Relaxation and timescales in disordered XX model with on site dephasing

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Stability of Quantum Matter in and out of Equilibrium at Various Scales

January 18, 2024

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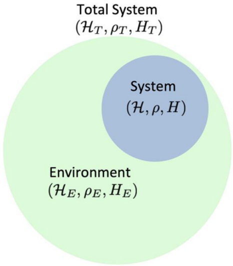
Model

Lindblad master equation



Von Neumann equation
$\frac{d\rho_T}{dt} = -i[\mathcal{H}_T, \rho_T]$

Need to trace out dof of environment. Approximations:-

- weak system-env coupling,
- No initial coupling,
- Env is in thermal state
- Sys-env correlations timescales much smaller than relaxation timescales, Markovian approximation.

Interaction picture, perturbation theory to second order $\rightarrow$ Redfield equation $\rightarrow$ not a positive map but trace preserving.
Want Positive trace preserving map in the space of density matrices , additional rotating wave approx and go back to Schrodinger picture $\rightarrow$ Lindblad equation. $\frac{\mathrm{d}\rho}{\mathrm{d}t} = i[\rho, H] + \mathcal{L}(\rho) = \mathcal{L}(\rho)$

Model

Disordered XX model :

$$H = -J \sum_{j=1}^{L-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y) + \sum_{j=1}^{L} h_j \sigma_j^z \tag{1}$$

Initial state:- Neel state.

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$\mathcal{L}^{\mathrm{deph}}(\rho) = \sum_k ([L_k \rho, L_k^\dagger] + [L_k, \rho L_k^\dagger])$. Choose $L_k = \sqrt{\frac{\gamma_k}{2}} \sigma_k^z$, kills off diagonal terms!

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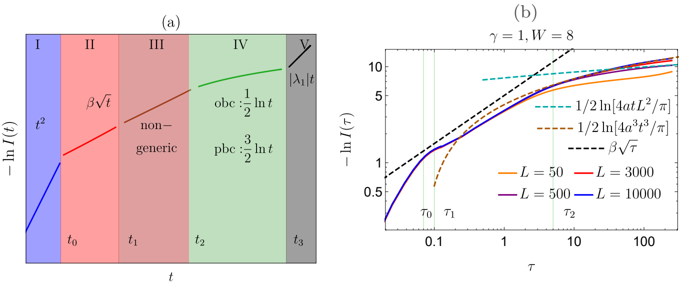
Quartic in fermion operators, typically solved using DMRG, but!!

Relaxation of Imbalance

How does $I(t)$ actually relax?



$\tau = \frac{8\gamma t}{W^2}$ As evident from the figure, five windows.

1. (I)
2. (II)
3. (III)
4. (IV)
5. (V)

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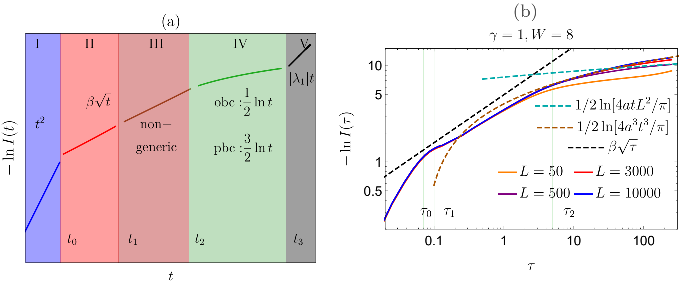

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1. $t < t_0 \sim \mathrm{Min}(1/\gamma, 1/W)$: $I(t) \sim 1 - t^2 \Rightarrow -\ln I(t) \sim t^2$. Off diagonal correlations first develop in the system and then start decaying after reaching a maxima. $W \ll \gamma$, $t_0 \to\sim 1/\gamma$. $W \gg \gamma \to t \sim 1/W$.
2.
3.
4.

