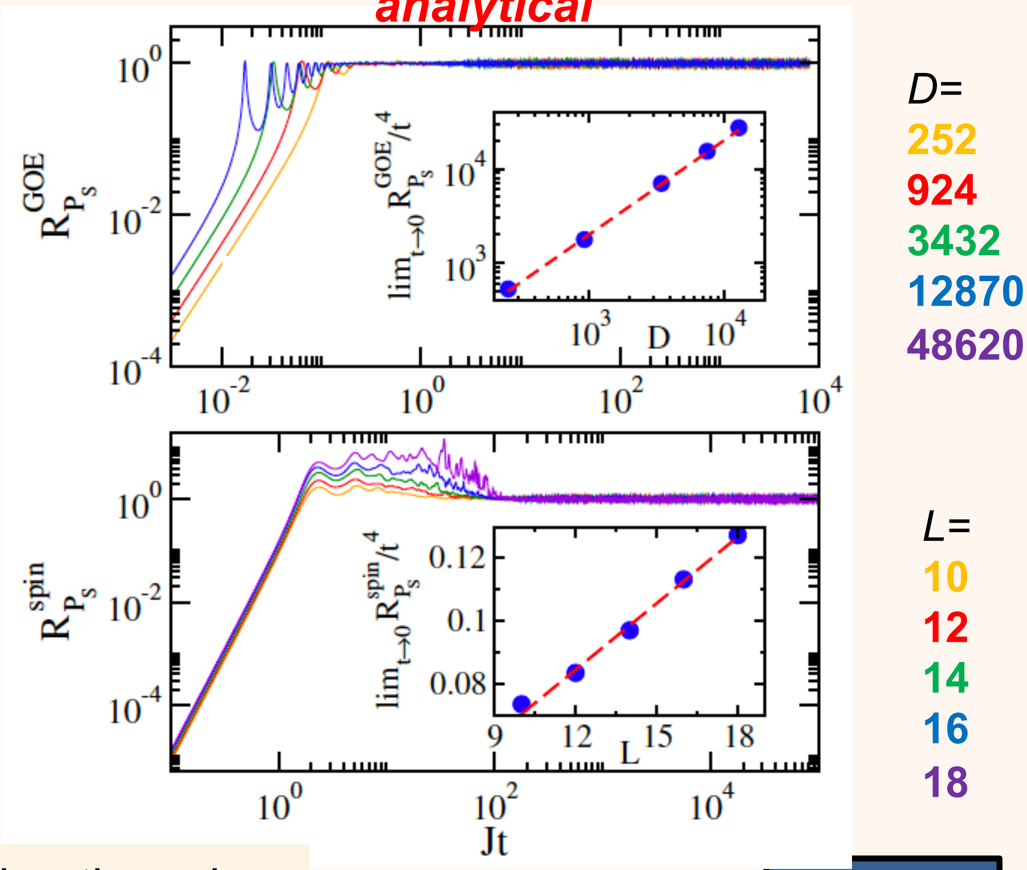
Survival Probability (NOWHERE self-averaging)

