Angular Momentum Densities Inside a Quark Dressed With a Gluon

Based on: R. Singh, S. Saha, A. Mukherjee and N. Mathur, [Phys. Rev. D **109** (2024) no.01, 016022.]

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Introduction

- Polarized deep inelastic scattering (DIS) experiments suggest that only one-third of nucleon spin is coming from the quark's intrinsic spin.

  [J. Ashman et al. (1988). Phys. Lett. B, 206:364]

- RHIC-spin experiments have provided important constraints on the contribution of gluon's helicity to the proton spin.

  [D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang (2014). Phys. Rev. Lett., 113(1):012001]

- These observations make the orbital angular momentum (OAM) of the quarks and gluons a good candidate.
- Future experiments like Jlab 12 GeV and Electron-Ion Collider (EIC) will provide valuable inputs on the OAM contribution of quarks and gluons.

  [J. Dudek et al. (2012). Eur. Phys. J. A, 48:187] [R. Abdul Khalek et al. (2022). Nucl. Phys. A, 1026:122447]

- It was believed that the angular momentum of gluons cannot be separated into intrinsic and orbital parts in a *gauge-invariant* way.
- In polarized electron-proton and proton-proton scattering experiments have been measuring $\Delta G(x)$, is referred to as the helicity distribution of the gluon in a nucleon.

  [S E. Kuhn, J P. Chen, and E. Leader (2009). Prog. Part. Nucl. Phys., 63:1–50]
- $\Delta G(x)$ coincides with a gauge non-invariant gluon spin in a particular gauge: light-front gauge $A^+ = 0$.

  [X D. Ji (1997). Phys. Rev. Lett., 78:610–613]
- Chen *et. al.* proposed a *gauge-invariant* decomposition of gluon angular momentum into spin and OAM parts!

  [X S. Chen, X F. Lu, W M. Sun, F. Wang, and T. Goldman (2008). Phys. Rev. Lett., 100:232002]

- Split the gauge field:

$$\boldsymbol{\mathcal{A}} = \boldsymbol{\mathcal{A}}_{\text{pure}} + \boldsymbol{\mathcal{A}}_{\text{phys}}, \tag{1}$$

$$\boldsymbol{A} - \boldsymbol{A}_{\text{phys}} = \boldsymbol{A}_{\text{pure}} = \boldsymbol{\nabla}\frac{1}{\boldsymbol{\nabla}^2}\boldsymbol{\nabla}\cdot\boldsymbol{A}, \quad \frac{1}{\boldsymbol{\nabla}^2}f(\boldsymbol{x}) = -\frac{1}{4\pi}\int d^3x' \frac{f(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}.$$

- Transformation properties:

$$\boldsymbol{\mathcal{A}}_{\text{pure}} \to U\boldsymbol{\mathcal{A}}_{\text{pure}}U^\dagger + \frac{i}{g}U\boldsymbol{\nabla}U^\dagger$$

$$\boldsymbol{\mathcal{A}}_{\text{phys}} \to U\boldsymbol{\mathcal{A}}_{\text{phys}}U^\dagger$$

- Chen *et. al.* decomposition for nucleon spin in QCD:

$$\boldsymbol{J}_{\text{Chen}} = \underbrace{\int d^3x\ \psi^\dagger\frac{1}{2}\boldsymbol{\Sigma}\psi}_{\boldsymbol{S}^q_{\text{Chen}}} + \underbrace{\int d^3x\ \psi^\dagger\left(\boldsymbol{x}\times i\boldsymbol{\mathcal{D}}_{\text{pure}}\right)\psi}_{\boldsymbol{L}^q_{\text{Chen}}} + \underbrace{\int d^3x\ \boldsymbol{E}^{\boldsymbol{a}}\times\boldsymbol{A}^{\boldsymbol{a}}{}_{\text{phys}}}_{\boldsymbol{S}^g_{\text{Chen}}}$$
$$+\underbrace{\int d^3x\ E^{ai}\left(\boldsymbol{x}\times\boldsymbol{\nabla}\right)A^{ai}_{\text{phys}}}_{\boldsymbol{L}^g_{\text{Chen}}}, \tag{2}$$

where $\boldsymbol{\mathcal{D}}_{\text{pure}} = \boldsymbol{\nabla} + ig\boldsymbol{\mathcal{A}}_{\text{pure}}$, $\boldsymbol{\mathcal{A}}_{\text{pure}} = \boldsymbol{A}^a_{\text{pure}}T^a$ and $\boldsymbol{\mathcal{A}}_{\text{phys}} = \boldsymbol{A}^a_{\text{phys}}T^a$.

- Wakamatsu's decomposition for nucleon spin in QCD:

$$\boldsymbol{J}_{\text{Wak}} = \underbrace{\int d^3x\ \psi^\dagger \frac{1}{2}\boldsymbol{\Sigma}\psi}_{\boldsymbol{S}^q_{\text{Wak}}} + \underbrace{\int d^3x\ \psi^\dagger \boldsymbol{x} \times (\boldsymbol{p} - g\boldsymbol{A})\,\psi}_{\boldsymbol{L}^q_{\text{Wak}}} + \underbrace{\int d^3x\ \boldsymbol{E}^a \times \boldsymbol{A}^a_{\text{phys}}}_{\boldsymbol{S}^g_{\text{Wak}}}$$
$$+ \underbrace{\int d^3x\ \left[E^{ai}\,(\boldsymbol{x} \times \boldsymbol{\nabla})\,A^{ai}_{\text{phys}} + \boldsymbol{x} \times \rho^a\,\boldsymbol{A}^a_{\text{phys}}\right]}_{\boldsymbol{L}^g_{\text{Wak}}}. \tag{3}$$

[M. Wakamatsu (2010). Phys. Rev. D, 81:114010]

- The term $\int d^3x\ \ \boldsymbol{x} \times \rho^a\boldsymbol{A}^a_{\text{phys}} = \int d^3x\ \ \psi^\dagger \boldsymbol{x} \times \boldsymbol{\mathcal{A}}_{\text{phys}}\psi$, is called "potential angular momentum". Here $\rho^a = g\psi^\dagger T^a\psi$.

