CMB and GW constraints on Primordial Black Holes

Lecture I: CMB constraints

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New York University

Less traveled path of dark matter, ICTS, Nov 10, 2020

Plan of this lecture

1- The CMB as a probe of energy injection in the early Universe

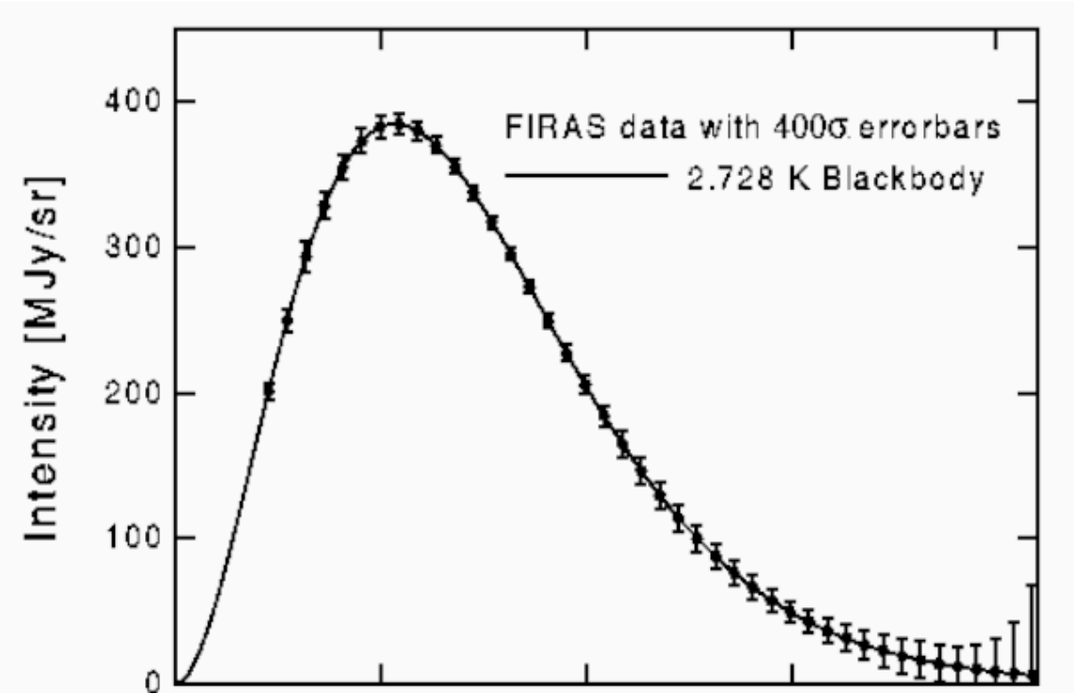
2- Energy-injection processes due to PBHs, and resulting CMB constraints

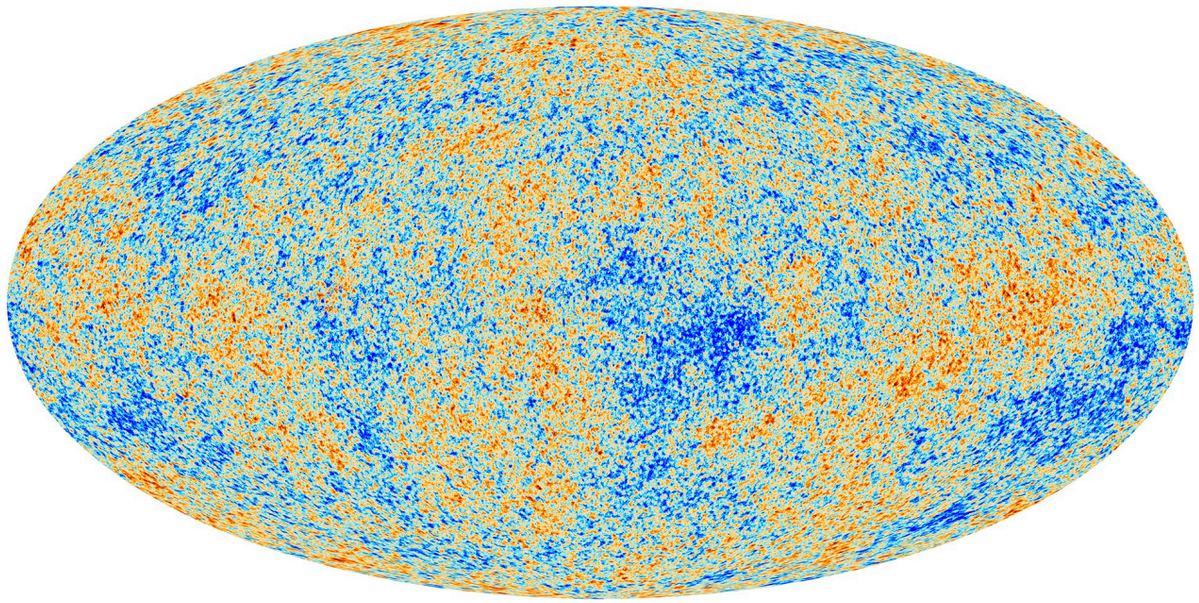
A- Dissipation of small-scale perturbations

B- Hawking radiation

C- Accretion

1- Brief review of CMB physics





$$I(\nu, n) \equiv \frac{2h\nu^3}{\exp(h\nu/T) - 1}$$

n: direction in the sky

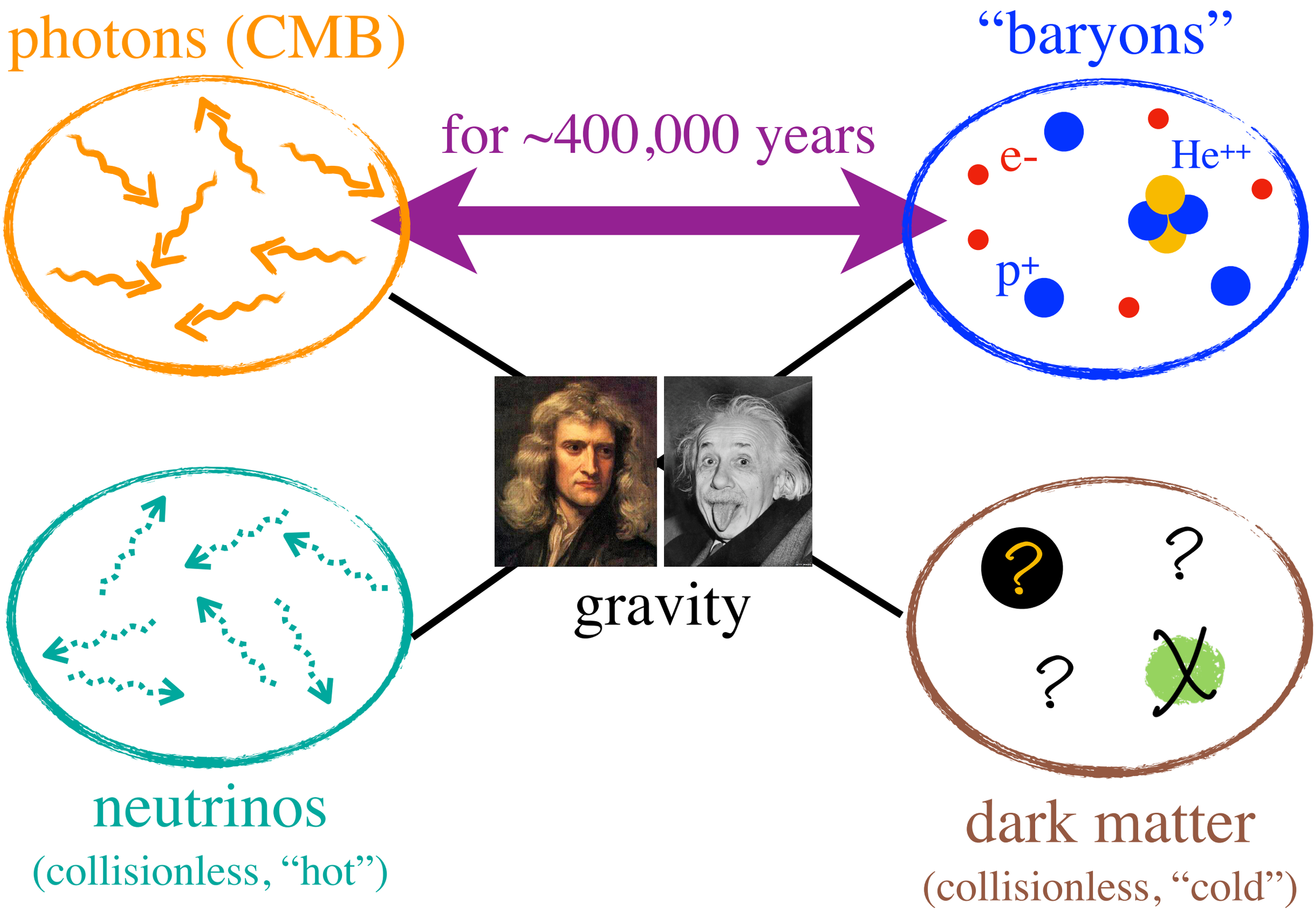
$$T(\nu, n) = T_0 \; + \Delta T(\nu) \; + \Delta T(n) \; + \delta T(\nu, n)$$

spectral distortions

anisotropies

spectral-spatial distortions (e.g. SZ)

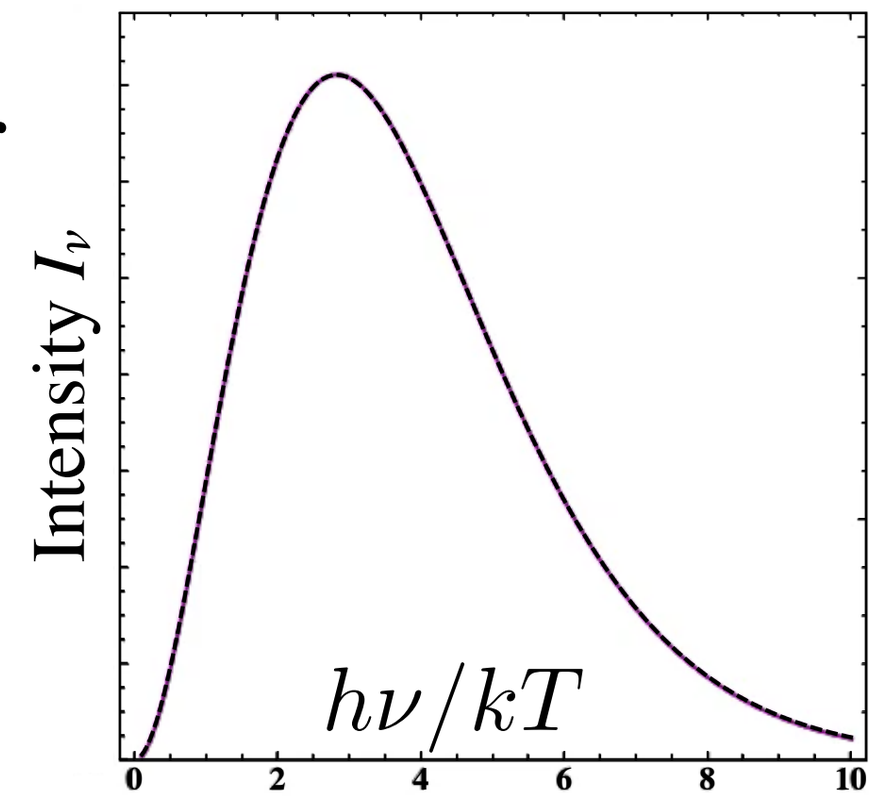
Protagonists and stage



Essential spectral distortion physics

- First 2 months ($z > 2e6$): photons are easily created and destroyed, and thermalized

⇨ **blackbody spectrum**, no matter what happens in first 2 months



Zeldovich & Sunyaev 1969,
Hu & Silk 1993, Chluba & Sunyaev 2012

After 2 months: **energy injection/extraction** would **distort** **photon spectrum**

$$\frac{\Delta I_\nu}{I_\nu} \sim \int_{t_*} dt \frac{\dot{\rho}_{\rm inj}}{\rho_\gamma}$$

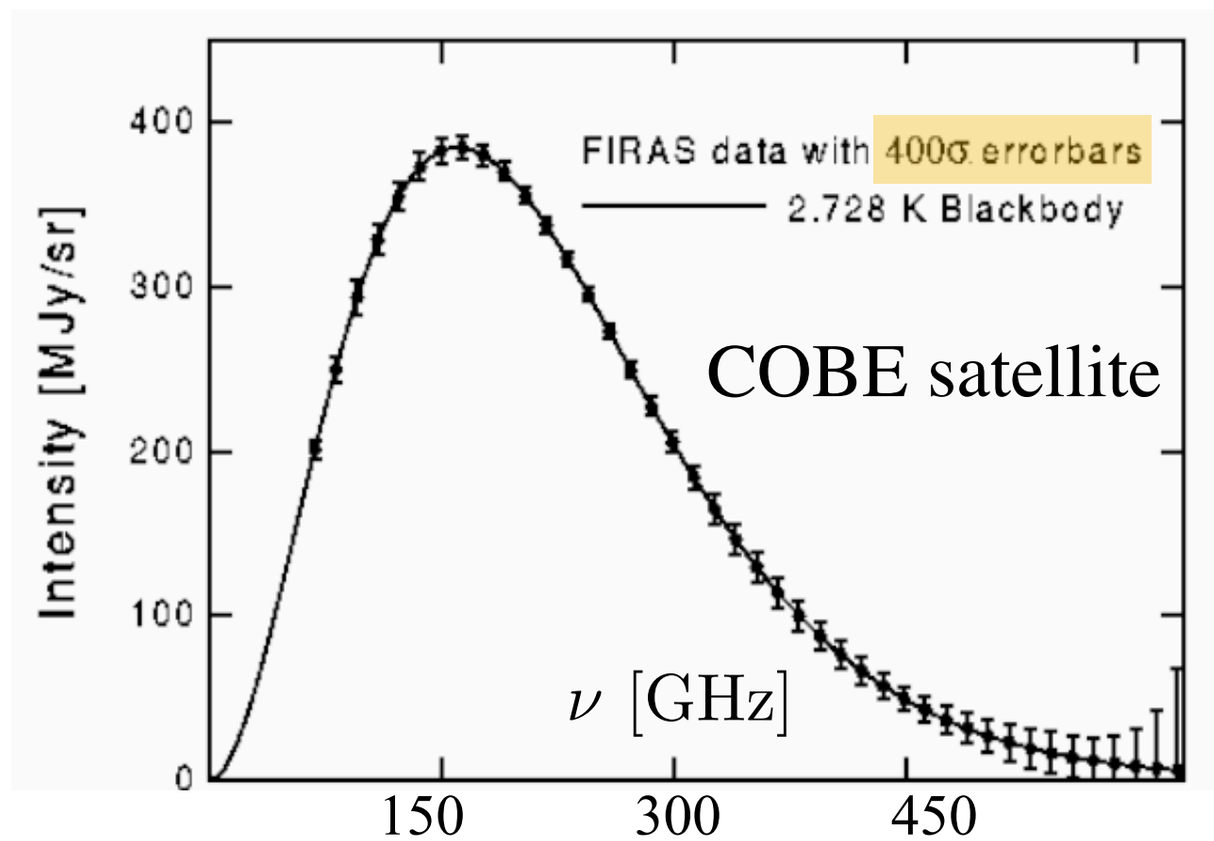
$$\dot{\rho}_{\rm inj} = \left.\frac{dE}{dtdV}\right|_{\rm injected}$$

$t_* \approx 2$ months

$[z_* \approx 2 \times 10^6]$

COBE FIRAS results (Fixsen et al. 1996):

$$\frac{\Delta I_\nu}{I_\nu} \lesssim 10^{-4}$$



=> Constrains the amount of non-standard energy injection in the early Universe

Fast-forward in time

~ 20,000 years: 1st Helium recombination

$$He^{++} + e^{-} \rightarrow He^{+} + \gamma$$

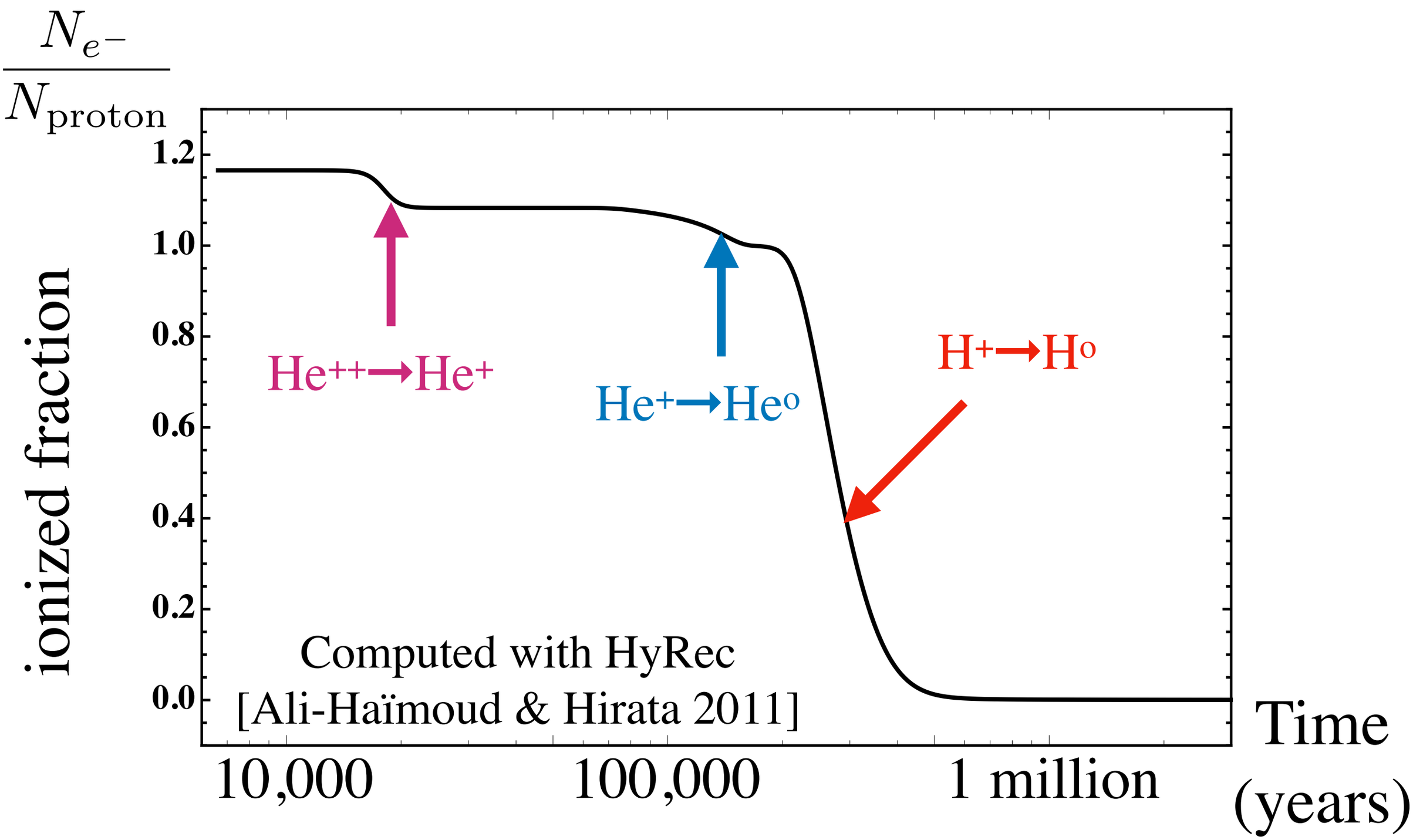
~ 130,000 years: 2nd Helium recombination

$$He^{+} + e^{-} \rightarrow He^{0} + \gamma$$

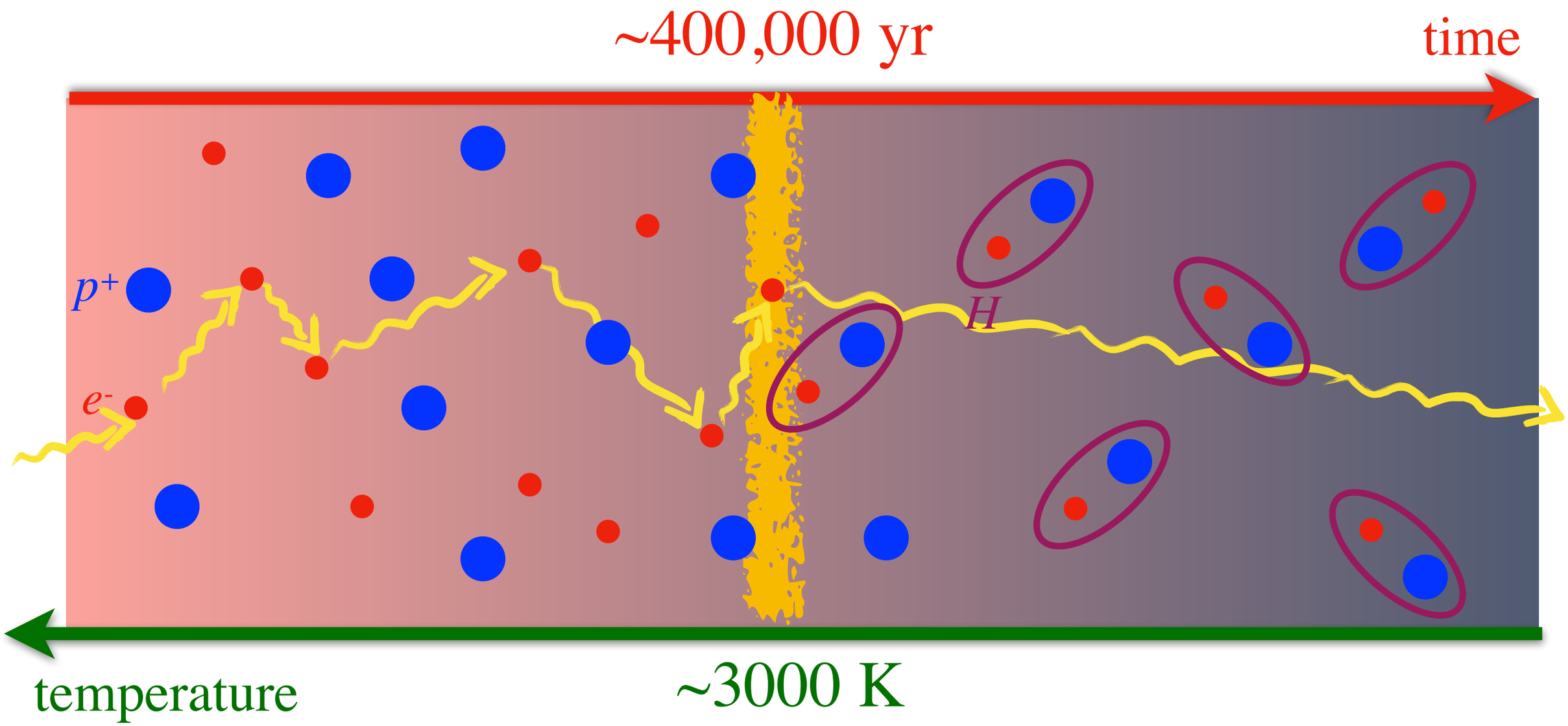
~ 400,000 years: hydrogen recombination

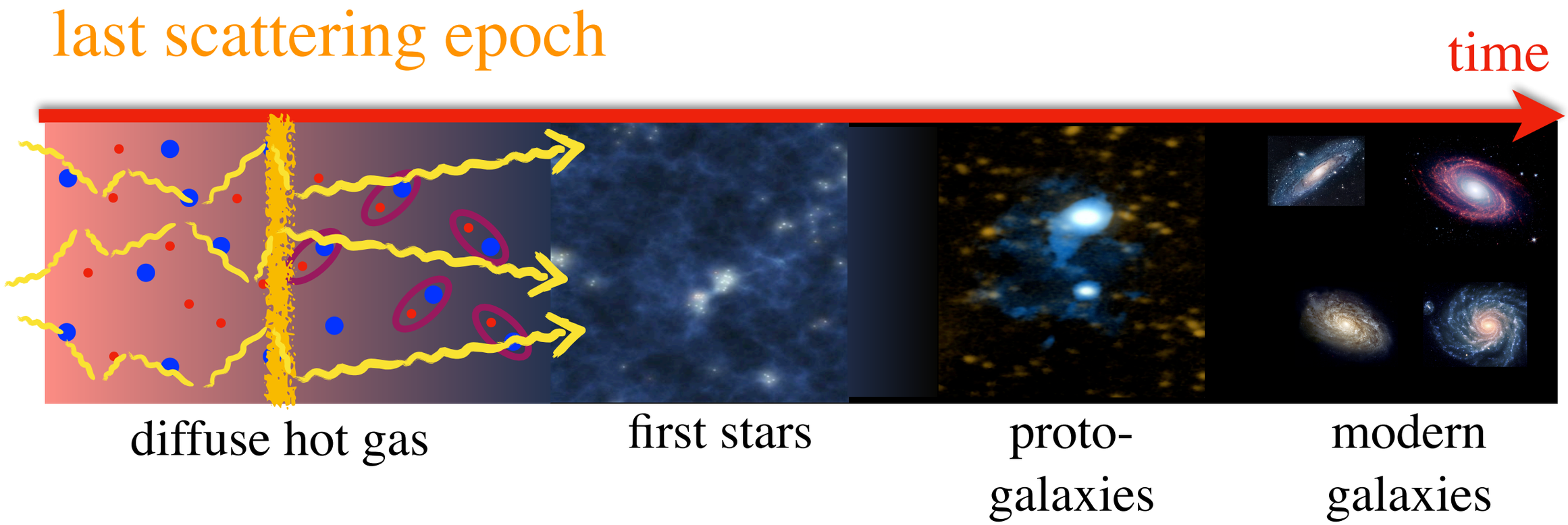
$$H^{+} + e^{-} \rightarrow H^{0} + \gamma$$

