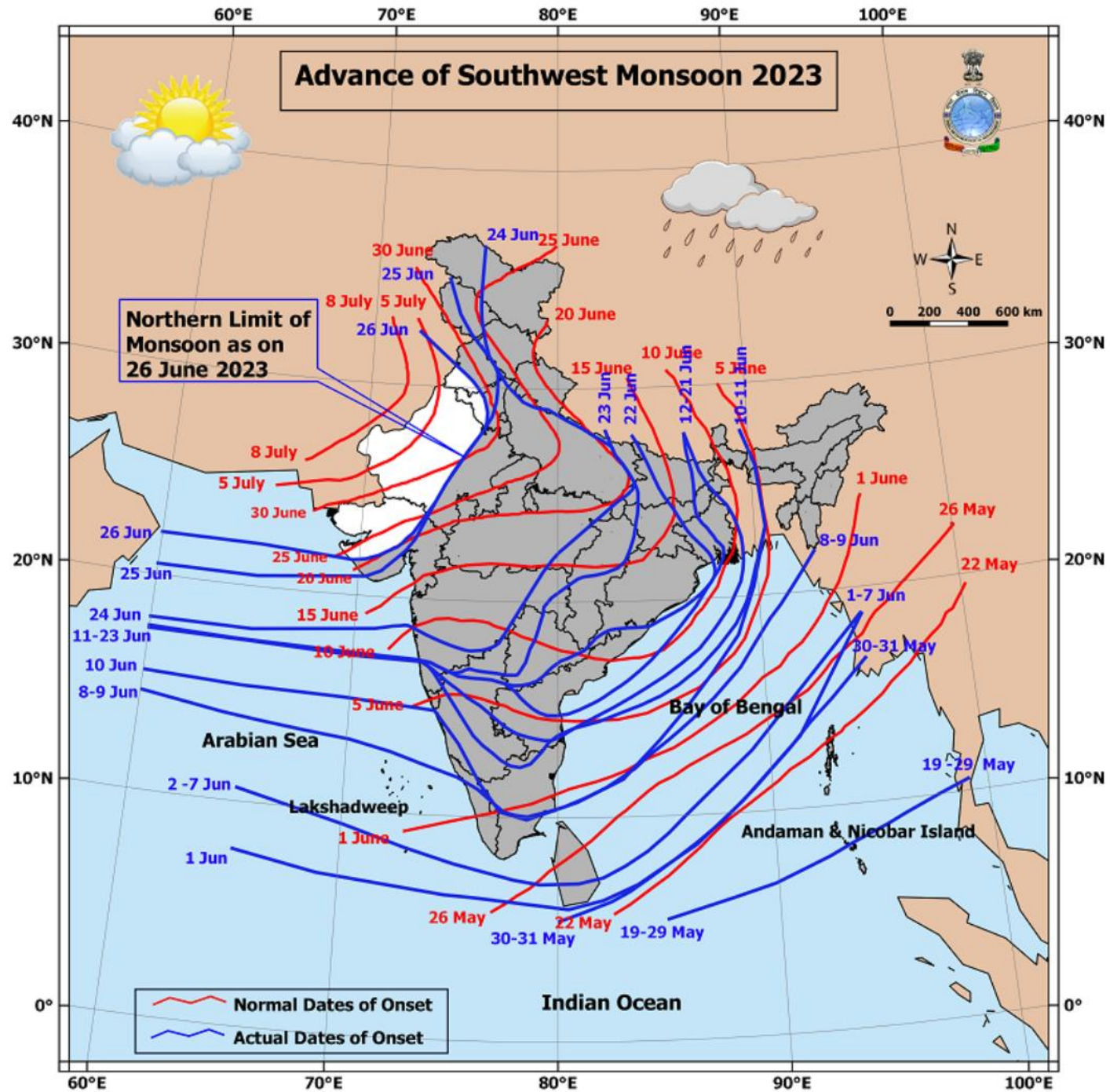


Simple Models of Monsoon Variability

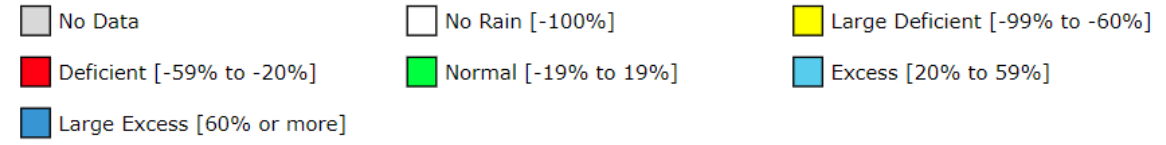
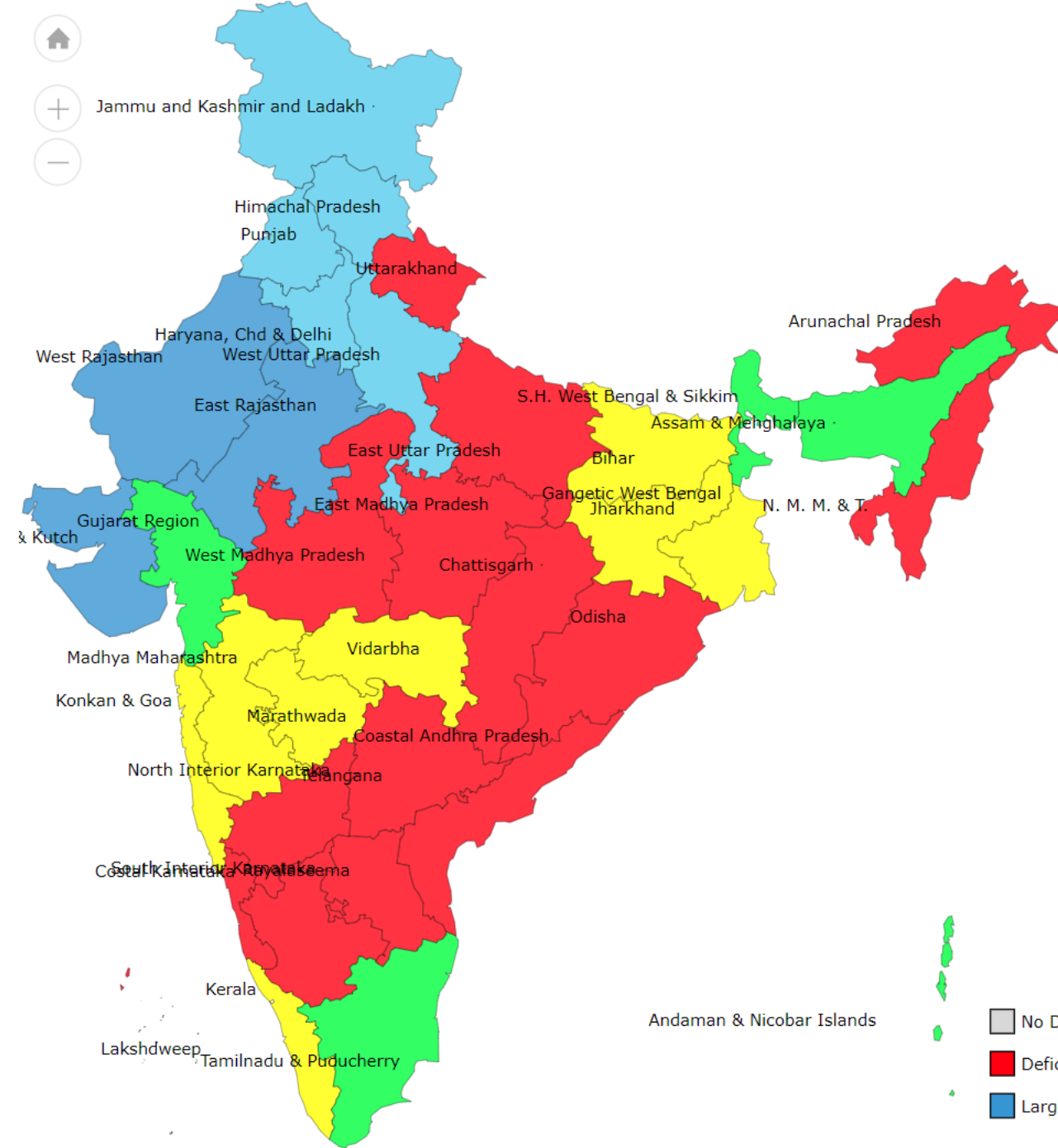
Vishal Dixit

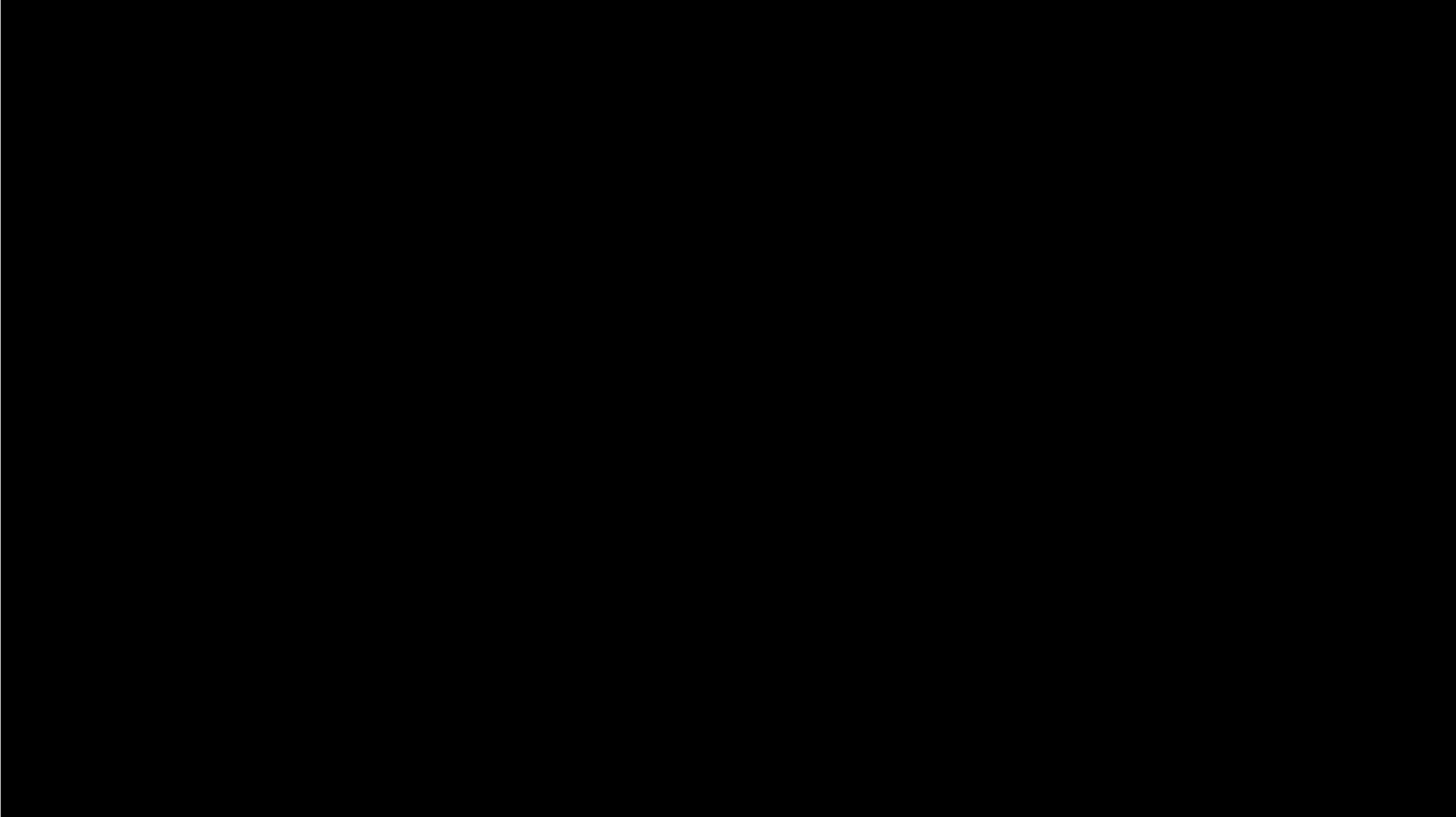
Assistant Professor

Interdisciplinary Program in Climate Studies, IIT Bombay

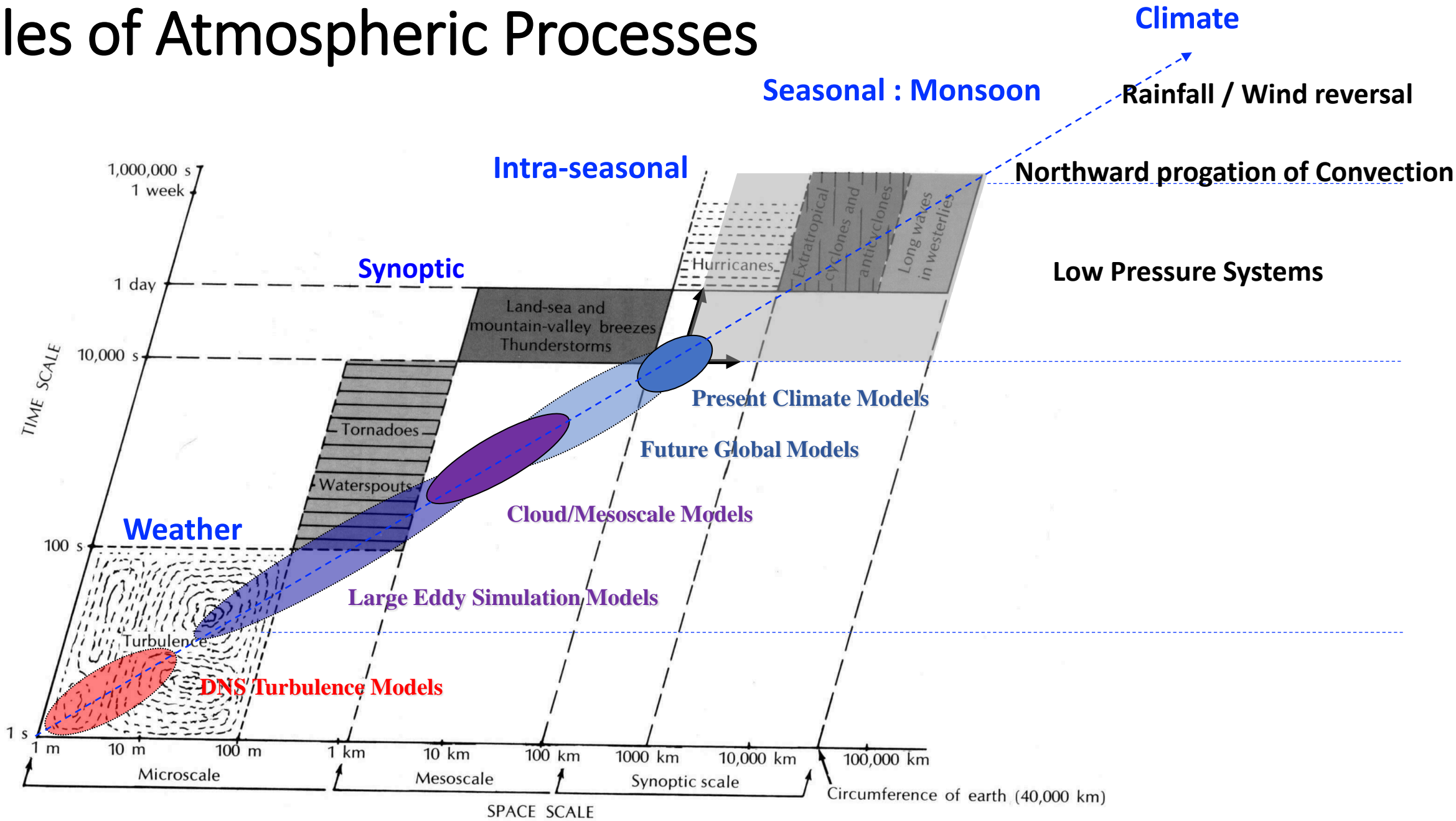


Rainfall until today (27 June 2023)