$$SP(t) = \left|\langle\Psi(0)|e^{-iHt}|\Psi(0)\rangle\right|^2$$

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$



analytical



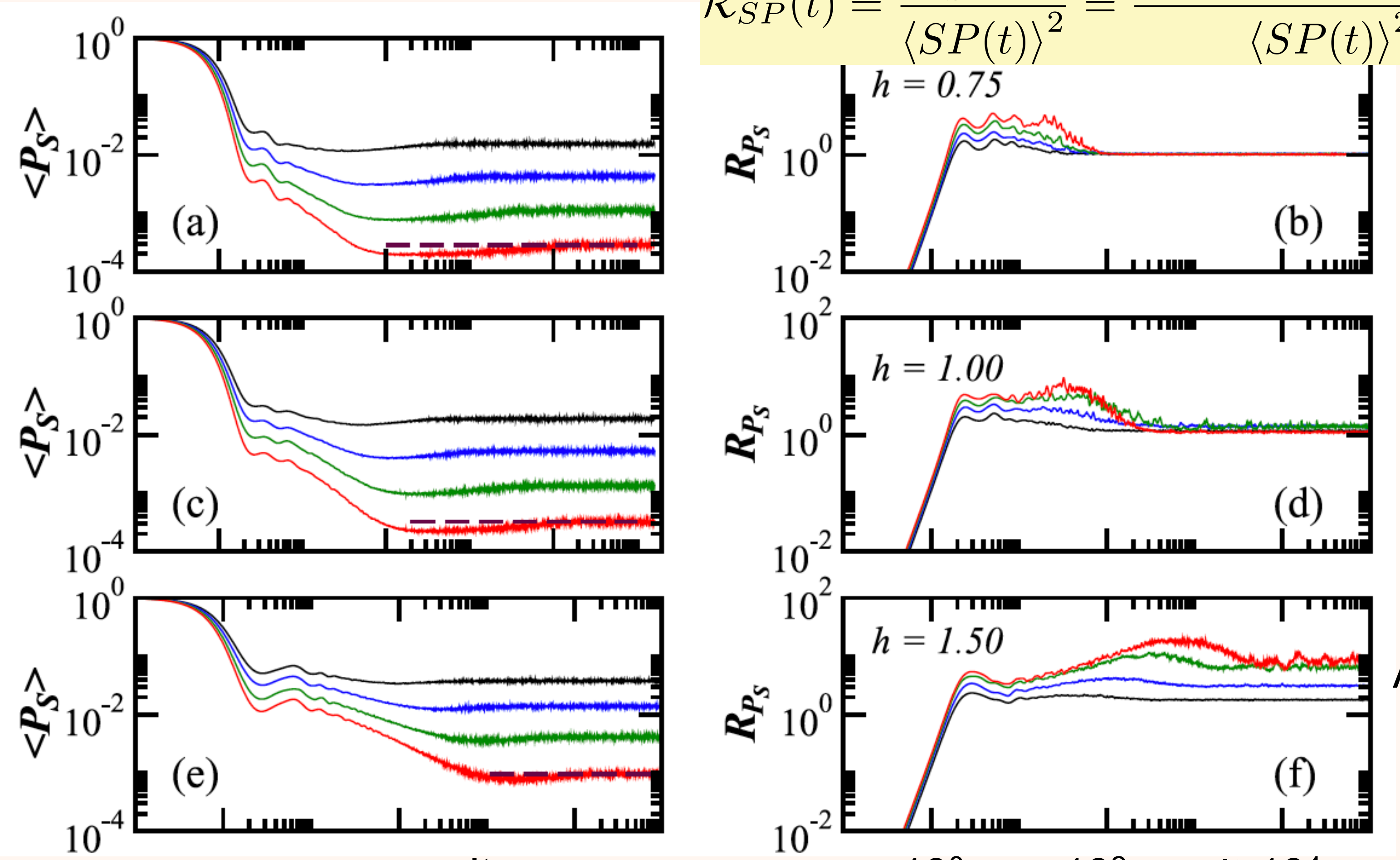
PRB **101**, 174312 (2020)

Chaotic regime

ICTS 2024

Survival Probability (NOWHERE self-averaging)

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$



L= 10 12 14 16

1

Above 1

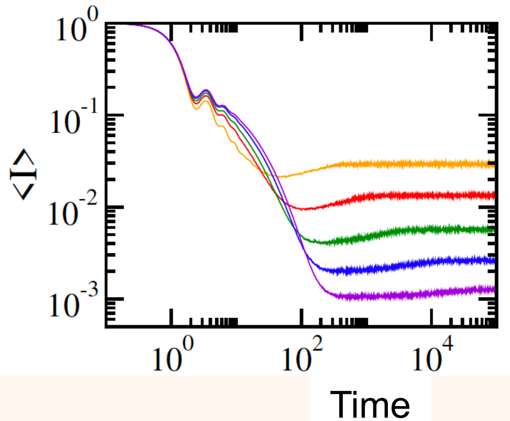
10^0 10^2 Jt 10^4

10^0 10^2 Jt 10^4

Increasing disorder

PRB **102**, 094310 (2020)