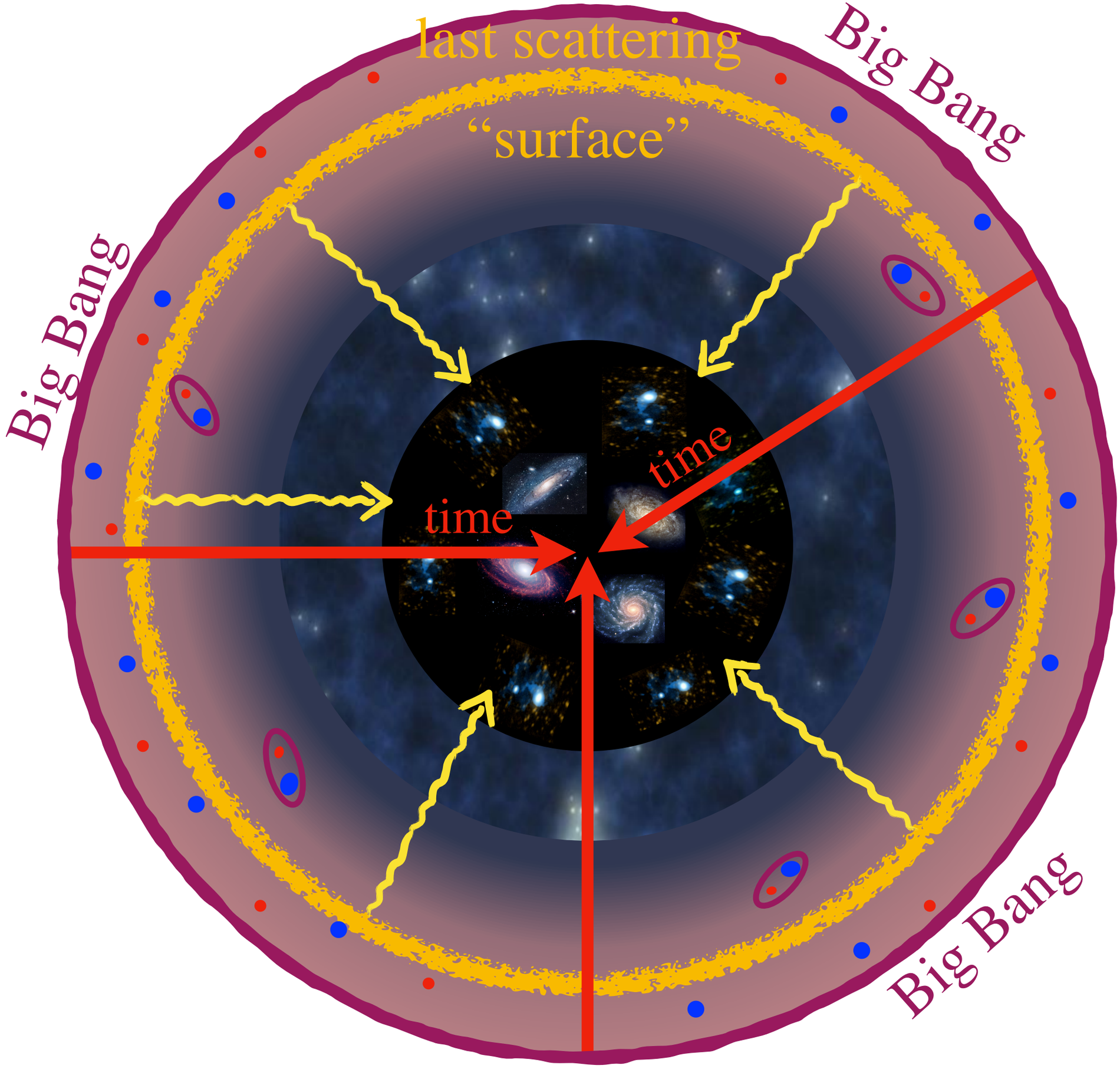
Recombination history

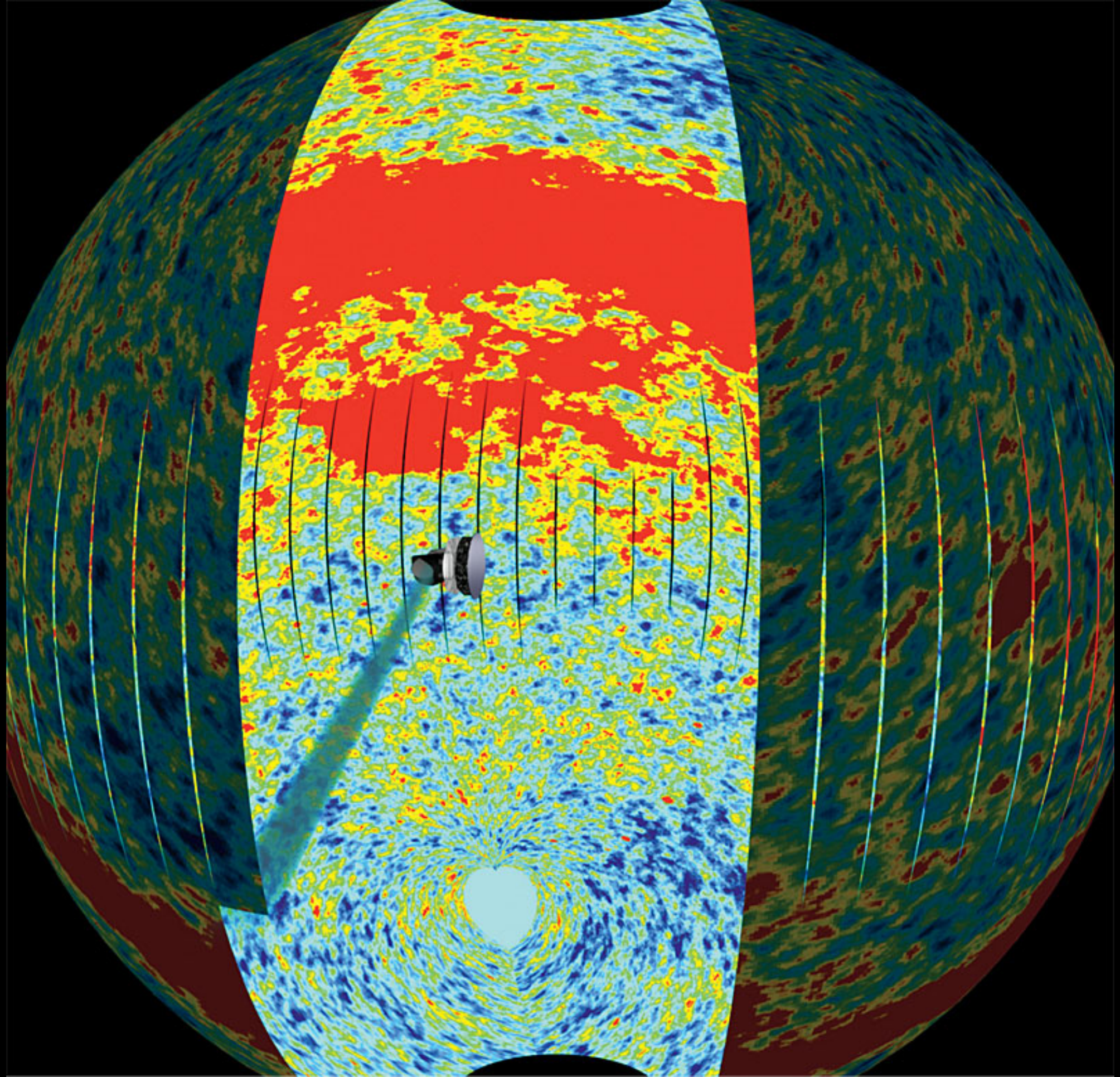


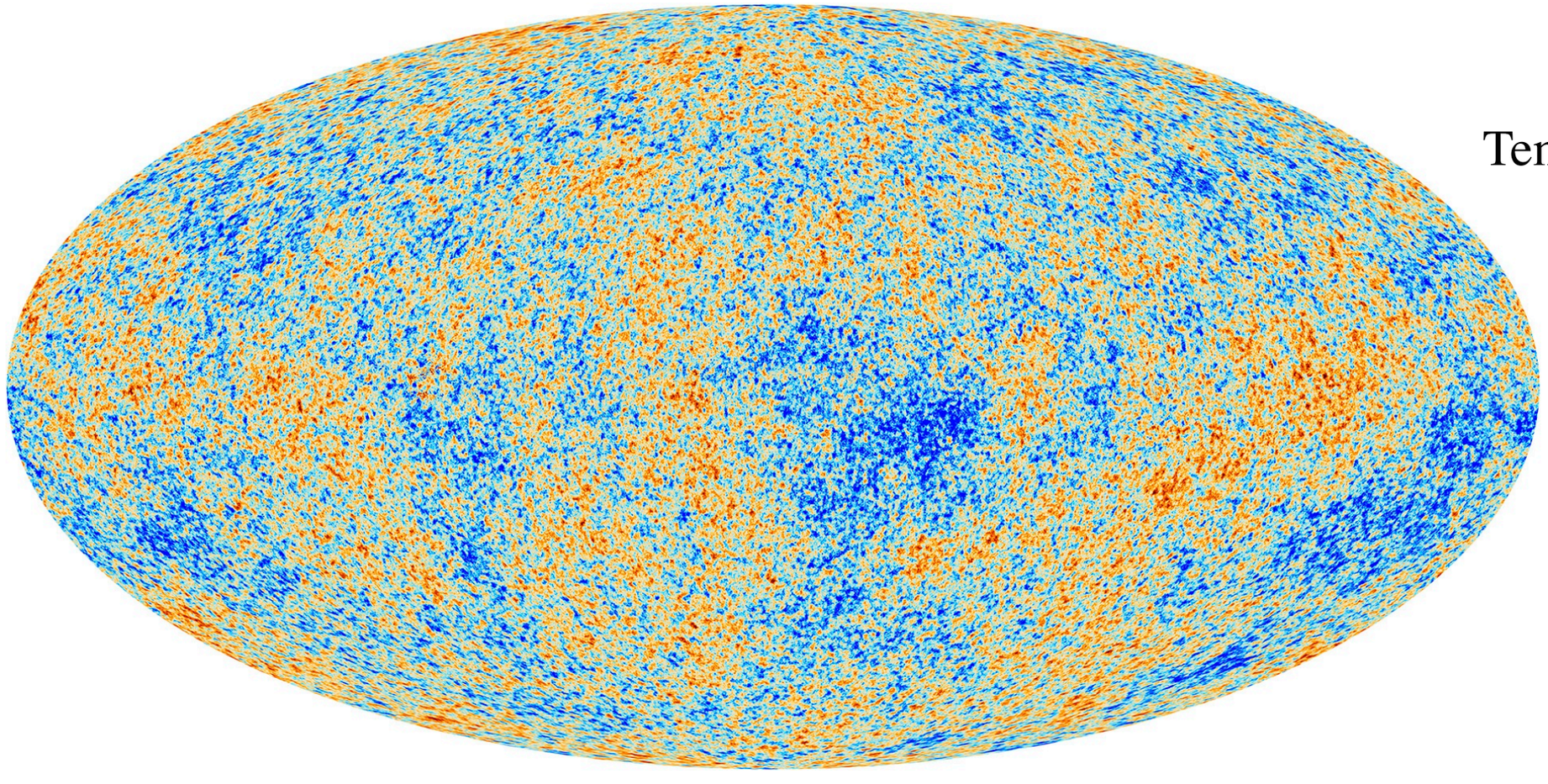
last scattering epoch

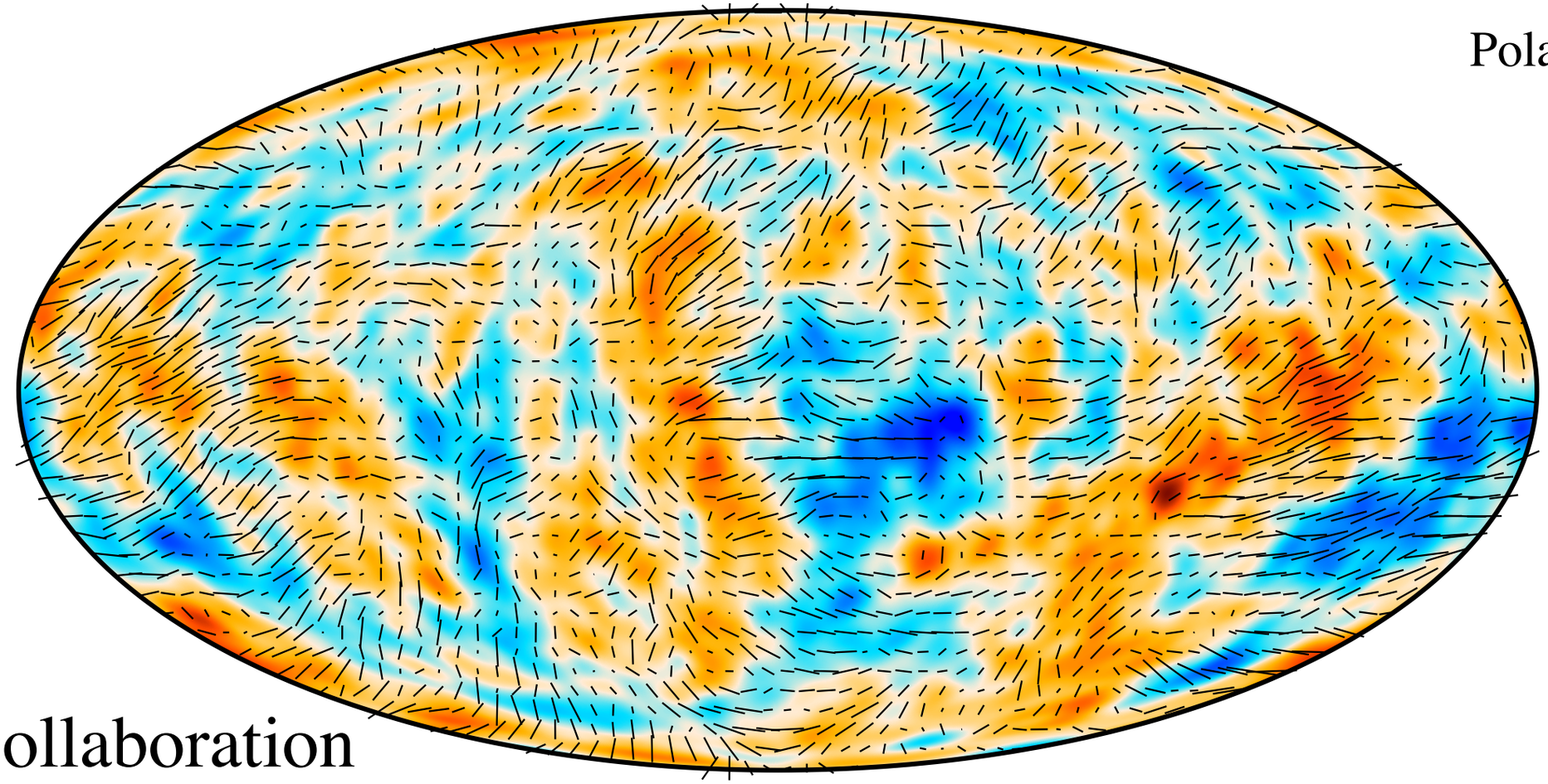



last scattering epoch
time
diffuse hot gas
first stars
proto-
galaxies
modern
galaxies


last scattering
"surface"
Big Bang
Big Bang
Big Bang
time
time

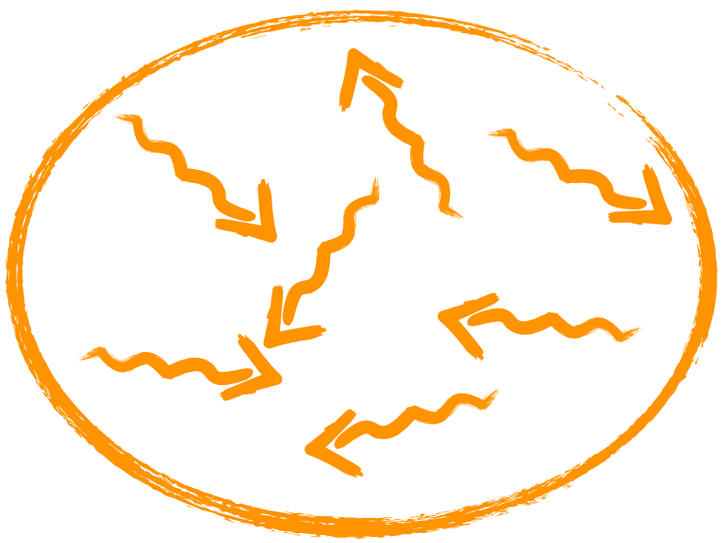




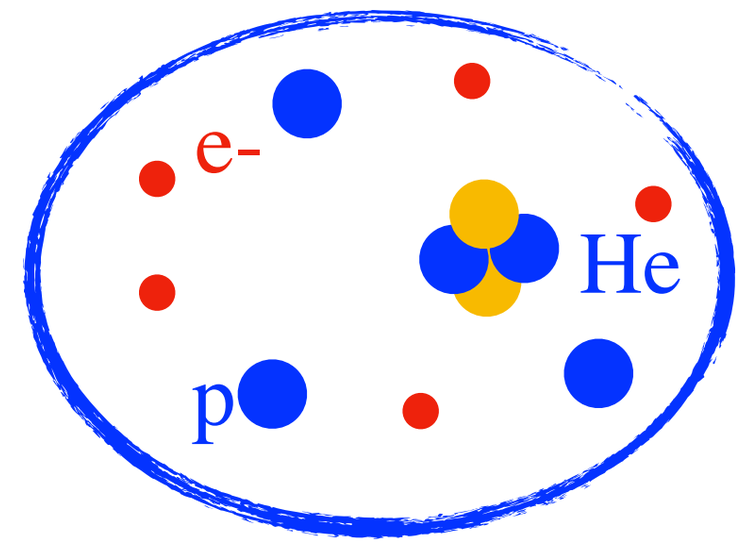


Planck collaboration

What are we seeing exactly?
before last scattering:



photons



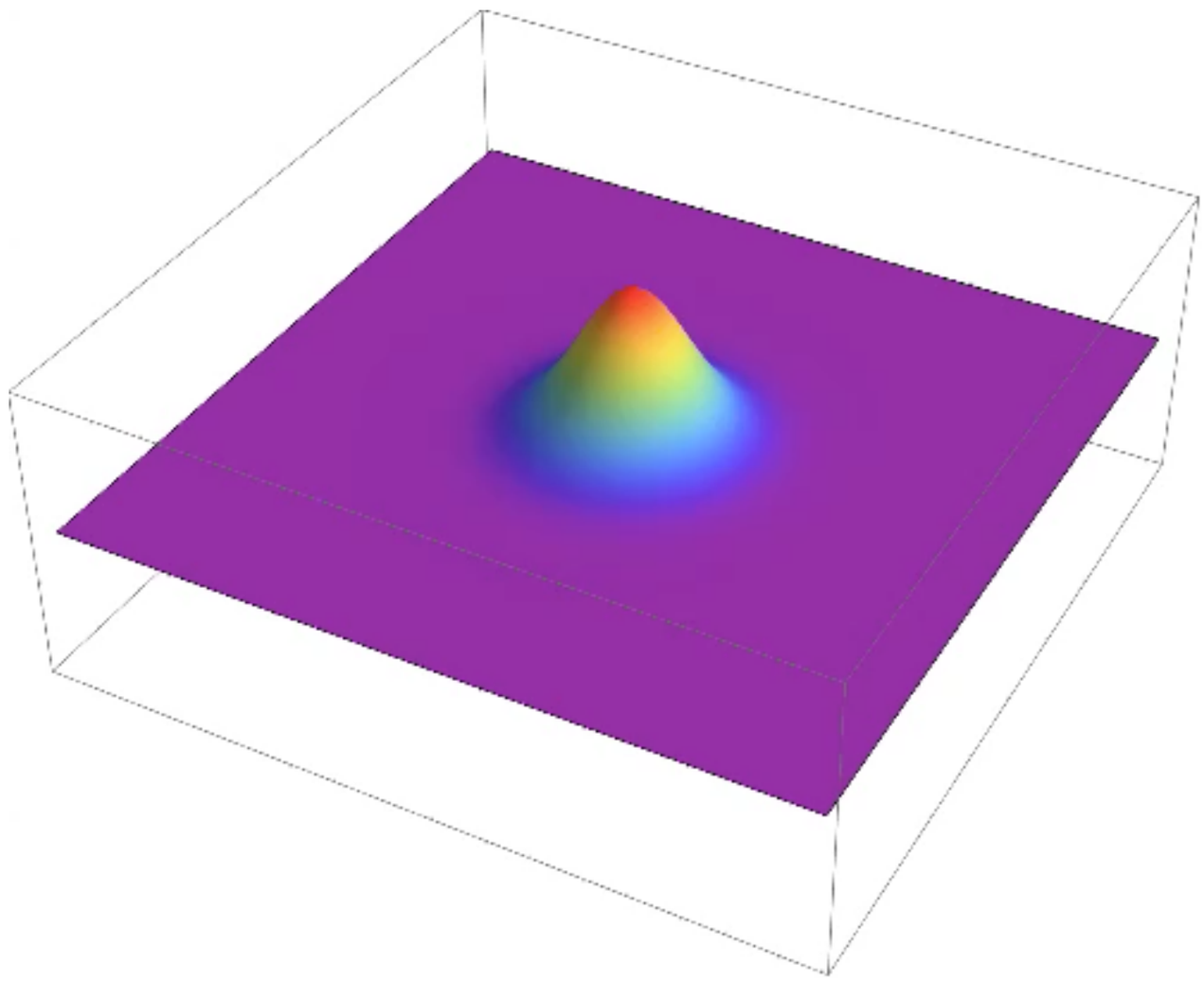
“baryons”

provide pressure

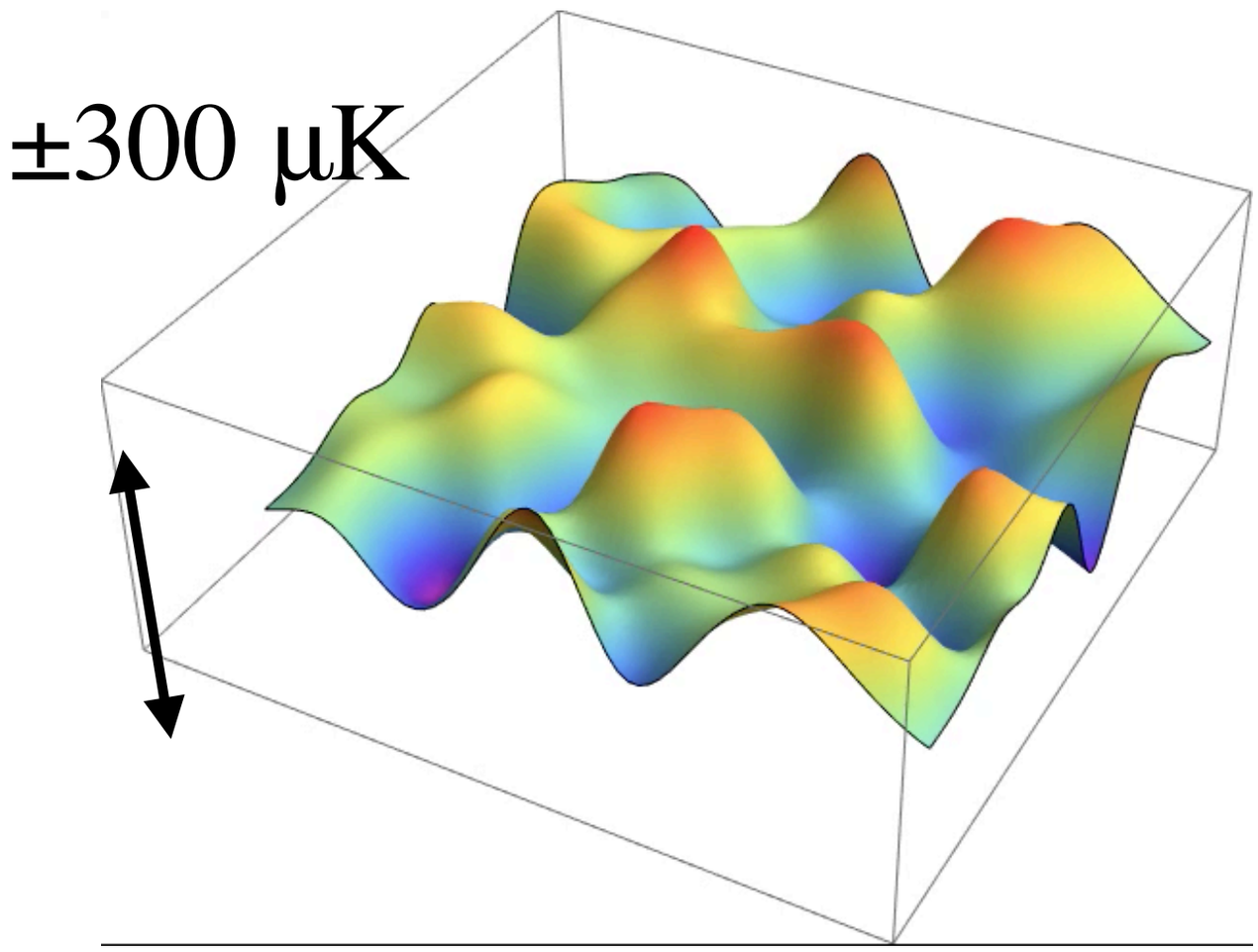
provide containment
(impede free streaming)

together: ideal fluid with large pressure

Small initial overdensities generate sound waves in photon-baryon fluid, propagating for ~400,000 years

