

LEPTON FLAVOUR VIOLATION (LECTURES 1 & 2)

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Future flavours: prospects for beauty, charm, and tau physics, ICTS, May 2 2022

OUTLINE

- lecture 1: lepton flavor in the SM
- lecture 2: LFV observables in muons
- lecture 3: LFV in taus; Higgs and flavor
- lecture 4: LFV searches and light new physics

OUTLINE LECTURES 1 & 2

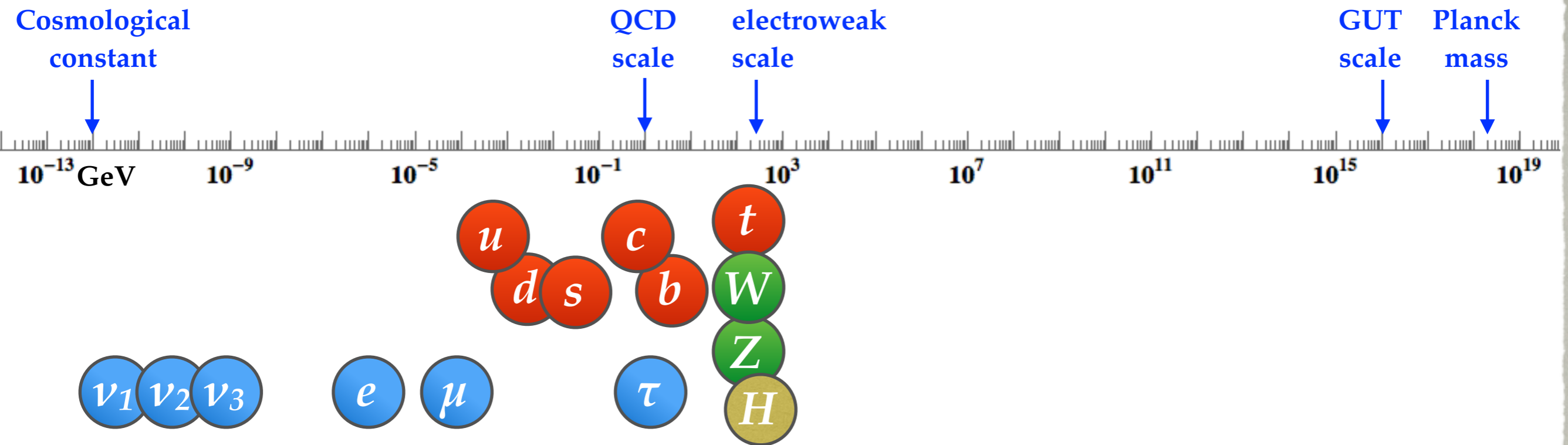
- in lectures 1 & 2:
 - leptonic vs. quark flavor structure in the standard model
 - observables sensitive to lepton flavor violation in muons

USEFUL REFERENCES

- some general introductions to flavor physics
 - Yuval Grossman's lectures in this school
 - Nir, 0708.1872, 1605.00433
 - Grossman, Tanedo, 1711.03624
 - JZ, 1903.05062
 - ...
- on lepton flavor violation
 - Calibbi, Sirognelli, 1709.00294
 - Ardu, Pezzullo, 2204.08220

QUARKS VS. LEPTONS

- when comparing quark and lepton sector of the Standard Model we observe:
- leptons of the same generation are lighter than quarks
 - smaller number of kinematically allowed decay modes for τ , μ than for t, b, c
 - *e.g.*, $B^- \rightarrow \tau^- \bar{\nu}_\tau$ allowed, while $\tau^- \rightarrow B^- \nu_\tau$ is not
- quarks carry color \Rightarrow bound inside hadrons
 - lepton decays are simpler to predict
- "up" leptons' ($=\nu$'s) mass \ll "down" leptons' ($=\ell$ ') mass
 - absolute neutrino masses not yet known
 - in many processes neutrino masses can be neglected



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QUARKS

- in the SM the flavor structure resides in the Yukawa interactions

$$\mathcal{L}_{\text{Yukawa}} \supset -Y_d^{ij} \bar{Q}_L^i H d_R^j - Y_u^{ij} \bar{Q}_L^i H^c u_R^j + \text{h.c.}$$

- $Y_{u,d}$ are 3×3 complex matrices
 - each can be diagonalized by a bi-unitary transform.
 - but not both at the same time (i.e. both can be made diagonal only after EWSB when we split Q_L into upper and lower components)

- keeping $SU(2)_L$ symmetry explicit perform quark field redefinitions

$$Q'_L = V_Q Q_L \quad u'_R = V_u u_R \quad d'_R = V_d d_R$$

- so that the Yukawa matrices transform to

$$Y'_u = V_Q^\dagger Y_u V_u \quad Y'_d = V_Q^\dagger Y_d V_d$$

- can choose a basis such that (down quark mass basis)

$$Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_u = V^\dagger \text{diag}(y_u, y_c, y_t)$$

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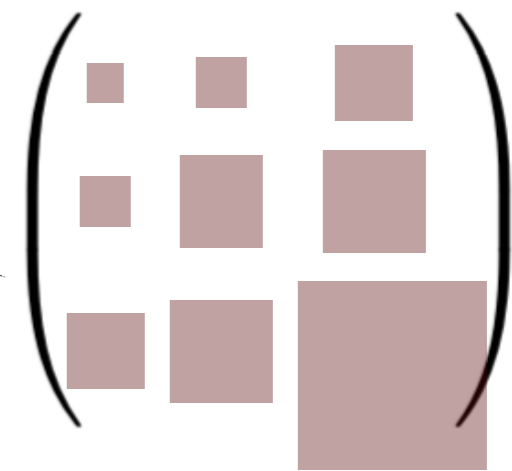
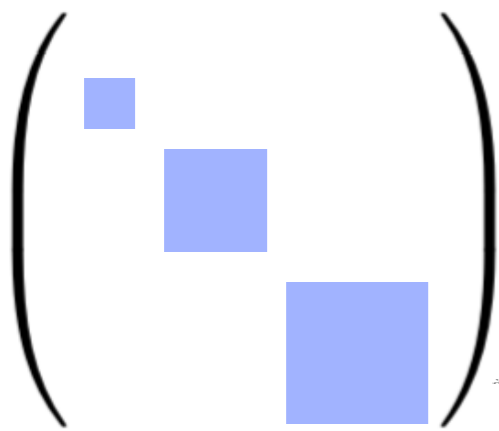
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CKM matrix

QUARKS

- quark Yukawa interactions in down mass basis

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$$Y_d = \text{diag}(y_d, y_s, y_b), \quad Y_u = V^\dagger \text{diag}(y_u, y_c, y_t)$$

- can move flavor changing interactions to kinetic term by field redefinition

$$Q_L \rightarrow \begin{pmatrix} V^\dagger u_L \\ d_L \end{pmatrix},$$

$$\mathcal{M}_q = Y_q \frac{(v+h)}{\sqrt{2}}.$$

$$H = \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix}$$

- in the so-called mass basis

$$\mathcal{L}_{\text{SM}} \supset (\bar{q}_i \not{D}_{\text{NC}} q_i) + \frac{g}{\sqrt{2}} \bar{u}_L^i W^+ V_{\text{CKM}}^{ij} d_L^j + m_{u_i} \bar{u}_L^i u_R^i \left(1 + \frac{h}{v}\right) + m_{d_i} \bar{d}_L^i d_R^i \left(1 + \frac{h}{v}\right) + \text{h.c.},$$

QUARKS

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- all flavor violation in charged currents (couplings to W^\pm)
- neutral currents are flavor conserving
 - no flavor viol. in Z couplings
 - couplings to Higgs are flavor diagonal

CKM MATRIX

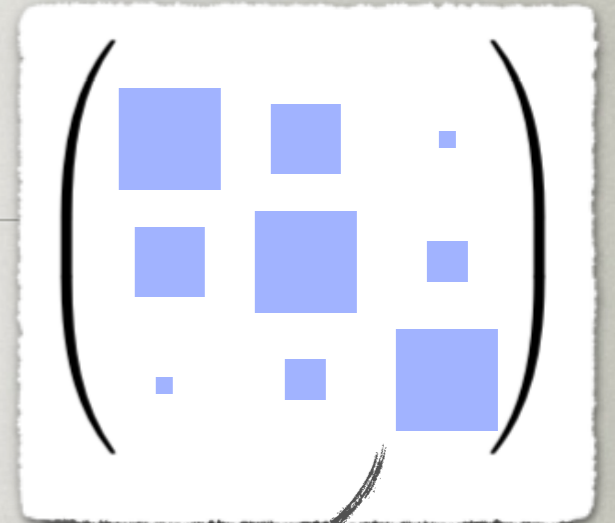
- hierarchical structure + unitarity
 - encoded in Wolfenstein parametrization

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$
$$\lambda \equiv |V_{us}| \simeq 0.22$$

- CKM matrix depends on 3 real params, 1 phase
 - 3 mixing angles, 1 phase
 - in Wolfenstein param. trade for
 - 3 real params: $\lambda, A, \rho,$
 - 1 imag. param: η

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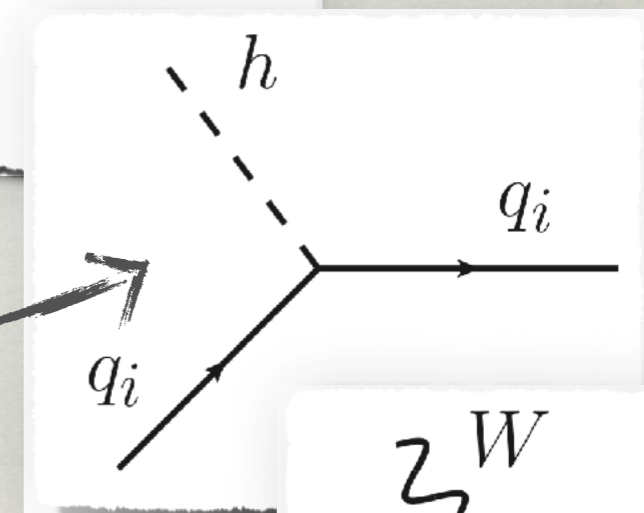
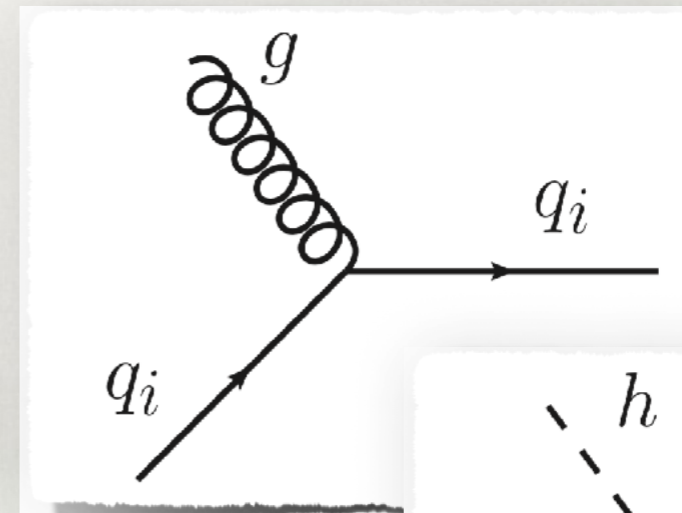
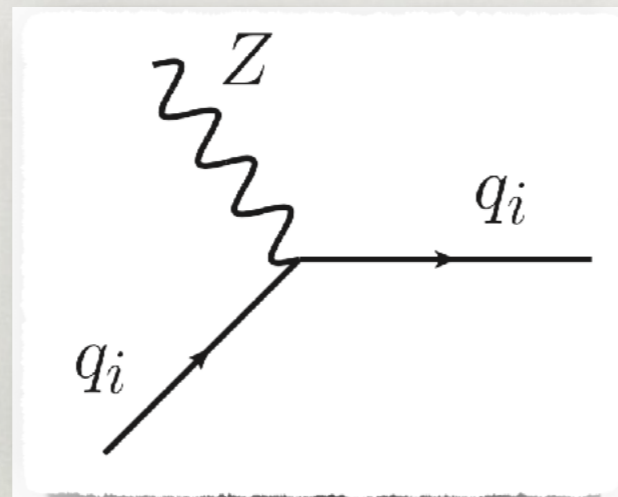
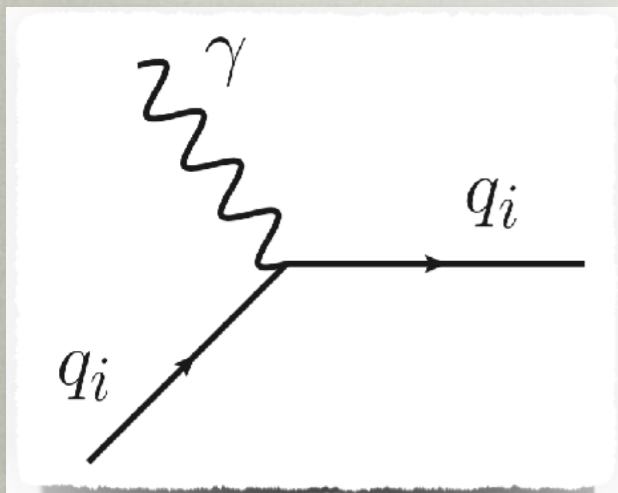
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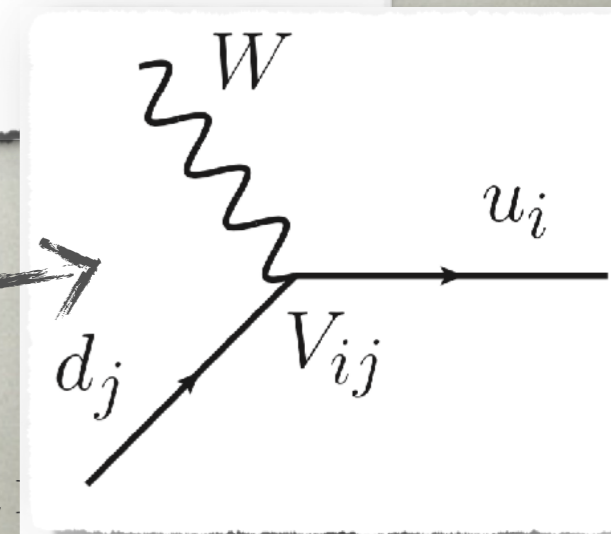
FLAVOR IN THE SM

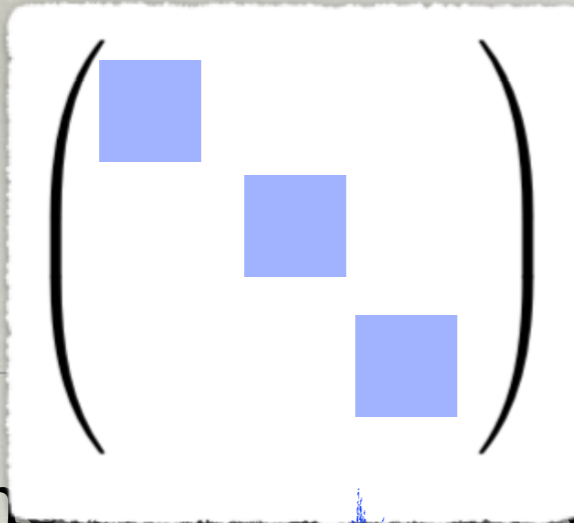
QUARK SECTOR

- neutral currents are flavor conserving (at tree level)
 - photon, gluon, Z: have *flavor (generation) universal* interactions



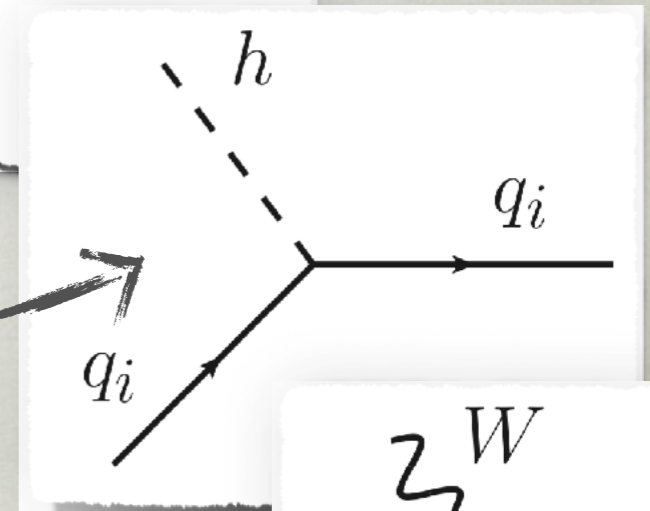
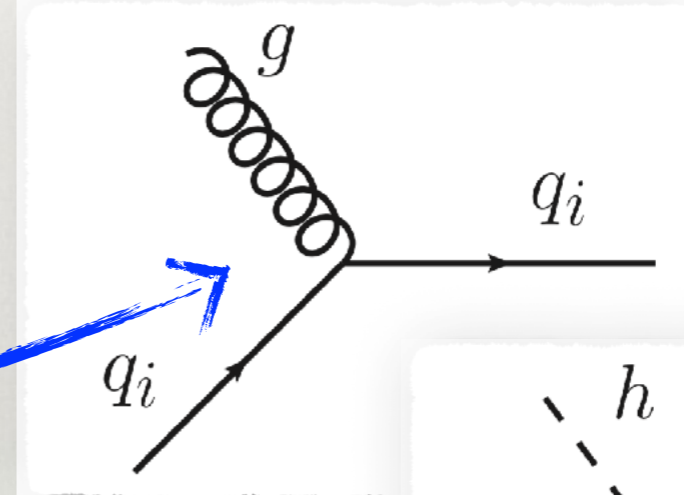
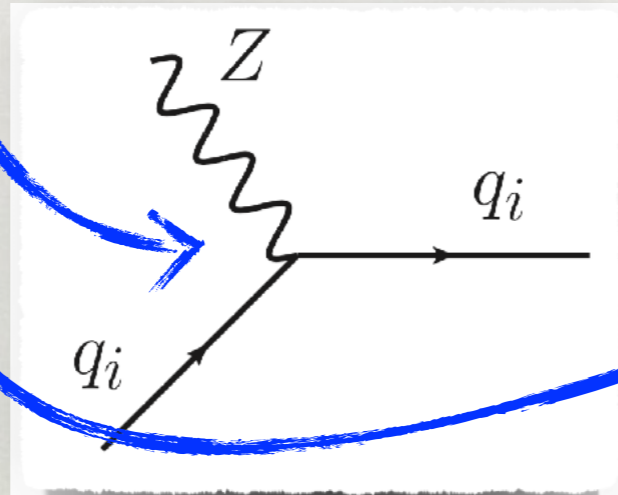
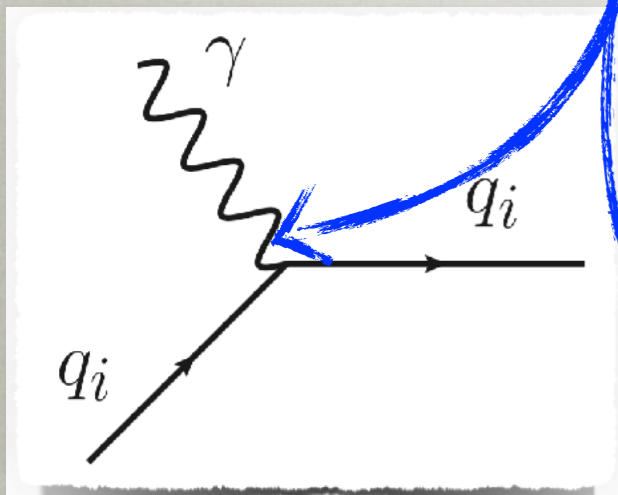
- Higgs has *flavor diagonal* interactions
 - proportional to quark mass
- charged currents are *flavor changing*
 - W couplings are flavor changing