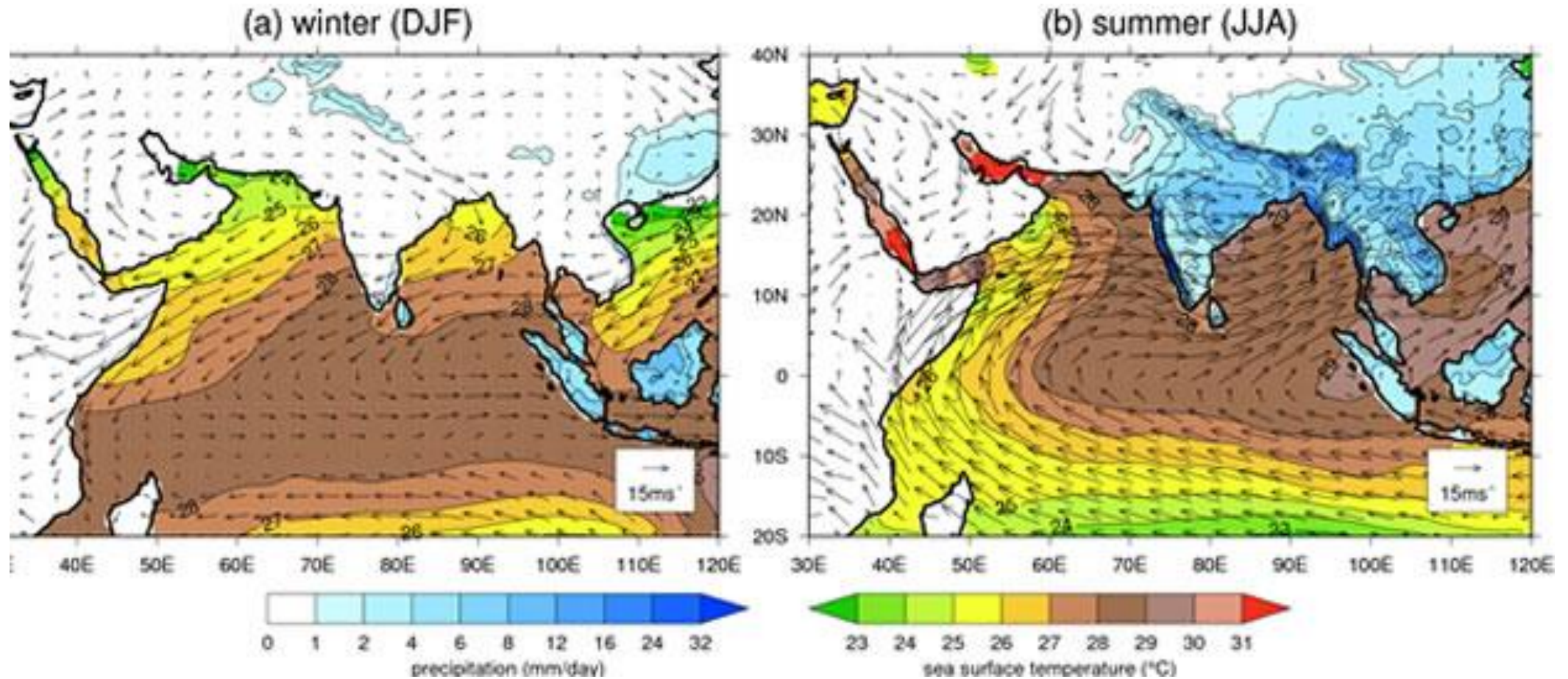


Scales of Atmospheric Processes



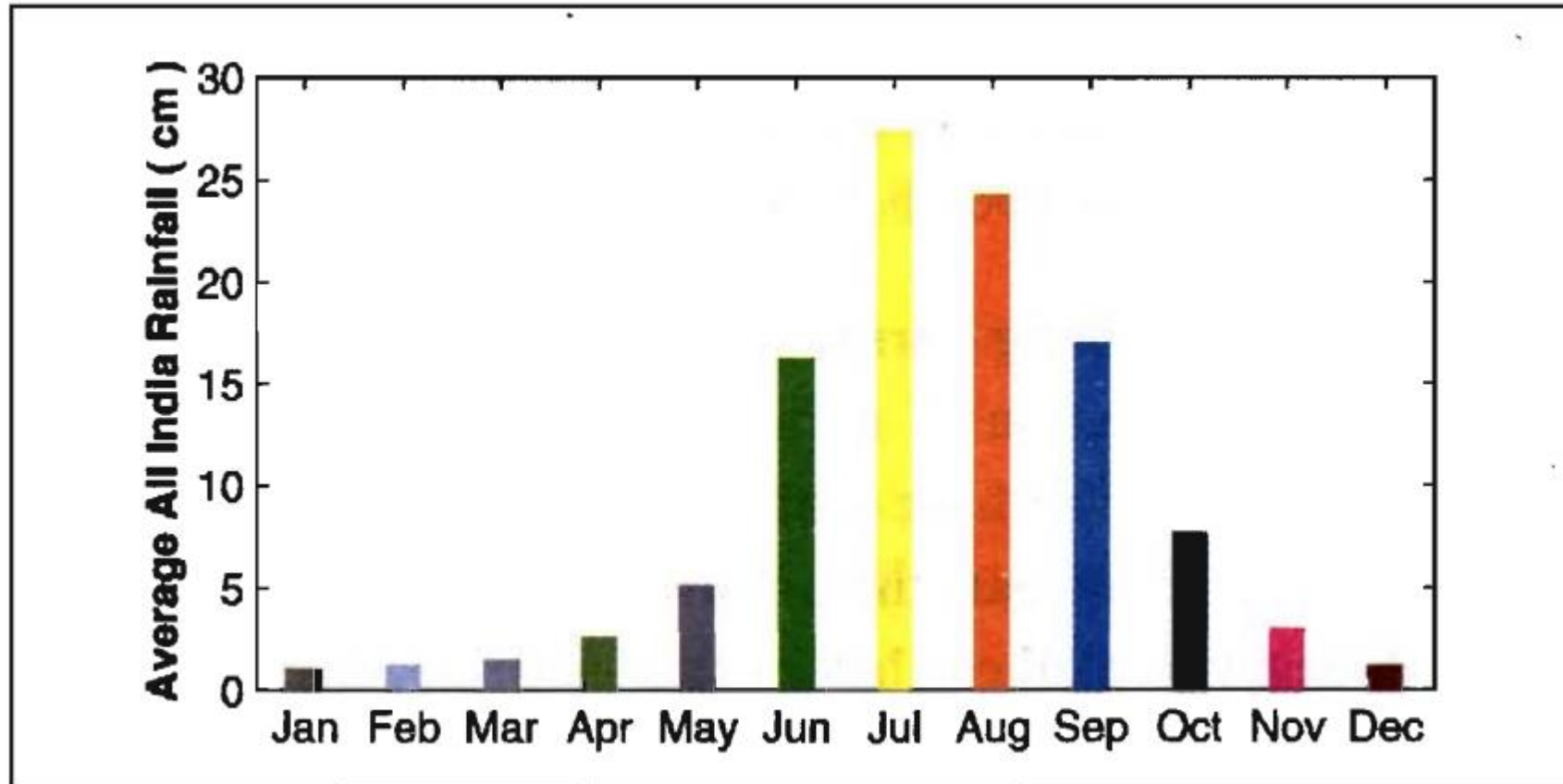
Modification of the figure from Andrew Gettleman, NCAR

Seasonal reversal of winds



Do Cloud-clusters actually migrate with the winds?

Average monthly All-India rainfall



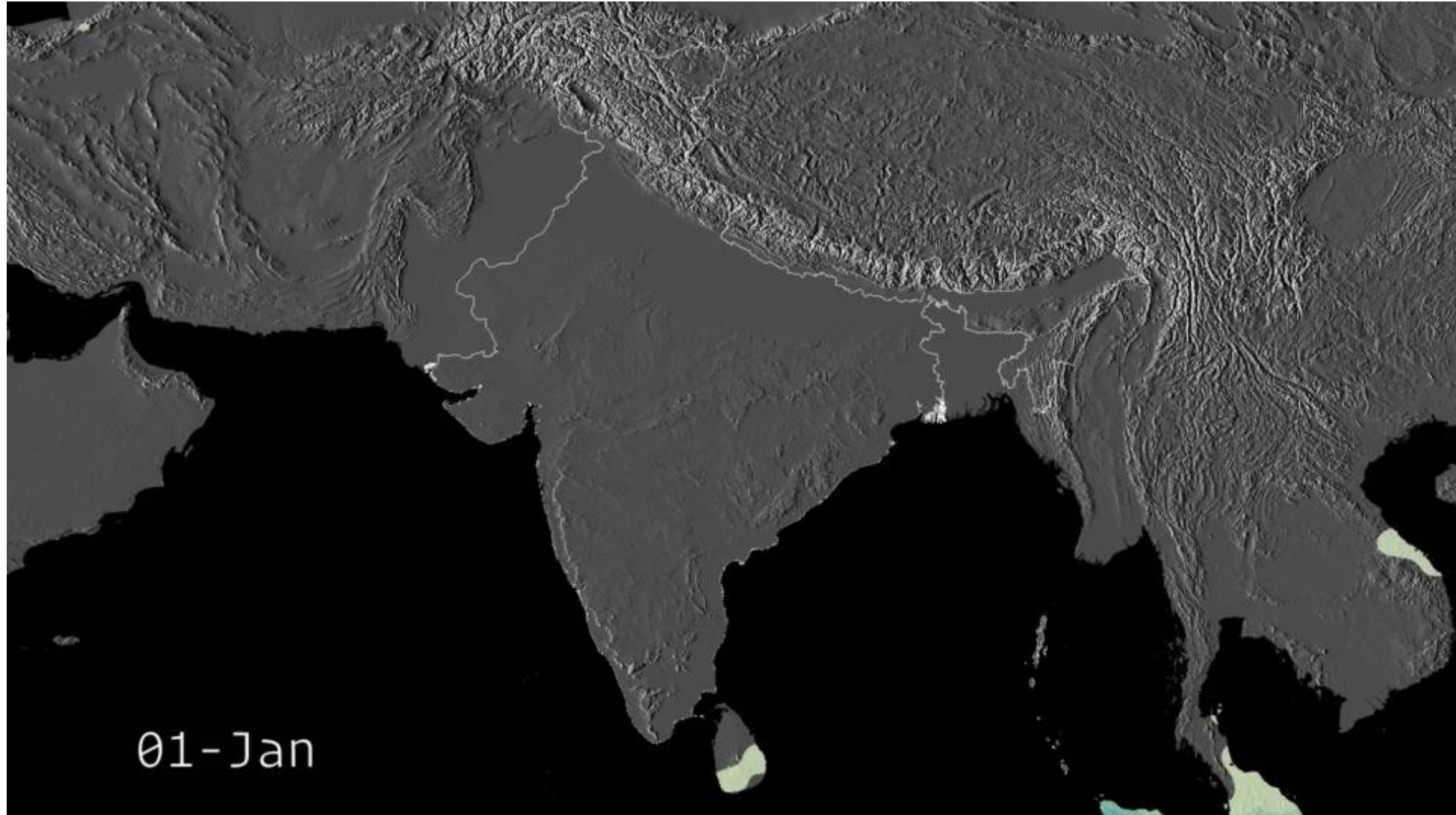
Late Hon. Pranab Mukherjee said “...Monsoons are the real finance minister of India..” – loosely quoted

Table 2.1: Temporal scale of monsoon variability along with factors affecting them.

	Intra-seasonal	Inter-annual Variability	Decadal and century	Millennium and Longer
Features	<i>Active Break cycle of monsoon;</i>	<i>Droughts and Floods</i>	<i>Changes in the frequency of droughts and floods</i>	<i>Changes in the areal extends of monsoon</i>
Factors	<ul style="list-style-type: none"> • <i>Sea surface temperatura /Sea ice</i> • <i>Snow cover</i> • <i>Soil Moisture</i> • <i>Stratospheric Initial conditions</i> • <i>The Madden-Julian oscillation</i> 	<ul style="list-style-type: none"> • <i>El Nino/Southern Oscillation</i> • <i>Local land surface conditions</i> • <i>Remote soil moisture/ snow cover</i> • <i>Sea ice anomalies</i> • <i>Dynamic memory of atmosphere</i> • <i>Stratospheric influences</i> 	<ul style="list-style-type: none"> • <i>Monsoon circulations variations</i> • <i>Deep ocean involvement</i> • <i>Human induced changes</i> • <i>Green house gases increase</i> • <i>Biospheric changes</i> • <i>Volcanic eruptions</i> 	<ul style="list-style-type: none"> • <i>Global climate excursions</i> • <i>Ice ages</i> • <i>Warm epochs</i> • <i>Extra-terrestrial factor (Sun-earth geometry)</i>

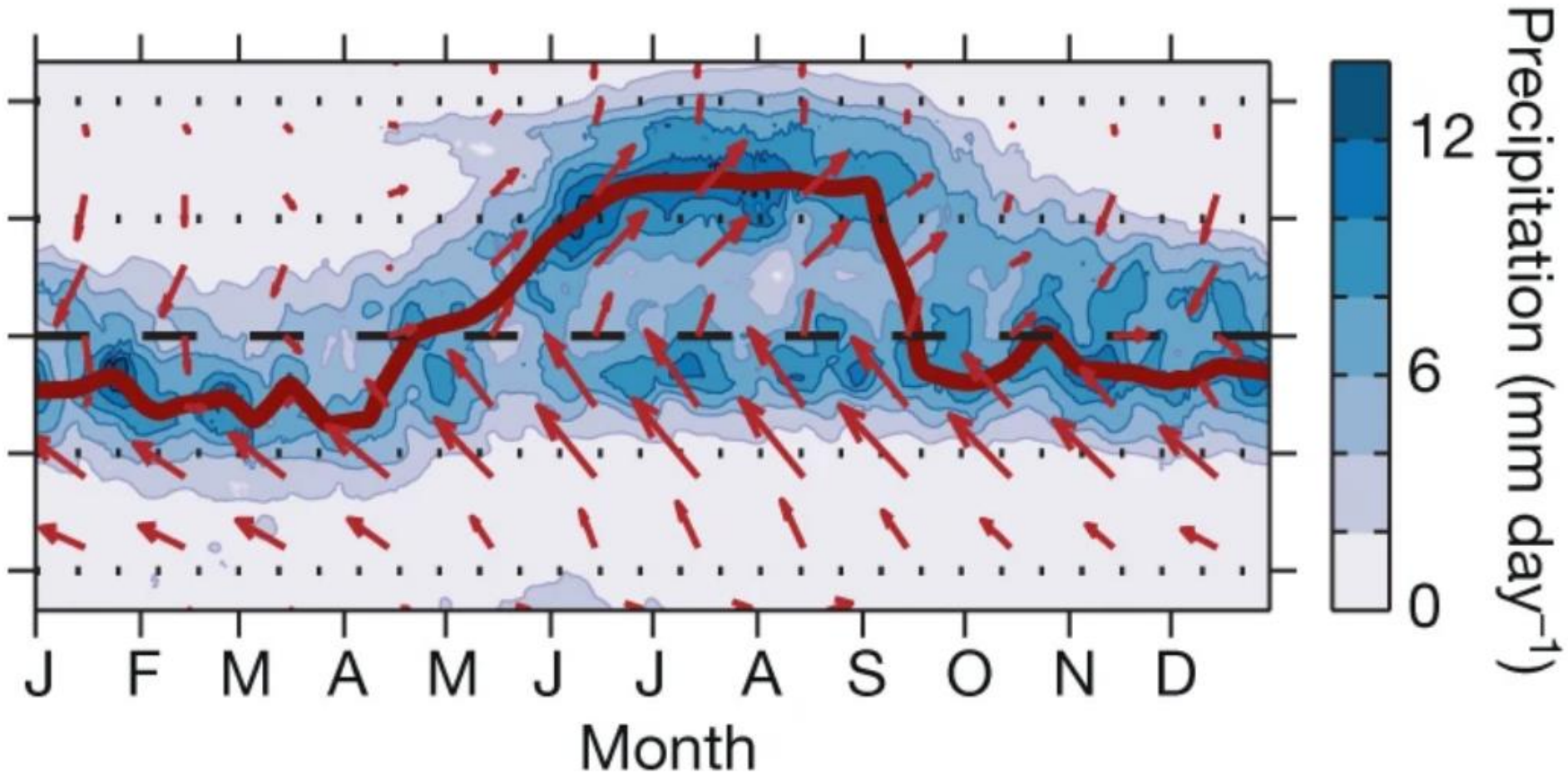
Simple Models for Seasonal Cycle

Monsoon advances from south-east to north-west



Seasonal cycle of Precipitation (65E-95E)

b



Simple Land-Sea Column Contrast Model

Columns under hydrostatic balance

$$\frac{\partial p}{\partial z} = -\frac{g}{R} \frac{p}{\bar{T}}$$

Integrating from $z=0$ to $z=z_1$

$$\Delta \ln p(z_1) = \frac{g}{R} z_1 \left(\frac{1}{\bar{T}_c} - \frac{1}{\bar{T}_w} \right) + \Delta \ln p(0)$$