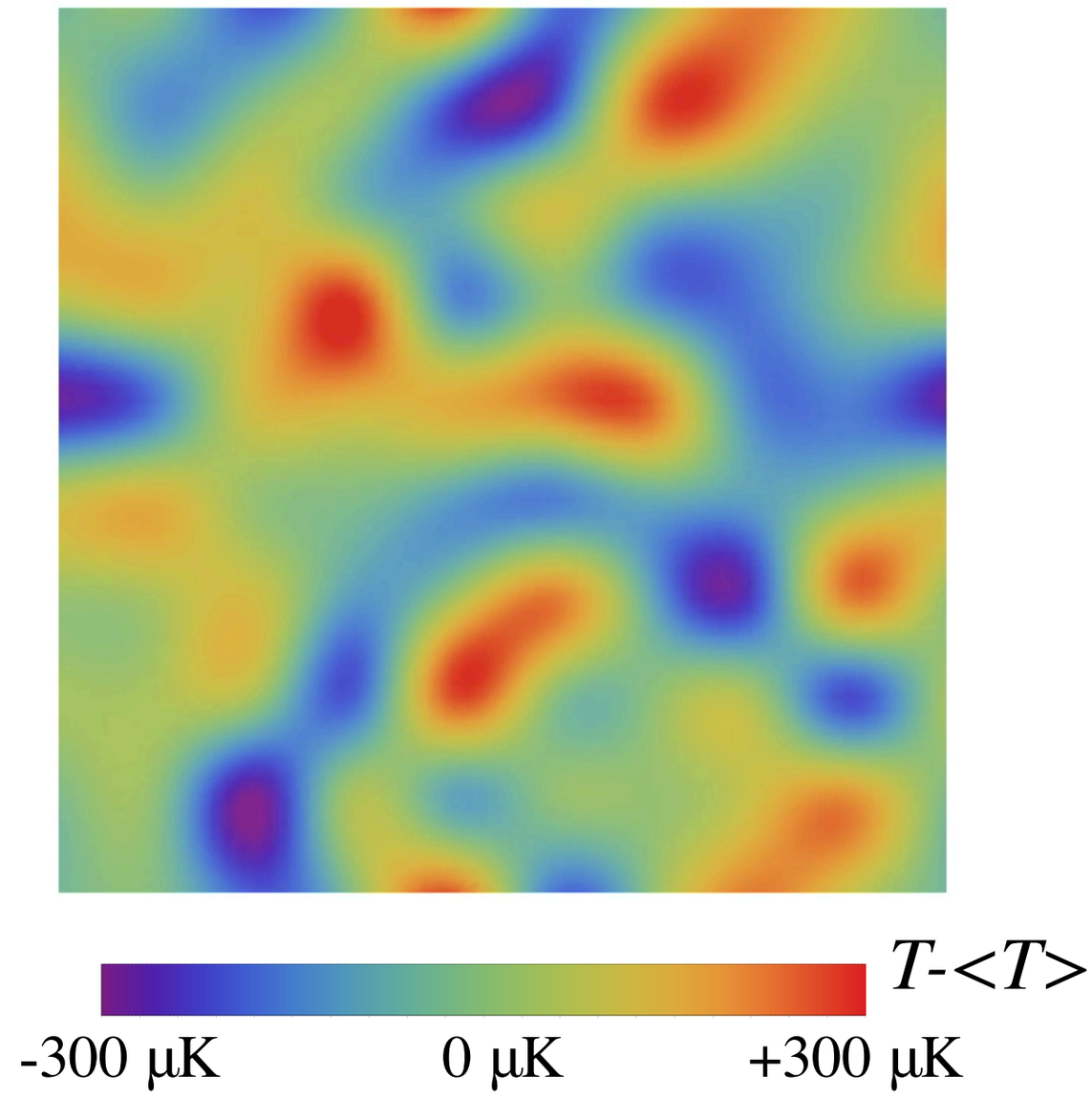


Superposition of many incoherent sound waves, oscillating for ~400,000 years



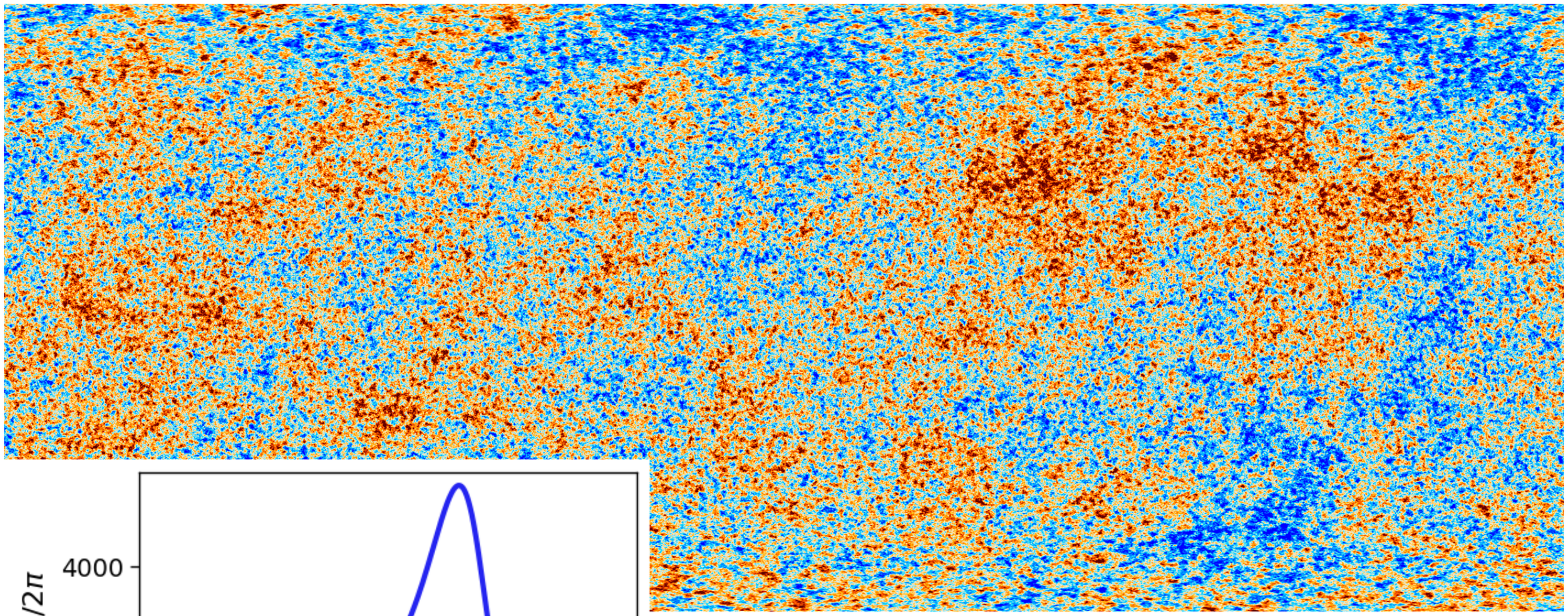
$$\delta(k,\eta) \sim \delta_0 \cos(k c_s \eta)$$

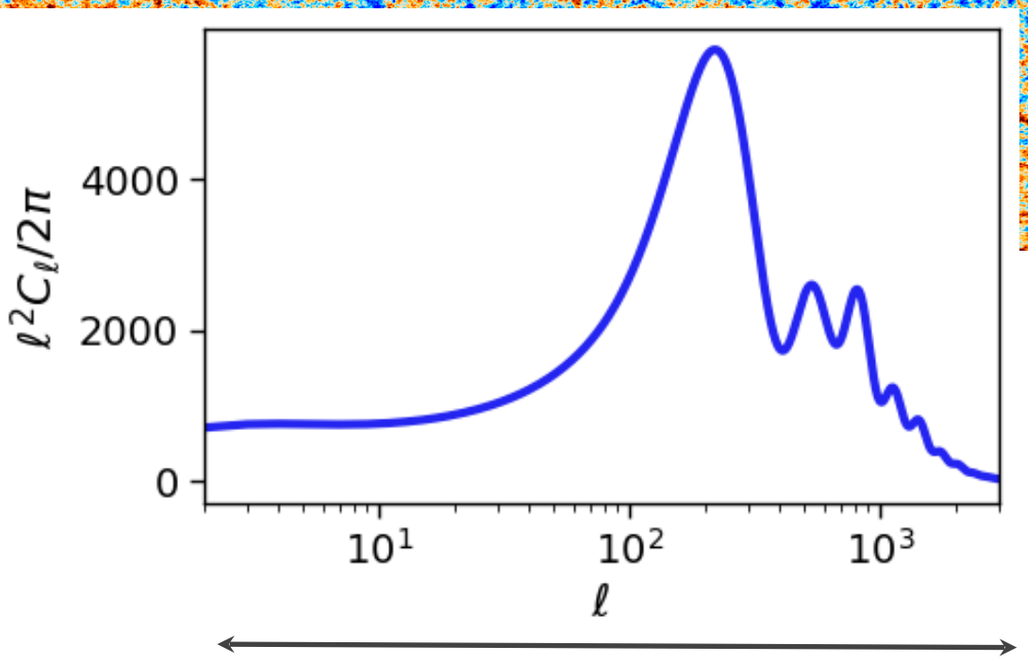
$$c_s \approx c/\sqrt{3}$$



Last ``snapshot" is imprinted on the last scattering surface

CMB Angular power spectrum = **variance of temperature fluctuations** as a function of **angular scale**





Courtesy of Mathew Madhavacheril

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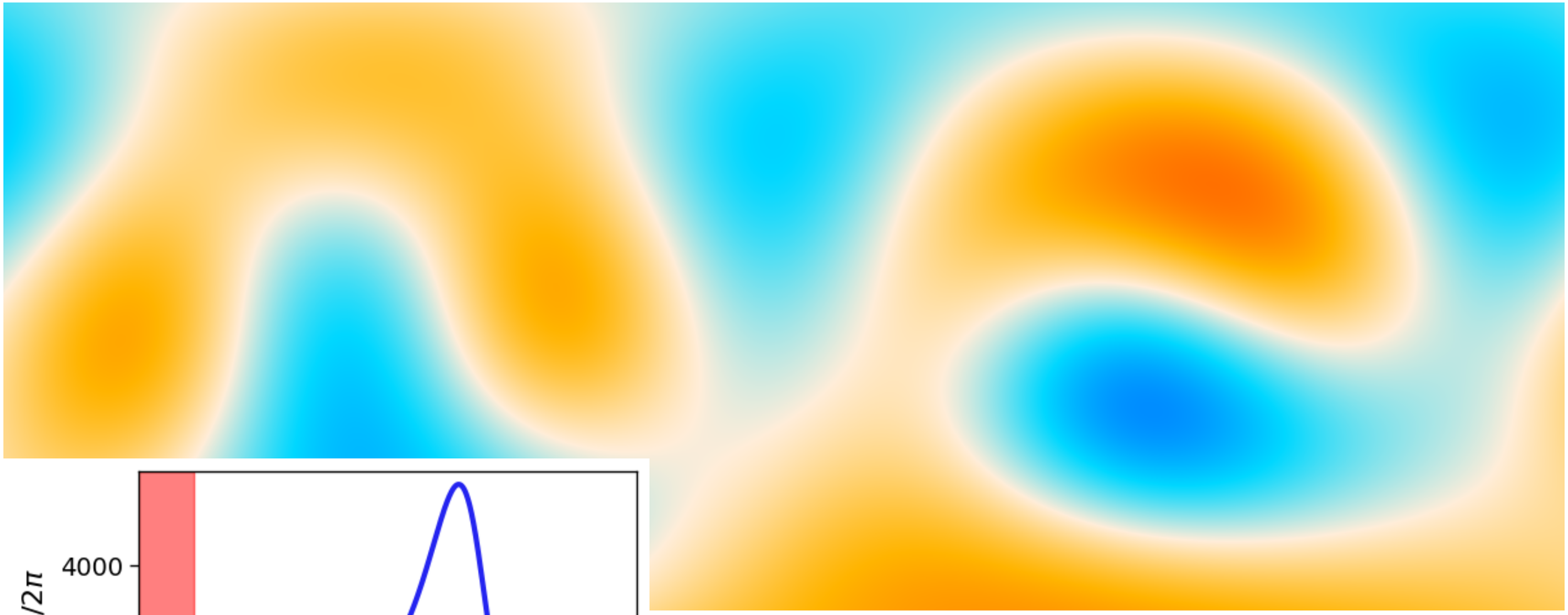

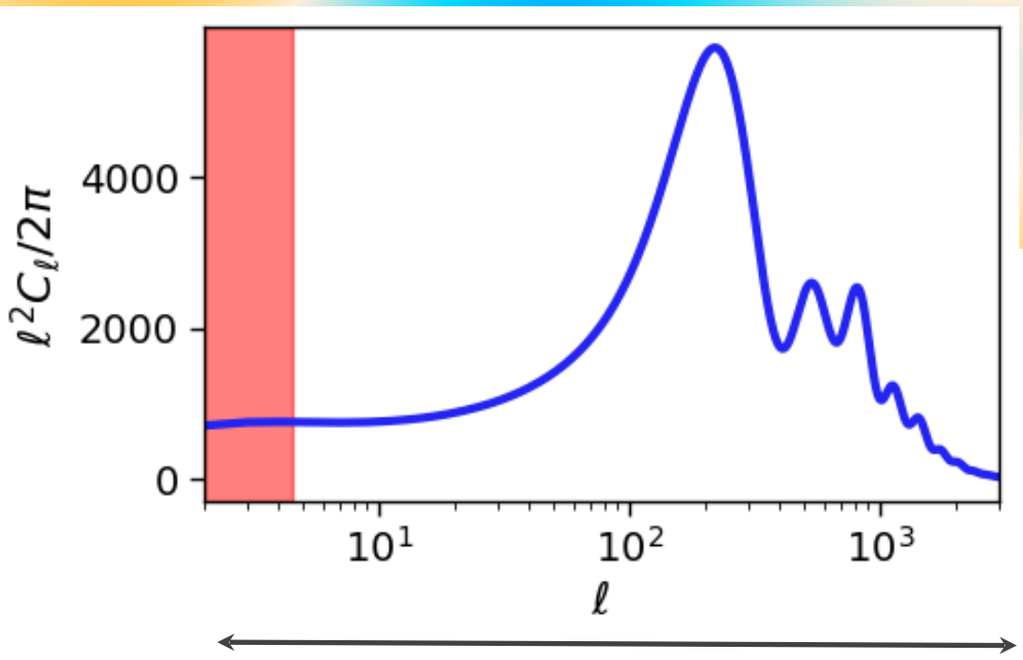

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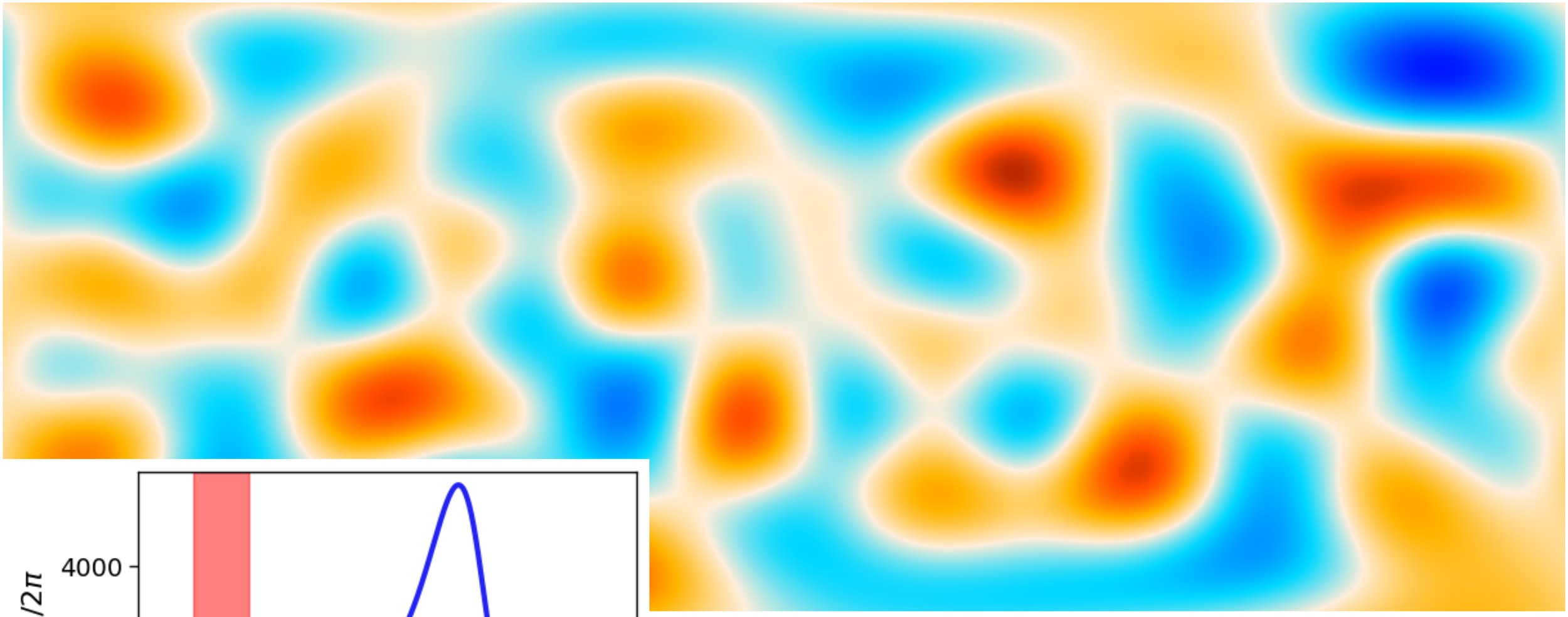

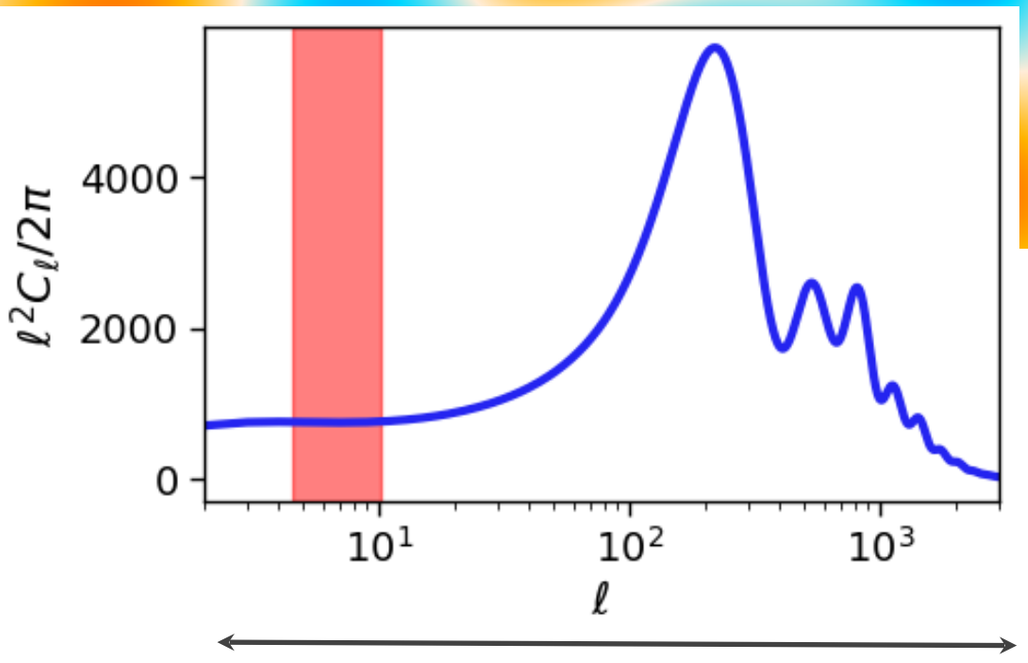

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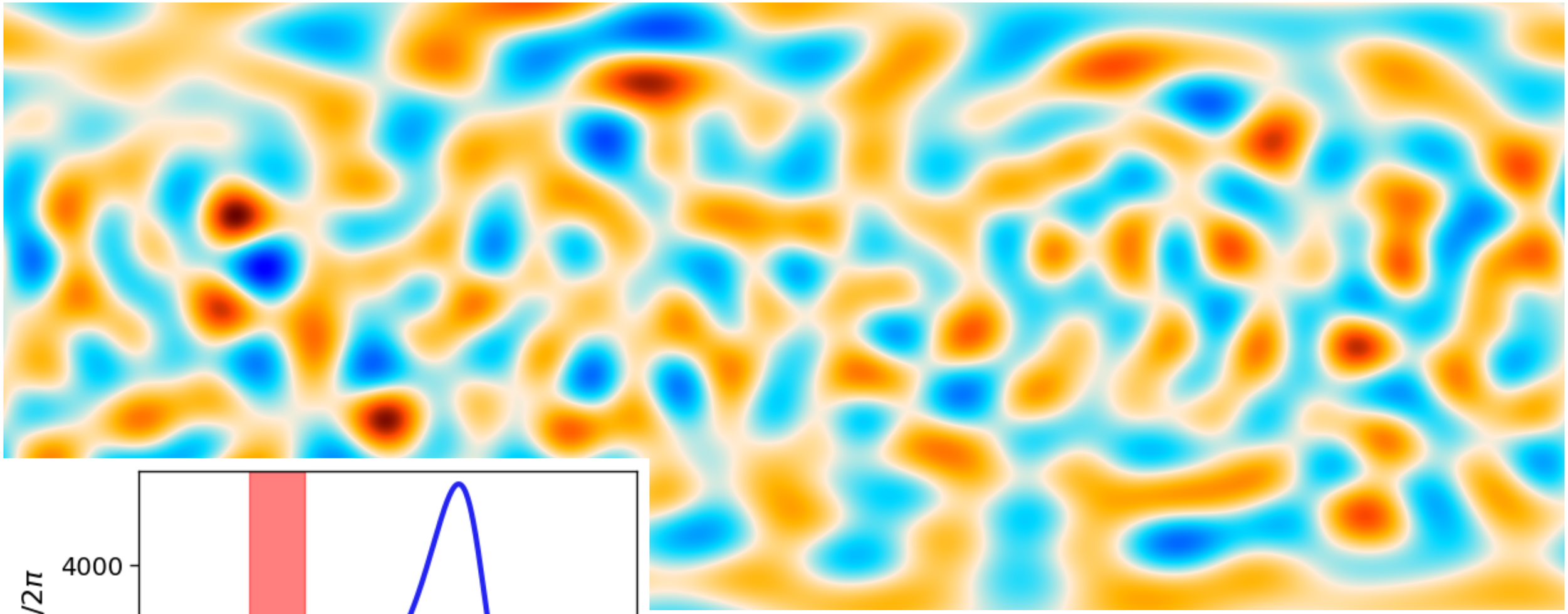