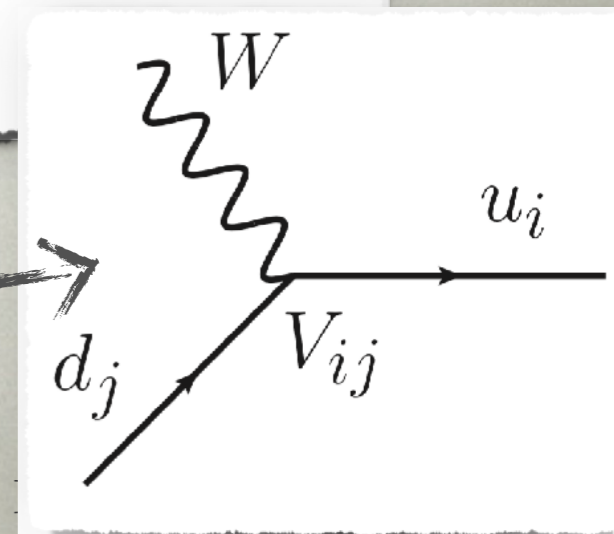


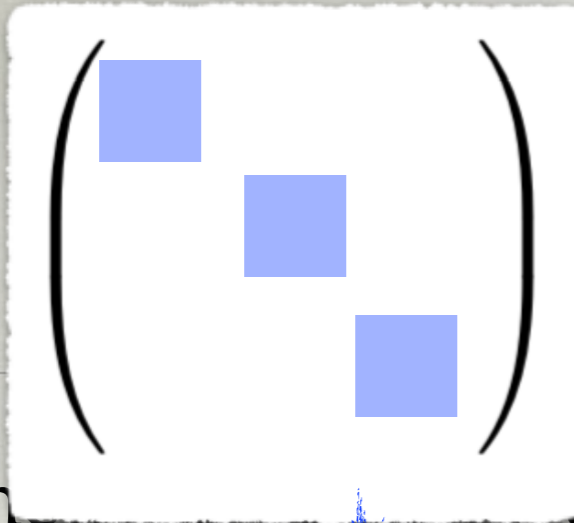
FLAVOR IN THE SM QUARK SECTOR

- n are flavor conserving (at tree level)
 - photon, gluon, Z: have *flavor (generation) universal* interactions

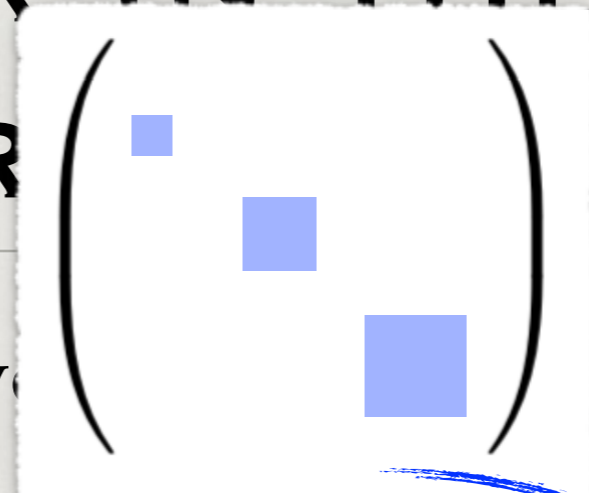


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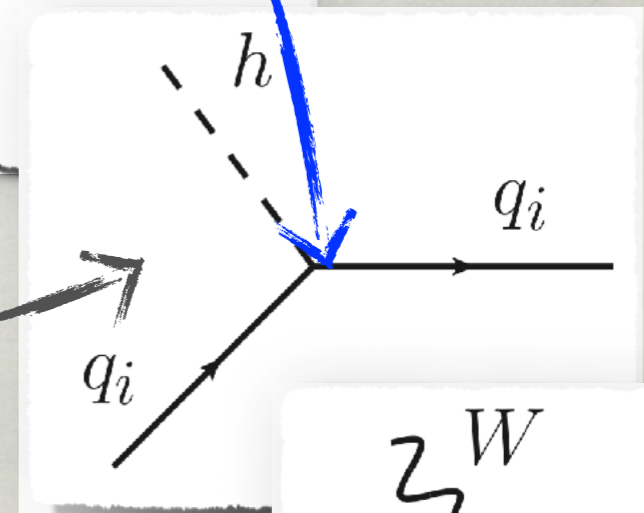
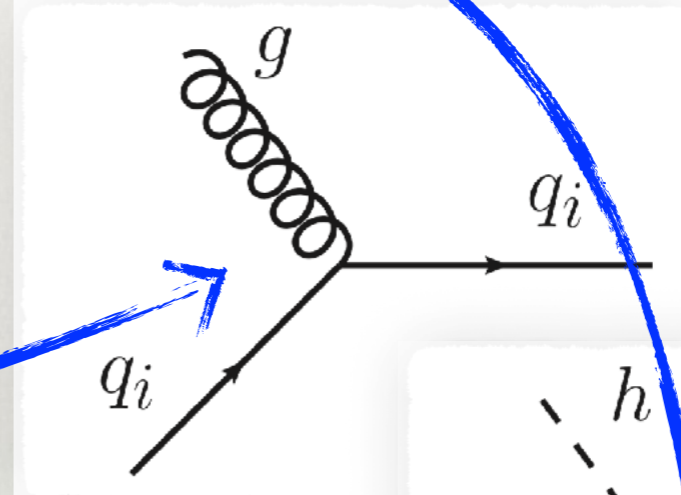
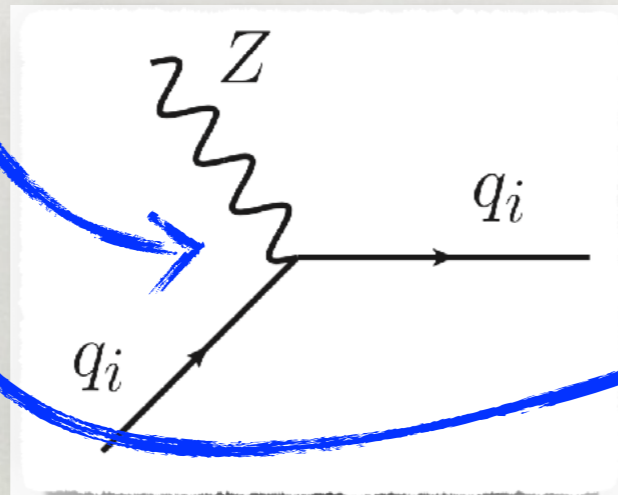
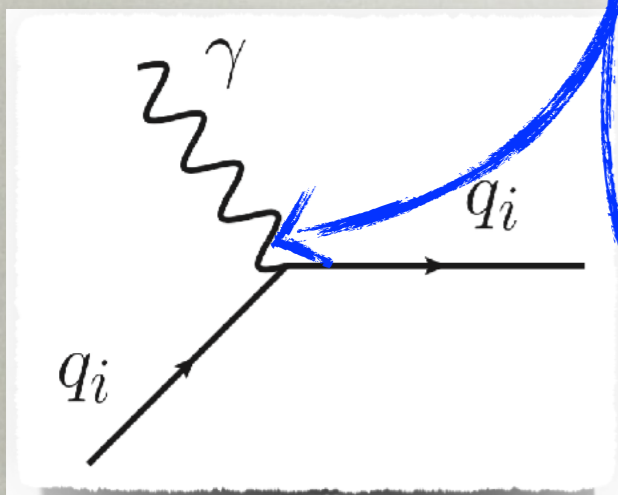




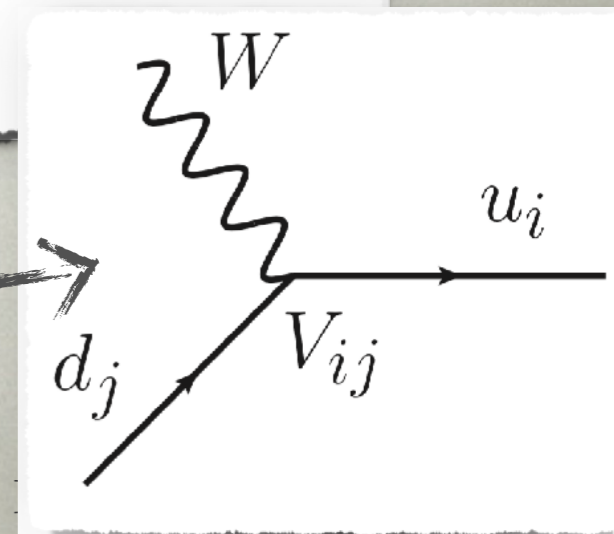
FLAVOR IN THE SM



- neutrinos are flavor (generation) universal (at tree level)
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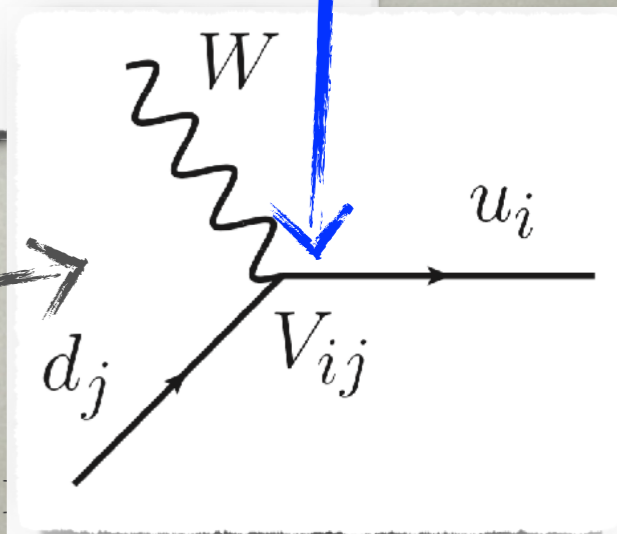
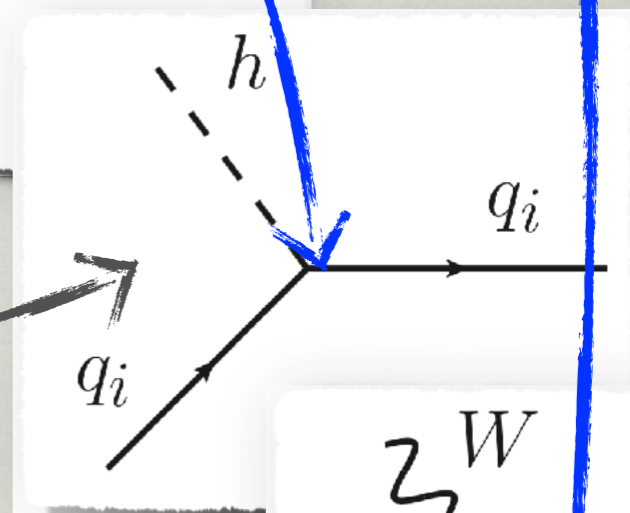
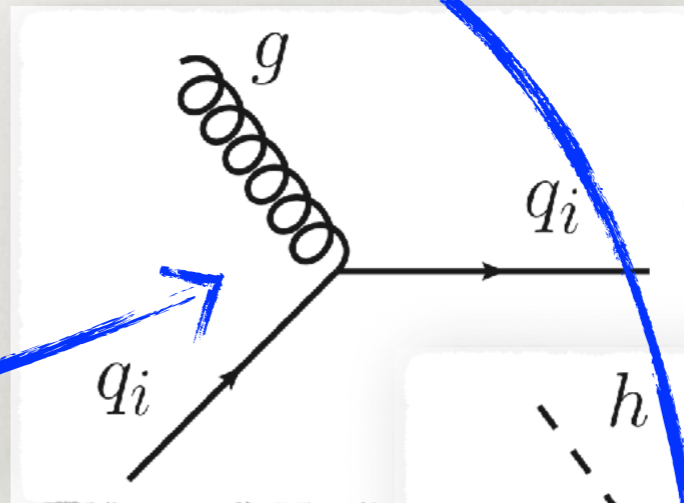
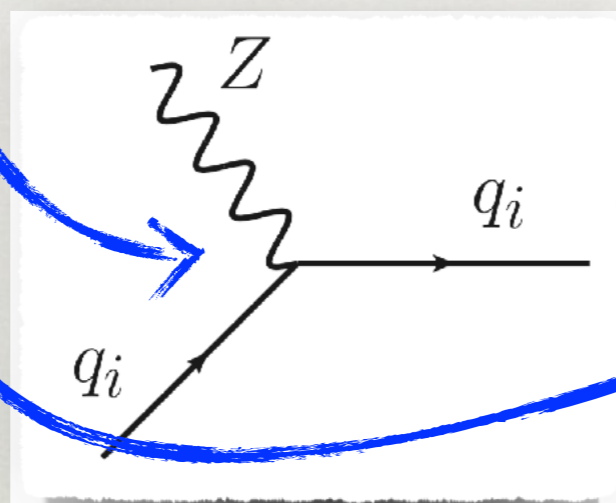
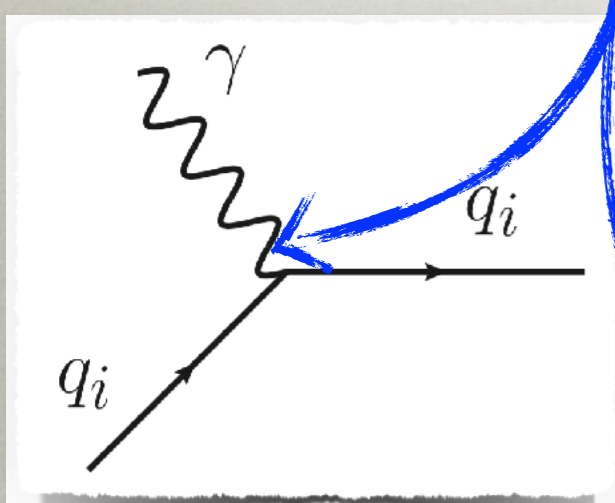




FLAVOR IN THE SM

FLAVOR DIAGONAL OR

- neutrinos are flavor changing at tree level
- photon, gluon, Z: have *flavor (generation) universal* interactions



- Higgs has *flavor diagonal* interactions
 - proportional to quark mass
- charged currents are *flavor changing*
 - W couplings are flavor changing

LEPTONS

- first assume that neutrino masses are zero
- extremely good approximation in
 - collider experiments, meson decays, charged lepton decays,...
 - in each of these: $E \gg m_\nu$

LEPTONS

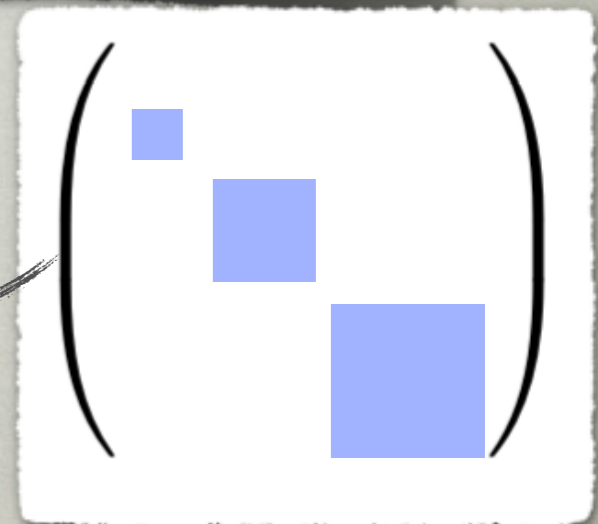
- in the limits of vanishing ν masses
 - a single lepton Yukawa
 - through field redefinitions can be made diagonal, real, positive

$$\mathcal{L}_{\text{Yukawa}} \supset -Y_\ell^{ij} \bar{L}_L^i H \ell_R^j + \text{h.c.}$$

$$L_L \rightarrow V_L L_L, \quad \ell_R \rightarrow V_\ell \ell_R,$$

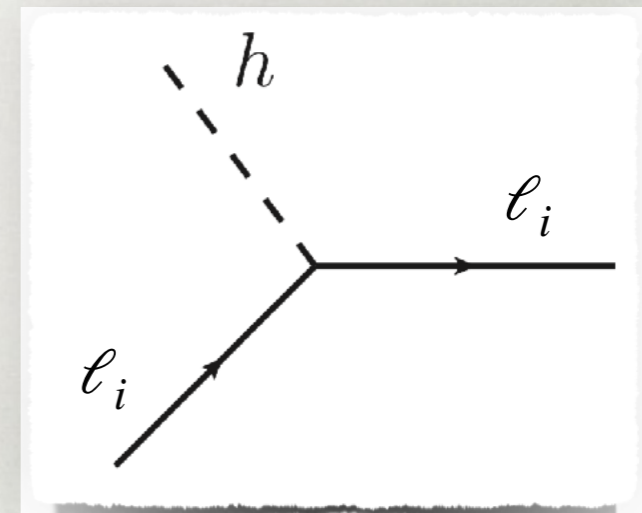
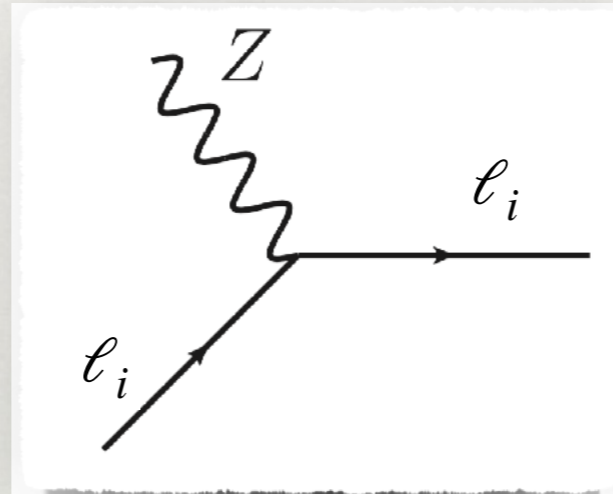
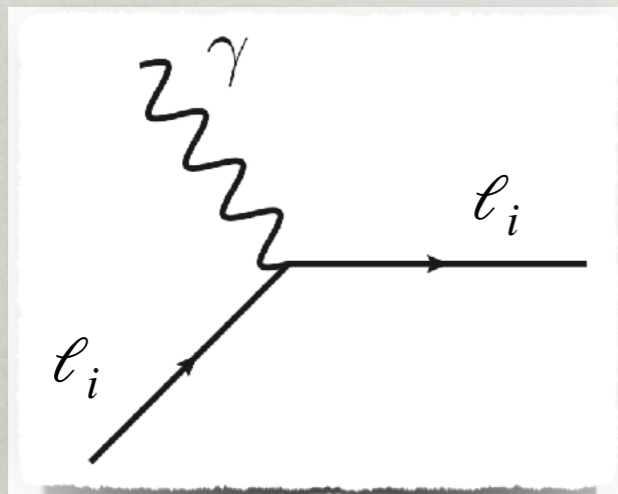
$$Y_\ell \rightarrow V_L^\dagger Y_\ell V_\ell = \text{diag}(y_e, y_\mu, y_\tau).$$