Threshold for return flow

$$\bar{T}_w > \frac{gz_1 \bar{T}_c}{gz_1 + R \Delta \ln p(0) \bar{T}_c}$$

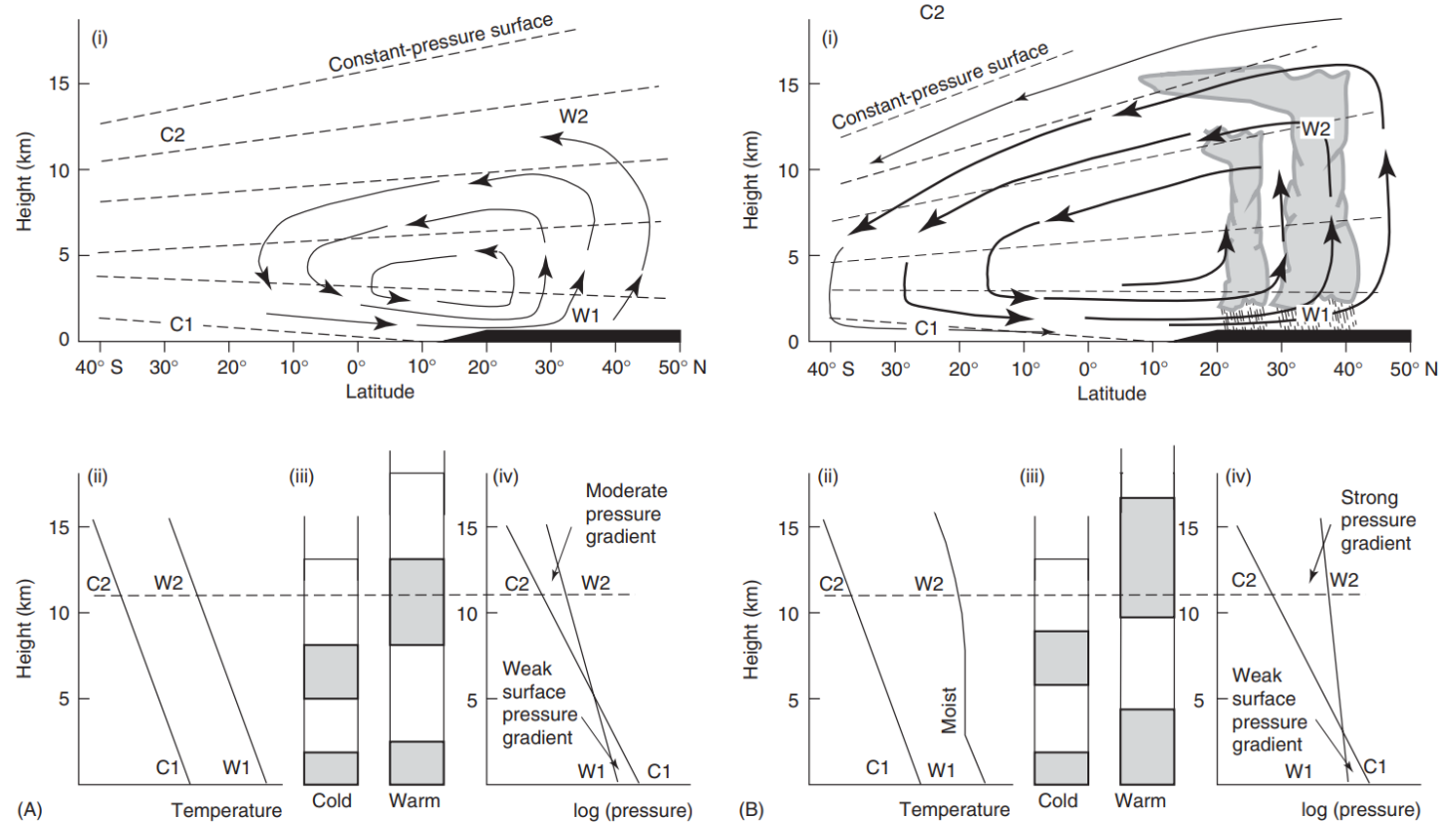
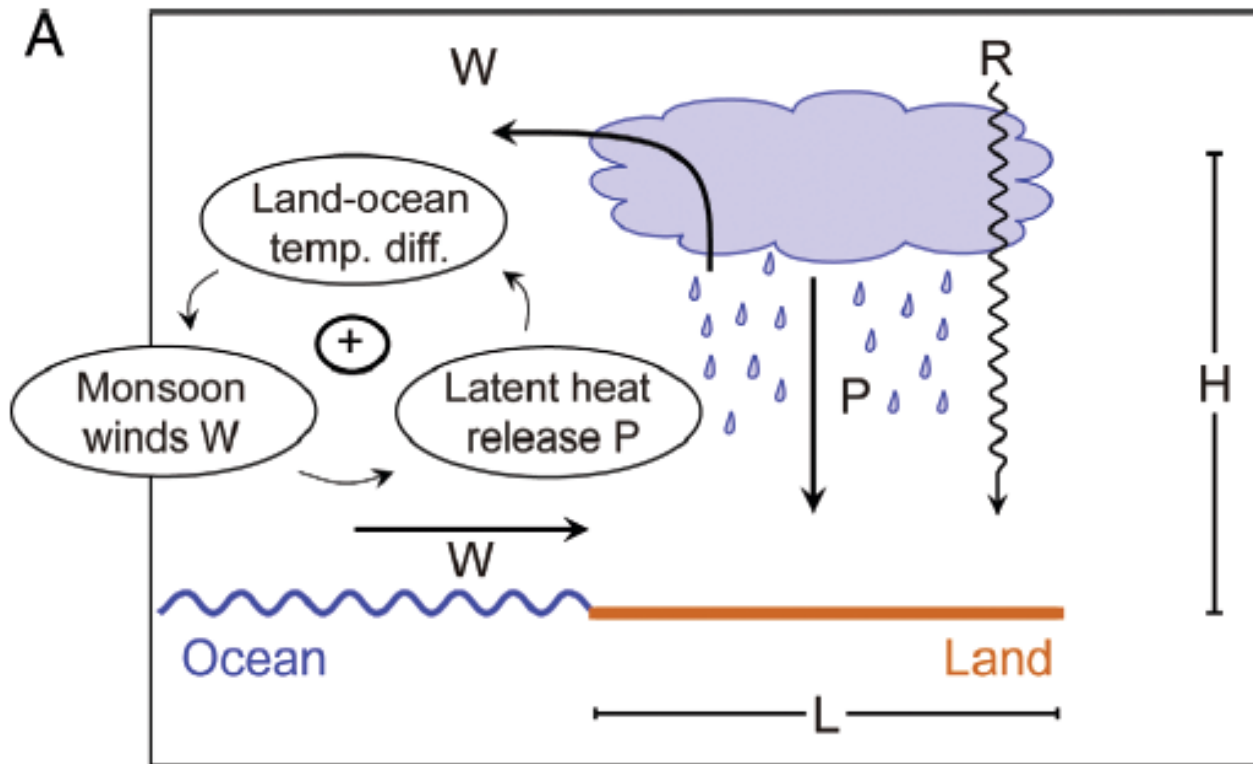


Figure 2 A mechanistic view of the development of the meridional monsoon circulation (A) when moist processes are ignored and (B) when moist processes are taken in to account. The panels show (i) the resultant circulation, (ii) the temperature profiles, (iii) the distribution of mass in the vertical columns, and (iv) the change of pressure with height. Dashed lines in panel (i) show constant-pressure surfaces. Dashed lines in panels (ii)–(iv) denote a constant height. In both examples it is assumed that the difference in temperature of the warm and cold columns is sufficient to generate a reversing pressure gradient with height in the presence of the surface pressure gradient as described in eqn [6]. The figure is discussed extensively in the text.

Major limitation: Precipitation? Condensation heating itself controls T_w

Empirical model to represent changes in Monsoon Rainfall?



[Anders Levermann](#) and his colleagues wrote a system of 4 equations to describe the relationship between Solar Energy input (R), Wind (W), Monsoon Precipitation (P) and lateral heat input (L)

The heat balance over India

- The Heat generated during Monsoon Cloud formation and Precipitation (P) is balanced by Heat emitted away (R) and heat given away to adjacent regions ($W \cdot \Delta T$).

$$\mathcal{L} \cdot P - \epsilon C_p W \cdot \Delta T + R = 0,$$

L, C_p, ϵ are constants

Wind generation

- Strong winds (W) generate when the temperature difference between Land and Ocean (ΔT) is large,
- Larger temperature difference imply larger winds

$$W = \alpha \cdot \Delta T.$$

α is a constant

Moisture balance between land and ocean

- The amount of moisture carried by winds from ocean to land proportional to the difference in humidity over land and Ocean ($q_O - q_L$) and wind strength (W)

$$\epsilon W \cdot \rho(q_O - q_L) - P = 0,$$

ρ, ϵ are constants

Monsoon Precipitation

- Monsoon rainfall (P) is proportional to humidity over land (q_L)

$$P = \beta q_L.$$

Beta is a constant

How do you solve system of equations?

$$\mathcal{L} \cdot P - \epsilon C_p W \cdot \Delta T + R = 0,$$

Energy equation

$$W = \alpha \cdot \Delta T.$$

Momentum equation

$$\epsilon W \cdot \rho(q_O - q_L) - P = 0,$$

Moisture equation

$$P = \beta q_L.$$

Precipitation parameterisation

The relation between wind (w) and Energy input (r)

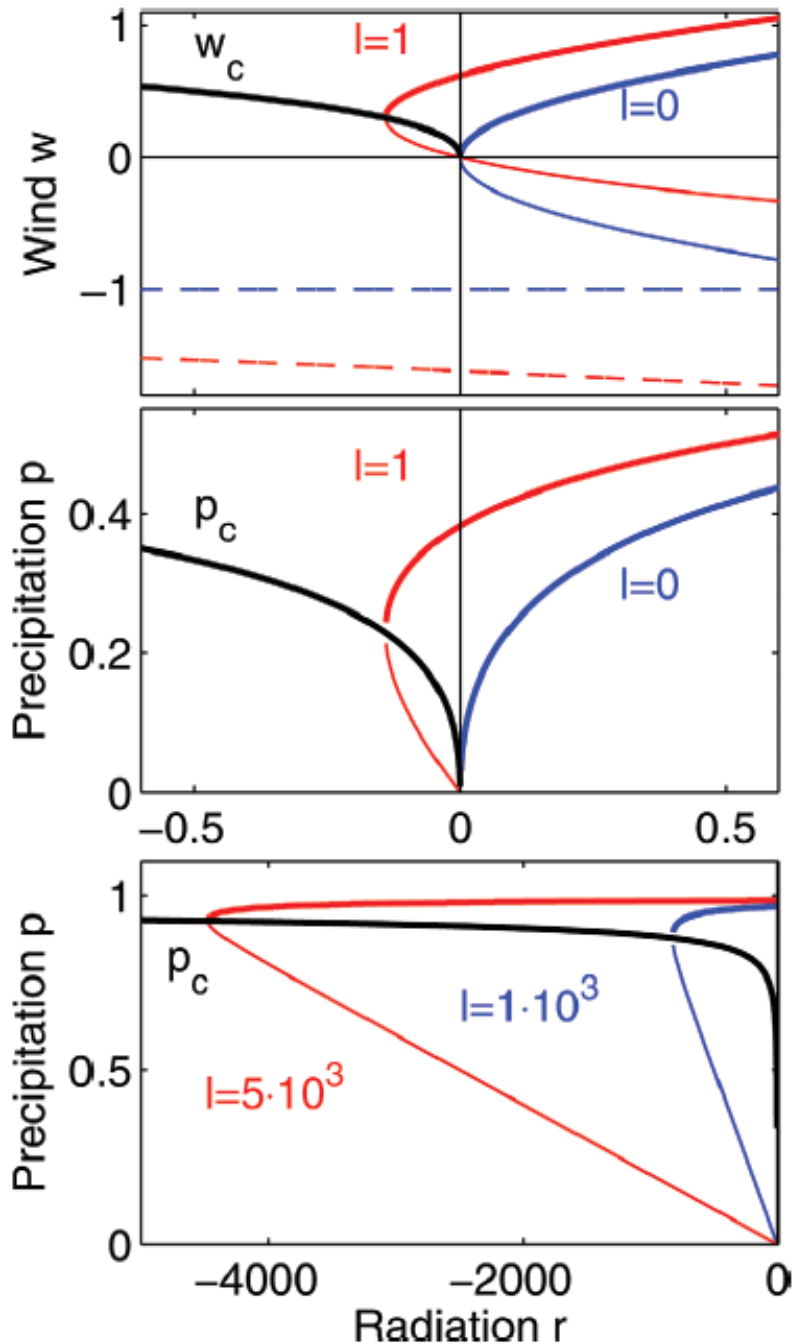
$$W^3 + \frac{\beta}{\epsilon\rho} W^2 - \frac{\alpha}{\epsilon C_p} (\mathcal{L}q_0\beta + R) \cdot W - \frac{\alpha\beta}{\epsilon^2\rho C_p} \cdot R = 0.$$

Clubbing different parameters together:

$$w^3 + w^2 - (l + r)w - r = 0,$$

Measure of ratio of moisture
to heat advection

Measure of radiative energy input



- The cubic equation has 3 solutions for each r , given the value of l .
- Not all solutions are physical (imaginary solutions)
- Quiz: Cubic polynomial has at least ___ real root(s).
- The Red curves suggest that Monsoon will fail below a critical value of r

Criticism on Levermann's model

- In tropical atmosphere the dominant balance is energy equation is not between horizontal advection and diabatic heat sources but that **between vertical advection and diabatic heat sources.**
- When appropriate changes are incorporated, the non-linearity disappears [[Ashwin Seshadri's talk](#)]
- The Climate Model experiments with large changes to radiative energy input do not show non-linear response

ENERGETICS FRAMEWORK

Based on the equations for conservation of moisture & Moist Static Energy.

$$\int_{P_t}^{P_b} \left(\vec{U} \cdot \nabla m + \omega \frac{\partial m}{\partial p} \right) dp = gQ_{net}$$

$$\int_{P_t}^{P_b} \left(\vec{U} \cdot \nabla q + \omega \frac{\partial q}{\partial p} \right) dp = g[E - P]$$

m - Moist Static Energy;

$$(m = C_p * T + g * Z + L_v * q)$$

F_b - Bottom Fluxes

F_t - Fluxes at Top of Atm.

u - x component of velocity

q - Specific Humidity (kg/Kg)

ω - Vertical velocity (Pa/s)

P_b - Surface Pressure

P_t - Top Pressure

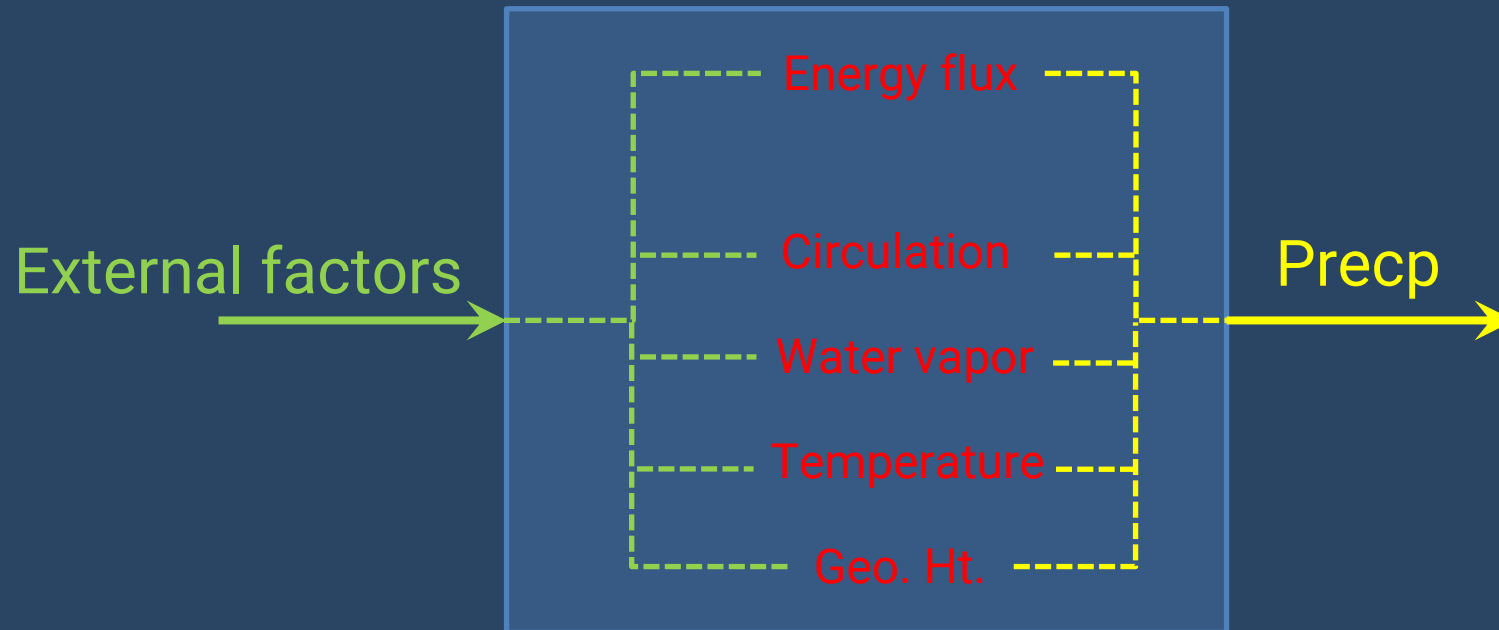
v - y component of velocity

ENERGETICS FRAMEWORK

(Neelin & Held 1987; Raymond 2009)

$$P - E = \frac{Q_{net}}{TGMS}$$

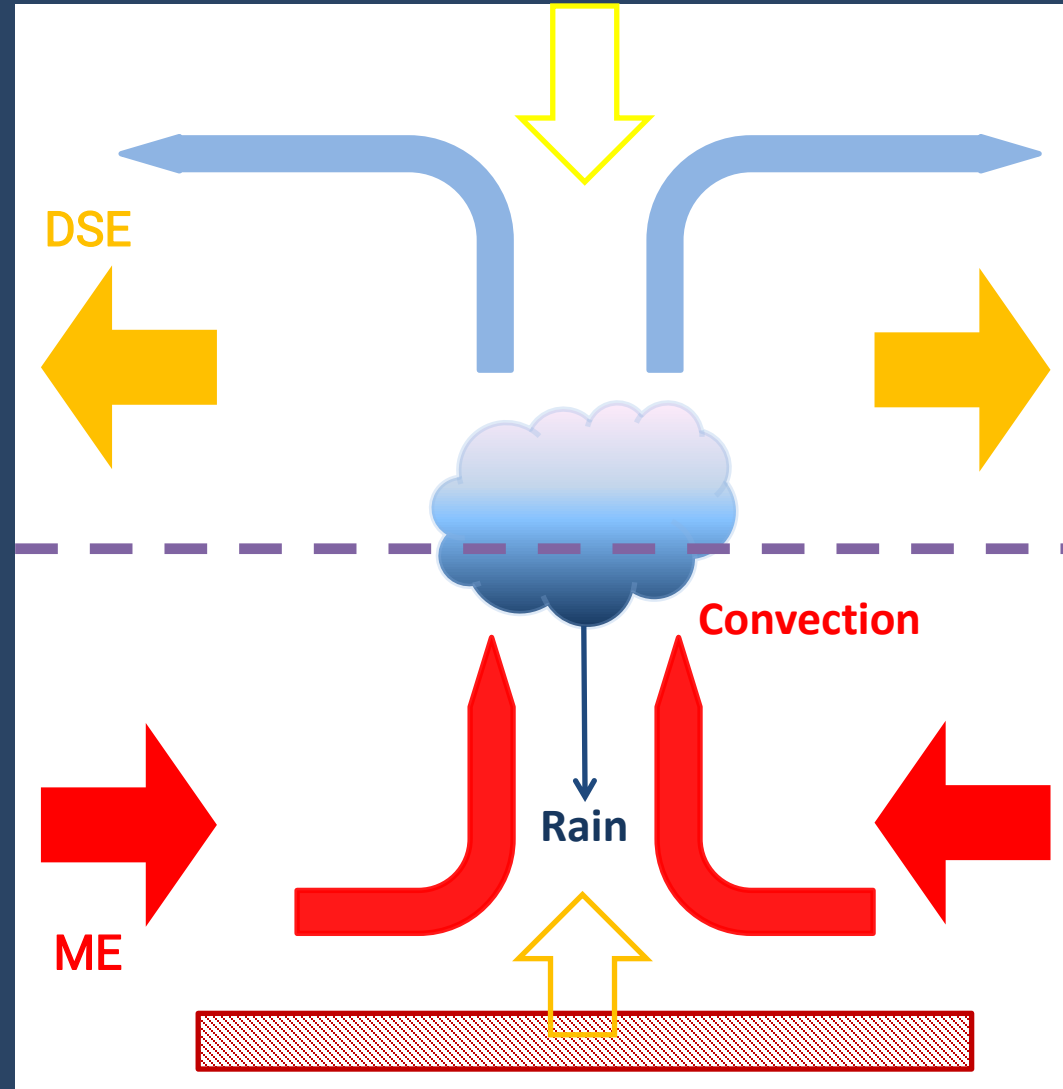
$$TGMS = - \frac{\int_{P_B}^{P_T} \vec{U} \cdot \nabla m + \omega \frac{\partial m}{\partial p} dp}{L_v \int_{P_B}^{P_T} \vec{U} \cdot \nabla q + \omega \frac{\partial q}{\partial p} dp}$$



WHAT ARE MONSOONS?