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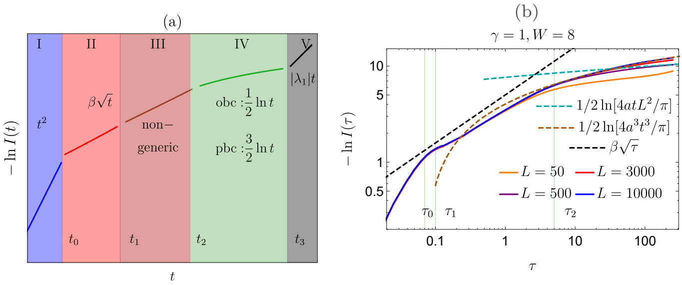
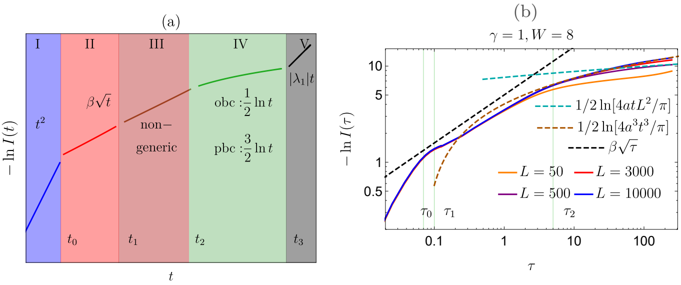


$\tau = \frac{8\gamma t}{W^2}$ As evident from the figure, five windows.

- (I)
- (II) $t_0 < t < t_1 \sim 0.1W^2/(8\gamma J^2)$: local relaxation of σ^z starts. $-\ln I(\tau) \sim -\ln(1-\beta\sqrt{\tau}) \sim \beta\sqrt{\tau}$ irrespective of the nature of disorder chosen . Short time, linear regime of the stretched exponential behavior of previous works, valid until $\tau \sim \tau_1 = (8\gamma J^2 t_1)/W^2 = 0.1$.
- (III)
- (IV)

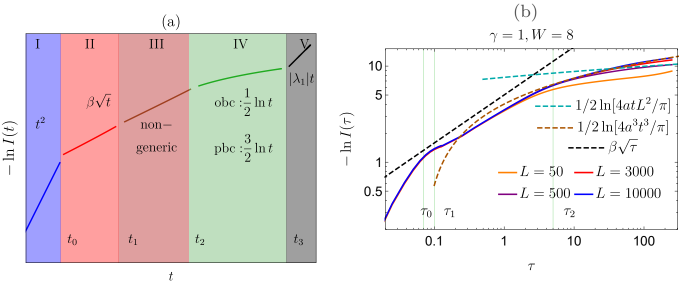
How does $I(t)$ actually relax?



$\tau = \frac{8\gamma t}{W^2}$ As evident from the figure, five windows.

1.
2.
3. $t_1 < t < t_2 \approx 5W^2/(8\gamma J^2)$: in this regime, which holds until $\tau \sim \tau_2 = (8\gamma J^2 t_2)/W^2 \approx 5$ a non generic decay dependent on the choice of disorder distribution is seen. t_2 also marks the end of local relaxation in the system.
4.

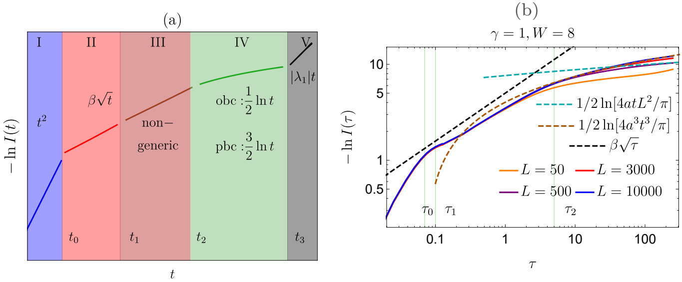
How does $I(t)$ actually relax?



$\tau = \frac{8\gamma t}{W^2}$ As evident from the figure, five windows.