- since Y_ℓ diagonal: no FV unless $m_\nu \neq 0$
- since Y_ℓ real: no CPV unless $m_\nu \neq 0$

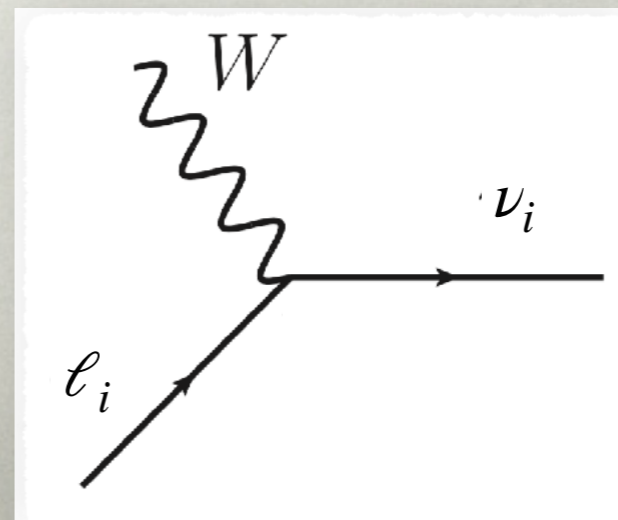


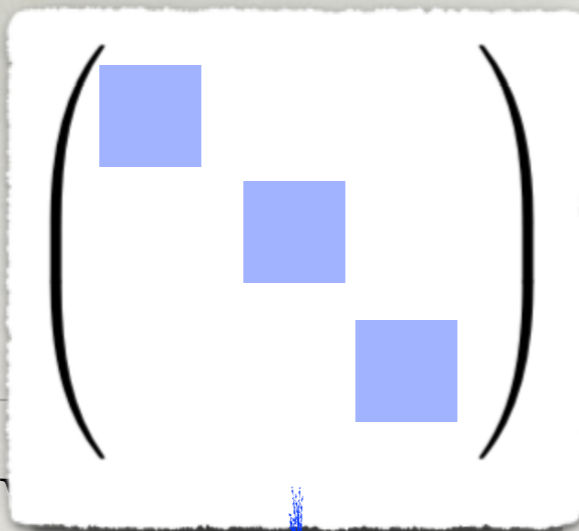
LEPTONS

- \Rightarrow in SM with massless ν no leptonic FCNCs
 - photon, Z : *flavor (generation) universal* interactions



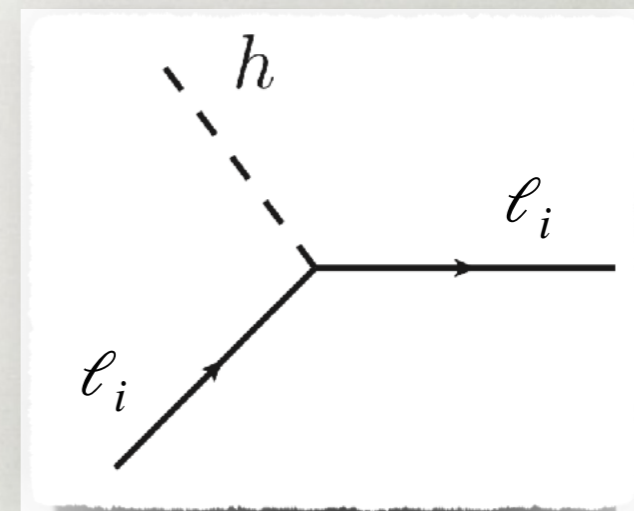
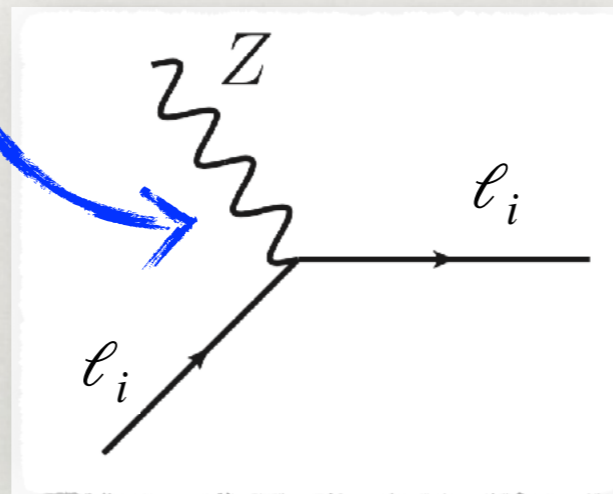
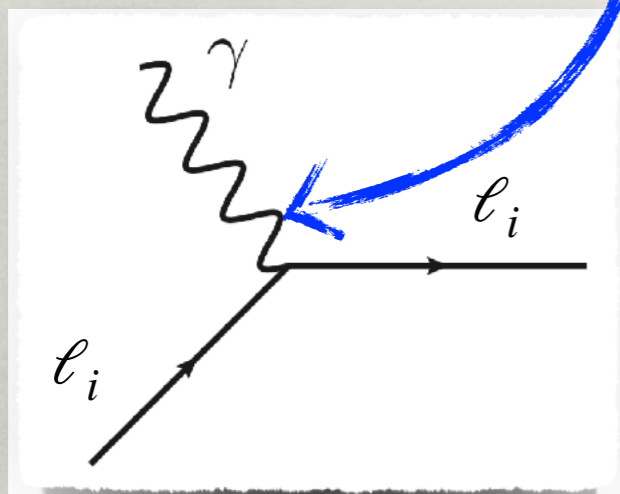
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 - proportional to lepton masses
- charged currents (W couplings) are *flavor universal*



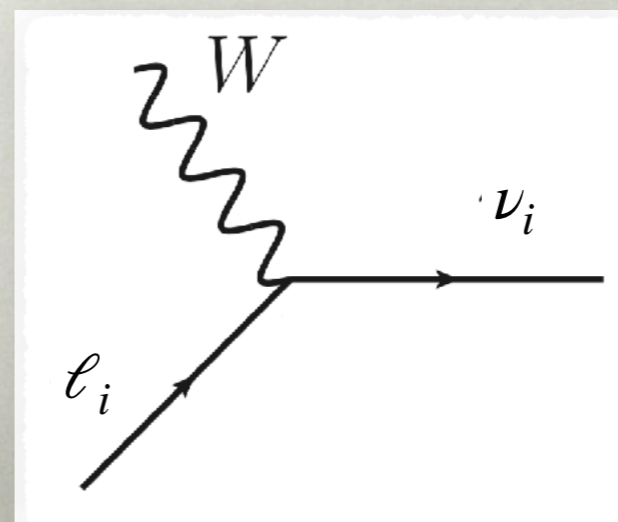


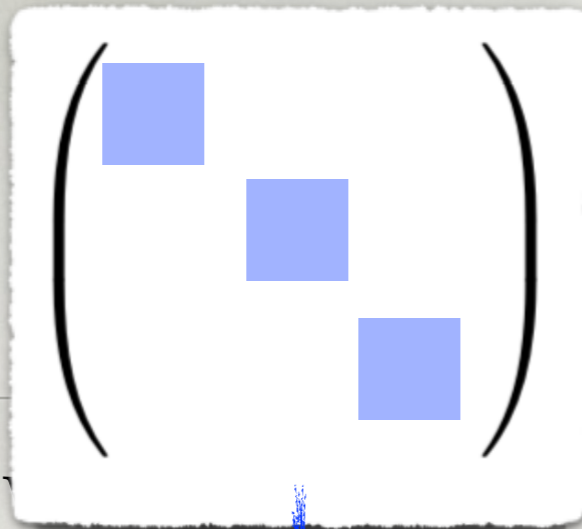
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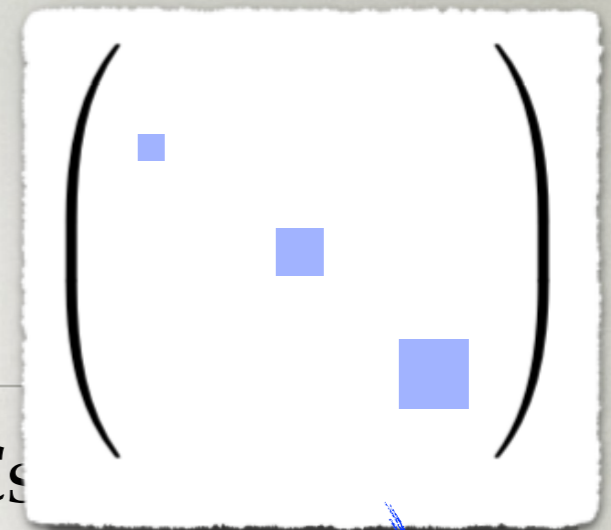


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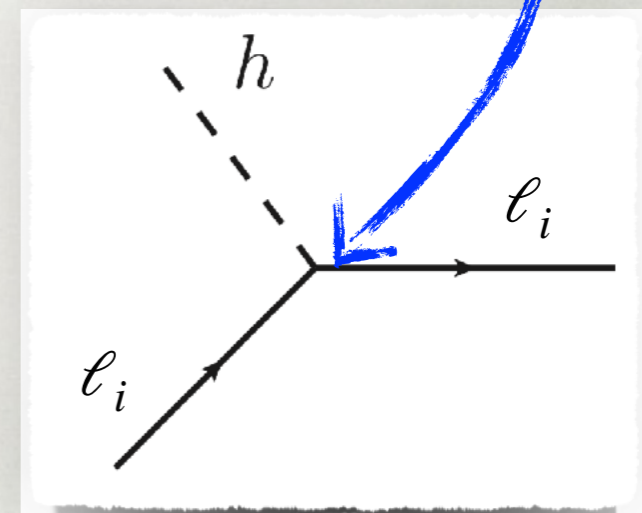
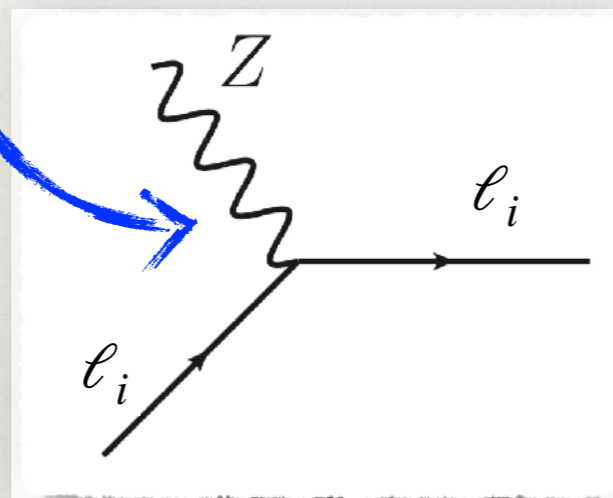
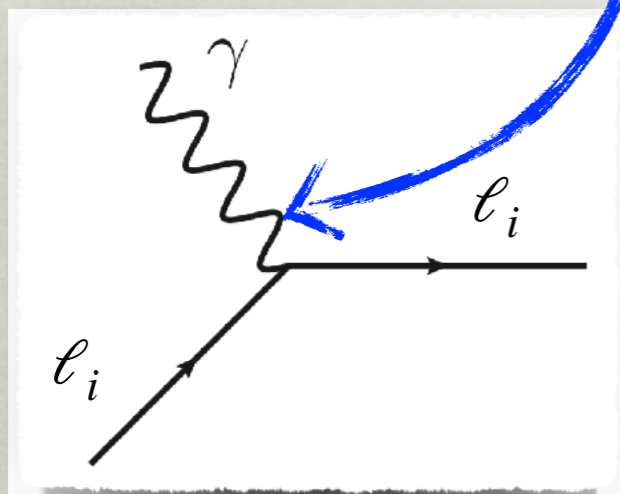




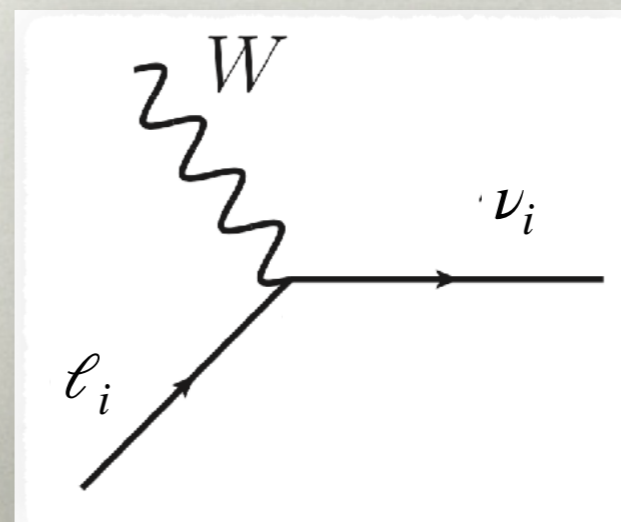
LEPTONS

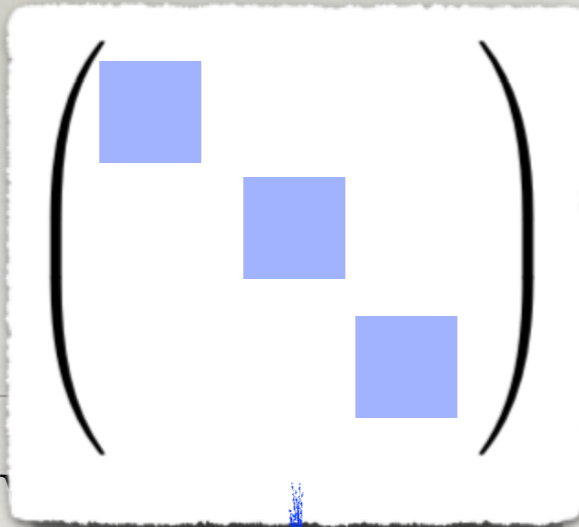


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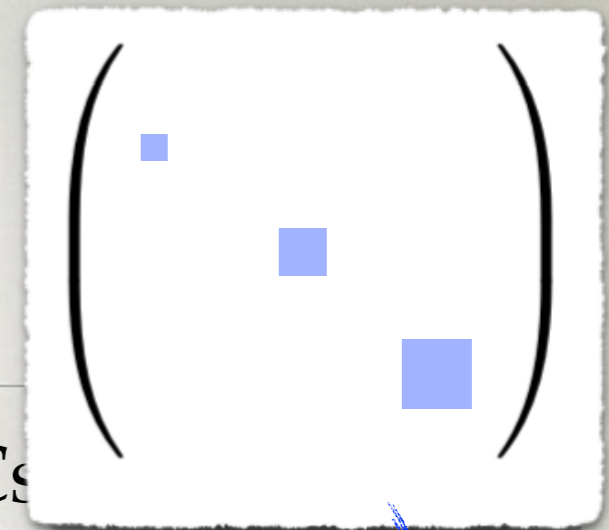