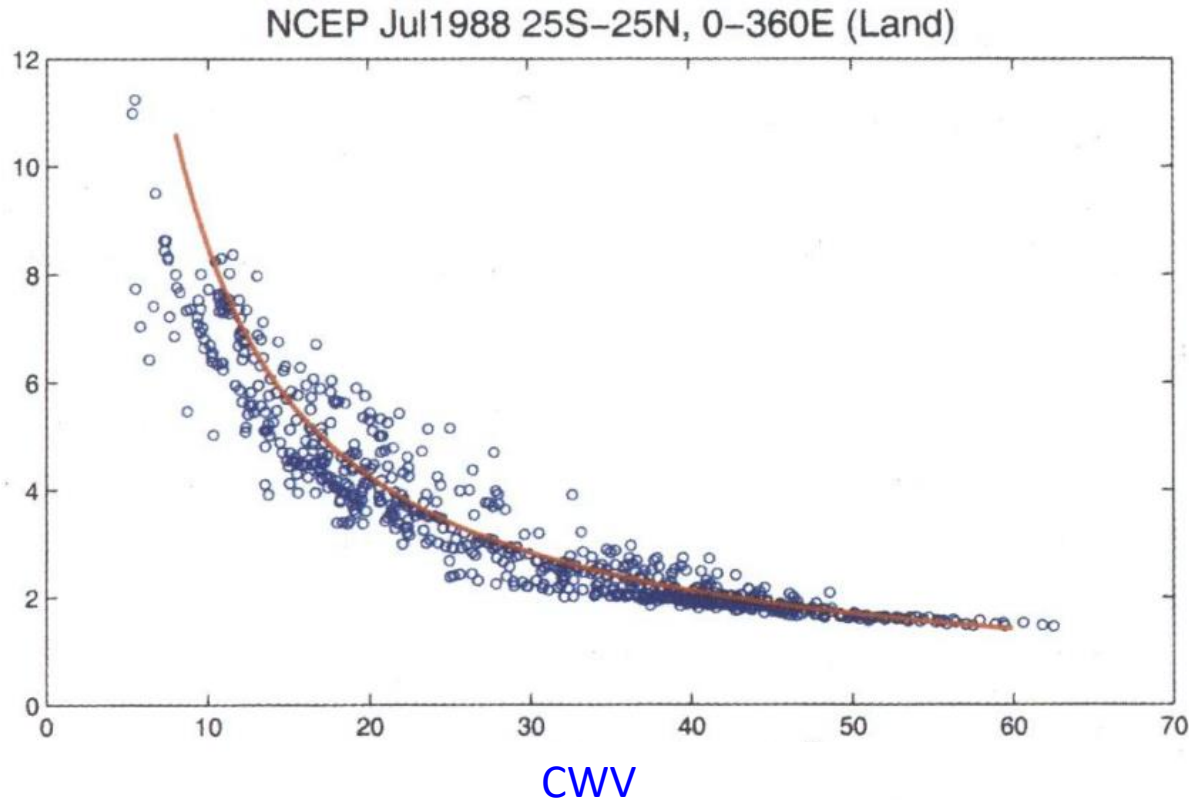
$$P - E = \frac{Q_{net}}{TGMS}$$

Monsoon are Systems that export energy.



Simple model from Energetics

TGMS



$$P = \frac{Q_{net}}{80.43 / CWV - 1.3} + E$$

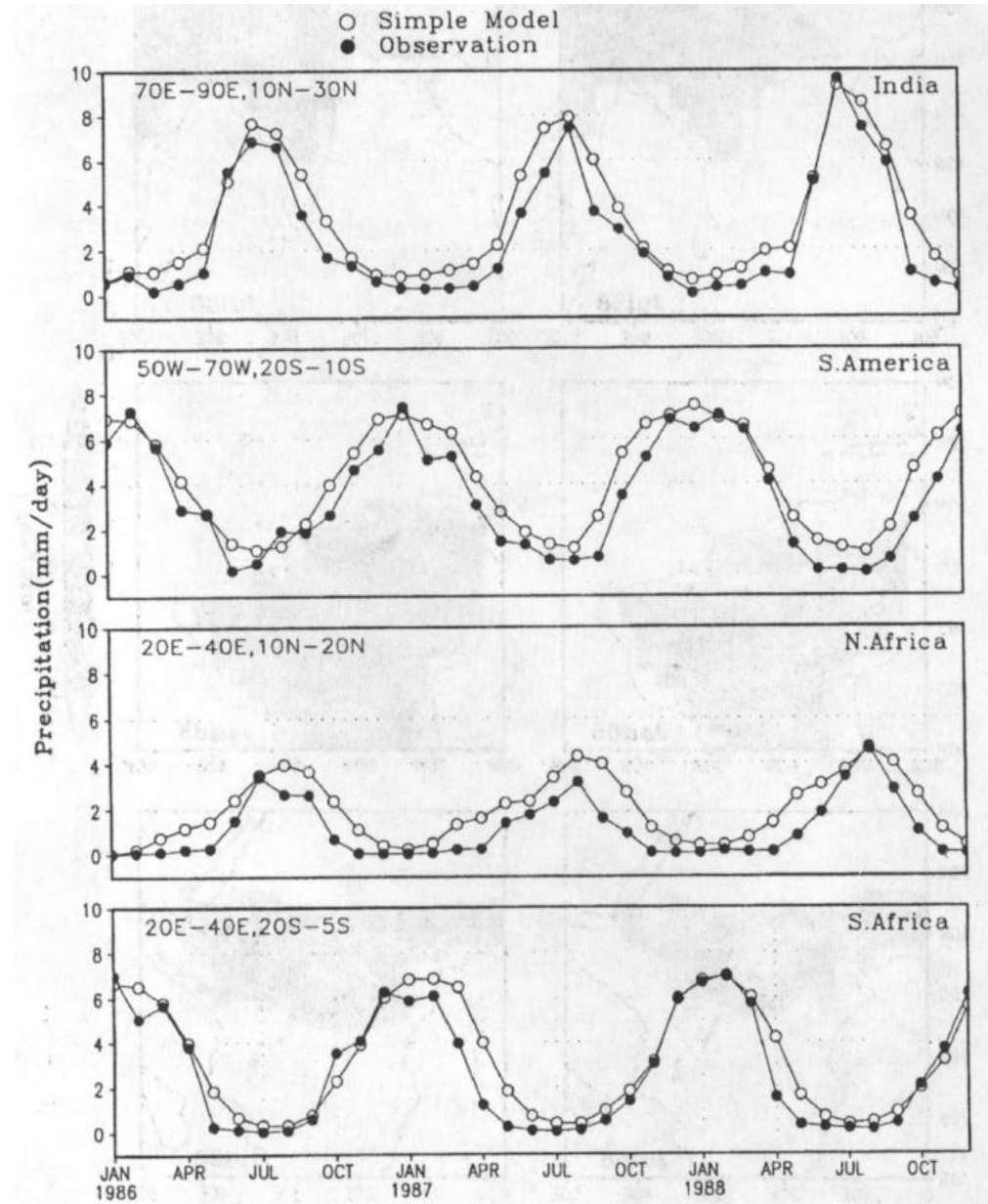
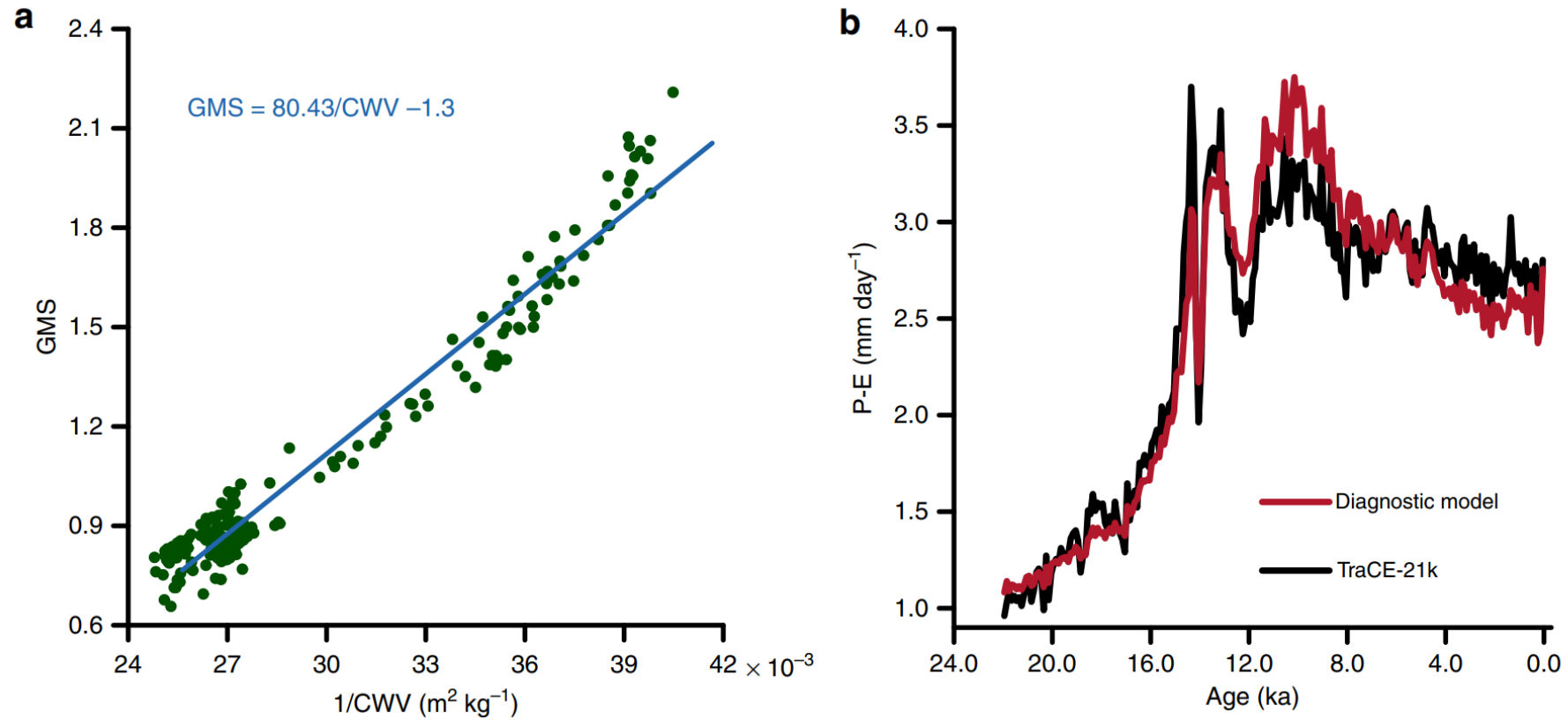


Figure 3. Seasonal variation of monsoon rainfall in tropical continents during 1986 to 1988 from simple model and observations.

Model captures Paleo-Monsoon Variability



Criticism: No dynamics in this model!

Neutral vs forced modes of the Tropical Atmosphere

Single layer simple models for ISO

$$\frac{\partial u}{\partial t} - \beta y v = -\frac{\partial \varphi}{\partial x} + M(u) + Fx + \varepsilon u$$

$$\frac{\partial v}{\partial t} + \beta y u = -\frac{\partial \varphi}{\partial y} + M(v) + Fy + \varepsilon v$$

$$\frac{\partial \varphi}{\partial t} = P + R + M(\varphi) + F\varphi + \mu\varphi$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial p} = 0$$

$$\frac{\partial q}{\partial t} + u \frac{\partial \bar{q}}{\partial x} + v \frac{\partial \bar{q}}{\partial y} + \omega \frac{\partial \bar{q}}{\partial p} = E - P + M(q) + Fq + \nabla^2 q$$

M : effects of mean flow (Seasonality ?),

F : Scale interaction terms (Convection and large-scale flow evolve at different scales ?)

Mean profile \bar{q} is given

Matsuno-Gill Model

$$\frac{\partial u}{\partial t} - \beta y u = -\frac{\partial \phi}{\partial x} + M(u) + Fx + \epsilon u$$

$$\frac{\partial v}{\partial t} + \beta y v = -\frac{\partial \phi}{\partial y} + M(v) + Fy + \epsilon v$$

$$\frac{\partial \phi}{\partial t} + S(p)\omega = P + R + M(\phi) + F\phi + \mu\phi$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial p} = 0$$

~~$$\frac{\partial q}{\partial t} + u \frac{\partial \bar{q}}{\partial x} + v \frac{\partial \bar{q}}{\partial y} + \omega \frac{\partial \bar{q}}{\partial p} = E - P + M(q) + Fq + \nabla^2 q$$~~

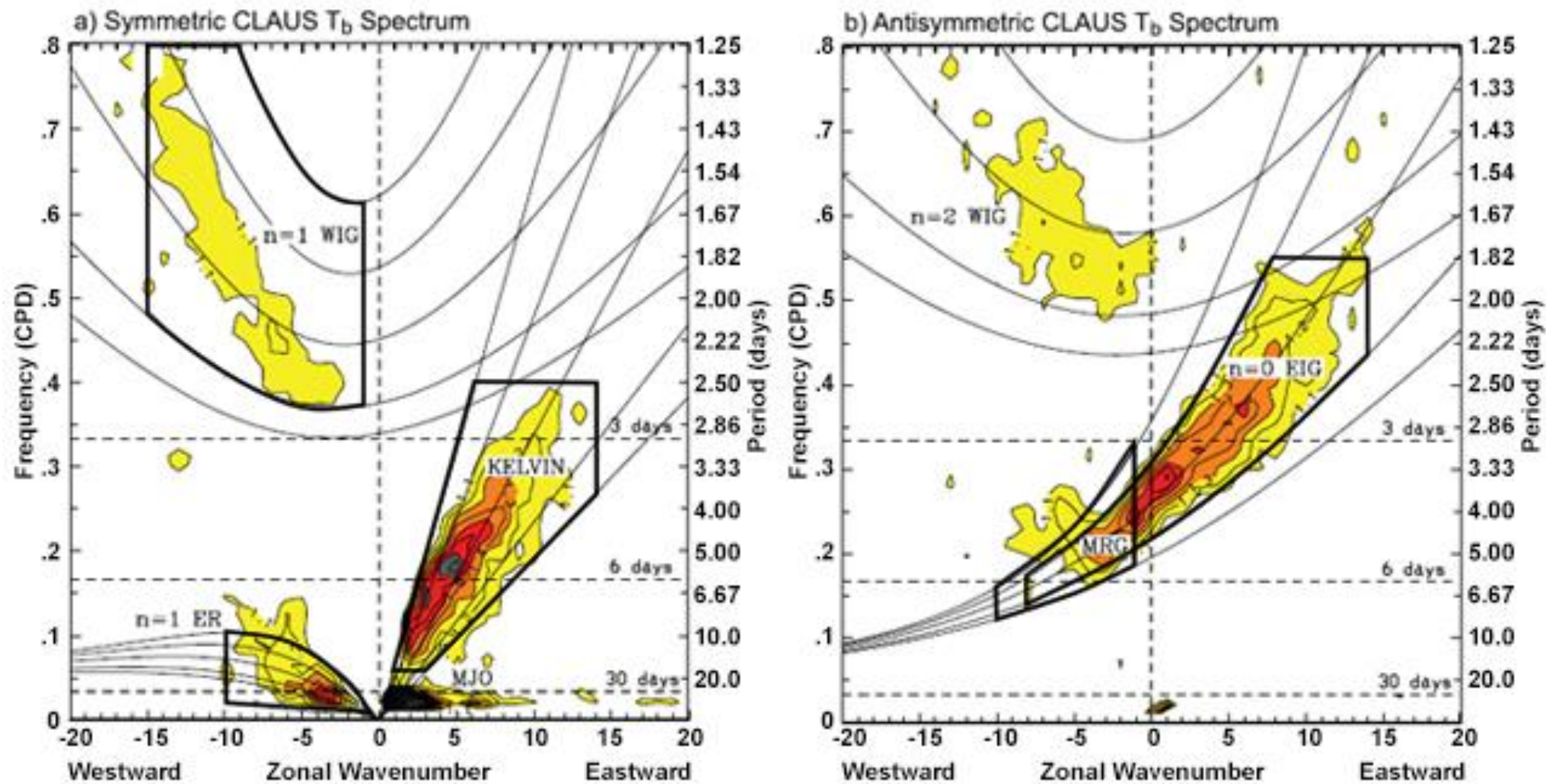
Rayleigh friction,
 Newtonian cooling
 No mean flow
 No scale interaction
 P+R considered as net “forcing”
 “Dry” modes