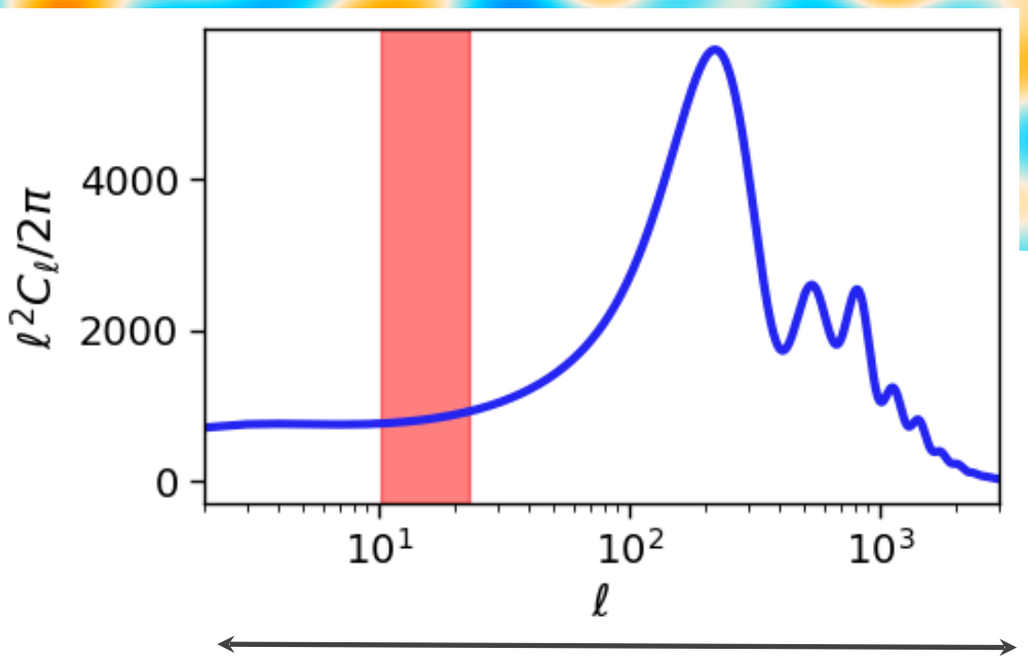

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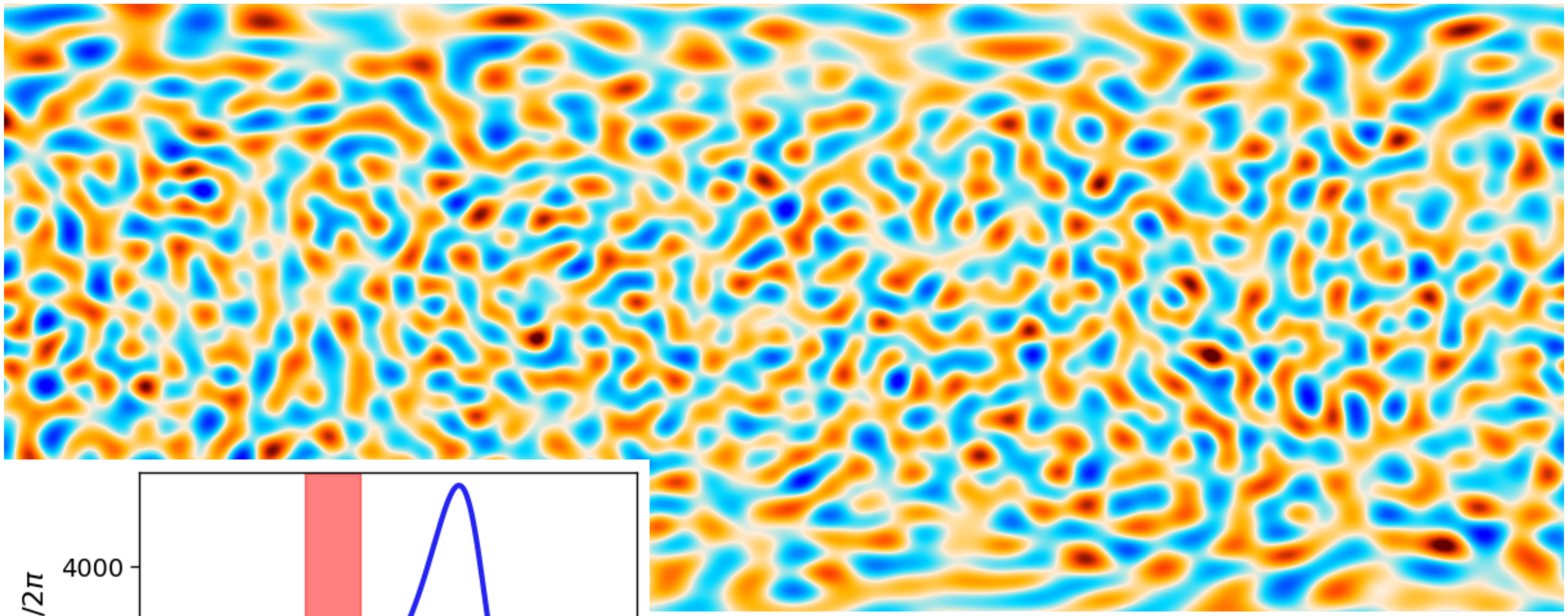

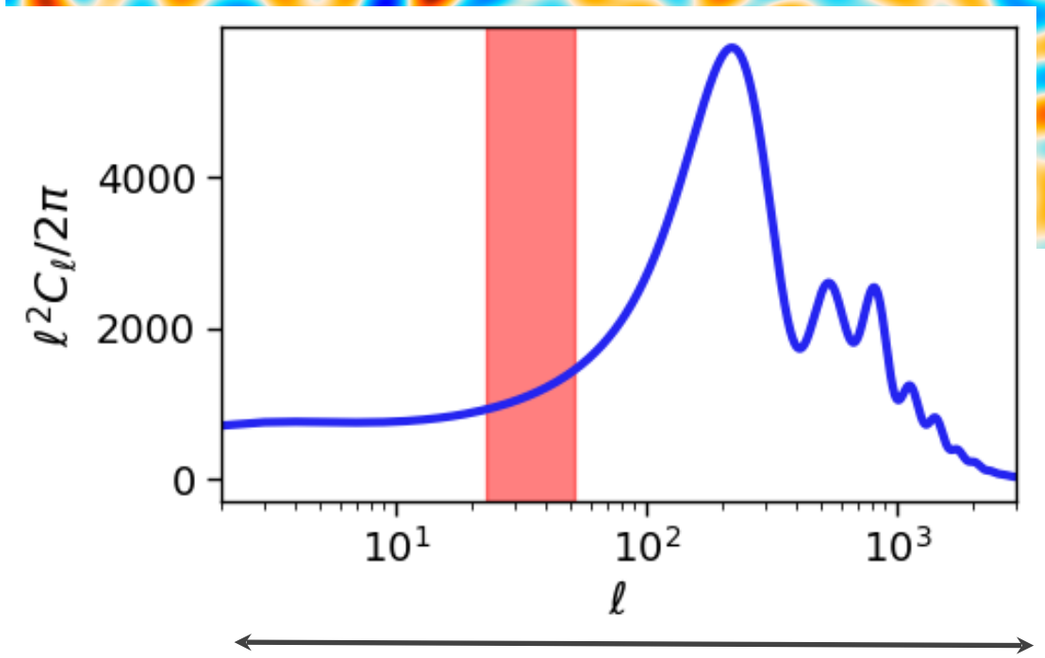

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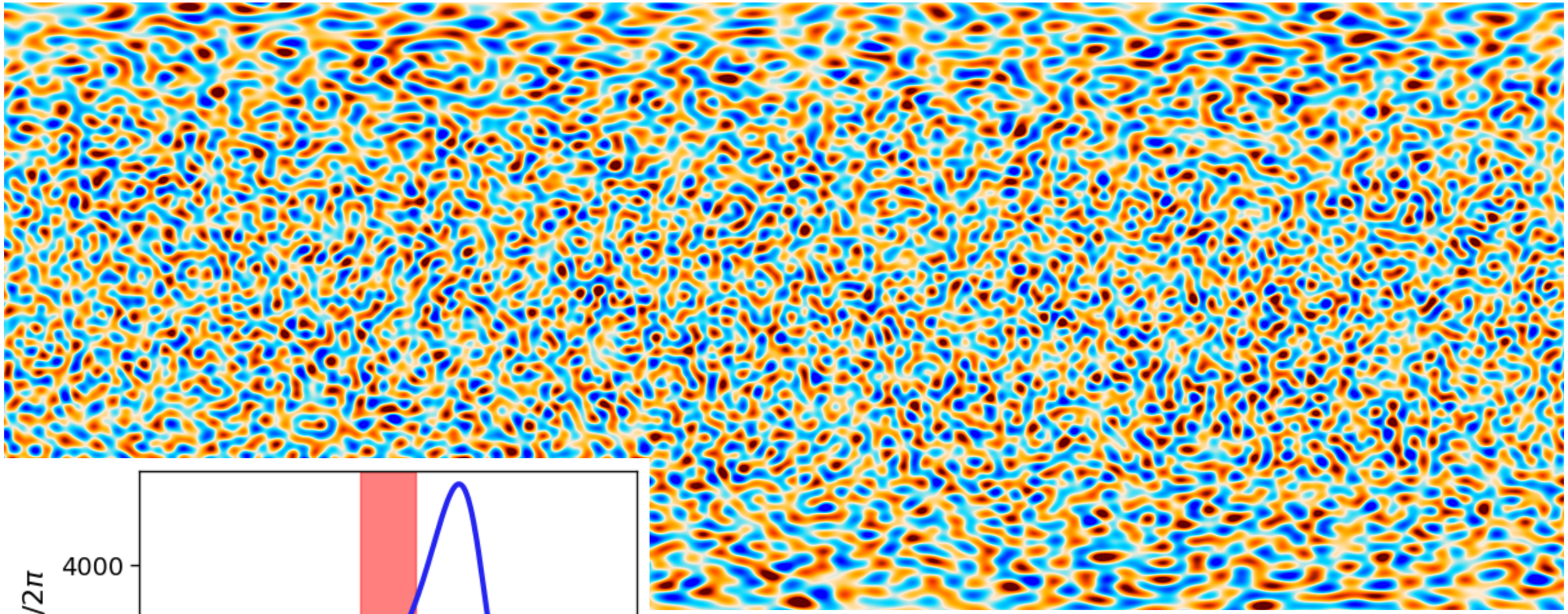

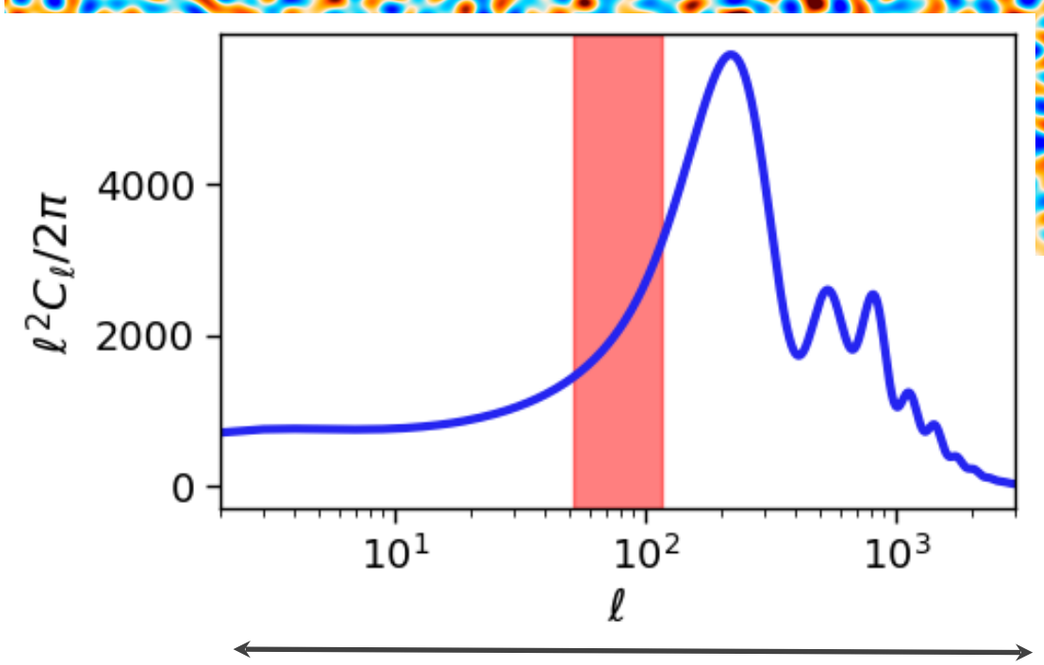

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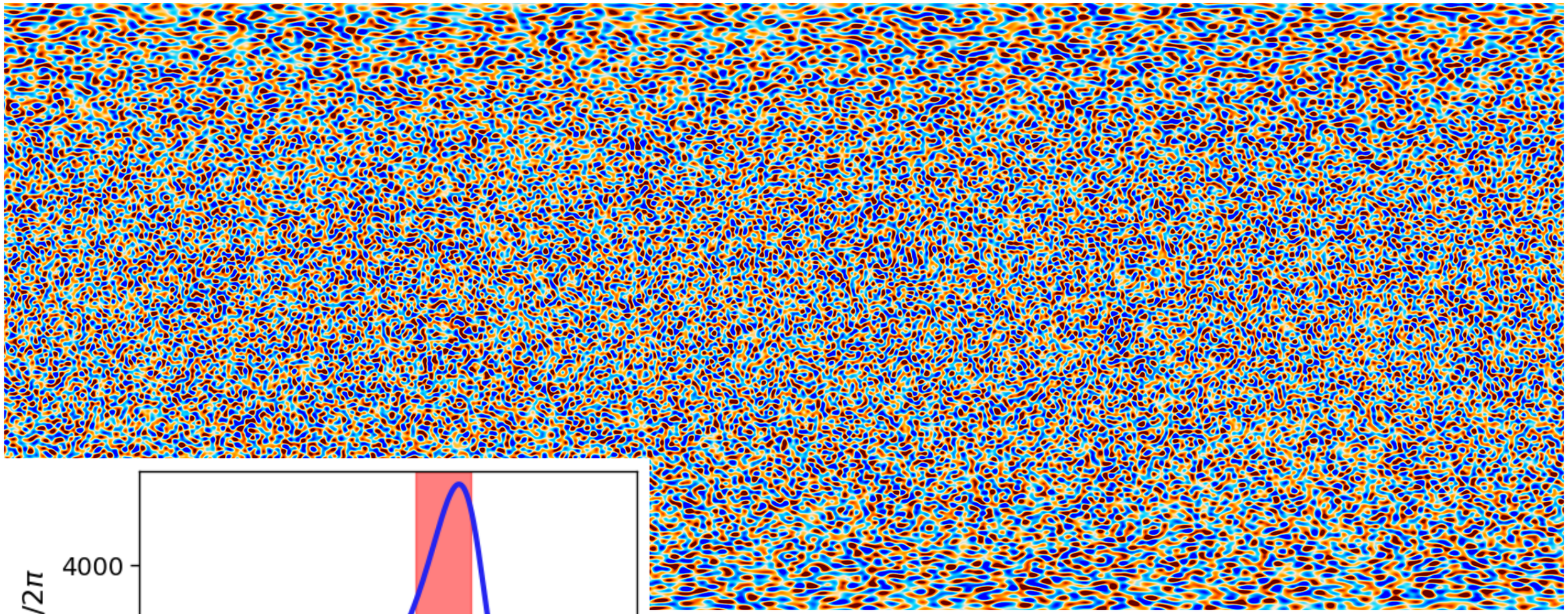

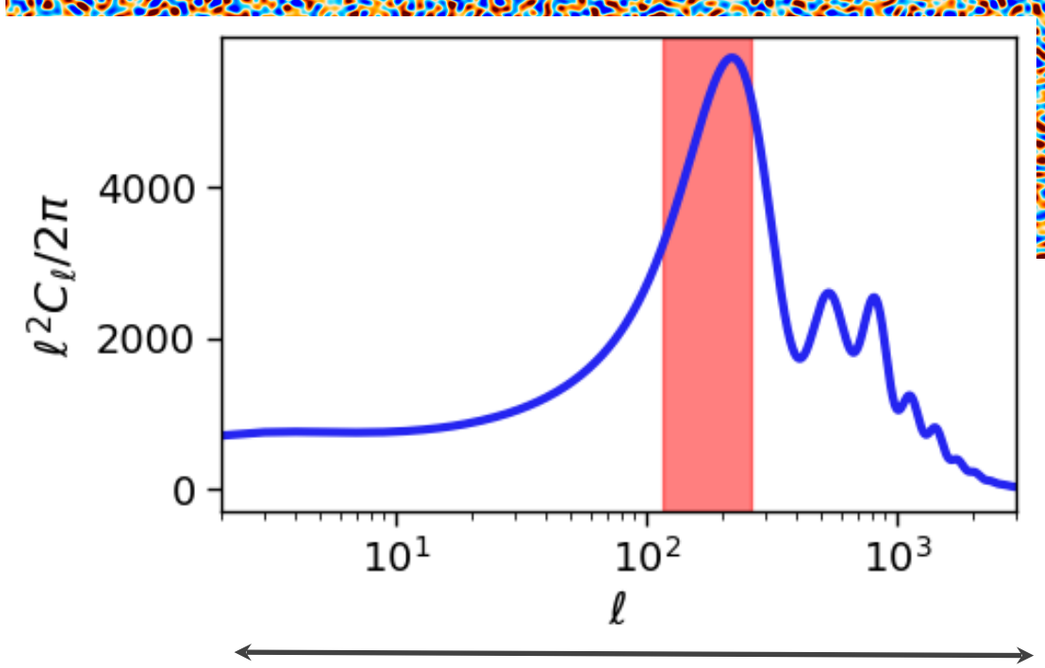

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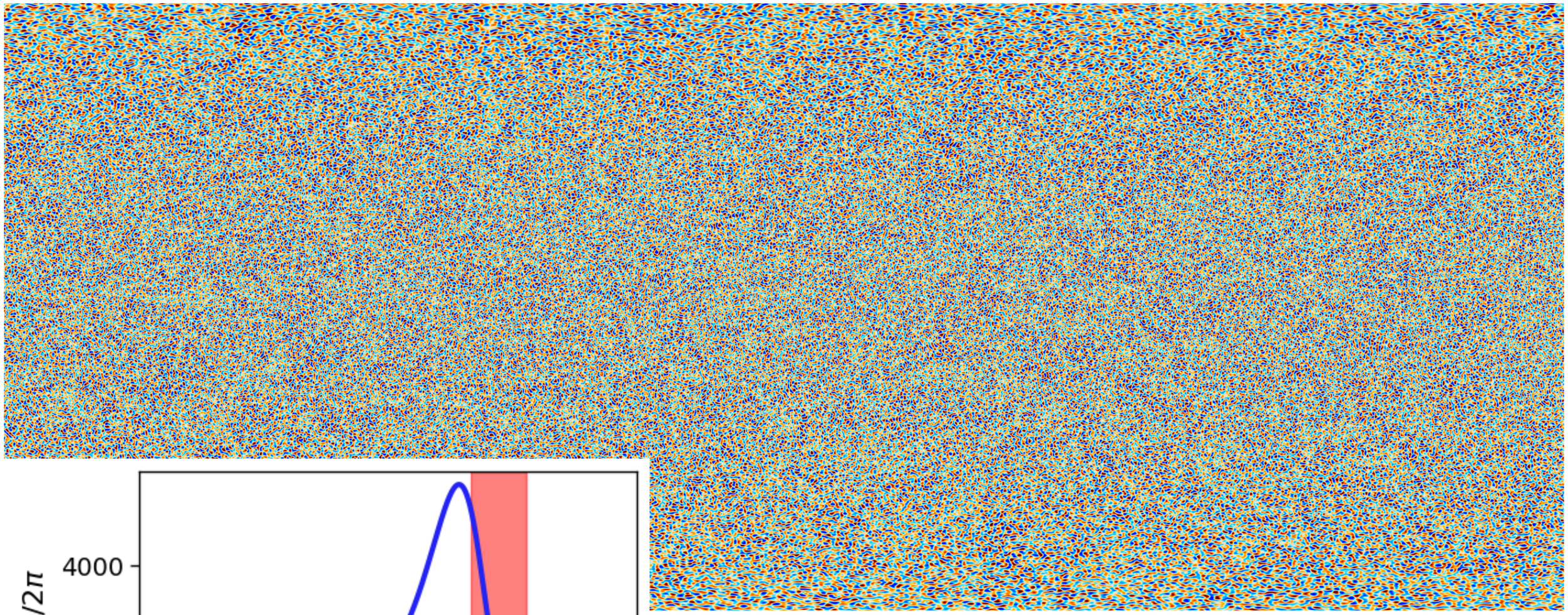

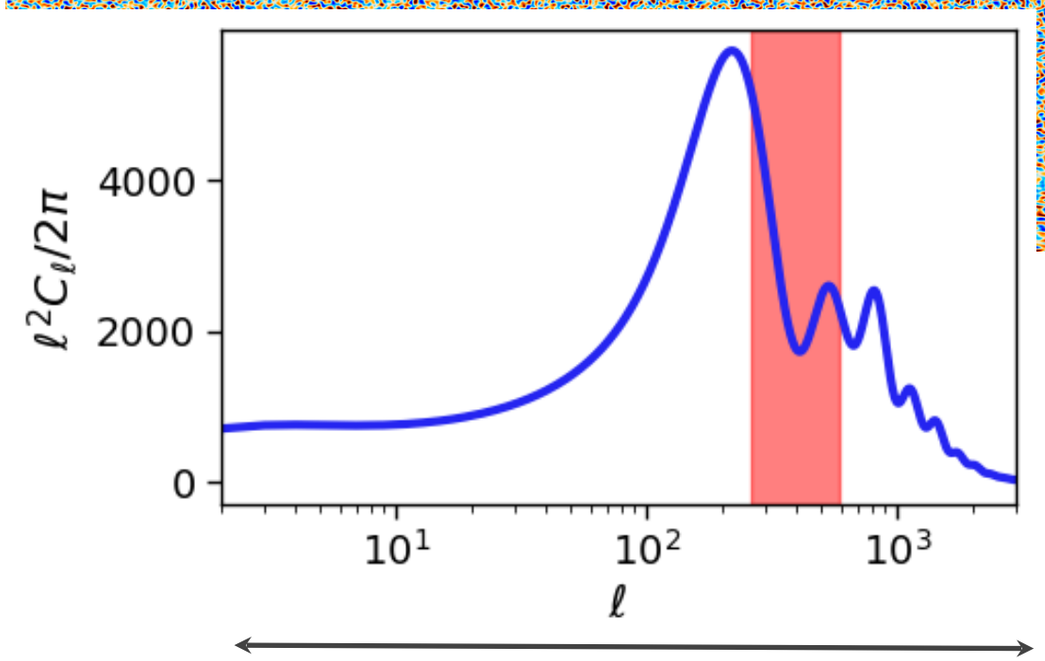

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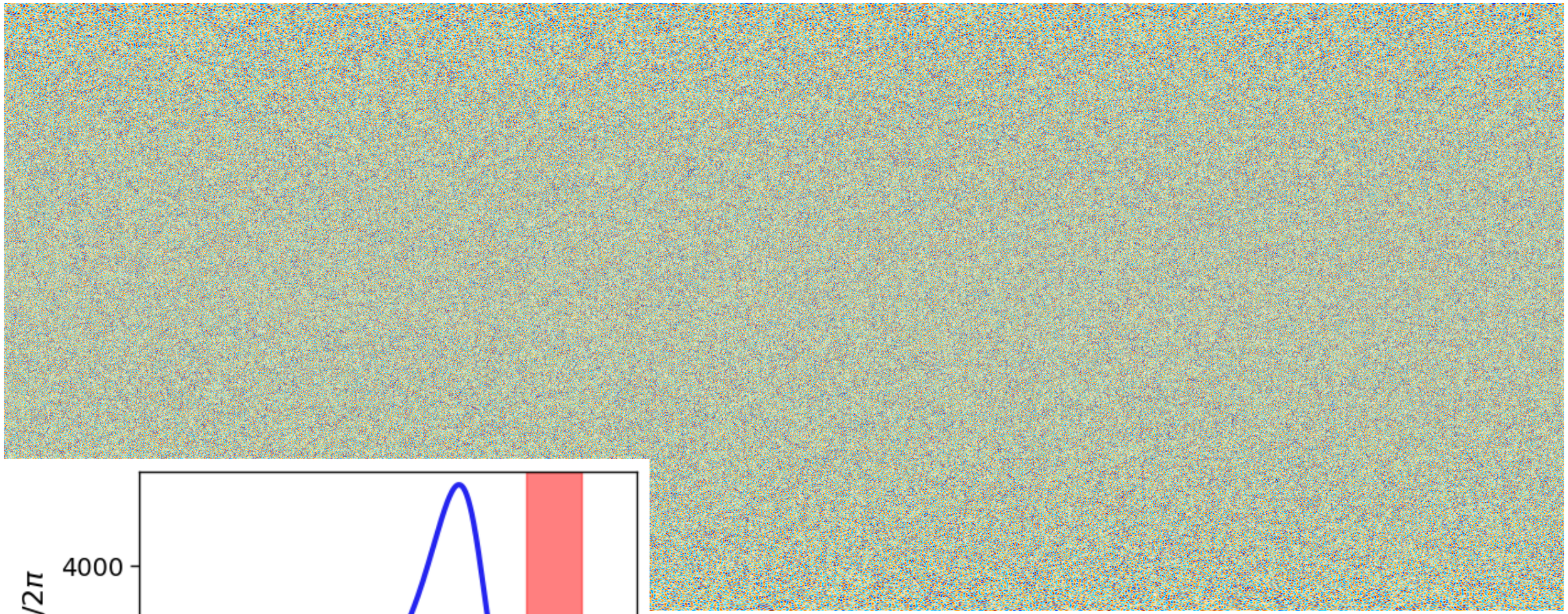