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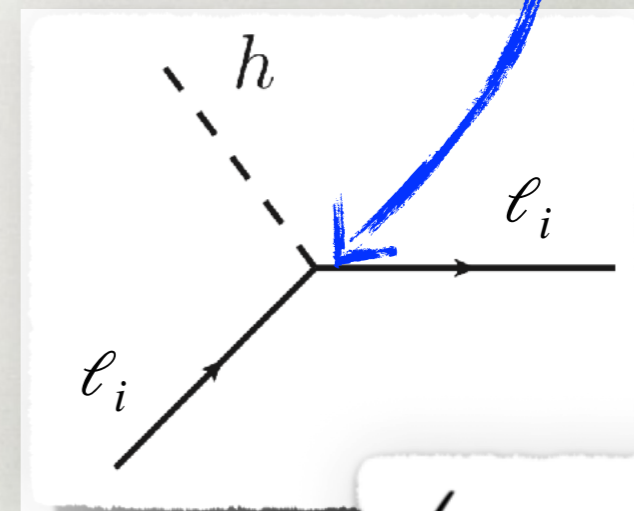
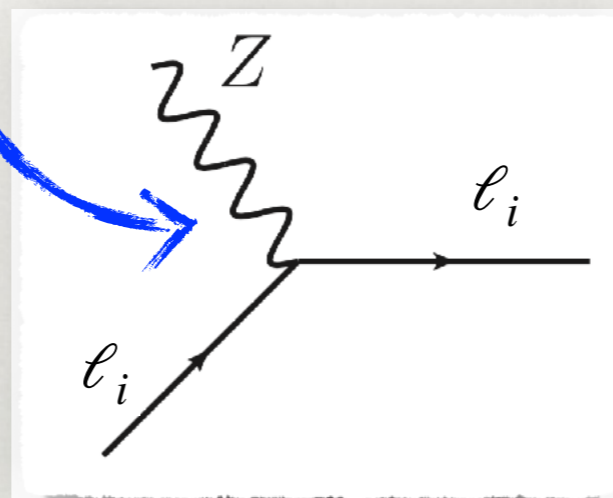
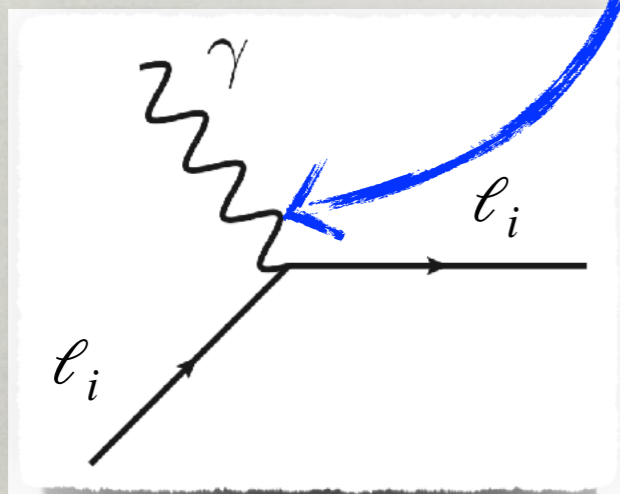




LEPTONS

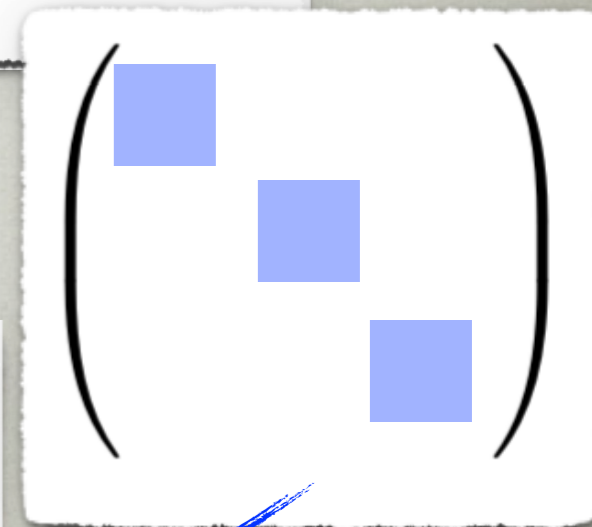
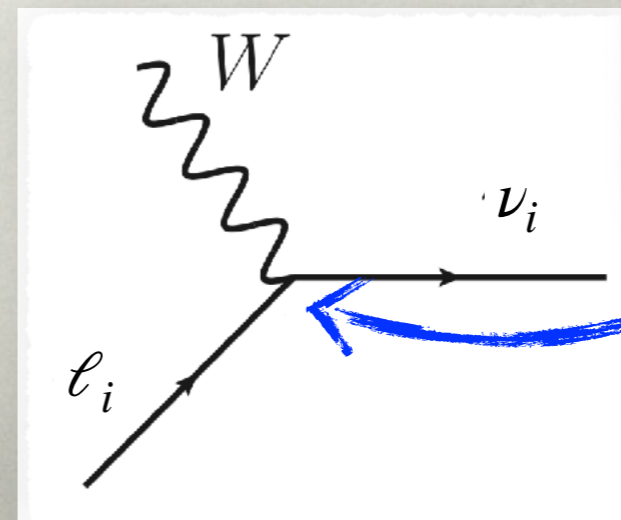


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LEPTONS

- this means that for $m_\nu = 0$ in the SM
 - $Br(\mu^+ \rightarrow e^+ e^- e^+) = 0$
 - $Br(\mu^+ \rightarrow e^+ \gamma) = 0$
 - $Br(\tau^+ \rightarrow \mu^+ \mu^- \mu^+) = 0$
 - $Br(\tau^+ \rightarrow \mu^+ \rho^0) = 0$
 - ...

LEPTONS

- agrees well with stringent experimental bounds in PDG
 - $Br(\mu^+ \rightarrow e^+e^-e^+) < 1.0 \times 10^{-12}$
 - $Br(\mu^+ \rightarrow e^+\gamma) < 4.2 \times 10^{-13}$
 - $Br(\tau^+ \rightarrow \mu^+\mu^-\mu^+) < 2.1 \times 10^{-8}$
 - $Br(\tau^+ \rightarrow \mu^+\rho^0) < 1.2 \times 10^{-8}$
 - ...

NEUTRINO MASSES

- however, neutrinos are not completely massless
 - at some level leptonic FCNCs will arise in the SM
- how much does $m_\nu \neq 0$ matter?
- in experiments we are interested in: not too much
 - corrections suppressed by $(m_\nu/E)^n \ll 1$
 - for instance for muon decays:
$$E \sim m_\mu \Rightarrow m_\nu/m_\mu < 10^{-9}$$

NEUTRINO MASSES

- with $QUDL$ field content m_ν forbidden in the SM
- two ways of introducing ν masses
 - *Dirac neutrinos*: add RH neutrino fields ν_R , singlets under SM + conserv. L

3 × 3 complex

$$\mathcal{L}_{\text{Yukawa}} \supset -Y_\nu^{ij} \bar{L}_L^i H^c \nu_R^j + \text{h.c.}$$

- *Majorana neutrinos*: m_ν from dimension 5 Weinberg operator, is $\Delta L = 2$

3 × 3 symm., complex

$$\mathcal{L}_{\text{dim. 5}} \supset -\frac{1}{2} \frac{Y'_\nu{}^{ij}}{\Lambda} (\bar{L}_L^{ci} H^c) (H^{c*} L_L^j) + \text{h.c.}$$

- counting of physical parameters slightly differs in the two cases
 - in both cases weak (flavor) eigenstates are linear superpositions of mass eigenstates

$$\nu_{aL} = \sum_{i=1}^3 U_{ai} \nu_{iL}, \quad a = e, \mu, \tau$$

PMNS matrix

PMNS MATRIX

- canonical form of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix

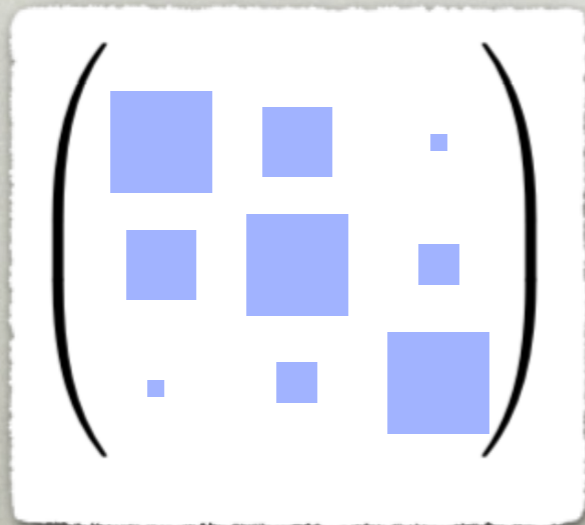
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times P$$

- P matrix takes the form:
 - $P = 1$ for Dirac neutrinos
 - $P = \text{diag}(1, e^{i\alpha_{21}}, e^{i\alpha_{31}})$ for Majorana ν 's

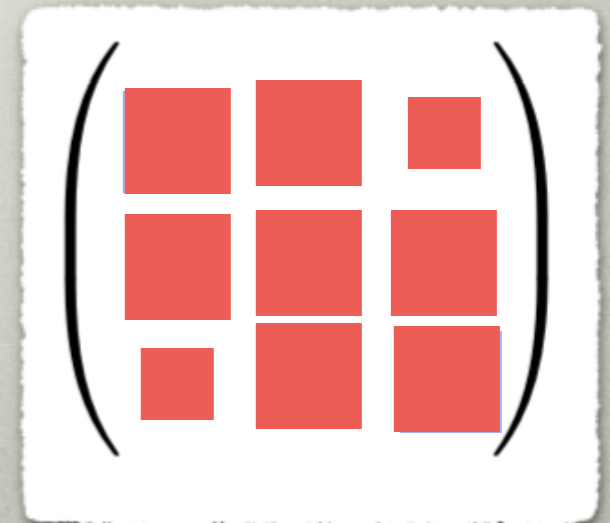
PMNS MATRIX

- assuming "normal ordering": $m_3 > m_2 > m_1$
 $m_2^2 - m_1^2 \sim (10^{-3} \text{ eV})^2$
 $m_3^2 - m_1^2 \sim (0.05 \text{ eV})^2$
 $\sin \theta_{12} \sim \sin \theta_{23} \sim 0.5, \sin \theta_{13} \sim 0.15$
 $\delta, \alpha_{12}, \alpha_{13} = ?$

CKM
matrix



PMNS
matrix



$\mu \rightarrow e\gamma$ IN THE SM

- we already know that $\mu \rightarrow e\gamma$ vanishes for massless neutrinos
 - GIM mechanism very effective in LFV transitions
 - amplitude proportional to $A(\mu \rightarrow e\gamma) \propto m_\nu^2$

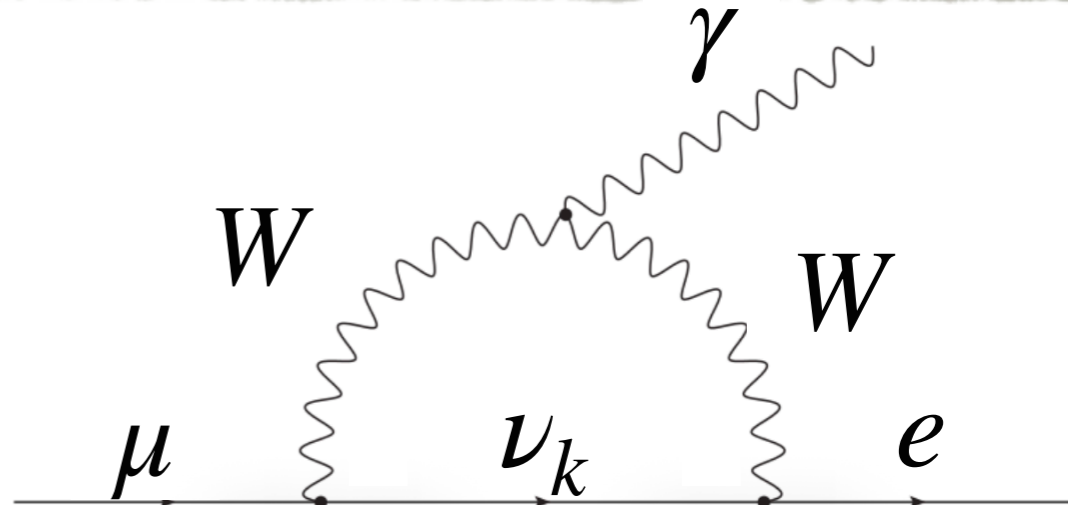
Very small !!!

$$\text{BR}(\mu \rightarrow e\gamma) \simeq \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \frac{U_{\mu k} U_{ek}^* m_{\nu k}^2}{M_W^2} \right|^2.$$

$$\text{BR}(\mu \rightarrow e\gamma) = 10^{-55} \div 10^{-54}$$

- similar suppressions for $\mu \rightarrow 3e, \tau \rightarrow 3\mu, \mu \rightarrow e, \dots$
- for charged LFV transitions SM is well below experimental reach
 - if found, a clear signal of new physics

$\mu \rightarrow e\gamma$ IN T



- we already know that $\mu \rightarrow e\gamma$ vanishes to $U_{\mu k}$ unless U_{ek}^* is non-zero
- GIM mechanism very effective in LFV transitions
- amplitude proportional to $A(\mu \rightarrow e\gamma) \propto m_{\nu}^2$

Very small !!!

$$\text{BR}(\mu \rightarrow e\gamma) \simeq \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \frac{U_{\mu k} U_{ek}^* m_{\nu k}^2}{M_W^2} \right|^2.$$

$$\text{BR}(\mu \rightarrow e\gamma) = 10^{-55} \div 10^{-54}$$

- similar suppressions for $\mu \rightarrow 3e, \tau \rightarrow 3\mu, \mu \rightarrow e, \dots$
- for charged LFV transitions SM is well below experimental reach
 - if found, a clear signal of new physics

NEW PHYSICS: SEE SAW EXAMPLE

- a simple example of new physics probed by $\mu \rightarrow e\gamma$
- a see-saw model for neutrino masses
 - allow for Majorana mass term for ν_R

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + i\bar{\nu}_R \not{\partial} \nu_R - \left(Y_\nu \bar{\nu}_R \tilde{\Phi}^\dagger L_L + \frac{1}{2} M_R \bar{\nu}_R \nu_R^c + \text{h.c.} \right).$$

Dirac mass term \Rightarrow mixing of ν_L and ν_R

- mass spectrum consists of Majorana neutrinos
 - 3 heavy states, mostly ν_R with masses $\sim M_R$
 - 3 light neutrinos, mostly ν_L , mass matrix

$$m_\nu = -\frac{v^2}{2} Y_\nu^T M_R^{-1} Y_\nu$$

SEE SAW AND $\mu \rightarrow e\gamma$

- due to ν_L and ν_R mixing
 - PMNS matr. does not diagonalize the full $\nu_{L,R}$ mass matrix
 - the mixing matrix \mathcal{U} entering the $W - \ell - \nu$ vertex is not unitary

$$\mathcal{U} = \left(1 - \frac{v^2}{2} Y_\nu^\dagger M_R^{-2} Y_\nu \right) U.$$

note: in m_ν we have Y_ν^T not Y_ν^\dagger

- modified prediction for $\mu \rightarrow e\gamma$

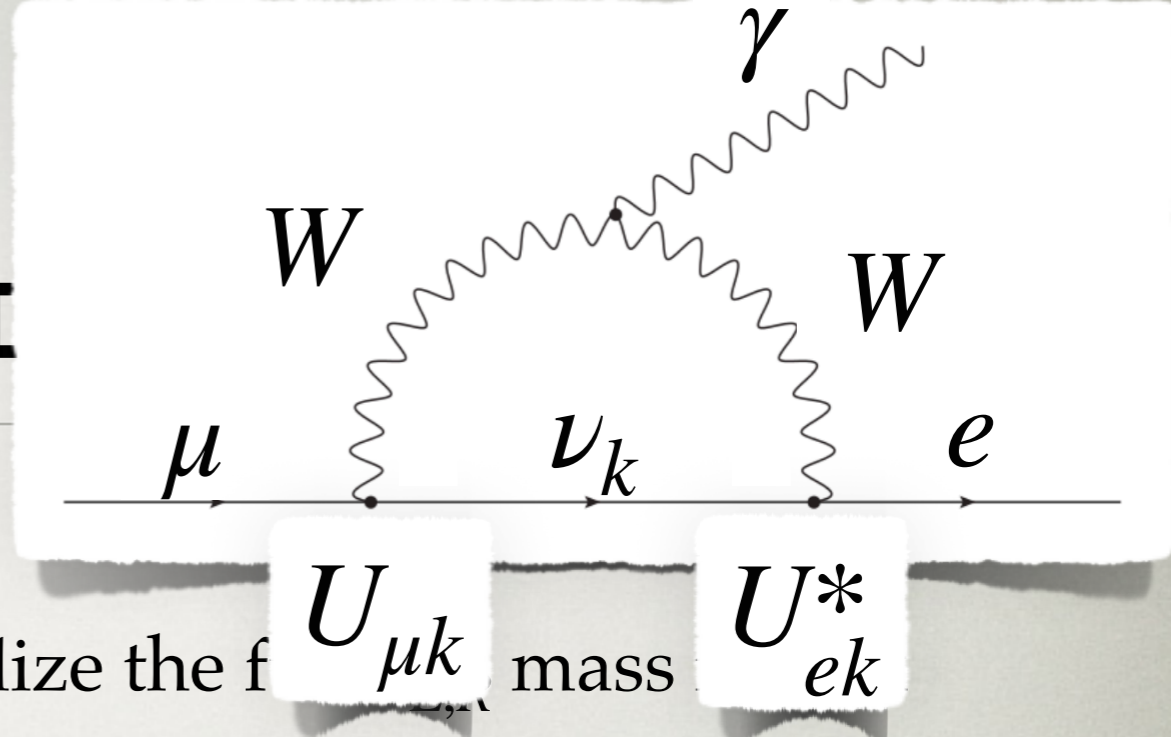
$$\text{BR}(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \frac{|\sum_k \mathcal{U}_{\mu k} \mathcal{U}_{ek}^* F(x_k)|^2}{(\mathcal{U}\mathcal{U}^\dagger)_{\mu\mu} (\mathcal{U}\mathcal{U}^\dagger)_{ee}},$$

$$F(x_k) = \frac{10}{3} - x_k + \mathcal{O}(x_k^2).$$

$$x_k = m_{\nu_k}^2 / M_W^2$$

- GIM mechanism no longer fully operational
- $Br(\mu \rightarrow e\gamma)$ not suppressed by light ν masses, can be larger

SEE SAW ANI



- due to ν_L and ν_R mixing
 - PMNS matr. does not diagonalize the f mass
 - the mixing matrix \mathcal{U} entering the $W - \ell - \nu$ vertex is not unitary

$$U = \left(1 - \frac{v^2}{2} Y_\nu^\dagger M_R^{-2} Y_\nu \right) U.$$

note: in m_ν we have Y_ν^T not Y_ν^\dagger

- modified prediction for $\mu \rightarrow e\gamma$

$$\text{BR}(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \frac{|\sum_k \mathcal{U}_{\mu k} \mathcal{U}_{ek}^* F(x_k)|^2}{(\mathcal{U}\mathcal{U}^\dagger)_{\mu\mu} (\mathcal{U}\mathcal{U}^\dagger)_{ee}},$$