- The gluon field is decomposed into two parts as

$$\begin{aligned} A^\mu &= A^\mu_{\text{phys}} + A^\mu_{\text{pure}}, \\ F^{\mu\nu}_{\text{pure}} &= \partial^\mu A^\nu_{\text{pure}} - \partial^\nu A^\mu_{\text{pure}} - ig\left[A^\mu_{\text{pure}}, A^\nu_{\text{pure}}\right] = 0, \\ A^\mu_{\text{phys}}(x) &\to U(x) A^\mu_{\text{phys}}(x) U^\dagger(x) \\ A^\mu_{\text{pure}}(x) &\to U(x)\left(A^\mu_{\text{pure}} + \frac{i}{g}\partial^\mu\right)U^\dagger(x). \end{aligned}$$

- Covariant version of the Chen *et. al.* decomposition [gauge-invariant canonical (gic)]

$$T^{\mu\nu}_{\text{gic}}(x) = \frac{1}{2}\overline{\psi}(x)\gamma^\mu i \overleftrightarrow{D}^\nu_{\text{pure}}\psi(x) - 2\text{Tr}\left[G^{\mu\alpha}(x) D^\nu_{\text{pure}} A^{\text{phys}}_\alpha(x)\right] \tag{4}$$

$$\begin{aligned} J^{\mu\nu\rho}_{\text{gic}}(x) = {} & \frac{1}{2}\overline{\psi}(x)\gamma^\mu x^{[\nu} i \overleftrightarrow{D}^{\rho]}_{\text{pure}}\psi(x) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}(x)\gamma_\sigma\gamma_5\psi(x) - 2\text{Tr}\left[G^{\mu[\nu} A^{\rho]}_{\text{phys}}(x)\right] \\ & - 2\text{Tr}\left[G^{\mu\alpha} x^{[\nu} D^{\rho]}_{\text{pure}} A^{\text{phys}}_\alpha(x)\right] \end{aligned} \tag{5}$$

where $D^{\text{pure}}_\mu = \partial_\mu - igA^{\text{pure}}_\mu$ for quark fields.

[M. Wakamatsu (2011). Phys. Rev. D, 83:014012]

[E. Leader and C. Lorcé (2014). Phys. Rept., 541(3):163–248]

- The gauge-invariant kinetic (gik) decomposition

$$T^{\mu\nu}_{\text{gik}}(x) = \frac{1}{2}\overline{\psi}(x)\gamma^{\mu} i \overleftrightarrow{D}^{\nu}\psi(x) - 2\text{Tr}\left[G^{\mu\alpha}(x) D^{\nu}_{\text{pure}} A^{\text{phys}}_{\alpha}(x) - \left(D_{\alpha} G^{\alpha\mu}(x)\right) A^{\nu}_{\text{phys}}(x)\right] \tag{6}$$

$$J^{\mu\nu\rho}_{\text{gik}}(x) = \frac{1}{2}\overline{\psi}(x)\gamma^{\mu} x^{[\nu} i \overleftrightarrow{D}^{\rho]}\psi(x) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}(x)\gamma_{\sigma}\gamma_5\psi(x) - 2\text{Tr}\left[G^{\mu[\nu} A^{\rho]}_{\text{phys}}(x)\right]$$
$$- 2\text{Tr}\left[G^{\mu\alpha}(x) x^{[\nu} D^{\rho]}_{\text{pure}} A^{\text{phys}}_{\alpha}(x) - \left(D_{\alpha} G^{\alpha\mu}(x)\right) x^{[\nu]} A^{\rho]}_{\text{phys}}(x)\right] \tag{7}$$

[M. Wakamatsu (2011). Phys. Rev. D, 83:014012]

- The Jaffe-Manohar decomposition:

$$T^{\mu\nu}(x) = \frac{1}{2}\overline{\psi}(x)\gamma^{\mu} i \overleftrightarrow{\partial}^{\nu}\psi(x) - 2\text{Tr}\left[G^{\mu\alpha}\partial^{\nu} A_{\alpha}\right] \tag{8}$$

$$J^{\mu\nu\rho}(x) = \frac{1}{2}\overline{\psi}(x)\gamma^{\mu} x^{[\nu} i \overleftrightarrow{\partial}^{\rho]}\psi(x) + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}(x)\gamma_{\sigma}\gamma_5\psi(x) - 2\text{Tr}\left[G^{\mu[\nu} A^{\rho]}\right]$$
$$- 2\text{Tr}\left[G^{\mu\alpha} x^{[\nu}\partial^{\rho]} A_{\alpha}\right] \tag{9}$$

[R L. Jaffe and A. Manohar (1990). Nucl. Phys. B, 337:509–546]

[E. Leader and C. Lorcé (2014). Phys. Rept., 541(3):163–248]

- The Belinfante-decomposition:

$$T^{\mu\nu}_{\text{Bel}}(x) = \frac{1}{4}\overline{\psi}(x)\left[\gamma^{\mu} i \overleftrightarrow{D}^{\nu} + \gamma^{\nu} i \overleftrightarrow{D}^{\mu}\right]\psi(x) - 2\text{Tr}\left[G^{\mu\alpha}G^{\nu}_{\alpha}\right], \tag{10}$$

$$J^{\mu\nu\rho}_{\text{Bel}}(x) = \frac{1}{4}\overline{\psi}\gamma^{\mu}x^{[\nu} i \overleftrightarrow{D}^{\rho]}\psi + \frac{1}{4}x^{[\nu}\overline{\psi}\gamma^{\rho]} i \overleftrightarrow{D}^{\mu}\psi - 2\text{Tr}\left[x^{\nu}G^{\mu\alpha}G^{\rho}_{\alpha} - x^{\rho}G^{\mu\alpha}G^{\nu}_{\alpha}\right] \tag{11}$$