- I
- II
- III
- IV $t_2 < t < t_3$: Power law decay due to a continuum of low magnitude eigenvalues of Liouvillian, boundary conditions, dependent. pbc-$I(t) \propto 1/t^{3/2}$, obc-$I(t) \propto 1/(L\sqrt{t})$. $I(t) \sim 1/t^{3/2}$, for large L, obc transition $I(t) \propto 1/(L\sqrt{t})$ at a time $\tau \approx L/2$.

How does $I(t)$ actually relax?



$\tau = \frac{8\gamma t}{W^2}$ As evident from the figure, five windows.

- (I)
- (II)
- (III)
- (IV)
- (V) $t > t_3 \sim L^2W^2/\gamma$: $I(t) \sim \exp(-|\lambda_1|t)$ where λ_1 is the Liouvilian gap. $\lambda_1 \sim \gamma/(L^2W^2)$, $t_3 \sim L^2W^2/\gamma$.

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We need $I(t)$ only, quadratic in fermion operators, huge simplification!
$4^L \to L^2$.

M. Žnidarič, J. Stat. Mech. (2010)

M. Žnidarič and M. Horvat, EPJB (2013)

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$$\frac{\mathrm{d}\mathbf{C}(t)}{\mathrm{d}t} + 2\mathrm{i}(\mathbf{P}\mathbf{C}(t) - \mathbf{C}(t)\mathbf{P}^T) + 2(\mathbf{\Gamma}\tilde{\mathbf{C}}(t) + \tilde{\mathbf{C}}(t)\mathbf{\Gamma}) = 0, \tag{3}$$

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$C_{jk}(t) = a_j^{(k-j+1)}(t) + \mathrm{i}b_j^{(k-j+1)}(t)$ for $k > j$, $C_{jj}(t) = a_j^{(1)}(t)$ and $C_{jk} = C_{kj}^*$. $\tilde{\mathbf{C}} = \mathbf{C} - \mathrm{diag}(\mathbf{C})$, a and $b \to$ two fermion observable expectations.

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$\mathbf{P} = \mathbf{W} - \mathbf{T}$, $W_{jk} = h_k \delta_{jk}$, $T_{jk} = J(\delta_{j,k-1} + \delta_{j,k+1})$

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$\Gamma_j^k = \gamma\delta_{jk}$

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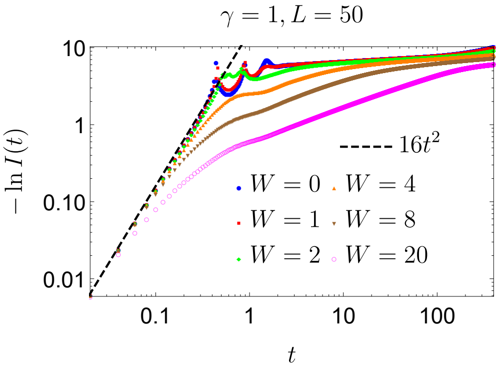
Another way:-

$$\frac{d\mathbf{f}}{dt} = \mathcal{Q}\mathbf{f} \quad (4)$$

where $\mathbf{f} = (C_{11}, C_{12}, \ldots, C_{1L}, C_{21}, \ldots, C_{LL})$

Region I



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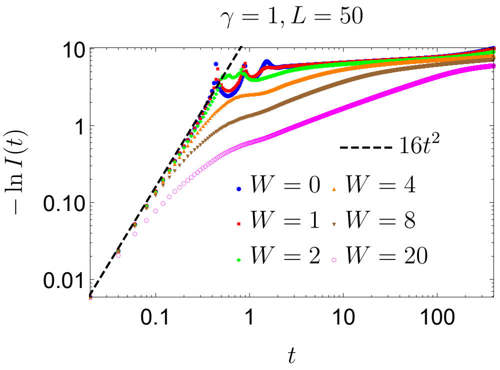

Power series $I(t)$ in t, the first nonzero term, $I(t) \sim 1 - \eta t^2$ behavior at the smallest timescale .