Matsuno (1966) => P+R = 0 Neutral modes

Gill (1980) => prescribed shape of P+R

M : effects of mean flow, F : Scale
 interaction terms

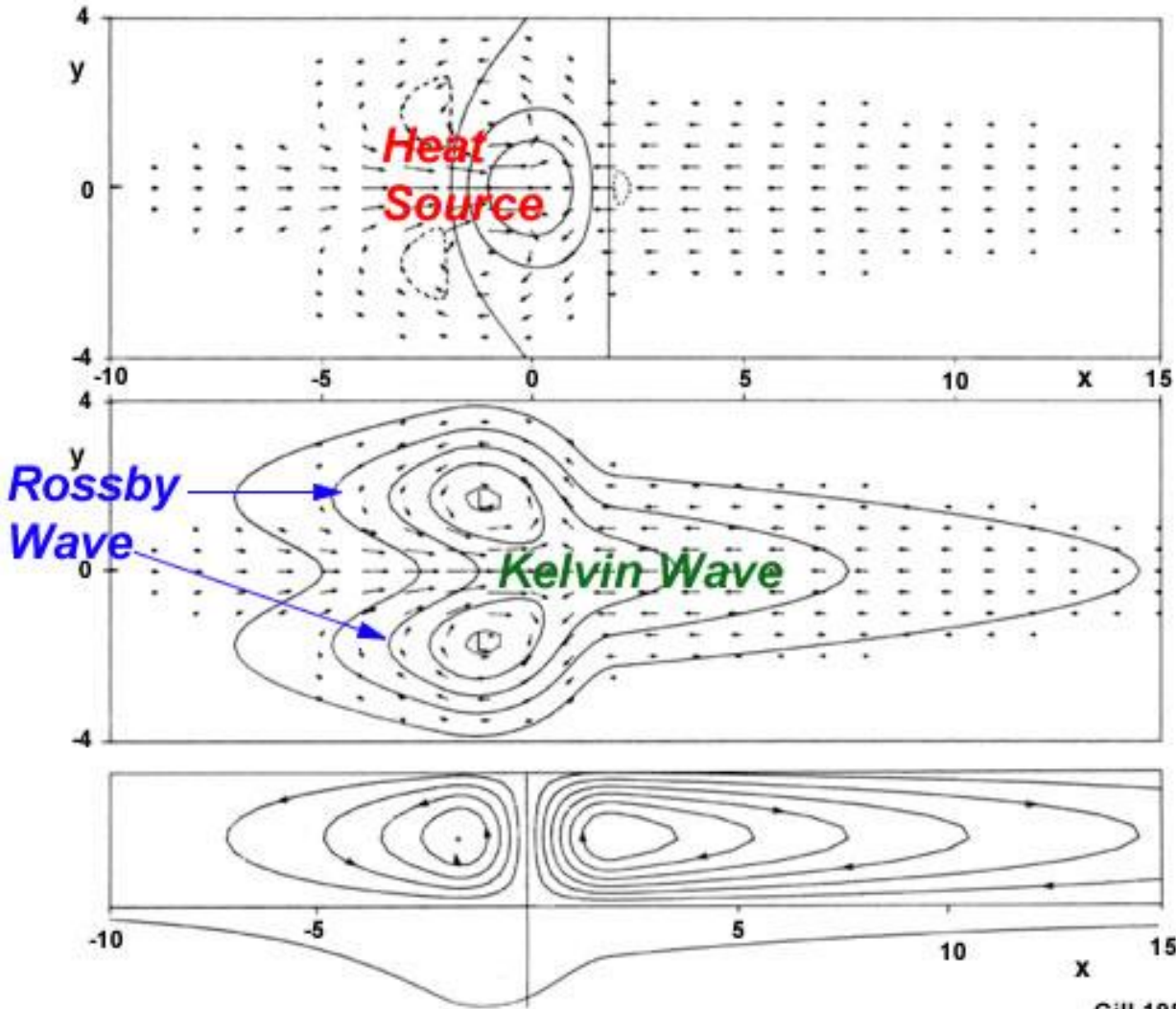
Wavenumber-Frequency Power Spectrum of Brightness Temperature 15N-15S for July 1983 to June 2005



Steady forced modes of tropical atmosphere

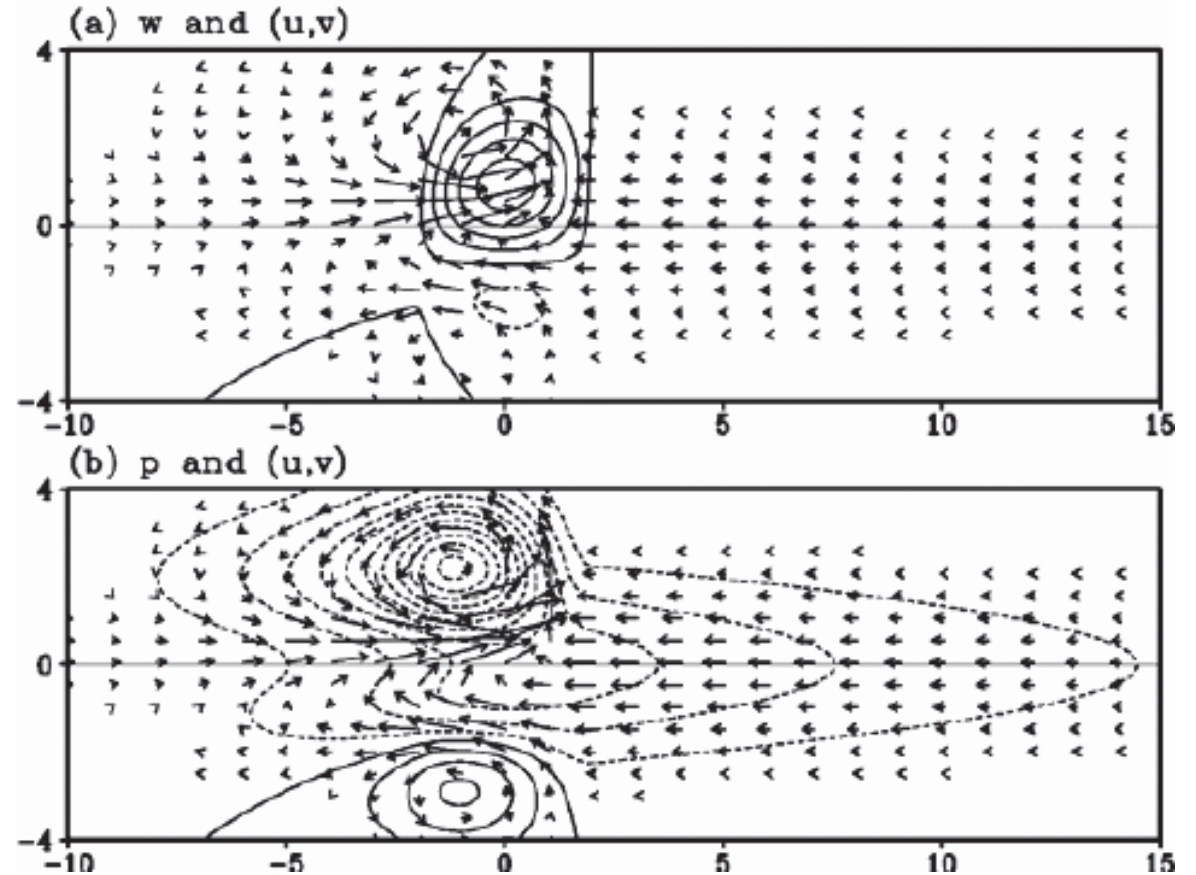
Symmetric forcing

Equatorial Heating and Dynamical Response



Gill 1980

Asymmetric forcing – Monsoon like solution



Simple models for Intra-seasonal Variability

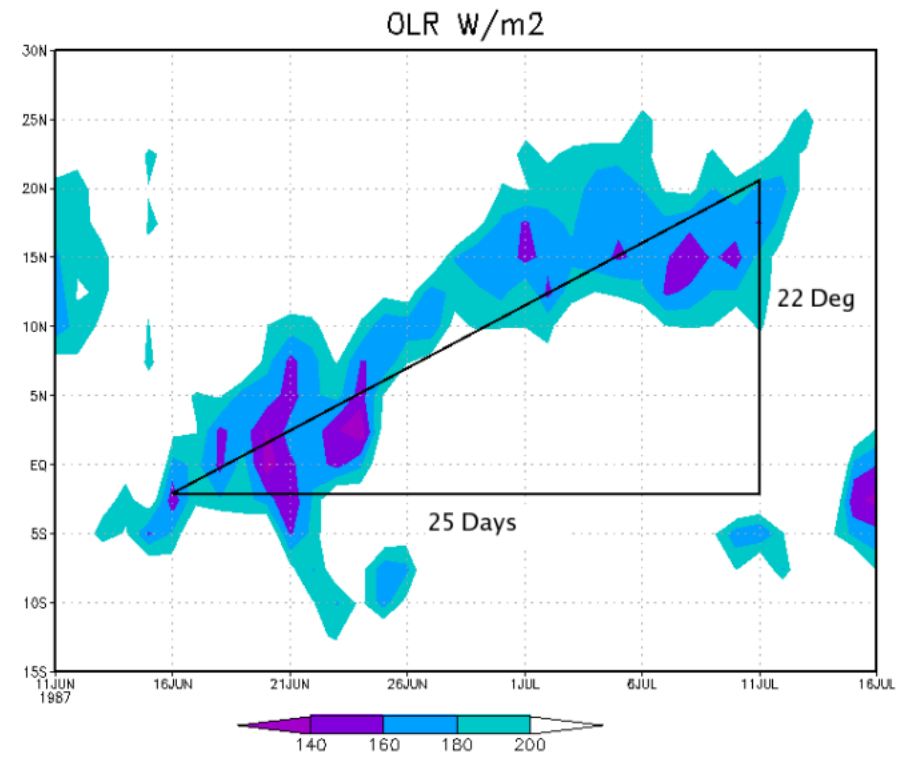
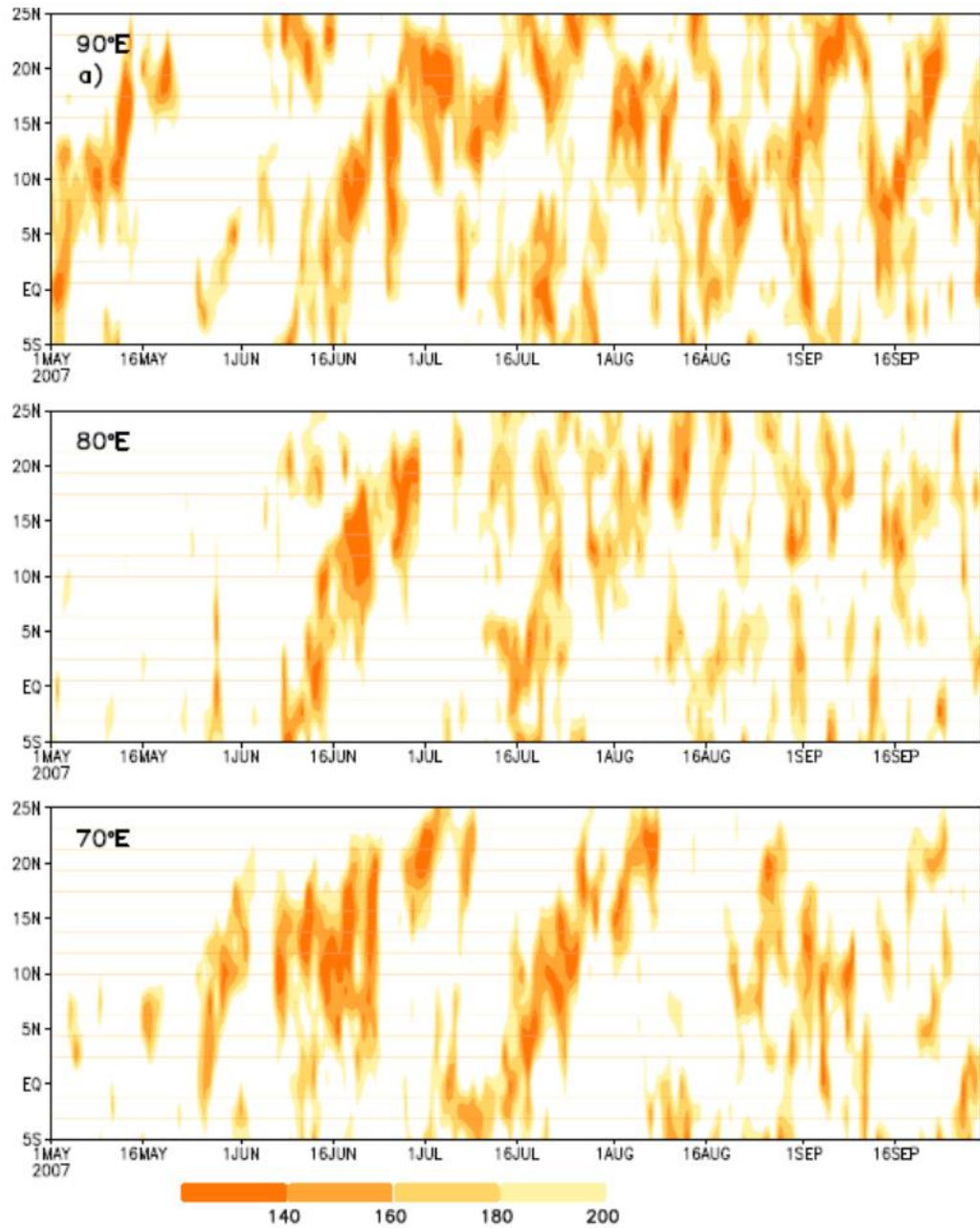
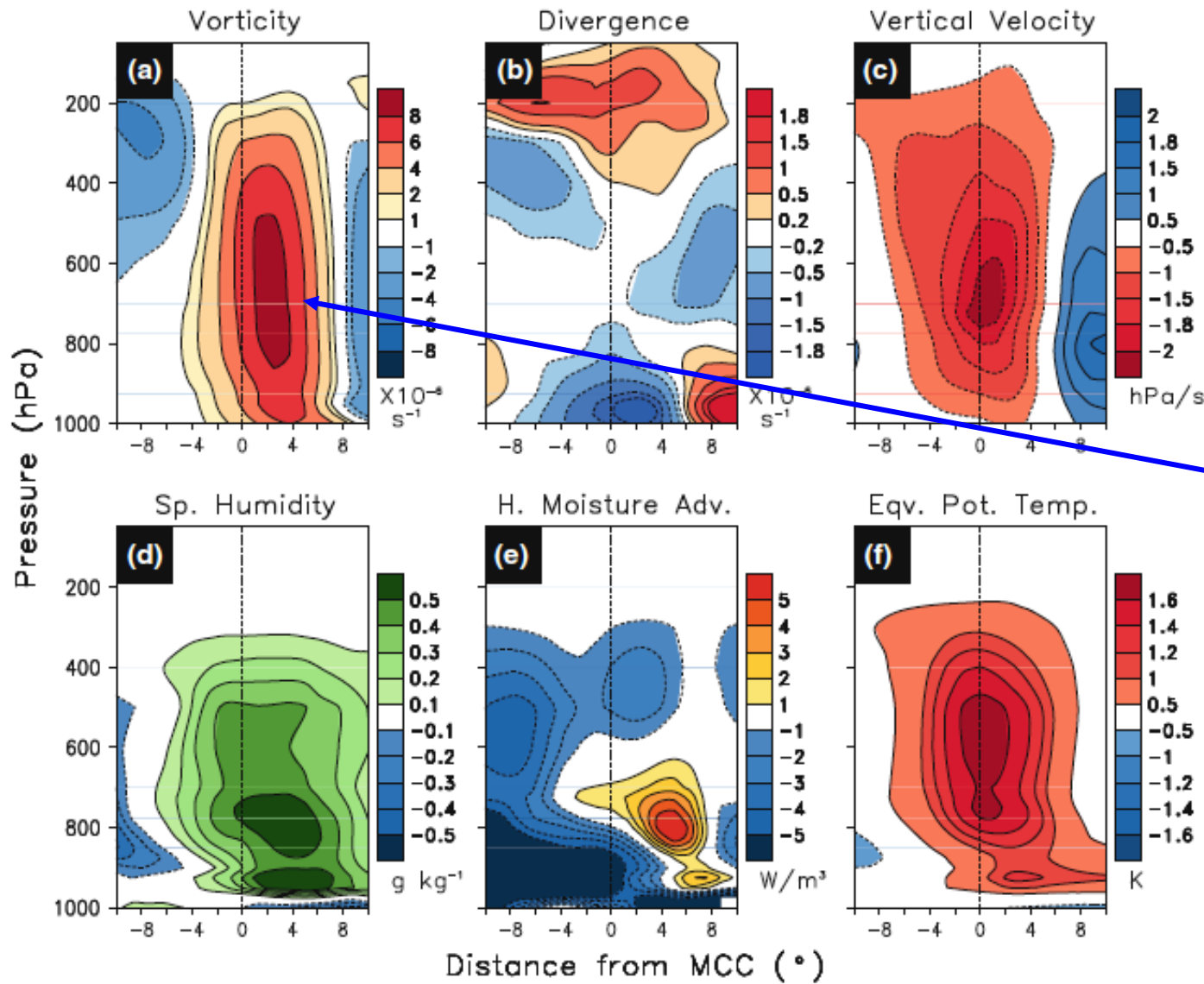


Figure 1.3: Spatial and temporal scale of northward propagation of cloud bands during boreal summer in 1987 at 80°E-90°E

Fig.4.3a : Variation of the low OLR region at 70°, 80°, 90° E during May-September 2007.