[F J. Belinfante (1939). Physica D: Nonlinear Phenomena, 6:887–898]

[E. Leader and C. Lorcé (2014). Phys. Rept., 541(3):163–248]

- The Ji's decomposition:

$$\begin{aligned} T^{\mu\nu}_{\text{kin}}(x) &= T^{\mu\nu}_{\text{kin,q}}(x) + T^{\mu\nu}_{\text{kin,g}} \\ &= \frac{1}{2}\overline{\psi}(x)\gamma^{\mu} i \overleftrightarrow{D}^{\nu}\psi(x) - 2\text{Tr}\left[G^{\mu\alpha}G^{\nu}_{\alpha}\right] \end{aligned} \tag{12}$$

$$\begin{aligned} J^{\mu\nu\rho}_{\text{kin}}(x) &= L^{\mu\nu\rho}_{\text{kin,q}}(x) + S^{\mu\nu\rho}_{\text{q}}(x) + J^{\mu\nu\rho}_{\text{kin,g}}(x) \\ &= \frac{1}{2}\overline{\psi}\gamma^{\mu}x^{[\nu} i \overleftrightarrow{D}^{\rho]}\psi + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\overline{\psi}(x)\gamma_{\sigma}\gamma_{5}\psi(x) - 2\text{Tr}\left[x^{\nu}G^{\mu\alpha}G^{\rho}_{\alpha} - x^{\rho}G^{\mu\alpha}G^{\nu}_{\alpha}\right] \end{aligned} \tag{13}$$

[X D. Ji (1997). Phys. Rev. Lett., 78:610–613]

[C. Lorcé, L. Mantovani, and B. Pasquini (2018). Phys. Lett. B, 776:38–47]

Class	EMT	AM densities	Gauge invariant	Follow $SU(2)$ algebra
Kinetic	Belinfante	$\mathbf{J}_{\text{Bel,q}}$	✓	×
		$\mathbf{J}_{\text{Bel,g}}$	✓	×
	Ji	$\mathbf{L}_{\text{Ji,q}}$	✓	×
		$\mathbf{S}_{\text{Ji,q}}$	✓	✓
		$\mathbf{J}_{\text{Ji,g}}$	✓	×
	Wakamatsu (gik)	$\mathbf{L}_{\text{gic,q}}$	✓	×
		$\mathbf{S}_{\text{gic,q}}$	✓	✓
		$\mathbf{L}_{\text{gic,g}}$	✓	×
		$\mathbf{S}_{\text{gic,g}}$	✓	×
Canonical	Jaffe-Manohar	$\mathbf{L}_{\text{JM,q}}$	×	✓
		$\mathbf{S}_{\text{JM,q}}$	✓	✓
		$\mathbf{L}_{\text{JM,g}}$	×	×
		$\mathbf{S}_{\text{JM,g}}$	×	×
	Chen et al. (gic)	$\mathbf{L}_{\text{gic,q}}$	✓	✓
		$\mathbf{S}_{\text{gic,q}}$	✓	✓
		$\mathbf{L}_{\text{gic,g}}$	✓	×
		$\mathbf{S}_{\text{gic,g}}$	✓	×

Table: Properties of all the angular momentum operators.

[R. Singh, S. Saha, A. Mukherjee, and N. Mathur (2024). Phys. Rev. D, 109(1):016022]

- Different decompositions of angular momentum differ from each other by surface terms.
- The kinetic and Belinfante-improved tensors in QCD are related as

$$T^{\mu\nu}_{\text{kin,q}}(x) = T^{\mu\nu}_{\text{Bel,q}}(x) - \frac{1}{2}\partial_\alpha S^{\alpha\mu\nu}_{\text{q}}(x), \quad T^{\mu\nu}_{\text{kin,g}}(x) = T^{\mu\nu}_{\text{Bel,g}},(x) \tag{14}$$

$$L^{\mu\nu\rho}_{\text{kin,q}}(x) + S^{\mu\nu\rho}_{\text{q}}(x) = J^{\mu\nu\rho}_{\text{Bel,q}}(x) - \frac{1}{2}\partial_\sigma \left[x^\nu S^{\sigma\mu\rho}_{\text{q}}(x) - x^\rho S^{\sigma\mu\nu}_{\text{q}}(x)\right], \quad J^{\mu\nu\rho}_{\text{kin,g}}(x) = J^{\mu\nu\rho}_{\text{Bel,g}}(x). \tag{15}$$

[C. Lorcé, L. Mantovani, and B. Pasquini (2018). Phys. Lett. B, 776:38–47]

- The total angular momentum density cannot be interpreted as a sum of the OAM and spin density only.

- Generalized parton distributions (GPDs) contain information about the spatial distribution of quarks and gluons.

  [M. Burkardt (2003). Int. J. Mod. Phys. A, 18:173–208]

- Polyakov showed how hard exclusive processes allow us to access the spatial distribution of angular momentum.

  [M V. Polyakov (2003). Phys. Lett. B, 555:57–62]

- Two-dimensional distributions in the light-front framework are free from relativistic corrections.
- We have investigated different decompositions of angular momentum and their contribution to the intrinsic spin and orbital angular momentum distributions.
- Relativistic spin$-\frac{1}{2}$ composite state of a quark dressed with a gluon at one loop in QCD.