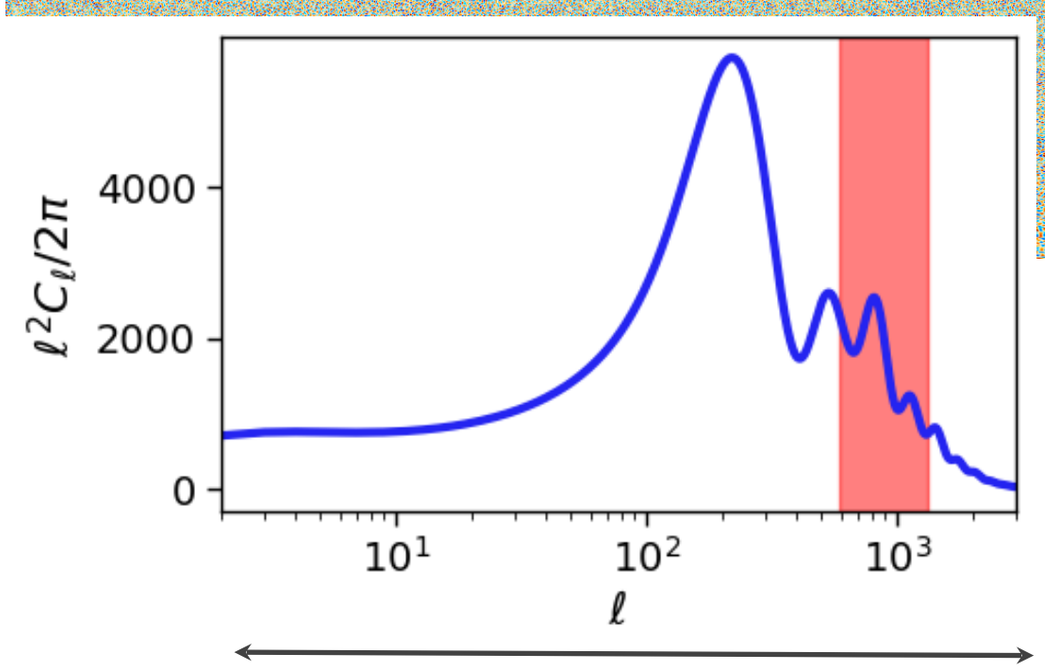

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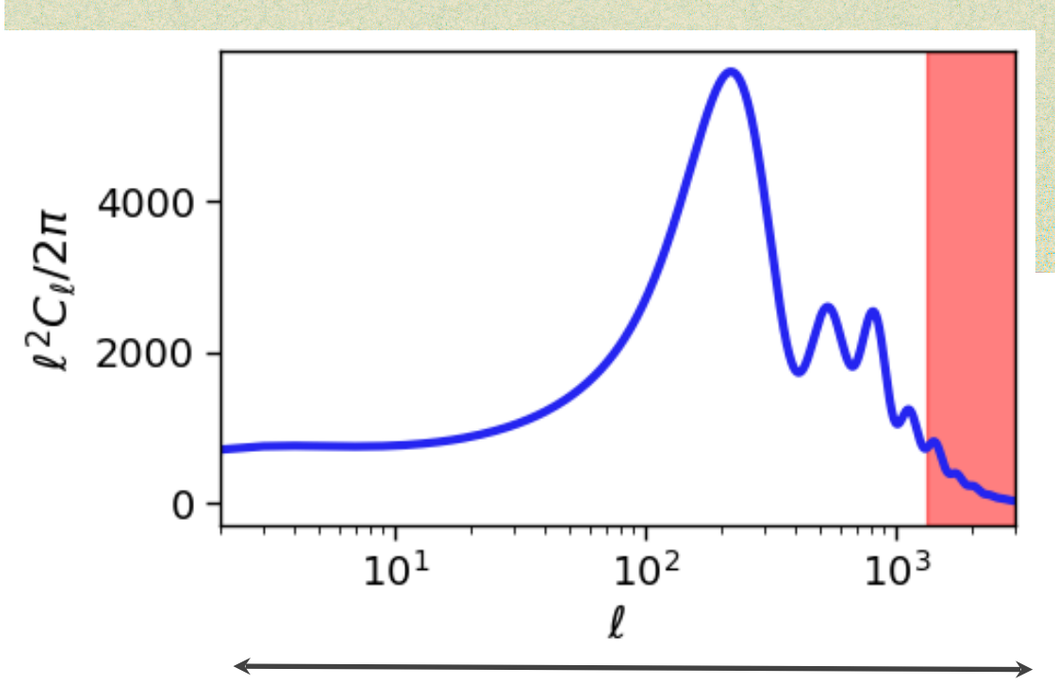
CMB Angular power spectrum = **variance of temperature fluctuations** as a function of **angular scale**



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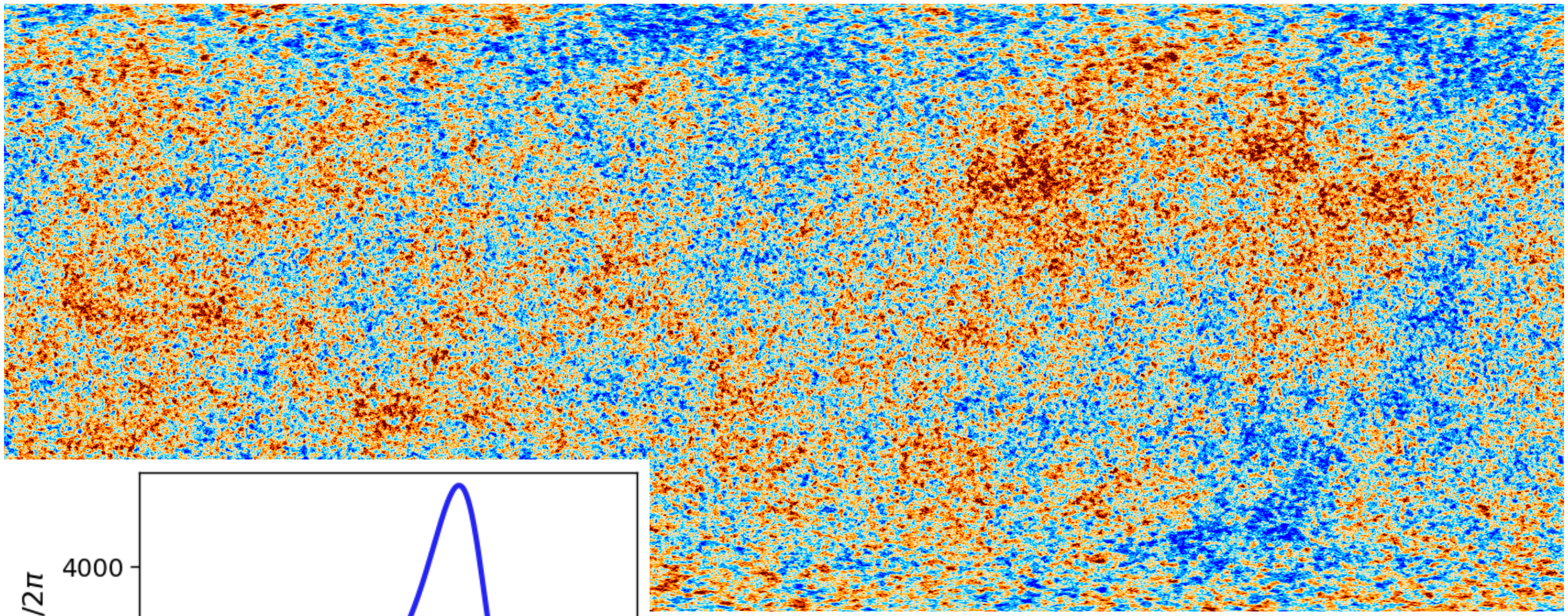
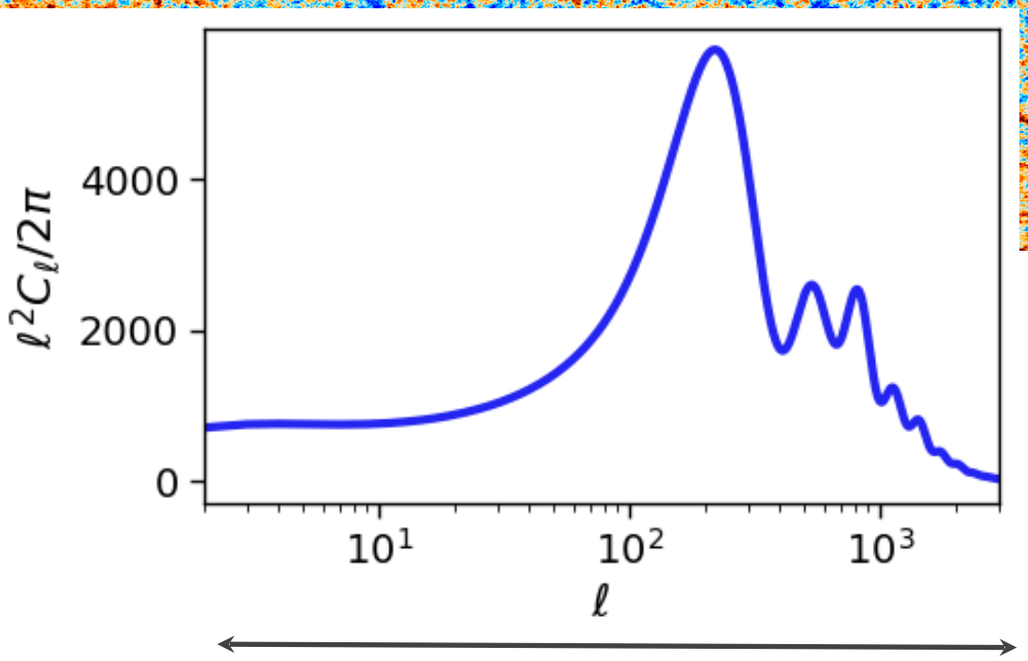
CMB Angular power spectrum = **variance of temperature fluctuations** as a function of **angular scale**





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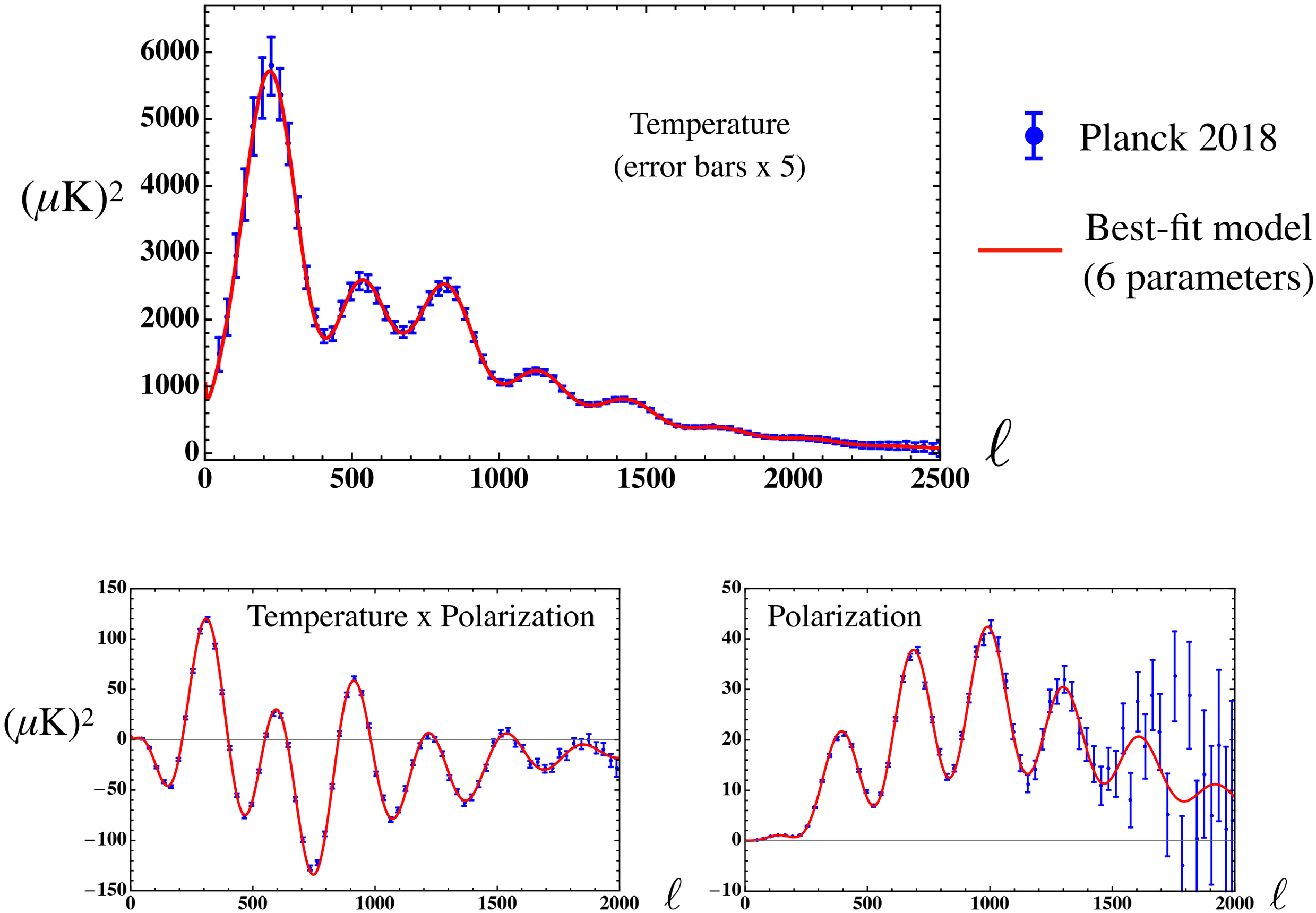
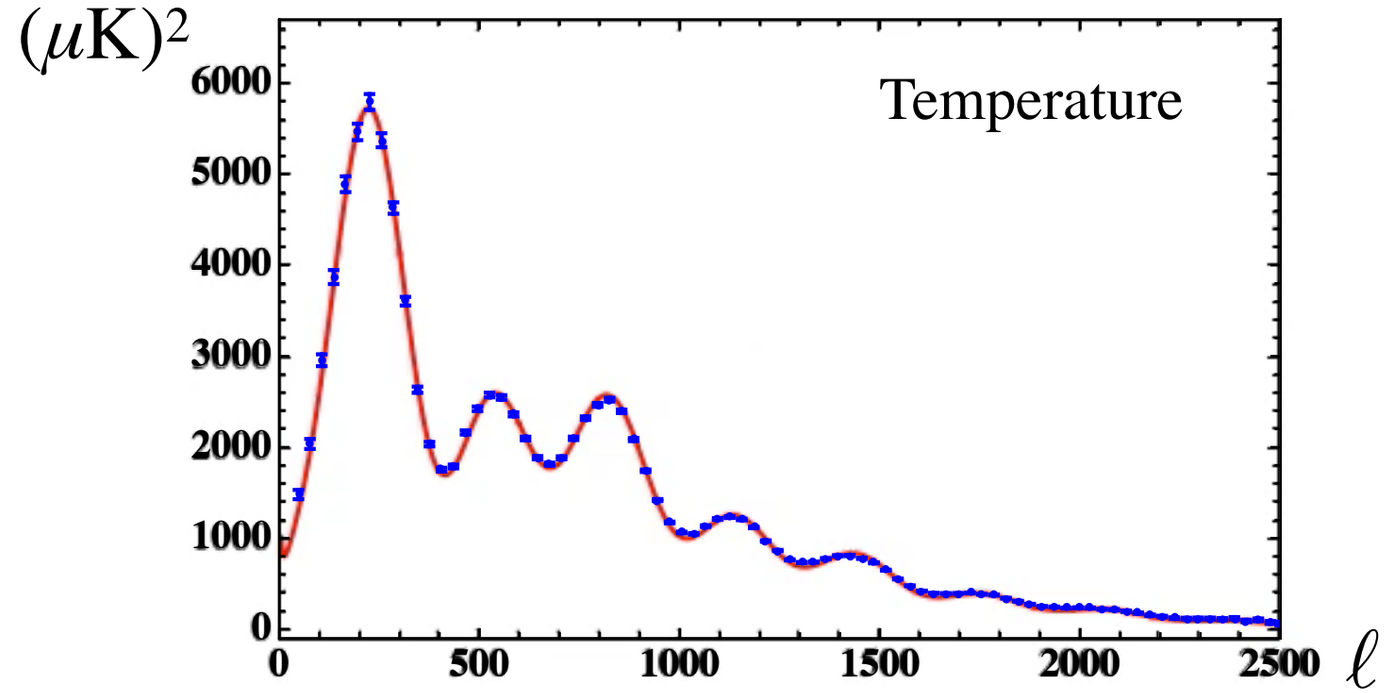
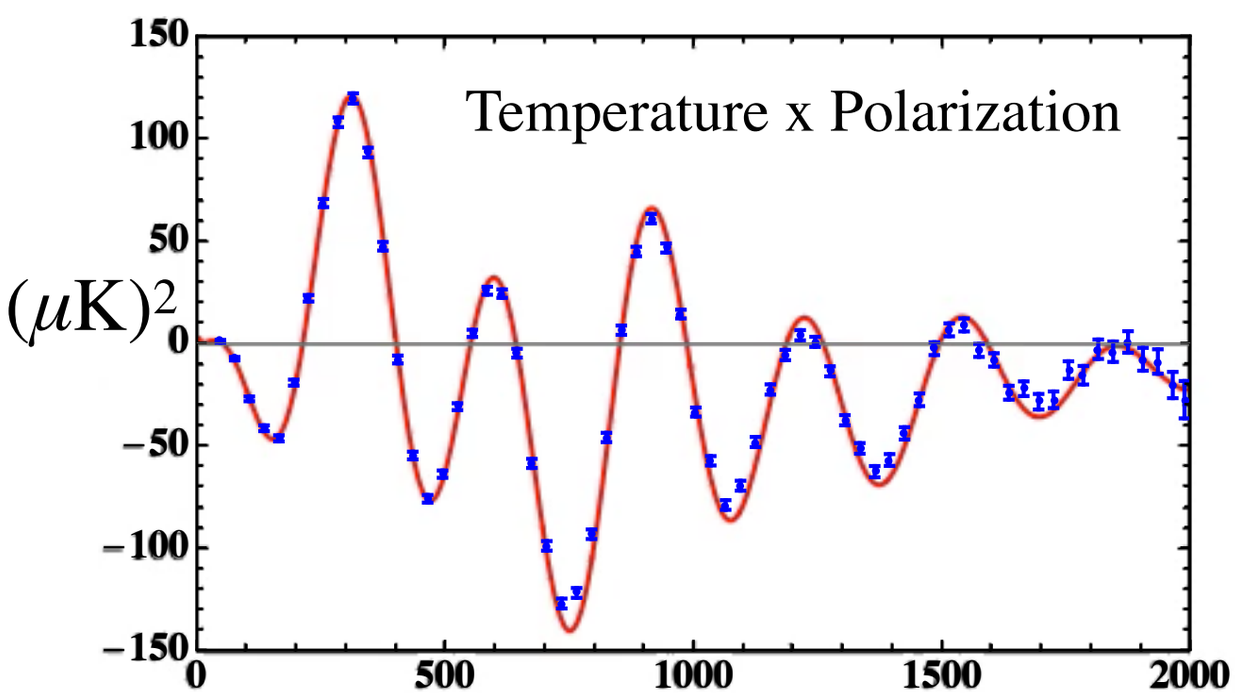
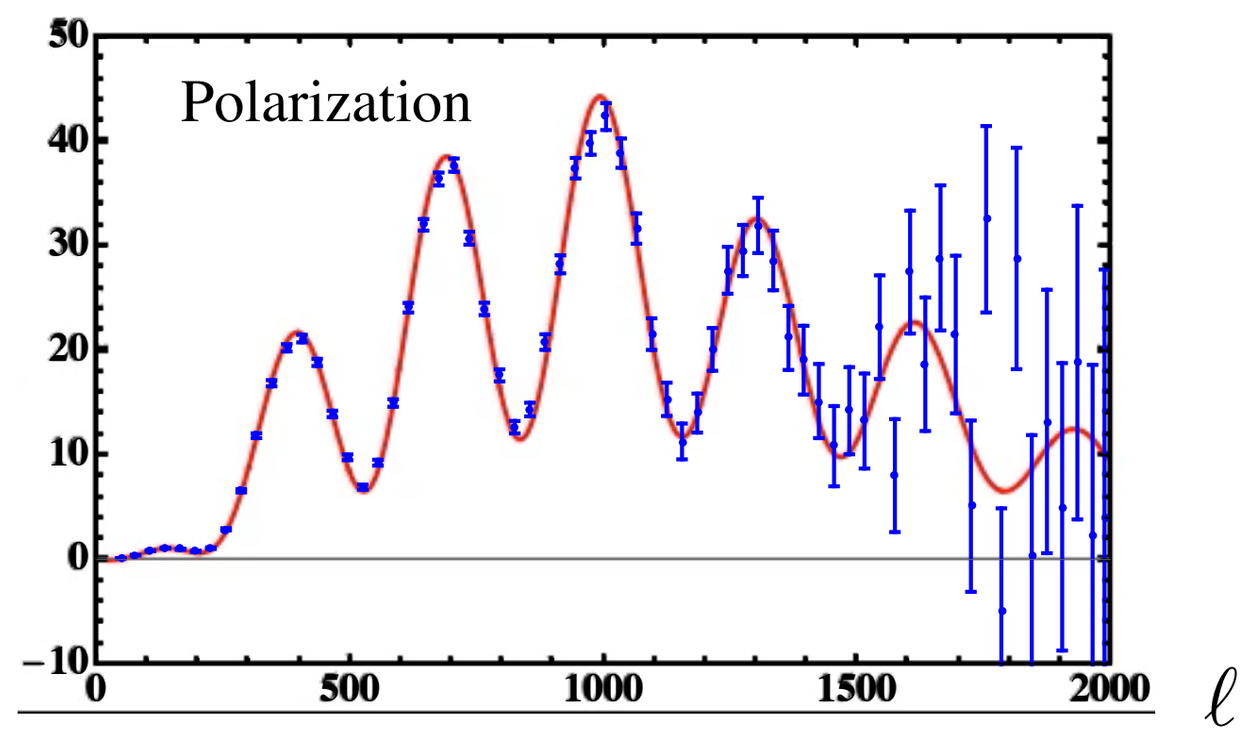

Temperature
(error bars x 5)
Planck 2018
Best-fit model
(6 parameters)
$(\mu K)^2$
ℓ
Temperature x Polarization
Polarization



$$\frac{\text{dark matter}}{\text{baryons}} \approx 5.36 \pm 0.05 \text{ (Planck 2018)}$$





CMB anisotropies are *very* sensitive to recombination

State-of-the-art recombination codes:

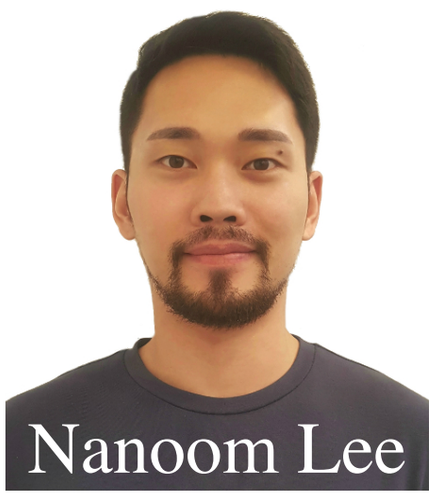
HyRec Ali-Haïmoud & Hirata 2010, 2011

CosmoRec Chluba & Thomas 2011

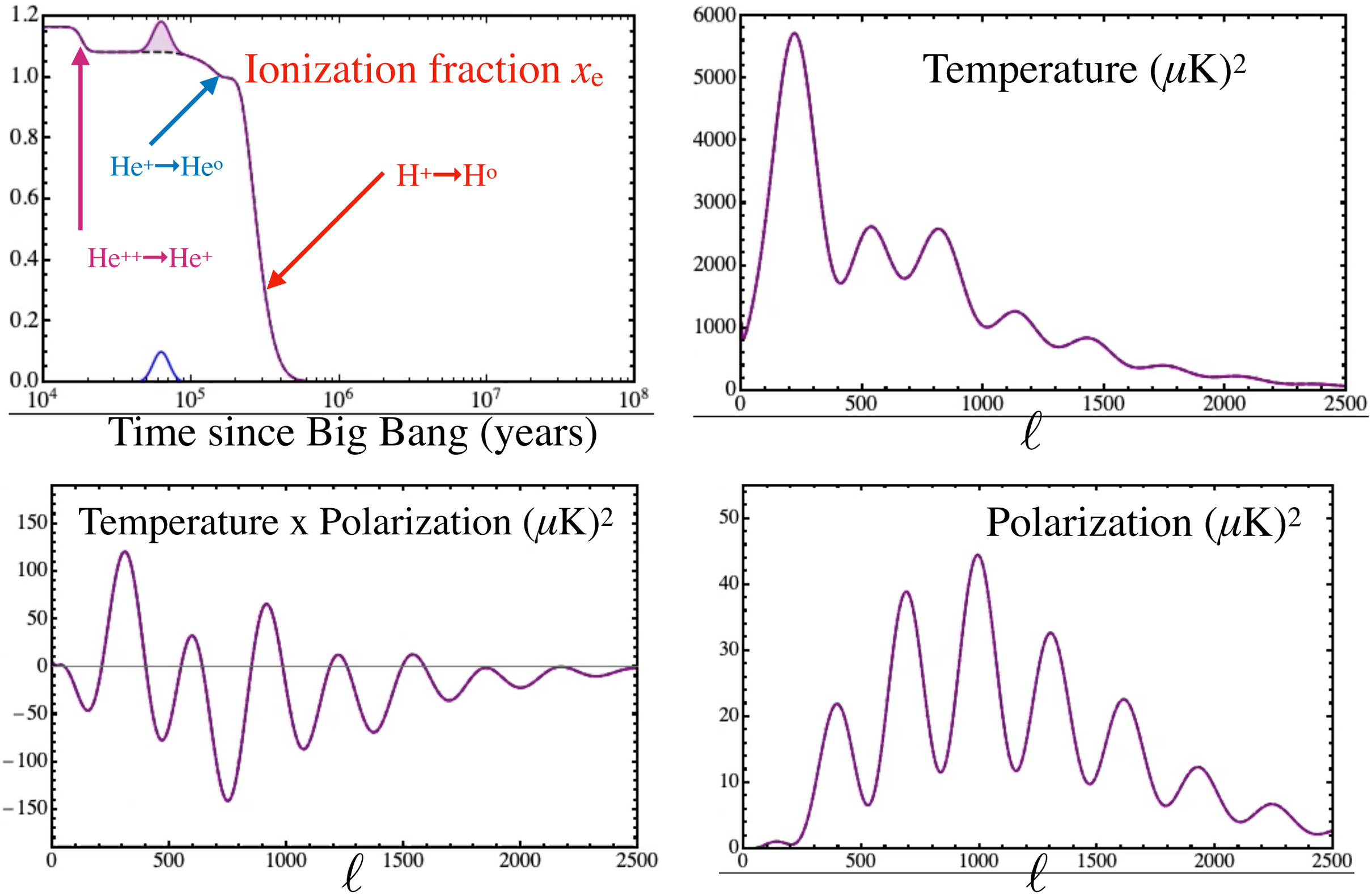
compute x_e[standard] with accuracy ~few parts in 1e4, in ~1 second / cosmology