Note: Convection centre defined based on max phase of ISO precip

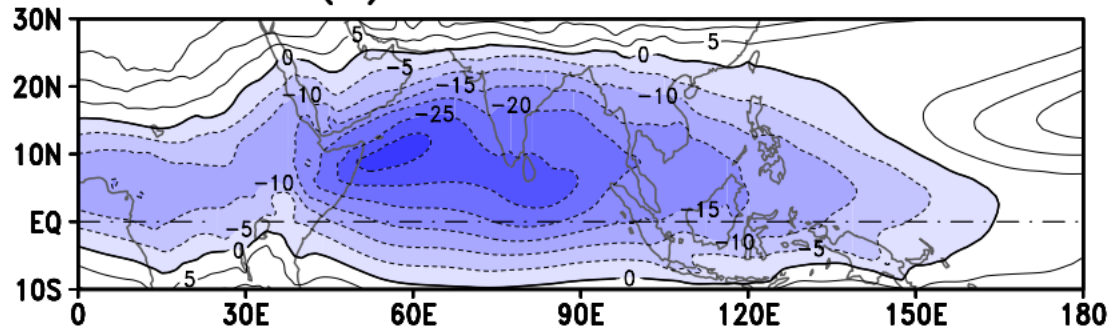
Notice that vorticity is to the north of convection center (x=0)

Fig. 6 Meridional-vertical structures of the northward propagating strong BSISO events for **a** vorticity ($\times 10^{-6} \text{ s}^{-1}$), **b** divergence ($\times 10^{-6} \text{ s}^{-1}$), **c** vertical velocity (hPa s^{-1}), **d** specific humidity (g kg^{-1}), **e** horizontal moisture advection (W m^{-3}) and **f** equivalent

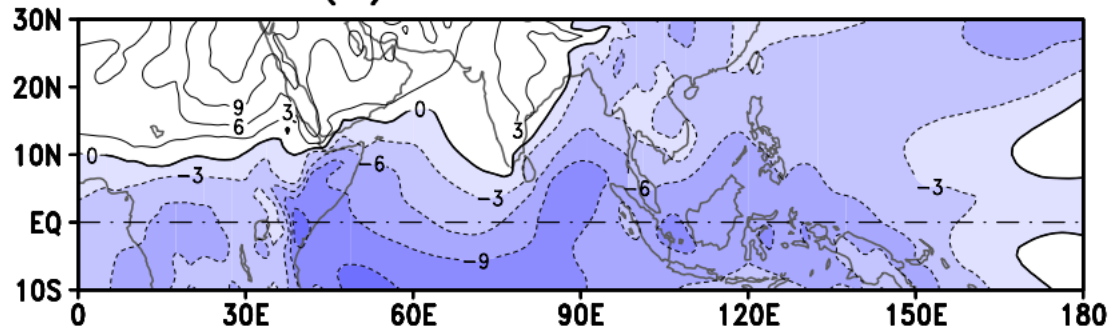
potential temperature (θ_e , K) from *MERRA Reanalysis*. Horizontal axis represents the meridional distance (degree) with respect to maximum convection center (MCC). The +ve (-ve) value means to the north (south) of MCC

Background state for northward propagations

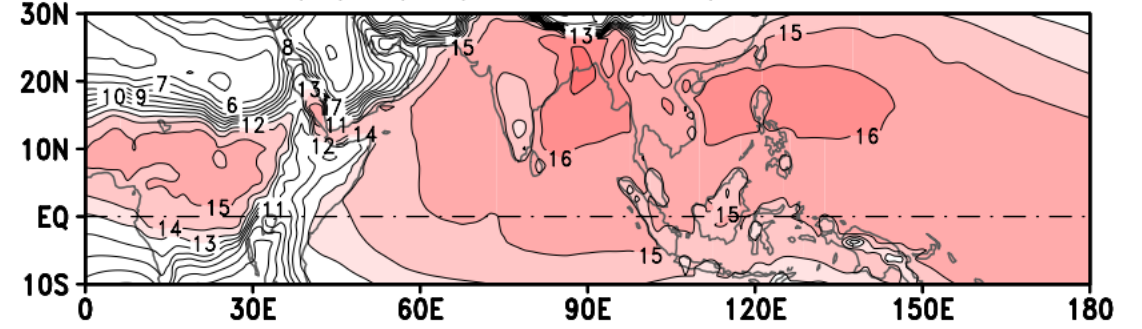
(a) U200-U850 JJASO



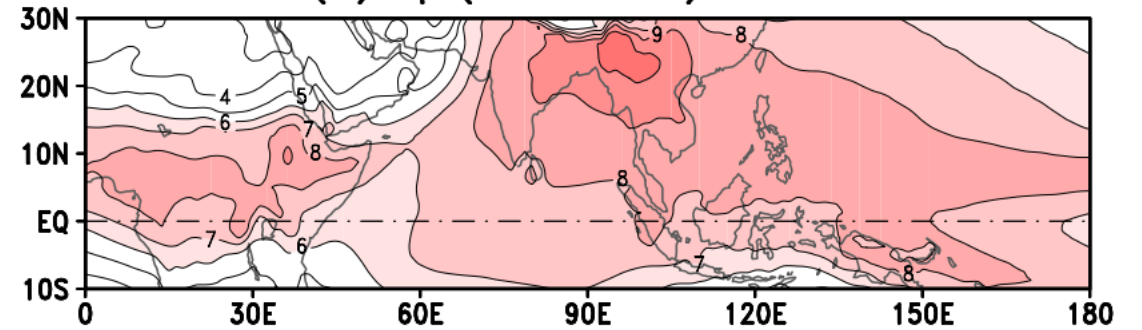
(b) V200-V850 JJASO



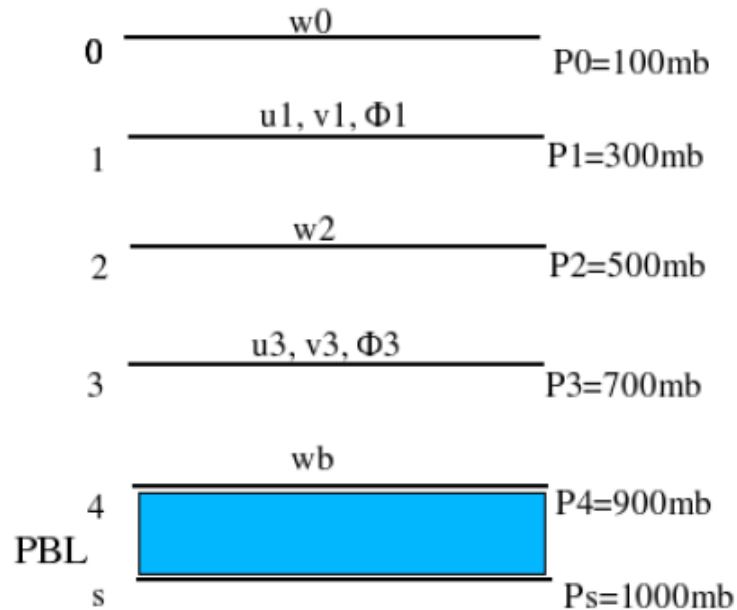
(d) q_B (900-1000) JJASO



(e) q (600-900) JJASO



2.5 layered model



$$\frac{\partial u_1}{\partial t} + \bar{V}_1 \frac{\partial u_1}{\partial y} + w_1 \left(\frac{\partial \bar{U}}{\partial p} \right)_1 - f_0 v_1 = K \nabla^2 u_1$$

$$\frac{\partial u_3}{\partial t} + \bar{V}_3 \frac{\partial u_3}{\partial y} + w_3 \left(\frac{\partial \bar{U}}{\partial p} \right)_3 - f_0 v_3 = K \nabla^2 u_3$$

$$\frac{\partial v_1}{\partial t} + \bar{V}_1 \frac{\partial v_1}{\partial y} + w_1 \left(\frac{\partial \bar{V}}{\partial p} \right)_1 - f_0 u_1 = -\frac{\partial \phi_1}{\partial y} + K \nabla^2 v_1$$

$$\frac{\partial v_3}{\partial t} + \bar{V}_3 \frac{\partial v_3}{\partial y} + w_3 \left(\frac{\partial \bar{V}}{\partial p} \right)_3 - f_0 u_3 = -\frac{\partial \phi_3}{\partial y} + K \nabla^2 v_3$$

Considering hydrostatic approximation,

$$\frac{\partial \phi}{\partial p} = -\frac{RT}{p}$$

Basic state thermal wind Balance,

$$f_0 \frac{\partial \bar{U}}{\partial p} = \frac{R}{p} \frac{\partial \bar{T}}{\partial y}$$

Barotropic and Baroclinic modes

$$\frac{\partial u_+}{\partial t} + \overline{V_T} \frac{\partial u_-}{\partial y} + \overline{U_T} (2D_+ + D_-) - f_0 v_+ = K \nabla^2 u_+$$

$$\frac{\partial u_-}{\partial t} + \overline{V_T} \frac{\partial u_+}{\partial y} - \overline{U_T} D_+ - f_0 v_- = K \nabla^2 u_-$$

$$\frac{\partial v_+}{\partial t} + \overline{V_T} \frac{\partial v_-}{\partial y} + \overline{V_T} (2D_+ + D_-) + f_0 u_+ = -\frac{\partial \phi_+}{\partial y} + K \nabla^2 v_+$$

$$\frac{\partial v_-}{\partial t} + \overline{V_T} \frac{\partial v_+}{\partial y} - \overline{V_T} (D_+) + f_0 u_- = -\frac{\partial \phi_-}{\partial y} + K \nabla^2 v_-$$

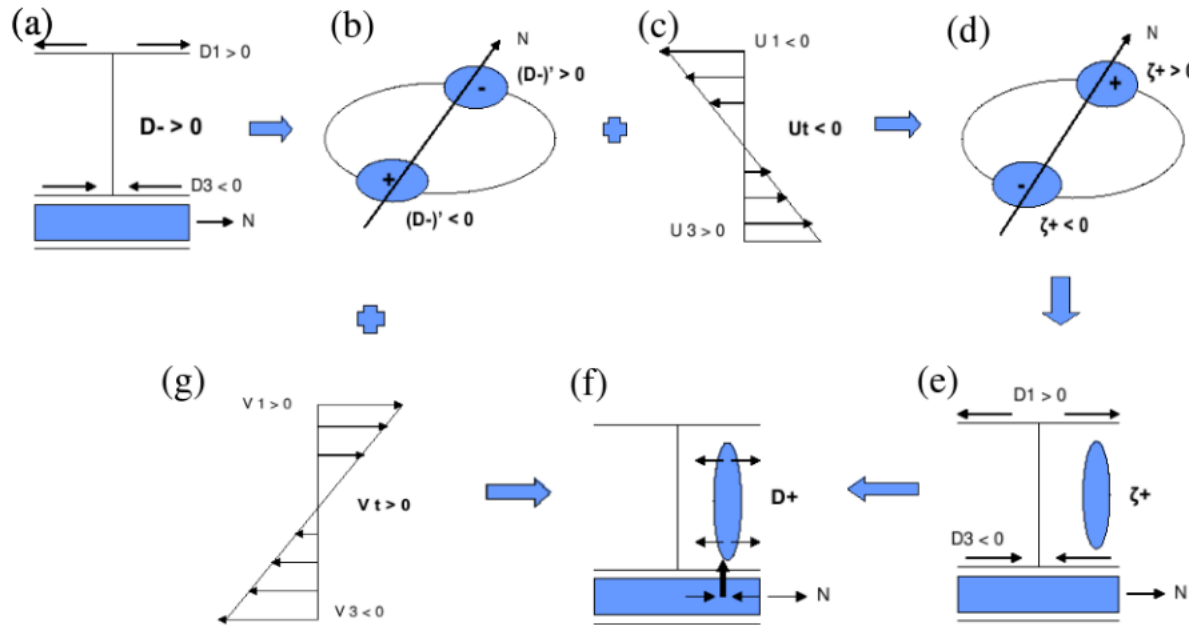
$$A_+ = \frac{A_1 + A_3}{2} \quad \text{Barotropic Mode}$$

$$A_- = \frac{A_1 - A_3}{2} \quad \text{Baroclinic Mode}$$

Factors leading to generation of Barotropic vorticity

$$\begin{aligned}
 & \frac{\partial \zeta_+}{\partial t} + \overbrace{\frac{\partial \zeta_-}{V_T \partial y}}^{\text{advection}} - \overbrace{\frac{\partial(2D_+ + D_-)}{U_T \partial y}}^{\text{tilting}} + \overbrace{f_0 D_+}_{\text{stretching}} = K \nabla^2 \zeta_+ \\
 & \frac{\partial \zeta_-}{\partial t} + \overline{V_T} \frac{\partial \zeta_+}{\partial y} + \overline{U_T} \frac{\partial D_+}{\partial y} + f_0 D_- = K \nabla^2 \zeta_- \\
 & \frac{\partial D_+}{\partial t} + \overline{V_T} \frac{\partial D_-}{\partial y} + \overline{V_T} \frac{\partial(2D_+ + D_-)}{\partial y} - f_0 \zeta_+ = -\nabla^2 \phi_+ + K \nabla^2 D_+ \\
 & \frac{\partial D_-}{\partial t} - f_0 \zeta_- = -\nabla^2 \phi_- + K \nabla^2 D_-
 \end{aligned}$$

Vertical shear mechanism



$$\frac{\partial \zeta_+}{\partial t} + \overline{U_T} \frac{\partial D_-}{\partial y} = 0$$

$$\frac{\partial D_+}{\partial t} + 2\overline{V_T} \frac{\partial D_-}{\partial y} - f_0 \zeta_+ = 0$$

Figure 4.4: Schematic diagram for the mechanism of instability: a) Consider the convection with baroclinic structure b) Negative gradient of baroclinic vorticity is generated to the north of the convection center c) Easterly shear in mean zonal winds d) induces barotropic vorticity to the north of the convection center e) With Coriolis force barotropic vorticity at the north f) generates barotropic divergence in free troposphere towards north of the convection g) Southerly shear in meridional winds acting on negative gradient of baroclinic vorticity produces barotropic divergence towards north of the convection center.

Alternate: BL Moisture advection mechanism

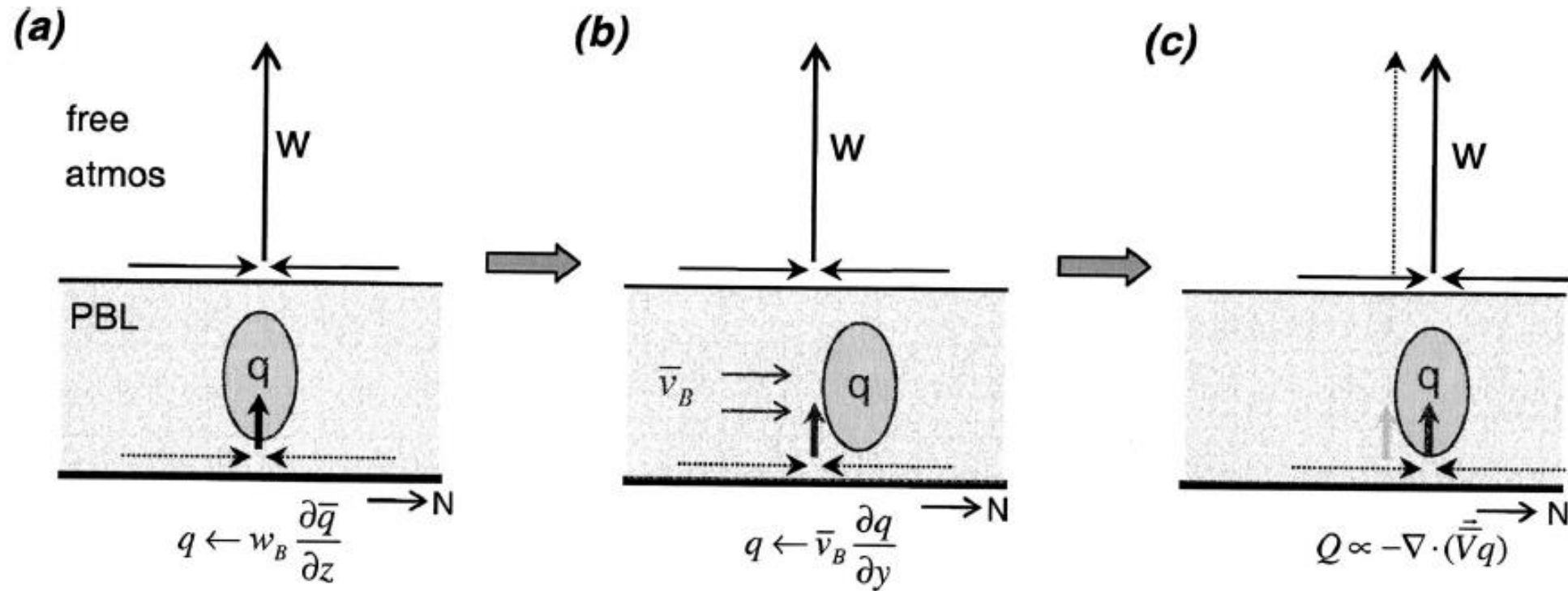
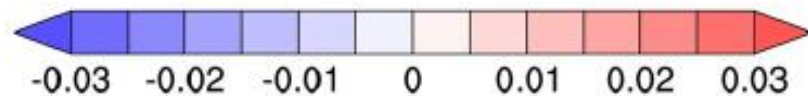
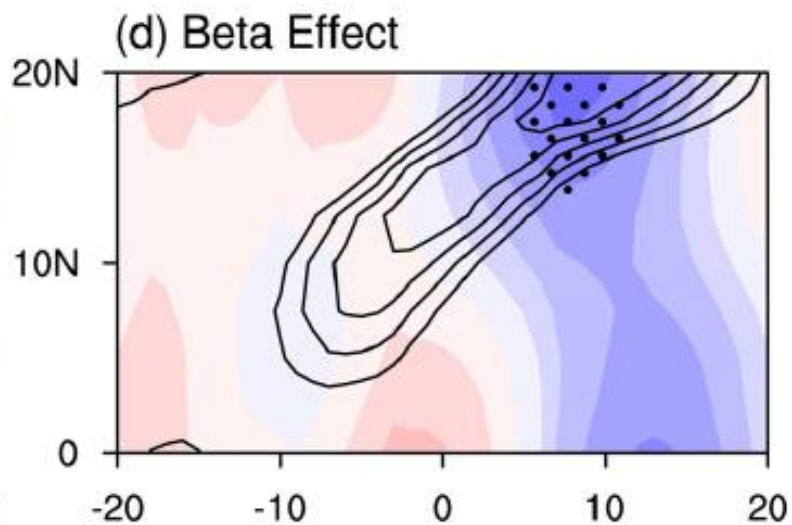
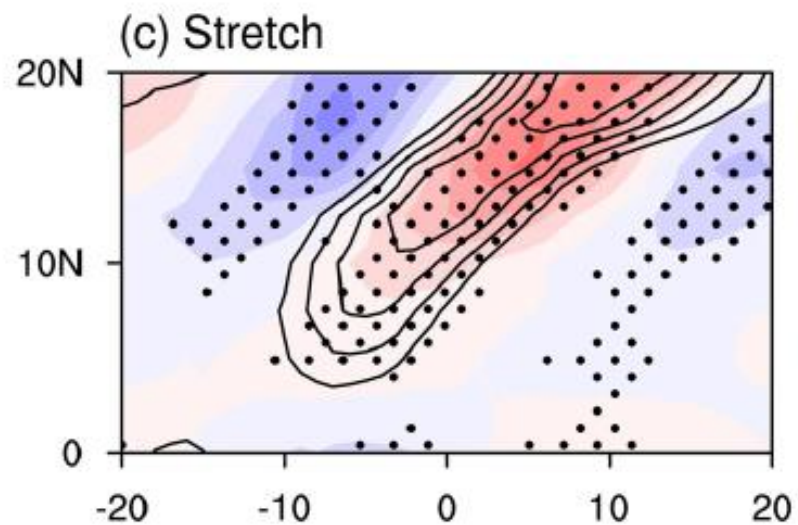
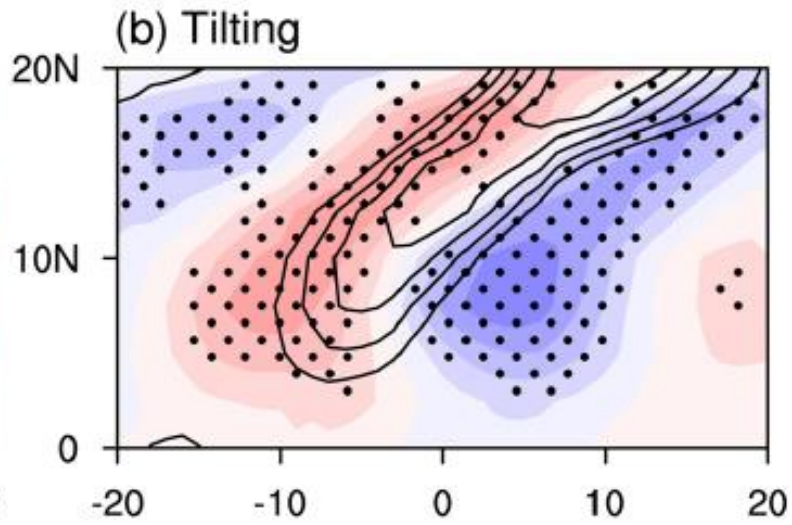
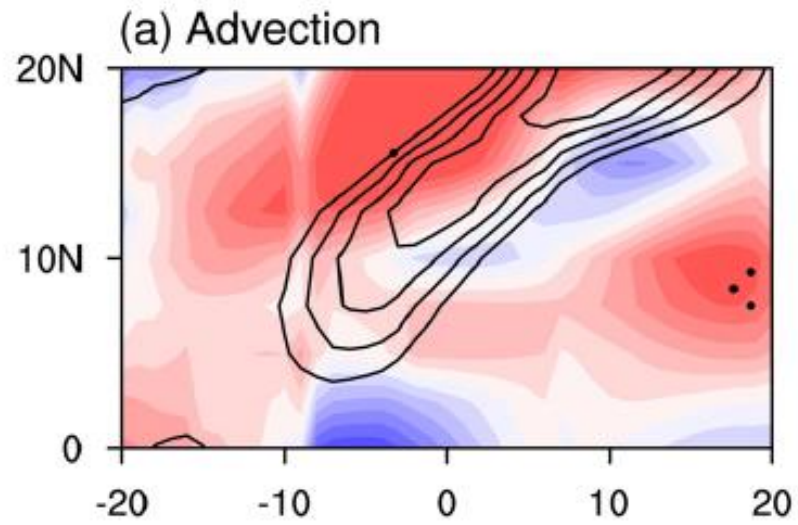