Work Methodology

- We are working in light-front (LF) coordinates

$$x^{\pm} = (x^0 \pm x^3)\,, \quad x^{\mu} = \left(x^{+}, x^{-}, \boldsymbol{x}^{\perp}\right) \tag{16}$$

 where x^+ is the light-front time, x^- and $\boldsymbol{x}^{\perp}$ are the longitudinal and transverse spatial coordinates respectively.
- Light-front four-momentum:

$$p^{\mu} = \left(p^{+}, p^{-}, \boldsymbol{p}^{\perp}\right), \tag{17}$$

 where p^+ is the longitudinal momentum, p^- and $\boldsymbol{p}^{\perp}$ are the energy and transverse momentum respectively.
- Drell-Yan frame defined by $\Delta^+ = 0$ and $\boldsymbol{P}^{\perp} = 0$.

$$p^{\mu} = \left(P^{+}, -\frac{\boldsymbol{\Delta}^{\perp}}{2}, \frac{1}{P^{+}}\left(m^2 + \frac{\boldsymbol{\Delta}^{\perp 2}}{4}\right)\right), \quad p'^{\mu} = \left(P^{+}, \frac{\boldsymbol{\Delta}^{\perp}}{2}, \frac{1}{P^{+}}\left(m^2 + \frac{\boldsymbol{\Delta}^{\perp 2}}{4}\right)\right), \tag{18}$$

$$\Delta^{\mu} = (p' - p)^{\mu} = \left(0,\ \boldsymbol{\Delta}^{\perp}, 0\right). \tag{19}$$

- The impact-parameter distribution of orbital angular momentum is

$$\langle L^z \rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk} \int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp \cdot \boldsymbol{b}^\perp} \left[\frac{\partial \langle T^{+k} \rangle_{\mathsf{LF}}}{\partial \Delta_\perp^j} \right], \tag{20}$$

- Spin distribution in light-front is

$$\langle S^z \rangle(\boldsymbol{b}^\perp) = \frac{1}{2}\epsilon^{3jk} \int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp \cdot \boldsymbol{b}^\perp} \langle S^{+jk} \rangle_{\mathsf{LF}}. \tag{21}$$

- Belinfante-improved total angular momentum distribution is

$$\langle J^z_{\mathsf{Bel}} \rangle(\boldsymbol{b}^\perp) = -i\epsilon^{3jk} \int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp \cdot \boldsymbol{b}^\perp} \left[\frac{\partial \langle T^{+k}_{\mathsf{Bel}} \rangle_{\mathsf{LF}}}{\partial \Delta_\perp^j} \right]. \tag{22}$$

where $\boldsymbol{b}^\perp$ is the impact parameter and $\langle T^{\mu\nu} \rangle_{\mathsf{LF}} = \frac{\langle p', \boldsymbol{s} | T^{\mu\nu}(0) | p, \boldsymbol{s} \rangle}{2\sqrt{p'^+ p^+}}$.

[C. Lorcé, L. Mantovani, and B. Pasquini (2018). Phys. Lett. B, 776:38–47]

- To make a proper comparison between Belinfante and Ji's decomposition, we need to include the correction term

$$L^{\mu\nu\rho}_{\text{kin,q}}(x) + S^{\mu\nu\rho}_{\text{q}}(x) = J^{\mu\nu\rho}_{\text{Bel,q}}(x) - \underbrace{\frac{1}{2}\partial_\sigma\left[x^\nu S^{\sigma\mu\rho}_{\text{q}}(x) - x^\rho S^{\sigma\mu\nu}_{\text{q}}(x)\right]}_{\text{Correction term}} \tag{23}$$

$$\langle M^z\rangle(\boldsymbol{b}^\perp) = \frac{1}{2}\epsilon^{3jk}\int\frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\Delta^l_\perp\frac{\partial\langle S^{l+k}\rangle_{\text{LF}}}{\partial\Delta^j_\perp}. \tag{24}$$

where $\boldsymbol{b}^\perp$ is the impact parameter.

[C. Lorcé, L. Mantovani, and B. Pasquini (2018). Phys. Lett. B, 776:38–47]

- A Gaussian wave packet state has been considered, where the system is confined in transverse momentum space with definite longitudinal momentum:

$$\frac{1}{16\pi^3}\int\frac{d^2\boldsymbol{p}^\perp dp^+}{p^+}p^+\ \delta(p^+ - p^+_0)\ e^{-\frac{\boldsymbol{p}^{\perp 2}}{2\sigma^2}}\ |\,p^+,\boldsymbol{p}^\perp,\lambda\rangle, \tag{25}$$

where σ is the width of the Gaussian wave packet.

[D. Chakrabarti and A. Mukherjee (2005). Phys. Rev. D, 72:034013]

The Dressed Quark State

- We consider a relativistic spin-$\frac{1}{2}$ state of a quark dressed with a gluon at one loop in QCD as a perturbative model with a gluon degree of freedom.
- The dressed quark state can be expanded in Fock space in terms of multiparton light-front wave functions (LFWFs) which can be calculated using the light-front Hamiltonian in perturbation theory.

  [A. Harindranath, A. Mukherjee, and R. Ratabole (2001). Phys. Rev. D, 63:045006]
- We can write the LFWFs in a boost invariant way in terms of relative momenta that are frame-independent.

  [S J. Brodsky, H C. Pauli, and S S. Pinsky (1998). Phys. Rept., 301:299–486]

- Up to two-particle sector, dressed quark state is given by

$$
\begin{aligned}
|p,\lambda\rangle =& \psi_1(p,\lambda) b^\dagger_\lambda(p)|0\rangle \\
& + \sum_{\lambda_1,\lambda_2} \int \frac{dk_1^+ d^2\boldsymbol{k}_1^\perp}{\sqrt{2(2\pi)^3 k_1^+}} \int \frac{dk_2^+ d^2\boldsymbol{k}_2^\perp}{\sqrt{2(2\pi)^3 k_2^+}} \psi_2\left(p,\lambda|k_1,\lambda_1;k_2,\lambda_2\right) \sqrt{2(2\pi)^3 P^+} \\
& \delta^3\left(p-k_1-k_2\right) b^\dagger_{\lambda_1}(k_1) a^\dagger_{\lambda_2}(k_2)|0\rangle, \qquad (26)
\end{aligned}
$$

$\psi_1(p,\lambda)$ is normalization, $\psi_2\left(p,\lambda|k_1,\lambda_1;k_2,\lambda_2\right)$ is the probability of finding a quark and gluon with momentum (helicity) $k_1(\lambda_1)$ & $k_2(\lambda_2)$ respectively.