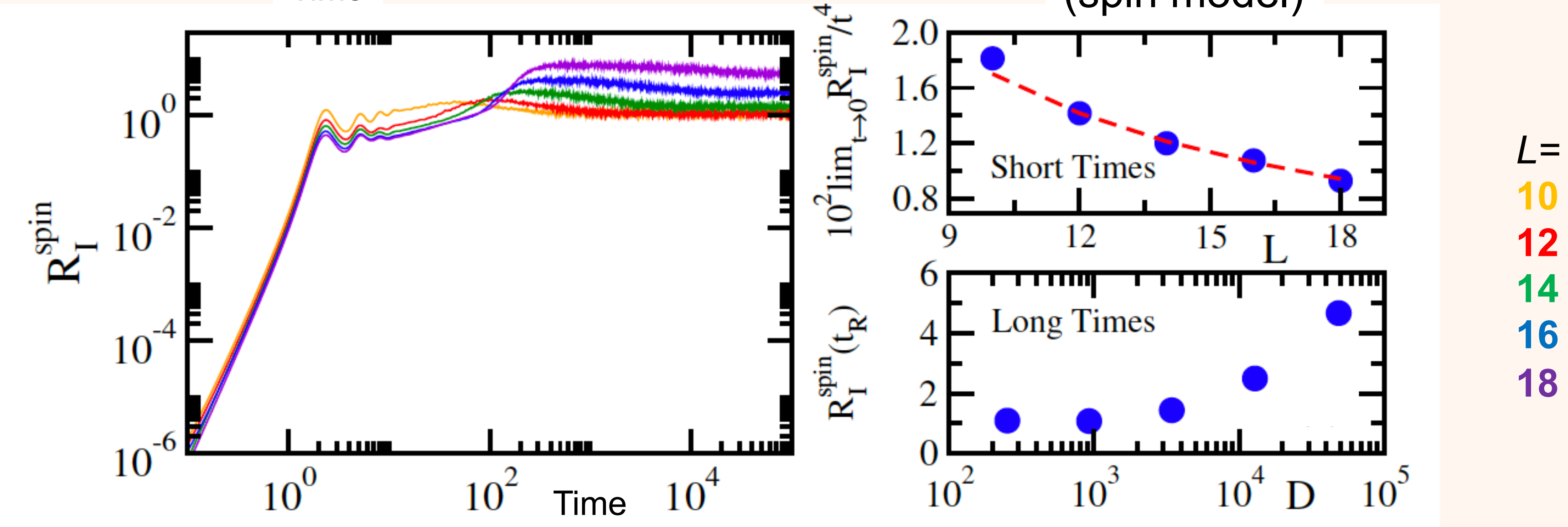
Spin Autocorrelation Function (lack of self-averaging at long times)



$$I(t) = \frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\sigma_k^z e^{iHt}\sigma_k^z e^{-iHt}|\Psi(0)\rangle$$

$$\mathcal{R}_I(t) = \frac{\sigma_I^2(t)}{\langle I(t)\rangle^2} = \frac{\langle I^2(t)\rangle - \langle I(t)\rangle^2}{\langle I(t)\rangle^2}$$

(spin model)