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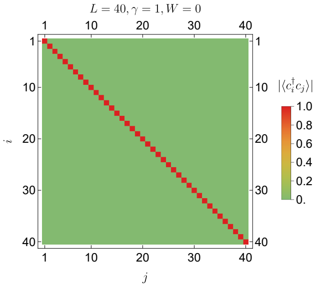

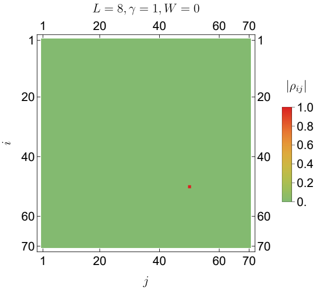

$W \to 0$. At $t = 0$,

Correlations then spread rapidly throughout the system, reach a maxima and start decaying around $t \sim 1/\sqrt{|\gamma^2 - 4|} \sim 1/\gamma$ for $\gamma \gg 1$. This behavior can be qualitatively extracted from a simple two-site model, we get, for small t, $I(t) \sim 1 - 8t^2 + O(t^3)$.

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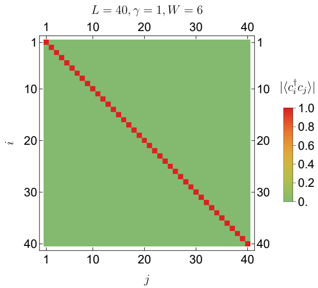
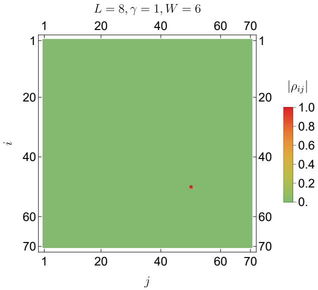
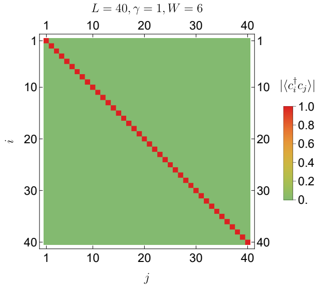
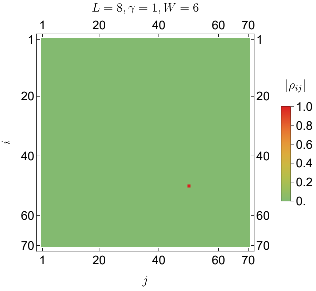




On addition of disorder?

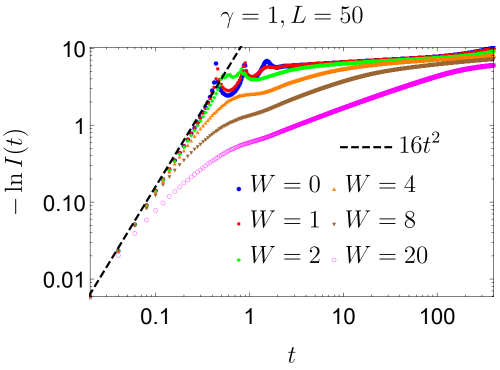
Region I





On addition of disorder?Correlations can develop in the system either due to **P**, or due to **Γ**. ⇒ dominating term?

Region I



Two site model gives will get $t_0 \ll 1/\gamma$. If we approximate the width of the distribution of δs as W, we can say that t_0 is around $\mathrm{Min}(1/\gamma, 1/W)$.

Region II and II—Three-site model

Assumptions- $W \gg \gamma$, take into account the influence of the neighboring sites, $j-1$ and $j+1$.

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Further ignore C_{ij}'s where $|j-i|>1$, We can then use the linearized equation with

$$\mathbf{f} = \begin{pmatrix} C_{11}, & C_{12}, & C_{21}, & C_{22}, & C_{23}, & C_{32}, & C_{33} \end{pmatrix}, \tag{5}$$