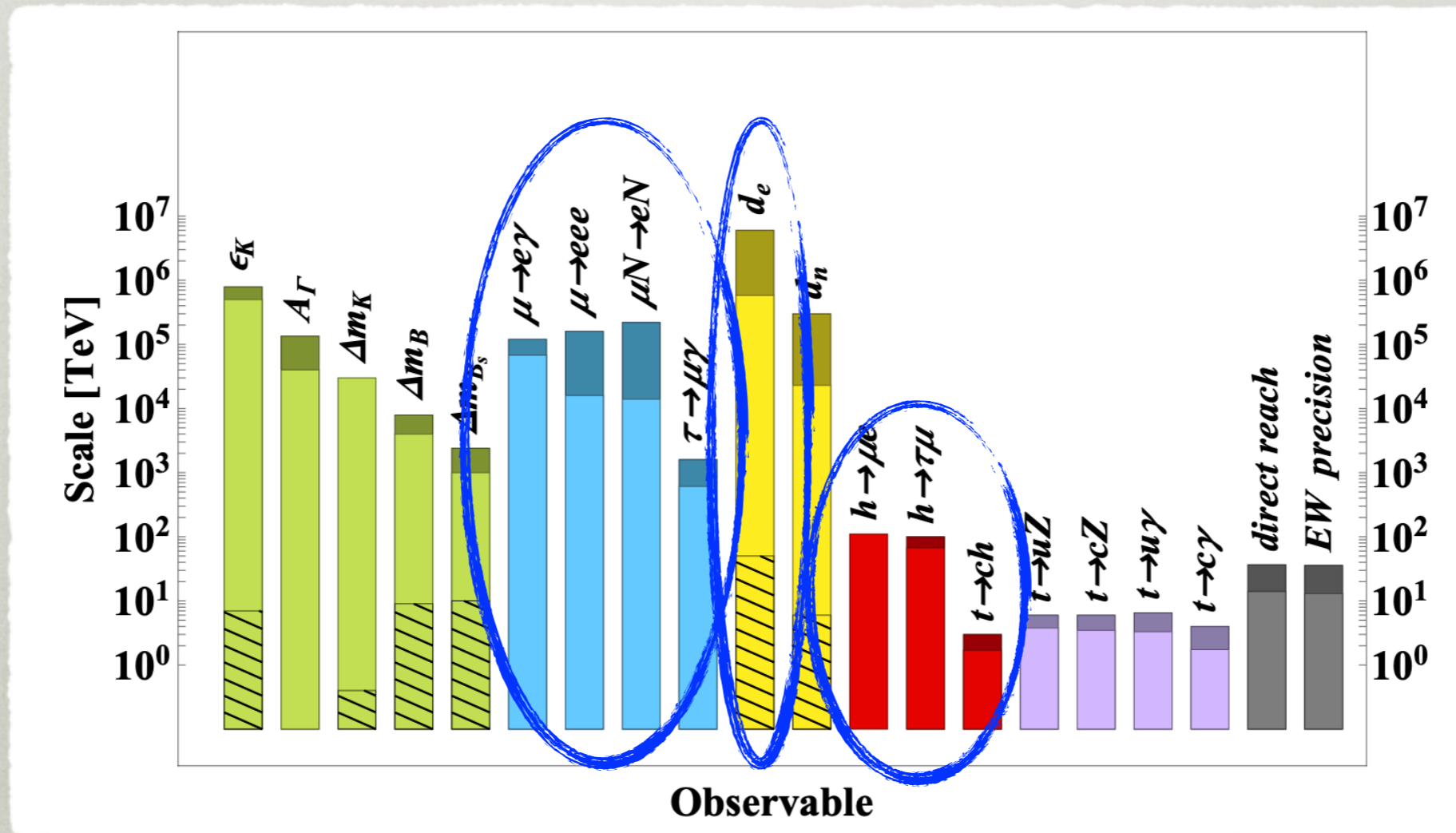
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$$x_k = m_{\nu_k}^2 / M_W^2$$

- GIM mechanism no longer fully operational
- $Br(\mu \rightarrow e\gamma)$ not suppressed by light ν masses, can be larger

SEARCHING FOR NEW PHYSICS

- LFV observables probe very high scales



- the rest of these lectures: focusing on the above observables

OBSERVABLES

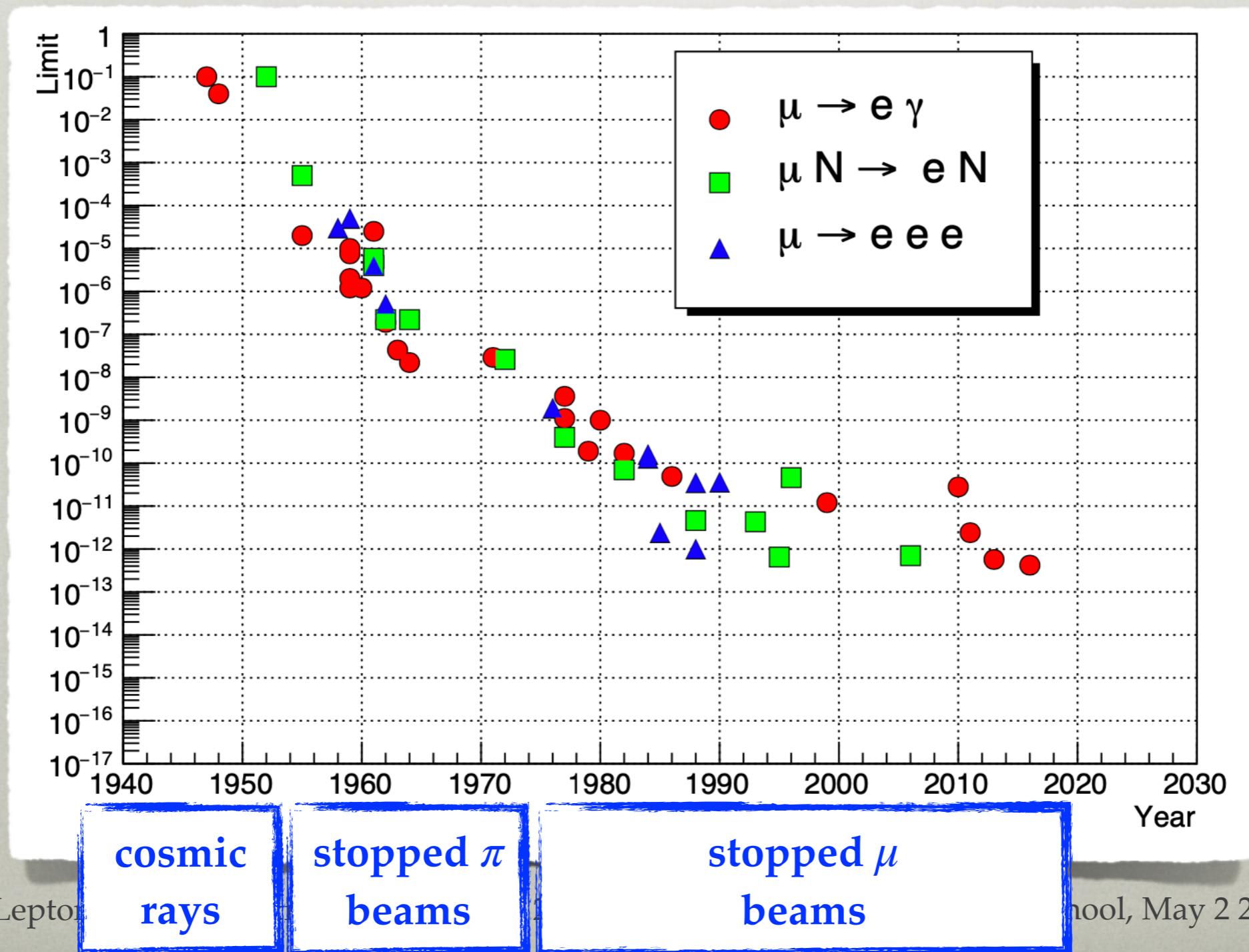
- CLFV transitions
 - $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma, \mu \rightarrow 3e, \mu \rightarrow e \text{ conv.}, \dots$
- CPV probes
 - electric dipole moment (EDM) of the electron
- Higgs decays
 - $h \rightarrow \tau\tau, h \rightarrow \mu\mu, h \rightarrow \tau\mu, \dots$

MUONS

- today: LFV muon decays
- tomorrow: all the other probes

EXPERIMENTAL PROGRESS

- steady experimental progress since 1940s



VERY RARE PROCESSES

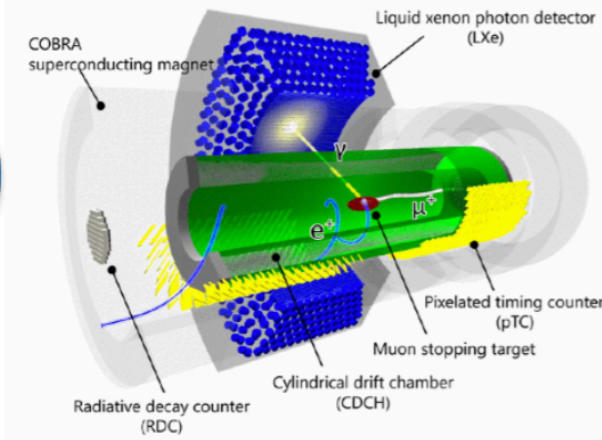
- planned experiments aim for $Br(\mu \rightarrow \dots) \sim 10^{-15} - 10^{-18}$
- very rare: if one were to use cosmic rays passing through your hand*
 - \Rightarrow would take more than the age of the Universe to collect a sample large enough for one event

* the flux of cosmic ray muons at sea level is $\sim 1 \text{ muon/cm}^2/\text{min}$

cLFV experiments in the world

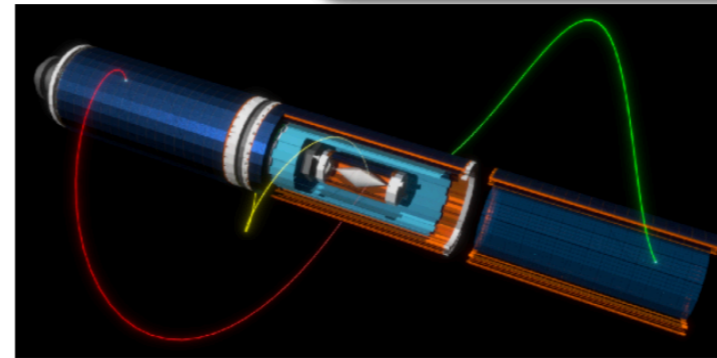
MEG II

$\mu^+ \rightarrow e^+ \gamma$



Mu3e

$\mu^+ \rightarrow e^+ e^+ e^-$



Coincidence measurement:
DC beam needed to minimize
backgrounds from accidental
coincidences

$BKG \propto (Rate)^2$

PSI



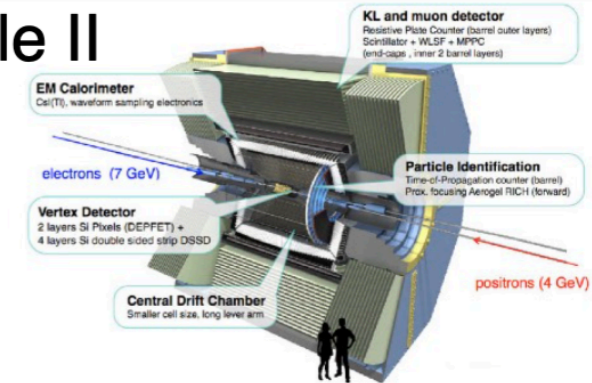
CERN

LHCb/ATLAS/CMS

$\tau \rightarrow 3\mu, \tau \rightarrow \mu\gamma$

KEK

Belle II

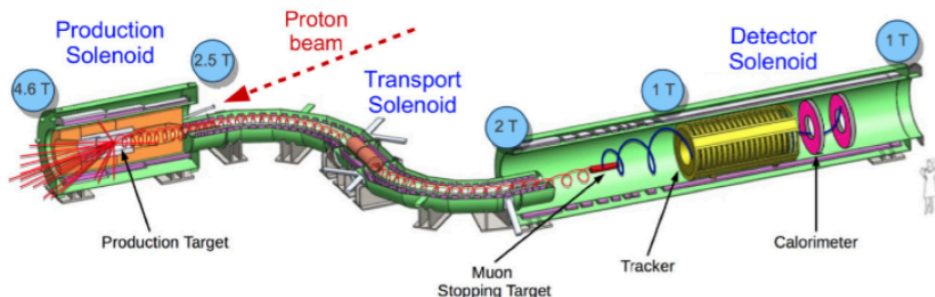


Fermilab

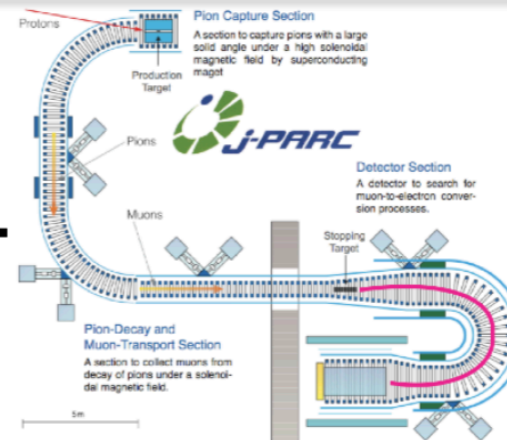
$\mu-N \rightarrow e-N$

J-PARC

Mu2e



DeeMe,
COMET



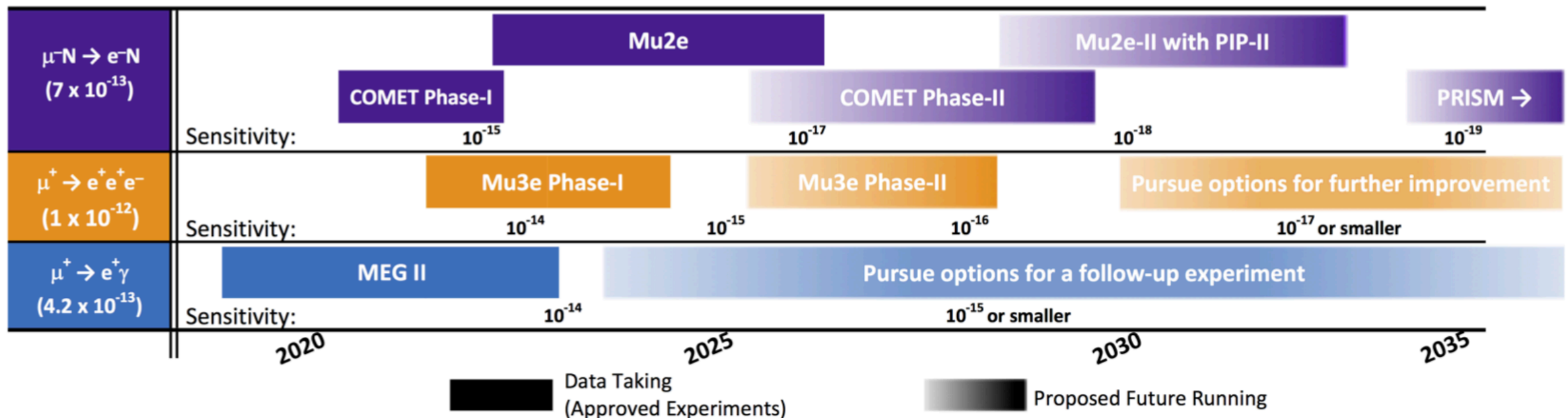
Single e^- measurement:
pulsed beam needed
Many pion-induced
backgrounds after
proton pulse
wait it out with 26 ns
lifetime

TIMELINE

- pre-Covid timeline from 2019

Physics Briefing Book, 1910.11775

Searches for Charged-Lepton Flavor Violation in Experiments using Intense Muon Beams



$$\mu \rightarrow e\gamma$$

$\mu \rightarrow e\gamma$ EXPERIMENTAL RESULTS

- present best bound

- MEG (2016):

MEG coll., hep-ex/1605.05081

$$Br(\mu^+ \rightarrow e^+\gamma) < 4.2 \times 10^{-13}$$

- future experiment (just started physics data taking)

present status in Meucci, 2201.08200

- MEG-II (~2025):

$$Br(\mu^+ \rightarrow e^+\gamma) < 6 \times 10^{-14}$$

HEAVY NEW PHYSICS

- if there is heavy NP, can be integrated out
 - results in SM Effective Field Theory (SMEFT)
 - renormalizable SM supplemented by higher dimensional operators

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \sum_a C_a^{(5)} Q_a^{(5)} + \frac{1}{\Lambda^2} \sum_a C_a^{(6)} Q_a^{(6)} + \dots$$

- $\mu \rightarrow e\gamma$ results in a dimension 6 operator

$$\mathcal{L} \supset -\frac{\sqrt{2}e v}{(4\pi\Lambda_{ij})^2} \bar{\ell}_L^i \sigma^{\mu\nu} \ell_R^j F_{\mu\nu} + \text{h.c.} ,$$