Chaotic regime

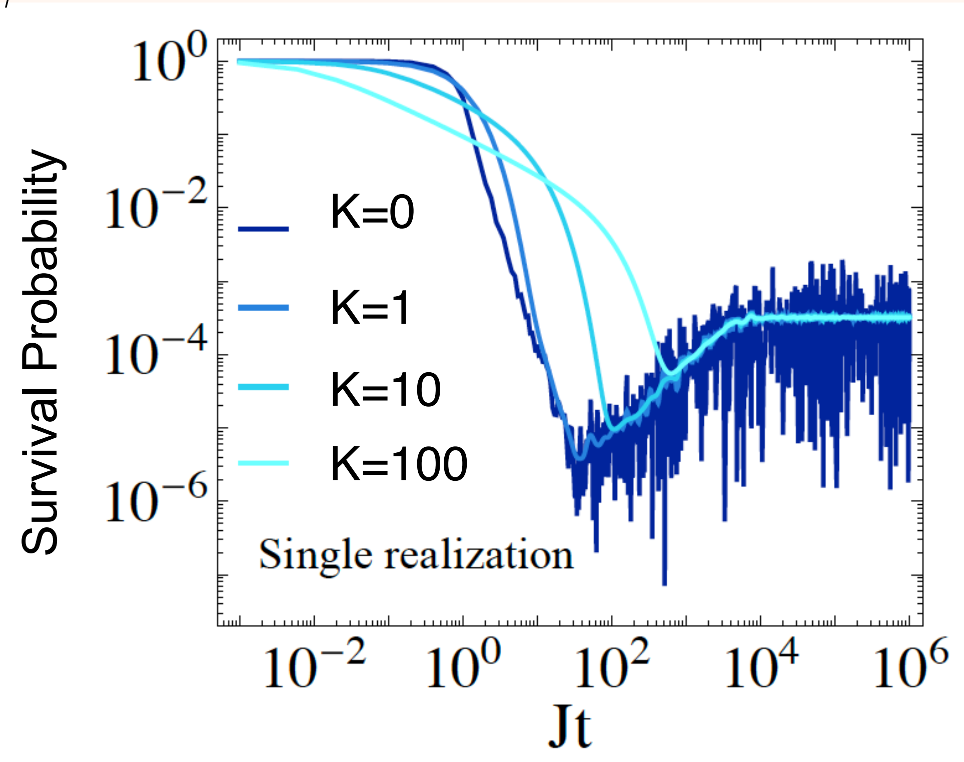
Avoiding averages with decoherence

$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

Phys. Rev. B **103**, 064309 (2021)

$$SP(t) = \sum_{\alpha,\beta} \frac{e^{-E_\alpha/T}}{Z}\frac{e^{-E_\beta/T}}{Z} e^{-2it(E_\alpha - E_\beta) - \kappa t(E_\alpha - E_\beta)^2}$$

decoherence coefficient



Adolfo del Campo

Self-averaging in open systems: GOE

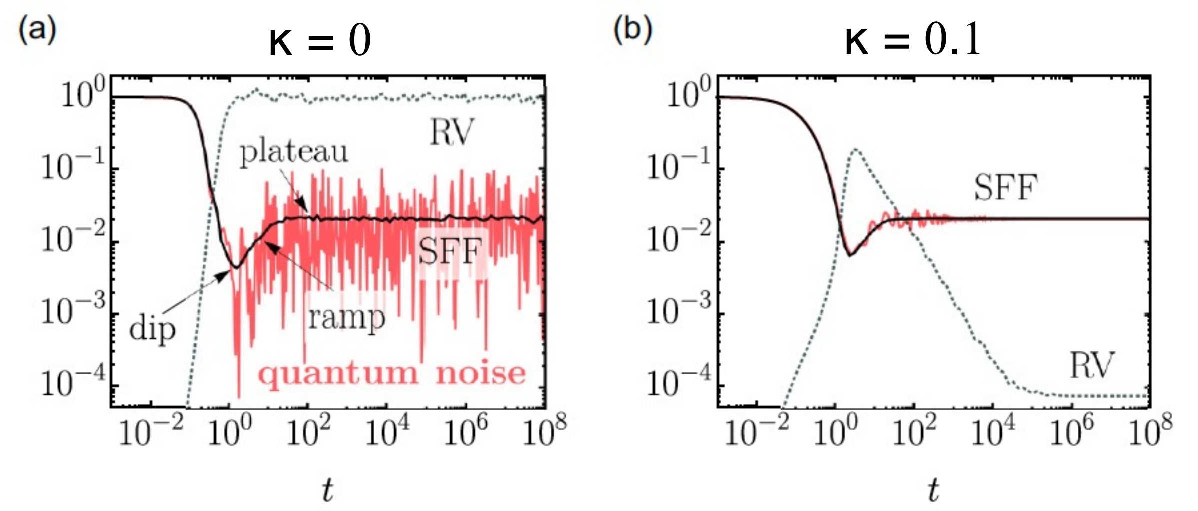
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