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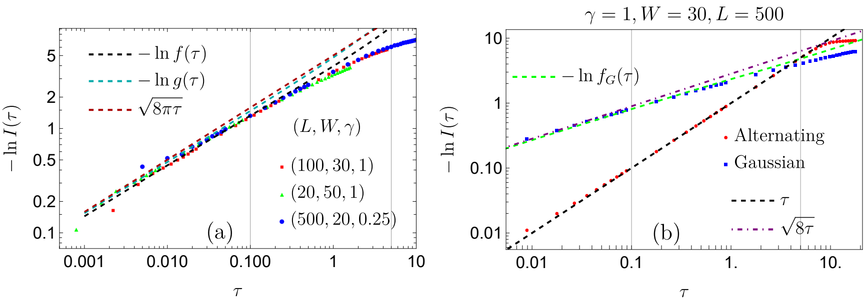
$\mathcal{Q}$ can be rearranged to a tractable form

$$\mathcal{Q} = \begin{pmatrix} \mathbf{O}_{3\times 3} & \mathbf{B}_{3\times 4} \\ \mathbf{C}_{4\times 3} & \mathbf{D}_{4\times 4} \end{pmatrix}. \tag{6}$$

Then using second order degenerate perturbation theory find the perturbation to **O** sector to get

$$\Delta^{(\pm)} \sim \frac{8\gamma(\delta_1^2+\delta_2^2 \pm \sqrt{\delta_1^4+\delta_2^4-\delta_1^2\delta_2^2})}{\delta_1^2\delta_2^2}$$

Region II and III

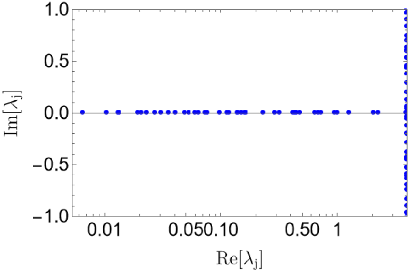


$$f(t, W, \gamma) = \lim_{L\to\infty} (1/L) \sum_{j=1}^{L} e^{-\Delta_j^{(+)} t},$$

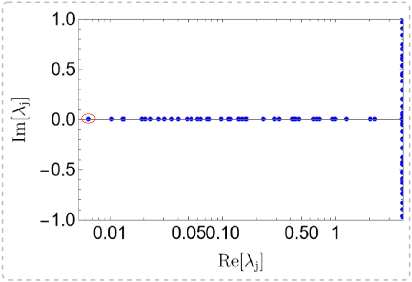
$$\Delta^{(\pm)} \sim \frac{8\gamma(\delta_1^2 + \delta_2^2 \pm \sqrt{\delta_1^4 + \delta_2^4 - \delta_1^2\delta_2^2})}{\delta_1^2\delta_2^2}$$

$$\begin{aligned} g(\tau) \quad &= \quad [-\sqrt{2\pi}\sqrt{\tau}\text{erfc}\left(\sqrt{\frac{\tau}{2}}\right) \\ &\quad + e^{-\frac{\tau}{2}} + \frac{1}{2}\tau\Gamma\left(0, \frac{\tau}{2}\right)]^2 \end{aligned}$$

$\rho(t) = e^{\mathcal{L}t}\rho(0)$. $-\lambda_j \to$ roots of Liouvillian $\mathcal{L}$ (one-particle sector).



Region V



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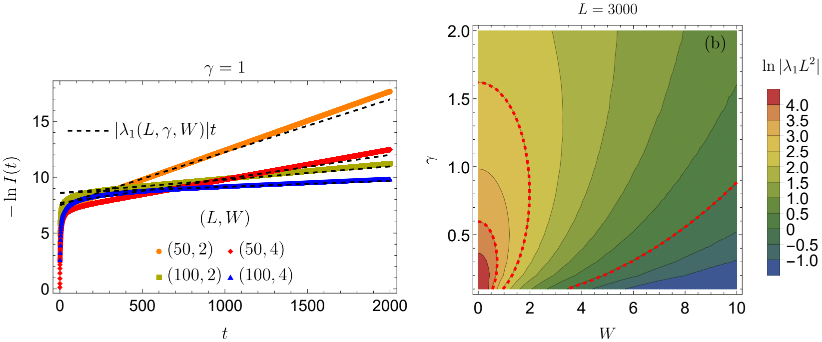
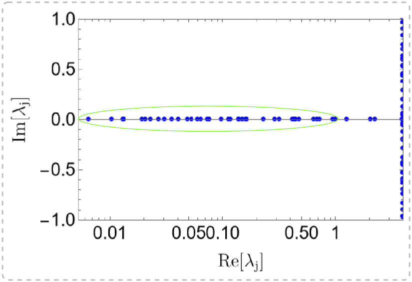