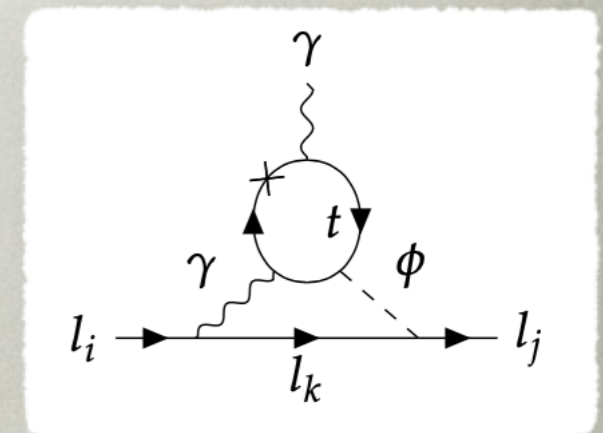
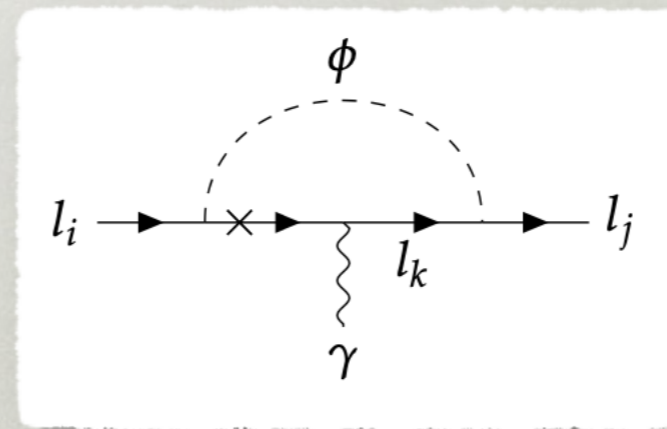
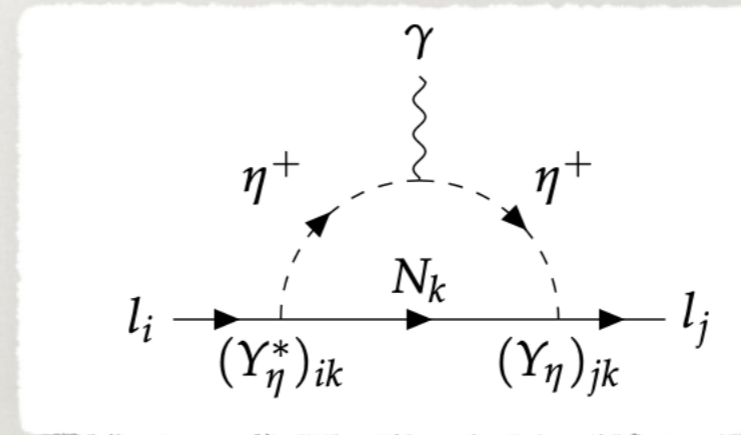
- exp. bounds imply that it is highly suppressed

$$\mu \rightarrow e\gamma \Rightarrow \Lambda_{21} \gtrsim 3500 \text{ TeV}$$

Greljo, Stangl, Thomsen, 2103.13991

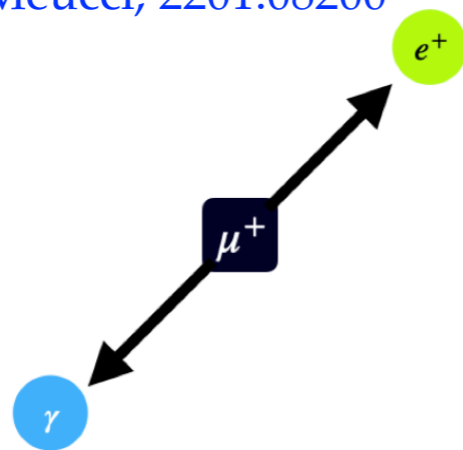
NEW PHYSICS EXAMPLES FOR $\mu \rightarrow e\gamma$

- any new states with FV couplings to SM leptons will contribute to $\mu \rightarrow e\gamma$
- a selection of examples
 - neutrino mass models
 - see-saw
 - loop generated neutrino masses
 - 2 Higgs Doublet Model
 - low energy supersymmetry
 - extra dimensional models

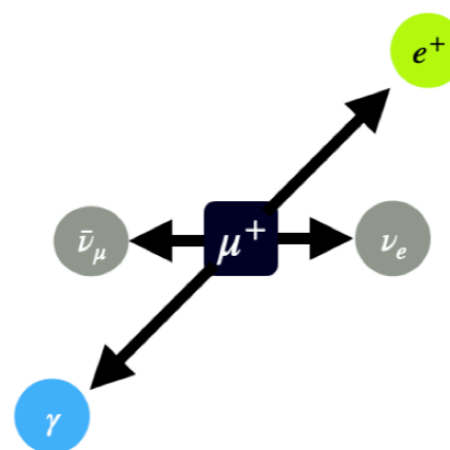


$\mu \rightarrow e\gamma$ EXPERIMENTS

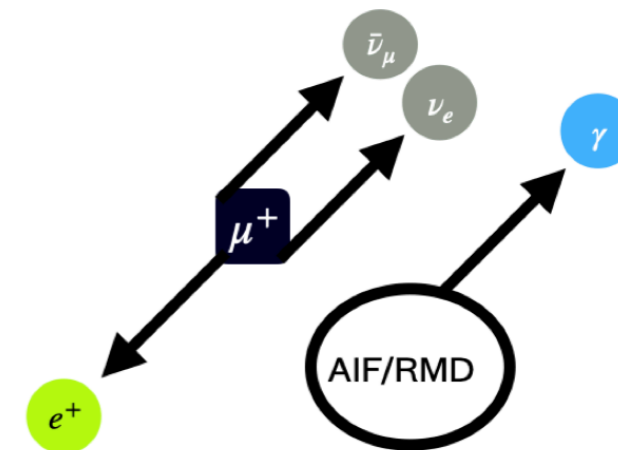
- in muon rest frame e and γ are monochromatic
 - $E_e = E_\gamma \simeq m_\mu/2 \simeq 52.8 \text{ MeV}$
- convenient to perform experiments with stopped muons
 - use μ^+ so that it does not get bound to nucleus, i.e., avoid the spread of line from decay in orbit
 - the measured process is thus $\mu^+ \rightarrow e^+\gamma$
- muons are stopped in the thinnest possible targets
 - so that the e^+ do not lose energy when escaping
 - search for monochromatic e^+ line at the kinematical edge of SM $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ decay (the "Michele edge")
 - require coincidence with a photon of the same energy
 - energy resolution very important to reduce SM background
 - irreducible background is the SM decay $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu\gamma$



SIGNAL



RADIATIVE MUON DECAY



ACCIDENTAL BACKGROUND

- i

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- use μ^+ so that it does not get bound to nucleus, i.e., avoid the spread of line from decay in orbit

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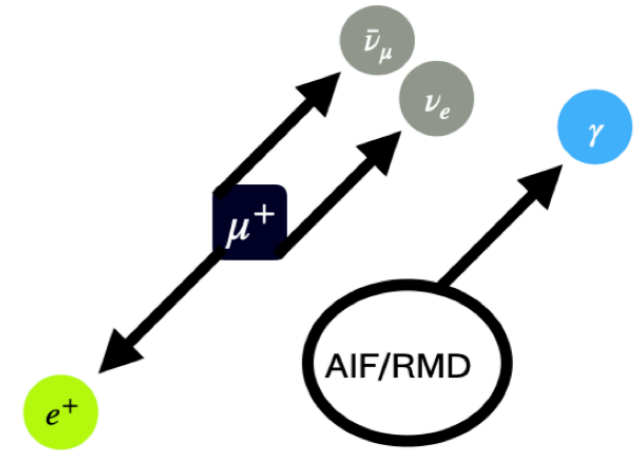
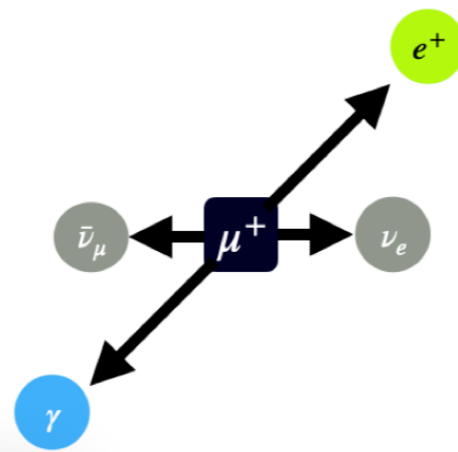
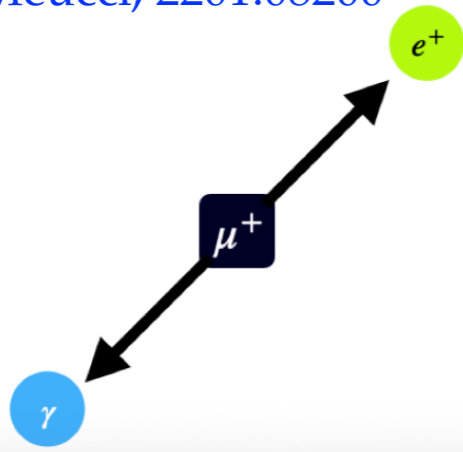
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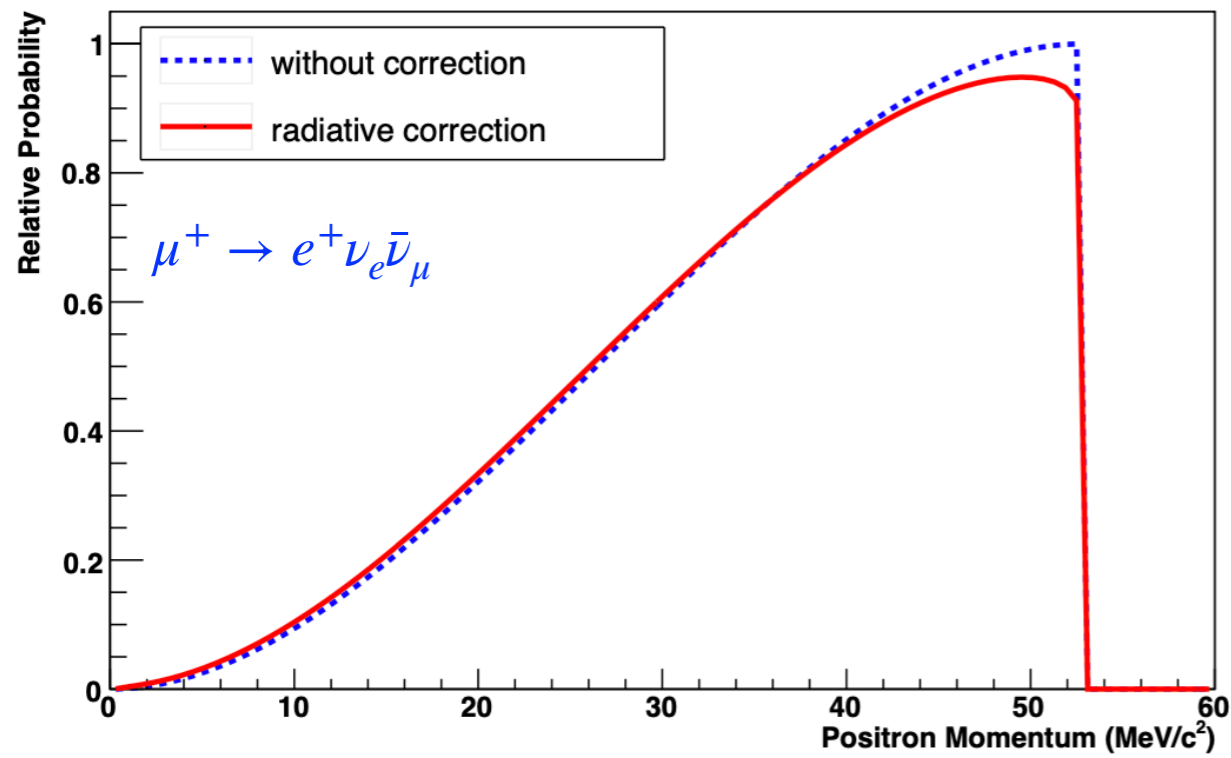
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RADIATIVE MUON DECAY

ACCIDENTAL BACKGROUND



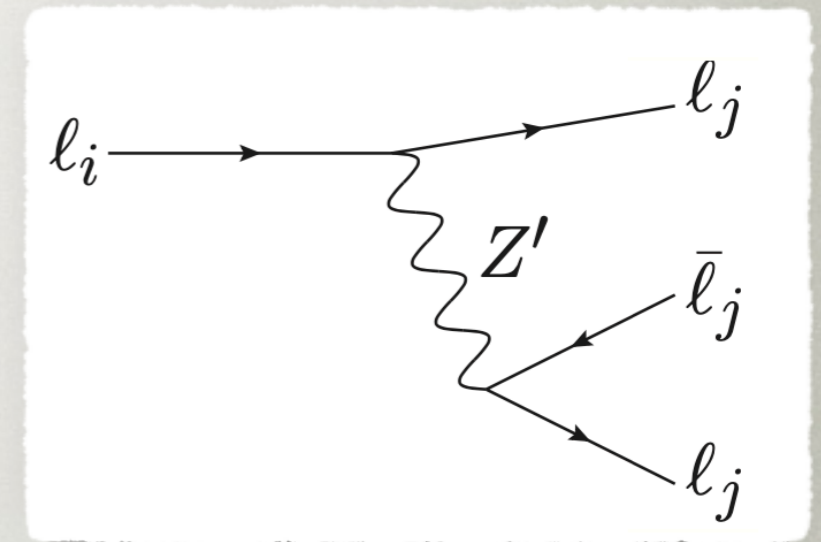
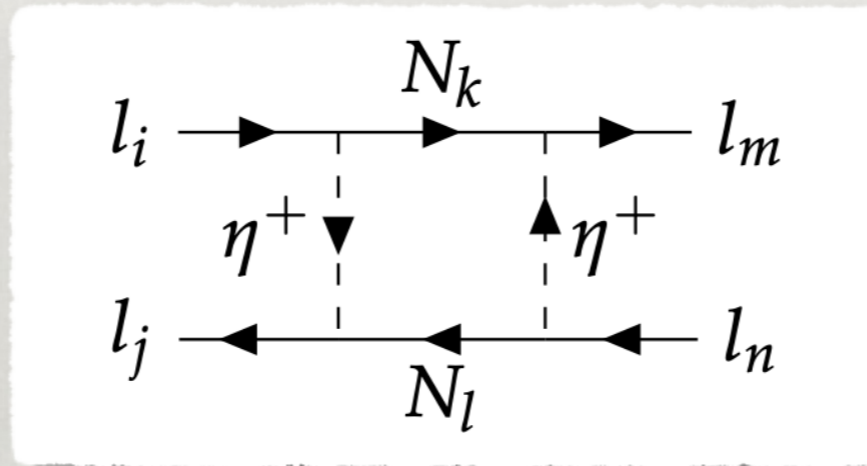
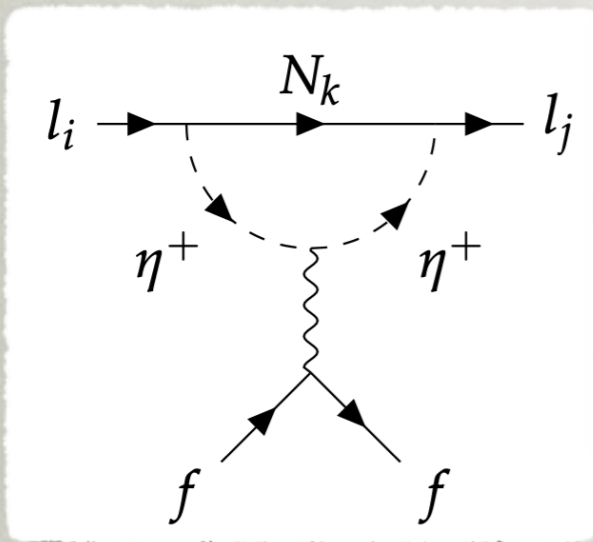
with stopped muons
 und to nucleus, i.e., avoid the spread of
 $\rightarrow e^+\gamma$
 possible targets
 gy when escaping

- search for monochromatic e^+ line at the kinematical edge of SM $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ decay (the "Michele edge")
- require coincidence with a photon of the same energy
- energy resolution very important to reduce SM background
 - irreducible background is the SM decay $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu\gamma$

$$\mu \rightarrow 3e$$

$\mu \rightarrow 3e$

- $\mu^+ \rightarrow e^+e^-e^+$: tree level or one loop NP contri. possible



- if NP heavy, can be integrated out
 - then the $\mu \rightarrow 3e$ transition described by an EFT with

- dipole operators

$$\bar{\ell}_L^i \sigma^{\mu\nu} \ell_R^j F_{\mu\nu}$$

- four fermion operators

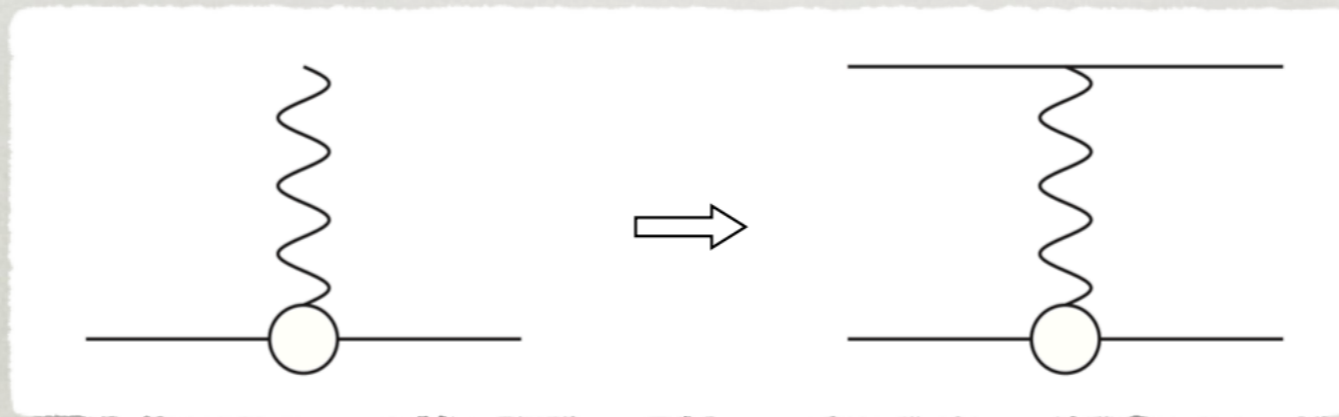
$$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_L \gamma^\mu L_L)$$

$$(\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$$

$$(\bar{L}_L \gamma_\mu L_L)(\bar{e}_R \gamma^\mu e_R)$$

DIPOLE LIMIT

- if NP such that the dipole contribution dominates
- then $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ rates are related



$$\text{BR}(\mu \rightarrow eee) \simeq \frac{\alpha}{3\pi} \left(\log \frac{m_\mu^2}{m_e^2} - 3 \right) \times \text{BR}(\mu \rightarrow e\gamma)$$