GOE

CLOSED

OPEN

Spectral form factor for a single realization (**red**) and upon Hamiltonian average (**black**), together with the RELATIVE VARIANCE (**dotted** line).



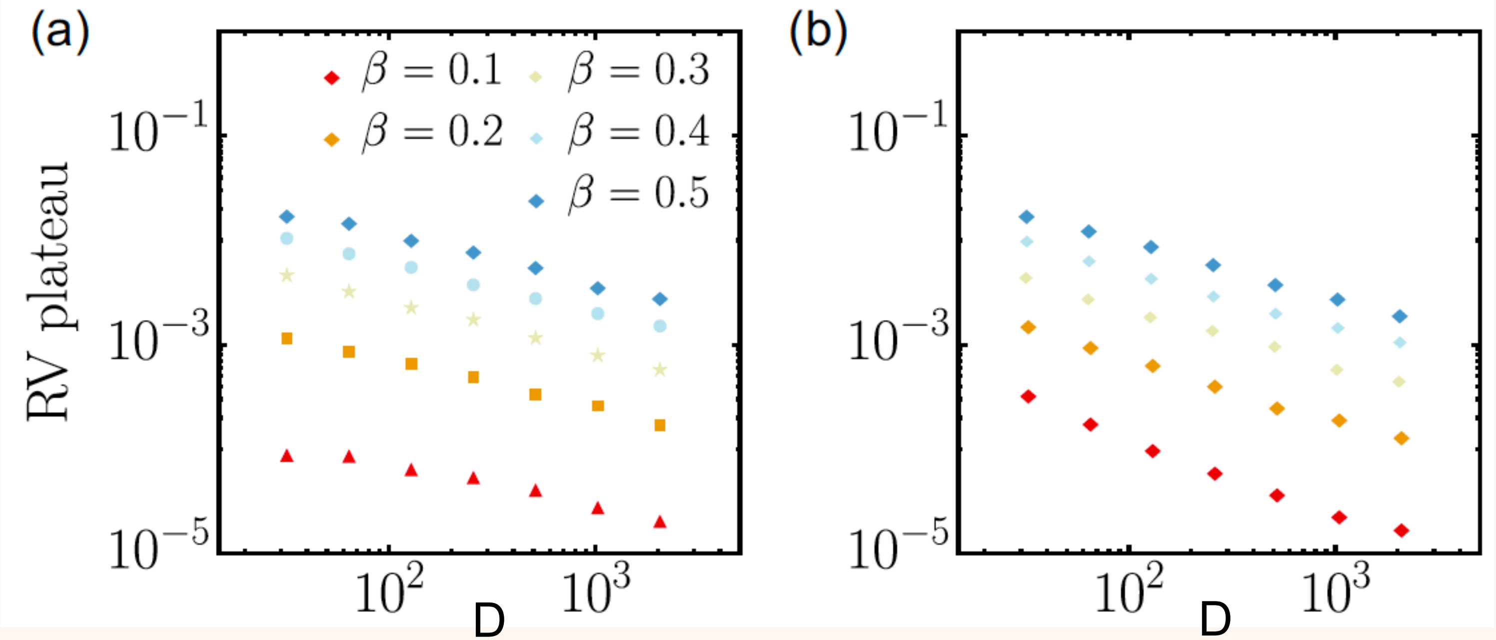
$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