Dashed lines from analytic computation via effective heisenberg hamiltonian

Region IV



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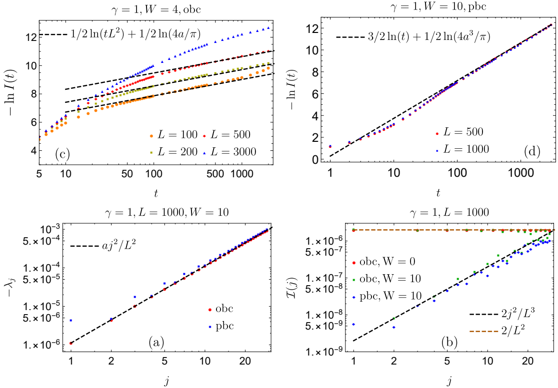

$$I(t) \quad = \quad I_{\text{NESS}} + \int dj \mathcal{I}(j) e^{\lambda_j t}, \mathcal{I}(j) = \frac{1}{L}\sum_{k=1}^{L}(-1)^{k+1}[\delta \mathbf{f}(j)]_{(k-1)L+k}$$

$$\lambda_j \quad \sim \quad -a(\gamma, W) j^2/L^2, a(\gamma, W) = \frac{8\pi^2\gamma}{(\frac{2W^2}{3} + 4\gamma^2)}$$

M. V. Medvedyeva, T. Prosen, M. Žnidarič, PRB(2016).

Various cases

$$\lambda_j \sim -j^{\alpha}, \quad \mathcal{I}(j) \sim j^{\beta} \tag{7}$$

Case	disorder	α	β	$I(t)$	$-\ln I(t)$
pbc, even L	0	2	no overlap	$e^{-4\gamma t}$	4γ
pbc, odd L	0	2	0	$\frac{1}{L\sqrt{8\pi t}}$	$\frac{1}{2}\ln(tL^2) + \frac{1}{2}\ln(8\pi)$
obc, any L	0	2	0	$\frac{1}{L\sqrt{8\pi t}}$	$\frac{1}{2}\ln(tL^2) + \frac{1}{2}\ln(8\pi)$
obc, any L	W	2	0	$\frac{\sqrt{\pi}}{2L\sqrt{at}}$	$\frac{1}{2}\ln(tL^2) + \frac{1}{2}\ln\frac{4a}{\pi}$
pbc, any L	W	2	2	$\frac{\sqrt{\pi}}{2(at)^{3/2}}$	$\frac{3}{2}\ln t + \frac{1}{2}\ln\frac{4a^3}{\pi}$

Conclusion

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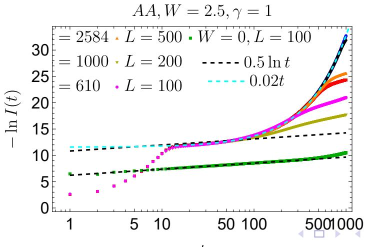
- pbc behaviour true asymptotic? Totally new relaxation!
- Asymptotics for quasiperiodic disorder? How does range of the model affect it? What about interactions?(possibly limited effect but deeper study needed). Non local dephasing can also be treated by same technique (M. Žnidarič arXiv:2311.07375).



THANK YOU

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$$\mathcal{H}_{\text{eff}} = \sum_{j=1}^{L} 2\frac{(\gamma_j + \gamma_{j+1})}{(h_j - h_{j+1})^2 + (\gamma_j + \gamma_{j+1})^2}(1 - \boldsymbol{\sigma}_j.\boldsymbol{\sigma}_{j+1}), \qquad (8)$$

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4. Add disorder, combine 1 and 2 to get eigenvalues. Eigenvectors? Part numerics part and part intuition.