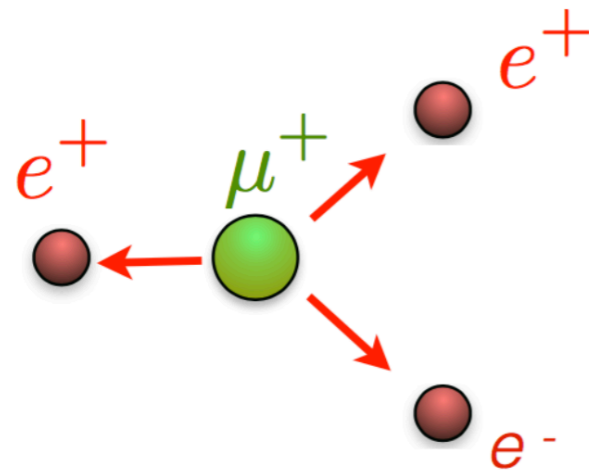
- in general all operators are present
 - the above operators mix under the RG

EXPERIMENTS

- also use stopped μ^+ so the lab frame is the muon rest frame
- $\mu^+ \rightarrow e^+e^-e^+$ is a 3-body decay, so no mono-energetic particle
 - maximal energy for each e is $E_{\max} \simeq m_\mu/2$
- the signature is
 - $2e^+$ and $1e^-$ coming from common vertex (and nothing else)
 - their energy adds up to m_μ
- the main "irreducible" SM background $\mu^+ \rightarrow e^+e^-e^+\bar{\nu}_\mu\nu_e$ decay
 - two neutrinos appear as missing energy E_{inv}
 - need very precise energy measurement to make sure
$$E_{e^+} + E_{e^-} + E_{e^+} = m_\mu$$

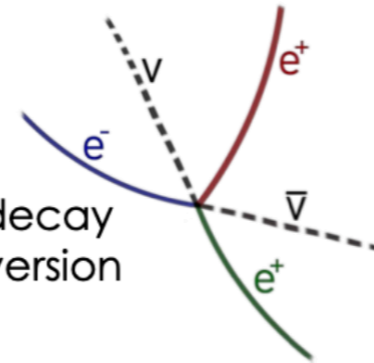
Signal

Background



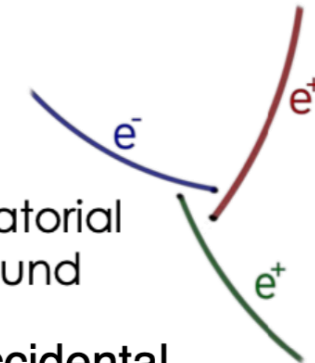
$\Sigma p_e = 0$
 $\Sigma E_e = m_\mu$
 Common vertex
 Coincident

Radiative SM decay
 + photon conversion
 $\mu^+ \rightarrow e^+e^-e^+\nu\bar{\nu}$



$\Sigma p_e \neq 0$
 $\Sigma E_e \neq m_\mu$
 Common vertex
 Coincident

Combinatorial background



Accidental

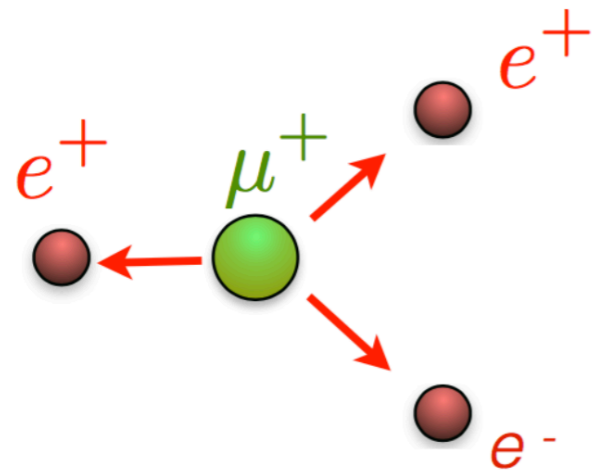
$\Sigma p_e \neq 0$
 $\Sigma E_e \neq m_\mu$
 No common vertex
 Not coincident

- $2e^+$ and $1e^-$ coming from common vertex (and nothing else)
- their energy adds up to m_μ
- the main "irreducible" SM background $\mu^+ \rightarrow e^+e^-e^+\bar{\nu}_\mu\nu_e$ decay
 - two neutrinos appear as missing energy E_{inv}
 - need very precise energy measurement to make sure

$$E_{e^+} + E_{e^-} + E_{e^+} = m_\mu$$

Signal

Iwamoto @ FPCP2021

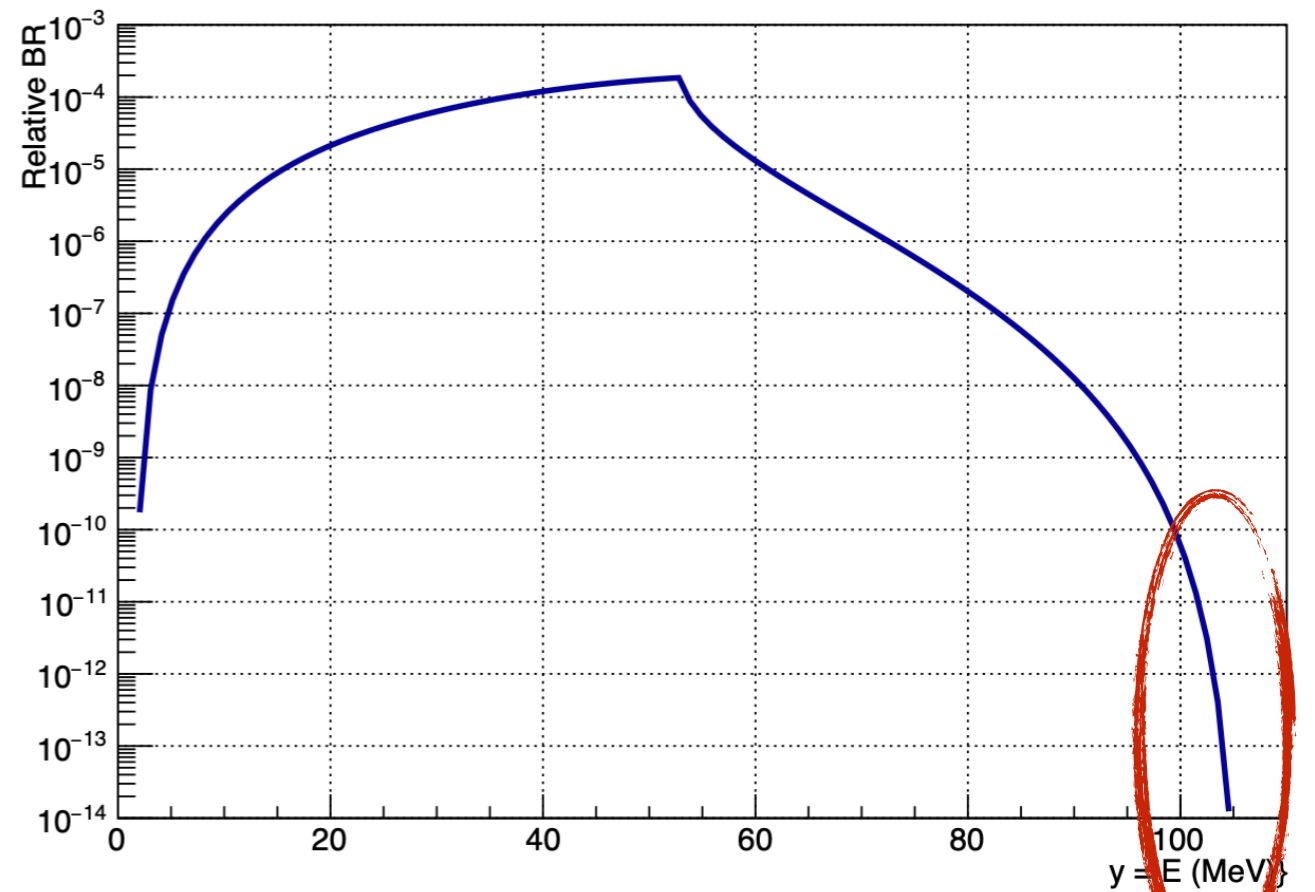


Radiative SM
+ photon cc
 $\mu^+ \rightarrow e^+e^-e^+$

$\Sigma p_e = 0$
 $\Sigma E_e = m_\mu$
Common vertex
Coincident

$\Sigma p_e = 0$
 $\Sigma E_e = m_\mu$
Common vertex
Coincident

Visible energy in the $\mu \rightarrow 3 e \nu \nu$ decay

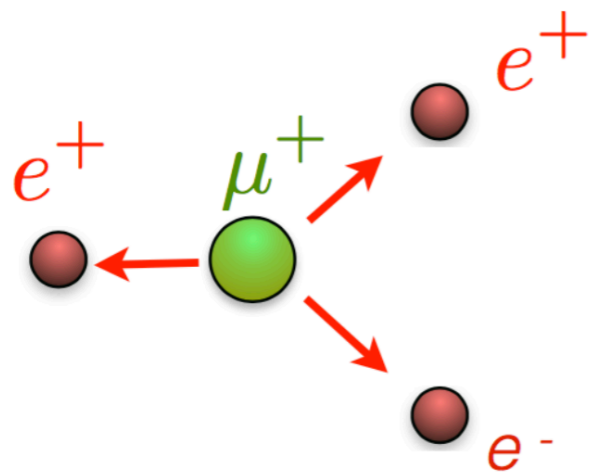


- $2e^+$ and $1e^-$ coming from common vertex (and nothing else)
- their energy adds up to m_μ
- the main "irreducible" SM background $\mu^+ \rightarrow e^+e^-e^+\bar{\nu}_\mu\nu_e$ decay
 - two neutrinos appear as missing energy E_{inv}
 - need very precise energy measurement to make sure

$$E_{e^+} + E_{e^-} + E_{e^+} = m_\mu$$

Signal

Iwamoto @ FPCP202



Radiative SM
+ photon cc
 $\mu^+ \rightarrow e^+e^-e^+$

$$\Sigma p_e = 0$$

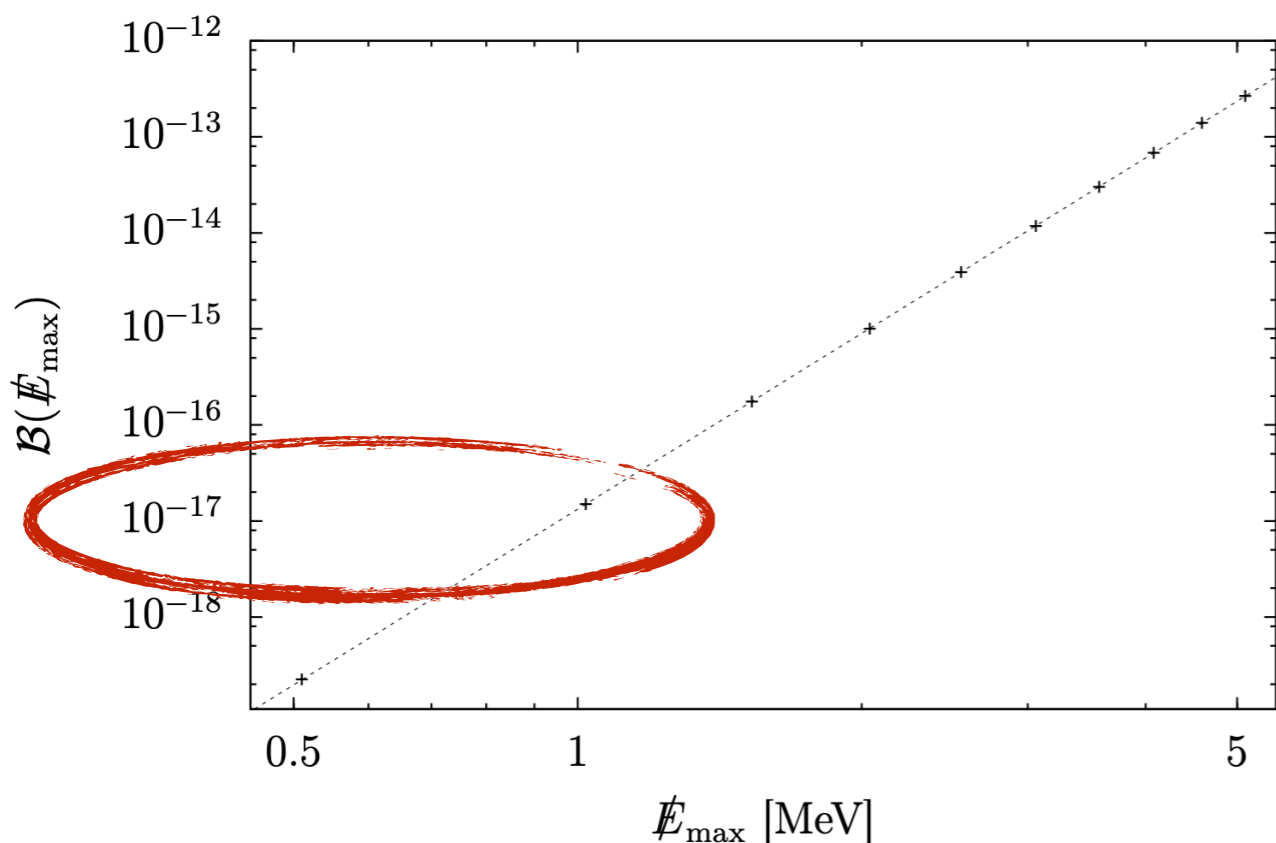
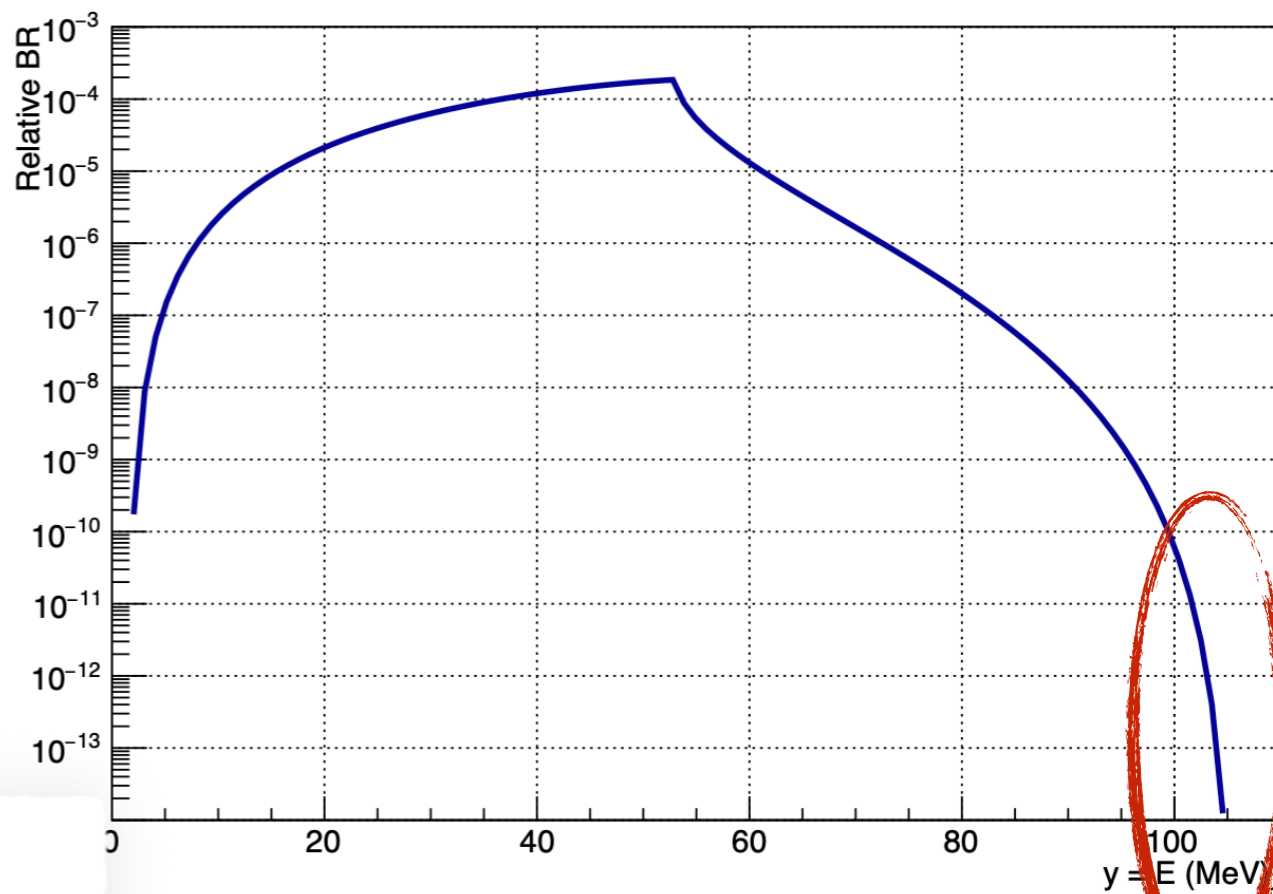
$$\Sigma E_e = m_\mu$$

Common vertex

$$\mu^- \rightarrow e^-(e^+e^-)\nu_\mu\bar{\nu}_e$$

Com

Visible energy in the $\mu \rightarrow 3 e \nu \nu$ decay



Common vertex (and nothing else)

Background $\mu^+ \rightarrow e^+e^-e^+\bar{\nu}_\mu\nu_e$ decay

using energy E_{inv}

measurement to make sure

-
- present best bound

- SINDRUM (1988):

Nuclear Physics B 1988, 299

$$Br(\mu^+ \rightarrow e^+ e^- e^+) < 1.0 \times 10^{-12}$$

- future

- Mu3e:

2009.11690

$$\text{Phase 1 (~2025): } Br(\mu \rightarrow 3e) < 2 \times 10^{-15}$$

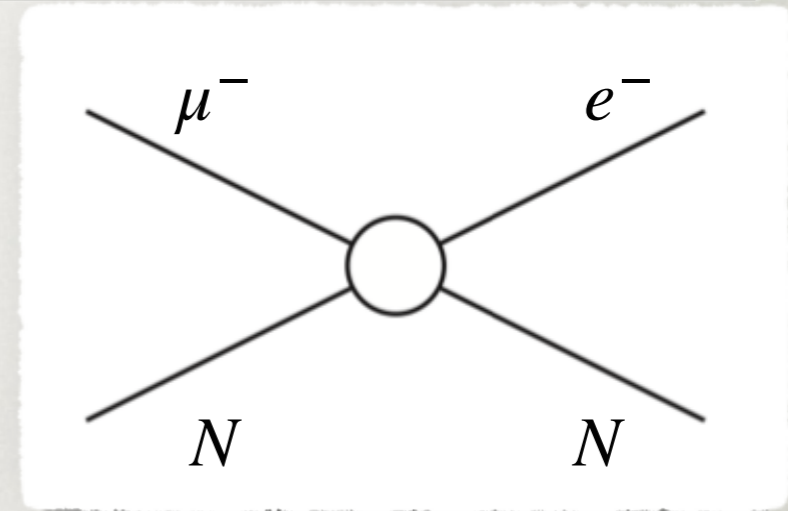
$$\text{Phase 2 (2030s): } Br(\mu \rightarrow 3e) \lesssim 10^{-16}$$