$$S_P(t) = \mathrm{Tr}[\rho(t)\rho(0)]$$

$$\rho_{\alpha\beta}(t) = \rho_{\alpha\beta}(0)e^{-i(E_\alpha - E_\beta)t - \kappa(E_\alpha - E_\beta)^2 t}$$

PRA **108**, 062201 (2023)

Self-averaging in open systems: GOE



$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

$$d_t\rho_t = -i(H_T\rho_t - \rho_t H_T^{\dagger}) + 2\mathrm{tr}(\Gamma\rho_t)\rho_t$$

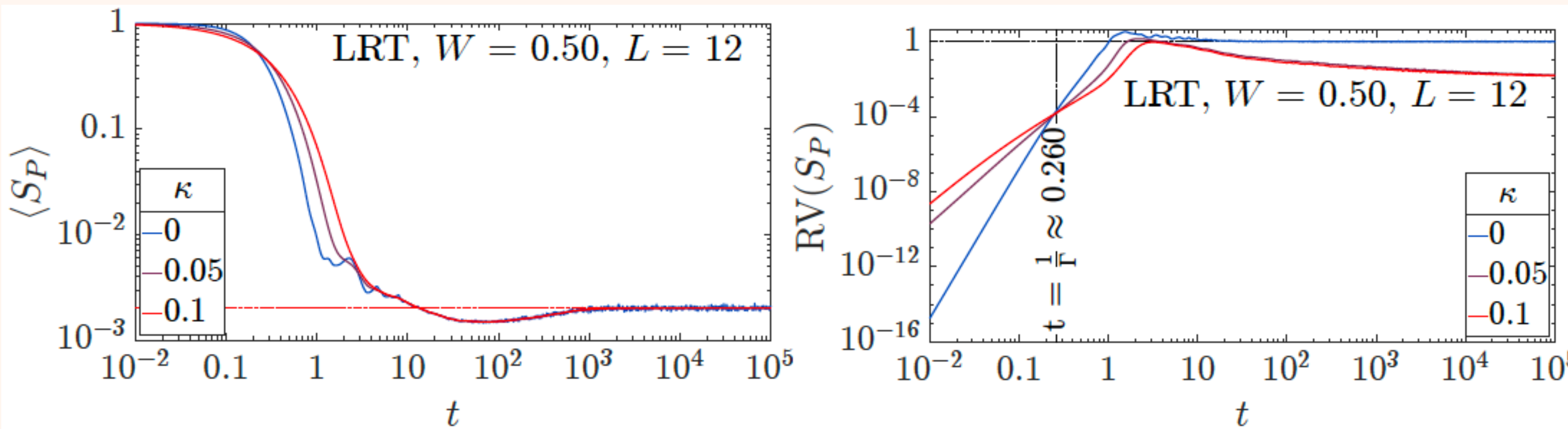
$$H_T = H - i\Gamma.$$

PRA **108**, 062201 (2023)

Self-averaging in open physical systems

$$S_P(t) = \mathrm{Tr}[\rho(t)\rho(0)]$$

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$



$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

Fulbright fellow graduating this year → Adway Kumar Das

Isaías Vallejo

soon in the arXiv…

Summary

- Many-body quantum chaos gets manifested in the evolution of physical quantities.

  It may be possible to detect experimentally.

- Quantities that exhibit dynamical manifestations of quantum chaos take an exponentially long time in L to equilibrate.

- Lack of self-averaging is a problem that can be circumvented by slightly opening the system to an environment.

Thank you!

ಧನ್ಯವಾದ