[A. Harindranath, R. Kundu, and W M. Zhang (1999). Phys. Rev. D, 59:094013]

- $\psi_2\left(p,\lambda|k_1,\lambda_1;k_2,\lambda_2\right)$ can be calculated from the following equation

$$
H|p,\lambda\rangle = \frac{m^2+\boldsymbol{p}^{\perp 2}}{p^+}|p,\lambda\rangle, \qquad (27)
$$

where H is the light-front QCD Hamiltonian.

[W M. Zhang and A. Harindranath (1993). Phys. Rev. D, 48:4881–4902]

- Boost invariant two-particle LFWF can be written as

$$\phi^{\lambda}_{\lambda_1,\lambda_2}(x_i,\boldsymbol{\kappa}_i) = \sqrt{p^+}\psi_2\left(p,\lambda|k_1,\lambda_1;k_2,\lambda_2\right) \tag{28}$$

$$\phi^{\lambda}_{\lambda_1,\lambda_2}(x_i,\boldsymbol{\kappa}_i) = \frac{g}{\sqrt{2(2\pi)^3}}\left[\frac{x(1-x)}{\boldsymbol{\kappa}^{\perp 2}+m^2(1-x)^2}\right]\frac{T^a}{\sqrt{1-x}}\times$$
$$\chi^{\dagger}_{\lambda_1}\left[\frac{-2\left(\boldsymbol{\kappa}^{\perp}\cdot\epsilon^{\perp *}_{\lambda_2}\right)}{1-x} - \frac{1}{x}\left(\tilde{\sigma}^{\perp}\cdot\boldsymbol{\kappa}^{\perp}\right)\left(\tilde{\sigma}^{\perp}\cdot\epsilon^{\perp *}_{\lambda_2}\right) + im\left(\tilde{\sigma}^{\perp}\cdot\epsilon^{\perp *}_{\lambda_2}\right)\frac{1-x}{x}\right]\chi_{\lambda}\psi^{\lambda}_{1}. \tag{29}$$

Jacobi momentum

$$k_i^+ = x_i p^+, \qquad \boldsymbol{k}_i^{\perp} = \boldsymbol{\kappa}_i^{\perp} + x_i\boldsymbol{p}^{\perp}, \tag{30}$$

$$x_1 + x_2 = 1, \qquad \boldsymbol{\kappa}_1^{\perp} + \boldsymbol{\kappa}_2^{\perp} = 0, \tag{31}$$

where m is quark mass, g is gluon coupling constant, T^a are $SU(3)$ colour matrices, $\epsilon^{\perp}_{\lambda_2}$ is gluon polarization vector and χ_{λ} is two-component spinor.

[A. Harindranath, R. Kundu, and W M. Zhang (1999). Phys. Rev. D, 59:094013]

Two-Component QCD

- In light-front coordinates, the unphysical degrees of freedom of the gauge field are eliminated by light-front gauge $A_a^+ = A_a^0 + A_a^3 = 0$.
- In the light-front Hamiltonian framework, the quark fields are decomposed as

$$\psi = \psi_+ + \psi_-, \tag{32}$$

where $\psi_\pm = \Lambda_\pm \psi$ and $\Lambda_\pm$ are the projection operators.
- The ψ_- component and longitudinal gauge field A_a^- are constrained fields and can be determined from the following equations

$$i\partial^+ \psi_- = \left(i\alpha^\perp \cdot \partial^\perp + g\alpha^\perp \cdot A^\perp + \beta m\right)\psi_+, \tag{33}$$

$$\frac{1}{2}\partial^+ E_a^- = \left(\partial^i E_a^i + g f^{abc} A_b^i E_c^i\right) - g\psi_+^\dagger T^a \psi_+, \tag{34}$$

where $\alpha^\perp = \gamma^0\gamma^\perp$, $\beta = \gamma^0$ and $E_a^{-,i} = -\frac{1}{2}\partial^+ A_a^{-,i}$, $(i = 1, 2)$.

[W M. Zhang and A. Harindranath (1993). Phys. Rev. D, 48:4881–4902]

- Independent dynamical degrees of freedom in light-front QCD is ψ_+ and the transverse gauge fields A_a^i.
- In light-front coordinates, four-component fermion field can be reduced to a two-component field in a light-front representation of gamma matrices defined as

$$\gamma^+ = \begin{pmatrix} 0 & 0 \\ 2i & 0 \end{pmatrix}, \quad \gamma^- = \begin{pmatrix} 0 & -2i \\ 0 & 0 \end{pmatrix} \tag{35}$$

$$\gamma^i = \begin{pmatrix} -i\sigma^i & 0 \\ 0 & i\sigma^i \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}. \tag{36}$$

- In this representation, the projection operators become

$$\Lambda_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \Lambda_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{37}$$

- The quark fields decompose as

$$\psi_+ = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \qquad \psi_- = \begin{bmatrix} 0 \\ \eta \end{bmatrix}. \tag{38}$$

[W M. Zhang and A. Harindranath (1993). Phys. Rev. D, 48:4881–4902]

- The two-component quark fields are given by

$$\xi(y) = \sum_\lambda \chi_\lambda \int \frac{[dk]}{\sqrt{2(2\pi)^3}}[b_\lambda(k)e^{-ik\cdot y} + d^\dagger_{-\lambda}(k)e^{ik\cdot y}], \tag{39}$$

$$\eta(y) = \left(\frac{1}{i\partial^+}\right)\left[\sigma^\perp \cdot \left(i\partial^\perp + gA^\perp(y)\right) + im\right]\xi(y). \tag{40}$$

- The dynamical components of the gluon field are given by

$$A^\perp(y) = \sum_\lambda \int \frac{[dk]}{\sqrt{2(2\pi)^3 k^+}}[\epsilon^\perp_\lambda a_\lambda(k)e^{-ik\cdot y} + \epsilon^{\perp *}_\lambda a^\dagger_\lambda(k)e^{ik\cdot y}], \tag{41}$$

where $[dk] = \frac{dk^+ d^2\boldsymbol{k}^\perp}{\sqrt{2(2\pi)^3 k^+}}$, χ_λ is the eigenstate of σ^3 and ϵ^i_λ is the polarization vector of transverse gauge field.