HyRec-2 Lee & Ali-Haïmoud 2020

accuracy ~few parts in 1e4, in ~1 millisecond / cosmology



Effect of non-standard x_e on CMB power spectra



Non-standard recombination from energy injection

Suppose some process injects energy with volumetric rate

$$\dot{\rho}_{\rm inj} = \left.\frac{dE}{dtdV}\right|_{\rm injected}$$

Not all energy is efficiently *deposited* in the plasma (e.g. if the process only injects neutrinos, no deposition)

$$\dot{\rho}_{\rm dep} = \left.\frac{dE}{dtdV}\right|_{\rm dep} = f \times \dot{\rho}_{\rm inj}$$

Deposition efficiency f depends on details of injected energy (type of particles and spectrum)

Non-standard recombination from energy injection

Deposited energy

Extra heating of the gas

Direct extra ionizations of Hydrogen (and Helium)

Extra excitations of Hydrogen (and Helium)

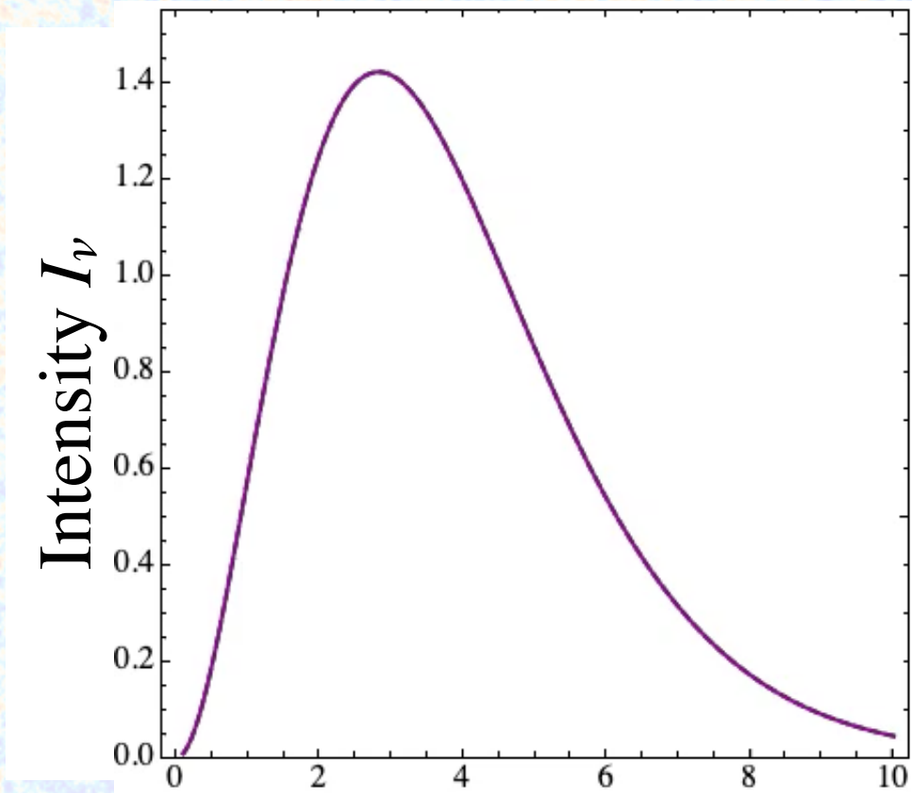
net effect: $$\dot{x}_e = \dot{x}_e[\text{standard}] + \Delta\dot{x}_e$$

$$\Delta\dot{x}_e \sim (1 - x_e)\frac{\dot{\rho}_{\text{dep}}}{n_{\text{H}} \times 13.6\ \text{eV}}$$

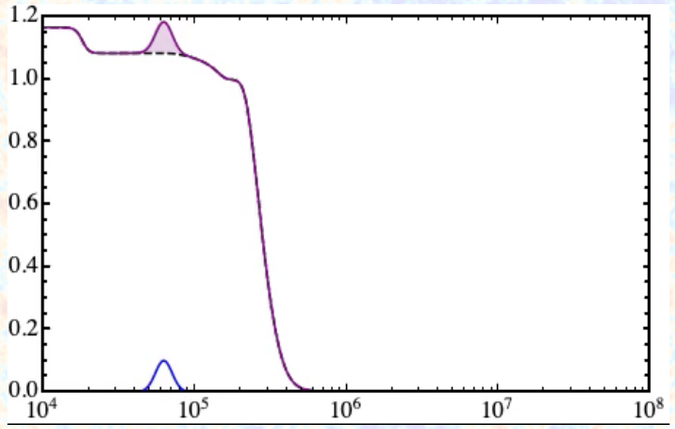
Summary of Part-1

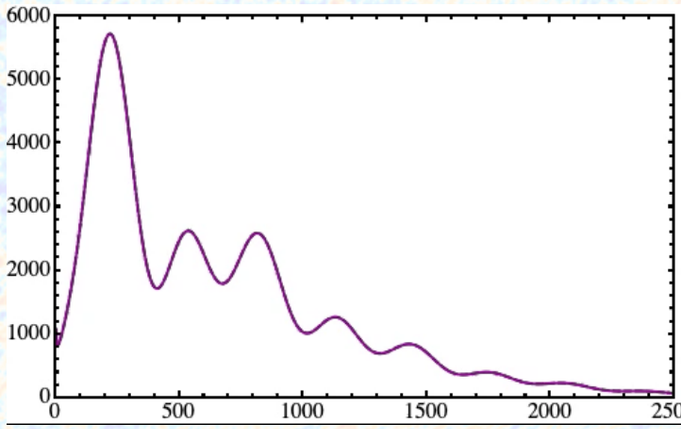
- CMB spectral distortions probe heat injection/extraction into/from photon-baryon plasma at z < 2e6

$$\frac{\Delta I_\nu}{I_\nu} \sim \int_{t_*} dt \frac{\dot{\rho}_{\rm dep}}{\rho_\gamma}$$

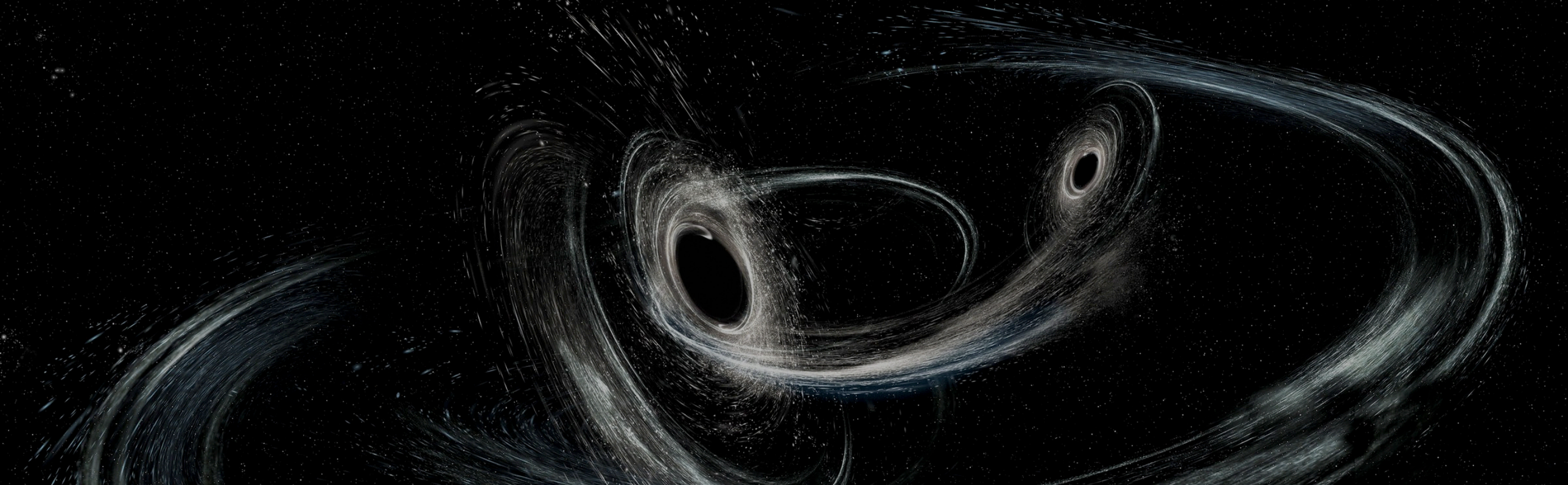


- Energy injection also affects Recombination hence CMB anisotropies





$$\Delta \dot{x}_e \sim (1 - x_e) \frac{\dot{\rho}_{\rm dep}}{n_{\rm H} \times 13.6 \text{ eV}}$$



2- Energy injection processes due to PBHs and resulting CMB constraints

A-Dissipation of small-scale perturbations

Ideal fluid: infinitely small mean-free path between collisions

Comoving mean-free path between Thomson scattering events:

$$\lambda = \frac{1}{n_e \sigma_{\rm T}} \qquad \lambda_{\rm com} = \frac{1}{a n_e \sigma_{\rm T}} \approx 3\ {\rm Mpc} \frac{(10^3/z)^2}{x_e(z)}$$

=>The photon-baryon plasma is not quite an ideal fluid.

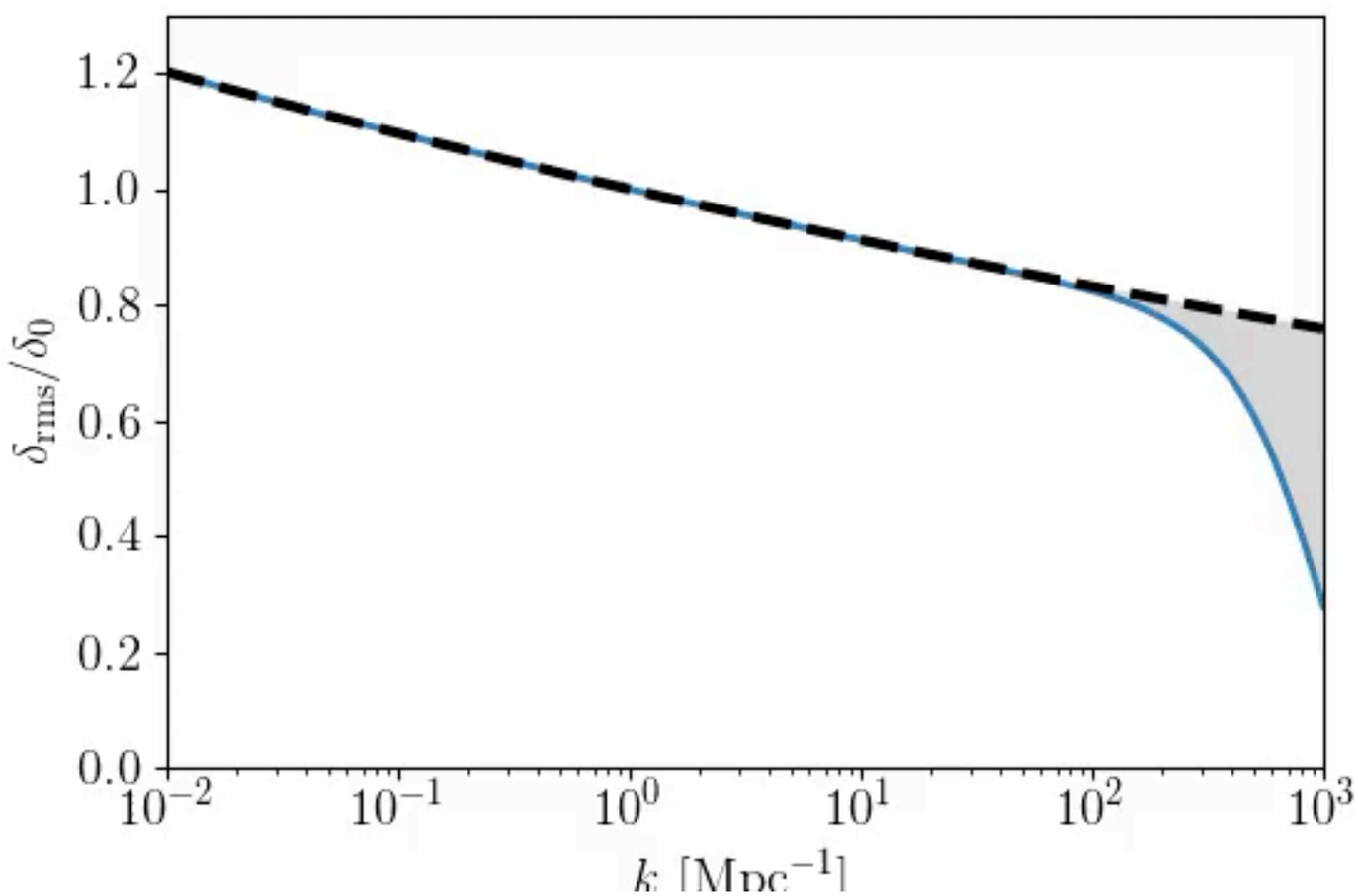
This induces a viscous force:

$$\left.\frac{d\vec{v}}{dt}\right|_{\rm visc} \sim -\lambda c_s \nabla^2 \vec{v} \qquad c_s \approx c/\sqrt{3}$$

=> Viscosity force exponentially damps acoustic oscillations at scales smaller than the Silk damping scale

$$\delta(\eta) \sim \delta_0 \cos(kc_s\eta) e^{-k^2/k_{\rm D}^2(\eta)}$$

$$k_{\rm D}^{-2}(\eta) \sim \int^{\eta} \frac{d\eta'}{a' n_e' \sigma_{\rm T}} \qquad k_D \sim 0.1(z/10^3)^{3/2}\ {\rm Mpc}^{-1}$$



Energy density in sound waves:

$$\rho_{\rm waves} \sim \rho_\gamma c_s^2 \langle \delta_\gamma^2 \rangle = \rho_\gamma c_s^2 \int^{k_{\rm D}} d\ln k \; \Delta_\gamma^2(k)$$

=> Energy dissipated

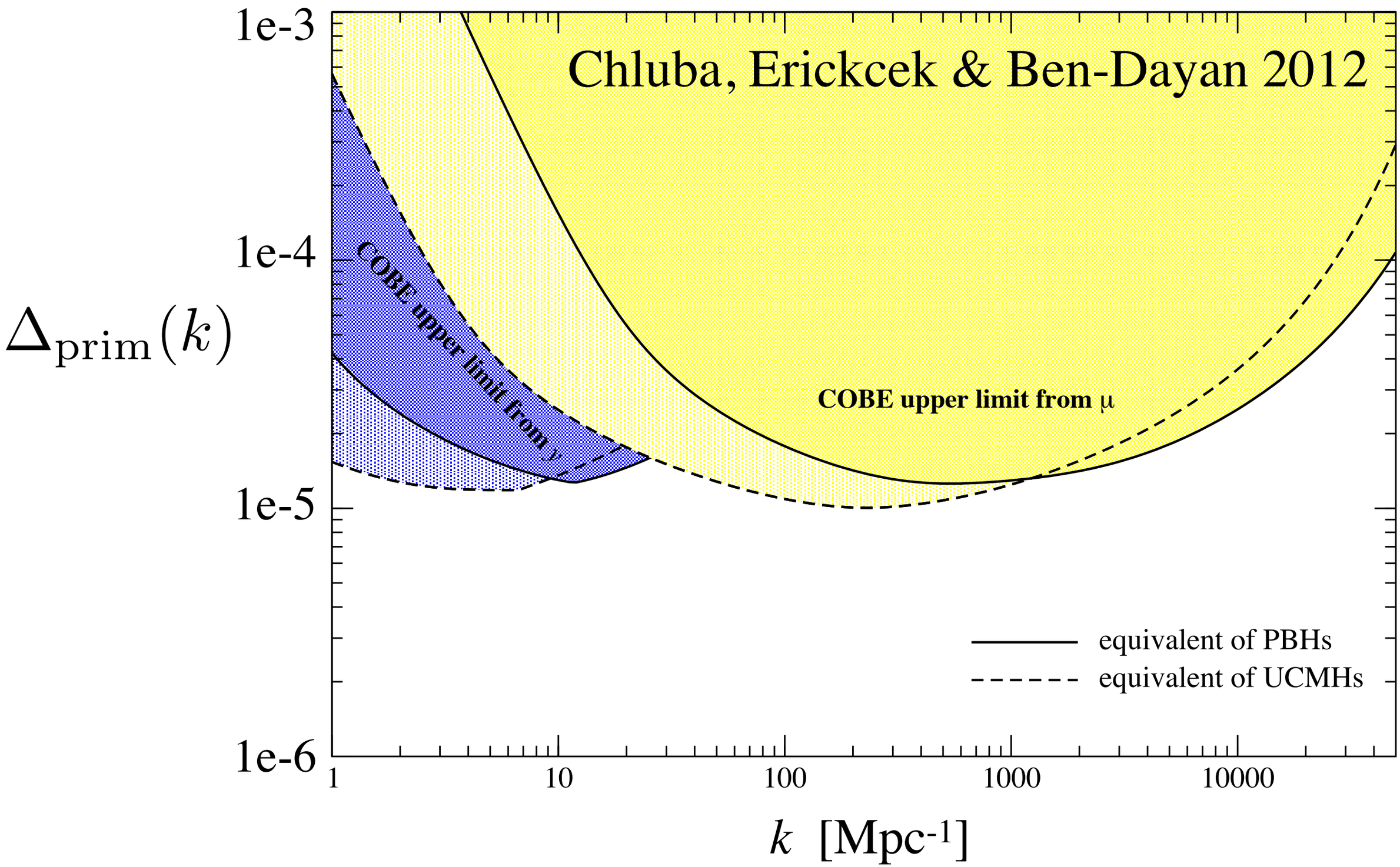
$$\frac{\dot\rho_{\rm diss}}{\rho_\gamma} \sim c_s^2 \frac{d\ln k_{\rm D}}{dt} \Delta_\gamma^2(k_{\rm D})$$

=> Spectral distortion

$k_D(z_*) \sim 10^4 \ {\rm Mpc}^{-1}$

$$\frac{\Delta I_\nu}{I_\nu} \sim c_s^2 \int^{k_D(z^*)} d\ln k_D \; \Delta_\gamma^2(k_D) \qquad [z_* \approx 2 \times 10^6]$$

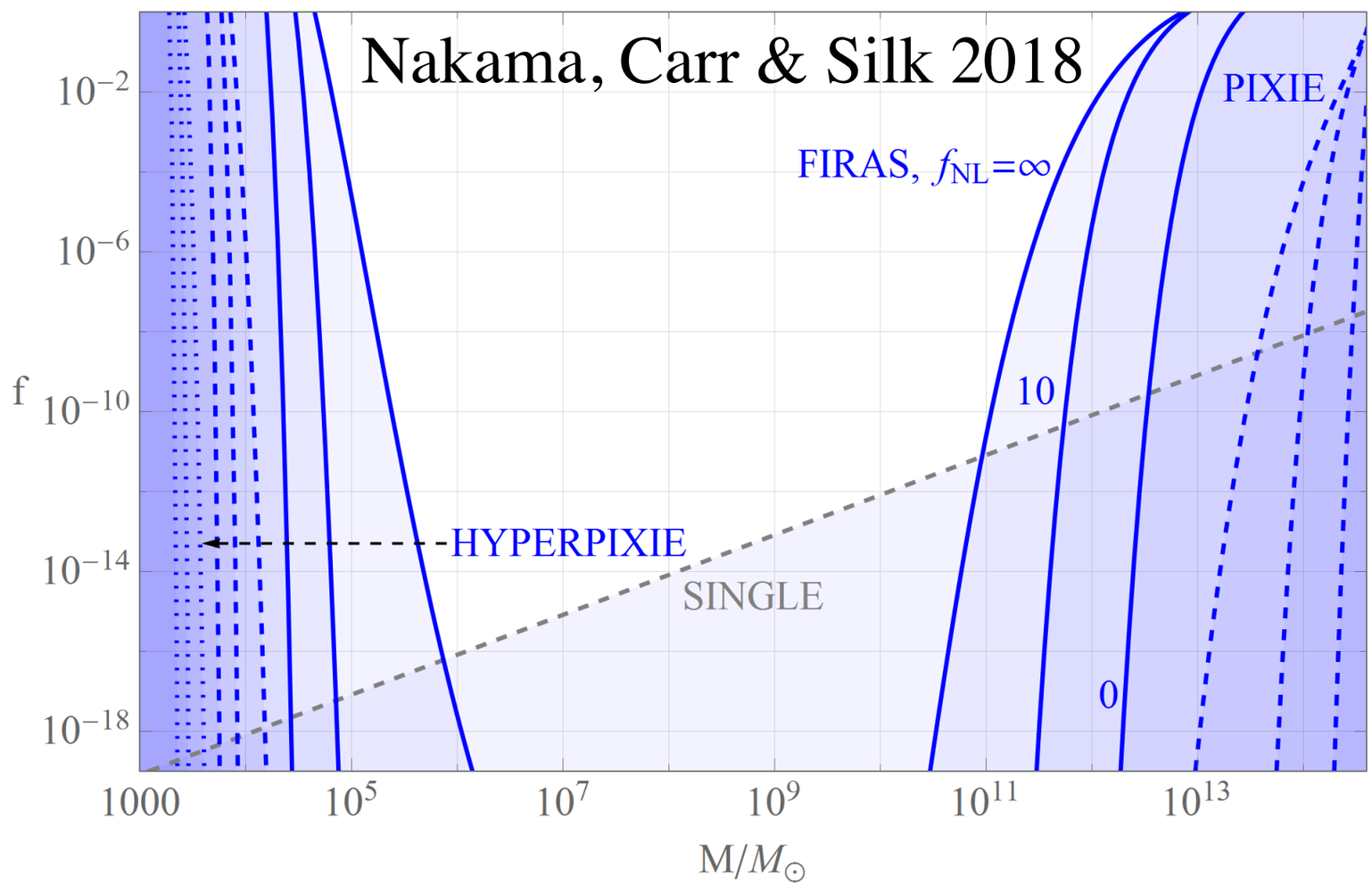
Upper limits on primordial (adiabatic) perturbations



Resulting indirect constraints on PBHs

If PBHs result from the collapse of order ~0.1 density perturbations and if primordial perturbations are Gaussian

=> Upper limits of ~1e-5 on Δ_{prim} essentially rule out PBHs with $\boldsymbol{M \gtrsim 2e4\, M_{sun}}$ from making any appreciable fraction of the DM



B-Hawking evaporation

$$\frac{dM}{dt} \sim -\frac{M_{\rm P}^3}{t_{\rm P} M^2} \qquad t_{\rm evap} \sim t_{\rm P} (M/M_{\rm P})^3$$

For ***M* ≲ 1e15** grams, PBHs would have **already evaporated by the present time**, thus cannot make more than ~1% of DM

For more massive PBHs, inject energy in the plasma

$$\dot{\rho}_{\rm inj} = n_{\rm PBH} \left| \frac{dM}{dt} \right| \sim f_{\rm pbh} \frac{\rho_{\rm DM}}{t_{\rm P}} (M_{\rm P}/M)^3$$

$$\left[n_{\rm PBH} = f_{\rm PBH} \frac{\rho_{\rm DM}}{M} \right]$$

Resulting effect on the CMB

- Effect on recombination: $$\Delta \dot{x}_e \sim (1 - x_e) \frac{\dot{\rho}_{\rm dep}}{n_{\rm H} \times 13.6 \text{ eV}}$$