FIG. 12. Schematic diagram for the mechanism of moisture advection by mean flow. (a) The specific humidity perturbation caused by Ekman pumping is advected (b) by the mean northward meridional wind in the PBL, (c) which leads to the northward shift of moisture convergence and thus convective heating to the convection center.



Contours: ISO Vorticity

Shade: Different terms

X-axis: Days lead-Lag

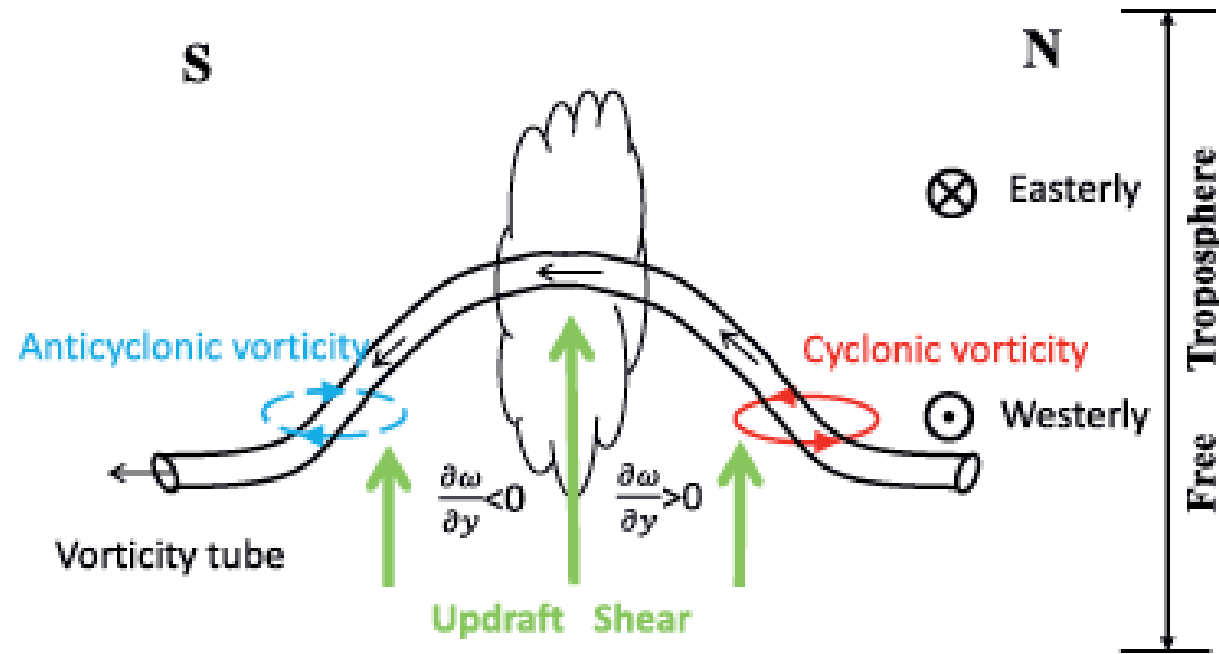


Figure 4. Sketch for the northward propagation of the monsoon intraseasonal oscillation. The green arrows denote the vertical velocities associated with the intraseasonal convection; the vortex tube is induced by the vertical shear of the background zonal wind, as westerly (outward arrow) in the lower troposphere and the easterly (inward arrow) in the upper troposphere; the black thin arrows within the tube denote the direction of the background horizontal vortex tube; The red (blue) cycle denotes the cyclonic (anticyclonic) vorticity tilted by the meridional vertical wind shear at the north (south) of the convection.

Notice that dw/dy structure is key to this mechanism

Proposed hypothesis from different studies

$$\frac{\partial \zeta_+}{\partial t} = \underbrace{U_T \frac{\partial(2D_+ + D_-)}{\partial y}}_{\text{Mean Baroclinic tilting} \quad \mathbf{1}} - \underbrace{V_T \frac{\partial \zeta_-}{\partial y}}_{\text{Mean Baroclinic adv.} \quad \mathbf{2}} + \underbrace{v_+ \frac{\partial \bar{\zeta}_+}{\partial y}}_{\text{Barotropic adv.} \quad \mathbf{3}} - \underbrace{f D_+}_{\text{Stretching} \quad \mathbf{4}}$$

1. Jiang et al. 2004, Drbohlav and Wang 2005, Li et al. 2021

2. Bellon and Sobel, 2008

1+2 Dixit and Srinivasan, 2011

3. Qin, Li, Zhou and Murtugudde 2021

Proposed hypothesis from different studies

$$\frac{\partial q}{\partial t} \sim \underbrace{-v' \frac{\partial \bar{q}}{\partial y}}_1 \underbrace{-\bar{v} \frac{\partial q'}{\partial y}}_2 \underbrace{-\bar{u} \frac{\partial q'}{\partial x}}_3 - u' \frac{\partial \bar{q}}{\partial x}_4$$

1. Jiang et al. 2004, Karmakar and Mishra, 2020, Srinivasan et al. 1993
(MSE budget framework)

2. Wang and Li, 2021, In MSE budget framework

1+2 Pillai and Sahai 2016

3. Li et al. 2012, For the first branch northward propagation of season

Srinivasan and Smith (1993) WISHE model

$$U_t + \bar{U}U_x - yV + \Phi_x = 0, \quad (1)$$

$$V_t + \bar{U}V_x + yU + \Phi_y = 0, \quad (2)$$

$$\begin{aligned} \Phi_t + \bar{U}\Phi_x + (U_x + V_y) \\ = B\{U_x + V_y\} + A\{U\}. \end{aligned} \quad (3)$$

$$V_{yy} - yDV_y + \left(G - \frac{y^2}{\chi}\right)V = 0,$$

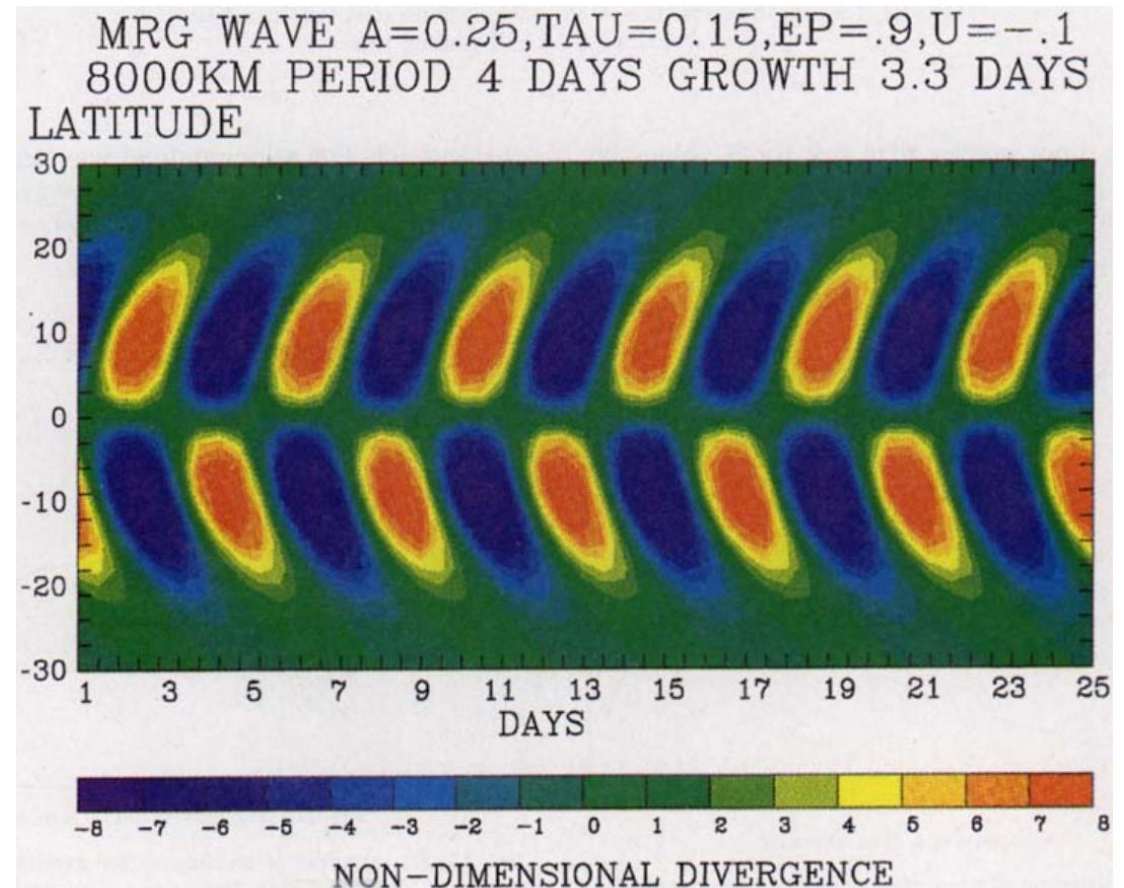
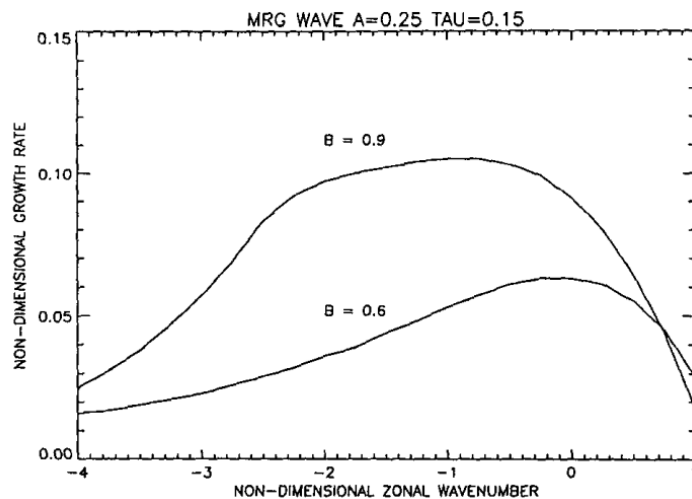
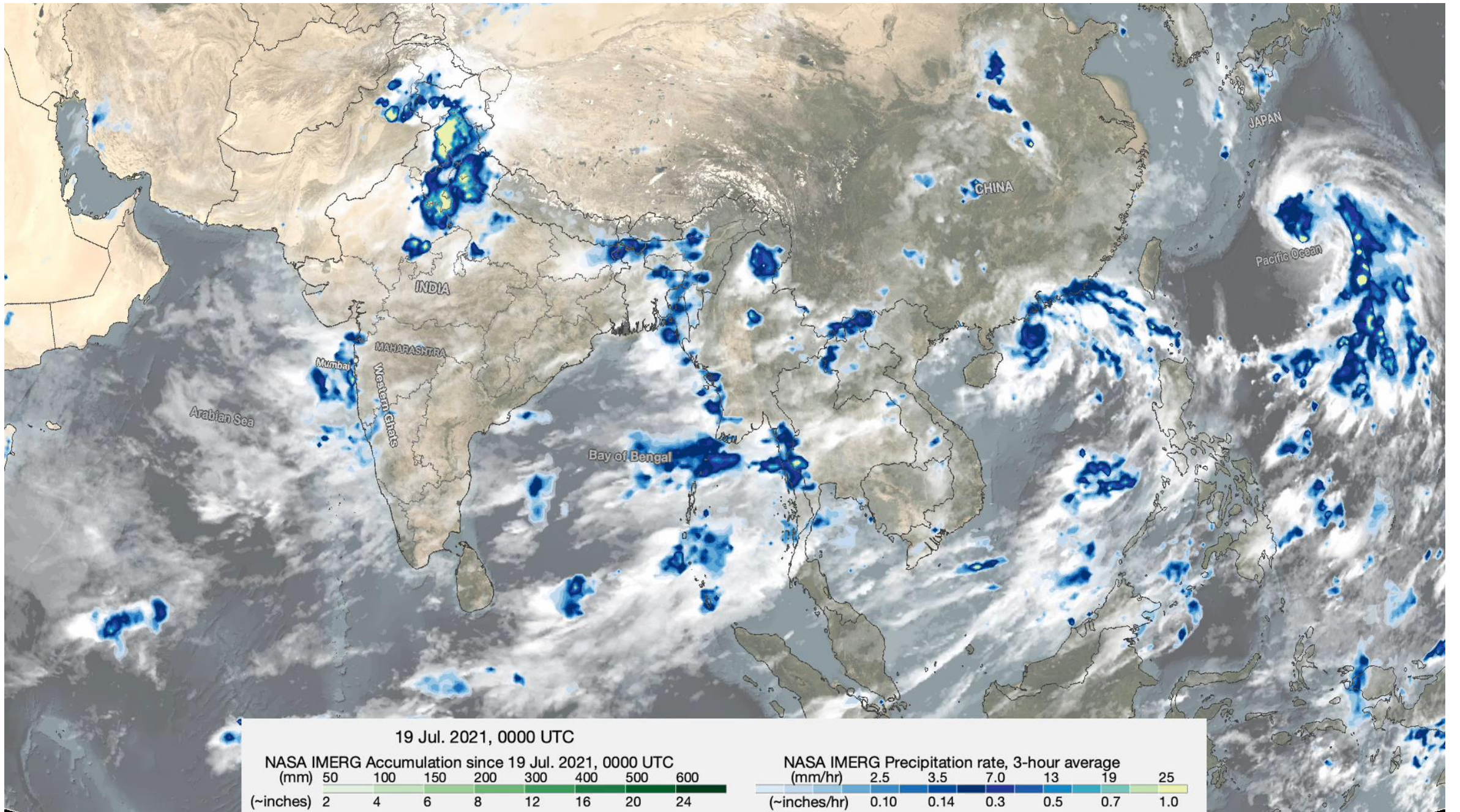


FIG. 15. Latitude-time variation of divergence induced by mixed Rossby-gravity wave with wavelength of 8000 km and period of 4 days.