[A. Harindranath, R. Kundu, and W M. Zhang (1999). Phys. Rev. D, 59:094013]

- Here, we have suppressed the colour indices.

Results

- We have taken only the quark part of the EMT.
- Quark part of the canonical EMT:

$$T^{+k}_{\text{kin,q}}(x) = \frac{1}{2}\overline{\psi}(x)\gamma^{+} i \overleftrightarrow{\partial}^{k}\psi(x) + \underbrace{g\overline{\psi}(x)\gamma^{+}A^{k}\psi(x)}_{(\text{Potential term } = 0)}, \tag{42}$$

$$T^{+k}_{\text{kin, q}} = \frac{1}{2}\overline{\psi}\gamma^{+} i \overleftrightarrow{\partial}^{k}\psi = T^{+k}_{\text{q}}. \tag{43}$$

The potential term is also found to be zero for an electron in QED at one loop level. [X. Ji, A. Schäfer, F. Yuan, J H. Zhang, and Y. Zhao (2016). Phys. Rev. D, 93(5):054013]

- Gauge-invariant canonical decomposition EMT:

$$T^{+k}_{\text{gic, q}} = \frac{1}{2}\overline{\psi}(x)\gamma^{+} i \overleftrightarrow{D}^{k}_{\text{pure}}\psi(x), \quad T^{+k}_{\text{gic,q}} = T^{+k}_{\text{q}}, \tag{44}$$

where, $\overleftrightarrow{D}^{\mu}_{\text{pure}} = \overleftrightarrow{\partial}^{\mu} - 2igA^{\mu}_{\text{pure}}$ and $A^{\mu}_{\text{pure}} = A^{\mu} - A^{\mu}_{\text{phys}}$.

- Gauge-invariant kinetic decomposition EMT:

$$T^{+k}_{\text{gik, q}} = T^{+k}_{\text{kin,q}}. \tag{45}$$

[R. Singh, S. Saha, A. Mukherjee, and N. Mathur (2024). Phys. Rev. D, 109(1):016022]

- longitudinal component of OAM distribution:

$$\langle L^z_{\text{kin, q}}\rangle(\boldsymbol{b}^\perp) = \frac{g^2 C_F}{72\pi^2}\int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{b}^\perp\cdot\boldsymbol{\Delta}^\perp}\Bigg[-7+\frac{6}{\omega}\left(1+\frac{2m^2}{\Delta^2}\right)\log\left(\frac{1+\omega}{-1+\omega}\right) - 6\log\left(\frac{\Lambda^2}{m^2}\right)\Bigg], \tag{46}$$

- Intrinsic spin distribution of quarks:

$$\langle S^z_{\text{kin, q}}\rangle(\boldsymbol{b}^\perp) = -\frac{g^2 C_F}{32\pi^2}\int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\int\frac{dx}{1-x}\times \Bigg[\omega(1+x^2)\log\left(\frac{1+\omega}{-1+\omega}\right) + \left(\frac{1-\omega^2}{\omega}\right)x\log\left(\frac{1+\omega}{-1+\omega}\right) - (1+x^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\Bigg]. \tag{47}$$

where $\omega = \sqrt{1+\frac{4m^2}{\Delta^2}}$ and Λ is the ultraviolet cutoff on the transverse momentum.

- Belinfante-improved total angular momentum distribution:

$$
\begin{aligned}
&\langle J^z_{\text{Bel, q}}\rangle(\boldsymbol{b}^\perp) \\
&= g^2 C_F \int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{b}^\perp\cdot\boldsymbol{\Delta}^\perp} \int \frac{dx}{16\pi^2} \frac{1}{(1-x)\Delta^4\omega^3} \\
&\quad \times \Big[\big(8m^4(1-2x)(1-x(1-x)) + 6m^2(1-(2-x)x(1+2x))\Delta^2 \\
&\quad + (1-(2-x)x(1+2x))\Delta^4 \big) \log\left(\frac{1+\omega}{-1+\omega}\right) \\
&\quad - \omega\Delta^2 \big(4m^2(1-(1-x)x) + (1+x^2)\Delta^2 \\
&\quad + (1-(2-x)x(1+2x))(4m^2+\Delta^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\big)\Big], \qquad (48)
\end{aligned}
$$

where $\omega = \sqrt{1+\frac{4m^2}{\Delta^2}}$ and Λ is the ultraviolet cutoff on the transverse momentum.

- Distribution of the superpotential term

$$\langle M^z_{\mathsf{q}}\rangle(\boldsymbol{b}^\perp) = \frac{g^2 C_F}{32\pi^2}\int \frac{d^2\boldsymbol{\Delta}^\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}^\perp\cdot\boldsymbol{b}^\perp}\int \frac{dx}{(1-x)}\frac{1}{\omega^3\Delta^4} \Big[\omega\Delta^2\left((4m^2+\Delta^2)(1+x^2)-4m^2x\right) - 2m^2\left((4m^2+\Delta^2)(1+x^2)-4m^2x-2x\Delta^2\right)\Big]. \quad (49)$$

where $\omega = \sqrt{1+\frac{4m^2}{\Delta^2}}$ and Λ is the ultraviolet cutoff on the transverse momentum.