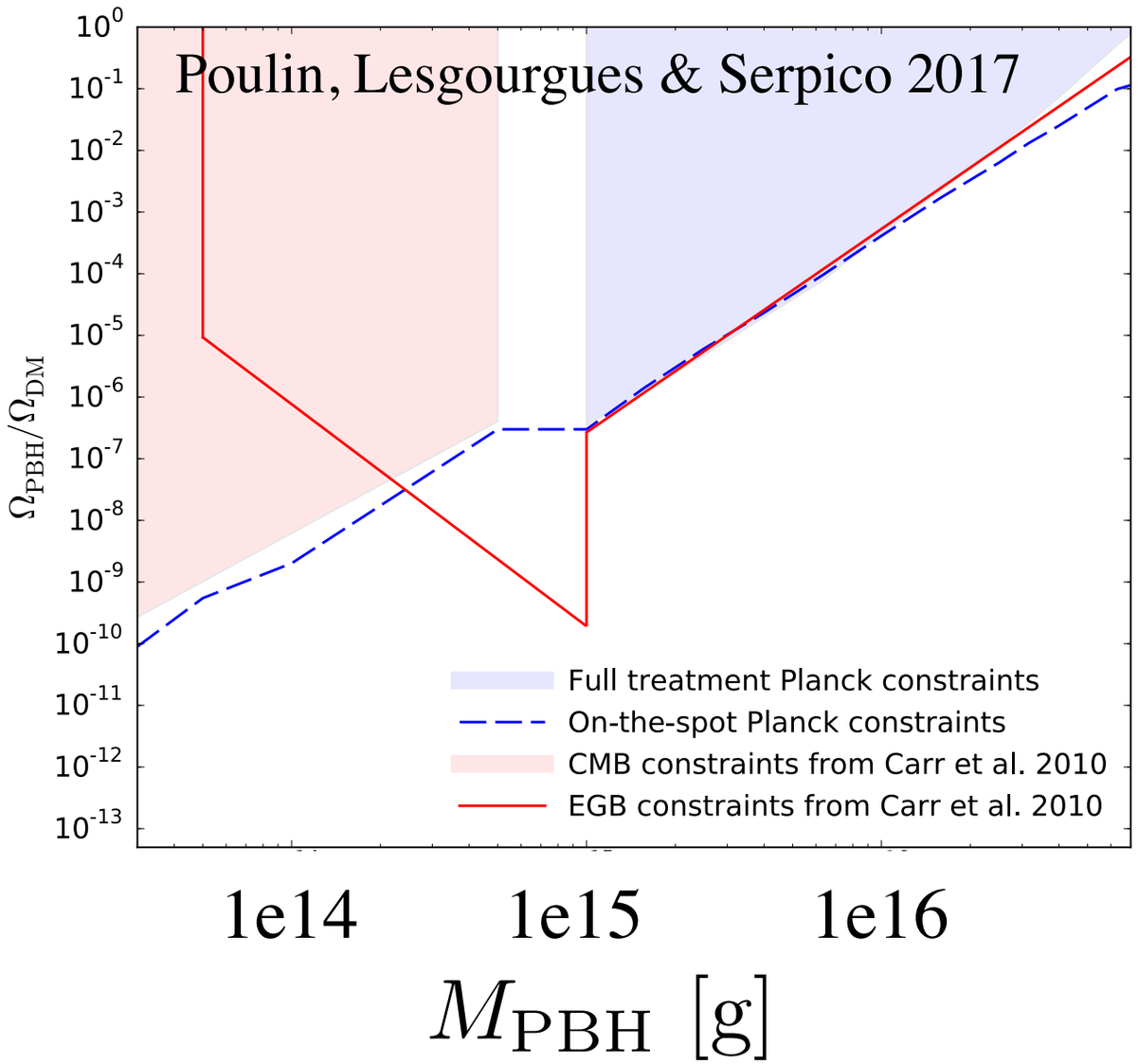
$$\Delta x_e \sim t_{\rm rec} \frac{\dot{\rho}_{\rm dep}}{10^{-8} \rho_b} \sim 10^8 f_{\rm dep} f_{\rm pbh} (M_P/M)^3 \frac{\rho_{\rm DM}}{\rho_b} \frac{t_{\rm rec}}{t_{\rm P}}$$

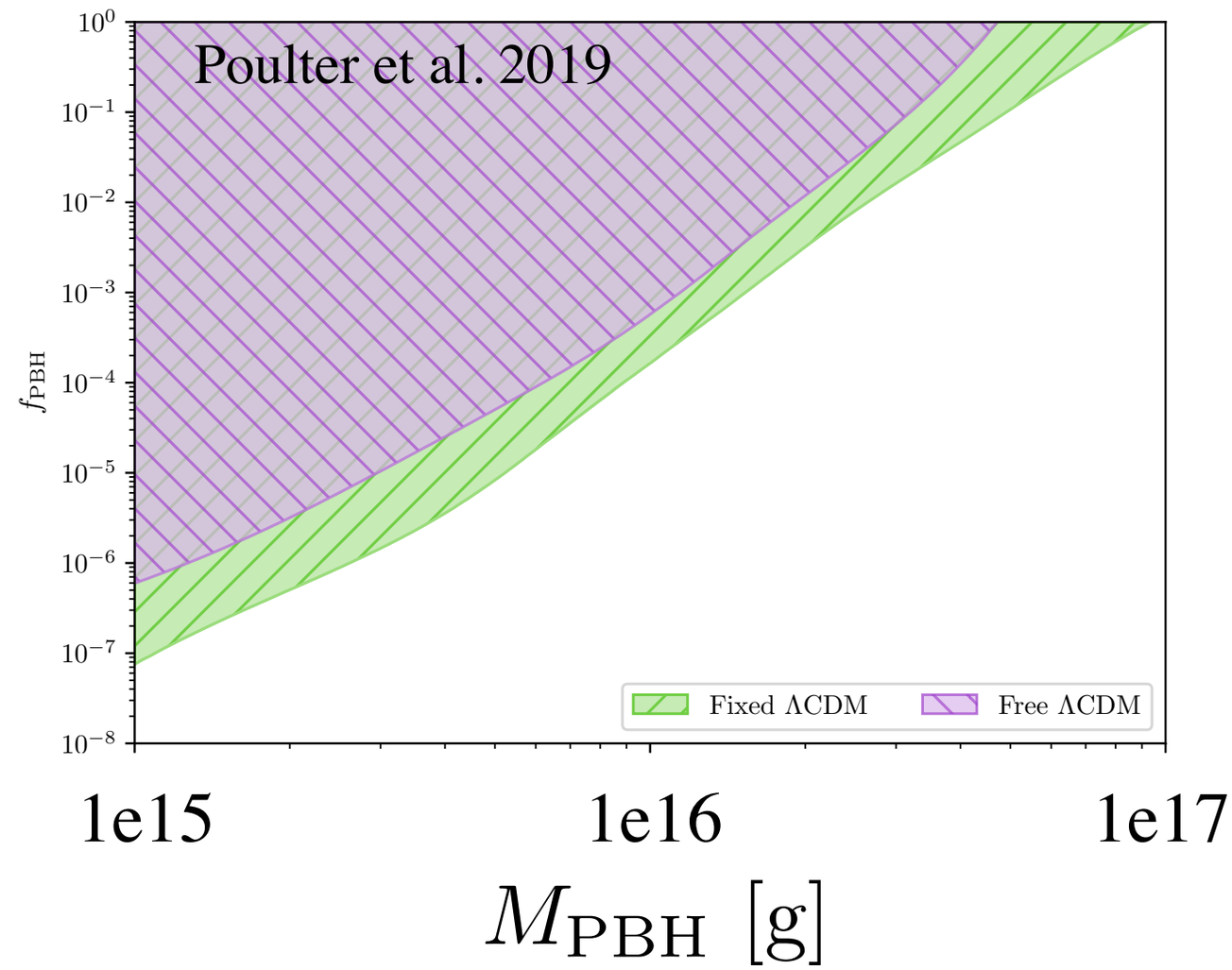
$$\sim 10^5 (f_{\rm dep}/0.1) f_{\rm pbh} (10^{15} {\rm g}/M)^3$$

A **very** rough estimate of the sensitivity of CMB anisotropies to recombination perturbations: $\Delta x_e \lesssim 0.01$

$$f_{\rm pbh} \lesssim 10^{-7} (M/10^{15} {\rm g})^3$$

The precise limits require a much more detailed treatment of course!





Exercise: estimate the amplitude of **spectral distortions** from PBH evaporation and show that SDs are much less constraining that CMB anisotropies. Same for PBH accretion.

C-Accretion

- PBHs accrete gas with accretion rate $\dot{M}$
- Part of the accreted rest-mass is re-radiated $L = \epsilon \dot{M} c^2$
- Volumetric rate of energy injection:

$$\dot{\rho}_{\rm inj} = n_{\rm PBH} L = f_{\rm PBH} \, \frac{\rho_{\rm DM}}{M} L$$

- Conversion from injection to deposition depends on type of particles and spectrum.
- Compute effect on recombination, then CMB anisotropies

first studies: Miller 2000 and Ricotti, Ostriker & Mack 2008

Accretion rate $\dot{M}$

- Simplest scenario: stationary black hole in a uniform gas with density $\varrho_{\rm b}$ with sound speed $c_{\rm s}$. Known solution: Bondi-Hoyle

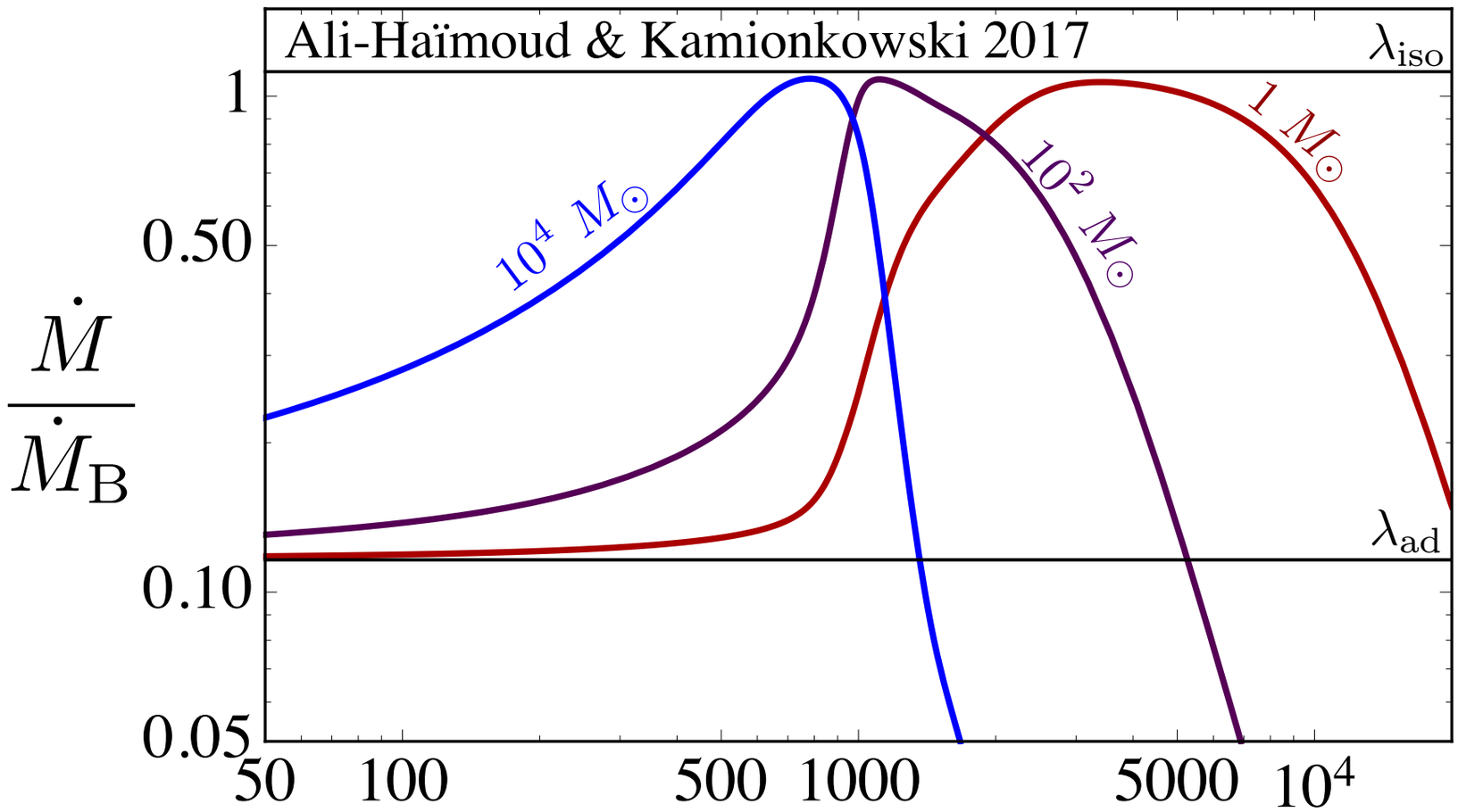
$$R_{\rm B} = \frac{GM}{c_s^2} \qquad \dot{M}_{\rm B} = 4\pi\rho_b c_s R_{\rm B}^2 = 4\pi\rho_b \frac{(GM)^2}{c_s^3}$$

- Complication 1): $c_{\rm s}$ = Adiabatic sound speed? Isothermal sound speed? => heat equation has to be solved self-consistently, accounting for Compton heating by CMB photons.

- Complication 2): Bondi-Hoyle solution results from balance of gravity and pressure forces only. In the early Universe, baryons feel Compton drag from CMB photons

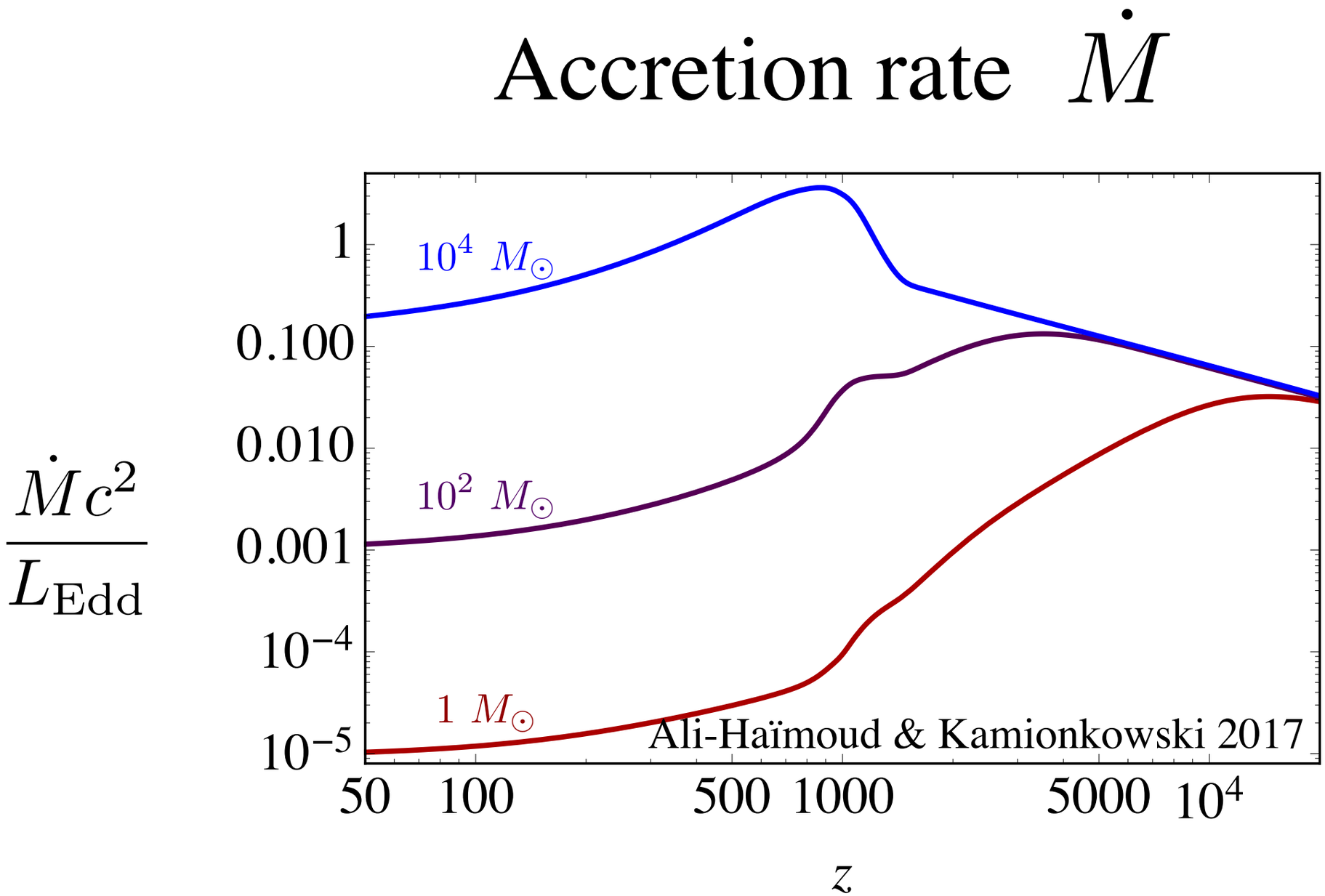
$$\dot{M} = \lambda\ \dot{M}_{\rm B}[{\rm iso}]$$

Accretion rate $\dot{M}$



- Complication 3): PBHs are not stationary: large relative velocity between DM and baryons (~ Mach 5 around z ~ 1e3).

Ad-hoc solution: $c_s \to \sqrt{c_s^2 + v_{\rm rel}^2}$ then average L over $v_{\rm rel}$

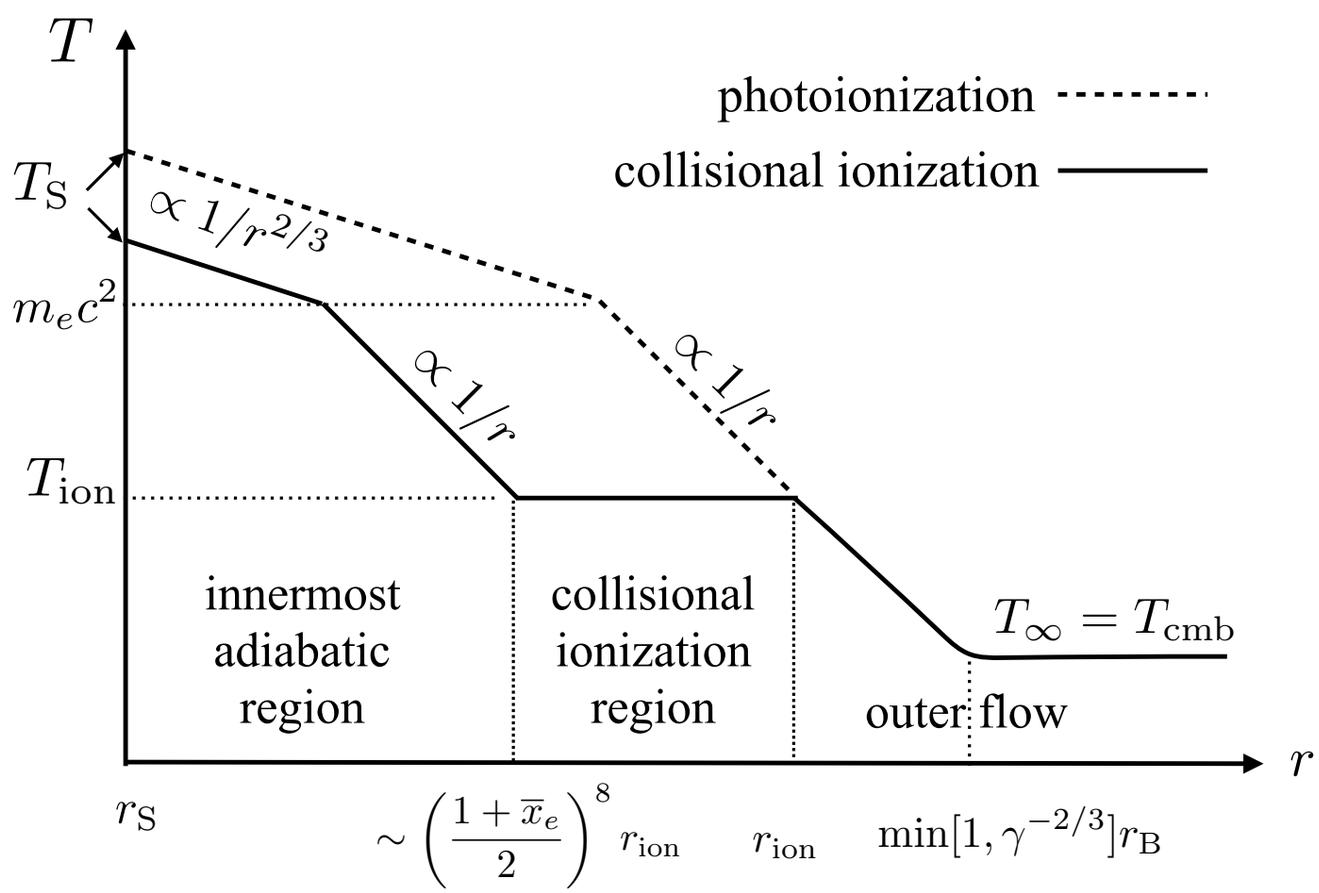


- Complication 4): an accretion disk might develop (Poulin et al. 2017). Caveat: does the flow cool fast enough to form a disk?

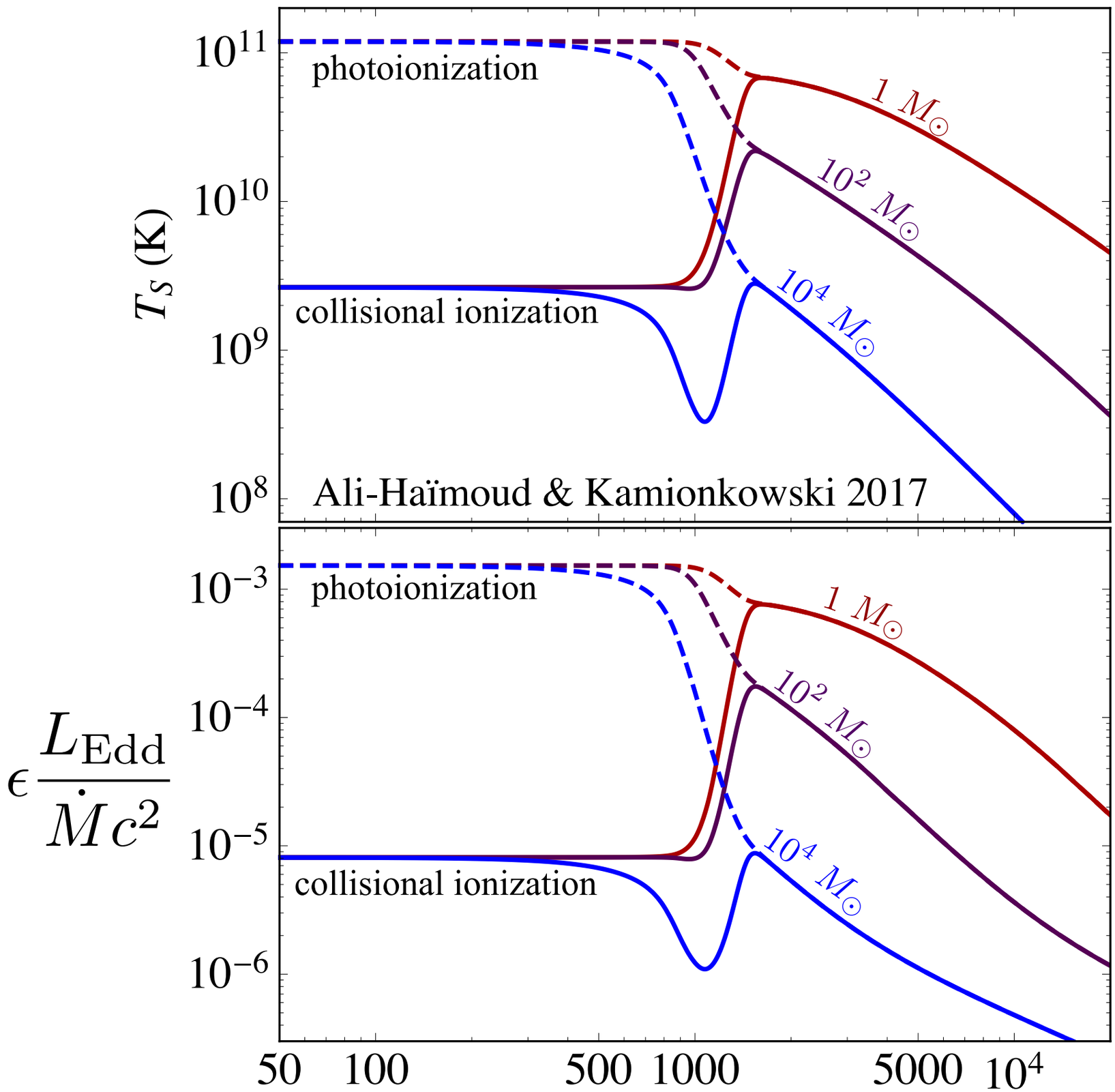
Radiative efficiency

This is a famously difficult problem
=> Let us try to estimate the minimum possible radiative efficiency, to derive a conservative bound on PBH abundance.