$\mu \rightarrow e$ CONVERSION

$\mu^- N \rightarrow e^- N$ CONVERSION

- $\mu^- N \rightarrow e^- N$ conversion:

- μ^- converts to e^- in the field of nucleus, without emission of ν 's



- the μ^- is first stopped in target, and quickly forms muonic atom
- the $\mu^- N \rightarrow e^- N$ conversion has two body kinematics
- results in monochromatic e^- with energy

$$E_e = m_\mu - E_b - \frac{m_\mu^2}{2m_N}$$

muonic binding energy

$$E_b \sim Z^2 \alpha^2 m_\mu / 2$$

kin. eng. of

the nucleon

- e.g. in Al (used in Mu2e), $E_e = 104.96 \text{ MeV}$

$$\mu^- N \rightarrow e^- N$$

CONVERSION

- if NP effects described by EFT, the relevant higher dim. ops. are

- dipole operators $\bar{\ell}_L^i \sigma^{\mu\nu} \ell_R^j F_{\mu\nu}$

- four fermion operators with quarks

$$(\bar{L}_L \gamma_\mu L_L)(\bar{Q}_L \gamma^\mu Q_L)$$

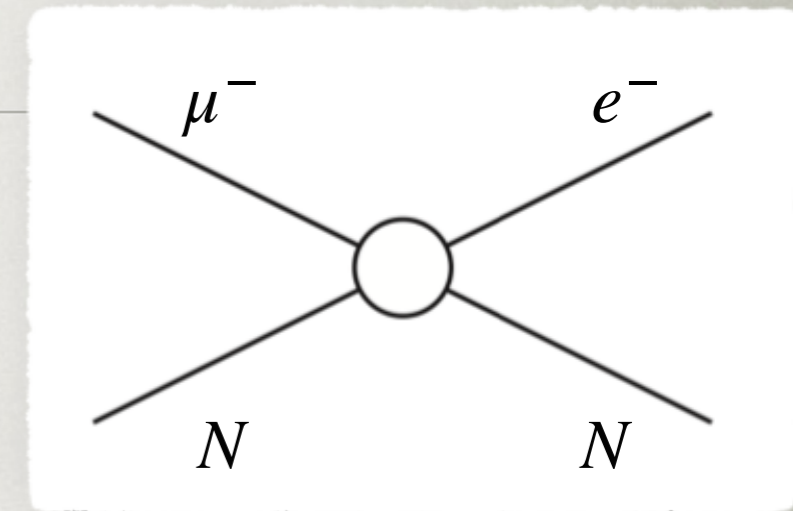
$$(\bar{L}_L \gamma_\mu \tau_I L_L)(\bar{Q}_L \gamma^\mu \tau_I Q_L)$$

$$(\bar{e}_R \gamma^\mu e_R)(\bar{Q}_L \gamma_\mu Q_L)$$

$$(\bar{L}_L \gamma_\mu L_L)(\bar{d}_R \gamma^\mu d_R)$$

$$(\bar{e}_R \gamma_\mu e_R)(\bar{d}_R \gamma^\mu d_R)$$

- general treatment rather complicated



$$(\bar{L}_L \gamma_\mu L_L)(\bar{u}_R \gamma^\mu u_R)$$

$$(\bar{e}_R \gamma_\mu e_R)(\bar{u}_R \gamma^\mu u_R)$$

$$(\bar{L}_L^a e_R)(\bar{d}_R Q_L^a)$$

$$(\bar{L}_L^a e_R) \epsilon_{ab} (\bar{Q}_L^b u_R)$$

$$(\bar{L}_i^a \sigma_{\mu\nu} e_R) \epsilon_{ab} (\bar{Q}_L^b \sigma^{\mu\nu} u_R)$$

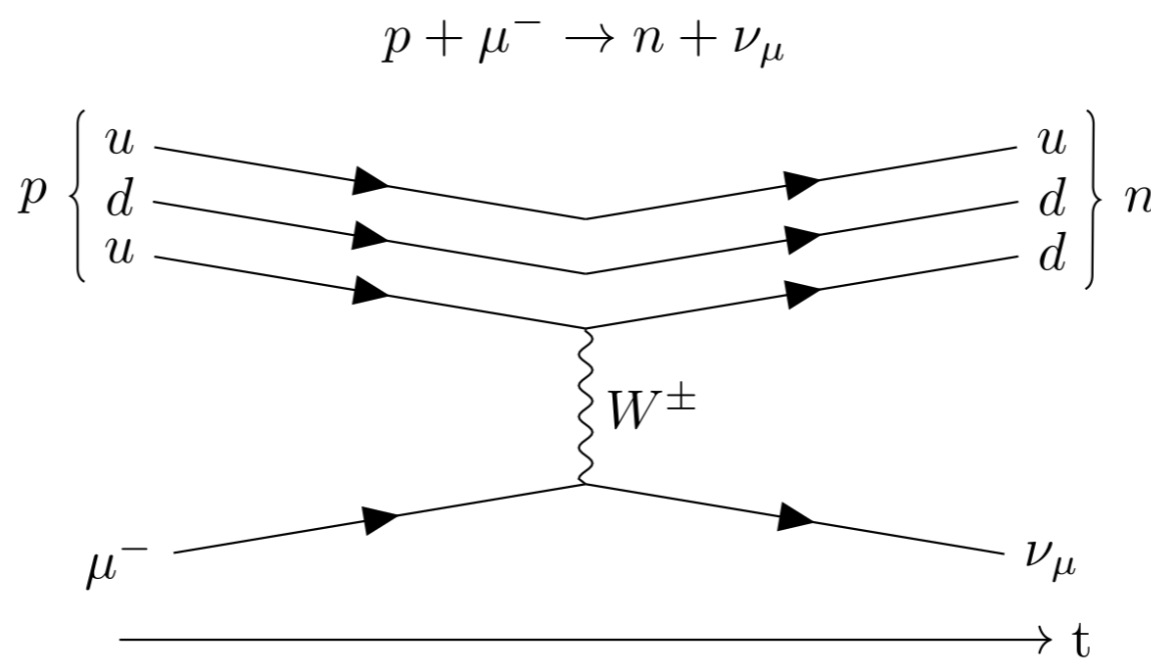
- requires predictions for nuclear matrix elements
- no first principle calculations for elements used as targets, approximate nucl-th treatments

$\mu^- N \rightarrow e^- N$ CONVERSION

- results are quoted in terms of normalized conversion rate

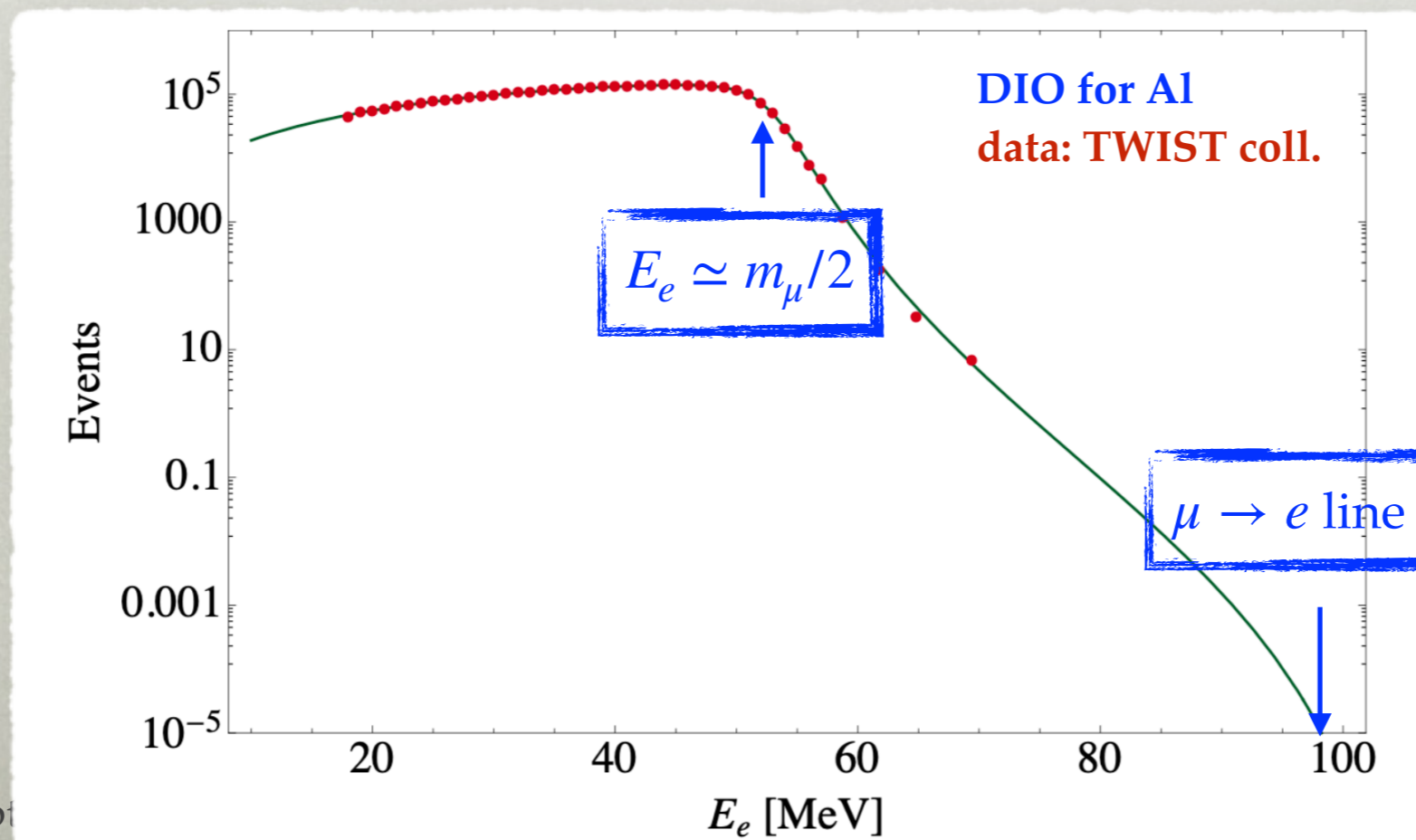
$$R_{\mu e} = \text{CR}(\mu N \rightarrow e N) \equiv \frac{\Gamma(\mu - e \text{ conversion})}{\Gamma(\text{nuclear capture})}$$

- normalization to nuclear capture rate reduces theoretical uncertainties



$\mu^- N \rightarrow e^- N$ CONVERSION

- experimentally $\mu \rightarrow e$ conversion offers many advantages over, e.g., $\mu \rightarrow e\gamma$
 - the only intrinsic bckgd is $\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$ decay in orbit
 - in $\mu^- N \rightarrow e^- N$ the e^- is at the kinematical edge of DIO



$\mu^- N \rightarrow e^- N$ CONVERSION

- present bound

- SINDRUM-II (1993, 2006):

$$R_{\mu e} < 6.1(7.1) \times 10^{-13} \text{ on Ti (Au)}$$

Physics Letters B 1993, 317, 631

Eur. Phys. J. C 2006, 47

- future (on C)

- DeeMee: $R_{\mu e} \lesssim 1(0.2) \times 10^{-13}$ on C (SiC)

- future (on Al)

- COMET Phase 1: $R_{\mu e} \lesssim 10^{-15}$
- Mu2e & COMET Phase-II: $R_{\mu e} \lesssim 10^{-17}$
- Mu2e-II: $R_{\mu e} \lesssim 10^{-18}$

COMPLEMENTARY PROBES

- complete list of dim 6 CLFV operators

4-leptons operators		Dipole operators	
Q_{ll}	$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_L \gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L \sigma^{\mu\nu} e_R) \tau_I \Phi W_{\mu\nu}^I$
Q_{ee}	$(\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L \sigma^{\mu\nu} e_R) \Phi B_{\mu\nu}$
Q_{le}	$(\bar{L}_L \gamma_\mu L_L)(\bar{e}_R \gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q_{lq}^{(1)}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{Q}_L \gamma^\mu Q_L)$	Q_{lu}	$(\bar{L}_L \gamma_\mu L_L)(\bar{u}_R \gamma^\mu u_R)$
$Q_{lq}^{(3)}$	$(\bar{L}_L \gamma_\mu \tau_I L_L)(\bar{Q}_L \gamma^\mu \tau_I Q_L)$	Q_{eu}	$(\bar{e}_R \gamma_\mu e_R)(\bar{u}_R \gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R \gamma^\mu e_R)(\bar{Q}_L \gamma_\mu Q_L)$	Q_{ledq}	$(\bar{L}_L^a e_R)(\bar{d}_R Q_L^a)$
Q_{ld}	$(\bar{L}_L \gamma_\mu L_L)(\bar{d}_R \gamma^\mu d_R)$	$Q_{lequ}^{(1)}$	$(\bar{L}_L^a e_R) \epsilon_{ab} (\bar{Q}_L^b u_R)$
Q_{ed}	$(\bar{e}_R \gamma_\mu e_R)(\bar{d}_R \gamma^\mu d_R)$	$Q_{lequ}^{(3)}$	$(\bar{L}_i^a \sigma_{\mu\nu} e_R) \epsilon_{ab} (\bar{Q}_L^b \sigma^{\mu\nu} u_R)$
Lepton-Higgs operators			
$Q_{\Phi l}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_L \gamma^\mu L_L)$	$Q_{\Phi l}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I \Phi)(\bar{L}_L \tau_I \gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R \gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R \Phi)(\Phi^\dagger \Phi)$

probed by

$\mu \rightarrow e\gamma$

$\mu \rightarrow 3e$

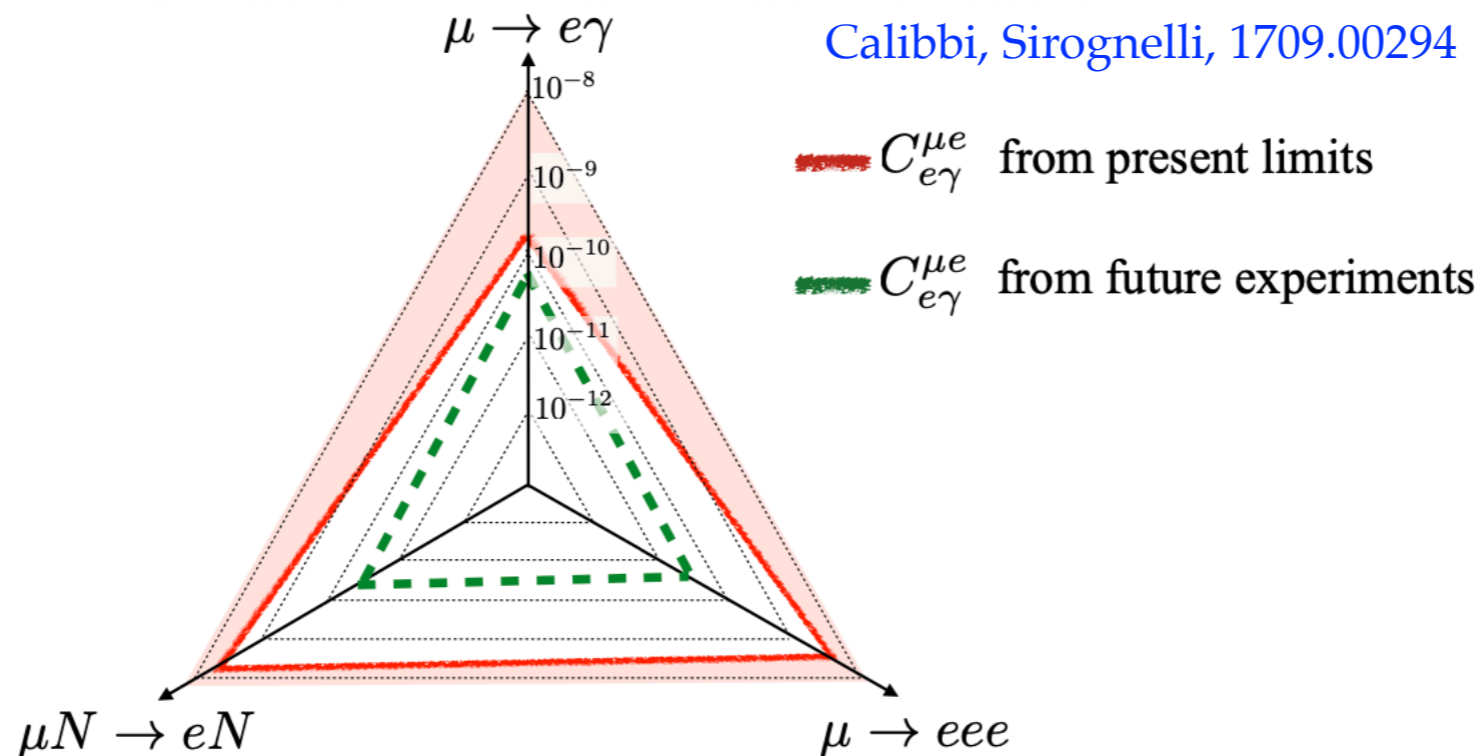
$\mu \rightarrow e$

DIPOLE OPERATOR DOMINANCE

- simplified scenario - assume the dipole operator dominates
- interesting to compare the reach of different experiments

$$\text{BR}(\mu \rightarrow eee) \simeq \frac{\alpha}{3\pi} \left(\log \frac{m_\mu^2}{m_e^2} - 3 \right) \times \text{BR}(\mu \rightarrow e\gamma),$$

$$\text{CR}(\mu N \rightarrow e N) \simeq \alpha \times \text{BR}(\mu \rightarrow e\gamma).$$



UPSHOT

- several different probes in rare muon decays
- can probe different types of new physics
- also disentangle different contributions
- significant improvements projected

BACKUP SLIDES