- Gravitational form factor D:

$$\begin{aligned} &\langle p', \boldsymbol{s}' | T^{\mu\nu}(0) | p, \boldsymbol{s} \rangle \\ =&\overline{u}\left(p', \boldsymbol{s}'\right)\left[\frac{P^\mu P^\nu}{M} A(t) + \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M}\left(A+B+D\right)(t) + \frac{\Delta^\mu\Delta^\nu - g^{\nu\nu}\Delta^2}{M} C(t) \right. \\ &\left. + m g^{\mu\nu}\overline{C}(t) + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M}\left(A+B-D\right)(t)\right] u\left(p, \boldsymbol{s}\right), \end{aligned} \tag{50}$$

where M is the nucleon mass, $\boldsymbol{s}, \boldsymbol{s}'$ denotes the rest frame polarization of the initial and the final states respectively, and

$$P = \frac{p' + p}{2}, \quad \Delta = p' - p, \quad t = \Delta^2.$$

- The antisymmetric part of the EMT is related to the divergence of the spin density operator $T^{\mu\nu} - T^{\nu\mu} = -\partial_\mu S^{\mu\alpha\beta}(x)$.

- The matrix element of the quark spin density operator is also parameterized by two form factors

$$\langle p', \boldsymbol{s}' | S_{\mathsf{q}}^{\mu\alpha\beta}(0) | p, \boldsymbol{s} \rangle = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda}\ \overline{u}(p', \boldsymbol{s}') \left[\gamma_\lambda \gamma_5 G_A^q(t) + \frac{\Delta_\lambda \gamma_5}{2M} G_P^q(t) \right] u(p, \boldsymbol{s}), \tag{51}$$

where $G_A^q(t)$ and $G_P^q(t)$ are axial vector and pseudoscalar form factors respectively.

- The antisymmetric part of the quark EMT is given by

$$\overline{\psi}(x) \left[\gamma^\alpha i \overleftrightarrow{D}^\beta - \gamma^\beta i \overleftrightarrow{D}^\alpha \right] \psi(x) = -\epsilon^{\mu\alpha\beta\lambda} \partial_\mu \left[\overline{\psi}(x) \gamma_\lambda \gamma_5 \psi(x) \right], \tag{52}$$

thus

$$D_q(t) = -G_A^q(t). \tag{53}$$

- To extract this form factor we have used the following relation

$$\frac{\langle p', \boldsymbol{s}|S_{\mathsf{q}}^{+jk}(0)|p, \boldsymbol{s}\rangle}{2p^+} = \frac{1}{2}\epsilon^{+jk-} s^z\ G_A^q(t), \tag{54}$$

where we have used the relation $\overline{u}(p', \boldsymbol{s}')\gamma^+\gamma_5 u(p, \boldsymbol{s})$ =4$P^+ s^z$ in Drell-Yan frame with light-front spinors by taking $n_{\mathsf{LF}} = (1, 0, 0, -1)$

- Thus

$$D_q(t) = -\frac{g^2 C_F}{16\pi^2}\int \frac{dx}{1-x}\left[\omega(1+x^2)\log\left(\frac{1+\omega}{-1+\omega}\right) + \left(\frac{1-\omega^2}{\omega}\right) x \log\left(\frac{1+\omega}{-1+\omega}\right)\right.$$
$$\left. - (1+x^2)\log\left(\frac{\Lambda^2}{m^2(1-x)^2}\right)\right], \tag{55}$$

where $\omega = \sqrt{1+\frac{4m^2}{\Delta^2}}$ and $t = \Delta^2$.

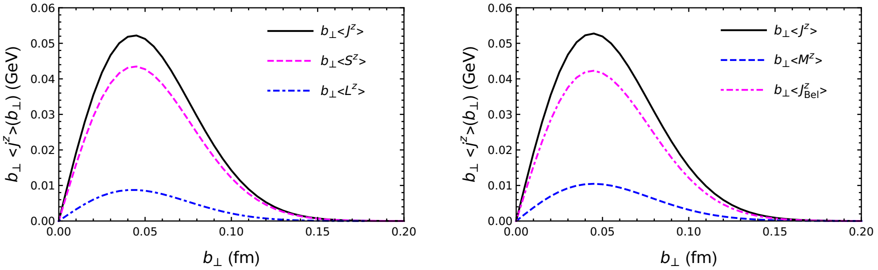

Figure: Plots of longitudinal angular momentum distribution of quarks as a function of impact parameter $b_\perp$. Left: Sum of the kinetic orbital AM $b_\perp \langle L^z \rangle$ (dot-dashed line) and spin AM $b_\perp \langle S^z \rangle$ (dashed line) given by kinetic total AM $b_\perp \langle J^z \rangle$ (solid line). Right: Kinetic total AM $b_\perp \langle J^z \rangle$ (solid line) is given by the sum of Belinfante total AM $b_\perp \langle J^z_{\text{Bel}} \rangle$ (dot-dashed line) and the correction term corresponding to the total divergence $b_\perp \langle M^z \rangle$ (dashed line). Here, $m = 0.3$ GeV, $g = 1$, $C_f = 1$, and $\Lambda = 1.7$ GeV. We choose the Gaussian width $\sigma = 0.1$ GeV. [R. Singh, S. Saha, A. Mukherjee, and N. Mathur (2024). Phys. Rev. D, 109(1):016022]

- Analysis done for proton using the basis light-front quantization spin contribution dominates over OAM.

  [Y. Liu, S. Xu, C. Mondal, X. Zhao, and J P. Vary (2022). Phys. Rev. D, 105(9):094018]

- Correction term has both positive and negative regions in a similar analysis using scalar diquark model.