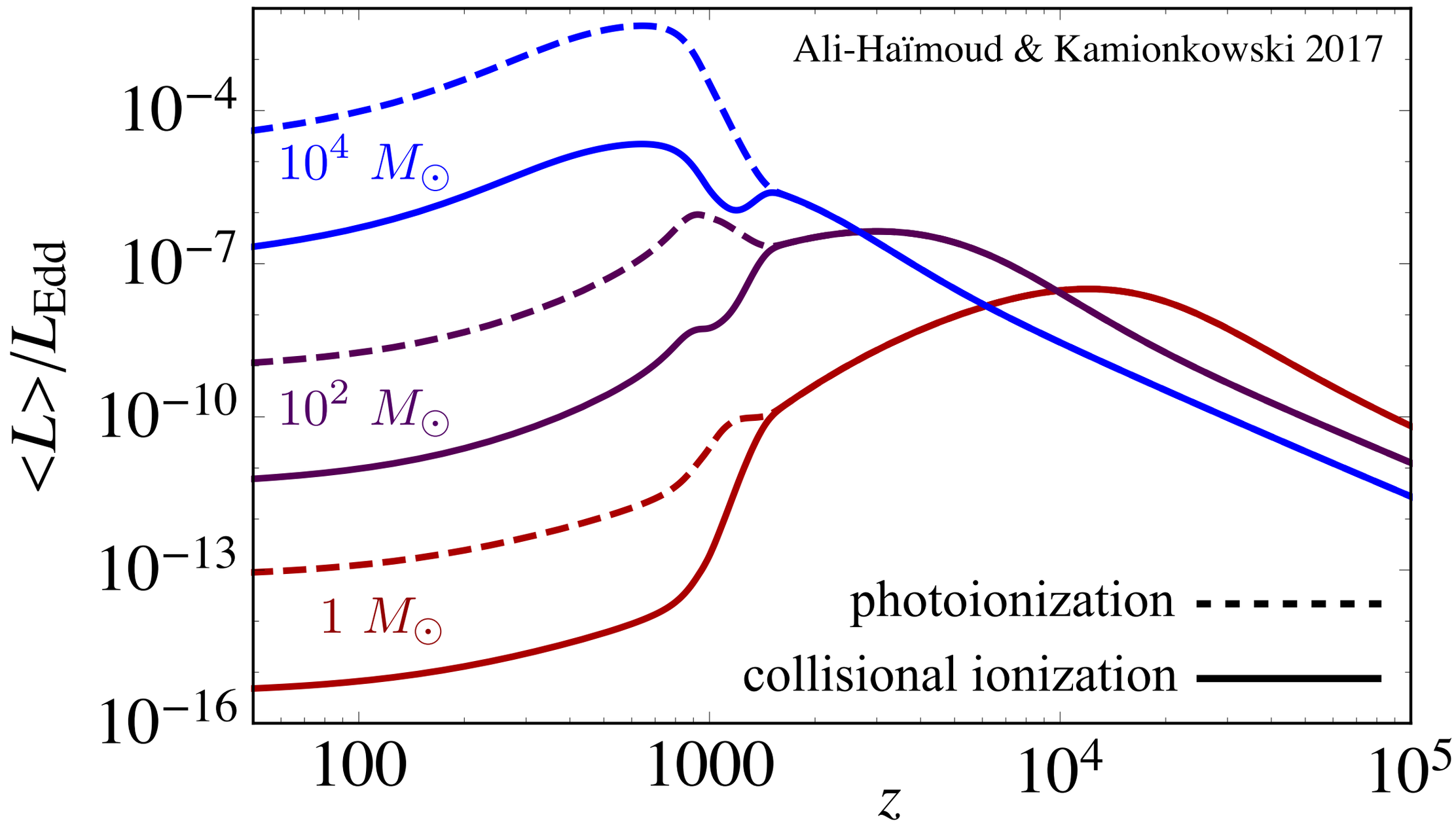
At minima, the infalling gas gets heated and ionized, and emits free-free radiation. => we need to estimate the density and temperature profile of the accreting gas



Radiative efficiency



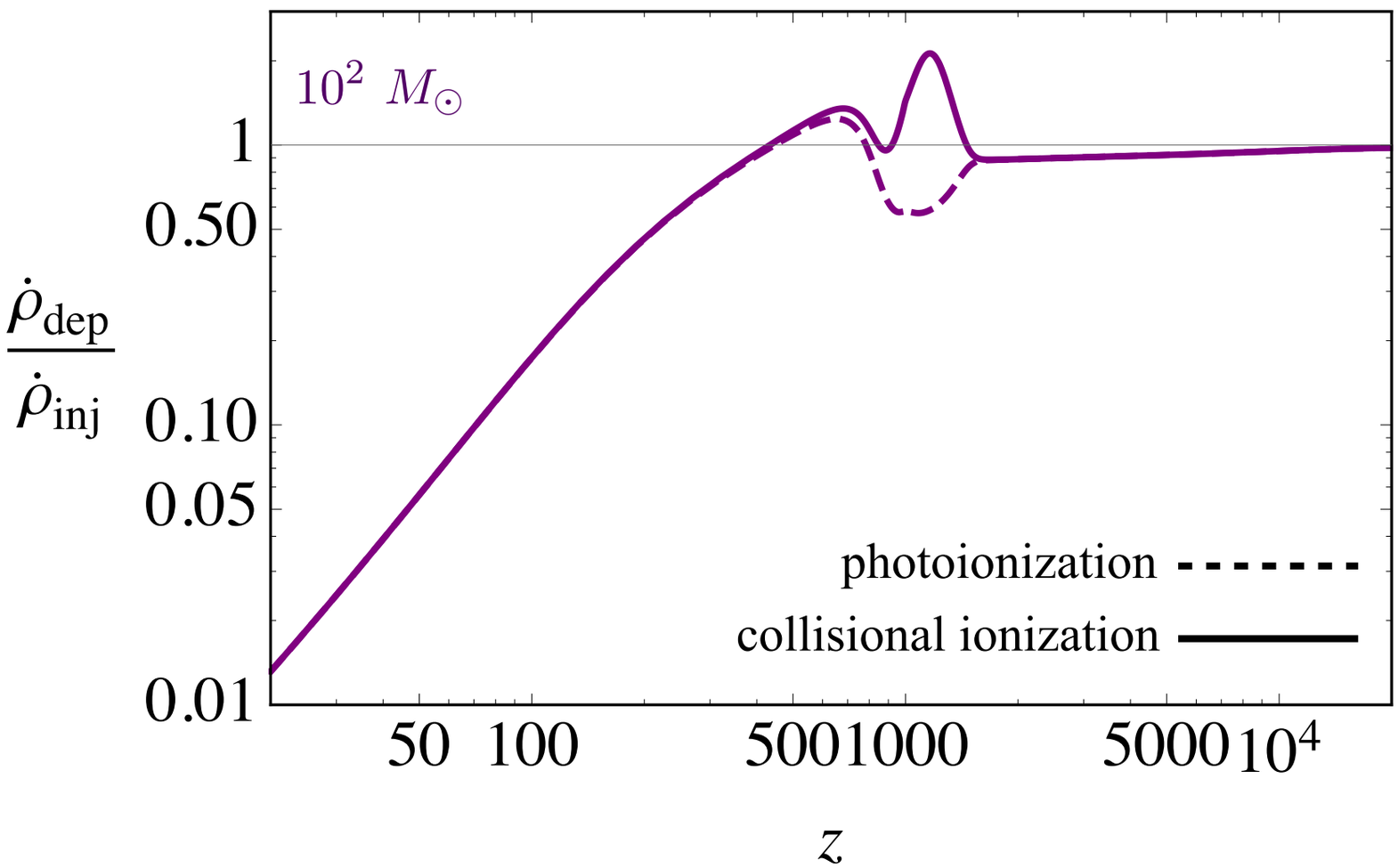
Radiative efficiency



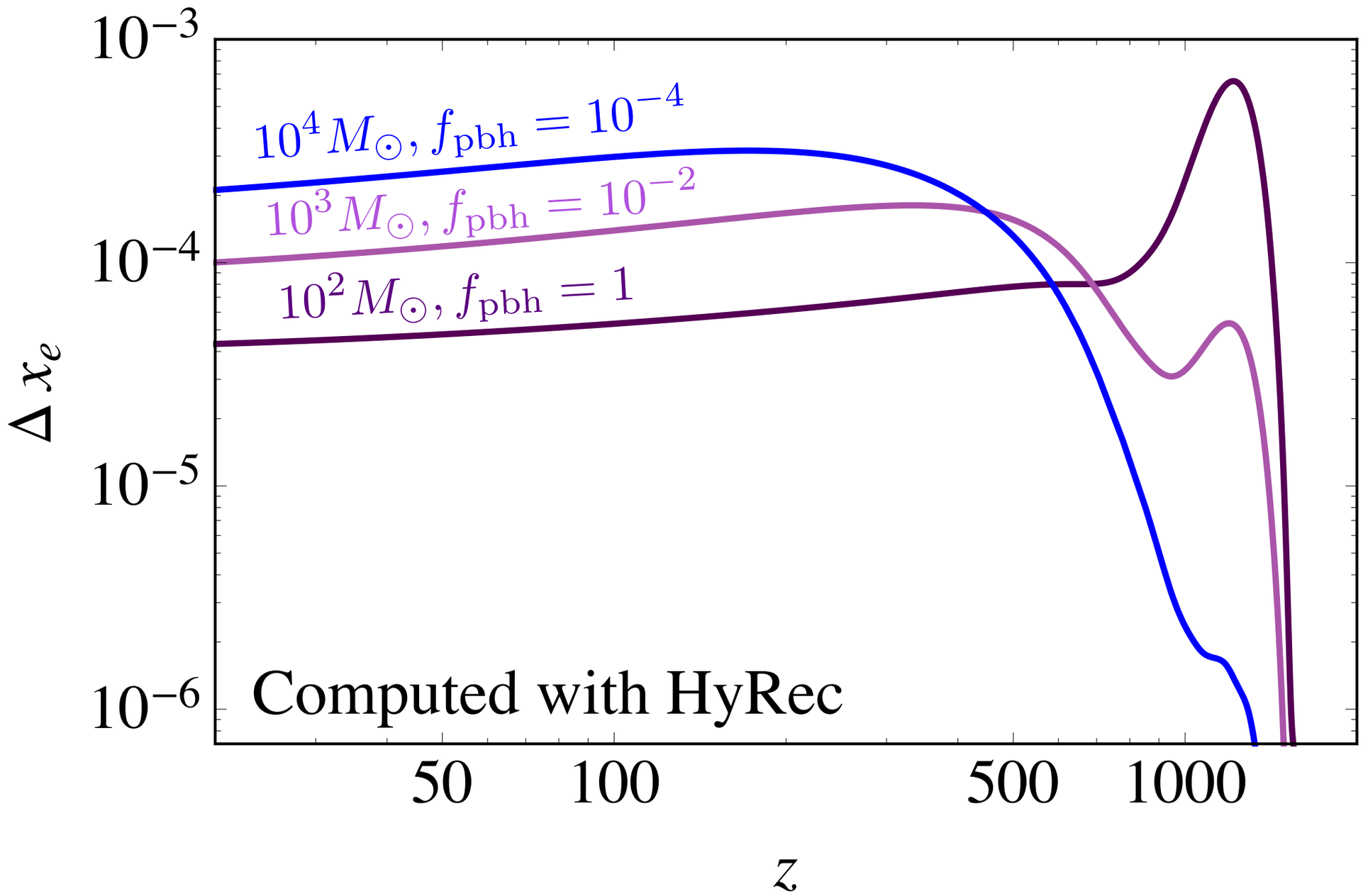
Note: this could be *substantially larger for disk-like accretion*
(see Poulin et al. 2017)

Efficiency of deposition

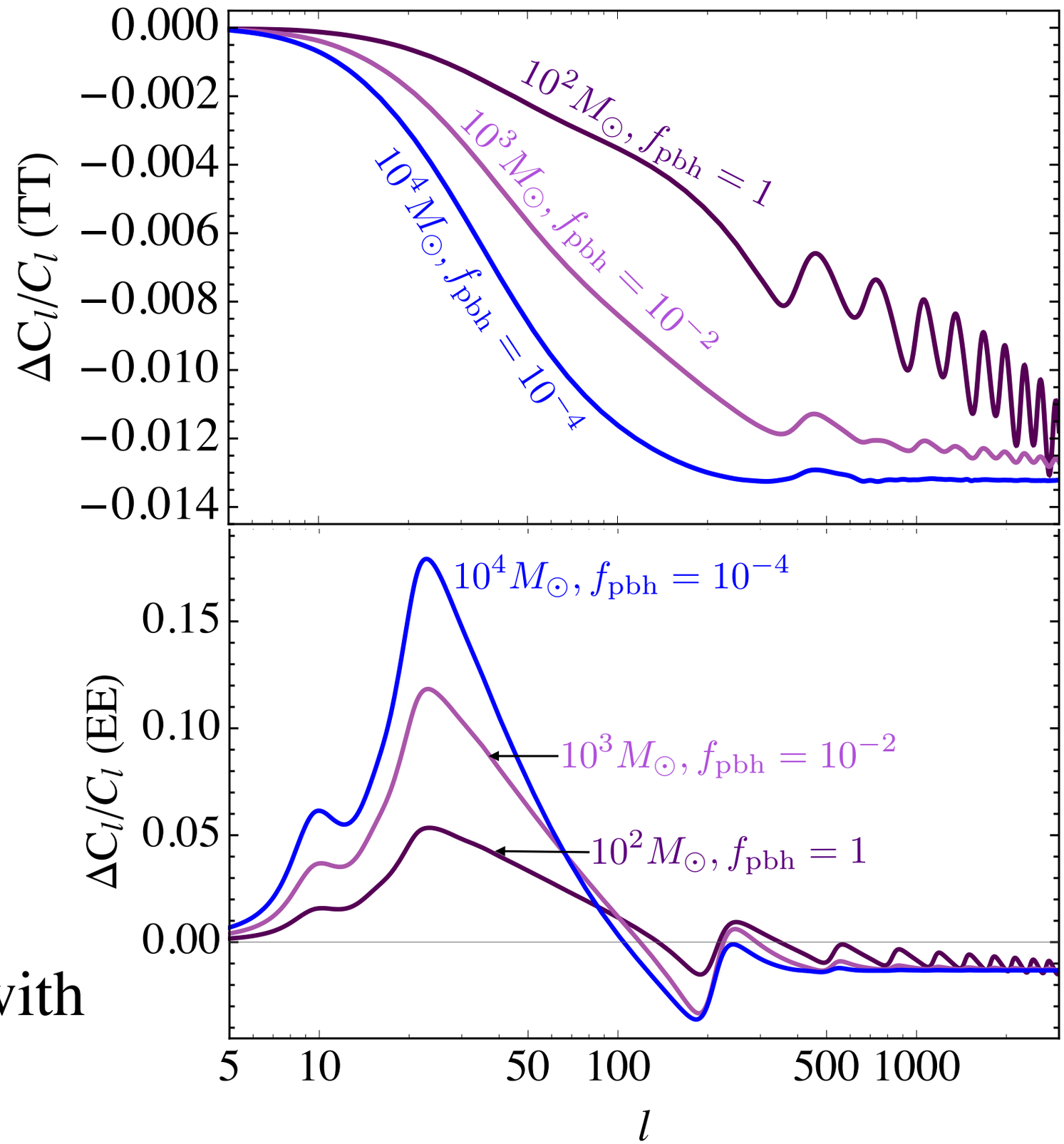
At the relevant photon energies (~0.1-10 MeV), main channel to deposit energy is through Compton scattering with free or bound electrons.



Effect on free-electron fraction…

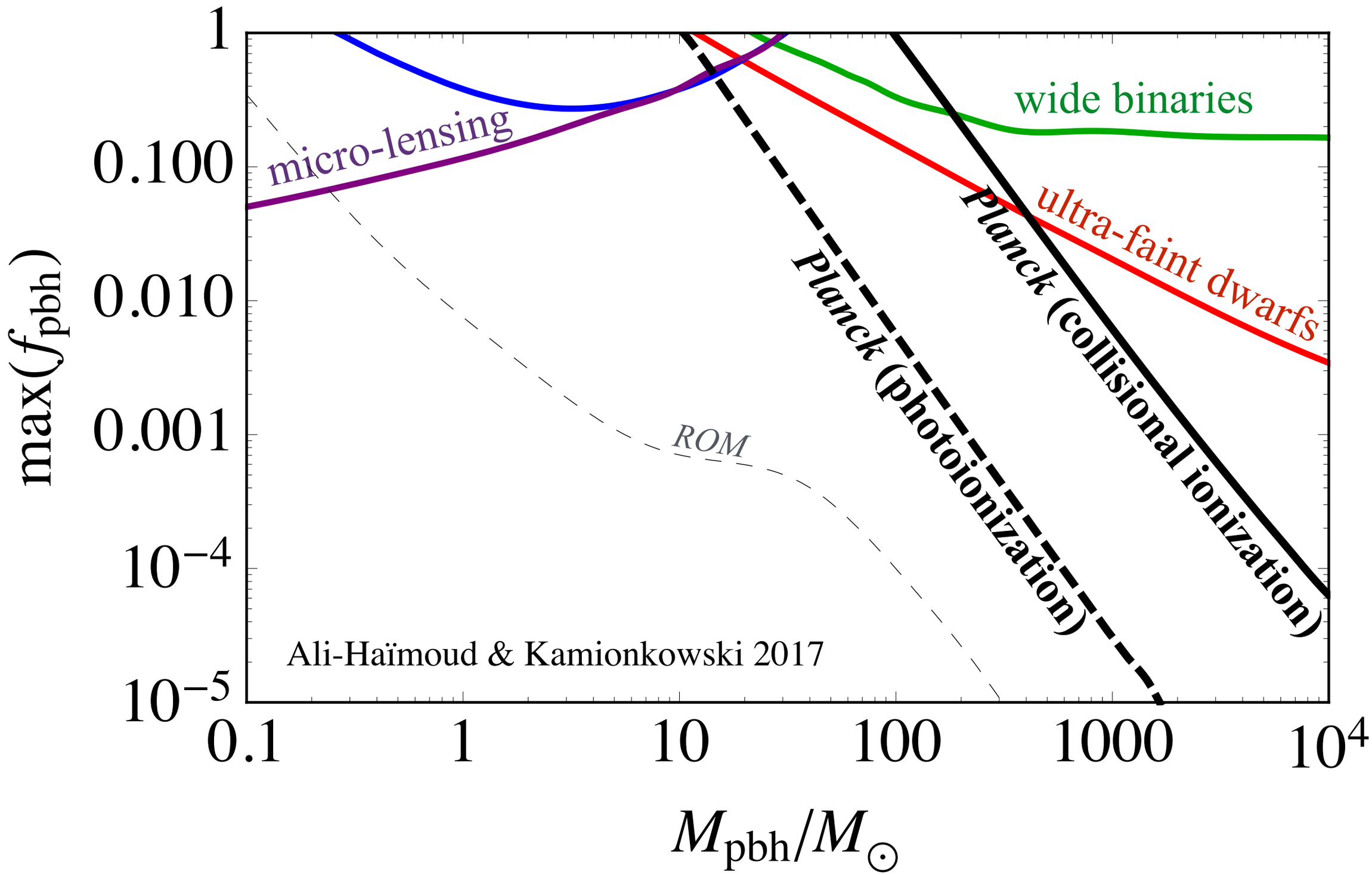


… and CMB anisotropies

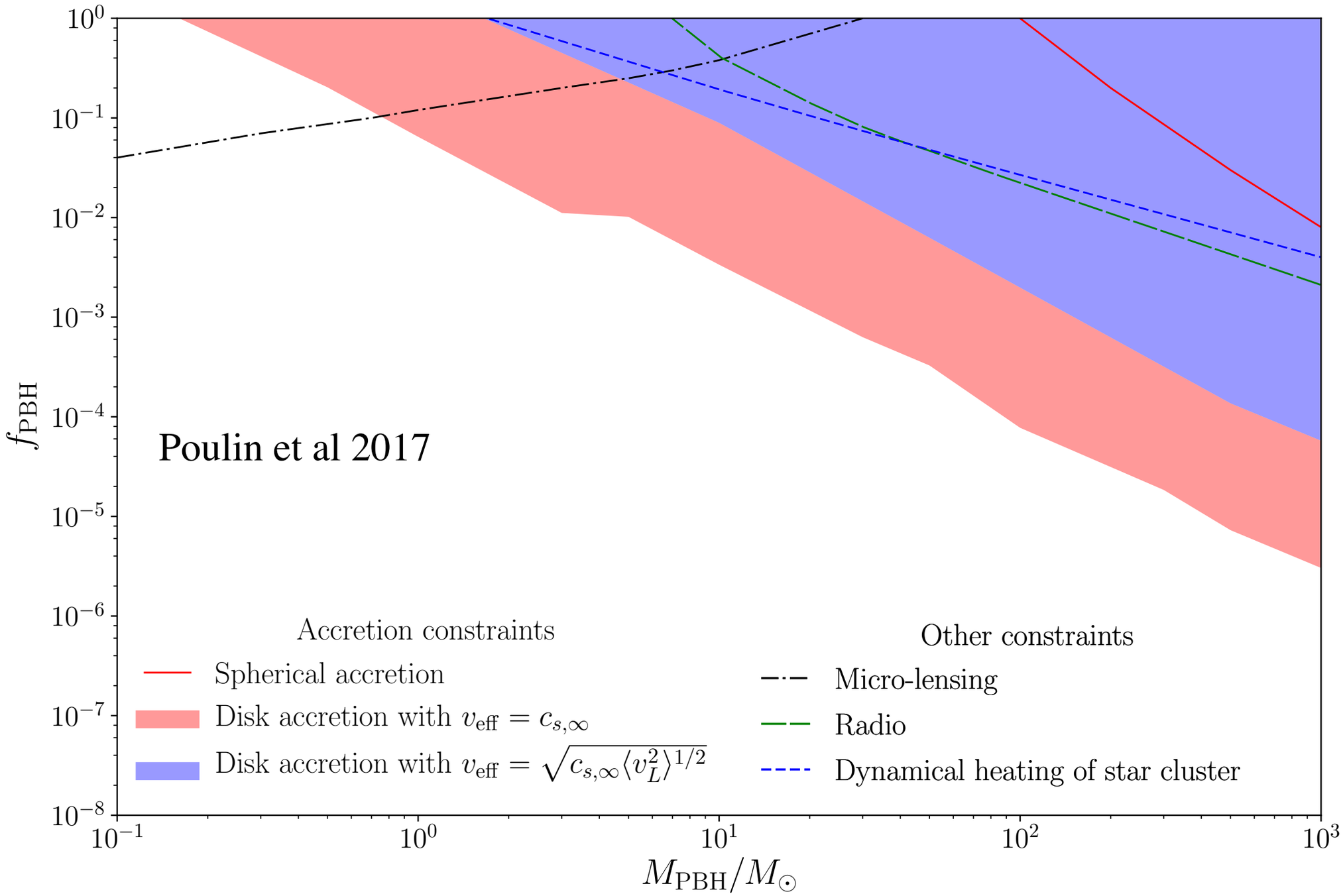


Computed with CLASS

Limits from Planck



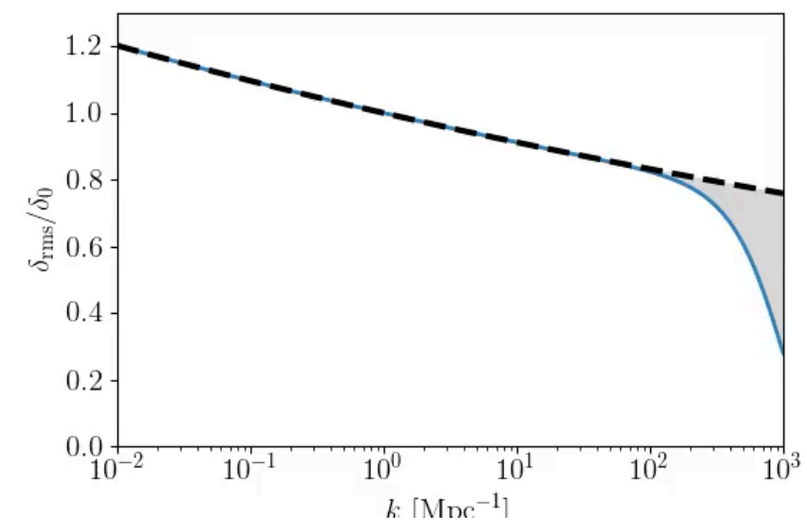
Potential reach if accretion was disk-like



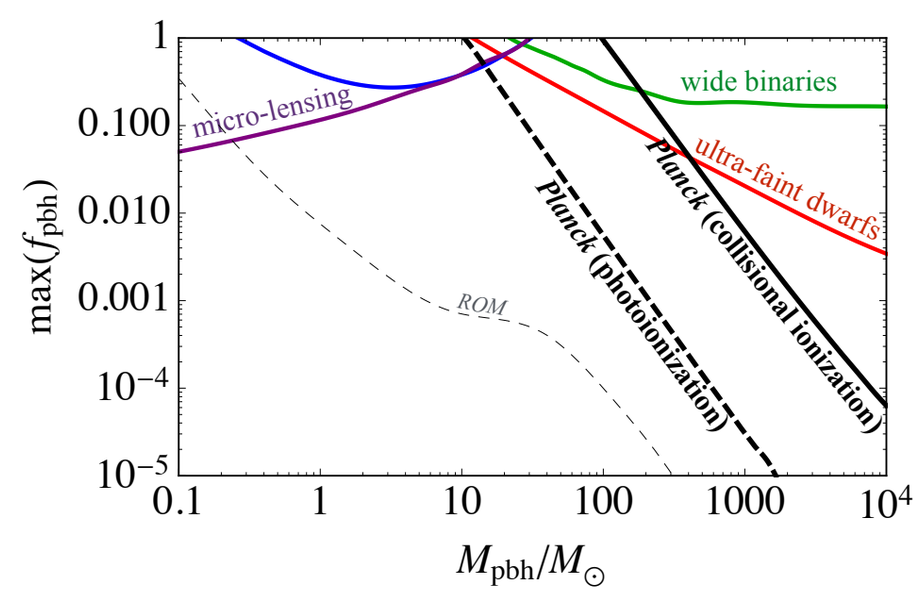
Summary of lecture 1

CMB spectrum and anisotropies probe energy injection in the early Universe. Here we have applied this to PBHs, and considered 3 mechanisms for energy injection:

- Dissipation of small-scale perturbations. Rule out PBHs with $M > 2e4\ M_{sun}$ if they are formed from the collapse of Gaussian primordial fluctuations



- Hawking evaporation: sets stringent bounds on PBH masses 1e15 g < M < 1e17 g



- Accretion: conservatively rules out PBHs with $M > 1e2\ M_{sun}$ from being all of the DM