Dynamics of Low Pressure Systems



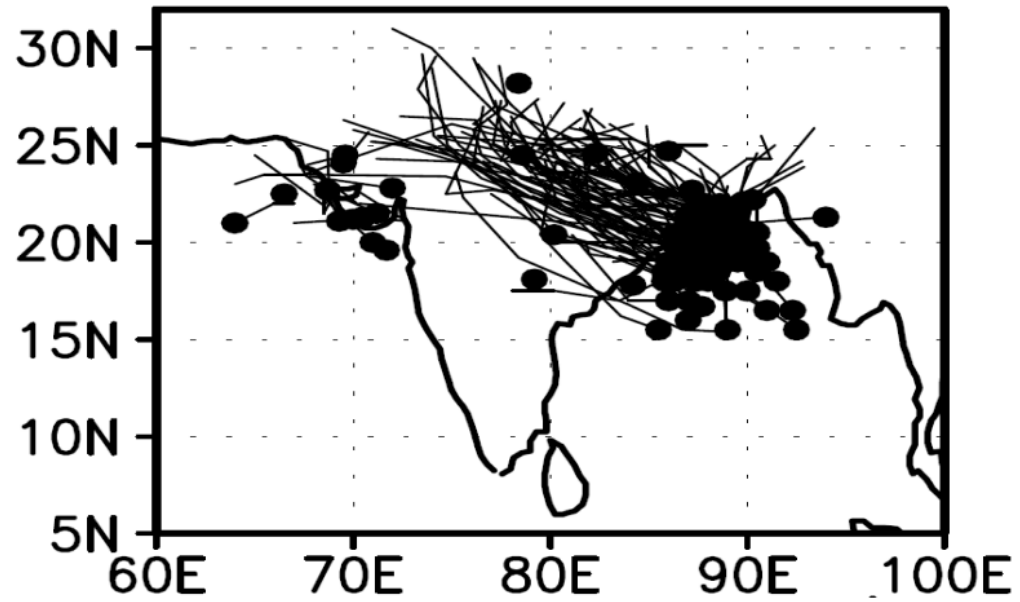


Fig.1.19a : Tracks of monsoon LPS during active phases of monsoon Intra Seasonal Oscillation during 1954-1983. Dots represent genesis point and lines show the tracks of LP

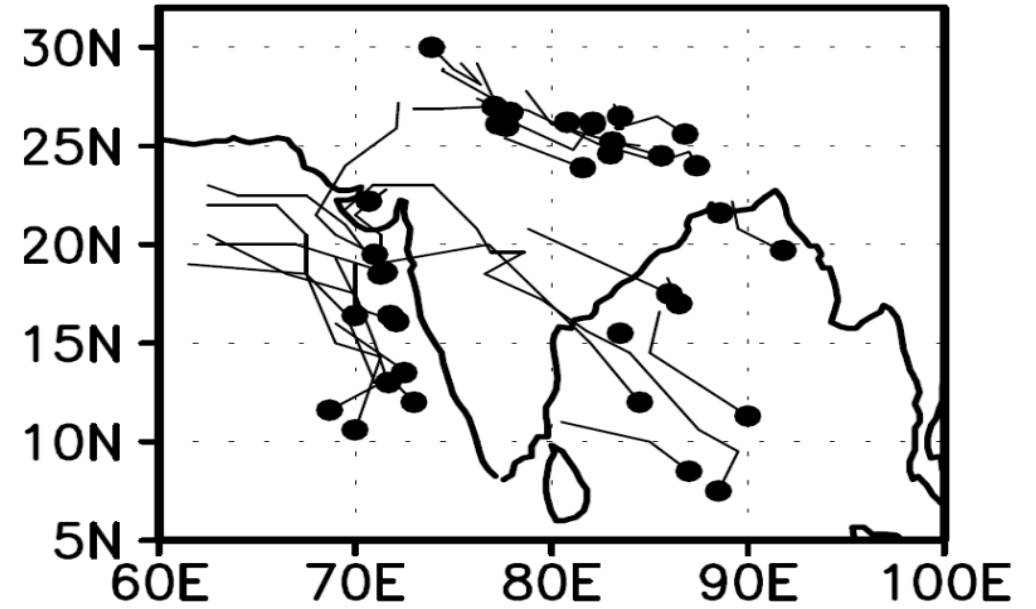


Fig.1.19b: Tracks of monsoon LPS during break phases of monsoon Intra Seasonal Oscillation during 1954-1983. Dots represent genesis point and lines show the tracks of LPS

Simple model for Low Pressure Systems

$$\frac{\partial \zeta}{\partial t} + \nabla \cdot (\mathbf{u} \zeta_a) = 0,$$

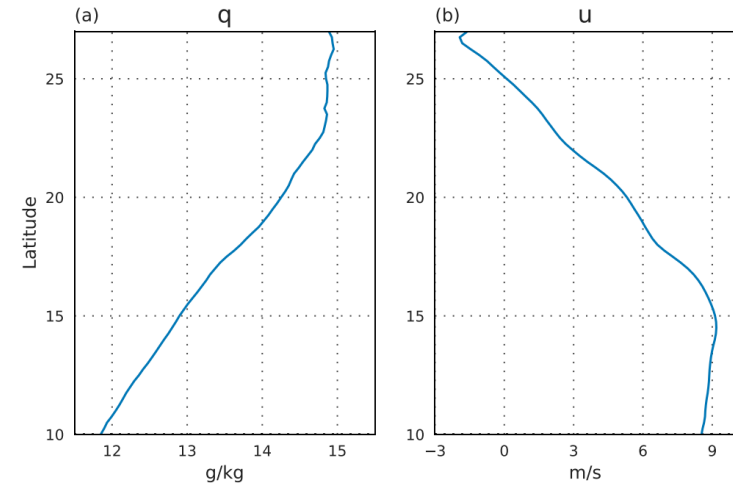
$$\frac{\partial \delta}{\partial t} - \mathbf{k} \cdot \nabla \times (\mathbf{u} \zeta_a) = -\nabla^2 \left(\frac{\mathbf{u} \cdot \mathbf{u}}{2} + gh \right),$$

$$\frac{\partial h}{\partial t} + \nabla \cdot (\mathbf{u} h) = -\chi \mathcal{P},$$

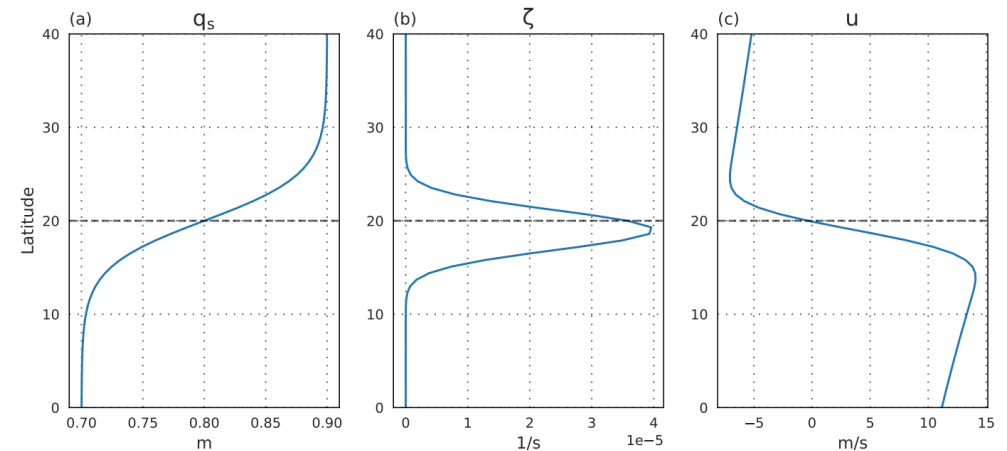
$$\frac{\partial q}{\partial t} + \nabla \cdot (\mathbf{u} q) = -\mathcal{P}.$$

$$\mathcal{P} = (q - q_s) \Theta(q - q_s) / \tau_c,$$

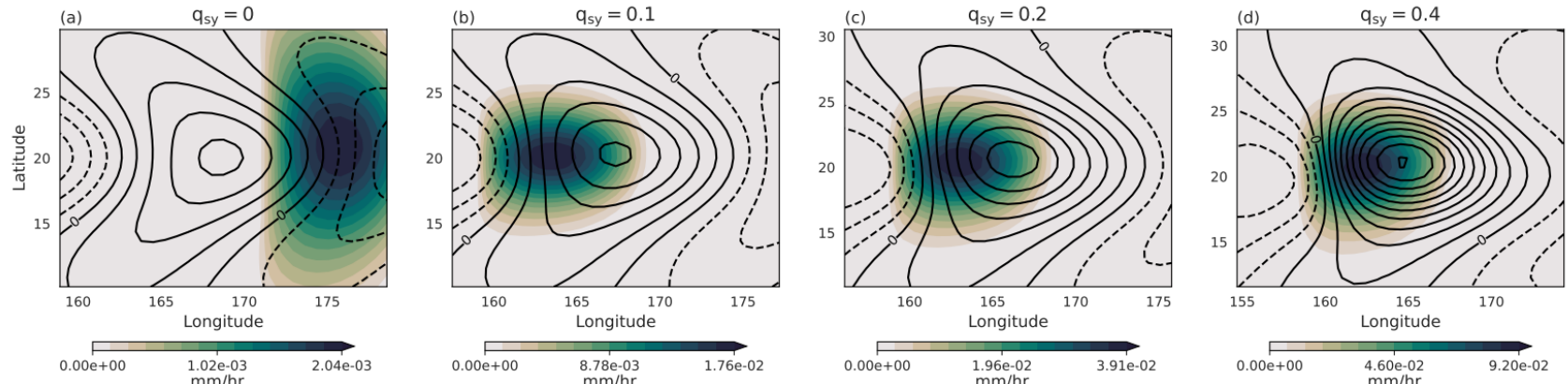
$\tau_c = 12$ h is the condensation timescale



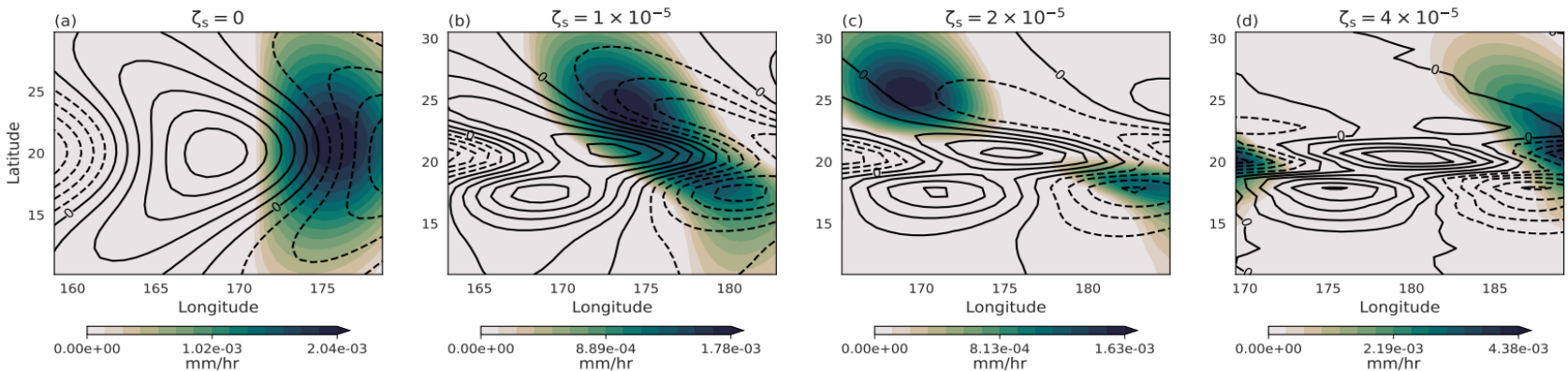
Observed humidity and meridional shear in zonal wind



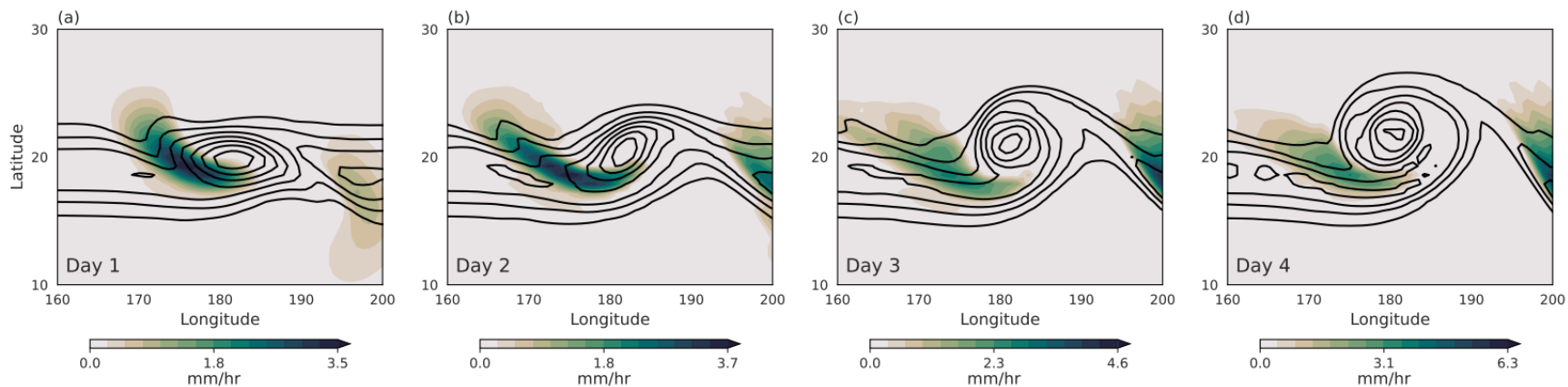
Background saturation humidity and Vorticity



No shear, Only moisture gradients



Only shear, No moisture gradients



Finite amplitude vortex initialisation

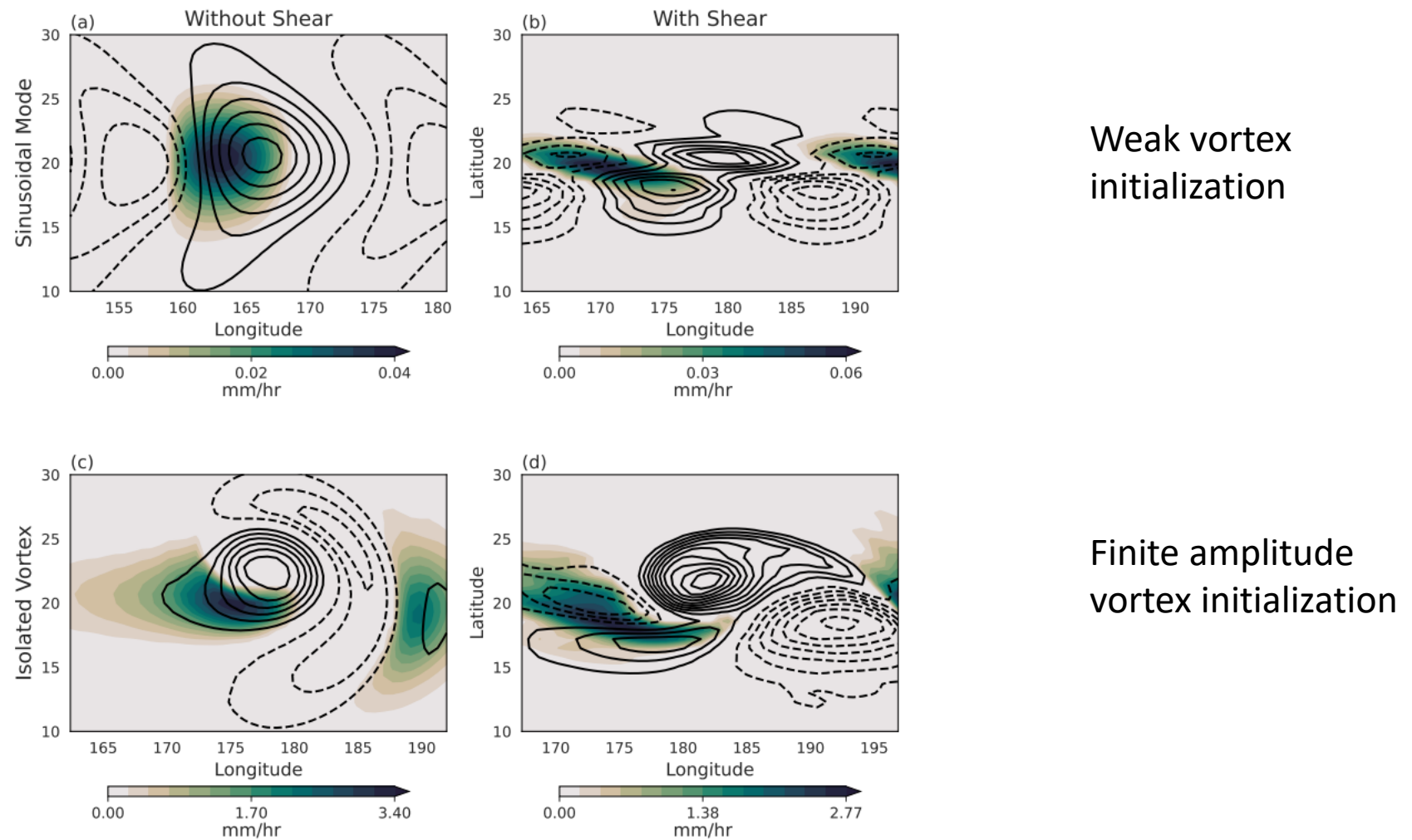


FIG. 12. Precipitation rate (color) and vorticity (black contours) at Day 3 for the experiments (a,c) without wind shear and (b,d) with a wind shear $\zeta_s = 4 \times 10^{-5} \text{ s}^{-1}$. All the plots have a basic-state moisture gradient of $q_{sy} = 0.2 \text{ m}$. Panels in the first row (a–b) correspond to runs with weak sinusoidal modes and in second row (c–d) to runs with a finite-amplitude isolated vortex. The basic-state vorticity strip is removed from the vorticity fields before plotting. The vorticity contour interval is (a) $2 \times 10^{-8} \text{ s}^{-1}$, (b) $5 \times 10^{-8} \text{ s}^{-1}$ and (c, d) $5 \times 10^{-6} \text{ s}^{-1}$.

Take home message

- Monsoon has significant variability on **Seasonal, intra-seasonal and synoptic time-scales**
- We **do not yet have a simple model** that explains seasonal monsoon features of both wind and precipitation.
- Models for intra-seasonal variability suggests that disturbances grow **deriving energy from background vertical shears in zonal and meridional winds**. Some theories also suggest that northward propagation of convection is driven by background moisture gradients
- We are beginning to use **shallow water models to understand Vorticity – Convection coupling** for low pressure systems