  [C. Lorcé, L. Mantovani, and B. Pasquini (2018). Phys. Lett. B, 776:38–47]

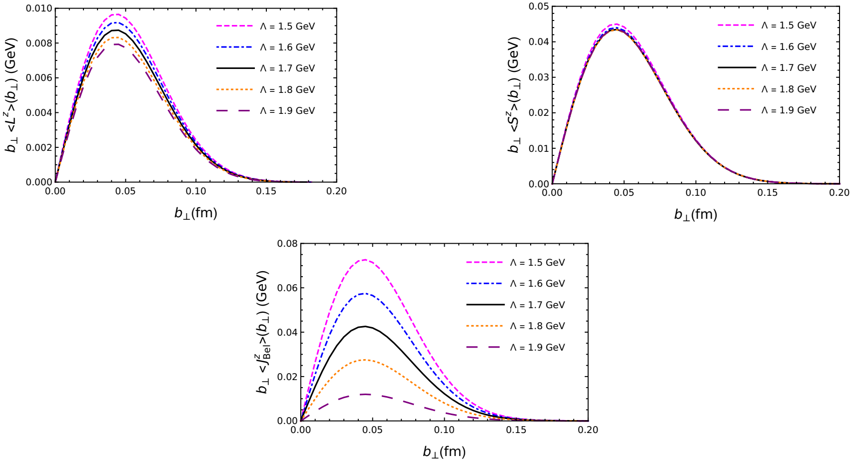

Figure: Plot of the dependence of different components of AM distribution on transverse momentum UV cutoff. Top-left: Variation of $b_\perp\langle L^z_{\text{kin, q}}\rangle$ with Λ. Top-right: Variation of $b_\perp\langle S^z_{\text{kin, q}}\rangle$ with Λ. Bottom: Variation of $b_\perp\langle J^z_{\text{Bel, q}}\rangle$ with Λ. Five different values of Λ are considered for the analysis: $\Lambda = 1.5, 1.6, 1.7, 1.8, 1.9$ GeV. The Gaussian width is $\sigma = 0.1$ GeV. **[R. Singh, S. Saha, A. Mukherjee, and N. Mathur (2024). Phys. Rev. D, 109(1):016022]**

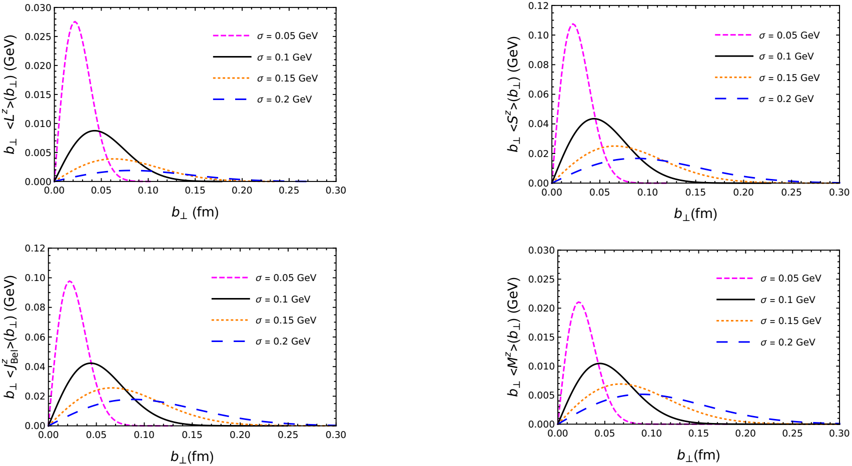

Figure: Plots showing the dependence of different components of AM distribution on the width of the Gaussian wave-packet, σ. Top-left: Variation of $b_\perp \langle L^z_{\text{kin, q}} \rangle$ with σ. Top-right: Variation of $b_\perp \langle S^z_{\text{kin, q}} \rangle$ with σ. Bottom-left: Variation of $b_\perp \langle J^z_{\text{Bel, q}} \rangle$ with σ. Bottom-right: Variation of $b_\perp \langle M^z_{\text{q}} \rangle$ with σ. Four different values of σ are considered for the analysis: $\sigma = 0.05, 0.10, 0.15, 0.20$ GeV. Here, $m = 0.3$ GeV, $\Lambda = 1.7$ GeV, and $g = C_f = N_f = 1$. [R. Singh, S. Saha, A. Mukherjee, and N. Mathur (2024). Phys. Rev. D, 109(1):016022]

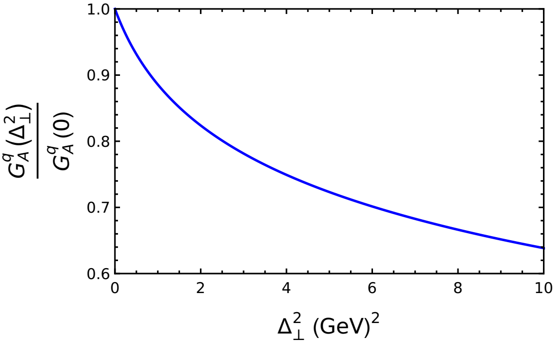

Figure: Plot of the axial vector form-factor $\frac{G_A^q(\Delta_\perp^2)}{G_A^q(0)}$ as a function of $\Delta_\perp^2$. Here $m = 0.3$ GeV, $g = 1$, $\Lambda = 1.7$ GeV and $C_f = 1$. [R. Singh, S. Saha, A. Mukherjee, and N. Mathur (2024). Phys. Rev. D, 109(1):016022]

- $D_q(\Delta_\perp^2) = -G_A^q(\Delta_\perp^2)$, $G_A^q(0) = 0.36$
- Qualitatively behaviour is similar to other model-based and lattice calculations.

  [A. Silva, H C. Kim, D. Urbano, and K. Goeke (2005). Phys. Rev. D, 72:094011], [Y C. Jang, R. Gupta, B. Yoon, and T. Bhattacharya (2020). Phys. Rev. Lett., 124(7):072002]

Conclusions

- We have calculated the spatial distribution of angular momentum for different decompositions for a relativistic spin-$\frac{1}{2}$ state, namely quark dressed with a gluon.
- We have calculated the gravitational form factor D related to the antisymmetric part of the energy-momentum tensor.
- We have calculated the superpotential term at the density level in the dressed quark state, which is responsible for the disparity between the total angular momentum density of Ji and Belinfante decomposition.
- The potential angular momentum which relates to the Jaffe-Manohar and Ji decomposition vanishes at one loop level in the dressed quark state.
- Decomposition of Wakamatsu and Chen *et. al.* gives the same result as Ji and Jaffe-Manohar.
- We have taken only the quark part of the EMT, we are calculating the gluon part contribution to these observables.

Thank you!